A STUDY OF RADAR ANGELS

by

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STATEMENT BY AUTHOR

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ABSTRACT

Radar echoes from a visibly clear atmosphere have been observed with the vertical looking pulsed doppler radar. Observations of these "radar angels" fall into two categories. The first consists of persistent narrow horizontal layers on the HTI scope. The second category consists of small transient "dot" echoes.

Radar theory demands very large gradients of refractive index to account for such observations. These gradients have not been observed with existing instruments. Recent work in radar theory suggests that the gradients required could be reduced if the surfaces from which radar scattering occurs can be considered as existing in the near zone of the radar rather than in the far zone.

This theory predicts detectable back-scattered energy during observations of a layer angel on 10 October 1962. It is further shown that oscillations in the back-scattered energy correspond to gravity wave oscillations on the inversion layer which existed that day.

The observations of dot angels on six different days are tabulated showing variations with time and height. Instrumentation was not available to make a detailed study of these observations, but correlations with general meteorological conditions are discussed.
1. INTRODUCTION

Scattering of radio energy by a volume of the atmosphere containing no visible scatterers (Figs. 1.1 and 1.2) has been observed with radio detection equipment since 1935 (Plank, 1956). Most studies of these echoes prior to World War II were performed by radio-physicists and radio-engineers. The equipment used for these studies operated at wavelengths above the microwave region (30 cm), used antennas with undirectional beams, and required rather large back-scattered signals to detect a target.

During World War II, cloud detection became a primary function of radio detection equipment. Under the influence of meteorologists, radar equipment has been designed with very short wavelengths (less than 10 cm), using highly directional antennas (1° or less) and capable of detecting very small back-scattered signals ($10^{-13}$ watts or smaller). As the performance of radar has improved, the frequency of occurrence of unexplained echoes has increased rapidly and these have come to be commonly known as "angels" to the meteorologist. Plank (1956, 1959) gives a complete historical summary of the observations and theory.

Most of the concentrated studies of angels have led to correlation of the observations to atmospheric phenomena such as sea breeze fronts, cold fronts, inversions, turbulent eddies, etc. However, the existing radar theory requires large gradients of refractive index (about 10 N units per cm, where $N = (n-1)x10^6$ and $n$ is the real index of
refraction), which in turn imply large gradients of moisture (about 40 mb per cm), and/or large gradients of temperature (about 15°C per cm). These have not been predicted by the atmospheric physicist nor supported by observation. The problem here may lie with the inability of the present instruments to make continuous measurements of refractive index gradients, temperatures or moisture over very small distances in the atmosphere. Studies at the University of Texas by Crain (1955a, 1955b) have aided considerably in this direction, but the space intervals of the best measurements still remain too large to verify the existence of the large gradients over small distances.

Because of this inability of theory or observation to show the existence of these large gradients, many have been convinced that angels are in fact produced by birds, insects, solid particulates, etc., which were not noted by the radar operators. This issue was raised repeatedly by Richardson et al. (1958), Harper (1958), and Mueller (1958) at the Seventh Weather Radar Conference. Such explanations are verified by some observations, but other observations exist in which none of these phenomena were present.

The observations themselves suggest that several different processes work to produce angels. The "dot angel" is a small radar echo appearing in the scopes as a dot which persists for only a few seconds. The "layer angel" is a continuous layer or a layer of recurring smaller echoes which persists at constant or slowly varying heights for several minutes. Observations of angels with other characteristics have been reported by researchers using horizontal-looking radar equipment.
In addition to efforts to observe and explain large refractive index gradients, Atlas (1960) has shown that the theory may be modified so that for certain special microwave propagation conditions the gradients required could be greatly reduced. Battan (1963a) discusses a layer angel observed at The University of Arizona on 10 October 1962. He further suggests that the back-scattered power detected from this layer could be predicted from the theory of Atlas for a refractive index gradient of only 0.1N unit per cm.

In this paper, a further study shall be made of this layer angel and of a large number of dot angels observed at The University of Arizona. It is the purpose of this paper to determine if a gravity wave on an inversion over Tucson could produce the necessary conditions to account for the layer angel and oscillations in back-scattered power detected. The course pursued will be:

1. To determine the factors which lead to detectable radar back-scattering from a layer of clear air.
2. To establish a model of an internal gravity wave.
3. To describe the observations of the layer angel.
4. To show that an inversion existed over Tucson during the observation of the angel.
5. To determine the characteristics of a gravity wave on this inversion.
6. To determine if a correlation exists between the gravity wave oscillations and the oscillations in back-scattered power from the angel.
7. Then, assuming the oscillations of the layer are caused by such a gravity wave, radii of curvature of the scattering surface will be determined from the amplitude and frequency of the oscillations of the layer angle.

8. Finally, the effects of focusing and interference by such a curved surface will be examined and power scattered to the radar by the surface will be computed.
2. RADAR THEORY

The general radar range equation has been derived by many writers (Battan, 1960; Probert-Jones, 1962). We use it here in the form:

\[ P_r = \left( \frac{P_t G}{4\pi r^2} \right) \left( \frac{\sigma}{4\pi r^2} \right) \left( \frac{\lambda^2}{4\pi} \right) \left( \frac{1}{2 \ln 2} \right) \]  

(1)

where

- \( P_r \) = back-scattered power in watts,
- \( P_t \) = power transmitted,
- \( G \) = antenna gain,
- \( \sigma \) = radar cross section,
- \( \lambda \) = wavelength,
- \( r \) = target range.

The first parenthesis is the power per unit area incident on the target, and when multiplied by the second, gives the power per unit area returned to the antenna. The third parenthesis is the power collection area of the antenna.

Atlas (1960) quotes a private communication by Holt which shows that the back-scattering cross section of a smooth sphere of radius \( \delta \gg \lambda \) is

\[ \sigma = \frac{\pi^2 (\pi \alpha^2 \gamma^2)}{(\alpha + \gamma)^2} \]  

(2)

where \( \gamma \) is the surface power reflection coefficient.
When \( r \) is much greater than \( a \) (the far zone problem),

\[
\sigma = \pi^2 \gamma \alpha^2
\]  

(3)

When \( r \) is much less than \( a \) (the near zone problem),

\[
\sigma = \pi^2 \gamma \gamma^2
\]  

(4)

McDonald (1962b) published a proof of an equivalent relation.

In this theory, it has been assumed that the laws of geometric optics are valid, and in particular the cross section of the target contains many Fresnel zones. Atlas has shown that the back-scattering cross section of the first Fresnel zone is four times that of a scatterer containing many zones, and according to theory presented by Jenkins and White (1950), the cross section of an infinite flat plane oscillates between zero and \( 4 \pi^2 \gamma \gamma^2 \) depending on the number of Fresnel zones illuminated. For a few zones, an even number of zones gives \( \sigma = 0 \), while the odd numbers of zones give \( \sigma = 4 \pi^2 \gamma \gamma^2 \).

A given number of Fresnel zones can be illuminated in two ways. A very large target may completely intercept the beam at a point where the beam width corresponds to an exact number of zones, or a smaller target which only partially fills the beam may correspond in size to an exact number of zones in the beam.

The curvature of the target surface also has an effect on the number of zones intercepted. Kerr (1951) shows that a sphere, completely
illuminated by a beam, intercepts an infinite number of zones. For a partially illuminated sphere, the number of zones intercepted increases with decreasing radius of curvature, resulting in smaller fluctuations of signal return due to interference.

In this light, Atlas suggests that rather than consider angels as small blobs with radii of curvature corresponding to their geometrical size, we should consider angels as segments of large smooth surfaces with large radii of curvature. This places the angels in the near zone. Very smooth surfaces must be assumed since any moderate degree of roughness will give a random phase of the reflected signal and completely destroy any constructive interference.

In the form of the radar range equation shown, \( \Sigma \) represents the radar cross section. For incoherent scattering from \( n \) targets with equal back-scattering cross sections, \( \Sigma = n \sigma \). If the reflections are indeed from smooth surfaces in the case of angel echoes, we have coherent scattering from a single large target and \( \Sigma = \sigma \).

It follows from Eq (4) that for a plane surface with many Fresnel zones, \( \sigma = \tau^2 \gamma^2 \). Substituting this in Eq (1) yields

\[
P_r = \frac{P_o G^2 \lambda^2 \tau^2}{64 \pi^2 \gamma^2 (2 \ell - 2)}
\]

Swingle (1950) has shown that for a zone of varying index of refraction

\[
\tau^2 = \left[ \frac{\Delta N}{\Delta Z} \left( \frac{\lambda_r}{8 \pi r} \right) 10^{-6} \right]^2
\]
where \( \frac{\Delta N}{\Delta z} \) is the gradient of refractive index, and

\[
\Delta N = -1.4 \Delta T + 4.2 \Delta \varepsilon .
\]

\( \Delta T \) is the change in temperature in degrees centigrade across the zone and \( \Delta \varepsilon \) is the change in vapor pressure in mb. If only small odd numbers of Fresnel zones are intercepted, \( P \) given by Eq (5) could be enhanced by a factor as large as four.

A curved surface whose radius of curvature may be considered constant over the illuminated portion may act to focus the back-scattered power in the same manner as a curved mirror in geometric optics. When the range from the antenna to the surface is equal to the radius of curvature of the portion of the surface illuminated, the back-scattered beam will be perfectly focused at the antenna. For ranges greater than the radius of curvature of the mirror, partial focusing will occur.

Atlas shows, using work by Ludlam (1958) that if dot angels are thermal (convective) bubbles, perfect focusing cannot occur at any altitude, thus \( P \) can only be enhanced by some degree of partial focusing. In the case of layer angels, oscillations on the layer could produce partial focusing in concave regions and increased divergence in convex regions.

The theory as presented indicates that in order to show that angel echoes are produced by regions where a refractive index gradient exists, it must be shown that one or more of the following conditions are met:
1. Very large gradients of refractive index, i.e., large gradients of temperature and/or vapor pressure.

2. Very large radii of curvature of the reflecting surface.

3. The existence of constructive interference.

Therefore, it will be the purpose of this paper to determine if gravity waves on a surface separating two media of different index of refraction can produce the necessary conditions giving rise to the angels.
3. THE GRAVITY WAVE

Radar theory discussed in Chapter 2 indicates that the highest reflectivity occurs at that layer where the gradient of refractive index of the air is highest. If such a layer is perturbed in the form of a wave and moving overhead, the power reflected to the radar from a given height would vary periodically even if other factors such as the refractive index gradients remained constant. All other factors being equal, the reflectivity of this layer would be greatest when the surface was concave downward and smallest when concave upward. Hence the surface could produce focusing and divergence effects. Further, the field of vertical motion associated with a gravity wave might induce changes in the temperature and water vapor distribution, thereby modifying the refractive index gradients. The magnitude of this effect must be considered, and will be evaluated in the next section.

A gravity wave is a disturbance in which buoyance acts as a restoring force on parcels displaced from hydrostatic equilibrium. A particular case is that of disturbances on the interface between two fluids of different density. Haurwitz (1941, 1951, 1961), Godske et al. (1957), Freeman (1948) and others have presented simple theoretical models of such waves. For the discussion to follow in this thesis, a model will be assumed for gravity waves on the interface between two incompressible, homogeneous fluids. The basic features of this model are:
1. The change of density from one fluid to the other occurs at the interface, i.e., density is assumed to be discontinuous there.

2. Both fluids are deep enough that other boundaries do not effect oscillations on the interface.

3. The interface is horizontal in its undisturbed state.

4. The wave is a small perturbation confined to a vertical plane.

5. The amplitude of the wave is constant.

Such a wave is stable when the upper fluid is less dense than the lower fluid and there is no wind shear across the interface. When the upper fluid is more dense, or the shear across the boundary is large, these waves will be unstable. In the observations to be discussed later, shear across the boundary is negligible.

Haurwitz (1951) discusses the same simple model permitting compressibility. The differential equation describing this case is a confluent hypergeometric equation. This equation has not been solved for the boundary between two fluids. Further, Freeman (1947, 1949) and Sekera (1939) evaluate the effects of compressibility on the wavelength and show that the effects are negligible at wave speeds much less than the speed of sound.

The observations to be studied in this thesis were taken with a vertical pointing doppler radar, discussed in more detail in the next section. Thus, the data are taken from a fixed point at periodic intervals as the wave passes overhead. If the radar data do describe an oscillation on the stable layer, it will only describe the component in the vertical plane containing the wind direction. Therefore, in the
discussion to follow, only the characteristics of the two-dimensional incompressible model will be considered.

Density at an inversion is continuous. However, a reasonably strong inversion may be considered to be a surface approximating the surface of density discontinuity of the simple model (Haurwitz, 1941). Therefore, the characteristics of such waves will be summarized and later it will be assumed that the results apply when steep inversions exist.

The solution of this simple model using perturbation techniques may be written:

\[ Z = H \cos \frac{2\pi r}{\lambda} (x - c t) \quad (8) \]

where \( Z \) is the height of the surface above its undisturbed position at time \( t \), \( H \) is the amplitude of the oscillation, \( \lambda \) is the wavelength of the gravity wave, \( c \) is the wave speed, and \( x \) is the horizontal distance from the phase reference point. The wave speed for this model is

\[ C = U \pm \sqrt{\frac{g \lambda}{2 \pi r} \left[ \frac{(\rho - \rho') \tanh \frac{2\pi r h}{\lambda}}{\rho + \rho' \tanh \frac{2\pi r h}{\lambda}} \right]} \quad (9) \]

where \( g \) is the acceleration of gravity, \( \rho \) is the density of the fluid below the surface, and \( \rho' \) is the density of the fluid above the surface, and \( h \) is the height of the surface above the ground.

Two special cases may be noted. When \( \frac{2\pi r h}{\lambda} \) is small, \( \tanh \frac{2\pi r h}{\lambda} \) is nearly equal to \( \frac{2\pi r h}{\lambda} \). When \( \frac{2\pi r h}{\lambda} \) is large, \( \tanh \frac{2\pi r h}{\lambda} \) is nearly equal to unity. Within 97 percent accuracy we can write:
(1) For \( h \leq 0.025 \lambda \)

\[
C = U \pm \sqrt{\frac{g h (1 - \frac{\rho'}{\rho})}{2 \pi}} = U \pm \sqrt{\frac{g h (T_{*'} - T_{*})}{2 \pi}},
\]  

(10)

where \( T_{*} \) is the virtual temperature below the surface (i.e., at the base of the inversion), and \( T_{*'} \) is the virtual temperature above the surface (at the top of the inversion).

(2) For \( h \geq 0.32 \lambda \)

\[
C = U \pm \sqrt{\frac{g \lambda}{2 \pi}} \left( \frac{\rho - \rho'}{\rho + \rho'} \right) = U \pm \sqrt{\frac{g \lambda}{2 \pi}} \left( \frac{T_{*'} - T_{*}}{T_{*'} + T_{*}} \right),
\]  

(11)

The disturbances on the surface between the fluids are assumed to extend some distance into the fluid media on either side. Thus, wave motion will exist over some finite vertical depth above and below the inversion.

The curvature of waves of the above type will now be examined. This will be needed in a later discussion of the effects of focusing on the back-scattered power. From the calculus, the radius of curvature of a curve \( y = f(x) \) is given by

\[
R = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} \frac{d^2 y}{dx^2}.
\]  

(12)

Equation 8 may be rewritten as \( Z = H \cos \frac{2 \pi s}{\lambda} \), if the phase reference point is chosen as the point where the radar beam intercepts the inversion, i.e., \( x = 0 \), and where \( ct = s \). The radius of this curve is
The absolute value of the radius of curvature of this wave varies from 
\[ \frac{\lambda^2}{4\pi^2\gamma} \] to \( \infty \).

In summary, the model that has been described in this section is a vertical gravity wave, originating as a perturbation at the base of an inversion, assumed to be a surface separating two media of different density, and causing the media above and below this surface to oscillate. The media have been assumed to be homogeneous, incompressible, with no shear existing across the base of the inversion. The absolute value of the radius of curvature of a surface under the influence of this wave takes on values from \( \frac{\lambda^2}{4\pi^2\gamma} \) to \( \infty \).
4. THE LAYER ANGEL

The pulsed doppler radar designed for cloud physics research at The University of Arizona has been described by Theiss (1963), and by Theiss and Kassander (1963). It was primarily designed for study of motions and precipitation in convective clouds. Work of this nature has been reported by Battan, Kassander, and Theiss (1963), Battan (1963b), and Wilson (1963). The characteristics of the radar are listed in Table I.

Table I. Important parameters of the radar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Wavelength = 3.25 cm (X band)</td>
<td></td>
</tr>
<tr>
<td>1.3° beam to symmetric half power points</td>
<td></td>
</tr>
<tr>
<td>First side lobes at -20 db</td>
<td></td>
</tr>
<tr>
<td>Nominal transmitted power = 40 kw</td>
<td></td>
</tr>
<tr>
<td>Minimum detectable power = 3.2 X 10^-14 w or -105 db</td>
<td></td>
</tr>
<tr>
<td>Pulse length = 0.25 μs (76.2 meters)</td>
<td></td>
</tr>
<tr>
<td>Pulse repetition frequency = 4000 cps</td>
<td></td>
</tr>
<tr>
<td>Velocity resolution = 1/2 mps</td>
<td></td>
</tr>
<tr>
<td>Antenna is 1.83 meter vertical-looking parabola</td>
<td></td>
</tr>
<tr>
<td>with emitter at 70 cm focal point</td>
<td></td>
</tr>
<tr>
<td>The main beam saturates the lower 306 meters</td>
<td></td>
</tr>
<tr>
<td>Antenna gain = 1.78 X 10^6</td>
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</tbody>
</table>

The observation schedules were set up to correspond to expected convective activity. Observations were taken from two sites, one on the valley floor near Tucson, the other on Mt. Bigelow. It was immediately recognized that dot angels could be observed with varying frequency at nearly all times during operation. The frequency varied from day to day and with time of day. Generally, angels occurred most frequently below
1220 meters with some occurring at all altitudes. Frequency and height of occurrence appear to increase with surface temperature and convective activity. Although this point has not been examined in detail, there does not appear to be a difference between the dot angel phenomena in the valley and that on the mountain. Battan, Kassander, and Theiss (1963) have summarized some of the early observations with the radar, showing that dot angels most often have upward motions.

Persistent layer angels were observed on a few occasions while operating in the valley. No such phenomenon has been recorded from the mountain site as of this writing.

A strong persistent layer angel was observed on the morning of 10 October 1962 with its center at 1670 meters above the ground. Its depth was approximately 305 meters. A-scope pictures were taken of this angel every 20 seconds and doppler velocities were recorded for ten velocities at 150 meter altitude intervals. The velocity channels were \( \pm 1/2, \pm 1, \pm 3, \pm 5, \) and \( \pm 7 \) meters per second.

Figure 4.1 is a plot of total return power from the center of the layer (1670 meters). Figure 4.2 is a plot of maximum power returned from a height of 1670 meters in any velocity channel, and Figure 4.3 is a plot of the velocity channel receiving maximum return. The power data plotted were taken from the digital record which measures the same samples as the A-scope.

It should be explained here that the A-scope data are continuous in height for all practical purposes, while the digital data are taken
FIGURE 4.1

Total signal power from the layer angel. Units are decibels with respect to a milliWatt for each 20 second frame period.
Power in the velocity channel with maximum signal. Units are decibels with respect to a milliwatt.
The velocity channels with maximum signal. Intervals are 20 sec.
at 150 meter intervals of height. The radar height resolution is one-half the pulse length, i.e., 38 meters.

The HTI (height-time indicator) record showed a distinct oscillatory nature at the base and top of the layer and in the bright bands within the layer. There were as many as three of these bright bands approximately 30 meters deep, at any given time. In addition, the digital data show that the power increased toward the center of the layer (Fig. 4.4).

4.1 Atmospheric conditions

The 1200Z Tucson sounding (Fig. 4.5) shows a moisture inversion from 765 mb to 754 mb, then a sharp decrease in dew point from 750 mb to above 737 mb. The radiosonde was motorboating between 737 mb and 697 mb. A temperature inversion appears at 720 mb in the middle of the dry layer. Examination of microfilm of the original radiosonde data shows a faint ink trace between 737 mb and 730 mb. Moisture was shown decreasing very rapidly and this trace is likely to be more indicative of instrument lag than real moisture decrease. The actual decrease, therefore, could be even greater than this trace. The original microfilm showed a constant moisture ordinate from 697 mb to 719 mb. This was below the instrument's lowest effective ordinate (the ordinate below which humidity data are not reliable).

It is realized that any estimates of humidity in a motorboating layer are of questionable value, but in order to calculate refractive index gradients for this zone, an estimate of the moisture content must
Height-time contours of power from the layer angel. Contour units are decibels with respect to a miliwatt in the velocity channel with maximum power (upper) and total power from the layer (lower). Battan (1963a)
FIGURE 4.5

Tucson sounding, 10 October 1962, 1200Z.
be made. Two distributions of dew-point temperature have been considered. First, the slope of the faint ink trace from 737 mb can be continued until it reaches the motorboating point on the trace, and then joined with the point at 697 mb, just above the dry layer. This is shown by the dashed line in Figure 4.5.

Second, we can continue the slope of the faint ink trace from 737 mb, and continue the slope of the curve between the first two points above the dry layer until they intersect. This is shown by the solid line in Figure 4.5.

The first assumed distribution of humidity gives an upper limit to the possible moisture content, since any value of dew point greater than those on this curve will not cause the instrument to motorboat. The second assumption probably is the most reasonable distribution of moisture in the layer.

Table II presents the data through the significant portions of the sounding, based on the assumptions described. Vapor pressure was obtained from the Smithsonian Tables. Values of refractive index, $N$, were calculated from the equation (Battan, 1960)

$$N = \frac{\gamma \gamma \gamma \gamma \gamma \gamma}{T} \left( \rho - \frac{e}{1 + \frac{4810 e}{T}} \right) \tag{14}$$

It is seen in Table II that the layer from which one would expect possible back-scattering is the layer from 740 to 730 mb. Assuming a constant gradient of index through this layer, we have

$$\frac{\Delta N}{\Delta z} = 1.9 \times 10^{-3} \frac{N \text{ units}}{\text{cm}}$$
Table IIa. Significant data from the modified sounding according to first assumption.

<table>
<thead>
<tr>
<th>h (m) above-ground</th>
<th>P (mb)</th>
<th>T (°C)</th>
<th>Td (°C)</th>
<th>T* (°C)</th>
<th>e (mb)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1590</td>
<td>765</td>
<td>12.6</td>
<td>0.7</td>
<td>13.5</td>
<td>6.42</td>
<td>239.6</td>
</tr>
<tr>
<td>1704</td>
<td>754</td>
<td>11.7</td>
<td>2.2</td>
<td>12.7</td>
<td>7.16</td>
<td>240.1</td>
</tr>
<tr>
<td>1750</td>
<td>750</td>
<td>11.6</td>
<td>2.3</td>
<td>12.6</td>
<td>7.21</td>
<td>239.3</td>
</tr>
<tr>
<td>1862</td>
<td>740</td>
<td>11.2</td>
<td>0.3</td>
<td>12.1</td>
<td>6.24</td>
<td>232.0</td>
</tr>
<tr>
<td>1966</td>
<td>730</td>
<td>10.8</td>
<td>-12.6</td>
<td>11.1</td>
<td>2.33</td>
<td>210.9</td>
</tr>
<tr>
<td>2019</td>
<td>725</td>
<td>10.4</td>
<td>-12.8</td>
<td>10.7</td>
<td>2.29</td>
<td>209.4</td>
</tr>
<tr>
<td>2072</td>
<td>720</td>
<td>10.7</td>
<td>-12.9</td>
<td>11.0</td>
<td>2.27</td>
<td>208.0</td>
</tr>
<tr>
<td>2083</td>
<td>719</td>
<td>10.8</td>
<td>-12.9</td>
<td>11.1</td>
<td>2.27</td>
<td>207.7</td>
</tr>
<tr>
<td>2184</td>
<td>710</td>
<td>9.9</td>
<td>-13.1</td>
<td>10.2</td>
<td>2.23</td>
<td>205.2</td>
</tr>
<tr>
<td>2298</td>
<td>700</td>
<td>8.9</td>
<td>-13.5</td>
<td>9.2</td>
<td>2.16</td>
<td>201.9</td>
</tr>
<tr>
<td>2414</td>
<td>690</td>
<td>8.0</td>
<td>-12.0</td>
<td>8.4</td>
<td>2.44</td>
<td>200.5</td>
</tr>
</tbody>
</table>

Table IIb. Significant data from the modified sounding according to second assumption.

<table>
<thead>
<tr>
<th>h (m) above-ground</th>
<th>P (mb)</th>
<th>T (°C)</th>
<th>Td (°C)</th>
<th>T* (°C)</th>
<th>e (mb)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>730</td>
<td>10.8</td>
<td>-12.6</td>
<td>11.1</td>
<td>2.33</td>
<td>210.9</td>
</tr>
<tr>
<td>2019</td>
<td>725</td>
<td>10.4</td>
<td>-18.5</td>
<td>10.6</td>
<td>1.43</td>
<td>205.3</td>
</tr>
<tr>
<td>2072</td>
<td>720</td>
<td>10.7</td>
<td>-18.0</td>
<td>10.9</td>
<td>1.49</td>
<td>204.4</td>
</tr>
<tr>
<td>2083</td>
<td>719</td>
<td>10.8</td>
<td>-17.8</td>
<td>11.0</td>
<td>1.52</td>
<td>204.0</td>
</tr>
<tr>
<td>2184</td>
<td>710</td>
<td>9.9</td>
<td>-16.0</td>
<td>10.2</td>
<td>1.76</td>
<td>202.7</td>
</tr>
<tr>
<td>2298</td>
<td>700</td>
<td>8.9</td>
<td>-14.0</td>
<td>9.2</td>
<td>2.08</td>
<td>201.4</td>
</tr>
</tbody>
</table>

The hourly surface observations, given in Table III for the period of observations, show that surface heating is not likely to have modified the sounding at the height of the angel by 0900M, assuming a
dry adiabatic lapse rate through a thermally mixed layer. It is also noted that no clouds or obscuring phenomena were reported. The original sounding for 11 October 0000Z (Fig. 4.6) shows that the dry adiabatic assumption was satisfactory, and the moisture discontinuity disappeared presumably because of mixing from beneath.

Table III. Hourly surface observations for 10 October 1962

<table>
<thead>
<tr>
<th>Time (MST)</th>
<th>T (°F)</th>
<th>T'w (°F)</th>
<th>WX</th>
<th>P (mb)</th>
<th>Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>64</td>
<td>50</td>
<td>Clear</td>
<td>27.37</td>
<td>SE 5</td>
</tr>
<tr>
<td>0100</td>
<td>62</td>
<td>49</td>
<td></td>
<td>.37</td>
<td>S 8</td>
</tr>
<tr>
<td>0200</td>
<td>62</td>
<td>48</td>
<td></td>
<td>.37</td>
<td>SSE 10</td>
</tr>
<tr>
<td>0300</td>
<td>60</td>
<td>48</td>
<td></td>
<td>.37</td>
<td>S 9</td>
</tr>
<tr>
<td>0400</td>
<td>59</td>
<td>47</td>
<td></td>
<td>.37</td>
<td>SSE 8</td>
</tr>
<tr>
<td>0500</td>
<td>59</td>
<td>47</td>
<td></td>
<td>.38</td>
<td>S 9</td>
</tr>
<tr>
<td>0600</td>
<td>58</td>
<td>47</td>
<td></td>
<td>.39</td>
<td>SSE 6</td>
</tr>
<tr>
<td>0700</td>
<td>59</td>
<td>47</td>
<td></td>
<td>.39</td>
<td>S 10</td>
</tr>
<tr>
<td>0800</td>
<td>65</td>
<td>50</td>
<td></td>
<td>.41</td>
<td>SSE 11</td>
</tr>
<tr>
<td>0900</td>
<td>71</td>
<td>53</td>
<td></td>
<td>.41</td>
<td>SSE 10</td>
</tr>
<tr>
<td>1000</td>
<td>77</td>
<td>55</td>
<td></td>
<td>.41</td>
<td>S 6</td>
</tr>
<tr>
<td>1100</td>
<td>81</td>
<td>56</td>
<td></td>
<td>.40</td>
<td>ENE 4</td>
</tr>
</tbody>
</table>

4.2 The boundary surface and the scattering layer

The previous discussion shows that at 1200Z there was an inversion layer with its base at 2050 meters on which a stable wave could form, i.e., \( \frac{T^*}{T^*} > T^* \). Table II shows that the largest gradients of refractive index occurred below this inversion.

The layer angel echo was observed with its center at 1670 meters between 1600 and 1700Z. The gravity wave model proposed in Chapter 3 shall be used and the variation of back-scattered power from the layer
FIGURE 4.6

Tucson sounding, 11 October 1963, 0000Z.
an angel shall be investigated to see if a correlation exists between these variations and the oscillations of the gravity wave. This procedure assumes that a gravity wave could originate on the temperature inversion and attenuates downward to the layer of maximum refractive index gradients, as previously discussed.

It has been assumed that the media above and below the boundary surface are incompressible. As long as we are interested in discussing the gravity wave, incompressibility may be assumed for velocities much less than the speed of sound. It will be seen later that this is a valid assumption for these observations. Now, however, an examination of the back-scattering of radar energy from the finite layer must be made and it is conceivable that compressibility of the air in the layer may appreciably influence the gradient of refractive index, i.e., of temperature and vapor pressure, in the layer as it undergoes periodic vertical motion. Thus, the gravity wave may be accurately considered for an incompressible atmosphere, whereas the atmosphere must be considered compressible to properly discuss the radar reflectivity. Haurwitz (1961) faces the same dilemma when discussing noctilucent clouds. He assumes the vertical transverse wave model on the boundary between two homogeneous incompressible fluids. After describing the wave this way, he considers compressibility in finding the divergence of the particles which make up the cloud in a layer below the stable boundary surface.

A parcel of air confined to a boundary surface undergoing gravity wave oscillations may be shown to oscillate in an elliptical
orbit about its undisturbed position (Lamb, 1945). The orbit will actually spiral toward or away from the undisturbed position when the wave is stable or unstable, respectively. The orbit will be closed for a wave in neutral stability. At different positions along the surface various parcels will be undergoing this orbital rotation, and will be periodically brought closer together and then separated. Parcels above or below the infinitesimal boundary surface may move toward or away from the interface. However, these parcels will still be periodically brought together and separated during the oscillations. This mechanism presents the problem of determining the effects of this periodic compression and expansion on the gradients of temperature and vapor pressure of the unsaturated oscillating layer from which radar energy is scattered.

Table II shows that the layer with maximum gradient of refractive index is the layer from 730 to 740 mb. In order to obtain a first estimate of the effects of vertical motion, changes in the gradients of temperature and vapor pressure will be examined when this layer is lifted and lowered 10 mb adiabatically and conserving its mixing ratio. The observed amplitude (H) of the oscillation of the angle when viewed on the A-scope was only 30 meters, i.e., about 3 mb. The following table shows the results of this vertical displacement:

<table>
<thead>
<tr>
<th></th>
<th>( \Delta T^\circ C )</th>
<th>( \Delta e (mb) )</th>
<th>( \Delta N )</th>
<th>( \Delta Z ) (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>740-750</td>
<td>0.4</td>
<td>3.98</td>
<td>22.2</td>
<td>109</td>
</tr>
<tr>
<td>730-740</td>
<td>.4</td>
<td>3.91</td>
<td>21.9</td>
<td>110</td>
</tr>
<tr>
<td>720-730</td>
<td>.4</td>
<td>3.86</td>
<td>21.5</td>
<td>111</td>
</tr>
</tbody>
</table>
These data show that lifting the entire layer through a given height does not produce significant changes in the gradient of refractive index. As a second estimate of the effect of compression, consider two parcels displaced vertically toward one another. One parcel originally at 750 mb will be lifted adiabatically to 740 mb, another at 720 mb will be considered to have descended to 730 mb.

<table>
<thead>
<tr>
<th></th>
<th>T°C</th>
<th>θ (mb)</th>
<th>ΔN</th>
<th>ΔZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>750 → 740 mb</td>
<td>10.8°C</td>
<td>7.0044</td>
<td>19.24</td>
<td>109 meters</td>
</tr>
<tr>
<td>720 → 730 mb</td>
<td>11.8°C</td>
<td>2.4804</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This process also produces refractive index gradients which do not differ greatly from the results of the sounding itself.

Lamb (1945) also considers the horizontal movement of the parcels bound to the interface with respect to one another. The qualitative picture is identical to that of the vertical movement, i.e., brought about by the same elliptical orbits. In order to make quantitative estimates of the effect of the vertical motion, the amplitude of the oscillations of the radar echo could be used. However, no such inference may be made from the observations concerning the horizontal motions. Furthermore, since the atmosphere has a tendency to be horizontally stratified (at least true over small areas) the horizontal differential motions would not be expected to have a greater effect on the gradients of refractive index than do the vertical motions. Other processes could be conceived which would lead to large changes of the original sounding. The possibilities involved are hopelessly numerous. Therefore,
Modification of the original sounding because of differential vertical motions will be neglected in this thesis.

4.3 Gravity waves on the inversion

Periods of the two special cases of gravity waves discussed in Chapter 3 can be computed from the data in Table II, the pibal wind, 6.9 meters per second, and equations 10 and 11.

(a) When \( h \leq 0.025 \lambda \),

\[
\lambda_{\text{minimum}} = 8.3 \times 10^4 \text{ meters.}
\]

\( C_1 = 12.1 \text{ mps} \) for waves moving in the same direction as the medium.

\( C_2 = 1.7 \text{ mps} \) for waves moving against the medium.

Eq (10) shows that in the case of long waves on a shallow medium, the phase speed is independent of wavelength, \( \lambda \). The minimum periods of this special case are:

\[
\tau_1 = 6.8 \times 10^3 \text{ sec for forward moving waves, and}
\]

\[
\tau_2 = 51.2 \times 10^3 \text{ sec for backward moving waves.}
\]

(b) When \( h \geq 0.32 \lambda \),

\[
\lambda_{\text{max}} = 6.4 \times 10^3 \text{ meters.}
\]

When \( \lambda = 6.4 \times 10^3 \text{ meters,} \)

\( C_1_{\text{max}} = 9.5 \text{ mps} \) for forward moving waves, and

\( C_2_{\text{min}} = 4.3 \text{ mps} \) for backward moving waves.
When $\Lambda = 0$

$$C_1 \min = 6.9 \text{ mps} \text{ for forward moving waves, and}$$

$$C_2 \max = 6.9 \text{ mps} \text{ for backward moving waves.}$$

Eq (11) shows that in the case of short waves on a deep medium, the phase speed is dependent on wave length,

$$\tau_1 \max = 675 \text{ sec} \text{ for forward moving waves, and}$$

$$\tau_2 \max = 1490 \text{ sec} \text{ for backward moving waves.}$$

The variations in the radar echo intensity clearly could not correspond to the long wave on shallow medium gravity waves since the sampling frequency of the radar is one per twenty seconds and the total sample period is only one hour (3600 sec). The next section, however, will show that the observed variations in echo intensity agree well with the short wave on deep medium.

4.4 Radar back scattering from the oscillating layer

So far, a model of a gravity wave on the surface at the base of an inversion has been discussed. The effects of the wave oscillations on the layer of air just below the inversion have been examined. Finally, a range of wavelength values has been determined for these oscillations. Now, the effects of this oscillating layer on the backscattered radar energy are to be studied.

Consider the horizontal layer across which a vertical gradient of refractive index exists as shown in Table II. The gradient of refractive index reaches a maximum value at some level within the layer.
The back-scattered power will also be a maximum at this level. Now if this layer is set to oscillating, the back-scattered radar energy from some fixed height will vary as the level of maximum gradient passes back and forth through that point. The A-scope photos and the digital data indicate that the maximum back-scattering does oscillate about the center of the angel echo at 1670 meters.

Figure 4.7 shows the theoretical relationship between the height oscillations of the gravity wave and the power variations of the back-scattered energy from a constant level near the center of a layer driven by the gravity wave on a surface above. This assumes the maximum refractive index gradient to be interior to the layer. When the constant reference level is near the undisturbed position of the maximum gradient, the level of maximum gradient will pass through the constant level twice each period. Thus, the frequency of the back-scattered power oscillations will be twice that of the wave at the base of the inversion.

Autocorrelation functions were computed for the back-scattered power for lags from one data interval to thirty. The techniques used are discussed by Panofsky (1958). Figure 4.8 shows autocorrelation coefficients ($r$) computed from

$$ r_L = \frac{\sum (x_i - \bar{x})(x_{i+L} - \bar{x})}{N s^2} \quad (15) $$

where

- $r_L = \text{autocorrelations coefficient for lag L}$,
- $s^2 = \text{variance of echo power} = 13.4 \text{ db}$,
Phase relationship between the oscillations of the layer (upper) and back-scattered energy (lower).
Autocorrelation of total signal power.

FIGURE 4.8
\[ N = \text{number of samples correlated} = 40 \text{ through } 69, \]
\[ \bar{X} = \text{mean power} = 18.2 \text{ db}, \]
\[ X_i = \text{power of } i\text{th sample from Figure 4.1}. \]

The plot of autocorrelation versus lag (Fig. 4.8) shows maximum correlations for various periods between four and sixteen intervals, dropping to negligible correlation for lag greater than eighteen intervals.

The significances of the correlations were computed by means of the F test (Panofsky, 1958).

\[ F = \frac{n \sum X_i^2}{1 - \gamma_i^2} \]

\( f_i \) = degrees of freedom of entire sample.

From a table of F-test limiting values such as Panofsky's Figure 19, the correlation coefficient for the lag of four periods is seen to have a significance above 95 percent. The coefficient for a lag of 16 periods has significance above 99 percent.

The measured vertical velocities of the layer angel (Fig. 4.3) might be expected to be correlated with the oscillations in back-scattered power. Such correlation coefficients were computed and found to be negligible. The following factors complicate such a correlation.

(a) Back-scattered power could be measured over a continuous range, while the velocities are measured in discrete increments.

(b) The velocities used in the correlation calculation were velocities in the channels with maximum signal. In many cases, there were other channels with signal power nearly as great as the one used,
and often these were not contiguous channels, but widely removed. Some measure of the total motion, i.e., some appropriate integral, might be a more proper indicator of correlation.

(c) We have already shown that the power variations have periods one-half that of the height oscillations on the inversion. Thus, we cannot be sure of the phase or lag between the two curves or the various components making up the curves. The methods used to determine correlations for such combinations of curves assume knowledge of the phases and lags.

4.5 Curvature of the wave surface

It has been shown that the oscillations of back-scattered power display preferred periods from four to sixteen intervals. It has also been shown that the gravity wave height oscillations have periods twice that of the corresponding power oscillations. Such a gravity wave therefore displays periods from 160 seconds (8 periods) to 640 seconds (32 periods). These periods are within the range already found in section 4.3 for backward or forward moving short gravity waves. Table IV summarizes the limits imposed on the period of the oscillating surface.

The lower limits of the radii of curvature R for a surface oscillating with amplitude of 30 meters in the range of frequencies with significant correlations are given in Table V. The upper limits were shown for each frequency to be infinite in section 3.
Table IV. Summary of various limits on the period.

<table>
<thead>
<tr>
<th>Determining factor</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity wave theory</td>
<td>1490 sec</td>
<td>0 sec</td>
</tr>
<tr>
<td>Frequency of observations</td>
<td>1800 sec</td>
<td>40 sec</td>
</tr>
<tr>
<td>Autocorrelation of observed power</td>
<td>320 sec</td>
<td>80 sec</td>
</tr>
<tr>
<td>Gravity waves implied by observed power</td>
<td>640 sec</td>
<td>160 sec</td>
</tr>
</tbody>
</table>

Table V. Lower limits of radii of curvature for the limits of significant frequencies (H = 30 m).

\[ T_1 = 160 \text{ seconds} \]
\[ \Lambda_1 = 1310 \text{ meters for forward moving wave} \]
\[ \Lambda_1 = 730 \text{ meters for backward moving wave} \]
\[ R_1 = 1410 \text{ meters forward} \]
\[ R_1 = 438 \text{ meters backward} \]
\[ T_2 = 640 \text{ seconds} \]
\[ \Lambda_2 = 6.2 \times 10^3 \text{ meters forward} \]
\[ \Lambda_2 = 3.1 \times 10^3 \text{ meters backward} \]
\[ R_2 = 3.2 \times 10^4 \text{ meters forward} \]
\[ R_2 = 8.0 \times 10^3 \text{ meters backward} \]

Any radius of curvature greater than one-half the height of the reflecting layer will give some degree of focusing from regions concave downward and divergence for regions convex downward for all radii. Most of the range of radii calculated above are greater than one-half the echo height. Shorter or longer radii are obtained when several oscillations are superimposed. It is reasonable to expect periodic focusing from such a wave.
5. THE RESULTS

5.1 Computed back-scattering from the layer

Calculations can be made of the back-scattered power from a plane layer by making use of the theory discussed in Chapter 2. This calculated power can then be modified by considering the curvature of the surface with the wave on it. Finally, the effects of interference phenomena can be taken into account.

Eqs (5) and (6) were written as:

\[ P_r = \left( \frac{P_x C^2 \lambda^2 \tau^2}{64 \pi \eta^2 \tau^2} \right) \left( \frac{1}{2 \ln \tau} \right) \]

\[ \tau^2 = \left[ \frac{\Delta N}{\Delta \tau} \left( \frac{\lambda_c}{8 \pi} \right) \right]^{10^{-6}} \]

From the parameters of the radar (Table I) and for \( \frac{\Delta N}{\Delta \tau} = 1.9 \times 10^{-3} \) N units/cm, it is found that

\[ \tau^2 = 6 \times 10^{-20}, \] and

\[ P_r = 3.2 \times 10^{-19} \text{ watt.} \]

This value of \( \frac{\Delta N}{\Delta \tau} \) was determined for \( \frac{\Delta N}{\Delta \tau} \) constant throughout the layer of largest refractive index gradient (114 meters thick). The appearance of bright bands in the layer (there were as many as three of these at one time) suggests that \( \frac{\Delta N}{\Delta \tau} \) was not constant in the layer and was much greater in the bright narrow bands of the layer.

The bright layers on the HTI film appeared to be 30 meters deep. If the entire gradient occurs in these smaller depths, \( P_r \) could be increased by a factor of 12 and
\[ r^2 = 7.2 \times 10^{-19} \]
\[ P_r = 3.9 \times 10^{-18} \]

5.2 Computed back-scattering from the oscillating layer

It was shown in section 4.5 that the range of radii of curvature for forward and backward moving waves in the range of periods measured was from 438 meters to infinity. When several waves are superimposed, the lower limit will be even lower, but the upper limit remains infinite.

The range equation (Eq (1)) was derived for back-scattering from a target with no focusing. The geometry of radiation reduces the returned signal from that transmitted by a factor of \(4r^2\). If a target is curved such that the antenna is at the focal point of the curved target, then there is no geometrical reduction.

Radii of curvature of the surface of separation have been calculated for different wavelengths of the gravity wave. These values can be used to determine the effect of focusing for each wavelength.

The curved surfaces in this problem cannot be compared to spherical mirrors from which perfect focusing could occur, but rather, should be compared to cylindrical mirrors which have a focal line rather than a focal point.

In deriving the range equation, the energy per unit area was reduced from antenna to target by a factor of \(\frac{1}{r^2}\). The returned energy was then further reduced by a factor of \(\frac{1}{r^2}\). The total reduction is \(\frac{1}{(2r)^2}\). If our antenna is illuminating a cylindrical reflector,
the factor from antenna to mirror remains $\frac{1}{r^2}$, but the return factor will be different. The cross section of the beam of returned energy will be nearly rectangular, rather than circular. The length of the rectangle will increase linearly with the distance from the mirror. The width, however, will increase or decrease with distance by a factor depending on the radius of curvature of the reflecting surface.

If the cylindrical surface's focal line lies within the plane of the antenna, the total reduction factor will simply be $\frac{1}{2\pi}$. This would represent an increase of the returned power computed in section 5.1 by a factor of $2\pi$. $P_\Sigma$ could be as great as $1.3 \times 10^{-12}$ watts for the greater values of $\frac{\Delta N}{\Delta z}$ used in section 5.1 in the presence of focusing by a cylindrical surface.

The geometric reduction of energy density in the transmitted beam remains $\frac{1}{r^2}$, while the reduction in the return beam depends on the shape of the surface and $r$. We must note the following points which arise from the oscillating nature of the wave.

(a) As the surface passes overhead (waves on the surface also pass overhead), it is found that the radius of curvature varies from $\frac{\lambda^2}{4 \pi^2 H}$ to $\infty$ to $-\frac{\lambda^2}{4 \pi^2 H}$ to $-\infty$ and so forth for the single wave.

(b) If the unperturbed position of the layer is horizontal, the focal point of the curved section illuminated by the radar will oscillate vertically and horizontally as the wave progresses.

(c) Superposition of several waves will extend the smaller limits of the radius of curvature to still smaller absolute values, thus extending the range of the geometric factor through which the focusing
oscillates. Superposition will also give reflector shapes other
than cylindrical.

These factors will combine during the observation to give
random oscillation of power from values much less than that for a flat
plane because of defocusing from convex surfaces, to values as large as
2π times that from a flat plane.

5.3 Computed back-scattering considering interference

In this discussion of interference effects on the power re-
turned from the layer, we follow the discussion of Atlas (1960) and
Jenkins and White (1957). The following symbols are employed:

\[ b = \text{cross section radius of Fresnel zone} \]
\[ r = \text{range} = 1670 \text{ meters} \]
\[ R = \text{radius of curvature of layer} \]
\[ F = \text{geometric cross section area of first Fresnel zone} \]
\[ S = \text{equivalent radius of curved surface which is illuminated}. \]

It can be shown that

\[ F = \frac{1}{2} \frac{\pi s \lambda}{\lambda}, \text{ and} \]
\[ S = \frac{R r}{R + r} \]

For \( R = 450 \text{ meters} \); \( F = 18.4 \text{ m}^2 \); \( b = 2.41 \text{ m} \)

For \( R = 3.06 \times 10^4 \text{ meters} \); \( F = 78 \text{ m}^2 \); \( b = 4.98 \text{ m} \)

For \( R \rightarrow \infty \), i.e., flat plane; \( b = 5.48 \text{ m} \)
The geometric cross section diameter of the beam at 1670 meters is 39.6 meters.

The radii of curvature of the oscillating surface are several orders of magnitude greater than the radii of the radar beam at 1670 meters, thus it will be assumed that the curvature of the section of the surface illuminated is constant.

Jenkins and White (1957) show that the intensity of reflected radiation from a small odd number of Fresnel zones is four times the average intensity of the incident beam. Whether a given number of zones is a "few" or "many" is determined by the angular width of the transmitted beam. The radar beam width is only 1.37°, thus it will contain only a few zones for surfaces whose radii of curvature are large when compared to the radar beam width. This is shown by:

\[ A_m = \text{const.} \times \frac{S_m}{d_m} \times (1 + \cos \theta) \]  
(17)

- \( A_m \) = amplitude of E vector from mth zone
- \( S_m \) = area of mth zone
- \( d_m \) = average distance of mth zone from antenna
- \( \theta \) = angle subtended by zone

For the case where the radius of curvature of the target is 450 meters, the illuminated section of the curved surface corresponds to over 60 zones. The plane surface at 1670 meters corresponds to 13 zones.

An odd number of zones has a radar cross section of

\[ \sigma = 4 \pi r^2 \tau^2 \]
for the near case (Atlas (1960)). Thus in the case of a small odd number of zones, the back-scattered power may be enhanced by a factor of four.

A summary of the effects of the various factors already discussed on the back-scattered power is given in Table VI.

Table VI. Summary of back-scattered power

<table>
<thead>
<tr>
<th>Condition on power</th>
<th>( \Delta Z = 109 ) meters</th>
<th>( \Delta Z = 30 ) meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_r ) from plane, no interference</td>
<td>( 3.2 \times 10^{-19} ) w</td>
<td>( 3.9 \times 10^{-18} ) w</td>
</tr>
<tr>
<td>( P_r ) from cylindrical reflector on focal line, no interference</td>
<td>( 1.1 \times 10^{-13} )</td>
<td>( 1.3 \times 10^{-12} ) w</td>
</tr>
<tr>
<td>( P_r ) from cylindrical reflector on focal line, Fresnel interference</td>
<td>( 4.4 \times 10^{-13} ) w</td>
<td>( 5.2 \times 10^{-12} ) w</td>
</tr>
<tr>
<td>( P_r ) from spherical reflector at focal point, no interference</td>
<td>( 3.6 \times 10^{-8} ) w</td>
<td>( 4.3 \times 10^{-7} ) w</td>
</tr>
<tr>
<td>( P_r ) from spherical reflector at focal point, Fresnel interference</td>
<td>( 1.4 \times 10^{-7} ) w</td>
<td>( 1.7 \times 10^{-6} ) w</td>
</tr>
</tbody>
</table>

Recalling that the minimum power detectable with the radar is \( 3 \times 10^{-14} \) watts, it is seen that any combined effects of curvature and interference will produce detectable signals when the curvature is equal to the target range. It is also seen from the magnitude of the predicted
returns that some variation of range about the value equal to the radius of curvature will still produce a detectable signal. It therefore seems reasonable to expect periodic oscillations in power returned to the radar from an oscillating surface as predicted in this paper.
6. THE DOT ANGELS

Dot angels are the most frequently observed echoes during daily observations made with the doppler system employed by The University of Arizona. These echoes, however, are far more difficult to explain than the layer angel observations. The layer angel characteristics suggest a persistent atmospheric process occurring over a relatively large area, therefore it seems justifiable to seek an explanation from soundings and data taken at regular observation sites, even though they may be several miles from the radar site. The characteristics of dot angels do not permit such freedom. They are very small (tens of meters) in diameter and appear for a few seconds on the radar scope (see Fig. 1.1). Any attempt to give a detailed explanation of these echoes would appear to require data collections on a micro-scale in the vicinity of the radar beam.

This type of data has not been taken, and no attempt will be made in this paper to correlate the dot angel observations to gradients of refractive index or to local micro-scale parameters as was done with the layer angel. This section will be limited to a presentation of some observations of dot angels and a qualitative discussion of these observations.

Figures 6.1 to 6.4 show the numbers of dot angels at different heights on four different days. The data for Figure 6.1 were taken from Mt. Bigelow (elevation 2440 meters); data for 6.2 to 6.4 were
Angel frequency with height, 1250-1345 MST, 22 July 1963. Radar was on Mt. Bigelow (2430 m). A total of 597 dot angels were observed.
Angel frequency with height, 1530-1627 MST, 28 July 1963. Radar was in the desert near Tucson (855 m). A total of 352 angels appeared.
Angel frequency with height, 1116-1332 MST, 3 July 1963. Airplanes flew near the beam during this observation. Radar was located on the floor of the desert near Tucson (855 m). A total of 415 dot angels appeared.
Angel frequency with height, 1118-1334 MST, 2 July 1963. Airplanes flew near the beam during this observation. Radar was located on the floor of the desert near Tucson (855 m). A total of 1465 dot angels appeared.
taken with the radar based on the desert floor (elevation 855 meters). The appearance of maxima and minima as a function of height on each day suggests the presence of an interference phenomenon. The maxima and minima, however, do not occur at the same heights on the separate days; thus it seems the interference phenomenon if it is real, might be influenced by meteorological parameters.

The frequency of echoes decreased with height on each day, but no pattern seems to exist on any given day for the rate of decrease. Figure 6.5, however, shows a composite plot of the observation from the four days, and above 520 meters the frequency seems to follow a \( \frac{1}{r} \) decrease with height.

In addition to these observations of frequency versus height, the following characteristics of dot angels are summarized.

1) During each observation period, echo frequency and height increased with increasing surface temperature, i.e., maximum frequency during midafternoon with minima just before sunrise (Fig. 6.6 and 6.7).

2) Echo frequency increases with increasing instability. This trend was not determined from measurements of instability, but by noting presence and intensity of convective clouds.

3) In addition to the general trend of (2) above, the frequency of dot angels increases rapidly as a cumulus cell moves near or overhead.

4) Airplanes were flown through the beam concurrently with the data taken in Figures 6.3 and 6.4 in an effort to test the possibility of observing turbulent wakes of aircraft. The tests were not highly controlled, and aircraft never approached lower than 914 meters above
Composite angel frequency with height. Dashed curve represents \(1/r^2\). Total of 2899 angels were observed.
FIGURE 6.6

Frequency of angels with height and time, 10 September 1963.
FIGURE 6.6

Frequency of angels with height and time, 23 September 1963.
the antenna. The radar operator had the immediate impression from scope
observation that angel activity did increase in presence of aircraft.
Further study of the record from the radar and the observer's log,
however, indicates the oscillations in frequency of echoes with time not
to be correlated with aircraft passage, and no different from periodic
oscillations observed during all observation periods.

These observations lead one to believe the dot angels to be
radar returns from refractive index gradients set up by thermal
"bubbles" or simply turbulent eddies, in and near which large tempera­
ture gradients probably exist. The detection of any particular eddy
is probably enhanced by constructive interference phenomena. The theory
of Chapter 2 further shows that the gradients required for such returns
are reduced by several orders of magnitude when we properly consider
the radius of curvature of the scattering surface rather than its
geometric radius, i.e., near zone scattering rather than far zone.
7. CONCLUSIONS

The refractive index gradients required to give detectable back-scattered radar energy from a clear volume of atmosphere are neither impossible nor unlikely when the scattering surface can be considered in the near zone and to be curved so that it focuses the back-scattered energy.

This characteristic of high resolution radar systems suggests that the structure of the atmosphere may further be studied by radar. Such a case was the inversion of temperature at 1670 meters over Tucson on 10 October 1962. The surface based radar detected the refractive index gradients associated with this inversion and the oscillations of a gravity wave on the inversion. The oscillations of the gravity wave were implied from oscillations of back-scattered power from 1670 meters. Further refinements in the instruments should make it possible to correlate observed vertical velocities to such a wave.

This same characteristic probably accounts for the numerous observations of dot angels; however, instrumentation on a three-dimensional micro-scale will be required to verify this point. We can imply from current observations that dot angel height and frequency of occurrence increase with increasing surface temperature and in the vicinity of convective turbulence.
REFERENCES


