FOURIER WAVEFORM SYNTHESIZER

by

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TABLE OF CONTENTS

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It is wished to express the writer's deep appreciation to Dr. R. E. Corby of the Physics Department for his generous assistance and encouragement during the design and construction of the waveform synthesizer which is the subject of this paper. His aid in the solution of problems attendant to the development and fabrication of such a device contributed to and is largely responsible for any degree of success which this project may have achieved.

The author's thanks also is given to Mr. Benjamin Cato of the Physics Department for his interest and effort in preparing the photographs which are to be found in this paper.

INTRODUCTION

The problem of synthesizing a waveform by adding sine waves as indicated by a Fourier analysis is not a new one. A combination mechanical and optical arrangement is described by Jenkins and White[1] wherein a system of mirrors is mounted as shown in Fig. 2. The mirrors are placed on vibrating strips of some sort of spring material, and the springs set into oscillation. A light beam is incident on the first mirror and reflected to the second, and from there to a rotating mirror. The mirror reflects the beam to a screen, and on this screen the beam of light will trace out a pattern which is the graphical sum of the component frequencies.

A combination mechanical and electrical instrument is available for producing synthesized periodic functions. In this instrument, a long shaft is driven by a small motor. Mounted on the shaft are disks, perpendicular to the length of the shaft, much like a cam (See Fig. 1). Placed near the disk is a coil of wire with a magnet in the center of it. As the disk approaches the magnet, it changes the permeability of the field surrounding the magnet, thus changing the flux linkage. In accordance with the equation relating emf produced in a coil,

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1. Jenkins and White, Fundamentals of Optics, 1950, pp 213, 214

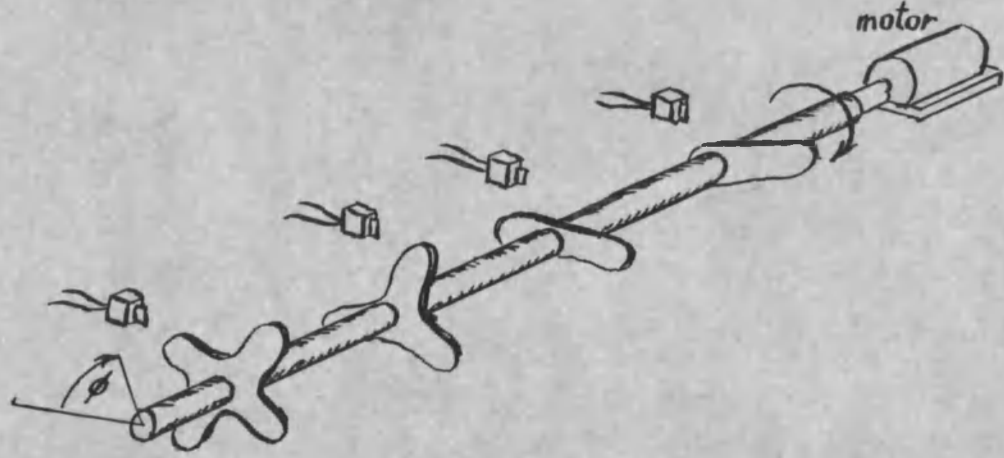

FIG. 1

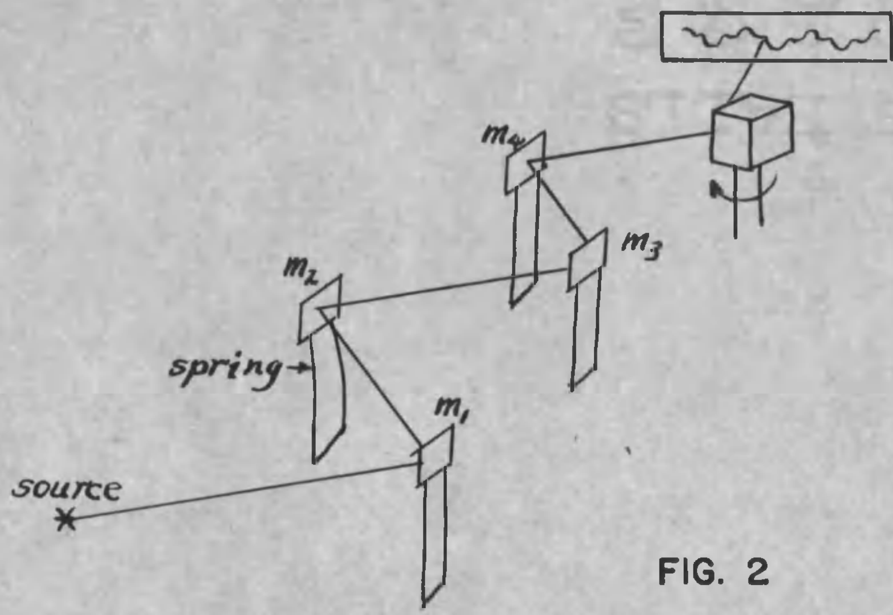

FIG. 2

$$e = -N\, d\phi/dt,$$

this change of field will set up an emf in the coil. Now if the shapes of the disks are such that the first passes the magnet once per shaft revolution, the second has two bulges and so nears the magnet twice per shaft revolution, and so forth, the effect will be to produce a fundamental wave plus a certain number of overtones, depending on the number of disks and pickups. Phase may be adjusted by changing the angle of the magnet position, as shown in Fig. 2.

The mechanical-electrical instrument described above is based on the same principles as an electric organ[2], in which a toothed wheel rotates, the teeth passing near a plate. The plate and wheel may be considered as two plates of a condenser wherein the capacitance changes periodically as the wheel rotates. This and many similar types of electromagnetic or electrostatic pickup tone wheels are rather common, and used extensively in commercial instruments.

Many such combinations of electrical, mechanical, and optical systems are in use, but nowhere in the literature is mention of a device to accomplish the same purposes but operating entirely electronically. The subject of this paper is such a device.

The idea for the construction of this instrument was

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2. S.K. Lewer, Electronic Musical Instruments, 1948 p 64

conceived by Dr. R. E. Corby of the University of Arizona. Dr. Corby felt that it would be desirable to have an instrument with which Fourier analysis could be demonstrated to his Electronics classes at the same time that the mathematics of Fourier analysis was being discussed. In keeping with the nature of the class, it would be well to have this instrument operate electronically. The instrument could also be used to demonstrate pictorially a beat frequency note, or to demonstrate to acoustics classes the fact that the phase of component frequencies of a complex sound wave has no effect with regard to the quality of a sound wave to the ear.[3]

In order to properly understand the operation inherent to any waveform synthesizer, something must be known of the mathematical principles on which the synthesizer is based. The subject of this paper is based on Fourier analysis, this being a less awkward series for most engineering applications than a power series.

According to a theorem due to Fourier, any periodic function may be represented as the sum of a number of sine and/or cosine terms. This is in most cases true, being subject to certain restrictions. It will be found that for most common periodic functions, the fundamental and lower numbered harmonics are of greater importance, i.e., have greater relative amplitude, than the higher numbered

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3. Alexander Wood, Acoustics, 1941, p 364

harmonics. Unless with a curve of unusual features, the trignometric representation is generally adequate for most purposes if the series is carried up to the tenth harmonic.[4] It has been decided that six harmonics will be adequate for the waveform synthesizer herein described.

Fourier analysis is discussed in most mathematics texts. For the reader's convenience, Appendix A of this paper contains a discussion of Fourier analysis. The material is presented in like manner to that of Reddick and Miller[5], although the writer has drawn material from Rojansky[6] and Woods[7] also.

Multivibrators play an important part in the frequency scaling of the subject of this paper. Multivibrator design and operation is discussed in most electronics texts. Some of the salient features are discussed in Appendix B of this paper, the information being much the same as that presented by Reich[8] and the Cruft Electronics Staff[9].

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4. G.P. Harnwell, Principles of Electricity and Magnetism, 1949, p 629

5. Reddick and Miller, Advanced Mathematics for Engineers, 1947, pp 191-197

6. V. Rojansky, Introductory Quantum Mechanics, 1938, p 34

7. F.S. Woods, Advanced Calculus, 1934, pp 295-301

8. H.J. Reich, Theory and Applications of Electron Tubes, 1944, pp 362-365

9. Cruft Electronics Staff, Electronic Circuits and Tubes, 1947, p 855

DEVELOPMENT OF THE FOURIER SYNTHESIZER

Originally it was decided that three main problems were to be solved in the construction of the waveform synthesizer, and that the rest of the circuit would develop itself about these three points. These three problems are (1) a circuit component which could give complete 360° phase control for each harmonic, (2) a waveform source which would give for each harmonic a sine waveform as exact as possible, and (3) synchronizing of waveform sources so that each harmonic frequency would be an exact multiple of the fundamental frequency, with no drift in phase of one frequency with respect to another.

Insofar as the first point is concerned, it was thought that Selsyns would serve satisfactorily. The use of Selsyns necessitated a three phase output from the sine wave generators, thus putting a restriction on the type of waveform source to be used in the second point above. In accordance with this restriction, a three phase feedback oscillator was developed. This oscillator is shown in Fig. 3.

Conditions of oscillation for any feedback oscillator are that the feedback signal must feed back in phase with the original signal. In order that oscillation be sustained, the total gain around the circuit must be equal to unity. Gain in this circuit is controlled by placing a potentiometer on the grid of each tube, and appropriately varying the grid to

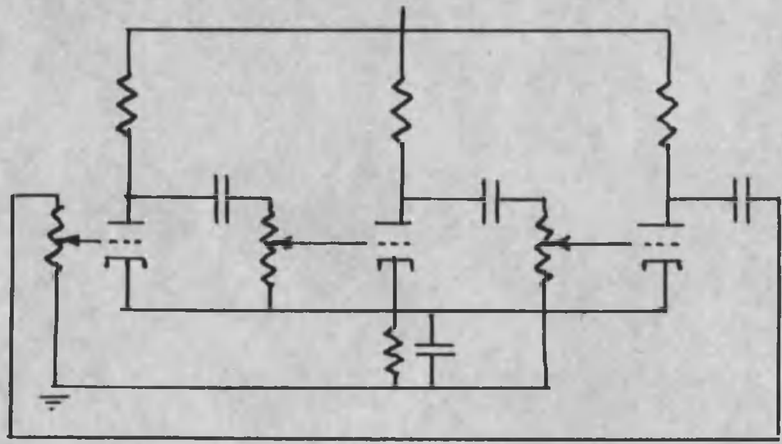


FIG. 3

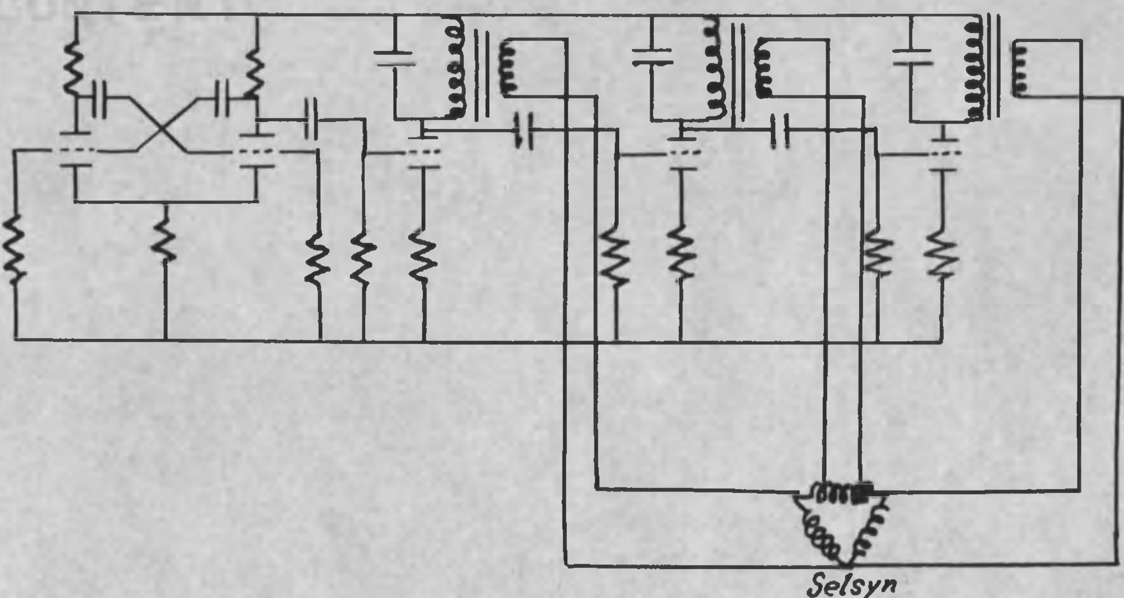


FIG. 4

ground resistance until oscillation is just sustained. The phase change throughout the circuit is controlled by the coupling capacitors. In each section of this oscillator, the grid to plate will change the phase by approximately 180°, and the coupling capacitor following should then drop the phase back by 60°, making the overall phase change equal to 120°. Through the three sections, then, the signal will change in phase three times at 120° shift each time, thus placing the input signal exactly in phase with the original signal when the total loop is considered. It may be developed that the gain and frequency equations of this circuit are

$$K = \frac{uR_1 R_g}{R^2}$$

and

$$f = \frac{R_1 \quad R_p}{2 \quad CR^2},$$

where

K is the gain
f is the frequency
u is the amplification factor of the tube
R_1 is the load resistance
R_g is the grid resistance
R_p is the plate resistance
C is the capacitance of the coupling condenser
R is the sum $R_1R_g \quad R_1R_p \quad R_pR_g$

A breadboard model of this circuit behaved in a very satisfactory manner, giving an acceptable waveform, a symmetrical three phase output, and close frequency control. Problems were encountered, however, when attempt was made to extract the signal and feed into a Selsyn. Since all available Selsyns were Y-wound rather than delta-wound, it was necessary to isolate each stage of the oscillator from the Selsyn winding. The Selsyn windings were of extremely low impedance, making impedance matching a problem. Impedance matching was accomplished by locating a step-down transformer in the plate of each tube, the transformer also acting to isolate the Selsyn from the oscillator. However, the transformer acted inductively in the circuit, disturbing the original symmetry with regard to the 120° phase shift per stage. At that time there was no instrument available in our laboratory to measure the inductance and Q of the transformers, making it difficult and rather haphazard to calculate appropriate circuit components to regain this 120° phase shift per stage.

Further than the difficulties already mentioned, it was found that frequency multiplication was somewhat of a problem. The oscillators had great tendency to operate at a particular frequency, this frequency being a function of tube characteristics, stray capacitance, and other minor factors in addition to the variables mentioned in the frequency equation above. Within certain close limits it was possible to synchronize one oscillator with another, but these limits were so close that minor chance fluctuations in circuit behavior were not tolerable. This problem was foreseen, of course, being point three of the original main problems of circuit design.

Multivibrators have long been in use in frequency scaling. By virtue of the ease and exactitude with which synchronization may be accomplished with multivibrators, it was felt that these provided the solution to the problem of synchronizing the various frequency generators. For the convenience of the reader, there is to be found in Appendix B of this paper information relative to the design and operation of multivibrators.

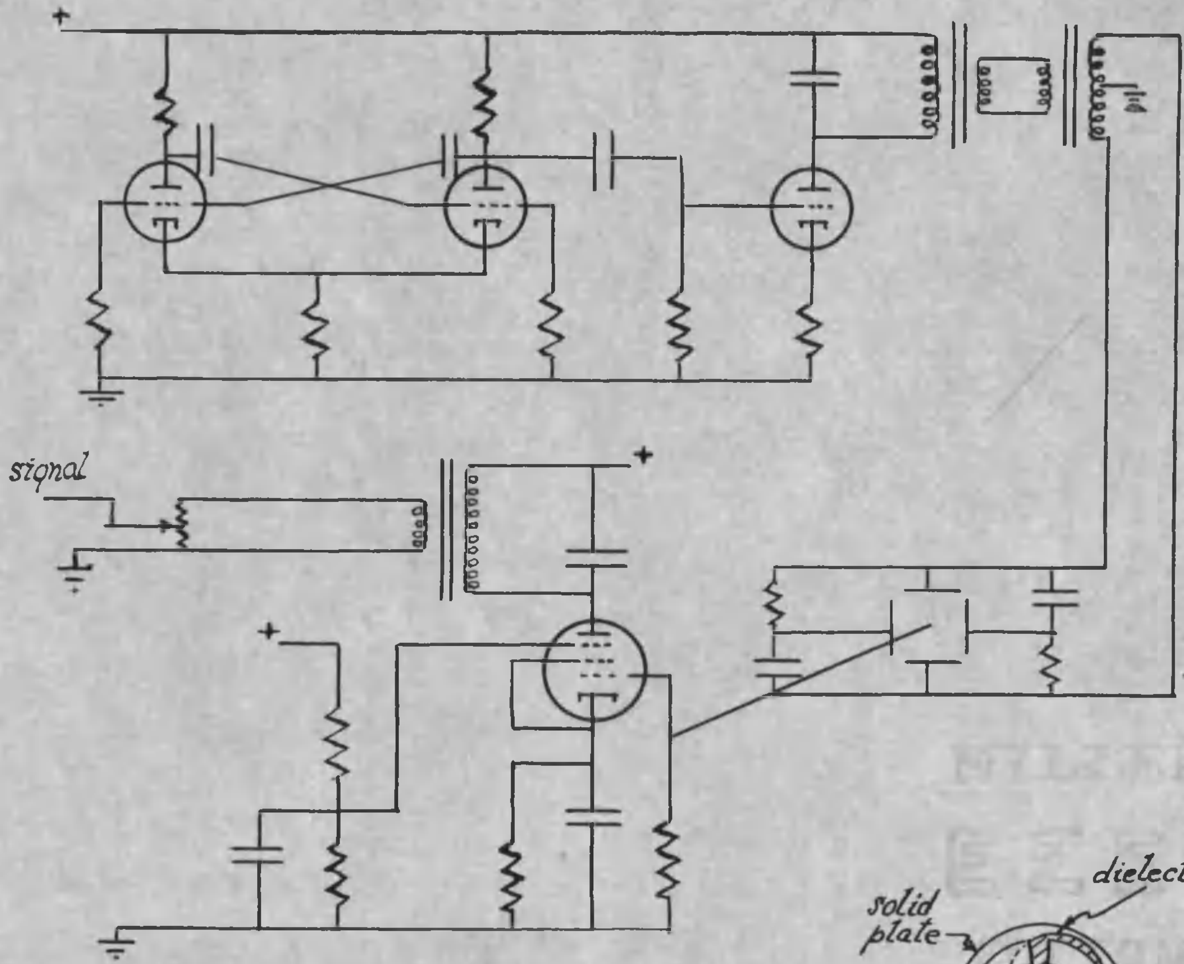
A sequence of multivibrator circuits was set up with the highest frequency being the lowest number which may be factored into the desired frequencies. At first it was thought that a fundamental frequency of 60 cycles would be satisfactory, and possibly the 60 cycle multivibrator could be synchronized with the line voltage available in the laboratory. This would probably lend a certain amount of stability to the circuit as a whole. This scheme, however, would complicate the circuit considerably, as it would require that all higher frequencies be exactly divisible by line frequency. The highest frequency multivibrator under these conditions would be operating at 3600 cycles. If this multivibrator were to be synchronized with line voltage, it would have to be free-running at precisely 60 times line frequency. An extremely small amount of drift would be tolerable, but the maximum allowable drift would have to be so small as to render this scheme undesirable for all practical purposes.

A circuit was constructed with a fundamental multivibrator operating at near 60 cycles, and it was found that this multivibrator tended to lock in with line frequency due to stray pickup only. This was certainly a disadvantageous feature, and accordingly the writer elected to change the fundamental frequency to 120 cycles. This necessitated changing the frequency of the highest frequency multivibrator to 7200 cycles, this number being the lowest number divisible by the first six harmonics of 120 cycles. The 7200 cycle multivibrator synchronized a multivibrator of 480 cycles, and also synchronized one of 3600 cycles, which in turn synchronized one of 1800 cycles and one of 1200 cycles. The 1800 cycles synchronized 360 cycles, and the 1200 cycles synchronized multivibrators of 120 cycles, 240 cycles, and 600 cycles. The 3600 cycle multivibrator also synchronized one of 720 cycles. There are available then, aside from the stepdown multivibrators, signals of 120, 240, 360, 480, 600 and 720 cycles, these signals being the fundamental and desired harmonics. The intermediate multivibrators were included as a safety factor so that at no time was the frequency division too great.

It now remained to construct signal generators which would be locked in frequency by the multivibrators. It was attempted to use the construction shown in Fig. 4, where a three phase output is obtained by a chain of three amplifiers, each one of which drops the phase back by 120° in the same manner as previously discussed. Two difficulties were encountered in this circuit. First, there was some tendency for a signal to be reflected from the Selsyn back through the transformer and the plate of a tube. Second, considerable difficulty was met in arranging the amplifiers so that exactly 120° phase shift was experienced by the signal in going through each amplifier. This problem was somewhat simplified in the case of the oscillator, where the only signal that sustained oscillation was that one which experienced 120° phase shift per stage in order to come back in phase with itself at the starting point.

It appeared at this time that a primary source of difficulty lay in the fact that a three phase output was required in order to feed correctly phased signals into the Selsyn. Another method of phase control suggested itself in the form of the phase shifter capacitor. This device requires for its operation four signals, each 90° out of phase with another.

The phase shifter capacitor consists of two plates seperated by a dielectric. The dielectric is disk shaped, and is mounted perpendicular to a shaft, but placed in an off-center manner with the shaft at the edge of the disk. One of the plates is a solid circular disk with a hole in the center of it to accomodate the shaft. The other plate is divided into four isolated quadrants, with a lead coming to each of the quadrants. This plate also has a hole in the center of it to accomodate the shaft. When the three components are assembled, with the dielectric lying between the two plates, it may be considered as four capacitors with one plate common to all four. The lines of electric flux will


signal

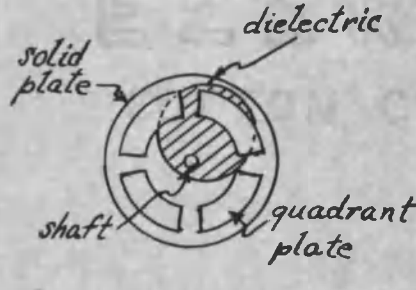

solid plate
dielectric
shaft
quadrant plate

FIG. 5

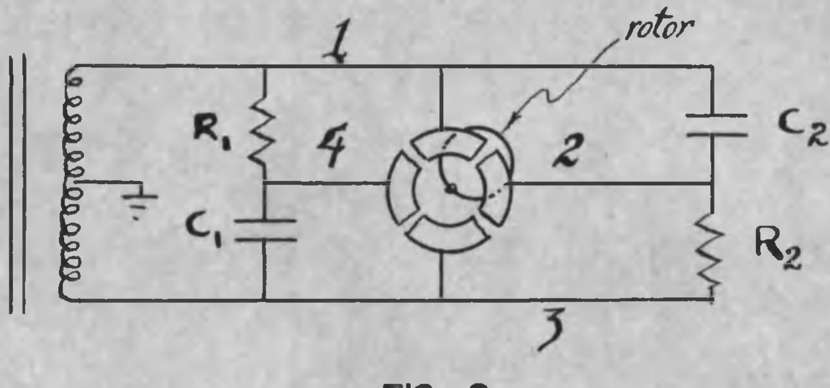

rotor
1
2
3
4
R1
C1
C2
R2

FIG. 6

tend to concentrate in the region of the dielectric. Accordingly that capacitor which is the dominant one will be that one which includes the dielectric between its plates. If the dielectric is shared by two adjacent quadrants, each will contribute to the output signal.

That the amplitude should be approximately constant with phase shift may be seen by inspection of Fig. 6. By appropriate calculation the magnitude of the voltage on each quadrant, i.e., points 1, 2, 3 and 4 of Fig. 6, was made equal by making the circuit symmetrical and requiring that the impedance of the capacitors be of the same magnitude as the resistance of the resistors. It may be noticed that since the dielectric diameter is equal to one complete quadrant length, the position of the dielectric at any one time will be such that only one quadrant, or enough of two adjacent quadrants to comprise one quadrant length, can be covered by the dielectric and thus contribute to the output signal. Accordingly, the signal output is necessarily approximately linear. This presentation makes a few broad assumptions regarding the shape of the electrical field configurations. A complete mathematical examination of the behavior of the phase shifter capacitor is not pertinent to the theory necessary for understanding the waveform analyzer, and is thus beyond the scope of this paper. For more complete information see Radio News (Eng. Edition) for June, 1947.

The circuit in which such a capacitor is placed is shown in Fig. 6. Its operation may be analyzed in the following

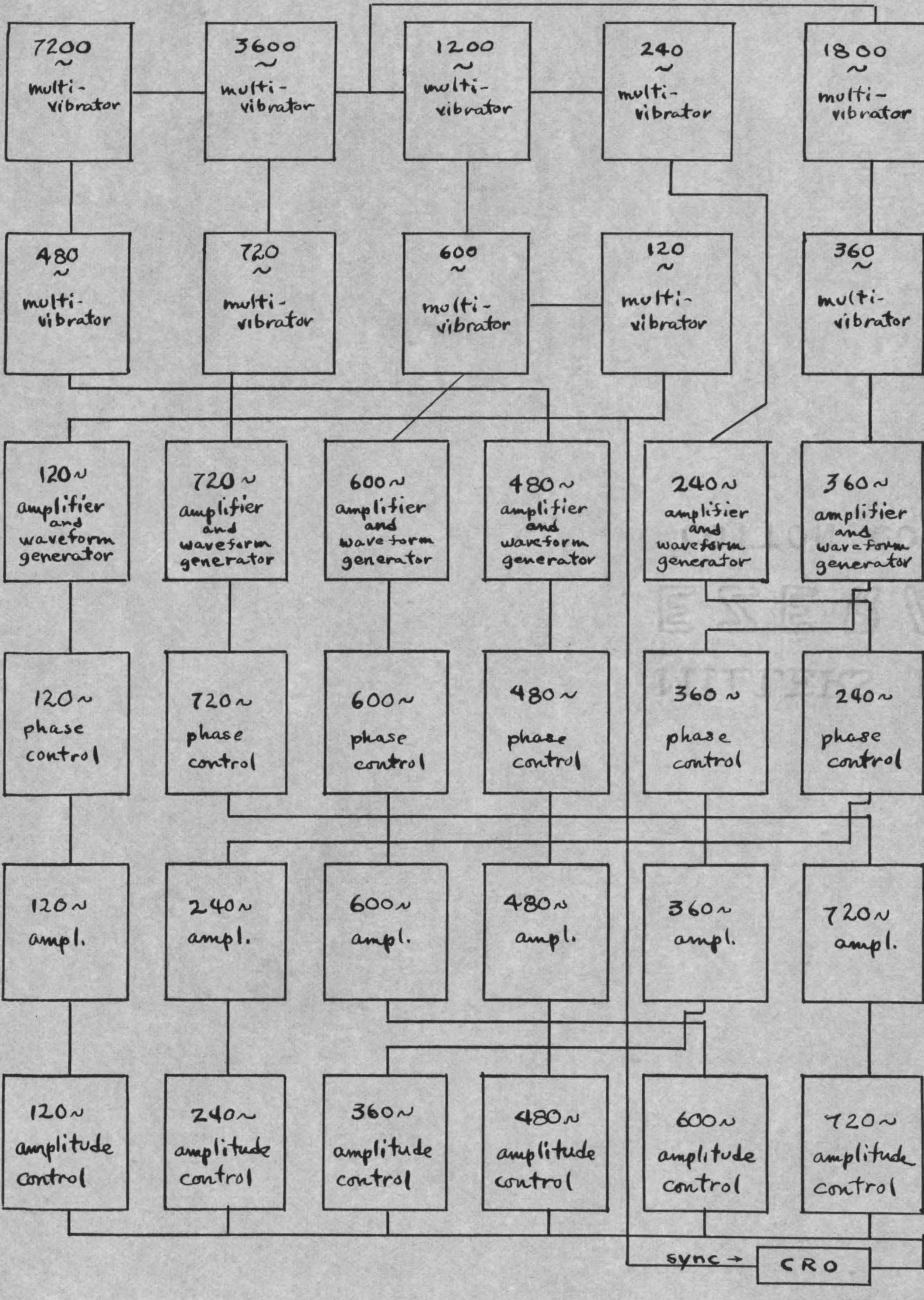
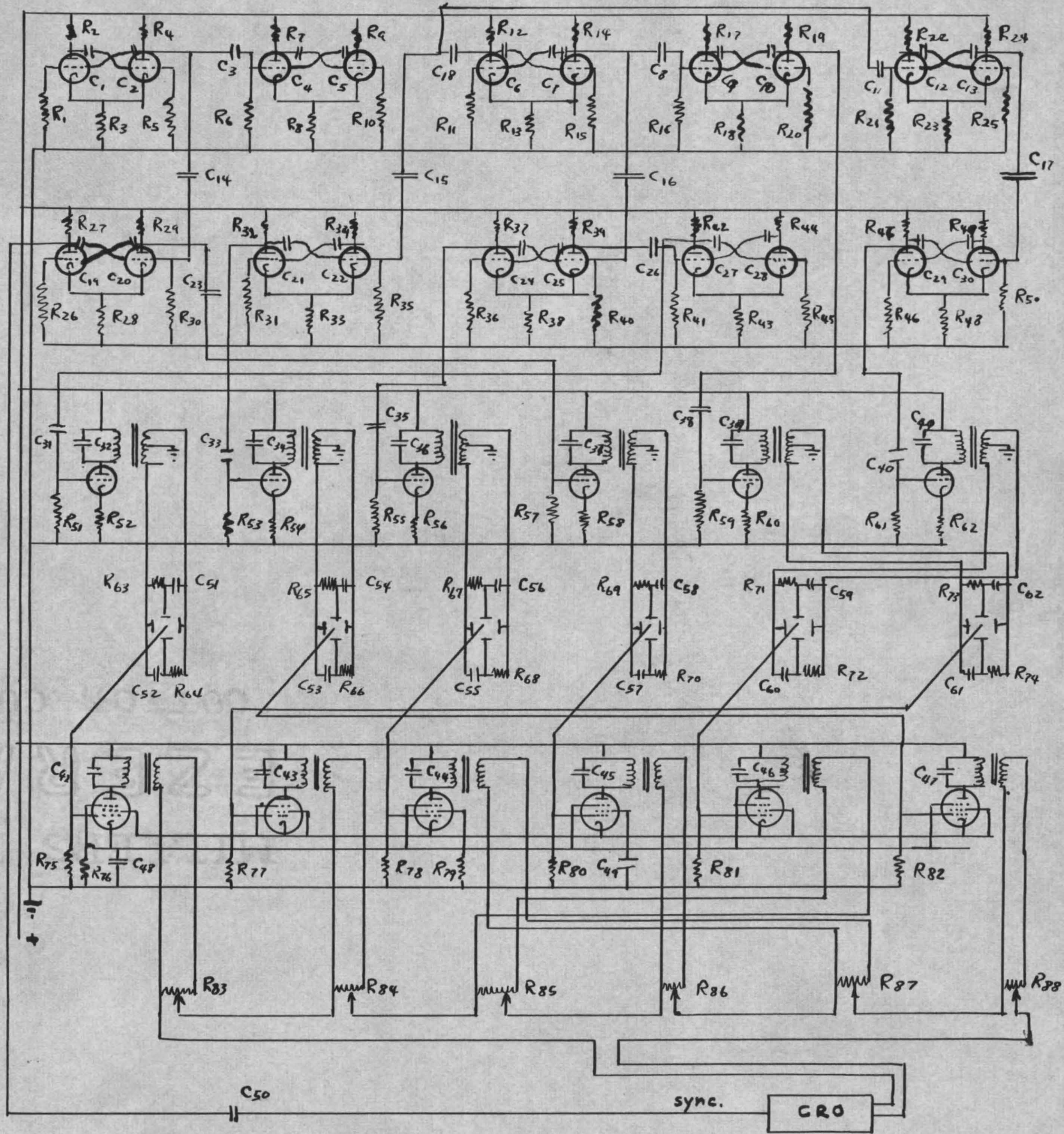

7200 ~ multi-vibrator
3600 ~ multi-vibrator
1200 ~ multi-vibrator
240 ~ multi-vibrator
1800 ~ multi-vibrator
480 ~ multi-vibrator
720 ~ multi-vibrator
600 ~ multi-vibrator
120 ~ multi-vibrator
360 ~ multi-vibrator
120~ amplifier and waveform generator
720 ~ amplifier and waveform generator
600~ amplifier and wave form generator
480~ amplifier and waveform generator
240~ amplifier and waveform generator
360~ amplifier and wave form generator
120~ phase control
720~ phase control
600~ phase control
480~ phase control
360~ phase control
240~ phase control
120~ ampl.
240~ ampl.
600~ ampl.
480~ ampl.
360~ ampl.
720~ ampl.
120~ amplitude control
240~ amplitude control
360~ amplitude control
480~ amplitude control
600~ amplitude control
720~ amplitude control
sync →
CRO

FIG. 7



R2
R4
R7
R9
R12
R14
R17
R19
R22
R24
C1
C2
C3
C4
C5
C18
C6
C7
C8
C9
C10
C11
C12
C13
R1
R3
R5
R6
R8
R10
R11
R13
R15
R16
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R35
R36
R38
R40
R41
R43
R45
R46
R48
R50
C31
C32
C33
C34
C35
C36
C37
C38
C39
C40
R51
R52
R53
R54
R55
R56
R57
R58
R59
R60
R61
R62
R63
C51
R65
C54
R67
C56
R69
C58
R71
C59
R73
C62
C52
R64
C53
R66
C55
R68
C57
R70
C60
R72
C61
R74
C43
C44
C45
C46
C47
R75
R76
C48
R77
R78
R79
R80
C49
R81
R82
R83
R84
R85
R86
R87
R88
C50
sync.
CRO

FIG. 8

manner. The center-tapped transformer is grounded on the center tap. Voltage at one end of the transformer is 180° out of phase with the voltage at the other end. Therefore the voltages at points 1 and 3 on the phase shifter capacitor schematic diagram of Fig. 6 are 180° apart. Now of course the voltages on the lower plate of C_1 or the upper plate of C_2 follow exactly the voltages of the points 3 and 1. The voltages on the other plates of these capacitors, however, are a function of the time required for the capacitors to charge, or discharge, as the case may be, through the resistors R_1 and R_2. If the impedance of the capacitor is made equal to the resistance of the resistor for the particular frequency range used, the voltage at the point 4 will be 90° out of phase with that of points 1 and 3, and the voltage of point 2 will be 180° out of phase with the voltage at point 4. The output signal will be taken with respect to ground.

The impedance of this capacitor is of the order of 50 megohms. Accordingly the input impedance to the next stage of amplification following such a capacitor must be extremely high, and very little stray capacity can be tolerated.

The phase shifter capacitor was employed in the final circuit as shown in Fig. 5. This type circuit proved to be most satisfactory from the standpoint of isolation of frequency generators, stability of frequency generators, and provision of a constant amplitude output with phase change. The complete circuit is shown in block diagram form in Fig.

7, and the complete schematic of the circuit is shown in Fig. 8. Fig. 5, to which we have previously made reference, shows the path of one particular frequency component, starting with the multivibrator of that frequency. The complete circuit, apart from the synchronizing multivibrators, is essentially six circuits similar to Fig. 5, each circuit supplying one of the harmonic frequencies in which we are interested.

It would be well to trace through the circuit of Fig. 8 in some detail. The multivibrator action is described elsewhere in this paper. In actual construction of the multivibrators, it was occasionally found desirable to add a high resistance in parallel with the place capacitors, in order to aid in draining off the charge on the capacitors. The multivibrators were made as symmetrical as possible originally, but in their final alignment no particular care was exercised towards absolute symmetry. It was further found desirable to add decoupling by-passed resistors and B plus. This was done in order to minimize any sharp fluctuations in B plus which might appear elsewhere in the circuit and tend to synchronize some other multivibrator. The signals developed by the multivibrators were quite similar in shape to those shown elsewhere in this paper as typical multivibrator signals. A 6SN7 double triode was used in the multivibrator. The signal taken from the plate of one of the multivibrators was fed through a coupling capacitor to the grid of an amplification stage. The tube used here was a 6SN7. In the plate of this amplifier was placed a step-down transformer, paralleled by a capacitor. The value of capacitance was varied so as to achieve resonance. The resonant point was difficult to identify, as the transformers used were extremely low Q. It was necessary to use the high winding side of the transformer in this plate load in order to have a high enough impedance to develop a signal. The low side had very small impedance. This side was also without a center tap. The phase shifter capacitor requires a center tapped input as shown in Fig. 6 for proper operation. Thus it was necessary to use a step-up transformer with a center tapped output. One desirable feature of using such a double transformer arrangement is the complete isolation afforded to the multivibrator. A second desirable feature is the smoothing action of the inductances on the waveform. A very acceptable sinusoidal waveform was obtained at this point.

The operation of the phase shifter circuit has been previously discussed. The output signal from the phase shifter was led to the grid of a 6AB7/1853 tube. A grid resistor of several megohms was used here, for reasons already mentioned. Another resonant tank was used on the plate of this tube, and the output from the transformer used in this tank was placed on a potentiometer. This potentiometer afforded the final amplitude control of the signal. From here the signal was added to the signals from the other chains, and the resultant signal fed into a cathode ray oscilliscope.

It was found necessary to use shielded cable from the output of the phase shifter capacitor to the grid of the pentode amplifier following this capacitor, in order to minimize any opportunity for this grid to receive stray pickup signals. It was further found that the 120 cycle and 240 cycle signals were so reduced in amplitude after coming through the phase shifter capacitor network, that an additional stage of amplification was desirable here.

A voltage regulated power supply constructed by the author was used in conjunction with the synthesizer, and it was found in a range of supply voltage from 260 volts to 310 volts the synthesizer was quite stable. A larger range of supply voltage was not available, and it is not known what the critical limits of supply voltage might be with regard to the synthesizer stability.

It was noted that filament supply voltage dropped from 6.3 volts to 5.0 volts when the synthesizer filament load was placed on the supply. This apparently did not seriously alter the tube operation.

The results of operation of this device are to be seen in the photographs comprising the final pages of this paper, together with the photograph of the finished instrument.

APPENDIX A

FOURIER ANALYSIS

A Frenchman named Joseph Fourier announced in 1807 that any recurrent curve may be analyzed into a unique series of simple harmonic components. Since that time the statement has been referred to as Fourier's Theorem.[10] It is this general statement which we shall now undertake to prove. Stated in more precise terms, the theorem is as given below.

If the series

$$\frac{a_0}{2} + a1 \cos x + a_2 \cos 2x + \dots + a_n \cos nx + \dots$$
$$+ b_1 \sin x + b_2 \sin 2x + \dots + b_n \sin nx + \dots \qquad (1)$$

is uniformly convergent in the closed interval $c \leq x \leq c + 2\pi$, and has the sum $f(x)$, then for $n = 0, 1, 2, \dots$ we have $a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x) \cos nx \, dx$,

$$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x) \sin x \, dx. \qquad (2)$$

Now it is noticed the series (1) is uniformly convergent in the indicated closed interval, and because of the periodicity of the functions sin nx and cos nx, (1) is uniformly convergent over every real x interval.

The following theorem is presented without proof.

Let the series of continuous functions

10. W.L. Taylor, Physics, p 142, 1941

$$\sum_{n=1}^{\infty} u_n(x) = u_1(x) + u_2(x) + \ldots + u_n(x) + \ldots \quad (3)$$

converge uniformly to a functions $s(x)$ in a closed interval $a \leq x \leq b$. Then if x_1 and x_2 are any two numbers between a and b, the series may be intergrated term by term between the limits of x_1 and x_2, and the series of intergrals is representable by an intergral of $s(x)$ over these limits.

The following two expansions are also now employed.

$$\int_c^{c+2\pi} \sin nx \, dx = \left(-\frac{1}{n} \cos nx\right)_c^{c+2\pi} = 0, \; n \neq 0 \quad (4)$$

$$\int_c^{c+2\pi} \cos nx \, dx = \left(-\frac{1}{n} \sin nx\right)_c^{c+2\pi} = 0, \; n \neq 0 \quad (5)$$

We may now represent the series (1) in the form

$$\int_c^{c+2\pi} f(x)\, dx = \int_c^{c+2\pi} \frac{a_o}{2}\, dx + \int_c^{c+2\pi} a_1 \cos x \, dx + \ldots\ldots$$

$$+ \int_c^{c+2\pi} a_n \cos nx \, dx + \int_c^{c+2\pi} b_1 \sin x \, dx + \ldots$$

$$+ \int_c^{c+2\pi} b_n \sin nx \, dx + \ldots$$

$$= a_o \pi. \quad (6)$$

a_o is given by setting $n = o$.

Let

$$s_n(x) = \frac{a_o}{2} + a_1 \cos x + a_2 \cos 2x + \ldots$$

$+ a_n \cos nx + b_1 \sin x$

$$+ b_2 \sin 2x + \ldots + b_n \sin nx. \qquad (7)$$

By definition of uniform convergence, where

$$\| f(x) - s_n(x) \| < \epsilon$$

for any x with n sufficiently large. Equation (1) is also uniformly convergent when multiplied by cos nx. This follows, since $\| \cos nx \| \leq 1$,

$$\| f(x) \cos nx - s_n(x) \cos nx \| \leq$$

$$\| f(x) - s_n(x) \| < \epsilon$$

Accordingly the series (7) may be integrated term by term. This gives

$$\int_c^{c+2\pi} f(x) \cos nx \, dx = \int_c^{c+2\pi} \frac{a_o}{2} \cos nx \, dx$$

$$+ \int_c^{c+2\pi} a_1 \cos x \cos nx \, dx + \ldots$$

$$+ \int_c^{c+2\pi} a_n \cos^2 nx \, dx + \ldots$$

$$+ \int_c^{c+2\pi} b_1 \sin x \cos nx \, dx + \ldots$$

$$+ \int_c^{c+2\pi} b_n \sin nx \cos nx \, dx + \ldots$$

$$= a_n \pi . \qquad (8)$$

This equality is seen when it is recalled that

$$\int_c^{c+2\pi} \sin mx \cos nx \, dx$$

$$= \frac{1}{2}\int_c^{c+2\pi} \sin(m-n)x + \sin(m+n)\, x \, dx$$

$$= 0, \tag{9}$$

$$\int_c^{c+2\pi} \cos mx \cos mx \, dx$$

$$= \frac{1}{2}\int_c^{c+2\pi} \cos(m-n)x \cos(m+n)\, x \, dx$$

$$= 0, \tag{10}$$

and $\int_c^{c+2\pi} \cos^2 nx \, dx$

$$= \frac{1}{2}\int_c^{c+2\pi} (1+\cos 2\, nx \, dx)$$

$$= \quad . \tag{11}$$

Now had (1) been multiplied by sin nx, by the same evaluation procedures it could be shown that

$$\int_c^{c+2\pi} f(x) \sin nx \, dx = b_n \pi . \tag{12}$$

Now it is not true that all continuous functions are representable by their Fourier series. Usually only highly discontinuous functions which do not satisfy the Dirichlet conditions cannot be expanded into a Fourier series. A general rule is that the function be single-valued, finite, and continuous except for finite discontinuities and has not an infinite number of maxima or minima. Fortunately most functions applicable to engineering problems satisfy this rule. Further information on the conditions under which a function is representable by its Fourier series is to be found in "Mathematical Analysis", Vol. I, by Goursat-Hedrick.

A few examples of Fourier developments are shown in Fig. 9. The corresponding functions are given below.

For Fig. 9 (a) $f(x) = \sum_{n=0}^{\infty} \frac{4}{\pi} \frac{1}{(2n+1)} \sin(2n+1)x$

Fig. 9 (b) $f(x) = \sum_{n=0}^{\infty} \frac{8}{\pi^2} \frac{(-1)^n}{(2n+1)} \sin(2n+1)x$

Fig. 9 (c) $f(x) = \sum_{n=0}^{\infty} \frac{1}{n} \sin nx$

Fig. 9 (d) $f(x) = \frac{\theta}{\pi} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{\sin n\theta}{n} \cos nx$

Fig. 9 (e) $f(x) = \frac{\theta}{\pi} + \frac{2}{\pi\theta} \sum_{n=0}^{\infty} \frac{(\sin n\theta)^2}{n} \cos nx$

Fig. 9 (f) $f(x) = \frac{1}{\pi} + \frac{1}{2} \cos x + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \cos 2nx}{4n^2 - 1}$

Fig. 9 (g) $f(x) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{4n^2 - 1} \cos 2nx$

It is of interest to examine the quality of a waveform with only six harmonics present. This is shown graphically in the sawtooth waveform of Fig. 10. The harmonic amplitude and phase were selected simply by application of the fore-

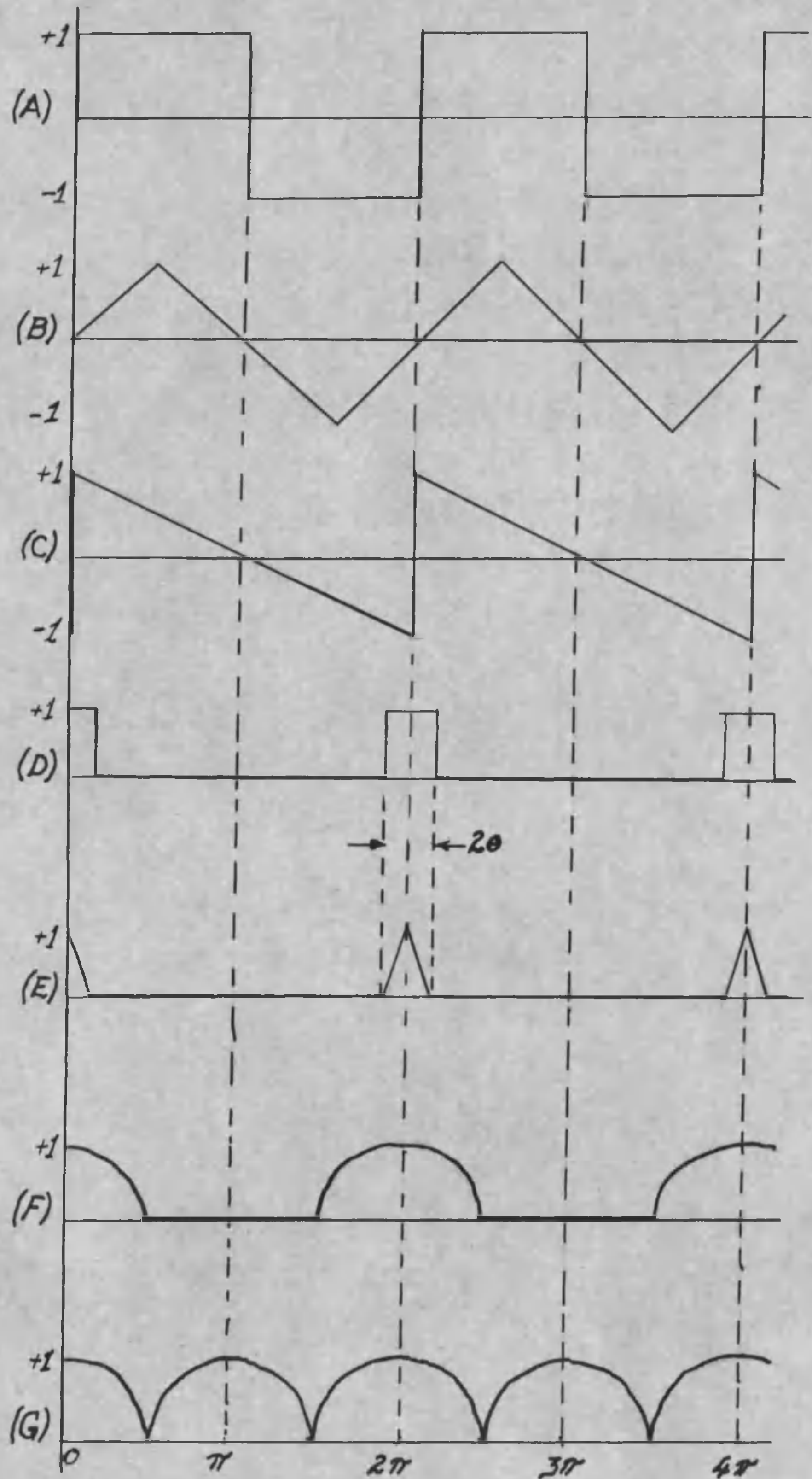


FIG. 9

going theory. The function was seen to be defined by

$$f(x) = x, \qquad -\pi < x < \pi.$$

In this case,

$$a_o = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} x\, dx = 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos nx\, dx$$

$$= 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx\, dx$$

$$= -\frac{2}{n} \cos n\pi,$$

fron which

$$f(x) = 2(\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \ldots).$$

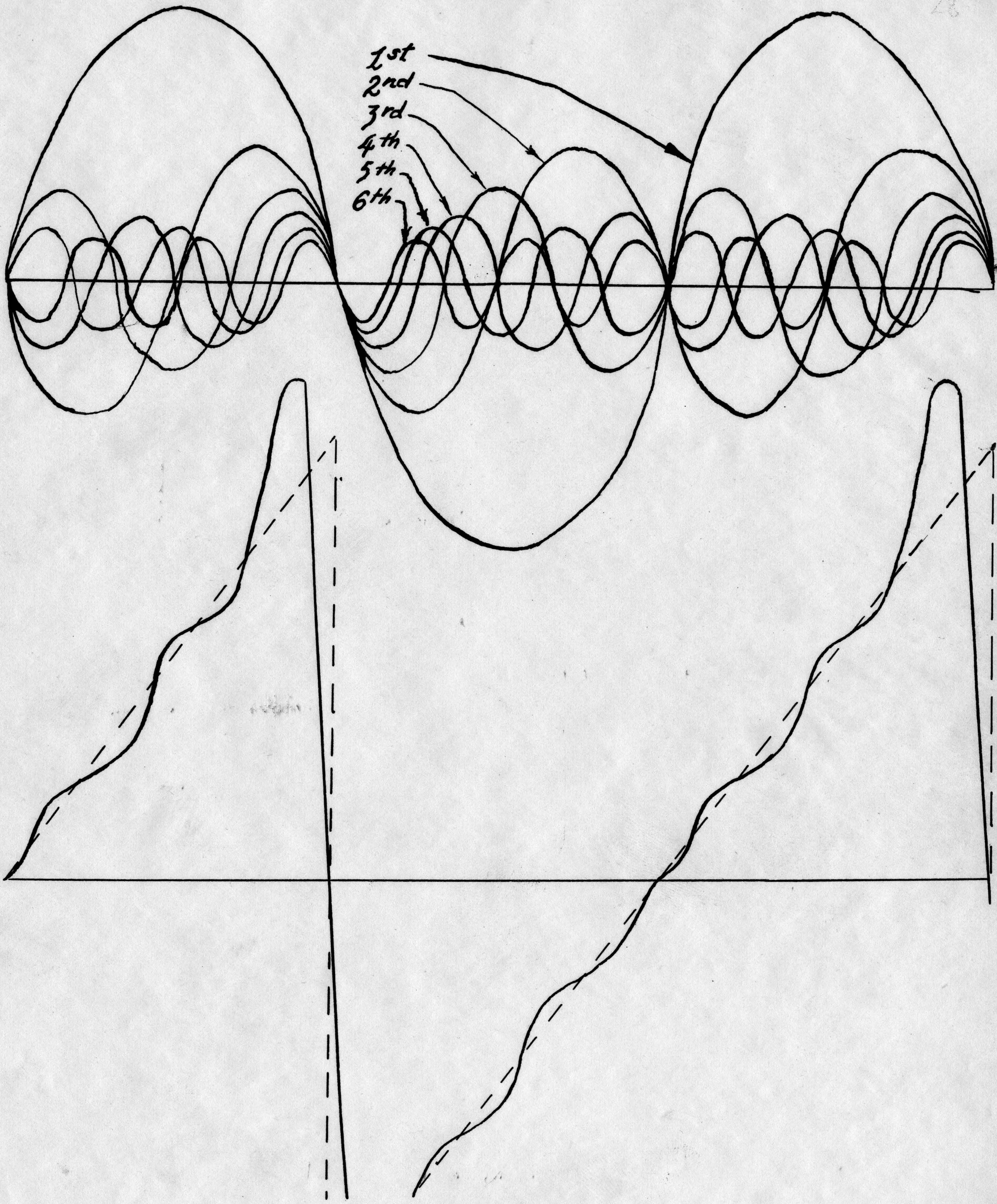


FIG. 10

APPENDIX B

MULTIVIBRATORS

The multivibrator is a self-tripping timing circuit. Its role is a little analogous to a self driven amplifier acting as an oscillator. Multivibrator circuitry is developed from the basic Eccles-Jordan trigger circuit, which, as a matter of fact, is the basis for most trigger circuits.

The Eccles-Jordan circuit is shown in Fig. 11. Suppose that at the initial time of our consideration of the circuit the two tubes 1 and 2 are carrying the same amount of plate current. In any tube conducting current there will be minor fluctuations in this current, usually called "hiss noise". This is a reasonable situation, as it is inconceivable that each current should have precisely the same electron density at all times. Let us suppose that the tube 1 at some instant departs from the initial condition of equal plate currents, and starts to conduct a slightly larger current than tube 2. More current through this plate resistor will cause point A to drop slightly in potential above ground. This causes point B to drop also, thus tending to limit the current through tube 2, which acts to increase point C in potential. This brings up the grid D, which causes more current to flow in tube 1. The effect is seen to be cumulative, and the result is that the tube 1 will skyrocket to a state of maximum conduction, and tube 2 will drop to near cutoff, or possibly

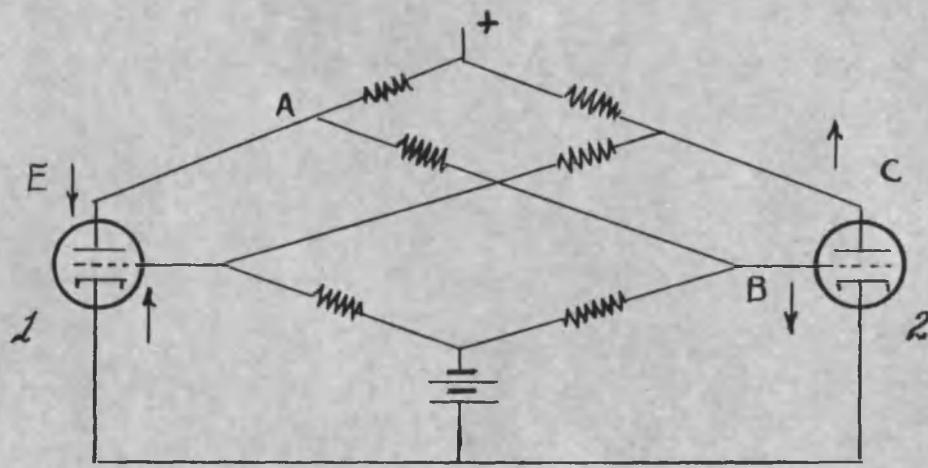


FIG. 11

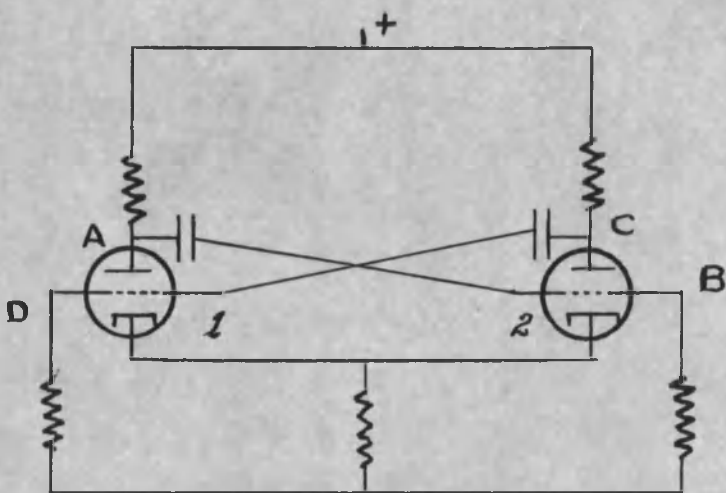


FIG. 12

below cutoff. This new conduction state is stable until some signal is applied to either tube which would tend to reverse the situation, for example a positive pulse at grid B. Then the tubes will reverse roles in a like manner to that described above. The circuit will then again be stable until "triggered" by an appropriate signal.

The basic multivibrator circuit is the same as the Eccles-Jordan with the bias supply removed and coupling capacitors replacing the plate-to grid resistors. The circuit is shown in Fig. 12. Whereas the Eccles-Jordan circuit was seen to flip only when excited by a signal, the multivibrator is free running. Qualitatively its operation may be examined with reference to Fig. 12. If tube 1 is originally conducting the same amount as tube 2, and a small change in operating conditions causes tube 1 to conduct a slightly larger amount, it will start to behave in the same manner as described for the Eccles-Jordan circuit. The point A falls in potential immediately, as does the point B. This reduces the plate current of tube 2, which reflects back through to further increase plate current in tube 1. All this happens almost instantly.

In order to cut off plate current in tube 2, the grid of that tube must be driven below cutoff. This was accomplished through the fact that for a quick surge of current a capacitor acts as a short circuit. Now, however, the capacitor on the plate of tube 1 starts to discharge, thus bringing up the potential on grid 2, point B. As soon as the potential at B reaches cutoff, tube 2 begins to conduct, and now the current in tube 2 rises to a maximum value immediately. Over a period of time, however, the capacitor on the plate of tube 2 will discharge and the same situation exists with tube 1 as previously existed with tube 2. This may be made somewhat clearer by an examination of Fig. 13, which shows the waveforms of a typical free-running balanced multivibrator. It may be seen that the frequency is controlled by the RC discharge time of the resistor and capacitor on each grid. The asymmetrical waveform of an unbalanced multivibrator is shown in Fig. 14. Here it may be noted that as before the respective RC discharge times govern the frequency, and this particular waveform was achieved by making the resistors and capacitors of different values for tubes 1 and 2 respectively.

Multivibrators properly synchronized provide an exact method of frequency division. Either sine waves or pulses may be used for synchronizing purposes, and the synchronizing signal may be applied to cathode or grid of the tube to be synchronized. Essentially, the method of synchronization is to apply the pulse at the time when the grid being pulsed is nearing cutoff from below. If the pulse is of sufficient magnitude to cause the tube to conduct, it will immediately start conducting maximum current. This maximum current is limited only by the resistance of the tube at saturation and the plate load resistance. From this point the multivibrator

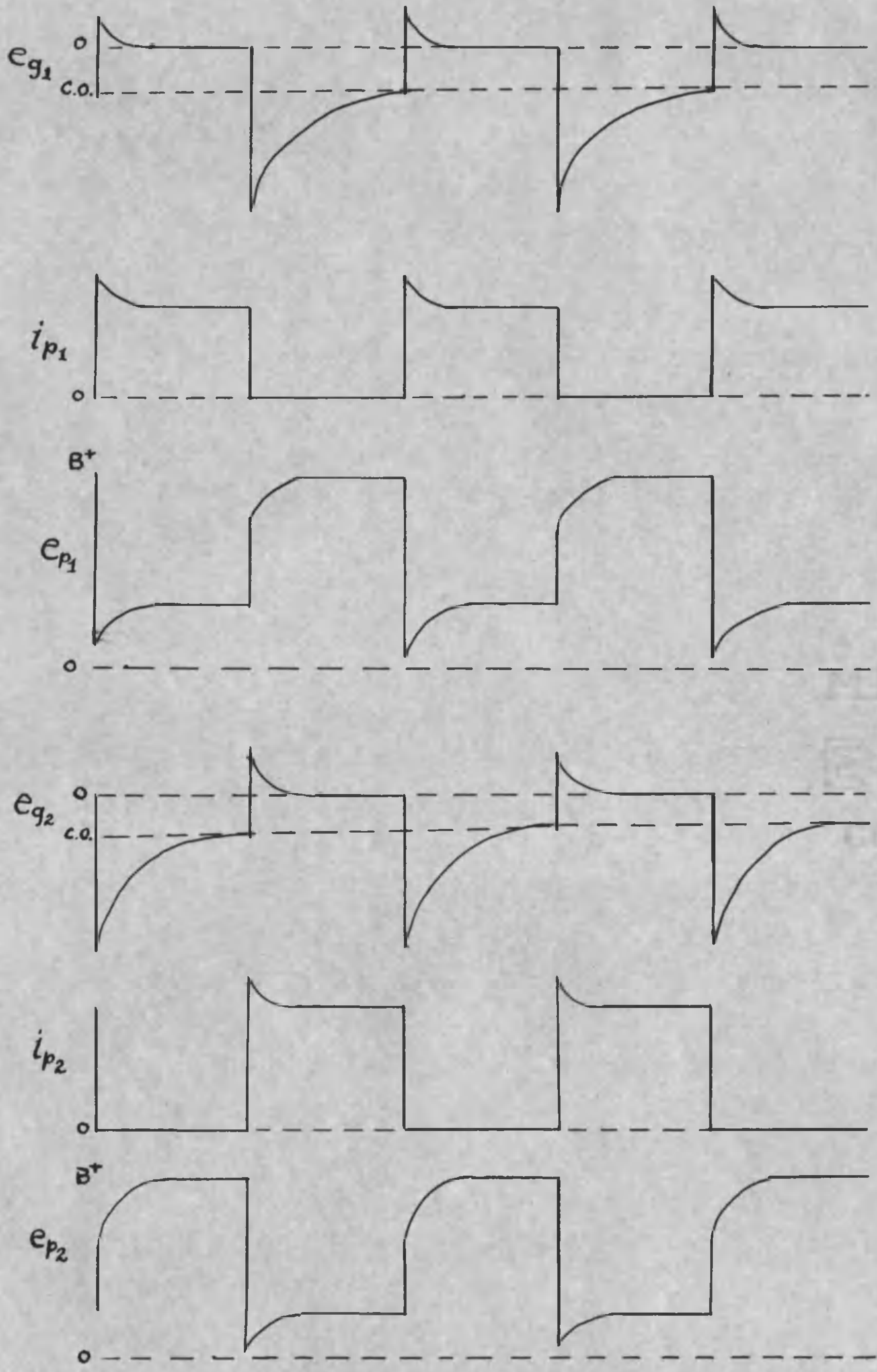


FIG. 13

behaves exactly as before, until the synchronized grid again nears cutoff from below, in which case it will be caused to rise above cutoff by a small positive synchronizing pulse. This is indicated graphically in Fig. 15. It may be noticed that for best frequency control the frequency of the free running multivibrator should be just slightly less than the desired frequency when the multivibrator is synchronized.

Dependent on the sharpness of the synchronizing pulse and the care and accuracy in the design and building of the multivibrator, it is possible to obtain accurate frequency division as high as twenty or thirty, although usually division by ten or less is used.

Either positive or negative pulses may be used for synchronization purposes. A positive trigger pulse applied to a conducting tube of a multivibrator has no effect on the action of the multivibrator, and if applied to a nonconducting tube can cause switching action only if the pulse is large enough to raise the grid above the cutoff voltage. Similarily, a negative trigger pulse applied to a nonconducting tube of a multivibrator has no effect on the operation of the multivibrator. If a negative pulse is applied to a conducting tube it will synchronize provided that when it is amplified through the tube the resultant positive pulse is large enough to raise the grid of the nonconducting tube up to cutoff.

Multivibrator design is approached by considering the

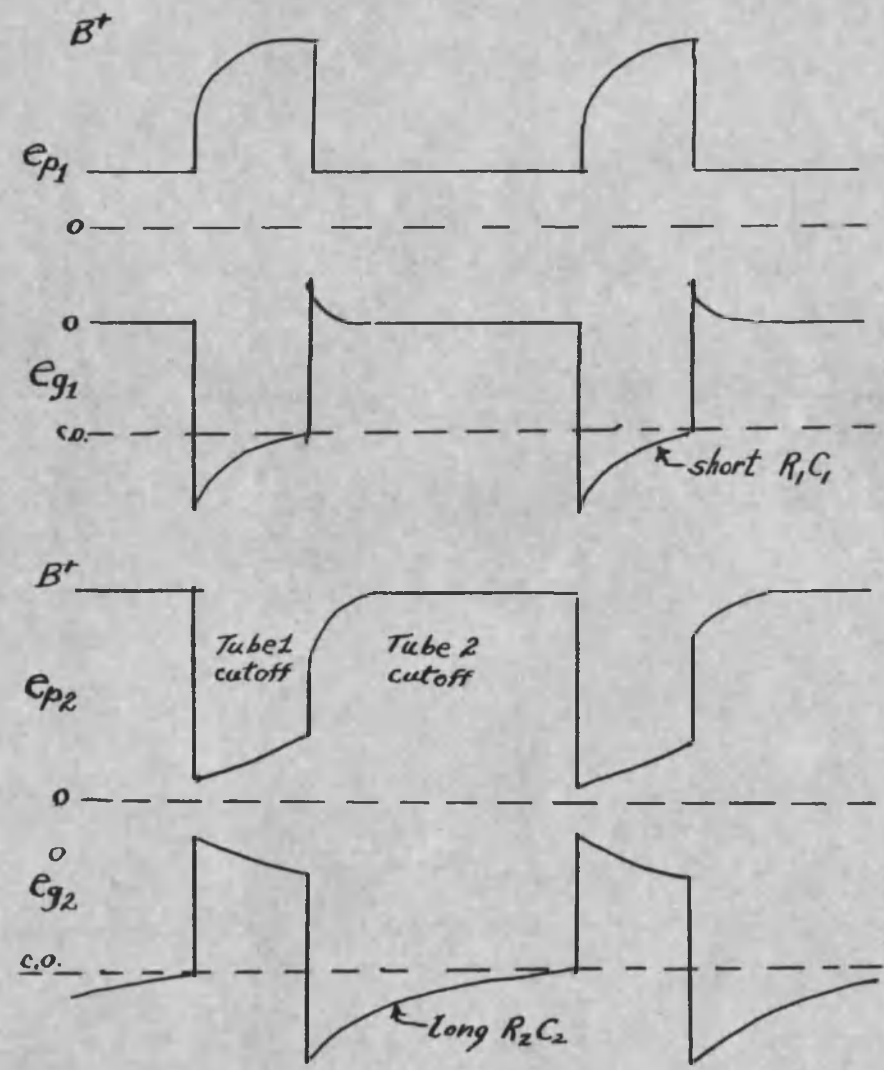

B+
ep1
o
o
eg1
c.o.
short R1C1
B+
Tube1
cutoff
Tube 2
cutoff
ep2
o
o
eg2
c.o.
Long R2C2

FIG. 14

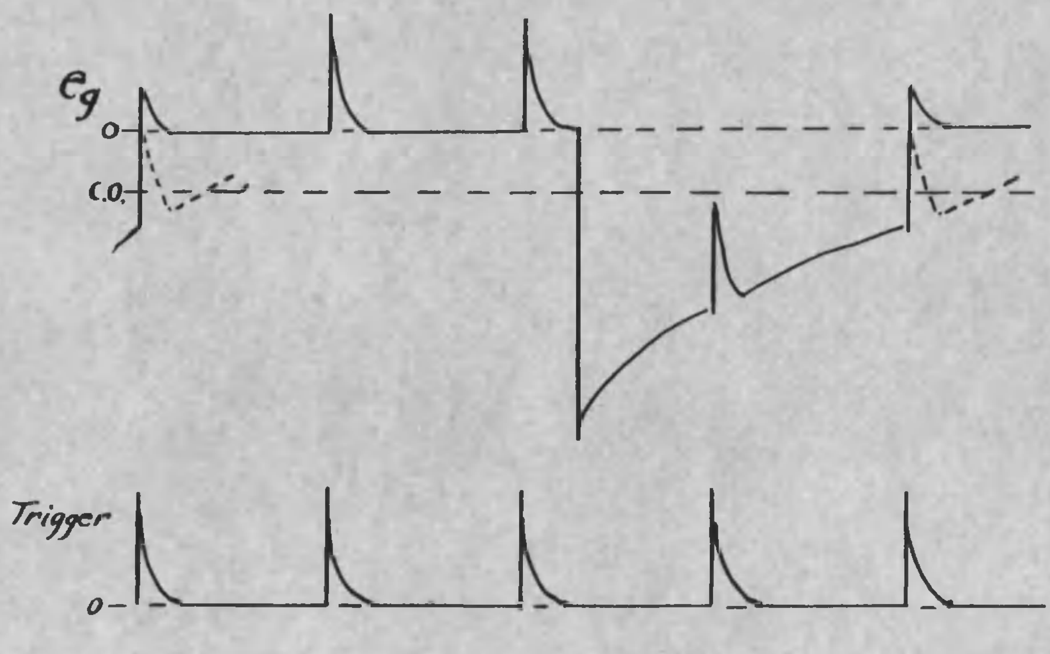
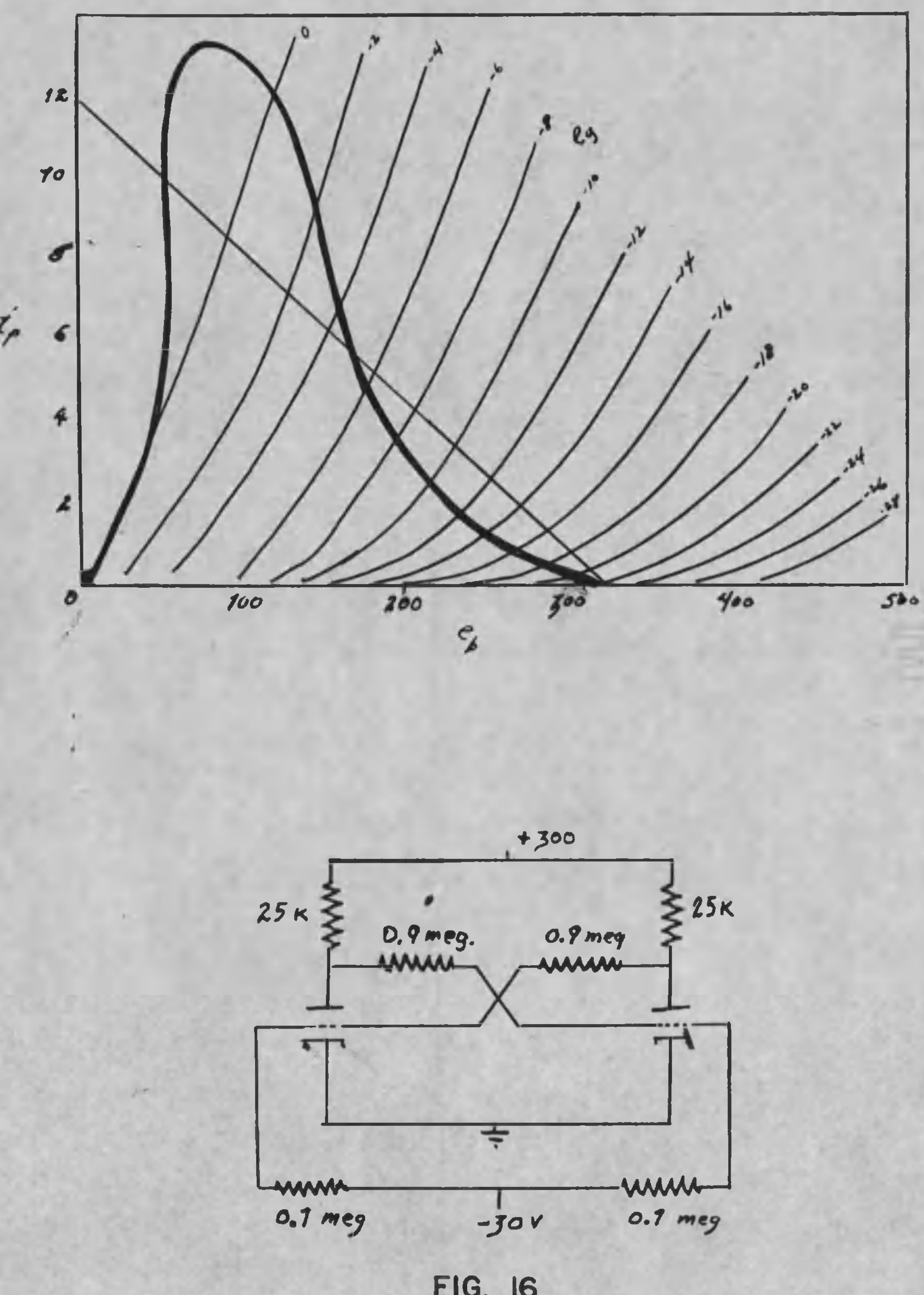

eg
o
c.o.
Trigger
o

FIG. 15

circuit first as an Eccles-Jordan circuit, working out a satisfactory design, and then inserting a capacitor in the plate to grid line. The tube to be used in the multivibrator might well be a 6C5 triode, the plate characteristic curves of which are shown in Fig. 16. Assume a value of plate resistor for the circuit, and then draw in the load line as shown in this same figure. Consider first the tube on the left. Remove the resistor from the plate of this tube, and assume some plate voltage E_{p1}. Assume some value of resistor network from plate to grid and grid to ground. Consider each side of the circuit to have the same value of circuit components. Compute the voltage on the grid of tube 2, E_{g2}, and then from the load line drawn in the characteristic curves, find the value of plate voltage on this tube, E_{p2}. From this, compute the potential of the grid of the first tube, E_{g1}. From the characteristic curves, not using the load line however, read the plate current I_{p1}. Repeat this whole procedure using a second assumed value of plate voltage on tube 1, and repeat with several such assumed values until enough information is obtained to plot a curve such as is shown superimposed over the load line and characteristic curves of Fig. 16.

From one consideration, the current-voltage relationship of tube one would be indicated by this newly drawn curve. But we know that also the load line must represent the current-voltage relationship of this tube. Accordingly



0
100
200
300
400
500
e_p
i_p
2
4
6
8
10
12
e_g
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
-24
-26
-28
+300
25K
25K
0.9 meg.
0.9 meg
0.1 meg
-30V
0.1 meg

FIG. 16

it is seen that there are only three points that satisfactorily represent conditions at the plate of this tube, these three points being the points of intersection of the load line and the plotted curve. These points are indicated in Fig. 16 as points A, B and C. Point B may be shown to be shown to be unstable, and thus the plate of tube 1 may be at either point A or point C. Due to the symmetry of the circuit, point C must represent the conditions at the plate of tube 2 when point A indicates the conditions on the plate of the first tube, and visa versa.

The curve of Fig. 16 was plotted by assuming that the grids were connected to a point 30 volts below ground, that the plate to grid resistance was 0.9 megohms, and that the grid to grid bias resistance was 0.1 megohm. A load resistor of 25000 ohms was selected. The following table was made out under those conditions:

E_{p1}	0	10	40	70	100	200
E_{g2}	-27	-26	-21	-20	-17	-7
E_{p2}	300	300	290	285	280	190
E_{g1}	0	0	.5	2	1	-8
I_{p1}	0	0	6 ma	12 ma	13 ma	3.5 ma

CONCLUSION

The number of harmonics contained in this particular synthesizer limits the smoothness of the manufactured waveform, but for the purposes to which this device was to be adapted it is the author's feeling that the quality of the waveform is adequate.

For use in lectures on Fourier analysis this apparatus should provide the lecturer with a satisfactory piece of demonstration equipment. Further than this it could be used to demonstrate certain phenomena having to do with acoustics, such as beat frequency tones or the audible effect of a change of phase of component frequencies of a complex waveform.

Complete calibration of the instrument is yet to be accomplished. Because of the haphazard order in which the chain of multivibrators may chance to lock in with each other when the instrument is turned on, the phase controls require a resetting each time the instrument is used. The writer sees no simple method whereby this condition might be corrected.

For an electronic synthesizer, the design employed in the construction of this instrument is apparently most satisfactory. It will be free from drift off frequency, and the phase lock-in will be stable. Not being aware of the existance of other wholly electronic synthesizers, the writer is not in a position to compare the results of operation of this one with the results of other investigators.

From the standpoint of practicability, it is doubtful that this instrument is an improvement on existant types of mechanical-electrical waveform synthesizers due to its weight, cost, and the details of its design, construction, and alignment in comparison to these same factors in other types of synthesizers.

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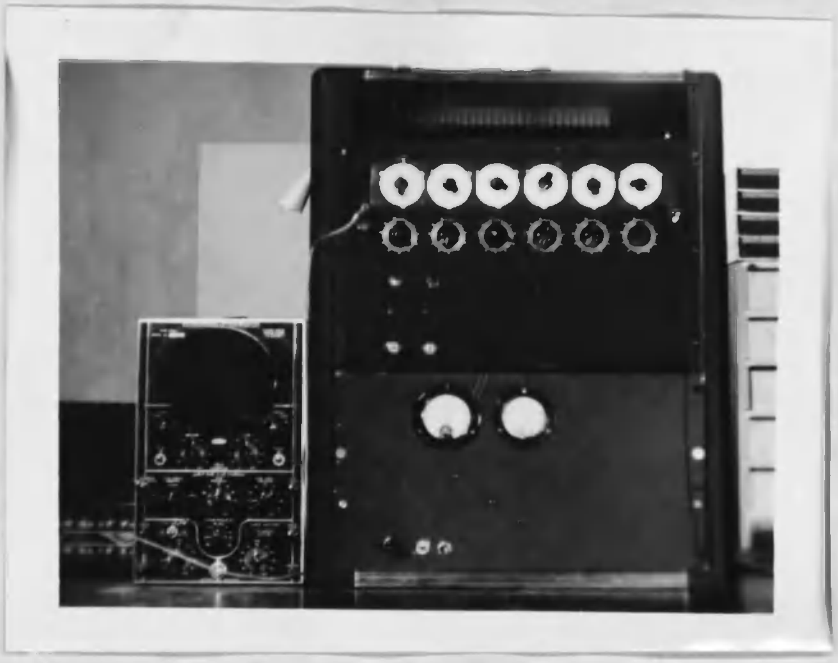
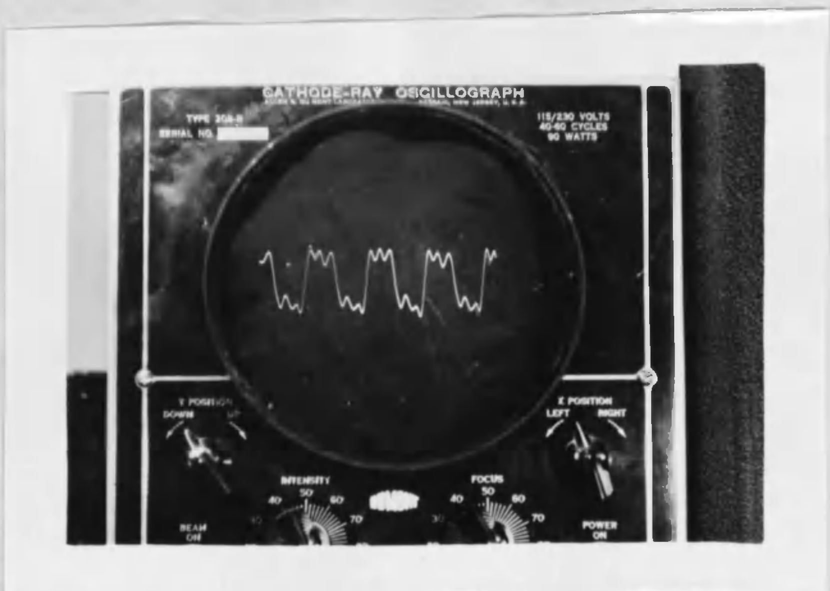


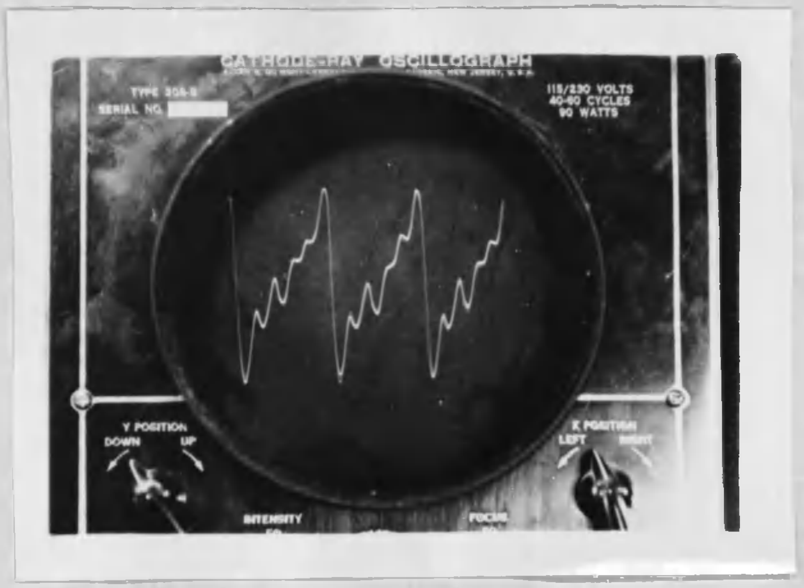
(a) The Fourier waveform synthesizer



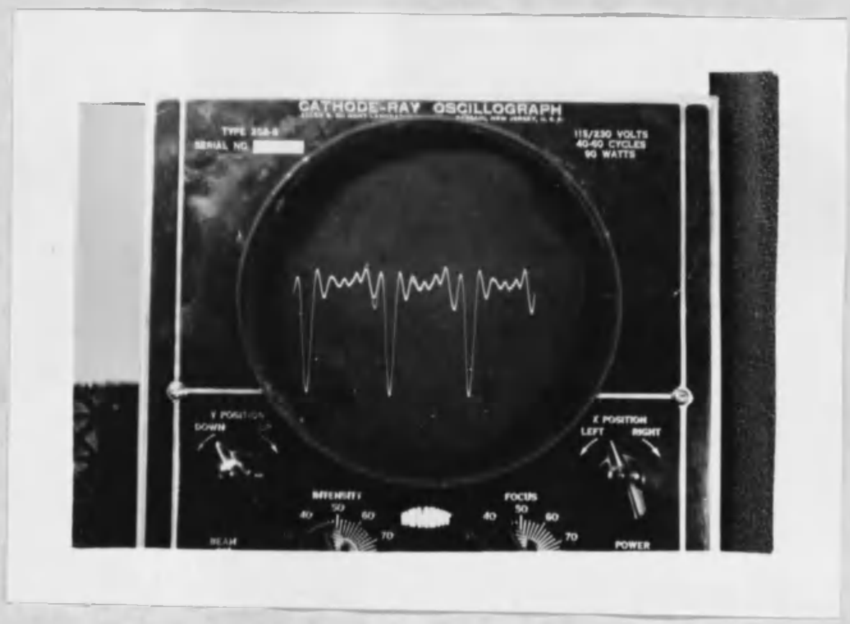
(b) Square wave from first, third, and fifth harmonics

PLATE ONE

Photographs by B.Cato



(a) Sawtooth wave from first six harmonics

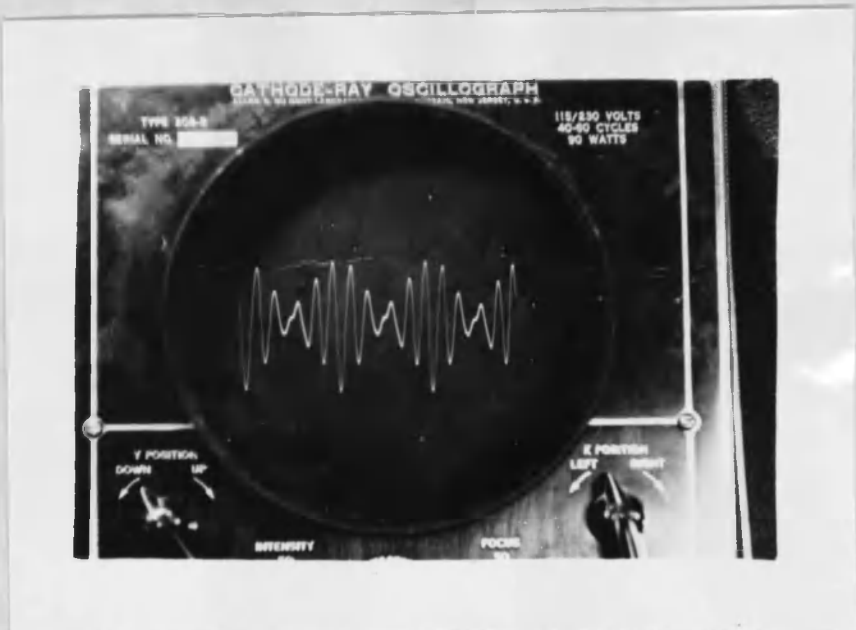


(b) Pulse wave from first six harmonics

PLATE TWO

Photographs by B.Cato

(a) Pulse wave from first three harmonics



(b) Addition of two adjacent harmonics

PLATE THREE

Photographs by B.Cato