DEVELOPMENT OF A COMMON EMITTER EQUIVALENT CIRCUIT FOR THE JUNCTION TRANSISTOR

by

David J. Sakrison

A Thesis

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DEVICE FOR THE JUXTAPOSITION TRANSDUCER

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INTRODUCTION

In the past five years the applications of the transistor in electronic circuitry have become extremely diverse. The use of the transistor is no longer confined to low power audio applications, but has been extended to include use in oscillators, switching and trigger circuits, and bandpass amplifiers. In spite of the expanding widespread usage of transistors, very little has been published on a satisfactory analytical method of design applicable to all types of electronic circuits in which it may be desirable to use transistors.

Dr. T. L. Martin\(^1\) has advanced a workable and extremely versatile method of electronic circuit design applicable to almost all types of electronic circuits now in use. The success of the method is dependent upon the representation of electronic components by suitable equivalent circuits. None of the transistor equivalent circuits in common use have proven to be at the same time both a workable and accurate representation of the transistor. A. W. Lo and others\(^2\) have


arrived at a fairly satisfactory equivalent circuit purely from a theoretical analysis of the mechanism of conduction in a transistor. This paper will be concerned with an attempt to develop empirically a workable equivalent circuit which accurately represents the junction transistor for all frequencies of interest.
Chapter 1

TRANSISTOR EQUIVALENT CIRCUITS

The purpose of this chapter is to introduce the reader to several of the more commonly used transistor equivalent circuits, particular emphasis being placed on the equivalent circuit's usefulness as a design tool. A basic understanding of the principles of electronic equivalent circuitry is assumed; for a basic discussion of the subject the reader is referred to chapter one of "Electronic Circuits." 3

In general any four terminal electronic component may be represented by an appropriate two-terminal-pair equivalent circuit. For the purposes of this discussion an equivalent circuit will be considered to be a two-terminal-pair network composed of linear active and passive elements such that the equivalent circuit viewed from the two pairs of terminals exhibits characteristics essentially the same as those of the device which it represents. A junction transistor, being a three terminal circuit element, may be considered to be a degenerate form of a two-terminal-pair

network in which one terminal of one pair is common to one terminal of the other pair. The selection of one of the terminals of a transistor as being common to both the input and the output terminals of a transistor amplifier gives rise to the three configurations of transistor amplifiers; the common emitter, the common base, and the common collector or emitter follower. In setting up an equivalent circuit for the transistor we may choose arbitrarily any one of the above three configurations for the equivalent circuit description of the transistor. To date only two of the three configurations, the common emitter and the common base, have been used for transistor equivalent circuits. From the author's point of view the common emitter appears to be the more desirable configuration for design use in view of the fact that the great majority of transistor circuits in use employ the common emitter amplifier most frequently.

Figure 1.1 shows the reference voltages and currents for the transistor to be used throughout this paper. For brevity, whenever the common emitter configuration is used, $V_{be}$ and $V_{ce}$ will be written as $V_b$ and $V_c$ respectively. The following notation will be used in describing the operation of the transistor:

- $V_{bb}$ = base bias supply voltage
- $V_{cc}$ = collector bias supply voltage
- $I_{bb}$ = quiescent component of the base current
Figure (1.1)
Reference Voltages and Currents for the Transistor
\[ i_b = \text{variational component of the base current} \]
\[ I_b = \text{total base current} = I_{bb} + i_b \]
\[ I_c = \text{total collector current} = I_{cc} + i_c \]
\[ V_b = \text{quiescent component of the base voltage} \]
\[ v_b = \text{variational component of the base voltage} \]
\[ V_c = \text{quiescent component of the collector voltage} \]
\[ v_c = \text{variational component of the collector voltage} \]

The transistor shown in figure (1.1) is an N-P-N type transistor. \( V_{cc}, V_{bb}, I_b, I_c, V_b, \) and \( V_c \) are all positive quantities; and \( I_e \) is a negative quantity with the reference polarities as shown in figure (1.1). For a P-N-P type transistor the reverse of the above situation is true, and the arrow on the emitter lead which indicates the direction of total emitter current would be in the opposite direction.
(1.1) **THE TEE EQUIVALENT CIRCUIT**

The tee equivalent circuit was the first transistor equivalent circuit to come into widespread use. As will be true for all of the equivalent circuits discussed in this paper, the tee equivalent circuit represents the transistor because it is equivalent for the variational components of the transistor currents and voltages in the linear region of operation. The tee equivalent circuit can be defined for either the grounded emitter or the grounded base configuration as shown in figures (1.2a) and (1.2b). It is to be noted that the parameters $r_b$, $r_e$, and $r_m$ are the same for both configurations while $r_d$, which appears in the common emitter configuration, is equal to $(r_c - r_m)$, $r_c$ being the collector resistance of the grounded base circuit.

The high frequency equivalent circuit shown in figure (1.2c) is derived in "Transistor Electronics." For circuit design, this circuit suffers from three main disadvantages. First, as noted in the reference, the circuit is not completely accurate at high frequencies. Second, the parameters of the equivalent circuit in this form would be difficult to obtain and would have to be measured indirectly. Third,

---

(A) GROUNDED Emitter Configuration

(B) GROUNDED BASE Configuration

(C) GROUNDED BASE HIGH FREQUENCY EQUIVALENT CIRCUIT

\[ \alpha' = \frac{\alpha_0}{1 + j\frac{\omega}{\omega_{ce}}} \]

(Figure 1.2)
The Tee Equivalent Circuit
and most important, this circuit is not at all applicable for the analysis and design of cascaded amplifiers. It may be shown that the equivalent circuit for a two stage cascaded amplifier with wiring capacitance neglected would have a transfer function whose denominator would be at least a sixth order polynomial in $s$. As the number of stages in the cascade is increased the complexity of the transfer function would correspondingly increase. It becomes readily apparent that this method is not convenient in the analysis or design of a cascaded amplifier of any complexity. It would be desirable to have an equivalent circuit whose output section is isolated from its input so that when the equivalent circuit is used to represent the transistor in a cascade of stages, the overall transfer function of the cascade may be found as the product of the transfer functions of the individual stages. This is the method which has been used and found to be most satisfactory in the analysis and design of vacuum tube circuits. It would be highly desirable to apply this same method to transistor circuits.
(1.2) **EQUIVALENT CIRCUIT PRESENTED IN "ELECTRONIC CIRCUITS"**

The equivalent circuit shown in figure (1.3), presented in chapter eight of "Electronic Circuits," is used to overcome the third disadvantage of the tee equivalent circuit discussed in section 1.1. The circuit of figure (1.3) lends itself readily to the analysis of cascaded amplifiers. The box marked "load" in figure (1.3) represents the interstage coupling network between two stages in a cascade and may consist of an RC coupling circuit, a transformer, a tuned coupling circuit, or any other type of interstage coupling network to be used in an amplifier. The voltage $E_o$ appearing in figure (1.3) is the voltage appearing between ground and the input terminal of the next transistor in the cascade. The equivalent circuit of the cascaded amplifier is then made up of a cascade of non-interacting stages, and the overall transfer function of the cascade is equal to the product of the transfer functions of the individual stages of the cascade.

The circuit used to derive the equivalent circuit presented in "Electronic Circuits" is shown in figure (1.4). It is seen that this circuit is essentially a grounded emitter amplifier using the grounded base configuration of the tee equivalent circuit shown in figure (1.2b). The method of

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**Figure (1.3)**
Equivalent Circuit Presented in "Electronic Circuits"

**Figure (1.4)**
Tee Equivalent Circuit Used to Derive the Circuit of Figure (1.3)
derivation was to solve for the loop currents $i_p$ and $i_c$ in each equivalent circuit under suitable conditions and equate the corresponding currents for the two circuits equal to one another. The impedance $z_t$ was defined as the impedance looking into the transistor from the collector terminal to the emitter terminal with the input terminals of the transistor short-circuited.

The method of attack presented in "Electronic Circuits" and outlined above is basically a good one for the analysis and design of cascaded transistor amplifiers but is not without its shortcomings. The tee equivalent circuit from which the equivalent circuit of figure (1.3) was derived is not an accurate representation of the transistor at high frequencies. A tee equivalent circuit of the form given in figure (1.2c) could have been used for the derivation but the expressions for $z_1$, $g_{ct}$, and $z_t$ would be extremely unwieldy. Also, if the circuit of figure (1.2c) is used for the derivation, the derived equivalent circuit will still suffer from the disadvantage that the parameters of the circuit of figure (1.2c) are difficult to measure.
(1.3) **THE COMMON BASE HYBRID PARAMETER EQUIVALENT CIRCUIT**

Until recently, the common base hybrid parameter equivalent circuit has probably been second only to the tee equivalent circuit in popular usage as a transistor circuit design tool. The circuit, shown in figure (1.5), derives its name from the fact that its four parameters, rather than being the four impedance of admittance parameters usually used to describe a two-terminal-pair network, are a hybrid mixture of impedances, admittances, and dimensionless ratios. The four variables used to describe the operation of the transistor are the base to emitter voltage, $v_{eb}$; the base to collector voltage, $v_{cb}$; the emitter current, $i_e$; and the collector current, $i_c$. Two of the four variables will be dependent and the other two independent. In defining the common base hybrid parameters, $v_{eb}$ and $i_c$ are taken to be the dependent variables and $v_{cb}$ and $i_e$ the independent variables. In terms of these variables the hybrid parameters are:

\[ h_{11} = \left( \frac{\partial v_{eb}}{\partial v_{cb}} \right) v_{cb} \quad (1.1a) \]
\[ h_{21} = \left( \frac{\partial i_c}{\partial i_e} \right) v_{cb} \quad (1.1c) \]
\[ h_{12} = \left( \frac{\partial v_{eb}}{\partial v_{cb}} \right) i_e \quad (1.1b) \]
\[ h_{22} = \left( \frac{\partial i_c}{\partial v_{cb}} \right) i_e \quad (1.1d) \]

The common base hybrid parameter equivalent circuit is presented here only because of its onetime widespread usage. As stated in the beginning, the common emitter configuration is more convenient for use in transistor circuit
Figure (1.5)

Common Base Hybrid Parameter Equivalent Circuit
design; also, the common base hybrid parameter equivalent circuit has not so far proven to be a convenient form for the high frequency description of the transistor.
(1.4) THE COMMON EMITTER IMPEDANCE PARAMETER EQUIVALENT CIRCUIT

The common emitter impedance parameter equivalent circuit shown in figure (1.6) is defined in terms of the four open circuit impedance parameters commonly used in the analysis of two-terminal-pair networks. The four variables used to describe the operation of the transistor are the base voltage, \( V_b \); the collector voltage, \( V_c \); the base current, \( I_b \); and the collector current, \( I_c \). For purposes of defining the common emitter impedance parameters \( V_b \) and \( V_c \) are taken as the dependent variables and \( I_b \) and \( I_c \) as the independent variables. The four impedance parameters are:

\[
\begin{align*}
\Xi_{11} &= \left( \frac{\partial V_b}{\partial I_b} \right) I_c \quad (1.2a) \\
\Xi_{21} &= \left( \frac{\partial V_c}{\partial I_b} \right) I_c \quad (1.2b) \\
\Xi_{12} &= \left( \frac{\partial V_b}{\partial I_c} \right) I_b \quad (1.2a) \\
\Xi_{22} &= \left( \frac{\partial V_c}{\partial I_c} \right) I_b \quad (1.2b)
\end{align*}
\]

The common emitter impedance parameter equivalent circuit has never received widespread usage. Although somewhat similar to the common emitter hybrid parameter equivalent circuit to be discussed in the next section, the impedance parameter circuit has not proven as adaptable to the description of the transistor as the hybrid parameter circuit.
Figure (1.6)
Common Emitter Impedance Parameter Equivalent Circuit
(1.5) **THE COMMON EMITTER HYBRID PARAMETER EQUIVALENT CIRCUIT**

Currently coming into widespread use in the analysis of transistor circuits is the common emitter hybrid parameter equivalent circuit shown in figure (1.7). This equivalent circuit uses the same variables to describe the operation of the transistor as does the impedance parameter equivalent circuit of section 1.4. The selection of independent variables, however, is different. In defining the common emitter hybrid parameters, $v_b$ and $i_c$ are the dependent variables and $i_b$ and $v_c$ the independent variables. It is this selection of independent variables which makes the hybrid parameters more suitable than the impedance parameters for describing the transistor at low frequencies. This is true because the impedance of the collector circuit is normally larger than the load connected to it and the collector voltage is more easily controlled than the collector current. The base circuit, on the other hand, normally has a lower impedance than the source connected to it and the base current is more easily controlled than the base voltage. The hybrid parameters in terms of the four variables are:

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6. To the best of the author's knowledge, the common emitter hybrid parameter equivalent circuit was first published in "Transistor Electronics" (see footnote 2). Prior to that time the circuit had been defined and used by Josef Gartner and Aladdin N. Perkins of the Electrical Engineering Dept. of the University of Arizona.
**Figure (1.7)**
Common Emitter Hybrid Parameter Equivalent Circuit

**Figure (1.8)**
Common Emitter High Frequency Equivalent Circuit
\[ a_{11} = \left( \frac{\partial V_e}{\partial i_b} \right) v_c \quad (1.3a) \]
\[ a_{21} = \left( \frac{\partial i_c}{\partial i_b} \right) v_c \quad (1.3c) \]
\[ a_{12} = \left( \frac{\partial V_b}{\partial i_b} \right) i_b \quad (1.3b) \]
\[ a_{22} = \left( \frac{\partial i_c}{\partial i_b} \right) i_b \quad (1.3d) \]

As will be discussed in chapter two, the d-c values of the hybrid parameters may be obtained from the d-c characteristics of the transistor operated in the grounded emitter configuration.

In view of the similarities between the two equivalent circuits, it would seem logical that the common emitter hybrid parameters could be expressed in terms of the common emitter impedance parameters. It may be shown by writing the loop equations around the different equivalent circuits that the common emitter hybrid parameters, the common emitter impedance parameters, the common base hybrid parameters, and the tee parameters may all be expressed in terms of one another. Using this approach, conversion formulas between the different parameters have been derived.7

Figure (1.8) shows the high frequency form of the hybrid parameter equivalent circuit developed in chapter five. Although certain secondary effects have been neglected in developing the equivalent circuit, it is still an accurate representation of the transistor for all conditions under

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7. For a list of the conversion formulas, see chapter two of "Transistor Electronics" (see footnote 2) and chapter two of "A Common Emitter Equivalent Circuit for Transistor Design" (Thesis) by A. M. Perkins, University of Arizona, Tucson, 1955.
which the transistor will normally be operated in the linear region. It is seen that the output section of the equivalent circuit is isolated from the input; hence, the equivalent circuit is adaptable to the method of analysis of cascaded transistor amplifiers discussed in section 1.2.

The attention of the remainder of this paper will be confined to a discussion of the common emitter hybrid parameter equivalent circuit.
Chapter 2

D-C CHARACTERISTICS OF THE TRANSISTOR

(2.1) D-C DESCRIPTION OF THE TRANSISTOR

As pointed out in the first chapter, the four variables $V_b$, $V_c$, $I_b$, and $I_c$ may be used to completely describe the operation of the transistor in the common emitter configuration. Two of the variables are dependent and the other two independent. We may then write two equations, each expressing one of the dependent variables in terms of the two independent variables. As pointed out in section 1.5, a logical choice of variables for one of the equations would be to express $I_c$ as a function of $V_c$ and either $I_b$ or $V_b$. Because in the class A linear region $I_c$ is nearly a linear function of $I_b$, we will express $I_c$ as a function of $V_c$ and $I_b$. The other equation will then express $V_b$ as a function of $V_c$ and $I_b$.

As in vacuum tube practice, we will represent each equation by a family of curves, the dependent variable and one of the independent variables being plotted along the two coordinate axes and the other independent variable being treated as a parameter. In the equation for $I_c$, $I_b$ will be treated as the parameter, and the family of curves will be referred to as the **mutual characteristics** of the transistor. In the other equation $V_c$ will be considered the parameter,
and the family of curves will be referred to as the input characteristics of the transistor.

The d-c characteristics of the transistor have two important uses. First, as will be shown in section 2.4, the linear equivalent circuit can be derived from the d-c characteristics. The low frequency values of the hybrid parameters for a transistor may thus be obtained from its d-c characteristics. Second, a knowledge of the d-c characteristics is essential in designing a transistor circuit. If the circuit is to be used for class A operation, the d-c characteristics must be used to properly determine a quiescent operating point for which the output of the circuit will not be excessively distorted. If the transistor is to be used in a switching or multivibrator circuit, the circuit must be designed around a knowledge of the d-c characteristics.
(2.2) **OUTPUT CHARACTERISTICS OF THE TRANSISTOR**

The d-c characteristics of the transistors to be used for the investigation were obtained with the use of a Librascope X-Y Plotter. Figure (2.1) shows the diagram of the circuit used to measure the output characteristics of the transistor. The four pole double throw switch serves to reverse the polarities of the power supplies and meters to change from the measurement of N-P-N to P-N-P type transistors. The single pole double throw switch changes the resistance in series with the base lead to provide flexibility in the range of base current. Manual regulation of the base current while the characteristics are being taken is accomplished with the 0-20 k ohm variable resistance in series with the base lead. The 0-1 k ohm resistance is a precision decade resistance used to obtain a voltage proportional to the collector current to feed to the X input of the X-Y Plotter. The X input of the plotter is connected directly from the collector terminal to ground; thus, the two inputs to the plotter are the collector voltage and the collector current. The input impedance of the plotter inputs is 200 k ohms; hence, the plotter does not appreciably affect the circuit being measured.

In obtaining the output characteristics, different values of the base current were obtained by varying $V_{bb}$ and
Figure (2.1)
Circuit Used to Measure the Output Characteristics of the Transistor
the resistance in series with the base lead. The collector supply voltage, $V_{cc}$, was then varied from zero to the maximum desired, and the plotter recorded a continuous plot of the collector current, $I_C$, versus the collector voltage, $V_C$. As the collector voltage was varied, the base current was held at a constant value by manually adjusting the 0-20 k ohm resistance in series with the base lead. Regulation of the base current was not difficult and was required only until the knee of the curve was reached.

Five Bell Telephone Laboratory type 2N27 and five Sylvania type 2N35 transistors were used in the investigation. Output characteristics were obtained as described above for all of the transistors. Figures (2.2) and (2.3) respectively show the output characteristics obtained for one of the 2N35 and one of the 2N27 type transistors.

Figures (2.4) and (2.5) are the idealized output characteristics for the transistors whose output characteristics are given in figures (2.2) and (2.3) respectively. It is seen from the figures that the idealized output characteristics are broken up into three regions. Region three is the linear or normal region of operation. In drawing the idealized characteristics, it was assumed that in the linear region the curves were parallel straight lines spaced equal distances apart. In section 2.4, the low frequency equivalent
Figure (2.2)
D-C Output Characteristic
SYLVANIA 2N35 No. 53

Collector Current (mA) vs Collector Voltage (volts)

Base Current (mA)
Figure 2.3

D-C Output Characteristic
B.T.L. 2N27 No. 26
circuit will be derived from the idealised input and output characteristics of the transistor.
Figure 2.4

Idealized Output Characteristic

Sylvania 2N35 No. 53

Collector Current (mA) vs Collector Voltage (Volts)
Figure (2.5)
Idealized Output Characteristic
B.T.L. 2N27 No. 26

Collector Current (mA)

2.0
1.6
1.2
0.8
0.4
0.0

Collector Voltage (volts)

0 2 4 6 8 10 12 14 16

Region 1
Region 2
Region 3
(2.3) **INPUT CHARACTERISTICS OF THE TRANSISTOR**

The circuit used to measure the input characteristics is shown in figure (2.6). With minor differences, it is the same circuit used to measure the output characteristics. The decade resistance in series with the collector lead has been replaced by an 0-100 k ohm precision decade resistance in series with the base lead. This resistance feeds a voltage proportional to the base current to the Y input of the X-Y Plotter. The X input of the plotter is connected directly from the base terminal to ground; thus, the plotter records base voltage as a function of base current.

In measuring the input characteristics, it was exceedingly difficult to hold the collector voltage constant. For this reason, the electro-mechanically regulated power supply shown in the lower part of figure (2.6) was built. The output of the regulated supply was connected from the collector lead to ground, ensuring that the collector voltage, \( V_C \), remained constant while the input characteristics were being measured.

The 0-100 ohm helipot provides a variable reference voltage for the regulating system. The difference between this reference voltage and some fraction of the collector voltage is feed to the input of the Brown Electronik Continuous Balancing Unit. This balancing unit consists of a
Figure (2.6)
Circuit used to measure the input characteristics of the transistor.
chopper amplifier and a two-phase motor. The rotational velocity of the motor is roughly proportional to the d-c input to the balancing unit. The balancing unit then provides constant collector voltage by varying the 0-100 ohm resistance in series with the load across the collector voltage.

Different constant values of collector voltage for measuring the input characteristics were obtained by varying the output voltage range switch and the reference voltage picked off the helipot. The base bias voltage, $V_{bb}$, was varied from zero to the maximum desired, and the plotter recorded a continuous curve of base voltage versus base current for each of the different values of collector voltage.

The input characteristics of all ten transistors were measured. Figures (2.7) and (2.8) are respectively the input characteristics for one of the 2N35 and one of the 2N27 type transistors. The input characteristics of the transistor were divided into the same three regions of operation as the output to draw the idealized characteristics. Region three again corresponds to the normal linear region of operation. The idealized characteristics were drawn assuming that in region three, the characteristics were parallel straight lines equidistant from one another.
Figure 2.7
D-C Input Characteristic
SYLVANIA 2N35 No.53
Base Voltage (Volts)

Base Current (mA)

Figure (2.8)
D-C Input Characteristic
B.T.L. 2N27 No. 26
Figure 2.9
Idealized Input Characteristic
Sylvania 2N35 No. 53
Figure (2.10)
Idealized input characteristic
B.T.L. 2N27 No. 26
(2.4) DERIVATION OF THE LOW FREQUENCY EQUIVALENT CIRCUIT FROM THE D-C CHARACTERISTICS

The same selection of dependent and independent variables was made for the d-c characteristics as was made in defining the common emitter hybrid parameters. The hybrid parameters could then be derived from an analytical expression for the curves of the input and output characteristics using the definitions of the 'a' parameters given on page 16. The idealized characteristics will be used in the derivation since the parameters of the resulting equivalent circuit will be linear.

Consider the idealized output characteristics. In region two the equation of the curve for zero base current may be written

\[ I_c = I_{co} + \frac{\Delta I_c}{\Delta V_c}(I_b = 0) \]

\( I_{co} \) is the intercept value of the collector current and \( \frac{\Delta I_c}{\Delta V_c}(I_b = 0) \) is the slope of the curve for zero base current. All the curves are drawn as parallel straight lines and, hence, the slope of all the lines is the same and will be denoted by \( \frac{\Delta I_c}{\Delta V_c}(I_b = 0) \). The curves for uniformly increasing values of base current were drawn equidistant; consequently, the intercept value of the current for different values of base current may be written as

\[ I_{co} + \frac{\Delta I_c}{\Delta I_b}(V_c = 0) \]
The expression for the collector current may then be written

$$I_c = I_{co} + \frac{\Delta I_c}{\Delta I_b} (v_c = 0) + \frac{\Delta I_c}{\Delta v_c} (I_b = 0) v_c$$  \hspace{1cm} (2.2)

For the linearised characteristics we may write

$$\frac{\Delta I_c}{\Delta I_b} (v_c = 0) = \left( \frac{\partial I_c}{\partial I_b} \right) v_c = a_{21}$$  \hspace{1cm} (2.3)

and

$$\frac{\Delta I_c}{\Delta v_c} (I_b = 0) = \left( \frac{\partial I_c}{\partial v_c} \right) I_b = a_{22}$$  \hspace{1cm} (2.4)

From equations (2.3) and (2.4) equation (2.2) may be rewritten

$$I_c = I_{co} + a_{21} I_b + a_{22} v_c.$$  \hspace{1cm} (2.5)

Consider the curves of the idealised input characteristics in region three. Denote the intercept value of the base voltage for the lowest value of $v_c$ in region three by $v_{be}$.

$$v_b = v_{bc} + \frac{\Delta v_b}{\Delta I_b} (V_c = 0) I_b.$$  \hspace{1cm} (2.6)

The curves of the idealised input characteristics are parallel straight lines; therefore, the slope of each line is the same and will be denoted by $\frac{\Delta v_b}{\Delta I_b} (V_c = 0)$. The curves for uniformly increasing collector voltage are spaced equi-distant, and the intercepts of the curves for different values of collector voltage may be written as

$$v_b = v_{bc1} + \frac{\Delta v_b}{\Delta v_c} (I_b = 0) (V_c = V_c1).$$  \hspace{1cm} $V_c1$ is the lowest value of collector voltage in region three, corresponding to the
base intercept voltage $V_{bcl}$. The equation for the base voltage may then be written

$$V_b = V_{bcl} + \frac{\Delta V_b}{\Delta V_c} \bigg| \left(I_b = c\right) \left(V_c - V_{cl}\right) + \frac{\Delta V_b}{\Delta I_b} \bigg| \left(V_c = c\right) \left(I_b\right).$$

(2.7)

For the idealized characteristics

$$\frac{\Delta V_b}{\Delta I_b} \bigg| \left(V_c = c\right) = \left(\frac{\partial V_b}{\partial I_b}\right)_{v_c} = \beta_{11}$$

(2.8)

and

$$\frac{\Delta V_b}{\Delta V_c} \bigg| I_b = c = \left(\frac{\partial V_b}{\partial V_c}\right)_{i_b} = \beta_{12}.$$

(2.9)

Using the above definitions of the $a^p$ parameters the equation for $V_b$ becomes

$$V_b = V_{bcl} + a_{12} \left(V_c - V_{cl}\right) + \beta_{11} I_b.$$

(2.10)

The equivalent circuit representing equations (2.5) and (2.10) is shown in figure (2.11). If the d-c terms are dropped from this circuit the variational equivalent circuit becomes the one shown in figure (1.7). The $a^p$ parameters of the circuits of figures (1.7) and (2.11) are defined in terms of the idealized graphical d-c characteristics by equations (2.3), (2.4), (2.8), and (2.9). The values of $a_{11}, a_{21},$ and $a_{22}$ for the ten transistors used in this investigation were evaluated from the input and output characteristics of the transistors as indicated by equations (2.3), (2.4), and (2.8). Measurements of $\Delta I_c, \Delta V_c,$
Figure (2.11)

Low Frequency Transistor Equivalent Circuit Derived from the D-C Characteristics
$\Delta I_b$ and $\Delta V_b$ used in calculating the values of the $a^3$ parameters were taken in the linear region of operation. It was impossible to evaluate $a_{12}$ from the input characteristics. This was caused by the extremely small size, as measured on the characteristics, of incremental changes in the base current due to changes in the collector voltage.
### Table I

**Values of the Hybrid Parameters Obtained from the D-C Characteristics**

<table>
<thead>
<tr>
<th>Transistor Type</th>
<th>No.</th>
<th>$a_{11}$ (ohms)</th>
<th>$a_{21}$</th>
<th>$a_{22}$ (megohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2N27</td>
<td>20</td>
<td>1900</td>
<td>59</td>
<td>10</td>
</tr>
<tr>
<td>2N27</td>
<td>26</td>
<td>1800</td>
<td>42</td>
<td>11</td>
</tr>
<tr>
<td>2N27</td>
<td>27</td>
<td>1100</td>
<td>20</td>
<td>5.5</td>
</tr>
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<td>28</td>
<td>3200</td>
<td>110</td>
<td>23</td>
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<td>15</td>
</tr>
<tr>
<td>2N35</td>
<td>53</td>
<td>1150</td>
<td>32</td>
<td>22</td>
</tr>
<tr>
<td>2N35</td>
<td>58</td>
<td>1500</td>
<td>29</td>
<td>19</td>
</tr>
<tr>
<td>2N35</td>
<td>59</td>
<td>1350</td>
<td>18</td>
<td>25</td>
</tr>
<tr>
<td>2N35</td>
<td>62</td>
<td>1400</td>
<td>33</td>
<td>14</td>
</tr>
<tr>
<td>2N35</td>
<td>63</td>
<td>1300</td>
<td>40</td>
<td>20</td>
</tr>
</tbody>
</table>
Chapter 3

LOW FREQUENCY MEASUREMENT OF THE HYBRID PARAMETERS

(3.1) THE LOW FREQUENCY EQUIVALENT CIRCUIT

As stated in the last section, the variational equivalent circuit shown in figure (1.7) may be obtained from the circuit derived in the preceding chapter and shown in figure (2.11). In this circuit $a_{11}$ was defined as a resistance, $a_{22}$ as a conductance, and $a_{21}$ and $a_{12}$ as dimensionless ratios which were independent of frequency. The variational equivalent circuit, using these values of the $a^p$ parameters obtained from the d-c characteristics, provides an accurate representation of the transistor for frequencies up to about 1 kc. For higher frequencies, the $a^p$ parameters must be expressed as functions of frequency for the variational equivalent circuit to be an accurate representation of the transistor. The variational equivalent circuit using the values of the $a^p$ parameters defined in terms of the d-c characteristics will be referred to as the low frequency equivalent circuit.

This chapter is concerned with a-c methods of measurement of the low frequency equivalent circuit parameters. There are two reasons for measuring the low frequency parameters by a-c methods. First, the d-c parameters were evaluated by taking the average values of the parameters for
a particular region of operation. Also, the ambient temperature of the cases of the transistors was not controlled in measuring the d-c characteristics. The a-c measurements determine values of the parameters for a particular quiescent operating point and ambient temperature. This was desirable so that, when other measurements of the transistor were carried out, they could be made at the same temperature and quiescent operating point and the information from the different measurements could be correlated. Another reason for making a-c measurements of the low frequency parameters was to determine values for $A_{\alpha 2}$, because it was not possible to evaluate it from the d-c characteristics.

The a-c measurements were made at a temperature of 32 degrees centigrade and a frequency of 400 cps. The temperature of the case of the transistor was controlled to within one degree centigrade by operating the transistor in an oven. The quiescent operating points stated below were used in making the low frequency measurements and all following work. The 2N35 transistors were operated at a quiescent collector voltage of 6 volts and a quiescent collector current of 1.2 ma. The 2N27 transistors were operated at 6 volts quiescent collector voltage and 1.4 ma. quiescent collector current. The collector current and collector voltage were controlled to within .01 ma. and 0.1 volts respectively.
(3.2) **LOW FREQUENCY MEASUREMENT OF $a_{11}$ AND $a_{12}$**

Low frequency values of $a_{11}$ and $a_{12}$ were obtained by measuring the input impedance of the transistor. This impedance, at the frequency used for the measurements, appears as a pure resistance. Consider the transistor variational equivalent circuit shown in figure (3.1). The Kirchhoff voltage equation around the input loop of the equivalent circuit may be written.

$$v_b = a_{11}i_b + a_{12}v_c.$$  \hspace{1cm} (3.1)

Define the base to collector voltage gain of the transistor as $A_{bc} = v_c/v_b$ and rewrite equation (3.1) as

$$v_b = a_{11}i_b + a_{12}A_{bc}v_b.$$  \hspace{1cm} (3.2)

The input resistance of the transistor may be defined as the ratio of the base voltage, $v_b$, to the base current, $i_b$. Solving equation (3.2) for the input resistance, we may write

$$r_i = \frac{v_b}{i_b} = a_{11}(1 - a_{12}A_{bc}).$$  \hspace{1cm} (3.3)

On the basis of this equation the variational equivalent circuit for the transistor is redrawn as shown in figure (3.2).

The circuit, shown in figure (3.3a), used to measure
Figure (3.1)
Standard Form of the Variational Equivalent Circuit

Figure (3.2)
Variational Equivalent Circuit Showing Input Impedance
the input resistance of the transistor is a common Wheatstone Bridge circuit. The equivalent circuit of this measuring circuit and the deflector circuit used with the Wheatstone Bridge are shown in figures (3.3b) and (3.3c) respectively. The band-pass filter was needed for sharp determination of bridge circuit null above noise voltage present in the circuit. This problem arose because the signal voltage applied to the input of the transistor was limited to approximately ten milli-volts to assume linear operation of the transistor.

In the measuring circuit, the load resistance, $R_l$, was variable. The base to collector voltage gain, $A_{bc}$, is nearly a linear function of the load resistance, and could be varied over a wide range by using different values of the load resistance.

From equation (3.3), it is seen that if the product $a_{12}A_{bc}$ is negligible with respect to unity, the input resistance of the transistor is equal to $a_{11}$. From the d-c input characteristics of the transistors it was possible to determine the order of magnitude of $a_{12}$ as being $10^{-3}$. Thus, if the value of the load resistance was such that the base to collector gain of the circuit of figure (3.3a) was of the order of magnitude of unity, the input resistance of the transistor would be equal to $a_{11}$ to a good approximation. A low value of base to collector voltage gain was used; and
So.

A T  B fllO & E  M U L  R i=  ft if.

(A) Circuit Diagram Used to Measure $a_{11}$ and $a_{12}$

(B) A-C Equivalent Circuit of the Circuit of (A)

(C) Detector Circuit

$$L_1C_1 = L_2C_2 = \frac{1}{(2\pi f)^2}$$

$\text{f} = 400 \text{ cps.}$

Figure (3.3)

Circuit Used to Measure $a_{11}$ and $a_{12}$

At Bridge Null $R_i = R \frac{R_1}{R_2}$
values of $a_{11}$ for the ten transistors being investigated, were determined from Wheatstone Bridge measurements of the input resistance of the transistors.

Equation (3.3) may be rewritten in the following form,

$$\frac{r_4}{a_{11}} = 1 - a_{12}A_{bc} .$$

(3.4)

If $r_4$ is measured for successively greater gains, $a_{11}/r_4$ may be plotted as a function of $A_{bc}$. Equation (3.4) then predicts that $a_{11}/r_4$ vs. $A_{bc}$ will have an intercept of 1 and a slope of $a_{12}$. It should be remembered that $A_{bc}$ is a negative quantity and hence $a_{11}/r_4$ increases with $A_{bc}$.

Measurements of $r_4$ for different values of base to collector voltage gain were made for a 2N27 and a 2N35 type transistor. Figure (3.5) shows a plot of $a_{11}/r_4$ vs. $A_{bc}$ for both transistors. It is seen that the curves hold very closely to linear functions for values of $A_{bc}$ less than 225. Greater values of gain would rarely, if ever, be achieved in an actual circuit. The divergence of the curves from linear functions for large values of gain will not be given further consideration for this reason. Figure (3.5) thus substantiates the form of the input circuit of the transistor and justifies taking the slope of the curve as the value of $a_{12}$ for the transistor. Values of $a_{12}$ for the
Figure (3.4)
Circuit Used for Alternate Method of Measurement of $a_{11}$
remaining transistors were then calculated by measuring $r_1$ for two different values of gain for each transistor.

An alternate method of measuring $a_{11}$ was used as a check on the method described above. The circuit used for this alternate method is shown in figure (3.6a). The load resistance used was small so that the product $a_2a_{bc}$ was small compared to unity and the input resistance of the transistor was equal to $a_{11}$ to a good approximation.

Figure (3.6b) shows the equivalent circuit of the measuring circuit. Considering the input part of the equivalent circuit as a voltage dividing network, we may write

$$\frac{E_1}{E_s} = \frac{R_2}{R_1 + R_2} \cdot \frac{a_{11}}{a_{11} + R_2}$$

Equation (3.5) may be solved for $a_{11}$ to give

$$a_{11} = \frac{R_2}{\frac{E_s}{E_1} - 1}$$

Measurements of $E_1$ and $E_s$ were made for each of the ten transistors and the value of $a_{11}$ for each transistor was then calculated from equation (3.5).

Table II, at the end of the chapter, gives the values of $a_{11}$ for the transistors as obtained by both methods. The two methods are in agreement, within the limits of experimental accuracy, for all ten transistors. A close correlation between the a-c measured values and the values obtained
Figure (3.5)
Normalized input admittance of the transistor versus base to collector voltage gain.

B.T.L. 2N27 No. 29

Sylvania 2N35 No. 59
from the d-c characteristics is not to be expected for two reasons. First, the d-c characteristics were not taken at a constant ambient temperature as were the a-c measurements. Second, the values of all obtained from the d-c characteristics were average values for a certain range of base current while the a-c measurements were made at one particular quiescent operating point.
(3.3) LOW FREQUENCY MEASUREMENT OF $a_{22}$

The low frequency value of $a_{22}$ for a transistor was obtained by measuring the admittance looking into the transistor from the base terminal to the emitter terminal. The same bridge circuit used to measure the input resistance of the transistor was connected as shown in figure (3.6a) to measure the output admittance of the transistor. At the frequency used for the measurement this output admittance appears a pure conductance.

The equivalent circuit of the measuring circuit is shown in figure (3.6b). Since $R_b$ is much greater than $a_{11}$, the loop equation around the input circuit may be written

$$a_{12}v_c = i_b (R_b + a_{11}) \quad \Rightarrow \quad i_b = \frac{a_{12}v_c}{R_b + a_{11}}. \quad (3.7)$$

Solving the above equation for $i_b$ and multiplying through by $a_{21}$, we may write

$$a_{21}i_b = \frac{a_{21}a_{12}v_c}{R_b}. \quad (3.8)$$

If we then define the "effective admittance" of the $a_{21}i_b$ current generator as the ratio of the current $a_{21}i_b$ to the voltage $v_c$, we see that this "effective admittance" is equal to $a_{12}a_{21}/R_b$. In the measuring circuit, a large enough value of the base biasing resistance, $R_b$, was used so that the "effective admittance" of the $a_{21}i_b$ was negligible with respect to $a_{22}$. The admittance seen looking to the left
(A) CIRCUIT DIAGRAM

AT BRIDGE NULL \( a_{22} + \frac{1}{R_L} = \frac{R_2}{R_1} \)

(B) A-C EQUIVALENT CIRCUIT OF (A)

Figure (3.6)
CIRCUIT USED TO MEASURE \( a_{22} \)
from terminals 1-1 of figure (3.6b) was then equal to $a_{22} + 1/R_1$.

The load resistance, $R_1$, was a precision resistance whose magnitude was approximately equal to $1/a_{22}$. It was thus possible to determine $a_{22}$ by measuring the admittance looking to the left at terminals 1-1 and subtracting the value of $1/R_1$.

Values of $a_{22}$ for the ten transistors being investigated were determined as described above and are given in Table II. For the same reasons discussed at the end of the preceding chapter, there is no reason to expect close correlation between these values of $a_{22}$ and the values determined from the d-c characteristics.
(3.4) LOW FREQUENCY MEASUREMENT OF THE SHORT CIRCUIT CURRENT GAIN

In terms of the circuit variables $a_{21}$ was given by the expression

$$a_{21} = \left( \frac{\partial i_c}{\partial i_b} \right) v_c$$

This defines $a_{21}$ as the short circuit current gain. This may be seen from the low frequency equivalent circuit if the collector voltage is reduced to zero by short circuiting the collector and emitter terminals. Thus, in measuring $a_{21}$, the a-c resistance from the collector to the emitter terminal must be extremely small. This is necessary for the base to collector voltage gain to be small and hence the voltage $a_{12}v_c$ negligible with respect to the voltage $a_{11}i_b$ across $a_{11}$. If this condition is met we may write

$$\frac{v_b}{a_{11}} \approx i_b.$$  \hspace{1cm} (3.9)

Denoting the a-c load resistance by $R_l$, and the parallel combination of $R_l$ and $1/a_{22}$ by $R$, the equation for the collector voltage may be written

$$\frac{v_c}{R} = a_{21} i_b.$$  \hspace{1cm} (3.10)

Dividing equation (3.10) by equation (3.9) we obtain

$$a_{21} = \frac{v_c}{v_b} \frac{F_{11}}{K} = \frac{A_{bc}}{K} \frac{F_{11}}{A}.$$  \hspace{1cm} (3.11)

Figure (3.7) shows the circuit used to measure $a_{21}$. 
Figure (3.7)
Circuit Used to Measure $a_{21}$
The load resistance, \( R_1 \), was small enough that equation (3.9) would be true to a good approximation. The measurements were made at a low frequency so that \( A_{bc} \), \( s_{11} \), and \( a_{22} \) were all real quantities. For each transistor, \( a_{21} \) was evaluated by measuring \( A_{bc} \) and calculating \( a_{21} \) from a knowledge of \( s_{11} \), \( a_{22} \), and \( R_1 \). \( R_1 \) was small enough that \( R = R_1 \); hence, inaccuracy in the value of \( a_{22} \) would not seriously affect the accuracy of the calculation of \( a_{21} \).

The values of \( a_{21} \) as evaluated by the above method are given for the ten transistors in Table II. In general the agreement between these values and the values obtained from the d-c characteristics is fairly good; although, as stated previously, complete agreement is not to be expected.
### Table II

**Low Frequency Values of the Hybrid Parameters**

<table>
<thead>
<tr>
<th>Transistor No.</th>
<th>Type</th>
<th>$a_{11}$ (ohms)</th>
<th>$a_{12}$</th>
<th>$a_{21}$</th>
<th>$a_{22}$ (micro-ohms)</th>
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<td></td>
<td>2350</td>
<td></td>
<td>0.20 \times 10^{-3}</td>
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<td>4000</td>
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<td>0.57 \times 10^{-3}</td>
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<td>0.82 \times 10^{-3}</td>
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<td>970</td>
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</table>
Chapter 4

THE HIGH FREQUENCY EQUIVALENT CIRCUIT

(4.1) APPROACH TO HIGH FREQUENCY MEASUREMENT

Up to this point, only the measurement of the equivalent circuit parameters from the d-c characteristics and directly at 400 c.p.s. has been discussed. Using constant values for the parameters, the equivalent circuit provides an accurate representation of the transistor in the frequency range from d-c to approximately one kc. Beyond this point, the parameters must be frequency dependent if the equivalent circuit is to represent the transistor at higher frequencies.

The equivalent circuit will be of little value if it cannot be used to predict the high frequency operation of a transistor circuit. The problem considered in this chapter is the determination of the four hybrid parameters of the transistor as functions of frequency and the synthesis of a two-terminal-pair equivalent circuit whose corresponding hybrid parameters are the same functions of frequency. Attention is then given to the measurement of values of these equivalent circuit elements for the ten transistors previously used.

Two of the four hybrid parameters were measured as
functions of frequency in the manner in which they are defined by equations (1.3a) and (1.3d). $a_{11}$ was measured as the input impedance of the transistor with the output short-circuited, and $a_{22}$ was measured as the output admittance of the transistor with the $a_{21}$ current generator open-circuited. The other two parameters were not measured as functions of frequency directly. $a_{21}$ was determined by measuring the voltage transfer characteristic of the transistor with its output virtually short-circuited. $a_{12}$ was partially measured by its effect on the voltage transfer characteristic.
The input impedance of the transistor was measured as a function of frequency with the circuit shown in figure (4.1a). The values of $C_d$, $C_b$, and $R_b$ were large enough so that in the a-c analysis of the measuring circuit the capacitances may be considered as short circuits and the resistance as an open circuit. The load resistance used in the output circuit was extremely small; hence, the voltage $a_{12}v_o$ in the input circuit may be considered negligible with respect to the voltage drop across $a_{11}$. Then the approximate equivalent circuit of the measuring circuit is as shown in figure (4.1b).

Considering the two impedances of figure (4.1b) as a voltage dividing network, we may write

$$\frac{E_1}{E_s} = \frac{a_{11}}{a_{11} + R_s} \quad (4.1)$$

If $R_s$, the resistance in series with the signal generator, is greater than 100 times $a_{11}$, the above equation may be written approximately as

$$\frac{E_1}{E_s} \approx \frac{a_{11}}{R_s} \quad (4.2)$$

For convenience, the voltage ratio $E_1/E_s$ expressed as a function of frequency will be referred to as the input characteristic of the transistor. From equation (4.2) it is clear that the input characteristic varies with frequency in the same manner as $a_{11}$ as long as $R_s$ is constant with respect to frequency.
(A) Circuit Diagram

(B) A-C Equivalent Circuit of (A)
Assuming that $a_{12} V_c << i_{b} a_{11}$

Figure (4.1)
Circuit used to measure the input characteristic of the transistor
The input characteristics were measured for all ten transistors. The solid curves of figures (4.2) and (4.3) show the results for one of the 2N27 and one of the 2N35 transistors respectively. To a good approximation these curves could be represented by functions of the form

\[ \frac{E_1}{E_s} = \frac{a_{11}}{R_s} = K \frac{S + a}{S + b}, \]

where \( a \) is larger than \( b \). The curves start to diverge from this form as the frequency increases beyond about one megacycle. This is caused by the shunting effect of the wiring capacitance of the circuit and the input capacitance of the vacuum tube voltmeter used to measure \( E_1 \).

We now wish to synthesize a network whose impedance has the form of equation (4.3). It may be shown that a network of the form shown in figure (4.4) has an impedance equal to

\[ Z = r_{11} \frac{S + \frac{1}{RC_{11}}}{S + \frac{1}{r_{12}C_{11}}}, \]

where \( R \) is the parallel combination of \( r_{11} \) and \( r_{12} \). Letting \( r_{11} = KR_s, \) \( RC_{11} = 1/a, \) and \( r_{12}C_{11} = 1/b; \) a network of this form may be used to represent \( a_{11} \), and we may write

\[ a_{11} = R_s \frac{E_1}{E_2} = r_{11} \frac{S + \frac{1}{RC_{11}}}{S + \frac{1}{r_{12}C_{11}}}. \]

The equivalent circuit is then drawn as shown in figure (4.5). The terminal between the two resistances \( r_{11} \) and \( r_{12} \).
Figure 4.2
Normalized Input Characteristic and Transfer Characteristic
B.T.L. 2N27 NO. 20

Normalized Voltage Ratio

- $E_2/E_1$
- $E_1/E_3$

Frequency (cps)

ML K 5K 10K 20K 50K 100K 200K 500K 1M 2M 5M
Figure (4.3)
Normalized Input Characteristic and Transfer Characteristic Sylvania 2N35 No. 53

Normalized Voltage Ratio

Transistor Equivalent Circuit

Frequency (c.p.s.)

2K  5K  10K  20K  50K  100K  200K  500K  1M  2M
Figure (4.9)
Network synthesized to represent $d_{11}$

Figure (4.5)
Equivalent circuit with $d_{11}$ represented by the network of Figure (4.9)
will be designated as $B^i$ and the variational component of the current through $r_{12}$ as $i_b^i$.

At low frequencies, the reactance of $C_{11}$ is much greater than $r_{12}$, and

$$r_{11} + r_{12} = \text{low frequency value of } a_{11}. \quad (4.6)$$

Denote the low frequency value of the input characteristic by $A_{11}$. $A_{11}$ will denote the constant value that the input characteristic takes on sufficiently beyond the frequency, $1/2 \pi R C_{11}$. At high frequencies, $C_{11}$ may be considered a short circuit; and it is easily seen that

$$\frac{r_{11}}{r_{11} + r_{12}} = \frac{A_{1h}}{A_{11}}. \quad (4.7)$$

From the input characteristics of the transistors and their low frequency values of $a_{11}$, it is a simple matter to calculate the corresponding values of $r_{11}$ and $r_{12}$ from equations (4.5) and (4.6). Values of $r_{11}$ and $r_{12}$ thus calculated for the ten transistors are given in table III at the end of the chapter. Evaluation of the capacitance $C_{11}$ will be discussed in the next section.
(4.3) MEASUREMENT OF THE TRANSFER CHARACTERISTIC

Figure (4.6a) shows the circuit used to measure the transfer characteristic of the transistor. As used here, the transfer characteristic will refer to the transistor base to collector voltage gain expressed as a function of frequency. The load resistance in the output of the measuring circuit is again very small, and the same simplifying assumptions made in the last section may be used to draw the equivalent circuit shown in figure (4.6b). From the equivalent circuit of the measuring circuit, it is seen that if $R_1$ is very much smaller than $a_{22}$, then the voltage $E_2$ may be expressed as

$$E_2 = R_1 a_{21} i_b.$$  \hfill (4.8)

Measurements were made of the transfer characteristics of the two transistors whose input characteristics were given in figures (4.2) and (4.3). The results of these measurements are also shown as solid curves in figures (4.2) and (4.3). It is seen that both these curves follow the form of simple R-C cut-off curves. We could then write

$$E_2/E_1 = K_1 \frac{c}{S + c}.$$  \hfill (4.9)

However, $E_1 = a_{11} i_b$, and equation (4.9) could be rewritten as

$$E_2 = K_1 \frac{c}{S + c} a_{11} i_b.$$  \hfill (4.10)

Comparing the above equation with equation (4.8), we may write

$$a_{21} = K_1/R_1 \frac{c}{S + c} a_{11}.$$  \hfill (4.11)
Figure (4.6)

Circuit used to measure the transfer characteristic of the transistor
Substituting from equations (4.2) and (4.3), equation (4.11) becomes

\[ a_{21} = K_2 \frac{c}{s + c} \frac{s + a}{s + b} \]  

(4.12)

Equation (4.12) states that, in general, \( a_{21} \) is a rather complicated function of frequency. If, however, it were true that \( a = c \), the expression for \( a_{21} \) would be considerably simplified. Let us investigate that possibility.

If \( a = c \), then equation (4.9) could be rewritten

\[ \frac{E_2}{E_1} = K_1 \frac{a}{s + a} = K_1 \frac{1/RC_{11}}{s + 1/RC_{11}} \]  

(4.13)

Let \( f_c \) denote the frequency at which the transfer characteristic has fallen off to 0.707 of its low frequency value. From equation (4.13),

\[ f_c = \frac{1}{2\pi f_c R} \quad \text{or} \quad C_{11} = \frac{1}{2\pi f_c R} \]  

(4.14)

For the two transistors whose characteristics appear in figures (4.2) and (4.3), \( r_{11} \) and \( r_{12} \) are known. \( R \) is the parallel combination of the two resistors and may be calculated for both transistors. From the transfer characteristics of the two transistors, \( f_c \) can be measured; hence, under the assumption that \( a = c \), \( C_{11} \) for the two transistors can be calculated from equation (4.14). In equations (4.5) and (4.13) all the quantities except \( K_1 \) are now known,
and the normalized input and transfer characteristics for the two transistors may be plotted from these equations. The characteristics as calculated from equations (4.5) and (4.13) are plotted as the dashed lines in figures (4.2) and (4.3).

For the 2N27 transistor it is clear that the curves calculated from the above analysis agree with those obtained experimentally to within the experimental accuracy of the measurements. The agreement for the 2N35 transistor is not quite as close, but the calculated curves still represent the transistor to the degree of accuracy required for all practical purposes. The transistor may then be correctly represented at high frequencies by the hybrid parameter equivalent circuit, where $a_{11}$ is defined by equation (4.5), and $a_{21}$ is defined by

$$a_{21} = K_3 \frac{b}{s + b} = K_3 \frac{1/r_{12}C_{11}}{s + 1/r_{12}C_{11}}.$$  \hspace{1cm} (4.15)

It would be more convenient if none of the parameters in the equivalent circuit were functions of frequency. Consider the circuit shown in figure (4.6b). The equivalent circuit for $a_{11}$ may be thought of as a voltage dividing network, and we may write

$$\frac{E_1}{E_1} = \frac{r_{12}/SC_{11}}{r_{12} + 1/SC_{11}} = \frac{r_{11} + 1/RC_{11}}{(r_{11} + r_{12})(s + 1/RC_{11})}.$$  \hspace{1cm} (4.16)
It was shown above that, when $C_{11}$ was calculated on the assumption that $a = c$, the equivalent circuit accurately represented the transistor. We are thusly justified in stating that $a = c$, and the equations based on this premise may be considered correct. Dividing equation (4.13) by equation (4.16), we have

$$
\frac{E_2}{E_1'} = K_1 \frac{r_{11} + r_{12}}{r_{11}}.
$$

(4.17)

Substituting for $E_2$ from equation (4.8) and for $E_1'$ from $E_1' = i_{b}\cdot r_{12}$, equation (4.17) may be rearranged and re-written as

$$
a_{21}i_b = B_1i_{b}',
$$

(4.18)

where $B_1$ is a constant. At low frequencies $i_{b}' = i_b'$ hence, $B$ is equal to the low frequency value of $a_{21}$. On the basis of equation (4.18), the equivalent circuit is drawn as shown in figure (4.7).

The capacitance $C_{11}$ is connected from the base prime to the emitter terminal and will henceforth be designated as $C_{b'e}$. The .707 cut-off frequency, $f_c$, of the transfer characteristic was measured for all ten transistors. $C_{b'e}$ was then calculated from equation (4.14) for each transistor, and the values are given in table III.
Figure (9.7)

High Frequency Equivalent Circuit of the Transistor with $a_{11}$ and $a_{21}$ determined as functions of Frequency
(4.4) **MEASUREMENT OF THE BASE PRIME TO COLLECTOR CAPACITANCE**

In the equivalent circuit shown in figure (4.7), \( a_{12} \) has not yet been determined as a function of frequency. Instead of determining \( a_{12} \) as a function of frequency directly, the effect of the \( a_{12} V_c \) generator in the equivalent circuit will be replaced by a feedback impedance. That is, a feedback impedance will be found such that when it is suitably connected in the equivalent circuit, the equivalent circuit will represent the transistor to within the desired degree of accuracy.

In determining the feedback impedance, the low frequency component of \( a_{12} \) will be neglected. This is justified for two reasons. First, considering figure (3.2) and the values of \( a_{12} \) given in table II, it is seen for normal values of base to collector voltage gain that the effect of the \( a_{12} V_c \) generator in the equivalent circuit is negligible. Second, the effect of the \( a_{12} V_c \) generator at high frequencies is relatively much more important and cannot be neglected.

It will be assumed that the equivalent circuit may be represented as shown in figure (4.8), that is, that the equivalent circuit will accurately represent the transistor when the feedback impedance is connected from the collector to the base prime terminal. Consider the admittance of the equivalent circuit looking to the right at terminals a-a. The node equation may be written as

\[ i_b = E_1' S_{b'e} + E_1' / r_{12} + (E_1' - E_2') / Z_{b'c} \]  \hspace{1cm} (4.19)
Figure (4.8)

Assumed form of the equivalent circuit

Figure (4.9)

Alternate form of the circuit of Figure (4.8)
The admittance looking to the left at terminals b-b is denoted as $y_{bb}$; $y_{22}$ is then defined by the expression

$$y_{22} = y_{bb} + a_{22} \quad (4.20)$$

Let $Z_m$ be defined as the parallel combination of $R_1$ and $1/y_{22}$. The voltage $E_2$ may then be expressed as

$$E_2 = -B_{ib} Z_m = -E_1 B Z_m / r_{12} \quad (4.21)$$

Substituting for $E_2$ from the above equation, equation (4.19) may be written

$$i_b = E_1 \left[ S C_{b'e} + 1/r_{12} + (1 + B Z_m / r_{12})/Z_{b'c} \right] \quad (4.22)$$

The admittance looking to the right at terminals a-a will be referred to as $y_{aa}$ and is given by the expression

$$y_{aa} = i_b / E_1 = S C_{b'e} + 1/r_{12} + (1 + B Z_m / r_{12})/Z_{b'c} \quad (4.23)$$

On the basis of equations (4.20) and (4.23), the equivalent circuit of the transistor may then be redrawn as shown in figure (4.9).

In the measurement to be carried out, only frequencies up to the cut-off frequency of the transfer characteristic will be of interest. If, in making the measurement, $R_1$ is always small with respect to $1/y_{22}$, we may write

$$Z_m = R_m, \quad (4.24)$$

where $R_m$ is the low frequency value of $Z_m$. It may be shown that the mid-band or reference gain of the circuit shown in
Figure (4.9) is

\[ A_r = \frac{E_2}{E_1} \text{ mid frequency} = \frac{BR_m}{(r_{11} + r_{12})} \]  \hspace{1cm} (4.25)

Equation (4.23) may then be written

\[ y_{aa} = \frac{S}{c} \left[ c_{b'e} + l/r_{12} + \left[1 + A_r (r_{12} + r_{11})/r_{12}\right]/z_{b'c} \right] \]

\[ \hspace{1cm} (4.26) \]

It will now be hypothesized that \( 1/z_{b'c} = SC_{b'c} \). If this is true, then

\[ y_{aa} = S \left[ c_{b'e} - c_{b'c} \left[1 + A_r (r_{12} + r_{11})/r_{12}\right]\right] + 1/r_{12} \]

\[ = SC_{in} + 1/r_{12} \]  \hspace{1cm} (4.27)

where

\[ C_{in} = c_{b'e} + c_{b'c} \left[1 + A_r (r_{12} + r_{11})/r_{12}\right] \]  \hspace{1cm} (4.28)

On the basis of equation (4.28), the equivalent circuit for the transistor may be redrawn as shown in figure (4.10). The expression for the transfer characteristic is then

\[ \frac{E_2}{E_1} = K \frac{1/RC_{in}}{S + 1/RC_{in}} \]  \hspace{1cm} (4.29)

where \( R \) is the parallel combination of \( r_{11} \) and \( r_{12} \). From equation (4.29), it is seen that the cut-off of the transfer characteristic is

\[ f_c \leq 1/2\pi RC_{in} \text{ or } C_{in} = 1/2\pi Rf_c \]  \hspace{1cm} (4.30)

Equation (4.28) predicts that the input capacitance, \( C_{in} \), is a linear function of the reference gain. The circuit of figure (4.6) was used to measure the cut-off frequency of
\[ C_{IM} = C_{b'e} + C_{bc} \left( 1 + \frac{a''}{R_{12}} \right) \]

**Figure (4.10)**

Circuit of Figure (4.9) assuming \( \frac{1}{Z_{bc}} = 5C_{bc} \)

**Figure (4.12)**

High frequency equivalent circuit with all parameters but \( a_{22} \) determined
Figure (4.11)
Input capacitance versus $\frac{3\pi}{\sqrt{2}}$ times the base to collector voltage gain.
the transfer characteristic as a function of the reference gain, \( A_r \). The reference gain was varied by changing the load resistance, \( R_1 \). In all cases \( R_1 \) was small with respect to \( 1/y_{22} \). From equation (4.30), the input capacitance was calculated from the cut-off frequency of the transfer characteristic for different values of gain. Figure (4.11) shows the input capacitance plotted as a function of 

\[
\left( \frac{r_{11} + r_{12}}{r_{12}} \right) \times A_r
\]

times the reference gain for a 2N27 and a 2N35 transistor. For both transistors the curves follow linear functions to well within the accuracy of the measurement; hence, the postulate that \( 1/Z_{b'c} = SC_{b'c} \) is substantiated, and the equivalent circuit may be redrawn as shown in figure (4.12).

Equation (4.10) states that the intercepts of the curves of figure (4.10) are equal to the values of \( C_{b'e} + C_{b'c} \) for the transistors, and the slopes of the curves are equal to the values of \( C_{b'c} \). Values of \( C_{b'c} \) were calculated by this method for the ten transistors and are given in table III.
DETERMINATION OF $a_{22}$

The determination of $a_{22}$ as a function of frequency was carried out by measuring the admittance of the transistor from the collector terminal to the emitter terminal with the base open-circuited. The circuit used for the measurement is shown in figure (4.13a). The resistances, $R_g$, are extremely small shunts used in conjunction with vacuum tube voltmeters to measure the indicated currents. Currents were measured instead of voltages so that the input capacitance of the vacuum tube voltmeters was not placed across the transistor. It may be assumed that the resistances, $R_g$, and the reactances of the capacitors were small enough that they could be considered short circuits in the equivalent circuit shown in figure (4.13b). The base terminal is shown open-circuited in the equivalent circuit because of the large value of the base resistance, $R_b$.

Let us define the admittance, looking to the left at the terminals c-c in the equivalent circuit, as $y_{cc}$. If the admittance looking into the transistor from collector to emitter is denoted by $Y_{in}$, it is easily seen that

$$Y_{in} = a_{22} + y_{cc}.$$  \hspace{1cm} (4.31)

It is possible to measure $Y_{in}$ with the circuit shown in figure (4.13a). In order to determine $a_{22}$, $y_{cc}$ will be calculated from the equivalent circuit shown in figure (4.13b).

Writing the node equation at the collector terminal, $I_{in}$ may be expressed as

$$I_{in} = E_2a_{22} + B^1_b + (E_2 - E_1')S_{b,c}.$$  \hspace{1cm} (4.32)
(A) CIRCUIT DIAGRAM

(B) A-C EQUIVALENT CIRCUIT OF THE MEASURING CIRCUIT

Figure (4.13)
Circuit used to measure the output characteristic of the transistor
The expression for the voltage ratio $E_1'/E_2$ may be written by considering the circuit to the left of the $B_{i^b}$ generator as a voltage dividing network:

$$E_1'/E_2 = \frac{r_{12}/SC_{b^e}}{r_{12} + 1/SC_{b^e}}$$

The above expression may be simplified and rewritten as

$$E_1'/E_2 = K \frac{S}{S + w_{12}}$$

where

$$K = \frac{C_{b^e}}{C_{b^c} + C_{b^e}}$$

$$w_{12} = \frac{1}{r_{12}(C_{b^c} + C_{b^e})}$$

From the equivalent circuit, it is seen that $E_1' = r_{12}i_{b'}$. $E_1'$ and $i_{b'}$ may then be written as functions of $E_2$ by rearranging equation (4.34):

$$E_1' = i_{b'}r_{12} = E_2 K \frac{S}{S + w_{12}}$$

Let us substitute for $E_1'$ and $i_{b'}$ from equation (4.36) into equation (4.32). If both sides of equation (4.32) are then divided by $E_2$, the result is

$$I_{in}/E_2 = Y_{in} = a_{22} + (KB/r_{12}) \frac{S}{S + w_{12}} + \left[1 + K \frac{S}{S + w_{12}}\right] SC_{b^e}$$

(4.37)
For all frequencies of interest, the third term of the 
right hand side of the equation will be negligible with re-
spect to the second. Equation (4.37) may then be written 
approximately as
\[ Y_{in} = a_{22} + \frac{KB}{r_{12}} \frac{S}{S + w_{12}}. \] (4.38)

In equation (4.31) it was stated that \( Y_{in} = a_{22} + y_{cc} \). The 
above equation may then be solved for \( y_{cc} \):
\[ y_{cc} = \frac{KB}{r_{12}} \frac{S}{S + w_{12}}. \] (4.39)

The reciprocal of \( y_{cc} \) will be referred to as \( z_{cc} \). Inverting 
the above equation, \( z_{cc} \) may be expressed
\[ z_{cc} = \frac{r_{12}}{KB} + \frac{1}{KB} \frac{r_{12}w_{12}}{S}. \] (4.40)

Substituting for \( K \) and \( w_{12} \) from equations (4.35), the above 
equation becomes
\[ z_{cc} = \frac{C_{b'c} + C_{b'e}}{BC_{b'c}} r_{12} + \frac{1}{SBC_{b'c}}. \] (4.41)

From the above equation, it is seen that the impedance looking 
to the left at terminals c-c appears as a resistance and capaci-
tance in series.

Referring to the equivalent circuit of the measuring 
circuit, we may write
\[ \frac{I_{in}}{I_1} = \frac{Y_{in}}{Y_{in} + 1/R_1}. \] (4.42)

In the measuring circuit used, \( 1/R_1 \) was very much greater
than $Y_{in}$; and the above equation may be written approximately

$$I_{in}/I_1 = Y_{in}R_1 \quad (4.43)$$

Solving for $1/Y_{in}$, the above equation becomes

$$I/Y_{in} = R_1 \frac{I_1}{I_{in}} \quad (4.44)$$

Equation (4.44) states that $Z_{in}$ is proportional to the current ratio $I_1/I_{in}$. This current ratio expressed as a function of frequency, will be referred to as the output characteristic of the transistor.

Using the circuit of figure (4.13a), the output characteristics of a 2N27 and a 2N35 transistor were measured. The normalized characteristics are shown as the solid lines in figure (4.14). It is seen that the characteristics could be approximately represented by functions of the form $K \frac{S+a}{S+b}$. The impedance of the parallel combination of a resistance and a resistance and capacitance in series is also represented by a function of this form. Since $Z_{cc}$ appears as a resistance and capacitance in series, and since $Z_{in}$ is the parallel combination of $Z_{cc}$ and $1/a_{22}$, it would then be logical to conclude that $a_{22}$ is a pure conductance.

All the quantities in equation (4.37), other than $a_{22}$, have been previously determined. Using the values for $a_{22}$ given in table II, $Y_{in}$ could be calculated for the two transistors whose output characteristics are shown in figure
Figure (419)

Normalized Output Characteristic

Sylvania 2N35 No. 53
B.T.L. 2N27 No. 20

Transistor
Equivalent Circuit
(4.14). The inverse of $Y_{in}'$ $Z_{in}'$ was then calculated for the two transistors. The normalized values of $Z_{in}$ are shown plotted against frequency by the broken curves in figure (4.14). It is seen that the curves calculated from the equivalent circuit on the assumption that $a_{22}$ is a pure conductance are in agreement with the curves measured for the transistor. The equivalent circuit then represents the transistor with $a_{22}$ being a pure conductance, $g_{22}$, whose value is equal to the low frequency value of $a_{22}$.

There was a slight peaking effect noticed in the output circuit of the transistor. This could be accounted for in the equivalent circuit by placing an inductance in series with $g_{22}$. This effect has only been noticed at frequencies higher than those of interest for practical use of the transistor, and will be neglected for this reason.

The determination of $a_{22}$ completes the empirical derivation of the high frequency equivalent circuit for the junction transistor. The final circuit is shown in figure (4.15). Values of $r_{11}$, $r_{12}$, $C_{b'c}$, and $C_{b'e}$ for the ten transistors measured are given in table III. The values of $B$ and $g_{22}$ for these transistors are the values given in table II for $a_{21}$ and $a_{22}$ respectively.
Figure (4.15)

Empirically Derived High Frequency Equivalent Circuit for the Junction Transistor
# TABLE III

VALUES OF THE HIGH FREQUENCY EQUIVALENT CIRCUIT PARAMETERS

<table>
<thead>
<tr>
<th>Transistor Type</th>
<th>No.</th>
<th>$r_{11}$ (ohms)</th>
<th>$r_{12}$ (ohms)</th>
<th>$C_{b'\text{e}} \times 10^{-12} \text{f}$</th>
<th>$C_{b'\text{c}} \times 10^{-12} \text{f}$</th>
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<td>20</td>
<td>690</td>
<td>1470</td>
<td>5,920</td>
<td>14</td>
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<tr>
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<td>26</td>
<td>520</td>
<td>1500</td>
<td>7,000</td>
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Chapter 5

APPLICATION OF THE EQUIVALENT CIRCUIT
TO ANALYSIS OF TRANSISTOR CIRCUITS

(5.1) SEPARATION OF THE EQUIVALENT CIRCUIT

Figure (5.1) shows the high frequency equivalent circuit, derived in the previous chapter, connected to an external load. As pointed out in section (1.2), to be useful in analyzing cascaded amplifiers, the equivalent circuit must have its output section separate from its input. This is not true of the equivalent circuit derived in the last chapter. In this section, the equivalent circuit will be modified so that its input and output sections are separated.

Considering the circuit shown in figure (5.1) the node equation at the collector terminal may be written

\[ v_c Z_1 + v_c g_{22} + B i_{b'} + (v_c - v_{b'}) S_{b'c} = 0 \]  

(5.1)

However, \( i_{b'} \) may be expressed as \( v_{b'}/r_{12} \), and equation (5.1) may be rewritten as

\[ v_c \left(1/Z_1 + g_{22} + S_{b'c}\right) = -v_{b'} \left(B/r_{12} - S_{b'c}\right) \]

(5.2)

For the transistors which were measured, \( B/r_{12} \) is much greater than \( S_{b'c} \) for all frequencies of interest.
**Figure (5.1)**

Transistor Equivalent Circuit and Connected Load

**Figure (5.2)**

Separated Equivalent Circuit

\[ G_{in} = \frac{1}{r_{12}} - \omega C_{bc} \text{Im}(A_b'C) \quad g_t = B/r_{12} \]

\[ C_{in} = C_{bc} + [1 - \text{Re}(A_b'C)] \quad C_{bc} \]

\[ C_{o} = C_{bc} + \text{CASE CAPACITANCE FROM C TO E} \]
Equation (5.2) may then be solved approximately for \( v_c \):
\[
v_c = -v_{b^i} \frac{B/r_{12}}{1/Z_1 + g_{22} + SC_{b^i c}}.
\]  
(5.3)

Let \( g_t \) be defined as \( B/r_{12} \); equation (5.3) may then be rewritten as
\[
v_c = -g_t v_{b^i} \frac{1}{1/Z_1 + g_{22} + SC_{b^i c}}.
\]  
(5.4)

We will return to this equation in a moment. Now consider the admittance looking to the right at terminals 1-1 of the circuit shown in figure (5.1).

The node equation at the \( B^i \) terminal may be written
\[
\begin{align*}
i_b &= v_{b^i}/r_{12} + v_{b^i}SC_{b^i e} + (v_{b^i} - v_c) SC_{b^i c}.
\end{align*}
\]  
(5.5)

If the voltage gain from the base prime to the collector terminal is referred to as \( A_{b^i c} \), equation (5.5) may be rewritten as
\[
\begin{align*}
i_b &= v_{b^i} \left[ \frac{1}{r_{12}} + SC_{b^i e} - (1 - A_{b^i c}) SC_{b^i c} \right].
\end{align*}
\]  
(5.6)

Dividing both sides of the above equation by \( v_{b^i} \), and defining the admittance looking to the right at terminals 1-1 as \( Y_{b^i} \), we may write
\[
\begin{align*}
Y_{b^i} &= \frac{i_b}{v_{b^i}} = \frac{1}{r_{12}} + SC_{b^i e} + (1 - A_{b^i c}) SC_{b^i c}.
\end{align*}
\]  
(5.7)

Consider the term \( (1 - A_{b^i c}) SC_{b^i c} \). In the steady state we may replace \( S \) by \( jw \) and write
\[
(1 - A_{b^i c}) SC_{b^i c} = jw \left[ 1 - Re(A_{b^i c}) \right] C_{b^i c} - wC_{b^i c} Im(A_{b^i c}).
\]  
(5.8)
Equation (5.6) may then be rewritten

\[ V_b' = G_{in} + SC_{in} \quad (5.9) \]

where

\[ G_{in} = \frac{1}{\tau_{12}} - \frac{w C_{b'c}}{2} \text{Im}(A_{b'c}) \quad (5.10) \]

and

\[ C_{in} = C_{b'c} e + \left[ 1 - \text{Re}(A_{b'c}) \right] C_{b'c} \quad (5.11) \]

In the input section of the equivalent circuit we may thus replace the elements to the right of terminals 1-1 with the admittance given by equation (5.9). Now consider equation (5.4). The voltage \( V_c \) may be considered to be caused by a current generator \( g_{b'} V_b' \), working into an impedance made up of the parallel combination of \( Z_1, 1/g_{22}, \) and \( 1/SC_{b'c} \). The equivalent circuit can then be redrawn as shown in figure 5.2. In this form the equivalent circuit can be used for the analysis of cascaded amplifiers as will be shown in the following section.
(5.2) ANALYSIS OF CASCADED AMPLIFIERS

By using the equivalent circuit developed in the preceding section to analyze a cascaded transistor amplifier, the overall transfer function of the cascade can be expressed as a product of the transfer functions of its individual stages. Figure (5.3a) shows the circuit diagram of a cascaded transistor amplifier. The bias circuit used is the standard one and the interstage coupling networks shown may be any three-terminal coupling network. The cascade shown in figure (5.3a) is considered to be made up of \( n \) transistors. In a cascade made up of \( n \) transistors there are \( n + 1 \) coupling networks. For this reason the cascade in figure (5.3) will be treated as if there were \( n + 1 \) stages.

The equivalent circuit of the amplifier is shown in figure (5.3b). In the equivalent circuit, the bias resistors, the load resistor, \( g_{22} \), and \( C_0 \), have been incorporated into the appropriate coupling network. Note that the equivalent circuit is broken up into \( n + 1 \) separate units; this allows the transfer function of the cascade to be written as a product of \( n + 1 \) stage transfer functions. The transfer function of a single stage will henceforth be denoted as the "stage gain."

There are two definitions of stage gain that are of use in the analysis of the cascade. The base to base
(A) CIRCUIT DIAGRAM

(B) HIGH FREQUENCY EQUIVALENT CIRCUIT

FIGURE (5.3)

CASCaded TRANSISTOR AMPLIFIER OF N TRANSISTORS
voltage gain is defined as the ratio of the voltage from base to ground at one transistor to the voltage from base to ground at the preceding transistor in the cascade. Using the notation shown in figure (5.3b), this is expressed as

\[ A_1 = \frac{E_2}{E_1}, \ldots A_k = \frac{E_{k+1}}{E_k}, \ldots A_n = \frac{E_0}{E_n}. \]  

(5.12)

The stage gain can also be defined as the ratio of the voltage from base prime to ground at one transistor to the voltage from base prime to ground at the preceding transistor. This may be referred to as the base prime to base prime voltage gain, and is written

\[ A_{1}' = \frac{E_{2}'}{E_{1}'}, \ldots A_{k}' = \frac{E_{k+1}'}{E_k'}, \ldots A_n = \frac{E_0}{E_n}. \]  

(5.13)

The overall voltage gain of the cascade can then be expressed as a product of either of the stage gains:

\[ A_t = \frac{E_o}{E_{in}} = \frac{E_1}{E_{in}} \cdot \frac{E_2}{E_1} \cdot \frac{E_3}{E_2} \cdots \frac{E_0}{E_n} \]

\[ = A_0 \cdot A_1 \cdot A_2 \cdot A_3 \cdots A_n \]  

(5.14)

or

\[ A_t = \frac{E_0}{E_{in}} = \frac{E_1'}{E_{in}} \cdot \frac{E_2'}{E_1'} \cdot \frac{E_3'}{E_2'} \cdots \frac{E_0}{E_n} \]

\[ = A_0' \cdot A_1' \cdot A_2' \cdots A_n' \]  

(5.15)

It is now desirable to develop general equations for the stage gains. Consider the combination of \( r_{ll}, G_{in}, C_{in} \), and the preceding coupling circuit as being one interstage network. The mutual impedance of this interstage network is
defined as the ratio of the voltage appearing across $G_{in}$ to the current entering the input of the coupling network.

Denoting the mutual impedance of the interstage network between the $K^{th}$ and $K+1^{th}$ transistors as $Z_{m_k}$, the base prime voltage gain of the $k^{th}$ stage can then be written

$$A_k' = \frac{E_{k+1'}}{E_k'} = \frac{-B_k' Z_{m_k}}{E_k'} = \frac{-B_k Z_{m_k} E_k' / r_{12k}}{E_k'}$$

$$= \frac{-B_k' Z_{m_k}}{r_{12k}}. \quad (5.16)$$

However, $B/r_{12}$ has been previously defined as $g_e$, and the above equation becomes

$$A_k' = -E_k Z_{m_k}. \quad (5.17)$$

The overall voltage gain of the cascade is then

$$A_t = \frac{E_1'}{E_{in}} (-g_{t1} Z_{m1})(-g_{t2} Z_{m2}) \ldots (-g_{tn} Z_{mn})$$

$$= \frac{E_1}{E_{in}} \frac{E_k}{E_k'} \frac{E_{k+1'}}{E_{k+1}'} \ldots \frac{E_{k+1}}{E_{k+1}'} \quad (5.18)$$

where $E_1'/E_{in}$ is the ratio of the voltage appearing from base prime to ground of the first transistor to the input voltage of the amplifier. This ratio can be expressed as the voltage dividing effect of a network of simple passive elements. An expression for the overall voltage gain of the cascade has thus been developed and is equal to the product of a series of terms. Each term in the series is a function only of the equivalent circuit parameters and the external elements in the circuit.

An expression for the base to base stage gain can be derived by writing

$$A_k = \frac{E_{k+1}}{E_k} = \frac{E_k' E_{k+1}'}{E_k E_{k+1}'} \frac{E_k}{E_k'} \frac{E_{k+1}}{E_{k+1}'} \quad (5.19)$$
The ratio $E_k'/E_k$ can be written by considering the input of the transistor equivalent circuit as a voltage dividing network. By this method, it can be shown that

$$E_k'/E_k = \frac{r_{12k}}{r_{12k} + 1/G_{\text{in}k}} \frac{1/R_{kC_{\text{in}k}}}{S + 1/R_{kC_{\text{in}k}}}$$  \hspace{1cm} (5.20)

where $R_k$ is the parallel combination of $r_{12k}$ and $1/G_{\text{in}k}$.

The ratio $E_{k+1}'/E_{k+1}'$ is found similarly to be the inverse of the expression for $E_k'/E_k$ with the subscripts $k$ replaced by $k+1$. Equation (5.19) can then be written

$$A_k = -g_{\text{tk}Z_{\text{m}k}} \left[ \frac{r_{12k}}{r_{12k} + 1/G_{\text{in}k}} \frac{1/R_{kC_{\text{in}k}}}{S + 1/R_{kG_{\text{ink}}}} \right] \cdot \left[ \frac{r_{12k+1}}{r_{12k+1} + 1/G_{\text{in}k+1}} \frac{1/R_{k+1G_{\text{in}k+1}}}{S + 1/R_{k+1G_{\text{in}k+1}}} \right].$$  \hspace{1cm} (5.21)

It is seen that if the stage gains $A_1, A_2, \ldots, A_n$ are substituted into equation (5.14), the bracketed terms in equation (5.21) cancel out; and we obtain the same result as expressed by equation (5.18). If the stages in the cascade are identical, the bracketed terms in equation (5.21) will drop out, and the base to base stage voltage gain is equal to the base prime to base prime stage gain. Objection may be raised to using the base prime to base prime stage gain on the grounds that the voltages involved do not exist physically in the actual transistor circuit. This is not necessarily a serious restriction for use in designing transistor circuits. If the
stages are not identical, the expression for the base prime stage gain is much less complicated than the expression for the base stage gain; hence, it is generally easier to design or analyze a cascade in terms of the base prime stage gain.
As an example of the method of analysis discussed in the previous section, we shall derive the stage gain for an R-C coupled grounded emitter amplifier. Only the case in which the impedance from emitter to ground is negligible will be considered. If a by-passed emitter resistance is used, the analysis will be valid for those frequencies at which the emitter by-pass capacitor can be considered a short circuit.

The circuit diagram of one stage in the R-C coupled amplifier is shown in figure (5.4a). Assuming that the reactance of the coupling capacitances is negligible in the mid and high frequency regions, the high frequency equivalent circuit may be drawn as shown in figure (5.4b). It has been assumed that, for all frequencies of interest, \( 1/G_{\text{in}} = r_{12} \).

\( C_t \) is the parallel combination of \( C_0 \) and the interstage wiring capacitance. \( R_b \) will designate the parallel combination of \( g_22 \) and the load resistance for one transistor in parallel with the parallel combination of the two bias resistors of the following transistor.

The base prime stage gain is expressed

\[
A' = -g_t Z_m .
\]

The problem is now to find the mutual impedance of the interstage network. The interstage network is shown in figure (5.5), in the form of a pi network. It can be shown that the mutual impedance of a pi network may be expressed as
(A) ONE STAGE OF AN R-C COUPLED TRANSISTOR AMPLIFIER

(B) HIGH AND MID FREQUENCY EQUIVALENT CIRCUIT

Figure (5.3)
ONE STAGE OF AN R-C COUPLED AMPLIFIER

\[ Z_M = \frac{E_o}{I_{in}} = -\frac{E_2'}{3tE_1'} \]

\[ \frac{1}{R_b} = g_{22} + \frac{1}{R_L} + \frac{1}{R_1} + \frac{1}{R_2} \]

Figure (5.4)
INTERSTAGE NETWORK OF AN R-C COUPLED STAGE
\[ Z_m = \frac{Z_1 + Z_2}{S_1 + Z_2 + Z_3} \quad (5.23) \]

where \( Z_1, Z_2, \) and \( Z_3 \) are the impedances of the three arms of the \( \pi \). From figure (5.5), it is seen that for the R-C coupled amplifier, \( Z_1, Z_2, \) and \( Z_3 \) are given by:

\[ Z_1 = \frac{R_b/SC_t}{R_b + 1/SC_t} = \frac{R_b}{S + \frac{w_t}{w_t}}, \quad (5.24) \]

where \( w_t = 1/R_bC_t \);

\[ Z_2 = \frac{r_{12}/SC_{in}}{r_{12} + 1/SC_{in}} = r_{12}\frac{w_{in}}{S + w_{in}} \quad (5.25) \]

where \( w_{in} = 1/r_{12}C_{in} \); \n
\[ Z_3 = r_{11}. \quad (5.26) \]

If the above expressions for \( Z_1, Z_2, \) and \( Z_3 \) are substituted into equation (5.23), and the resulting equation simplified, \( Z_m \) is expressed as

\[ Z_m = \frac{r_{12}R_b}{r_{11}} \frac{w_{in}w_t}{S^2 + bS + c} \quad (5.27) \]

where

\[ b = (1 + r_{12}/r_{11})w_{in} + (1 + R_b/r_{11})w_t \quad (5.28) \]

and

\[ c = (1 + r_{12}/r_{11} + R_b/r_{11})w_{in}w_t. \quad (5.29) \]

Factoring the quadratic, equation (5.27) becomes

\[ Z_m = \frac{r_{12}R_b}{r_{11}} \frac{w_{in}w_t}{(S + S_1)(S + S_2)} \quad (5.30) \]

where

\[ S_1 = \frac{b}{2} \left[ 1 + \sqrt{1 - 4c/b^2} \right] \quad (5.31) \]

and

\[ S_2 = \frac{b}{2} \left[ 1 - \sqrt{1 - 4c/b^2} \right]. \quad (5.32) \]
Using typical values of the transistor parameters given in table II and III, it can be shown for values of $R_b$ from 1k to 30 k-ohms that $4c/b^2$ is always less than .01. If the radical is then expanded in a power series, we may write to a very good approximation:

$$\sqrt{1 - \frac{4c}{b^2}} = 1 - \frac{1}{2} \frac{4c}{b^2} = 1 - \frac{2c}{b^2} \quad (5.33)$$

If the result of the above equation is substituted into equations (5.31) and (5.32), the expressions for $S_1$ and $S_2$ become

$$S_1 = \frac{c}{b} \quad \text{and} \quad S_2 = b(1 - \frac{c}{b^2}) \quad (5.34)$$

However, $\frac{c}{b^2}$ is less than .0025, and $S_2$ may be approximately written as

$$S_2 = b \quad (5.35)$$

Substituting the values for $b$ and $c$ given by equations (5.28) and (5.29), the expressions for $S_1$ and $S_2$ become

$$S_1 = \frac{(1 + \frac{r_{12}}{r_{11}} + \frac{R_b}{r_{11}})w_{in}w_t}{(1 + \frac{r_{12}}{r_{11}})w_{in} + (1 + \frac{R_b}{r_{11}})w_t} \quad (5.36)$$

and

$$S_2 = (1 + \frac{r_{12}}{r_{11}})w_{in} + (1 - \frac{R_b}{r_{11}})w_t \quad (5.37)$$

For the values of parameters given in tables II and III, and for values of $R_b$ from 1 to 30 k-ohms, it is true that $(1 + \frac{R_b}{r_{11}})w_t$ is greater than 80 $(1 + \frac{r_{12}}{r_{11}})w_{in}$. $S_1$ and $S_2$ are then approximately:

$$S_1 = \frac{r_{12} + r_{11} + R_b}{R_b + r_{11}}w_{in} \quad (5.38)$$

and

$$S_2 = (1 + \frac{R_b}{r_{11}})w_t \quad (5.39)$$
If the above expressions for $S_1$ and $S_2$ are examined, it is seen that for the values of transistor parameters given in the tables, and values of $R_b$ from 1 to 30 k-ohms, $S_2$ is much greater than $S_1$. This is extremely important because it means that the response of the interstage network is almost entirely determined by $S_1$. This being the case, $S$ may be neglected compared to $S_2$ in the expression for the mutual impedance of the interstage network. The stage gain can then be expressed as

$$A' = -\frac{v_0 Z_m z_{in}}{S_2 (S + S_1)}$$

(5.40)

If the values of $S_1$ and $S_2$ given by equations (5.38) and (5.39) are substituted into the above equation, it can be shown after some simplification that

$$A' = -A_r' \frac{w_c}{S + w_c}$$

(5.41)

where

$$A_r' = \frac{v_0 r_{12} R_b}{r_{12} + r_{11}}$$

and

$$w_c = \frac{r_{12} + r_{11} + R_b}{r_{11} + R_c} w_{in}.$$

In the above expression for $A_r'$, $R$ denotes the parallel combination of $R_b$ and $(r_{12} + r_{11})$.

From equation (5.15), the overall voltage gain of a cascaded $R-C$ coupled transistor amplifier could then be written

$$A_t = \frac{E_1'}{E_{in}} A_1' A_2' A_3' \ldots A_n'$$

(5.42)

It is a simple matter to solve for $E_1'/E_{in}$. Considering the
input of the equivalent circuit shown in figure (5.4b) as a voltage dividing network the expression for $E_{1'}/E_{in}$ may be written

$$E_{1'}/E_{in} = \frac{r_{12}/SC_{in}}{r_{11} + r_{12}/SC_{in}} = \frac{r_{12}}{r_{12} + r_{11}} \frac{1/r_{12}SC_{in}}{S + 1/r_{12}SC_{in}}$$

(5.43)

Let $A_{ro'}$ denote $\frac{r_{12}}{r_{12} + r_{11}}$ and $W_{co}$ denote $1/r_{12}C_{in}$.

The overall voltage gain of the cascade may finally then be written

$$A_t = A_{ro'} \times (-A_{r1'}) \times (-A_{r2}) \times \cdots \times (-A_{rn'}) \times \frac{W_{co}}{S + W_{co}} \times \frac{W_{c1}}{S + W_{c1}} \times \frac{W_{c2}}{S + W_{c2}} \times \cdots \times \frac{W_{cn}}{S + W_{cn}}$$

(5.44)

The above equation completely describes the overall voltage gain of a cascaded R-C coupled amplifier in the mid and high frequency region. The low frequency response of the cascade could have been determined by means of the same type of analysis. In the equivalent circuit the coupling capacitances would have been included and the transistor and shunt wiring capacitances would have been neglected. The high frequency response is of somewhat more interest because it is determined by the transistor itself while the low frequency response is determined by the external circuit.
elements, i.e. the coupling capacitances.

The importance of the above analysis is not the analytical result but rather the method that was used to obtain it and the form in which the result is expressed. By means of the equivalent circuit shown in figure (5.2), any transistor circuit can be analyzed by the same method and its overall gain expressed as a product of individual stage gains. That the overall gain may be expressed in this manner greatly simplifies the analytical analysis and design of transistor circuits.
(5.4) **TRANSISTOR FIGURE OF MERIT**

It is desirable to have some coefficient to express the relative merit of transistors. In vacuum tube practice, a common term used for such a purpose is the figure of merit. The figure of merit of a tube is defined as the gain-bandwidth product of an R-C coupled amplifier employing that tube. In this section a similar expression will be derived for the transistor.

In the previous section it was shown that for an R-C coupled transistor amplifier the stage gain could be expressed as

\[ A' = \frac{r_{12}}{r_{12} + r_{11}} \frac{g_t R}{\frac{w_c}{S + w_c}} \]  

(5.41)

where

\[ w_c = \frac{R_b + r_{12} + r_{11}}{R_b + r_{11}} \frac{1}{r_{12}C_{in}} \]

The .707 cutoff frequency of the stage gain in radians per second is \( w_c \), and the reference gain is \( g_t R \frac{r_{12}}{r_{12} + r_{11}} \). The quantities \( R \) and \( R_b \) are as defined in the previous section.

If the lower cut-off frequency of the stage gain is much smaller than the upper cut-off frequency, the bandwidth of the stage gain is approximately equal to the upper cut-off frequency. The gain-bandwidth product of the stage gain is then

\[ F_s = g_t \frac{r_{12}}{r_{12} + r_{11}} \frac{R_b (r_{12} + r_{11})}{R_b + r_{12} + r_{11}} \frac{R_b + r_{12} + r_{11}}{R_b + r_{11}} \frac{1}{r_{12}C_{in}} \]

\[ = g_t / C_{in} \frac{R_b}{R_b + r_{11}} \]  

(5.45)
$F_s$, the stage figure of merit, has thus been defined as the gain-bandwidth product of the stage gain.

Inspection of equation (5.45) shows that $F_s$ increases with $R_b$ and is a maximum when $R_b$ is infinite. The gain-bandwidth product of the stage gain may thus be increased by increasing $R_b$. This is practical only up to the point where $R_b$ is roughly ten times $r_{11}$. Beyond this $F_s$ increases very little as $R_b$ is increased.

The transistor figure of merit, $f_t$, will be considered to be the maximum value of $F_s$. Since, by the definition of $R_b$, the maximum value it may have is $1/g_{22}$; the transistor figure of merit could be expressed as

$$f_t = \frac{g_t}{C_{in}} \frac{1/g_{22}}{1/g_{22} + r_{11}}.$$  \hspace{1cm} (5.46)

However, $1/g_{22}$ is much greater than $r_{11}$, and the above equation could be approximately written as

$$f_t \approx \frac{g_t}{C_{in}}.$$  \hspace{1cm} (5.47)

The transistor figure of merit is then a function of $g_t$, $C_{be}$, and $C_{bc}$. Values for the figure of merit of the transistors used in the measurements were calculated assuming a base to collector voltage gain of 25. These values are given along with values of $g_t$ in table IV.

The transistor figure of merit may be used in circuit design as a general guide in selecting a transistor. Other factors must be considered; but, for two otherwise similar
transistors, the one with the higher figure of merit should generally be preferred.

In deriving the expression for the stage figure of merit of an R-C coupled transistor amplifier, an important difference between a transistor R-C coupled stage and a vacuum tube R-C coupled stage should have been noted. Although the stage gain-bandwidth product of a vacuum tube R-C coupled amplifier is equal to a constant, both the reference gain and the upper cut-off frequency may be varied over a wide range. For the transistor R-C coupled amplifier this is not true. Inspection of equation (5.41) shows that although the reference gain is variable from zero to a maximum of \( g_t r_{12} = B \), the upper cut-off frequency of the stage gain varies only between the limits of \( 1/r_{12}C_{in} \) and \( \frac{r_{11} + r_{12}}{r_{11}} \cdot \frac{1}{r_{12}C_{in}} \). It is thus seen that the characteristics of a transistor amplifier are more nearly determined by the transistor than is the case with the vacuum tube; hence, the design of a transistor amplifier will not be as flexible as the design of a vacuum tube amplifier.
TABLE IV

VALUES OF $f_t$ AND $g_t$

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Chapter 6

SUMMARY

As stated in the introduction, the purpose of the investigation described herein was to develop an equivalent circuit for the transistor which would represent the transistor for all frequencies of interest and which would lend itself to use in circuit design. Chapter one points out how each of the transistor equivalent circuits now in use fails in some respect to meet these requirements. The rest of the paper was concerned with developing a satisfactory equivalent circuit using the hybrid parameter equivalent circuit as a starting point.

In Chapter two, it was shown how the transistor, from its d-c characteristics, may be represented by a linear equivalent circuit of the hybrid parameter form, and in what region of operation such a representation is justified. Exact measurement of the parameters of this equivalent circuit was then discussed in Chapter three.

The work described to this point is more or less routine, and the ideas used may be found in the present literature. It is in Chapter four that the investigation diverges from methods which have been discussed elsewhere. Here, as a
result of a series of experimental measurements, the hybrid parameter equivalent circuit is modified to explain the behavior of the junction transistor at high frequencies. With minor exceptions, the resulting equivalent circuit, shown in figure (4.15), is in agreement with the circuit derived from a theoretical analysis in "Transistor Electronics." The circuit shown in figure (4.15) does not contain several elements included in the circuit presented in "Transistor Electronics." As pointed out in Chapter four, these elements are secondary in describing the operation of the transistor and may be neglected in normal operation.

In Chapter five, a more useful form of the equivalent circuit is derived. It is then indicated how this form of the equivalent circuit, shown in figure (5.1), can be used in the analysis of transistor amplifiers. Extension of the method to various types of transistor circuits can then be carried out by techniques similar to those used in the analysis of vacuum tube circuits.

It is to be remembered that the investigation described above was carried out for only the junction type transistor. The useful frequency range of the junction transistors used in this investigation had an upper limit of approximately 100 kc. For applications requiring higher frequencies the

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circuit designer must resort to surface barrier or P-N-I-P type transistors. Further study is thus required to determine equivalent circuits for these types of transistors. Extension of the methods used to investigate the junction transistor could be used, although the measuring techniques would have to be extended to higher frequencies. A possible first step would be to insert cathode followers between the circuit being measured and the vacuum tube voltmeters. Another possibility would be to modulate the signal source used in the measurements and use a band-pass amplifier tuned to the frequency of the signal source in conjunction with a detector circuit in place of the vacuum tube voltmeters.
BIBLIOGRAPHY

Books


Articles
