DEVELOPMENT OF RELIABILITY PREDICTION TECHNIQUES FOR LONG MISSION SPACECRAFT

by

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TABLE OF CONTENTS

TABLE OF CONTENTS
(Cont'd)

TABLE OF CONTENTS
(Cont'd)

LIST OF SYMBOLS

$2r$ or
$2r + 2$ = Degrees of freedom of Chi Square Distribution

4_{sa} = Acquiescent mode of the sensing device for #4 standby unit

4_{sc} = Cyclic mode of switching for the sensing device for #4 standby unit

4_{so} = Inadvertently opening mode of the sensing device for #4 standby unit

5_{sa} = Acquiescent mode of the sensing device for #5 standby unit

5_{sc} = Cyclic mode of switching for the sensing device for #5 standby unit

5_{so} = Inadvertently opening mode of the sensing device for #5 standby unit

B = Bad

E_{CN} = Average number of corrective replacements

E_N = Total number of spares

E_{PN} = Average number of preventive replacements

$E(T')$ = Mean debugging time in hours

$f(t)$ = Failure density function

$F_w(T')$ = Wearout distribution function

G = Good

$H = \left[e^{-T/E(T')} \times (1 - e^{-t/E(T')}) \right]$

K_A = Dependent upon type of component part being replaced

K_B = Dependent upon location of component part being replaced

K_C = Dependent upon inherent abilities of the particular astronaut

K_D = Dependent upon environment conditions to which the astronaut is being subjected

K_E = Dependent upon the amount of radiation deterioration to the astronaut's brain

K_F = Dependent upon psychological condition of the astronaut

M = Mean of the normal wearout distribution

M_p = Mean time between failures of P components

M_T = Mean time between failure for maintenance every T hours

MTBF = Mean time between failure

P_S = Probability of survival

Q = Unreliability

r = Failure rate

r = Number of failures occurring during total logged test time

R_C = Component reliability within useful life

R_e = Component reliability within early life

R_p = Reliability for p components in parallel or 1 operating component with p-1 components in standby

R_S = System reliability

R_{Sw} = Sensing and switching reliability

LIST OF SYMBOLS
(Cont'd)

R_w = Component reliability within wearout

t = Mission time in hours

T = Initial age of component part at the beginning of mission in hours

T' = The age of a component in hours at the beginning of a mission (T) and at the end of a mission (T + t)

T_o = Time between preventive maintenance actions in hours

T_p = Age in hours at point P

T_x = Total logged test time

X^2 = Chi Square refers to Chi Square Confidence Tables

$Z = \frac{T' - M}{\sigma_T}$

λ_C = Useful life failure rate

$\hat{\lambda}_C$ = Estimated useful life failure rate

λ_o = Failure rate at zero age in failures per million hours

σ_T = Standard deviation of the normal wearout distribution

(t) = Distributions of wearout failures

τ = Repair time

ABSTRACT

In this thesis, generalized mathematical models are developed for long mission spacecraft flights, which incorporate series, parallel, and standby components in early, random and wearout failure periods. Mathematical models are established also which give the system reliability with various preventive maintenance practices and for component reliabilities incorporating wearout. The results of models are compared for possible application to manned and unmanned flights.

CHAPTER 1

INTRODUCTION

Reliability is the quantitative ingredient most vital to the economic utility of space vehicles. Originally, reliability was geared to the need of a man/machine team, knowing that what the machine lacked in reliability, the man could make up with inflight adjustments and post-flight repair. With the advent of the Aerospace Age and unmanned orbital and lunar missions, it became necessary to have a high reliability designed and built into the spacecraft before it was launched. With long mission manned spacecraft flights becoming a reality in the near future, the man/machine concept of reliability under space conditions must be investigated. Since various forms of maintenance and spare parts provisioning can be adapted to manned long mission flights and not to unmanned flights, the complexity of the prediction techniques will be much greater for manned flights. For the purpose of this study, long mission will be defined as a manned or an unmanned flight from three months or 2,190 hours to two years or 17,520 hours. Thus an earth orbiting space station or an extended lunar exploratory mission,

as well as manned flights to neighboring planets, would come under this study.

This thesis is intended to be a reference for predicting reliability for long mission spacecraft flights. The math models used herein will be kept as general as possible so that with certain modifications, the equations will apply to all electronic, electrical, mechanical and fluid networks under a space environment. The techniques used in this thesis will be developed in theory on the component, sub-system, and system level.

To this author's knowledge, no studies have been made on reliability prediction techniques for long mission manned and unmanned spacecraft flights. Bazovsky (1)* did a study of long mission reliability prediction techniques for shipboard machinery in which a general equation incorporating early, random, and wearout failure rates was derived.

The only spacecraft flight to date which could be classed as a long mission spacecraft flight was the Mariner R Venus probe which yielded no specific failure rate data or application factors (2).

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* Numbers in parentheses refer to REFERENCES.

The Langley Research Center Staff (3) did a study in which the technological problems of manned rotating spacecraft were investigated. This study defines fully the environmental conditions which must be considered in many of the long mission spacecraft reliability math models. This thesis does not contain space environment application factors. All of the equations developed herein are for ideal laboratory conditions. The numerical values obtained from these equations must be modified by application factors which allow for differences between laboratory and space environment.

In Silent Satellites Speak (4), the effects of radiation on the electronic equipment of Telstar were analyzed and some estimates of the actual radiation levels beyond the Van Allen Belt were made. There is a possibility that long exposures of six months or more to such high levels of radiation might impair the mental facilities of an improperly shielded astronaut. Until such time as the possibility of human brain deterioration can be completely eliminated, a deterioration factor should be incorporated into all equations relating to manned spacecraft flights. This deterioration factor multiplied by the human reliability factor would give the complete probability of an astronaut being able to repair and restore to operation equipment which has failed. The areas in

which human contributions play major parts have been defined by Gross (5). McCormick (6) has made a study of the human motor activities and the speed and accuracy with which these activities can be accomplished. Wolman and Okano (7) have derived an equation in which the astronaut's abilities to perform manual overrides under various environmental conditions were evaluated. In (7), the probability that an astronaut would be able to perform the proper overrides at the proper times was evaluated from the average of five independent estimates of the astronaut's performance, by individuals, professionally qualified to access man's performance capabilities. In the text on page 62, the independent estimates method is used for evaluating the performance of an astronaut with an overpressurized space suit.

Madison (8) found that engineering repairmen were able to achieve the same reliability as military repairmen using far less parts. This is an indication that the level of technical training of the astronaut who will be performing technical overrides should be considered when predicting reliability with preventive maintenance, and should also be taken into account when applying the equations for predicting the number of spare parts required for a long mission flight as described in Chapter 5.

Boechal (9) compared the levels of reliability needed for manned and unmanned orbital satellites and for military satellites and came to the conclusion that manned and military missions require a considerably higher degree of reliability than do scientific missions. In this thesis, an attempt will be made to establish reasonable risk levels for manned and unmanned flights.

Eagle (10) came up with a graph plotting, Failures versus Time, which illustrates the wearout failure distribution. Eagle also came up with an equation for the standardized normal wearout distribution which expresses a mission time in units of the standard deviation and the mean of the wearout distribution. This factor can be found in tables expressing the standard deviation as a percentage of area under the wearout curve. This area subtracted from one gives the wearout reliability as a percentage.

ARINC Research Corp. (11) has drawn a relationship between system failure rates and parts "removal rates." These procedures were based on the premise that the failure rate of a new system can be predicted from a knowledge of part "removal-rates" in predecessor systems of comparable design and functional complexity. This premise cannot be tested at this time in long mission flights unless the space vehicle can be recovered and the failed part

pinpointed. As in the case of the Mariner R Venus flight, many of the failures could only be pinpointed at the system level and some failures could not be pinpointed at all.

Pieruschka (12) states that the distribution curve of first wearout failures is nearly normal. Second failures are distributed according to one-half of the normal distribution curve. The distribution curves of third and higher order failures look more and more like a chance distribution. As a result, higher order wearout failures occur almost "by chance" and the force of mortality curve (failure rate curve) with repair levels off as more and more components are replaced and becomes horizontal. In application, if the components are replaced as they fail, after the third or fourth replacement, they would begin failing randomly with failure rate considerably higher than the initial chance failure rate. Thus, repair actions would have to be made more often.

McLaughlin and Voegtlin (13) found that in a study of radar, communications, and navigational equipment, there were 498 true failures and 1570 total replacements. The disparity was due to secondary or cascading failures, replacement of marginal parts suspected of impending failures, parts damaged in maintenance, parts removed unnecessarily in a sequential search for the real trouble,

etc. In a long mission flight, this unnecessary replacement rate must be reduced to nearer unity.

Gordy (14) presents a chart which illustrates how families of failure-rate curves enable designers to predict failure rates as a function of an environmental stress (such as temperature) for various operating stresses (such as the ratio of operating to rated power).

Pieruschka (15) has an excellent discussion on the theory of maintenance, (15) also contains a figure relating the various areas of maintenance, and a figure comparing failure rate with ideal repair maintenance. These figures are included in Chapter 5.

The Reliability Data Book, Martin (16), contains figures illustrating the effect on reliability of increasing system redundancy for various combinations of series and parallel sub-systems. Several of these figures are included in Chapter 3.

Meyers (17) has written an excellent discussion giving the pros and cons of sequential and functional redundancy, which are discussed in Chapter 3, and in a paper by Aroian and Meyers (18), the mathematical advantages of sequential redundancy are presented.

Bazovsky (19) contains math models for functional and sequential redundancy used in Chapter 3 and equations for predicting the Chi Square Confidence Limits used in Chapter 5.

Reliability prediction techniques involve the calculation of the reliability of all components involved in a system or sub-system for a specified time under a specified load and environment. The contribution of early, random, and wearout failure rates must be calculated for each component when they apply and the product of the contributions gives the component reliability. With a knowledge of component reliabilities, system reliabilities for systems of various configurations and purpose can be calculated for the given mission time. The manipulating of the system configurations and the methods of sustaining a specified reliability by various maintenance practices constitutes the science of reliability prediction.

CHAPTER 2

MATH MODELS FOR PART RELIABILITY

2.1 Differences between Short and Long Mission Prediction Techniques

In a short mission spacecraft flight, there is an excellent probability that the length of the mission will be within the useful life of some of the component parts in the spacecraft systems. As long as the mission time is shorter than the useful life of a particular component part, then the reliability of that part will be based upon a constant failure rate and the reliability of the component part within its useful life will be given by

$$R_c = e^{-\lambda ct}. \qquad (2.1)$$

If the length of the mission does not force any of the component parts into wearout, then the failure rate of the component parts not entering wearout is much lower than the failure rate of the component parts in a system whose mission time is much longer than the constant

failure rate time interval on the bathtub curve. See Fig. 1. Since system reliability is an inverse function of component part failure rate, it can be concluded that for identically redundant systems without replacement, the reliability is lower for long missions than for short missions. And, on a posteriori basis, the reliability for a time interval, t, taken during the useful life of a component part is greater than the reliability for the same time interval taken during wearout.

In a manned long mission spacecraft flight, there are many possible ways to gain additional reliability on a stock piece of equipment that are not available to unmanned long mission flights. Maintenance, preventive maintenance, spare parts provisioning and redundancy (with manual switching) can be used to increase system reliability. But, if the spacecrafts' weight is a critical design parameter, then the added weight of the astronaut, the weight of his life support equipment, and the weight of the spare parts and repair equipment will reduce the possible weight of the original systems and their backup or redundant systems which could have been included in the spacecraft. A great deal of redundant circuitry could be provided in the equaling of just the astronaut's basic equipped weight. To evaluate the various alternatives fairly, it will be assumed that the

added weight of the astronaut or astronauts does not present a problem and all of the circuitry which is needed for the various reliability practices can be included.

For an unmanned flight, parts replacement or preventive maintenance practices cannot be used because they require an astronaut to make the replacement. Of course, components can be added or subtracted from a circuit at intervals electronically, but this form of replacement must be considered standby redundancy. Then only sequential and functional redundancy techniques can be used to increase the Mean Time Between Failure of a circuit aboard an unmanned spacecraft. The MTBF for both a sequential and functionally redundant circuit is given by (19, pp. 100, 113)

$$M_p = \int_0^\infty R_p \, dT \tag{2.2}$$

where R_p is the reliability of p components in parallel for functional circuits, and 1 component with p-1 components in standby for a sequential circuit. In Sec. 3.3, it is shown that for the same number of equal component parts, sequential redundancy assuming perfect sensing and switching will give a higher reliability (R_p) than will functional redundancy. This means that since the MTBF is a direct function of R_p, the MTBF of the sequential circuit will be greater than the MTBF for a comparable

functionally redundant circuit. If the reliability of the sensing and switching unit decreases, R_p for sequential redundancy decreases and a point is reached where the inequality expressed above reverses.

2.2 Identification of the Total Failure Rate of Components

In predicting reliability from time T = 0 in the life of a component to a time T + t, the following three failure rates must be taken into account. See Fig. 1.

1. In the region which extends from T = 0 to T + t where $T + t \leq T_1$, the failure rate at any point T_p, where $0 \leq T_p \leq T_1$, is the summation of the early failure rate and the chance or random failure rate at T_p. It is assumed that at $T_p = T_1$, debugging has been completed and early failures are no longer contributing to the total failure rate. In the debugging process, it is assumed that perfect repair is practiced, i.e., no new early failures are introduced into the system when failed components are being replaced. To accomplish perfect repair, the replacement components must be taken from a population in which the substandard components have already been eliminated (19, p. 69).

The probability of a component part surviving infant mortality from T to T + t where $0 \leq T \leq T + t \leq T_1$ is given by (19, p. 73)

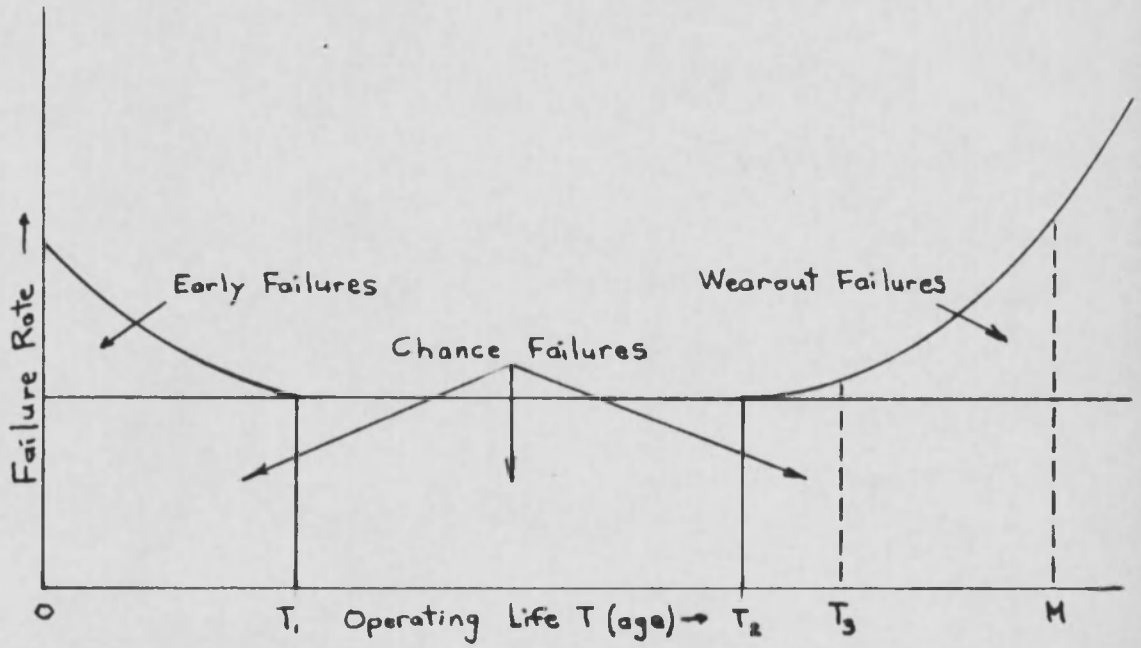

Figure 1. Component Failure Rate as a Function of Age (Bathtub Curve)

$$R_e(T,t) = \exp\left[-\lambda_o E(T') \; e^{-T/E(T')}(1-e^{-t/E(T')})\right] \qquad (2.3)$$

where $E(T')$ = mean debugging time in hours

λ_o = failure rate at zero age in failures per million hours

T = initial age of component part at the beginning of mission in hours

t = mission time in hours

It should be noted that for a mission from

$$T \text{ to } T + t$$

where

$$T_1 \ll T \leq T + t$$

$t/E(T')$ approaches zero faster than $T/E(T')$. Therefore, as

$$(1 - e^{-t/E(T')})$$

is approaching one,

$$E^{-T/E(T')}$$

is approaching zero, and

$$\left[\lambda_o E(T') \; e^{-T/E(T')}(1 - e^{-t/E(T')})\right]$$

approaches zero. And since T becomes large in comparison to $E(T')$, the probability of a component part surviving infant mortality for T to T + t

where $T_1 \leq T \leq T + t$ becomes

$$R_e\ (T \gg T_1,\ t) = \exp\ \left[\text{zero}\right] = 1 \qquad (2.4)$$

As an example, for a component part which has been debugged until time T where T is 5 times as large as E(T'), the reliability that the component will not fail due to early failures for a mission of 2190 hours beginning at time T is (Assuming $E(T') \leq 100$ hours)

$$R_e\ (T, t = 2190) \geq \exp\ \left[- \lambda_o 100\ e^{-5}(1-e^{-2190/100})\right]$$

$$\geq \exp\ \left[- \lambda_o (100)(.00674)(1-)\right]$$

$$\geq \exp\ \left[- .674\ \lambda_o\right]$$

Now even with a very large initial failure rate, say $\lambda_o = 100 \times 10^{-6}$

$$R_e\ (T, t = 2190) \geq \exp\ \left[- .674 \times 10^{-4}\right]$$

$$= \exp\ \left[-0.0000674\right]$$

$$R_e\ (T, t = 2190) \geq .9999326$$

In equation (2.3) if $\left[e^{-T/E(T')}(1-e^{-t/E(T')}\right]$ is defined as H, then $R_e = \exp\left[\lambda_o\ E(T')\ H\right]$. See Table I for values of H for determining R_e for various ratios of $T/E(T')$ where $t \geq 2190$ hours and $E(T') \leq 100$ hours. In actual practice, many component parts receive little or

TABLE I. Values of H for Determining R_E

The Probability of Surviving Early Failures Is Given in Terms of λ_o, $E(T')$, and H in the Equation $R_E = \exp\left[-\lambda_o E(T')\, H\right]$, where $H = \left[e^{-T/E(T')}(1-e^{t/E(T')})\right]$ In the Table, Values of H Are Given for Different Ratios of $T/E(T')$, Given that $t \geq 2190$ Hours and $E(T') \leq 100$

Ratio of T/E(T')	Value of H	Ratio of T/E(T')	Value of H
0.00	1.00	2.00	0.135335
0.02	0.980199	3.00	0.049787
0.04	0.960789	4.00	0.018316
0.06	0.941765	5.00	0.006738
0.08	0.923116	6.00	0.0024788
0.10	0.904837	7.00	0.0009119
0.20	0.818731	8.00	0.0003355
0.40	0.670320	9.00	0.0001234
0.60	0.548812	10.00	0.0000454
0.80	0.449329	15.00	0.0000003058
1.00	0.367879	20.00	0.00000000206

no debugging, so R_E for ratios of $T/E(T') < 5$ is less than one for long mission spacecraft flights.

2. In the area which extends from between T_1 and T_2 in Fig. 1, the failure rate at any point T_p, for $T_1 \leq T_p \leq T_2$ is given by the chance or random failure rate only. To illustrate the approximate areas of the bathtub curve, it was assumed on Fig. 1 that the wearout failure rate does not begin to contribute to the total failure rate until after $T + t > T_2$. In evaluating the contribution of wearout to the system unreliability in actual calculations, the integral of the wearout failure density function is evaluated from $T = 0$ to $T + t$. The probability of a component part surviving the chance failure from T to T + t where $T_1 \leq T < t \leq T_2$ is given by (20, p. 53)

$$R_c(T,t) = e^{-\lambda c(t)} \tag{2.5}$$

The exponential for the probability of surviving chance failure from $T \geq 0$ to $T + t \leq T_1$ can be combined with the exponential for surviving chance failure from time T_1 to $T_1 + t$ and the resulting exponential $e^{-\lambda c(t)}$ gives the probability of surviving chance failure from $T \geq 0$ to $T + t$. Therefore, the probability of a component part surviving both infant mortality and chance failure from

$T \geq 0$ to $T + t$ where $T < T + t \leq T_2$ is given by

$$R_e(T,t) \times R_c(T,t) = \exp\left[-\lambda_o E(T') e^{-T/E(T')} (1-e^{-t/E(T')})\right] \times e^{-\lambda_c(t)} \tag{2.6}$$

R_e is multiplied by R_c because the component part must survive both early and random failures.

3. In the area which lies beyond time T_2 in Fig. 1, the failure rate at any point T_p, where $T_p \geq T_2$ is the summation of the chance and wearout failure rate at T_p. The probability of a component part surviving chance and wearout failure from T to T + t where $T_2 \leq T < T + t$ is given by

$$R_c(T,t) \times R_w(T,t) = e^{-\lambda_c(t)} \times \frac{R_w(T + t)}{R_w(T)} \tag{2.7}$$

where

$$\frac{R_w(T + t)}{R_w(T)} = \frac{\int_{T + t}^{\infty} F_w(T') \, dT}{\int_{T}^{\infty} F_w(T') \, dT} \quad \text{where } T \geq T_2 \tag{2.8}$$

and $F_w(T')$ is the wearout distribution which is approximated by a normal distribution.

The most general reliability equation for the probability of a component part surviving early, random, and wearout failure from a time T to T + t becomes

$$R\ (T,t) = R_e\ (T,t) \times R_c\ (T,t) \times R_w(T,t) \qquad (2.9)$$

or,

$$R\ (T,t) = \exp\left[-\lambda_o E(T')\ e^{-T/E(T')}\left(1-e^{-t/E(T')}\right)\right] \times e^{-\lambda c(t)} \times \frac{\int_{T+t}^{\infty} F_w(T')\ dT}{\int_{T}^{\infty} F_w(T')\ dT} \qquad (2.10)$$

2.3 Effect of Wearout on Reliability for Extended Space Flights

In evaluating the effect of wearout on a component, the approximate wearout distribution and the mathematical properties of that distribution must be known. If the wearout distribution approximates a normal distribution for a particular component, then the mean of this distribution as well as the standard deviation must be known before the effect of wearout can be calculated.

To be able to compare the effect of the wearout failure rate with the effects of early and random failure rates for an extended space flight, $E(T')$, λ_o, λ_c, T, t, M, and σ must be known for the particular component part under investigation. As an example, let us say that for a given component $E(T') = 100$ hours,

λ_o = 10 failures/10^6 hours, T = 100 hours, λ_c = 1 failure/10^6 hours, t = 2190 hours, and the wearout distribution is normal with a mean (M) = 2400 hours and a standard deviation (σ) = 100 hours. If in a spacecraft, the given component was used in 10,000 identical applications, from T = 100 to T + t = 2290 hours, as will be seen in the following calculation, 1379 out of the 10,000 original components will fail on the average between T = 100 and T + t = 2290 hours.

As previously stated,

$$R\ (T = 100,\ t = 2190) = R_e\ (T = 100,\ t = 2190)$$

$$\times\ R_c\ (T = 100,\ t = 2190) \times R_w(T = 100,\ t = 2190)$$

$$= \exp\left[-\lambda_o E(T')\ e^{-T/E(T')}(1-e^{-t/E(T')})\right]$$

$$\times\ e^{-\lambda c(t)} \times \frac{\int_{T+t}^{\infty} F_w(T')\ dT}{\int_{T}^{\infty} F_w(T')\ dT}$$

where $R_e = \exp\left[-\lambda_o E(T')\ e^{-T/E(T')}(1-e^{-t/E(T')})\right]$

$$= 0.99632$$

$$R_c = e^{-\lambda c(t)} = 0.997810$$

$$R_w = \frac{\int_{2290}^{\infty} F_w(T')\ dT}{\int_{100}^{\infty} F_w(T')\ dT} = 0.8643 +$$

Then the probability of a component surviving from T = 100 to T = 2290 is R (T = 100, t = 2290) = 0.999632 x 0.997810 x 0.8643 = 0.8621.

The calculation of R_e (T = 100, t = 2190) and R_c (T = 100, t = 2190) are accomplished by straight substitution into equations 2.3 and 2.1 for the parameters given. The wearout reliability is the most difficult to calculate and an explanation of the method follows. From equation 2.8

$$R_w(T,t) = \frac{\int_{T+t}^{\infty} F_w(T')}{\int_{T}^{\infty} F_w(T')}$$

Assuming that the wearout failure distribution is described by a normal distribution, $F_w(T')$ is the curve of the normal distribution and is described by

$$F_w(T') = \frac{1}{\sqrt{2\pi}\,\sigma_T} \; e^{-\frac{1}{2}\frac{(T'-M)^2}{\sigma^2_T}} \qquad (2.11)$$

where $F_w(T')$ is the ordinate of the curve, π is a constant (3.1412), e is a constant (2.718), M is the mean of the distribution, σ_T is the standard deviation of the distribution, and T' is the lower limit of the integrals shown above, i.e., either T or T + t.

The calculation of the wearout reliability involves finding the fraction of area of the normal wearout distribution lying to the right of $T' = T + t = 2290$ hours in Fig. 2 and dividing that by the fraction of area of the normal distribution lying to the right of $T' = T = 100$ hours. Table II gives the areas under the normal distribution to the right of any point T' in terms of the standard deviation of the distribution. To use Table II, the distance between T' and the mean of the normal distribution M must be expressed in units of the standard deviation σ_T. This standardized distance from T' to the mean of the distribution is called Z in Table II and is equal to

$$Z = \frac{T' - M}{\sigma_T} \qquad (2.12)$$

For $T' = T = 100$ hours $Z = -23$

and $T' = T + t = 2290$ hours $Z = -1.1$

In Table II,

The area to the right of $T = 100$ hours for which $Z = -23$ is equal to 1.00 -

The area to the right of $T + t = 2290$ hours for which $Z = -1.1$ is equal to 0.8643, and

$$R_W(T = 100,\ t = 2190) = \frac{0.8643}{1.0 -} = 0.8643 +$$

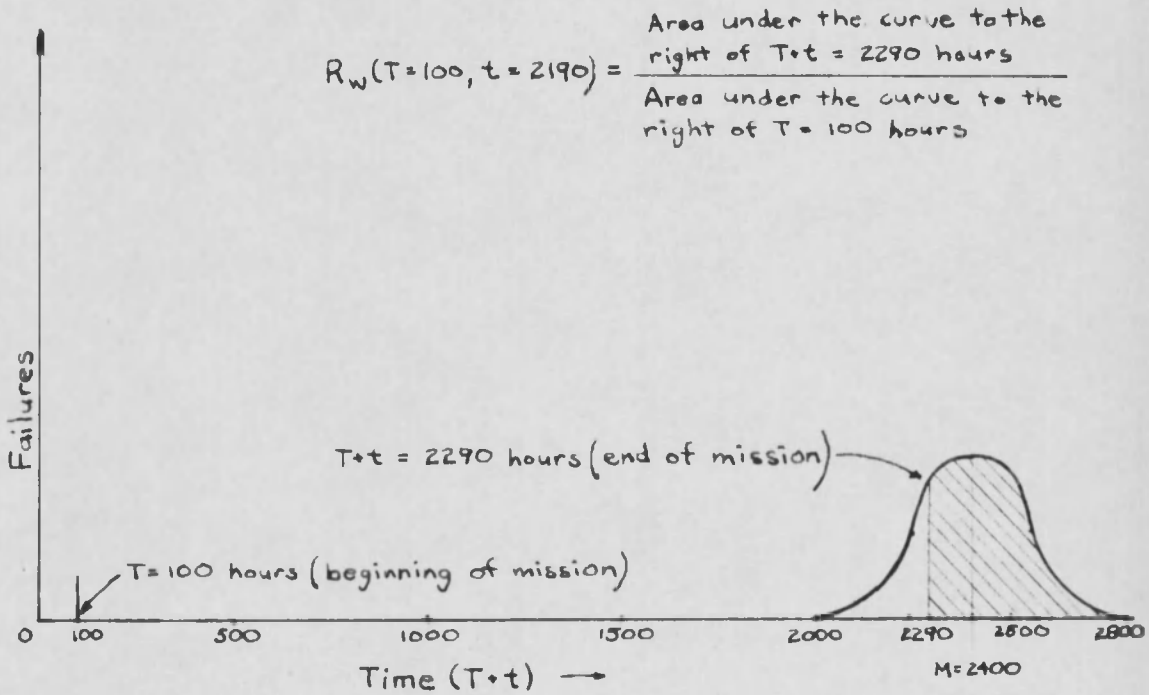

Figure 2. Normal Distribution Curve with a Mean = 2400 hours, and a Standard Deviation = 100 hours.

(Wearout Failure Distribution)

TABLE II. The Probability of No Failures from Age 0 to Age T' under the Normal Wearout Distribution Function

$$\Phi(Z) = \frac{1}{\sqrt{2\pi}} \int_{Z}^{\infty} e^{\frac{-Z^2}{2}} \, dZ \quad \text{for } -4.99 \leq Z \leq +4.99$$

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.9	$.9^{6}5208$	$.9^{6}5446$	$.9^{6}5673$	$.9^{6}5889$	$.9^{6}6094$	$.9^{6}6289$	$.9^{6}6475$	$.9^{6}6652$	$.9^{6}6821$	$.9^{6}6981$
-4.8	$.9^{6}2067$	$.9^{6}2453$	$.9^{6}2822$	$.9^{6}3173$	$.9^{6}3508$	$.9^{6}3827$	$.9^{6}4131$	$.9^{6}4420$	$.9^{6}4696$	$.9^{6}4958$
-4.7	$.9^{5}8699$	$.9^{5}8761$	$.9^{5}8821$	$.9^{5}8877$	$.9^{5}8931$	$.9^{5}8983$	$.9^{6}0320$	$.9^{6}0789$	$.9^{6}1235$	$.9^{6}1661$
-4.6	$.9^{5}7888$	$.9^{5}7987$	$.9^{5}8081$	$.9^{5}8172$	$.9^{5}8258$	$.9^{5}8340$	$.9^{5}8419$	$.9^{5}8494$	$.9^{5}8566$	$.9^{5}8634$
-4.5	$.9^{5}6602$	$.9^{5}6759$	$.9^{5}6908$	$.9^{5}7051$	$.9^{5}7187$	$.9^{5}7318$	$.9^{5}7442$	$.9^{5}7561$	$.9^{5}7675$	$.9^{5}7784$
-4.4	$.9^{5}4587$	$.9^{5}4831$	$.9^{5}5065$	$.9^{5}5288$	$.9^{5}5502$	$.9^{5}5706$	$.9^{5}5902$	$.9^{5}6089$	$.9^{5}6268$	$.9^{5}6439$
-4.3	$.9^{5}1460$	$.9^{5}1837$	$.9^{5}2199$	$.9^{5}2545$	$.9^{5}2876$	$.9^{5}3193$	$.9^{5}3497$	$.9^{5}3788$	$.9^{5}4066$	$.9^{5}4332$
-4.2	$.9^{4}8665$	$.9^{4}8723$	$.9^{4}8778$	$.9^{4}8832$	$.9^{4}8882$	$.9^{4}8931$	$.9^{4}8978$	$.9^{5}0226$	$.9^{5}0055$	$.9^{5}1066$
-4.1	$.9^{4}7934$	$.9^{4}8022$	$.9^{4}8106$	$.9^{4}8186$	$.9^{4}8263$	$.9^{4}8338$	$.9^{4}8409$	$.9^{4}8477$	$.9^{4}8542$	$.9^{4}8605$
-4.0	$.9^{4}6833$	$.9^{4}6964$	$.9^{4}7090$	$.9^{4}7211$	$.9^{4}7327$	$.9^{4}7439$	$.9^{4}7546$	$.9^{4}7649$	$.9^{4}7748$	$.9^{4}7843$
-3.9	$.9^{4}5190$	$.9^{4}5385$	$.9^{4}5573$	$.9^{4}5753$	$.9^{4}5926$	$.9^{4}6092$	$.9^{4}6253$	$.9^{4}6406$	$.9^{4}6554$	$.9^{4}6696$
-3.8	$.9^{4}2765$	$.9^{4}3052$	$.9^{4}3327$	$.9^{4}3593$	$.9^{4}3848$	$.9^{4}4094$	$.9^{4}4331$	$.9^{4}4558$	$.9^{4}4777$	$.9^{4}4988$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.7	$.9^{3}8922$	$.9^{3}8964$	$.9^{4}0039$	$.9^{4}0426$	$.9^{4}0799$	$.9^{4}1158$	$.9^{4}1504$	$.9^{4}1838$	$.9^{4}2159$	$.9^{4}2468$
-3.6	$.9^{3}8409$	$.9^{3}8469$	$.9^{3}8527$	$.9^{3}8583$	$.9^{3}8637$	$.9^{3}8689$	$.9^{3}8739$	$.9^{3}8787$	$.9^{3}8834$	$.9^{3}8879$
-3.5	$.9^{3}7674$	$.9^{3}7759$	$.9^{3}7842$	$.9^{3}7922$	$.9^{3}7999$	$.9^{3}8074$	$.9^{3}8146$	$.9^{3}8215$	$.9^{3}8282$	$.9^{3}8347$
-3.4	$.9^{3}6631$	$.9^{3}6752$	$.9^{3}6869$	$.9^{3}6982$	$.9^{3}7091$	$.9^{3}7197$	$.9^{3}7299$	$.9^{3}7398$	$.9^{3}7493$	$.9^{3}7585$
-3.3	$.9^{3}5166$	$.9^{3}5335$	$.9^{3}5499$	$.9^{3}5658$	$.9^{3}5811$	$.9^{3}5959$	$.9^{3}6103$	$.9^{3}6242$	$.9^{3}6376$	$.9^{3}6505$
-3.2	$.9^{3}3129$	$.9^{3}3363$	$.9^{3}3590$	$.9^{3}3810$	$.9^{3}4024$	$.9^{3}4230$	$.9^{3}4429$	$.9^{3}4623$	$.9^{3}4810$	$.9^{3}4991$
-3.1	$.9^{3}0324$	$.9^{3}0646$	$.9^{3}0957$	$.9^{3}1260$	$.9^{3}1553$	$.9^{3}1836$	$.9^{3}2112$	$.9^{3}2378$	$.9^{3}2636$	$.9^{3}2886$
-3.0	$.9^{2}8650$	$.9^{2}8694$	$.9^{2}8736$	$.9^{2}8777$	$.9^{2}8817$	$.9^{2}8856$	$.9^{2}8893$	$.9^{2}8930$	$.9^{2}8965$	$.9^{2}8999$
-2.9	$.9^{2}8134$	$.9^{2}8193$	$.9^{2}8250$	$.9^{2}8305$	$.9^{2}9359$	$.9^{2}8411$	$.9^{2}8462$	$.9^{2}8511$	$.9^{2}8559$	$.9^{2}8605$
-2.8	$.9^{2}7445$	$.9^{2}7523$	$.9^{2}7599$	$.9^{2}7673$	$.9^{2}7744$	$.9^{2}7814$	$.9^{2}7882$	$.9^{2}7948$	$.9^{2}8012$	$.9^{2}8074$
-2.7	$.9^{2}6533$	$.9^{2}6636$	$.9^{2}6736$	$.9^{2}6833$	$.9^{2}6928$	$.9^{2}7020$	$.9^{2}7110$	$.9^{2}7197$	$.9^{2}7282$	$.9^{2}7365$
-2.6	$.9^{2}5339$	$.9^{2}5473$	$.9^{2}5604$	$.9^{2}5731$	$.9^{2}5855$	$.9^{2}5975$	$.9^{2}6093$	$.9^{2}6207$	$.9^{2}6319$	$.9^{2}6427$
-2.5	$.9^{2}3790$	$.9^{2}3963$	$.9^{2}4132$	$.9^{2}4297$	$.9^{2}4457$	$.9^{2}4614$	$.9^{2}4766$	$.9^{2}4915$	$.9^{2}5060$	$.9^{2}5201$
-2.4	$.9^{2}1802$	$.9^{2}2024$	$.9^{2}2240$	$.9^{2}2451$	$.9^{2}2656$	$.9^{2}2857$	$.9^{2}3053$	$.9^{2}3244$	$.9^{2}3431$	$.9^{2}3612$
-2.3	.98928	.98956	.98983	$.9^{2}0097$	$.9^{2}0358$	$.9^{2}0613$	$.9^{2}0863$	$.9^{2}1106$	$.9^{2}1344$	$.9^{2}1576$
-2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
-2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
-1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
-1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
-1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
-1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
-1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
-1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
-1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
-1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.90147
-1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
-1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
- .9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
- .8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
- .7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
- .6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
- .5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
- .4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
- .3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
- .2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
- .1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
- .0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
+ .0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
+ .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
+ .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
+ .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
+ .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
+ .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2176
+ .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
+ .7	.2420	.2389	.2358	.2327	.2397	.2266	.2236	.2206	.2177	.2148
+ .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
+ .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
+1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
+1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
+1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.09853

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
+1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
+1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
+1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
+1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
+1.7	.04456	.04362	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
+1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
+1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
+2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
+2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
+2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
+2.3	.01072	.01044	.01017	$.0^{2}9903$	$.0^{2}9643$	$.0^{2}9387$	$.0^{2}9137$	$.0^{2}8894$	$.0^{2}8656$	$.0^{2}8424$
+2.4	$.0^{2}8198$	$.0^{2}7976$	$.0^{2}7760$	$.0^{2}7549$	$.0^{2}7344$	$.0^{2}7143$	$.0^{2}6947$	$.0^{2}6756$	$.0^{2}6569$	$.0^{2}6387$
+2.5	$.0^{2}6210$	$.0^{2}6037$	$.0^{2}5868$	$.0^{2}5703$	$.0^{2}5543$	$.0^{2}5386$	$.0^{2}5234$	$.0^{2}5085$	$.0^{2}4940$	$.0^{2}4799$
+2.6	$.0^{2}4661$	$.0^{2}4527$	$.0^{2}4396$	$.0^{2}4269$	$.0^{2}4145$	$.0^{2}4025$	$.0^{2}3907$	$.0^{2}3793$	$.0^{2}3681$	$.0^{2}3573$
+2.7	$.0^{2}3467$	$.0^{2}3364$	$.0^{2}3264$	$.0^{2}3167$	$.0^{2}3072$	$.0^{2}2980$	$.0^{2}2890$	$.0^{2}2803$	$.0^{2}2718$	$.0^{2}2635$
+2.8	$.0^{2}2555$	$.0^{2}2477$	$.0^{2}2401$	$.0^{2}2327$	$.0^{2}2256$	$.0^{2}2186$	$.0^{2}2118$	$.0^{2}2052$	$.0^{2}1988$	$.0^{2}1926$
+2.9	$.0^{2}1866$	$.0^{2}1807$	$.0^{2}1750$	$.0^{2}1695$	$.0^{2}1641$	$.0^{2}1589$	$.0^{2}1538$	$.0^{2}1489$	$.0^{2}1441$	$.0^{2}1395$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
+3.0	$.0^{2}1350$	$.0^{2}1306$	$.0^{2}1264$	$.0^{2}1223$	$.0^{2}1183$	$.0^{2}1144$	$.0^{2}1107$	$.0^{2}1070$	$.0^{2}1035$	$.0^{2}1001$
+3.1	$.0^{3}9676$	$.0^{3}9354$	$.0^{3}9043$	$.0^{3}8740$	$.0^{3}8447$	$.0^{3}8164$	$.0^{3}7888$	$.0^{3}7622$	$.0^{3}7364$	$.0^{3}7114$
+3.2	$.0^{3}6871$	$.0^{3}6637$	$.0^{3}6410$	$.0^{3}6190$	$.0^{3}5976$	$.0^{3}5770$	$.0^{3}5571$	$.0^{3}5377$	$.0^{3}5190$	$.0^{3}5009$
+3.3	$.0^{3}4834$	$.0^{3}4665$	$.0^{3}4501$	$.0^{3}4314$	$.0^{3}4189$	$.0^{3}4041$	$.0^{3}3897$	$.0^{3}3758$	$.0^{3}3624$	$.0^{3}3495$
+3.4	$.0^{3}3369$	$.0^{3}3248$	$.0^{3}3131$	$.0^{3}3018$	$.0^{3}2909$	$.0^{3}2803$	$.0^{3}2701$	$.0^{3}2602$	$.0^{3}2507$	$.0^{3}2415$
+3.5	$.0^{3}2326$	$.0^{3}2241$	$.0^{3}2158$	$.0^{3}2078$	$.0^{3}2001$	$.0^{3}1926$	$.0^{3}1854$	$.0^{3}1785$	$.0^{3}1718$	$.0^{3}1653$
+3.6	$.0^{3}1591$	$.0^{3}1531$	$.0^{3}1473$	$.0^{3}1417$	$.0^{3}1363$	$.0^{3}1311$	$.0^{3}1261$	$.0^{3}1213$	$.0^{3}1166$	$.0^{3}1121$
+3.7	$.0^{3}1078$	$.0^{3}1036$	$.0^{4}9961$	$.0^{4}9574$	$.0^{4}9201$	$.0^{4}8842$	$.0^{4}8496$	$.0^{4}8162$	$.0^{4}7841$	$.0^{4}7532$
+3.8	$.0^{4}7235$	$.0^{4}6948$	$.0^{4}6673$	$.0^{4}6407$	$.0^{4}6152$	$.0^{4}5906$	$.0^{4}5669$	$.0^{4}5442$	$.0^{4}5223$	$.0^{4}5012$
+3.9	$.0^{4}4810$	$.0^{4}4615$	$.0^{4}4427$	$.0^{4}4247$	$.0^{4}4074$	$.0^{4}3908$	$.0^{4}3747$	$.0^{4}3594$	$.0^{4}3446$	$.0^{4}3304$
+4.0	$.0^{4}3167$	$.0^{4}3036$	$.0^{4}2910$	$.0^{4}2789$	$.0^{4}2673$	$.0^{4}2561$	$.0^{4}2454$	$.0^{4}2351$	$.0^{4}2252$	$.0^{4}2157$
+4.1	$.0^{4}2066$	$.0^{4}1978$	$.0^{4}1894$	$.0^{4}1814$	$.0^{4}1737$	$.0^{4}1662$	$.0^{4}1591$	$.0^{4}1523$	$.0^{4}1458$	$.0^{4}1395$
+4.2	$.0^{4}1335$	$.0^{4}1277$	$.0^{4}1222$	$.0^{4}1168$	$.0^{4}1118$	$.0^{4}1069$	$.0^{4}1022$	$.0^{5}9774$	$.0^{5}9345$	$.0^{5}8934$
+4.3	$.0^{5}8540$	$.0^{5}8163$	$.0^{5}7801$	$.0^{5}7455$	$.0^{5}7124$	$.0^{5}6807$	$.0^{5}6503$	$.0^{5}6212$	$.0^{5}5934$	$.0^{5}5668$
+4.4	$.0^{5}5413$	$.0^{5}5169$	$.0^{5}4935$	$.0^{5}4712$	$.0^{5}4498$	$.0^{5}4294$	$.0^{5}4098$	$.0^{5}3911$	$.0^{5}3732$	$.0^{5}3561$
+4.5	$.0^{5}3398$	$.0^{5}3241$	$.0^{5}3092$	$.0^{5}2949$	$.0^{5}2813$	$.0^{5}2682$	$.0^{5}2558$	$.0^{5}2439$	$.0^{5}2325$	$.0^{5}2216$
+4.6	$.0^{5}2112$	$.0^{5}2013$	$.0^{5}1919$	$.0^{5}1828$	$.0^{5}1742$	$.0^{5}1660$	$.0^{5}1581$	$.0^{5}1506$	$.0^{5}1434$	$.0^{5}1366$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
+4.7	$.0^{5}1301$	$.0^{5}1239$	$.0^{5}1179$	$.0^{5}1123$	$.0^{5}1069$	$.0^{5}1016$	$.0^{6}9680$	$.0^{6}9211$	$.0^{6}8765$	$.0^{6}8339$
+4.8	$.0^{6}7933$	$.0^{6}7547$	$.0^{6}7178$	$.0^{6}6827$	$.0^{6}6492$	$.0^{6}6173$	$.0^{6}5869$	$.0^{6}5580$	$.0^{6}5304$	$.0^{6}5042$
+4.9	$.0^{6}4792$	$.0^{6}4554$	$.0^{6}4327$	$.0^{6}4111$	$.0^{6}3906$	$.0^{6}3711$	$.0^{6}3525$	$.0^{6}3348$	$.0^{6}3179$	$.0^{6}3019$

2.3.1 Determining the Number of Components Needed in the Original Population

To find the number of components that would have to be debugged from time = 0 to have 10,000 remaining at T = 100 hours (the beginning of the mission), the following equation must be solved.

Original population

$$= \frac{10{,}000}{R_E(T=0,\ t=100)} \tag{2.13}$$

$$= \frac{10{,}000}{\exp\left[-\lambda_o E(T')_e^{-T/E(T')}(1-e^{-t/E(T')})\right]}$$

= 10,006.3 or 10,007 components.

2.3.2 Determining the Effect of Early, Chance, and Wearout Reliabilities on System Reliability for Different Mission Times

In Table III are given the early, chance, and wearout reliabilities for T = 100 hours, t = 100, 200, 300, ..., 3000 hours and the product of early, chance, and wearout reliabilities for the same time intervals. The results of Table III are plotted as components surviving versus mission time in Fig. 3.

TABLE III

Early, Chance, and Wearout Reliabilities for T = 100 hours, t = 100, 200, 300, --- 3000 hours and the Product of Early, Chance, and Wearout Reliabilities for the Same Time Intervals. $R_{ss}(T,t) = R_e(T,t) \times R_c(T,t) \times R_w(T,t)$

λ_o = 10 failures/10^6 hours λ_c = 1 failure/10^6 hours

E(T') = 100 hours M = 2400 hours

T = 100 hours σ = 100 hours

t(hours)	$R_e(T,t)$	$R_c(t,t)$	$R_w(T,t)$	$R_{ss}(T,t)$
0	1.000000	1.0000	1.000000	1.0000
100	.999767	.9999	1.000000	.9997
200	.999682	.9998	1.000000	.9995
300	.999650	.9997	1.000000	.9994
400	.999639	.9996	1.000000	.9992
500	.999635	.9995	1.000000	.9991
600	.999632	.9994	1.000000	.9990
700	.999632	.9993	1.000000	.9989
800	.999632	.9992	1.000000	.9988
900	.999632	.9991	1.000000	.9987
1000	.999632	.9990	1.000000	.9986
1100	.999632	.9989	1.000000	.9985
1200	.999632	.9988	1.000000	.9984
1300	.999632	.9987	1.000000	.9983
1400	.999632	.9986	1.000000	.9982
1500	.999632	.9985	1.000000	.9981
1600	.999632	.9984	1.000000	.9980
1700	.999632	.9983	1.000000	.9979
1800	.999632	.9982	1.000000	.9978
1900	.999632	.9981	.999968	.9977
2000	.999632	.9980	.99865	.9976
2100	.999632	.9979	.9772	.9762
2190	.999632	.997810	.8643	.8621
2200	.999632	.9978	.8413	.8391
2300	.999632	.9977	.5000	.4987
2400	.999632	.9976	.1587	.1583
2500	.999632	.9975	.0228	.0227
2600	.999632	.9974	.00135	.0013
2700	.999632	.9973	.00003	.00003
2800	.999632	.9972	.00000	.00000
2900	.999632	.9971	.000000	.00000
3000	.999632	.9970	.000000	.00000

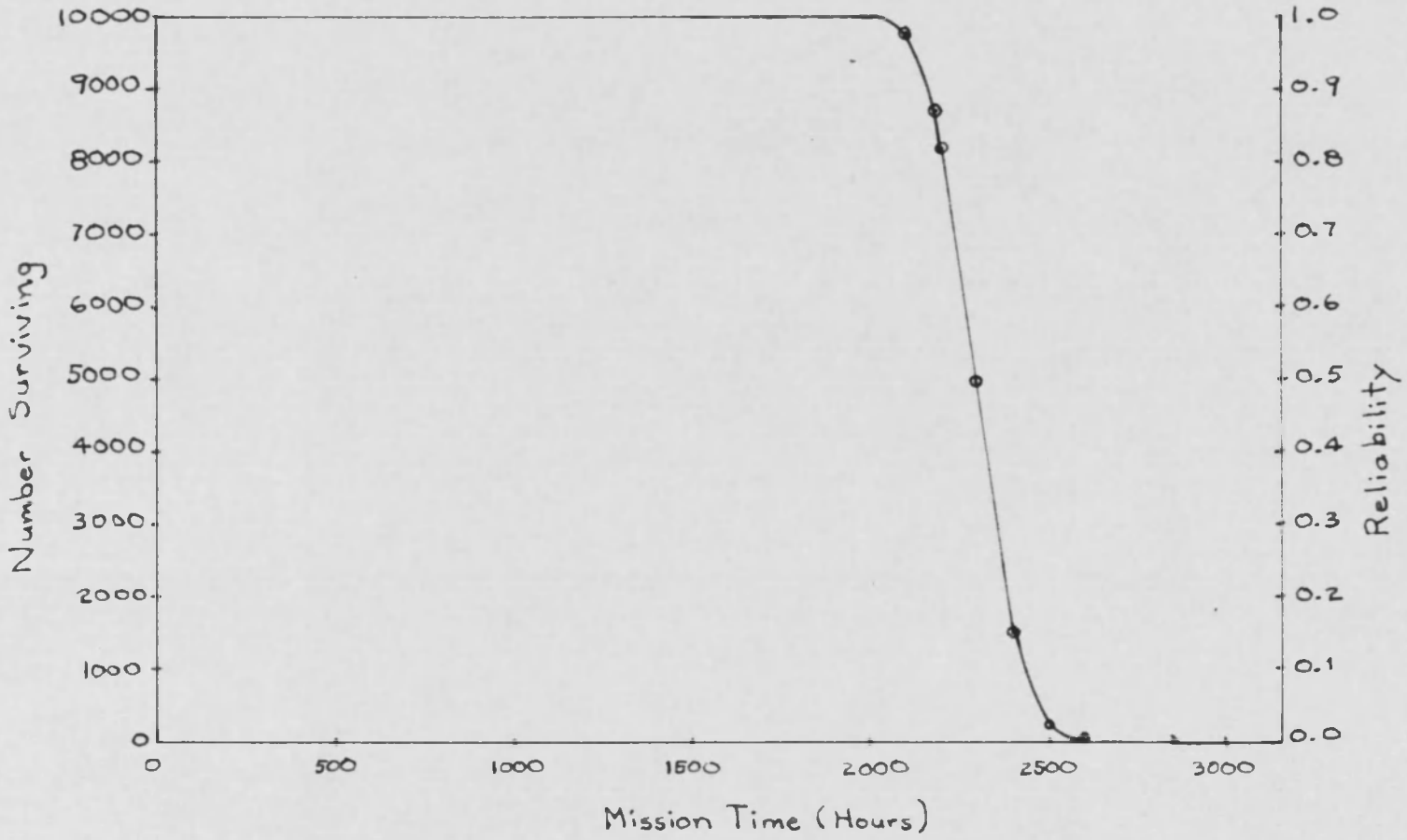

Figure 3. Components Surviving Versus Mission Time for Problem Outlined in Section 2.3

2.4 Standard Wearout Definition

In establishing a standard wearout definition, a clear distinction must be made between the first, second, third, and fourth wearout failure frequency distributions. The distribution curves for the first, second, and third, etc. failures for chance failures are identical as can be seen in Fig. 4a. The chance failure distribution curves should be compared with the wearout failure distribution curves for first, second, third, etc., which can be found in Fig. 4b. In Fig. 4b it can be seen that the distribution curve of first wearout failures is nearly normal while the second wearout failures are distributed according to one-half of the normal distribution curve. The distributions curves of third and subsequent failures look more and more like the exponential curve for chance distribution as can be seen in Fig. 4c. The similarity of those two distribution curves indicates that higher order wearout failures occur almost "by chance." (12, p. 10). As can be seen in the bathtub curve for first order wearout failures in Fig. 1, the failure rate curve begins a marked increase as wearout is entered. If the worn out parts are replaced as they fail, the failure rate curve begins to level off in a mixed zone, and then flattens out for the higher order failures. See Fig. 4d. This gives us three zones for wearout failures which are repaired. The

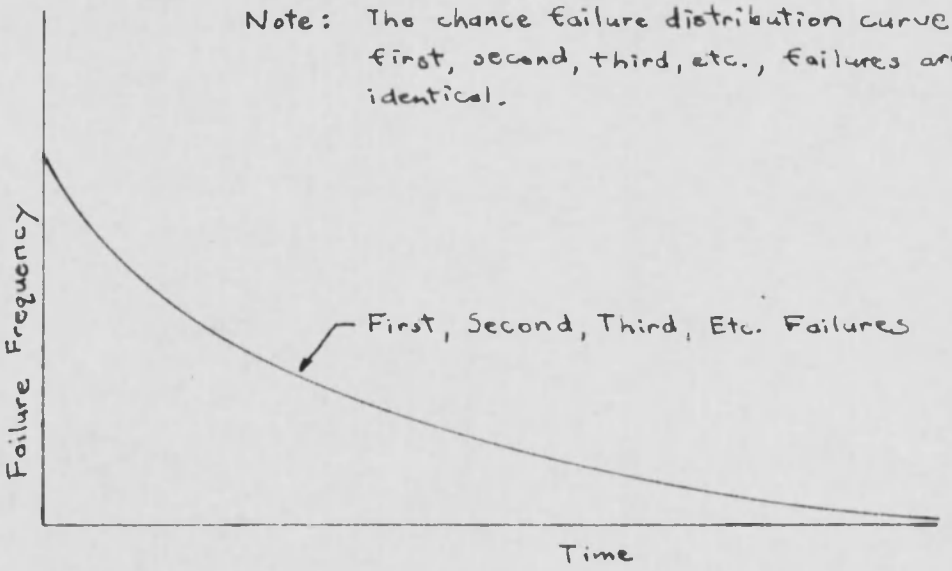

Figure 4(a). Chance Failure Distribution for First, Second, Third, etc., Failures (12, p 10)

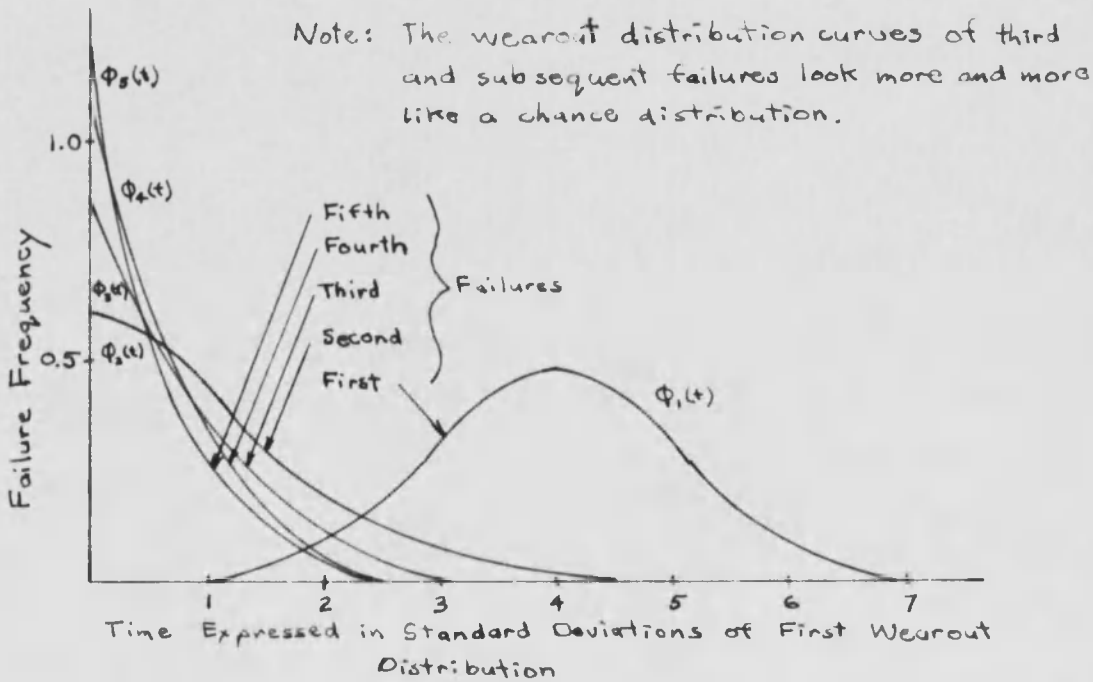

Figure 4(b). Wearout Failure Distribution for First, Second, Third, etc., Failures (12, p11)

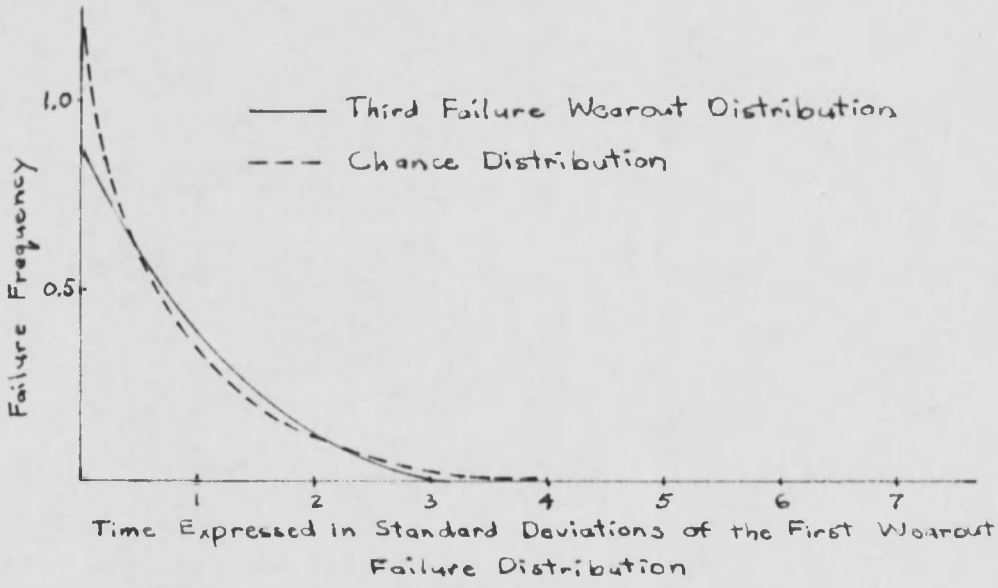

Figure 4 (c). Comparison of Third Wearout Distribution and Chance Distribution (12, p11)

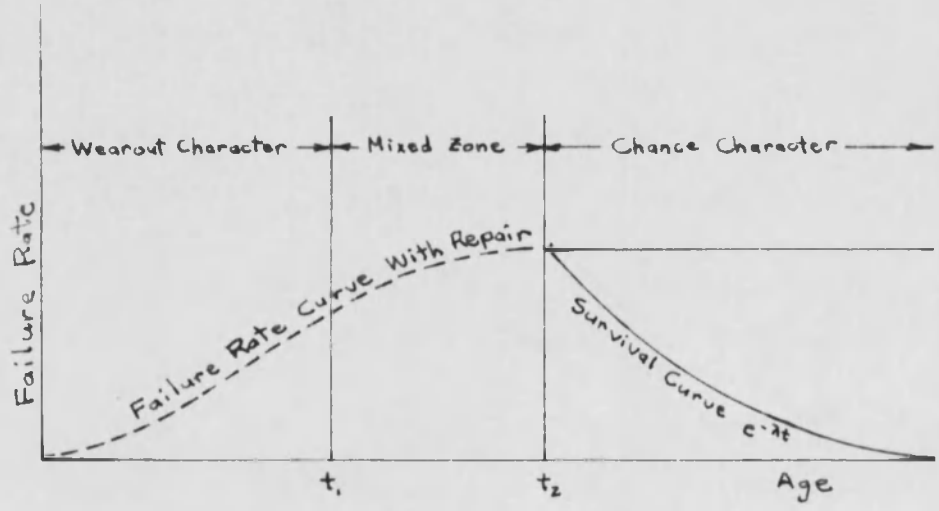

Figure 4 (d). Wearout Distribution, Mixed Distribution, and Chance Distribution

failure rate in zone one is of first order and has a wearout character. The failure rate in the second zone between T_1 and T_2 has a mixed wearout and chance characteristic because of the injection of second and third order distributions, and the failure rate in the third zone is of the higher order failures and has a chance character.

2.5 Risk Levels for Manned and Unmanned Flights

In an unmanned flight, the component part cannot be replaced before it fails, because in both sequential and functional redundancy the transfer of load is not made until after the component part fails. This means that when a mission time is specified for a component part in an unmanned flight, that part is required to operate for the full mission time, and the reliability for its operating the mission time can be calculated using equation 2.10. Let us say for the component discussed in Section 2.3 a target reliability had been set at 0.995 for an unmanned flight. The risk level is defined as being equal to 1 - target reliability. And in this example, we will risk or tolerate 5 component failures per 1000 components operated for 2190 hours each. This is the same as allowing 5 failures per 2.19×10^6 hours of component operation. As was calculated in Section 2.3 and shown in Table III, the component reliability for t = 2190 hours was 0.8621.

Assuming for the moment that the parameters for the component in Section 2.3, i.e., $E(T')$, M, and σ_T were fixed, and T was established by the vendor, the mission time (t) is the only variable. This means that there are only three methods of raising the mission reliability from 0.8621 to the target reliability of 0.995 for an unmanned flight.

1. The component can be returned to the manufacturer with a statement that the reliability must be increased to 0.995.

2. If the component reliability must remain fixed at 0.8621 then redundancy techniques discussed in Chapter 3 can be used to increase the reliability of having a functioning component, for a mission t, to 0.995.

3. The third possibility for increasing the component reliability would be to shorten the mission time t, that the component was required to operate. If the mission time for this component part could be shortened to 2000 hours, it can be seen from the reliability scale in Fig. 3 and from the tabulation for t = 2000 in Table III that the reliability for the new mission time would be 0.9976.

In a manned spacecraft flight, there are many avenues available for increasing component part reliability in addition to the unmanned techniques. As an example, if we let the component operate for 2000 hours and then replace it with a new component which is required to operate from 2000 hours to 2190 hours, then the probability of the system operating until t is given by

$$R(T=100,\ t=2190) = R(T=100,\ t=2000)$$

$$\times\ R(T=100,\ t=\ 190) = 0.9976 \times 0.9996 = 0.9975$$

The second part or the replacement part, which was installed new at T = 100 hours in its life will be required to operate only 190 hours or until the end of the mission. Then while the overall mission is 2190 hours, the mission time for the second component is only 190 hours. If i components will be used during the mission (the original plus i-1 replacements), then in general the probability of a component (with replacement) lasting from T = 100 to T + t = 2290 is

$$R(T,t) = R(T,t_1) \times R(T,t_2) \times R(T,t_3) \times \ldots R(T,t_i) \qquad (2.14)$$

where $t = \sum_{i=1}^{i} t_i$

$$R(T=100,\ t=2190) = R(T=100,t_1) \times R(T=100,t_2) \times R(T=100,t_3)$$

$$\times \circ \circ \circ \times R(T=100,t_i)$$

$$\text{where } t = 2190 = \sum_{1}^{i} t_i$$

The component reliability for a particular value of t_i can be found in Table III for the component part parameters given in Section 2.3. In a normal application, the mission times t_1 to t_i would be equal and the general equation for i-1 replacements at equal time intervals becomes

$$R(T,t) = R(T,t_1 = \text{typical time interval})^{\text{i power}}$$

In Chapters 3 and 4, components with mission reliabilities established by equation 2.10 are arranged in various redundant configurations to raise the component reliability to a desirable level. In Chapter 5, components with mission reliabilities established by equation 2.1 are preventively replaced every T_o hours to prevent the components from entering wearout. Replacement before wearout assures a high component reliability dependent only upon the chance failure rate given by equation 2.1.

CHAPTER 3

MATH MODELS FOR SYSTEM RELIABILITY INCORPORATING SEQUENTIAL AND FUNCTIONAL REDUNDANCY

3.1 Definition of Sequential Redundancy and Functional Redundancy

In an unmanned space flight, reliability may be increased only through the use of functional or sequential redundancy. The degree of reliability improvement depends upon the arrangement of the spares. When additional components are coupled through a switching device, so that they do not come into play until the original one fails, we have sequential or standby redundancy. With functional or full-time redundancy, all items operate continuously with (ideally) each one capable of carrying the load (17, pp. 43-45). In functional redundancy, since the items are in parallel, the probability of all items failing is much less than the probability of failure of one item used alone.

3.2 Functional Redundancy

The calculations for functionally redundant circuits are not very complex unless various modes of

component operation must be considered. For an example, a 24-part functionally redundant resistor circuit would require 3^{24} or about 300 billion combinations to evaluate the possible circuit output. The three comes from the three possible modes of a resistor can be in: normal, failed open, failed short (17, pp. 43-45).

The system reliability obtained by using functional redundancy is illustrated by equations 3.1 to 3.5 (21, pp. 4-9).

For a system with "K" parallel sub-systems, where each sub-system has "n" items in series (assuming independent failures) the reliability for a mission between T and T + t is given by:

$$R_S(T,t) = 1 - \Big[1 - R_{11}(T,t) \cdot R_{12}(T,t) \cdots R_{1n}(T,t)\Big]$$

(Series Sub-system 1)

$$\Big[1 - R_{21}(T,t) \cdot R_{22}(T,t) \cdots R_{2n}(T,t)\Big]$$

(Series Sub-system 2)

$$\vdots \qquad \vdots \qquad \vdots$$

$$\Big[1 - R_{K1}(T,t) \cdot R_{K2}(T,t) \cdots R_{Kn}(T,t)\Big] \qquad (3.1)$$

(Series Sub-system K)

If the failures are independent and the reliability of all items are equal, the system reliability for "K" parallel sub-systems where each sub-system has "n" items in series is given by:

$$R_S(T,t) = 1 - \left[1 - \left(R(T,t)\right)^{n}\right]^{K} \qquad (3.2)$$

If the failures are independent, the system reliability for "K" parallel sub-systems where each sub-system has "n" items in parallel is given by:

(Parallel Sub-system 1)

$$R_S(T,t) = 1 - \left[\left(1 - R_{11}(T,t)\right)\left(1-R_{12}(T,t)\right) \ldots \left(1-R_{1n}(T,t)\right)\right]$$

(Parallel Sub-system 2)

$$\left[\left(1-R_{21}(T,t)\right)\left(1-R_{22}(T,t)\right) \ldots \left(1-R_{2n}(T,t)\right)\right]$$

$$\begin{array}{cccccc} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{array}$$

(Parallel Sub-system "K")

$$\left[\left(1-R_{K1}(T,t)\right)\left(1-R_{K2}(T,t)\right) \ldots \left(1-R_{Kn}(T,t)\right)\right] \qquad (3.3)$$

It can be seen readily that the preceding system composed of parallel sub-systems composed of parallel components is equivalent to a system where all the components are in parallel. Then equation 3.3, the system reliability for "K" parallel sub-systems where each sub-system has "n" items in parallel, becomes

$$R_S(T,t) = 1 - \left[\left(1 - R(T,t)\right)^n\right]^K \qquad (3.4)$$

where the reliabilities of all items are equal and the failures are independent.

If the "n" parallel sub-systems each of "K" items are now connected in series (with independent failures), the system reliability becomes:

$$R_S(T,t) = 1 - \left[1 - R_{11}(T,t)\right] \cdot \left[1-R_{21}(T,t)\right] \cdots \left[1-R_{K1}(T,t)\right]$$
$$\cdot\ 1 - \left[1 - R_{12}(T,t)\right] \cdot \left[1-R_{22}(T,t)\right] \cdots \left[1-R_{K2}(T,t)\right]$$
$$\vdots \qquad\qquad \vdots \qquad\qquad\qquad \vdots$$
$$\cdot\ 1 - \left[1 - R_{1n}(T,t)\right] \cdot \left[1-R_{2n}(T,t)\right] \cdots \left[1-R_{Kn}(T,t)\right] \qquad (3.5)$$

If the reliabilities of all the items in equation 3.5 are equal, equation 3.5 becomes:

$$R_S(T,t) = \left[1 - \left(1 - R(T,t)\right)^k\right]^n \qquad (3.6)$$

To evaluate $R_S(T,t)$ in equations 3.1 to 3.6, the reliability $R_{Kn}(T,t)$ of each of the components in the system must be calculated using equation 2.10. In the systems where the components are all <u>equal</u> (equations 3.2 and 3.6), only one calculation must be made, using the given component parameters. If for a series - system of "3" parallel sub-systems each of "3" items, the component parameters from 2.3 were chosen, equation 3.6 becomes (see Fig. 5(a))

$$R_S(T=100,\ t=2190) = \left\{1 - \left[1 - R(T=100,\ t=2190)\right]^3\right\}^3$$

where $R(T=100,\ t=2190) = .8621$

and $R_S(T=100,\ t=2190) = \left\{1 - \left[1 - 0.8621\right]^3\right\}^3 = 0.9901$

0.9901 is considerably higher than the reliability of a system composed of 3 series sub-systems where each sub-system has only one component without any other components in parallel. See Fig. 5(b). The system reliability for 3 components in series with the same component parameters is equal to

$$R_S(T=100,\ t=2190) = (0.8621)^3 = 0.6407$$

If the sub-systems in Fig. 5(a) are arranged in parallel instead of series, the system reliability becomes

$$R_S(T=100,\ t=2190) = 1 - \left\{\left[1 - R(T=100,\ t=2190)\right]^3\right\}^3$$

$$= 1 - \left[0.1379\right]^9 = 0.99999998$$

For the systems with unequal components, $R_{Kn}(T=100,\ t=2190)$ would have to be calculated for each component. Since the components are unequal, it should be noted that while a mission time of 2190 hours may push a great many of the components into wearout, in the same system there may also be components which have not entered wearout at t = 2190 hours. $R(T=100,\ t=2190)$ for components

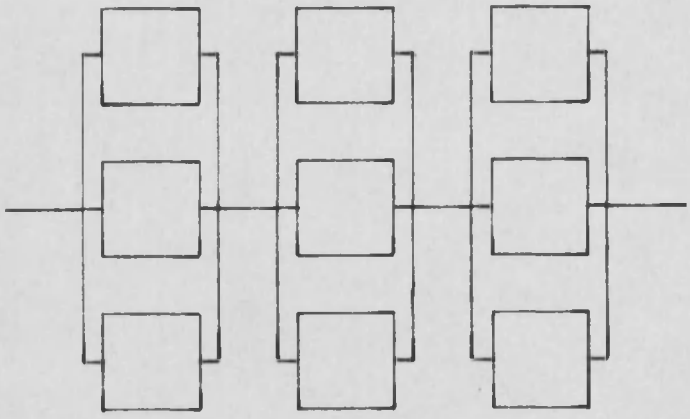

Figure 5(a). Series-system of "3" Parallel Sub-systems each of "3" components

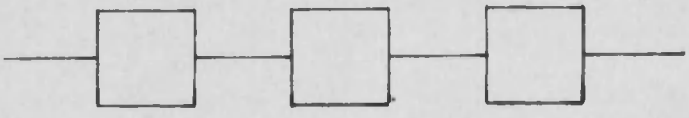

Figure 5(b). Series-system of "3" components

not entering wearout can be calculated using equation 2.6.

The effects on reliability of parallel redundancy considering X operative elements for various values of λt are given in Fig. 6. The effects on reliability of parallel series redundancy considering two complete systems cut into n equal sub-systems both systems under power throughout mission (operative redundancy) for various values of λt are given in Fig. 7. In Figs. 6 and 7, it can be seen that the probability of surviving chance failures for a given value of λt increases with the number of components in parallel. Fig. 6 is a special case of equation 3.6 where n=1 and "K" takes on values from 1 to 5. Fig. 7 is a special case of equation 3.6 where K=2 and n takes on values from 1 to 32.

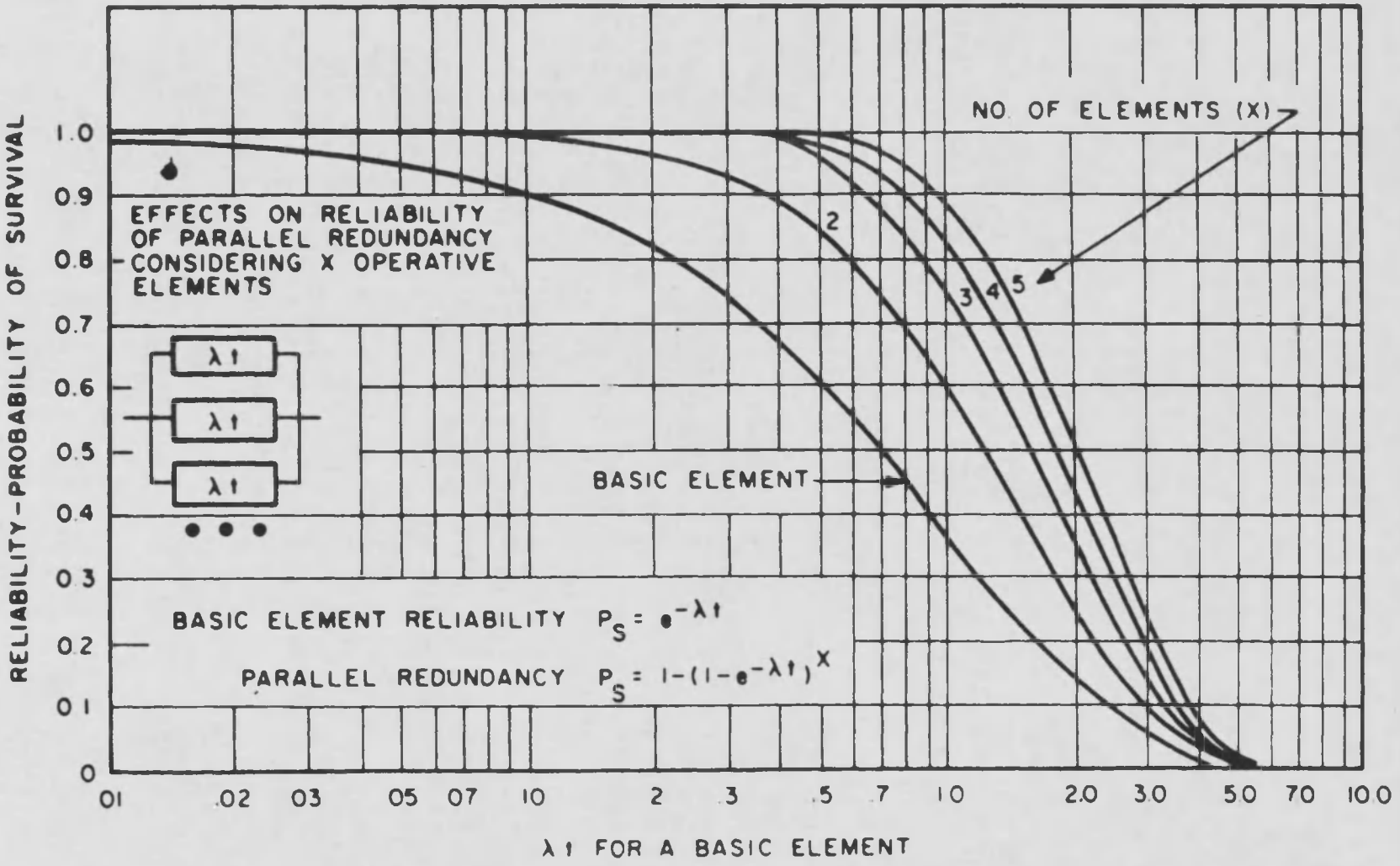

Figure 6. Effects on Reliability of Parallel Redundancy Considering x Operative Elements (16, p.25)

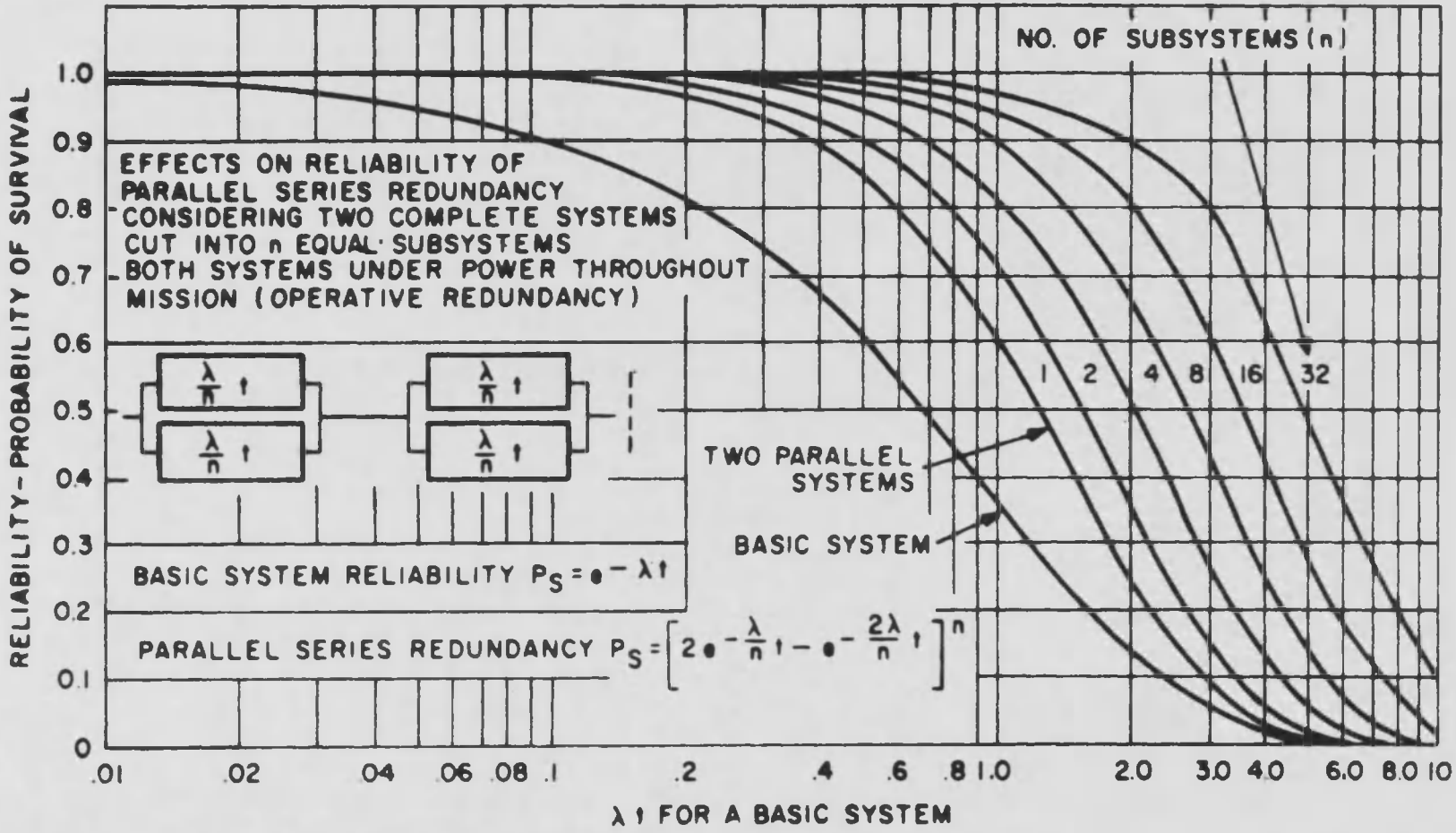

Figure 7. Effects on Reliability of Parallel Series Redundancy using Two Systems with Both Systems under Power throughout Mission (Operative Redundancy) (16, p. 27)

3.3 Sequential Redundancy Exponential Case

In sequential or standby redundancy, the MTBF is determined by the sum of the MTBF of the initial element and the MTBF's of the standby elements. The system reliability obtained by using sequential redundancy where the items are failing exponentially is illustrated by equations 3.13 to 3.17 (21, pp. 4-9).

The MTBF of a sequentially redundant system of "n" items with perfect sensing-switching is given by:

$$M_S = M_1 + M_2 + \ldots + M_i + \ldots + M_n \qquad (3.13)$$

When the "n" items of equation 3.13 are identical, the system MTBF becomes:

$$M_S = nM \qquad (3.14)$$

The reliability of the standby redundant system composed of two identical items, one operating and one in standby (assuming perfect sensing-switching) is given by:

$$R_S(T,t) = (1 + \lambda t)\, e^{-\lambda t} \qquad (3.15)$$

and, the reliability of the standby redundant system composed of n identical items, one operating and n-1 in standby (assuming perfect sensing-switching) is given by:

$$R_S(T,t) = e^{-\lambda t}\left(1 + \lambda t + \frac{\lambda^2 t^2}{2!} + \ldots \frac{\lambda^n t^n}{n!}\right) \qquad (3.16)$$

If the reliability of the sensing and switching device is equal to R_{Sw}, equation 3.15 becomes:

$$R_S(T,t) = (1 + \lambda t \hat{R}_{Sw}) e^{-\lambda t} \qquad (3.17)$$

For standby redundancy, "n" identical items, and the reliability of sensing-switching device = R_{Sw}, equation 3.16 becomes:

$$R_S(T,t) = e^{-\lambda t} \left(1+R_{Sw}\lambda t + \frac{R^2_{Sw}\lambda^2 t^2}{2!} + \ldots + \frac{R^n_{Sw}\lambda^n t^n}{n!}\right) \qquad (3.18)$$

The effects on reliability of parallel redundancy considering X standby elements for various values of λ t are given in Fig. 8. The effects on reliability of parallel series redundancy considering two complete systems cut into n equal sub-systems for various values of λ t are given in Fig. 9.

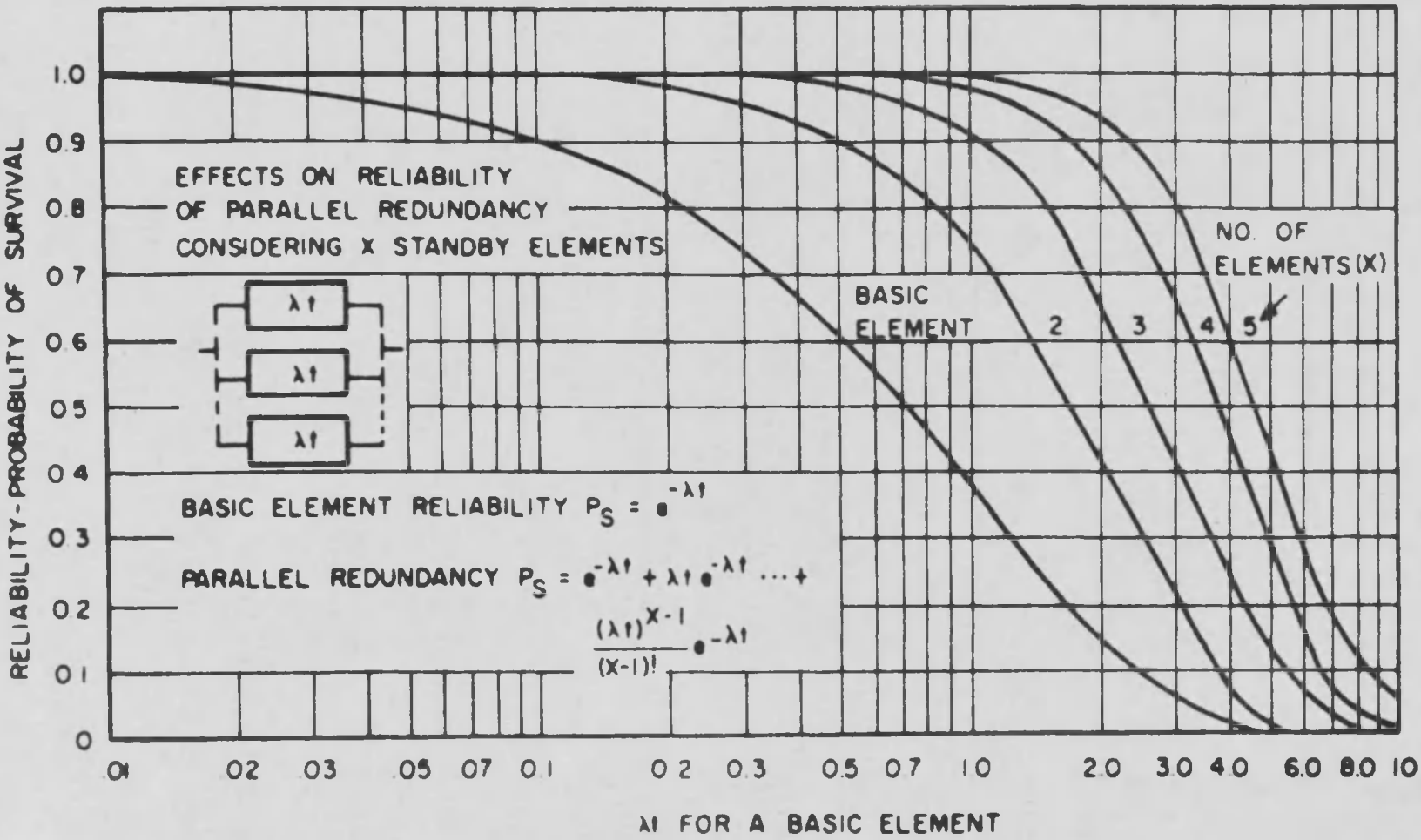

Figure 8. Effects on Reliability of Standby Redundancy Considering x Standby Elements (16, p. 26).

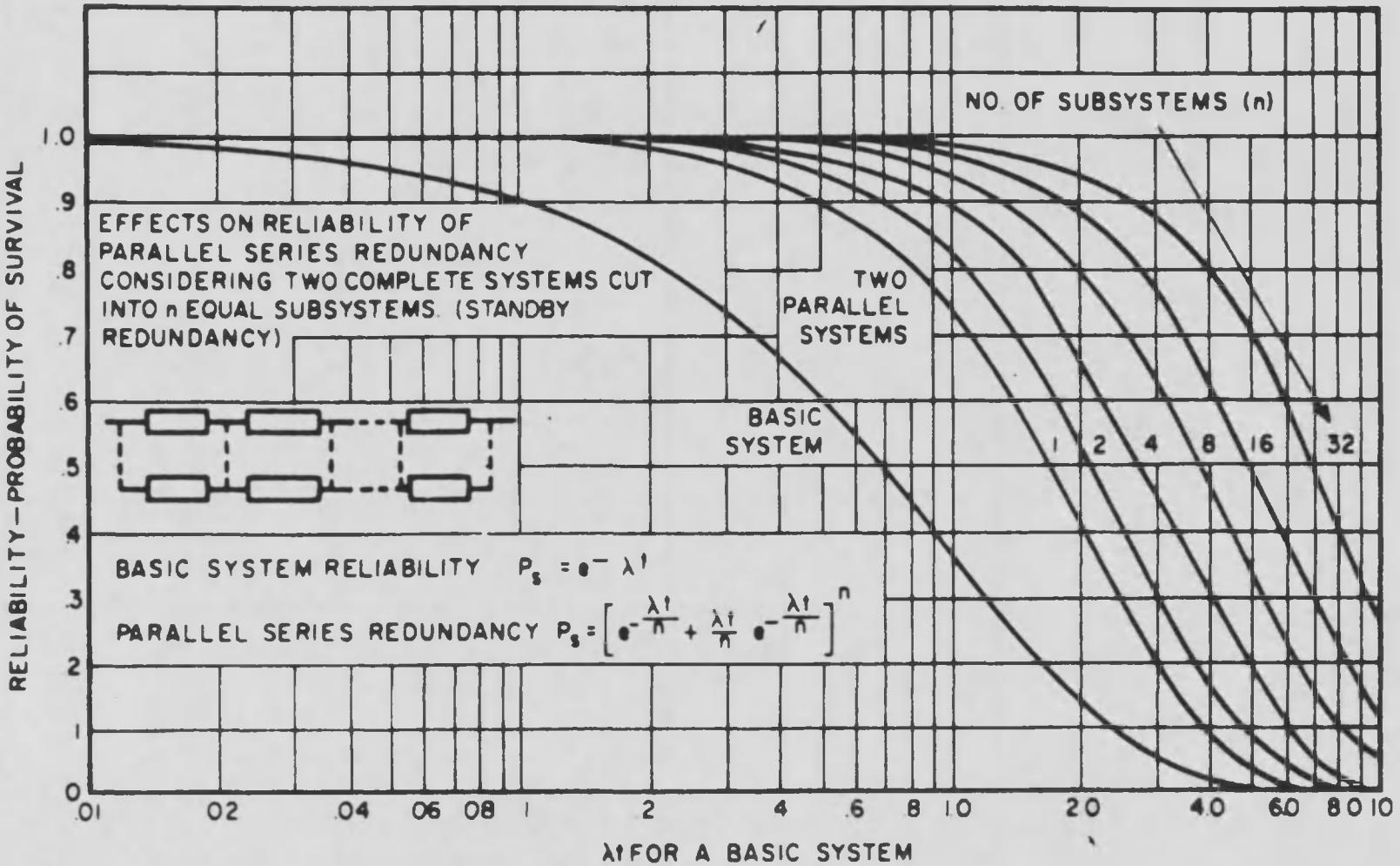

Figure 9. Effects on Reliability of n Systems in Series with each System Composed of a Functioning Unit with a Failure Rate of $\frac{\lambda}{n}$ and an Equal Standby Unit (16, p. 28).

If $E(T')$ for the component outlined in Section 2.3 is changed to 20 hours and the rest of the parameters are left unchanged, the ratio of $\frac{T}{E(T')} = 5$ and we can assume the reliability of surviving early failures from T=100 to T=2290 hours is equal to one.

If we compare the reliability obtained by using functional and sequential redundancy for the horizontal portions of the failure rate curve for the given component, i.e., from T=100 to T + t = 2100 hours, for two component parts, we find that equation 3.2 for n=1 and K=2 will give us a functional system reliability of:

$$R_{S}(T,t)_{\text{Funct.}} = 1 - (1-R_c(t))^2 = 1 - (1-e^{-0.002})^2$$

$$= 0.9^5\ 60079906734$$

Fig. 6 shows that for a λt value of 0.002 the system reliability for 2 parallel operative elements is approximately one which agrees with the results obtained by using equation 3.2, and equation 3.15 will give us a sequential system reliability of

$$R_S(T,t) = (1 + \lambda t)\ e^{-\lambda t} = (1 + 0.002)\ e^{-.002}$$

$$= 0.9^5800266466773.$$

Fig. 8 shows that for a λt value of 0.002 the system reliability for one standby element is

approximately one which agrees with equation 3.15.

The sequential reliability for n parts, n-1 of which are in standby (assuming perfect sensing and switching) will always be higher than the functional reliability for n operative parts in parallel for $n>1$. For $n=1$, the reliabilities will be equal. Or, in general,

$$1-(1-e^{-x})^n \leq e^{-x}\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^{n-1}}{(n-1)!}\right) \quad \text{for } n \geq 1, \; X \geq 0 \tag{3.19}$$

where $x = \lambda t$.

The inequality of equation 3.19 can be proved for specific ranges of x and n but a rigorous solution to the general inequality could not be found. Table 4 illustrates that the inequality does hold for large and small values of x for n=1, 2, 3 and 4. The input data in Table 4 was accurate to seventeen places for $x = .0001$, $x = 0.001$, $x = .01$, and $x = .1$, and was accurate to five places for $x = 1$.

TABLE 4. ANALYSIS OF THE INEQUALITY

$$1 - (1-e^{-x})^n \leq e^{-x}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^{n-1}}{(n-1)!}\right)$$ for

values of n = 1, 2, 3, and 4 for the range of values x = 0.0001, 0.001, 0.01, 0.1, and 1

Where the right member is the sequential reliability for n parts, n-1 of which are in standby (assuming perfect sensing and switching) and the left member is the functional reliability for n operative parts in parallel.

Number of Components (n)	Values of λt (x)	Functional Reliability	Sequential Reliability
1	0.0001	$0.9^{4}0000499983$	$0.9^{4}0000499983$
2	0.0001	$0.9^{8}0000999941$	$0.9^{8}500032998$
3	0.0001	$0.9^{12}0001499874$	$0.9^{12}830007949$
4	0.0001	$0.9^{16}0001999782$	$0.9^{16}9094985$
1	0.001	$0.9^{3}00049983337$	$0.9^{3}00049983337$
2	0.001	$0.9^{6}0009994700$	$0.9^{6}5002998$
3	0.001	$0.9^{9}0014987508$	$0.9^{9}80004990$
4	0.001	$0.9^{12}001997835165$	$0.9^{12}1839829668$

TABLE 4
(Contd)

1	0.01	$0.9^{2}004983374917$	$0.9^{2}004983374917$
2	0.01	$0.9^{4}009941915775$	$0.9^{4}50332086662$
3	0.01	$0.9^{6}01487574641$	$0.9^{6}834578349158$
4	0.01	$0.9^{8}01978498989$	$0.9^{8}916671294611$
1	0.1	0.90483741803596	0.90483741803596
2	0.1	$0.9^{2}094408299471$	$0.9^{2}532115980$
3	0.1	$0.9^{3}13821555578$	$0.9^{3}845346890$
4	0.1	$0.9^{4}179903719697$	$0.9^{5}6183287581$
1	1	0.36788	0.36788
2	1	0.60042	0.73576
3	1	0.74742	0.91970
4	1	0.84034	0.98101

3.3.1 Sequential Redundancy, Unequal Items, Exponential Case

Standby redundancy, two unequal items, perfect sensing-switching

$$R_S(T,t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_2-\lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \tag{3.20}$$

Standby redundancy, three unequal items, perfect sensing-switching

$$R_S(T,t) = \frac{\lambda_2\lambda_3\, e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)} + \frac{\lambda_1\lambda_3\, e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)} + \frac{\lambda_1\lambda_2\, e^{-\lambda_3 t}}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} \tag{3.21}$$

Standby redundancy, four unequal items, perfect sensing-switching

$$R_S(T,t) = \frac{\lambda_2\lambda_3\lambda_4\, e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)} + \frac{\lambda_1\lambda_3\lambda_4\, e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)(\lambda_4-\lambda_2)} + \frac{\lambda_1\lambda_2\lambda_4\, e^{-\lambda_3 t}}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)(\lambda_4-\lambda_3)} + \frac{\lambda_1\lambda_2\lambda_3\, e^{-\lambda_4 t}}{(\lambda_1-\lambda_4)(\lambda_2-\lambda_4)(\lambda_3-\lambda_4)} \tag{3.22}$$

Standby redundancy, n unequal items (where one is operating and n-1 are standing by) perfect sensing-switching.

$$R_S(T,t) = \frac{\lambda_2\lambda_3\lambda_4 \ldots \lambda_n e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_4-\lambda_1)\ldots(\lambda_n-\lambda_1)}$$

$$+ \frac{\lambda_1\lambda_3\lambda_4 \ldots \lambda_n e^{-\lambda_2 t}}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)(\lambda_4-\lambda_2)\ldots(\lambda_n-\lambda_2)}$$

$$+ \ldots + \frac{\lambda_1\lambda_2 \ldots \lambda_{i-1} \lambda_{i+1} \ldots \lambda_n e^{-\lambda_i t}}{(\lambda_1-\lambda_i)(\lambda_2-\lambda_i)\ldots(\lambda_{i-1}-\lambda_i)(\lambda_{i+1}-\lambda_i)\ldots(\lambda_n-\lambda_i)}$$

$$+ \ldots + \frac{\lambda_1\lambda_2\lambda_3 \ldots \lambda_{n-1} e^{-\lambda_n t}}{(\lambda_1-\lambda_n)(\lambda_2-\lambda_n)\ldots(\lambda_{n-1}-\lambda_n)} \qquad (3.23)$$

The calculation of the system reliability for a sequentially redundant system where there is one operating component and any number of equal or unequal standby components for the exponential case is accomplished by direct substitution of the given components chance failure rates (λ), into equations 3.15 to 3.24.

In the case where there is more than one operating component and n standby components, the possible success modes become quite involved and the evaluation of the system reliability for even the exponential case is complex. Dr. Dimitri Kececioglu has devised a method for calculating the reliability of such a system which is covered in paragraph 4.1. Such a system which involves both chance and wearout is discussed in paragraph 4.2.

3.3.2 Calculation of R_{Sw} for Manual Switching in a Manned Flight.

In sequential redundancy, if the sensing-switching reliability R_{Sw} is dependent upon the astronaut's ability to locate the failure and make a manual override, R_{Sw} becomes:

$$R_{Sw} = 1.0 \times K_A \times K_B \times K_C \times K_D \times K_E \times K_F$$

where K_A, K_B, K_C, K_D, K_E, and K_F are equal to or less than one, K_A is dependent upon the type of component part being replaced.

K_B is dependent upon the location of the component part being replaced.

K_C is dependent upon the inherent abilities of the particular astronaut being considered.

K_D is dependent upon the environmental conditions to which the astronaut is being subjected at the time the component part replacement must be made.

K_E is a factor evaluating the deterioration of the astronaut's brain due to prolonged exposure to radiation.

K_F is dependent upon the psychological condition of the astronaut.

K_A, and K_B can be determined from time and motion studies of the particular astronaut in question. Some work in evaluating K_C and K_D was done for Project Mercury .. 3-Orbit Manned and Unmanned Mission (7, pp. 1-24). In this case study, the ability of an astronaut to perform certain overrides with a normal, or an overpressurized space suit, was evaluated by panel members who made independent estimates of the astronaut's performance. The average for five estimates for each override was computed, and then the probabilities of having an overpressurized or normally pressurized suit were calculated. These served as weights to the two averages for each override, and an estimated probability for the astronaut's ability was then computed. There are many more environmental factors K_D which can be calculated and weighted in the above manner.

To date, no studies have been published which give values for K_E versus time for varying levels of radiation.

One of the values of K_F which has been difficult to evaluate is the reaction of different astronauts to being alone in cramped quarters for sustained periods of time. Short duration studies have been made, but no values for missions between 3 months and 2 years are available to date.

3.4 Comparative Disadvantages

3.4.1 Disadvantages of Functional Redundancy (17, pp. 43-45)

1. Partial failures are not detectable
2. Failed components may change operating characteristics

Functionally redundant circuits must be able to tolerate wide swings in circuit values.

3. Survivors must carry an increased load

Load will either stay the same or increase. If it increases, the remaining elements will fail prematurely.

4. More heat is generated

Since all components operate continually, heat dissapation will increase in proportion to the number of redundant parts.

5. Design is more difficult, more costly.

Because failures change operating characteristics, the design must be versatile enough to permit these changes without affecting performance.

6. Failures may not be independent.

Since functionally redundant parts are grouped together with operating parts, they are not as resistant to environmental failures. If vibration, temperature, or humidity causes one element to fail, it will probably cause the others to fail also. Standbys in sequential, on the other hand, can be widely separated to minimize the hazard of environmental failures.

7. Wearout failure is not substantially improved.

With all the elements operating full time, the only advantage of functional redundancy is that the system MTBF is determined by the

$M_p = \frac{1}{\lambda} + \frac{1}{2\lambda} + \frac{1}{3\lambda} + \ldots + \frac{1}{n\lambda}$ when n is the number of functionally redundant components.

8. The degree of redundancy is not easily changed after design.

If additional redundancy is needed, a functional design will demand extensive design effort. In sequential designs, additional redundancy can be inserted or removed to adjust the tradeoff between weight and reliability.

9. Life tests are more difficult to make and interpret.

Because component failures do not cause circuit failures, it is difficult to determine the lifetime of individual parts. This is not the case with sequential failures where part failures show up whenever a new element goes into action.

10. Power supply must be larger.

Controlling effect in satellite where power supply size is limited by the present state of technological development, not by size, weight, or cost.

11. Reliability analysis is difficult.

Calculations can be difficult since it is necessary to analyze all possible modes of failure.

12. There is no protection against degradation failures.

The design must anticipate the effect of parts that do not actually fail, but have performance degraded to the point where it impairs circuit operation. There will be wide swings in the effective value of each multiple component. Sequentially operated circuits, by contrast, provide backup protection for both degradation and random failures.

3.4.2 Disadvantages of Sequential Redundancy (17, pp. 43-45)

1. Failure detection and switching devices are required.

Two jobs of switching devices are to sense a failure in the operating component and to switch in its replacement. One method is to provide a circuit that compares performance of identically operating components and automatically switches in a substitute for those that do not measure up. Another way is to monitor performance individually, manually turning on a new element whenever an operating element fails. Either way, the switching device must have an inherently high reliability to prevent degradation of the system's over all reliability.

2. Shelf life reduces anticipated reliability.

If performance of an inactive element degrades with time, there may be little advantage to redundancy of any kind. The effectiveness of replacement may be partially lost even before it is switched into the circuit. This is in addition to environmental effects such as radiation which would destroy both active and inactive elements.

3. Downtime during switchover can be significant.

With manual switching, it may take some time to discover a failure, switch in a replacement, and make the necessary adjustments. If it is important that the equipment remain on continuously choose functional redundancy.

4. Stored data may be lost.

If the system contains a volatile memory, a failure in power supply, for example, could destroy the stored information. Functional redundancy is preferred under these conditions. If sequential redundancy is used, there must be an internal or external means of restoring the stored information.

CHAPTER 4

WEAROUT REDUNDANCY TECHNIQUES

4.1 Kececioglu's Probabilistic Method for Calculating the Reliability of a Sequentially Redundant System

To illustrate this method, the reliability of the system shown in Fig. 10 and described below will be calculated for a mission time t.

There are two functioning units and two standby units. Two out of two functioning units are required for system success. In the sensing and switching device for #4 standby unit, the acquiescent mode of the sensing device is 4_{sa}, the cyclic mode of the switching device is 4_{sc}, and the inadvertently opening mode for the switching device is 4_{so}.

5_{sa}, 5_{sc}, and 5_{so} will be the corresponding modes for the sensing and switching device for #5 standby unit.

Using the probabilistic language, 13 successful modes of function are found. The first six of these modes are written out in long form below and corresponded to the first six combinations in Table 5. In Table 5, the units and success modes and time domains of function of each unit in Fig. 10 for all of the 13 successful combinations of function are listed (22).

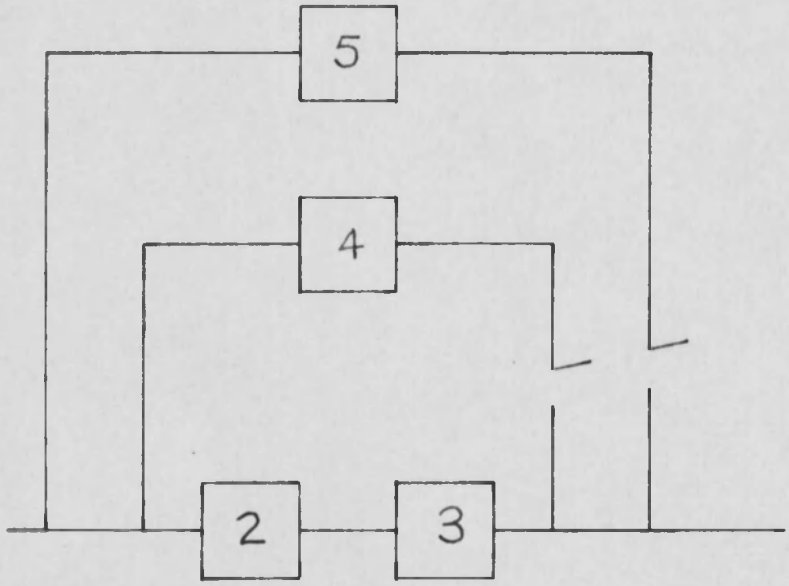

Figure 10. Standby System: Units 2 and 3 are functioning with Units 4 and 5 in standby.

TABLE 5. THE UNITS AND SUCCESS MODES AND TIME DOMAINS FOR THE SYSTEM SHOWN IN FIGURE 10

NO. OF COMBINATIONS	UNITS AND SUCCESS MODES AND TIME DOMAINS									
	#2	#3	4_{sa}	4_{sc}	#4	4_{so}	5_{sa}	5_{sc}	#5	5_{so}
1	G t	G t								
2	G t	B t_1	G t_1	G lc	G $t-t_1$	G $t-t_1$				
3	G t	B t_1	G t_1	G lc	G t_2-t_1	B t_2-t_1	G t_2	G lc	G $t-t_2$	G $t-t_2$
4	G t	B t_1	G t_1	G lc	B t_2-t_1	G t_2-t_1	G t_2	G lc	G $t-t_2$	G $t-t_2$
5	G t	B t_1	G t_1	B lc			G t_1	G lc	G $t-t_1$	G $t-t_1$
6	G t	B t_1	B t_1				G t_1	G lc	G $t-t_1$	G $t-t_1$
7	B t_1	G t	G t_1	G lc	G $t-t_1$	G $t-t_1$				
8	B t_1	G t	G t_1	G lc	G t_2-t_1	B t_2-t_1	G t_2	G lc	G $t-t_2$	G $t-t_2$
9	B t_1	G t	G t_1	G lc	B t_2-t_1	G t_2-t_1	G t_2	G lc	G $t-t_2$	G $t-t_2$

TABLE 5. THE UNITS AND SUCCESS MODES AND TIME DOMAINS FOR THE SYSTEM SHOWN IN FIGURE 10 (Contd)

NO. OF COMBINATIONS	UNITS AND SUCCESS MODES AND TIME DOMAINS									
	#2	#3	4_{sa}	4_{sc}	#4	4_{so}	5_{sa}	5_{sc}	#5	5_{so}
10	B t_1	G t	G t_1	B $1c$			G t_1	G $1c$	G $t-t_1$	G $t-t_1$
11	B t_1	G t	B t_1				G t_1	G $1c$	G $t-t_1$	G $t-t_1$
12	B t_1	B t_2	G t_1	G $1c$	G $t-t_1$	G $t-t_1$	G t_2	G $1c$	G $t-t_2$	G $t-t_2$
13	B t_2	B t_1	G t_1	G $1c$	G $t-t_1$	G $t-t_1$	G t_2	G $1c$	G $t-t_2$	G $t-t_2$

1. #2 succeeds for t and #3 succeeds for t.
2. #2 succeeds for t and #3 fails @ t_1 and 4_{sa} succeeds to t_1 and 4_{sc} works and #4 succeeds from t_1 to t and 4_{so} succeeds from t_1 to t.
3. #2 succeeds for t and #3 fails @ t_1 and 4_{sa} succeeds to t_1 and 1_{sc} succeeds and #4 succeeds from t_1 to t_2 and 4_{so} fails at t_2, and 5_{sa} succeeds to t_2 and 5_{sc} succeeds and #5 succeeds from t_2 to t and 5_{so} succeeds from t_2 to t.
4. #2 succeeds for t and #3 fails @ t_1 and 4_{sa} succeeds to t_1 and 4_{sc} succeeds and #4 fails @ t_2 and 4_{so} succeeds from t_1 to t_2 and 5_{sa} succeeds to t_2 and 5_{sc} succeeds and #5 succeeds from t_2 to t and 5_{so} succeeds from t_2 to t.
5. #2 succeeds for t and #3 fails @ t_1 and 4_{sa} succeeds to t_1 and 4_{sc} fails at t_1 and 5_{sa} succeeds to t_1 and 5_{sc} succeeds and #5 succeeds from t_1 to t and 5_{so} succeeds from t_1 to t.
6. #2 succeeds for t and #3 fails @ t_1 and 4_{sa} fails @ t_1 and 5_{sa} succeeds to t_1 and 5_{sc} succeeds and #5 succeeds from t_1 to t and 5_{so} succeeds from t_1 to t.

Then the reliability for the system is of the reliabilities of combinations 1 through 13 in Table 5 and is equal to

$$R_S = R_2(t)R_3(t) + R_2(t)\int_{o}^{t_1=t} \Big[f_3(t_1) \times R_{4sa}(t_1) \times R_{4sc}(1 \text{ cycle}) \times R_4(t-t_1) \times R_{4so}(t-t_1)\, dt_1 \Big]$$

$$+ R_2(t)\int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_3(t_1) \times R_{4sa}(t_1) \times R_{4sc}(1 \text{ cycle}) \times R_4(t_2-t_1) \times f_{4so}(t_2-t_1) \times R_{5sa}(t_2) \times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R_2(t)\int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_3(t_1) \times R_{4sa}(t_1) \times R_{4sc}(1 \text{ cycle}) \times f_4(t_2-t_1) \times R_{4so}(t_2-t_1) \times R_{5sa}(t_2) \times R_{5sc}(1 \text{ cycle})$$

$$\times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R_2(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_3(t_1) \times R_{4sa}(t_1) \times (1-R_{4sc})(1 \text{ cycle})$$

$$\times R_{5sa}(t_1) \times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_2)$$

$$\times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_3(t_1) \times f_{4sa}(t_1) \times R_{5sa}(t_1)$$

$$\times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R_3(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times R_{4sa}(t_1)$$

$$\times R_{4sc}(1 \text{ cycle}) \times R_4(t-t_1) \times R_{4so}(t-t_1)\, dt_1\, dt_2 \Big]$$

$$+ R_3(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times R_{4sa}(t_1) \times R_{4sc}(1 \text{ cycle})$$

$$\times R_4(t_2-t_1) \times f_{4so}(t_2-t_1) \times R_{5sa}(t_2) \times R_{5sc}(1 \text{ cycle})$$

$$\times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R_3(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times R_{4sa}(t_1) \times R_{4sc}(1 \text{ cycle})$$

$$\times f_4(t_2-t_1) \times R_{4so}(t_2-t_1) \times R_{5sa}(t_2) \times R_{5sc}(1 \text{ cycle})$$

$$\times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ R_3(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times R_{4sa}(t_1) \times (1-R_{4sc})(1 \text{ cycle})$$

$$\times R_{5sa}(t_1) \times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_1)$$

$$\times R_{5so}(t-t_1)\, dt_1\, dt_2 \Big]$$

$$+ R_3(t) \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times f_{4sa}(t_1) \times R_{5sa}(t_1)$$

$$\times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_1) \times R_{5so}(t-t_1)\, dt_1\, dt_2 \Big]$$

$$+ \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_1) \times f_3(t_2) \times R_{4sa}(t_1)$$

$$\times R_{4sc}(1 \text{ cycle}) \times R_4(t-t_1) \times R_{4so}(t-t_1) \times R_{5sa}(t_2)$$

$$\times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

$$+ \int_{t_2=o}^{t_2=t} \int_{t_1=o}^{t_1=t_2} \Big[f_2(t_2) \times f_3(t_1) \times R_{4sa}(t_1)$$

$$\times R_{4sc}(1 \text{ cycle}) \times R_4(t-t_1) \times R_{4so}(t-t_1) \times R_{5sa}(t_2)$$

$$\times R_{5sc}(1 \text{ cycle}) \times R_5(t-t_2) \times R_{5so}(t-t_2)\, dt_1\, dt_2 \Big]$$

(4.1)

4.2 Evaluation of a General Equation for a Sequentially Redundant System Entering Wearout

To evaluate the reliability of the system shown in Fig. 10 and given by equation 4.1, inputs for the different values of f and R must be found. The f's under the integral signs in equation 4.1 are density functions.

$$\text{Where } f(t) = \frac{-dR(t)}{dt} \text{ and } R(t) = R_c \times \frac{R_w(T+t)}{R_w(T)}$$

The R's found in equation 4.1 stand for the actual component reliabilities which can be evaluated by equation 2.10. The development of the failure density functions which appear in equation 4.1 is extremely complex for the combination of chance and wearout failures, and beyond the scope of this thesis. Equation 4.1 can be used as it is, however, when components used during their useful life only are considered. In that case, $f(t) = \lambda e^{-\lambda t}$.

CHAPTER 5

MATH MODELS FOR SYSTEM RELIABILITY WITH PREVENTIVE MAINTENANCE

5.1 Areas of Maintenance

The failure rate curve with ideal repair maintenance is a significant dividing line between the two basic areas of maintenance: the area of preventive maintenance and the area of repair maintenance. See Fig. 11. In Chapter 4, it was shown that with ideal repair, the failure rate curve for the wearout distribution became purely exponential after a state of mixed wearout was reached. Ideal repair maintenance is defined as the replacing of a worn out failed part with a new part with no damage and no error during the repair operation, and preventive maintenance is the replacing of components prior to the time when malfunction might occur.

At the bottom of the failure rate curve in Fig. 11, a constant failure rate line has been added to cover the failures due to chance. The chance failure rate cannot be decreased by any form of maintenance but can be increased by mishandling and damaging of equipment during maintenance (15, pp. 141-142).

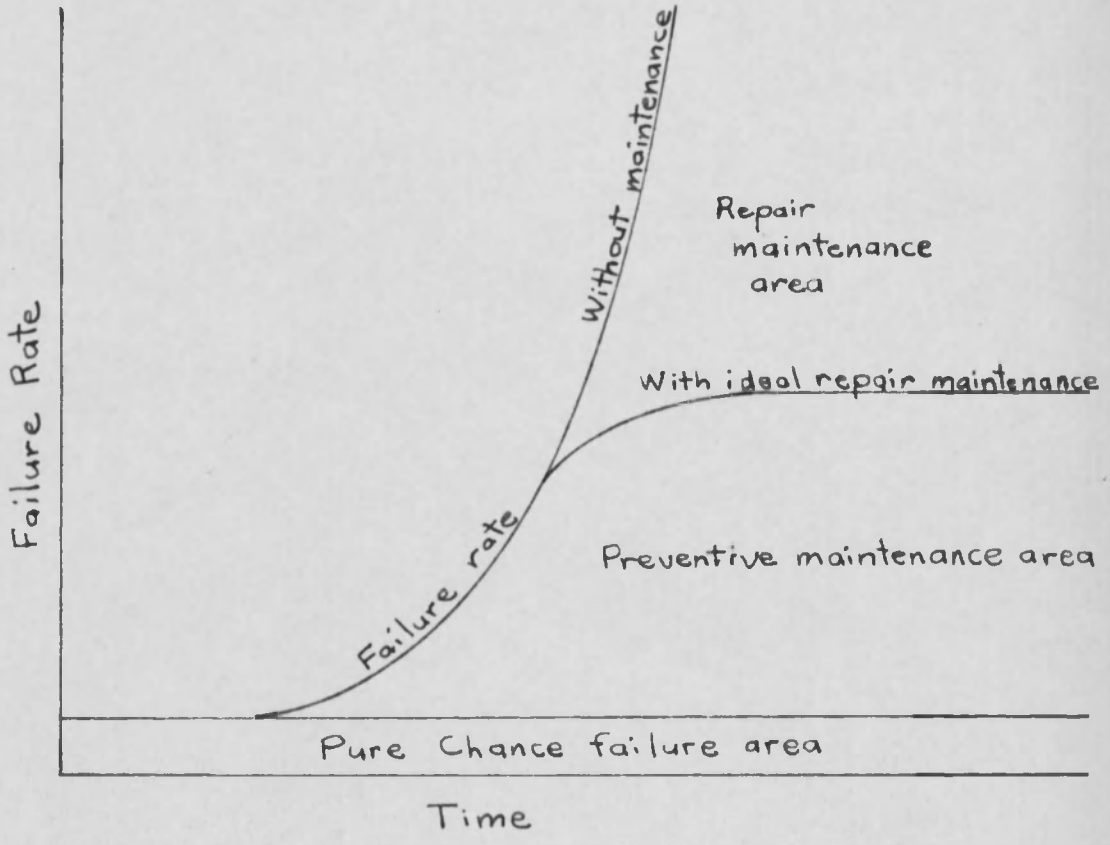

Figure 11. Areas of maintenance (15, p. 141)

As time increases without limit, the failure rate with ideal maintenance approaches the reciprocal of the mean value of the basic wearout function. After each device has been replaced several times, the devices fail in wearout according to chance, and the mean time between failure must equal the mean of the basic wearout function of the devices. See Fig. 12. In Fig. 12, a normal failure function with a mean M = 3 and a standard deviation = 1 is used as the basic wearout density function, and the damped oscillations of the failure rate with ideal repair maintenance should be noted.

In actual conditions, ideal repair is rarely encountered. Defective components must be replaced under unfavorable circumstances which diminish the life expectancy of the new component and cause new failures. In this case, the wearout distribution mean of the spare component is smaller than the original component wearout mean. And, since the failure rate for repair maintenance, as time goes to infinity, is equal to the reciprocal of the mean of the basic failure distribution, it can be concluded as the mean of the wearout distribution for a replaced component is shortened, the failure rate becomes greater than with ideal repair maintenance. In Fig. 12, the failure rate with repair maintenance is computed by dividing the failure rate with ideal repair maintenance by a factor of efficiency.

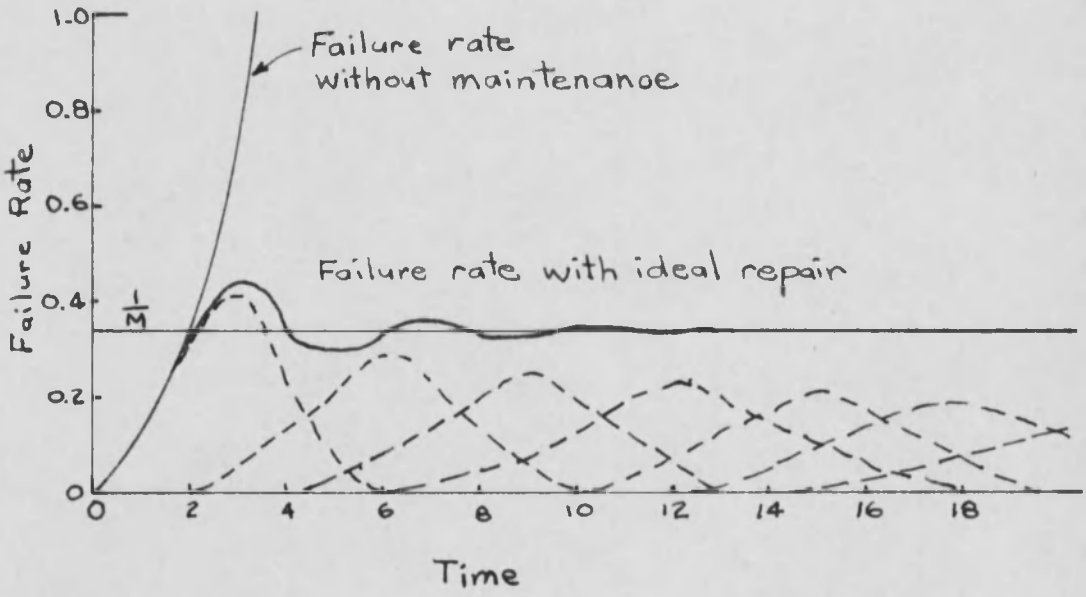

Figure 12. The failure rate with ideal repair maintenance for the normal wearout density function: $\sigma = 1$, $M = 3$ (15, p 148)

5.2 Preventive Maintenance Methods

5.2.1 Type 1

In type 1 preventive maintenance, the component is preventively replaced always after it has been in operation for T_o hours and is correctively replaced when it fails. Thus, if a component failed at some time less than T_o, it would be replaced and the next preventive replacement would be made T_o hours after the corrective replacement. This type of preventive maintenance is useful when individual preventive replacements time logs can be kept for each component and when it is necessary to minimize the number of spare parts that will be needed for a mission time t. If the reliability for a component is $R(T_o)$ for the replacement period T_o, then the mean time to failure of a part subject to type 1 preventive maintenance every T_o hours is: (1, p. 17)

$$M_T = \frac{\int_o^{T_o} R(t)\,dt}{1 - R(T_o)} = \frac{\int_o^{T_o} R(t)\,dt}{Q(T_o)} \qquad (5.1)$$

and the reciprocal of M_T is the constant long-term stabilized average failure rate of a part subject to type 1 preventive maintenance every T_o hours:

$$\lambda_{Average}(T_o) = \frac{Q(T_o)}{\int_o^{T_o} R(t)\,dt} \qquad (5.2)$$

The denominator of equation 5.2 is the mean time between both preventive and corrective replacements. Therefore, the total replacement rate r is the sum of the preventive and corrective replacements, and since the rate of corrective replacements equals the failure rate $\lambda_{AV}(T_o)$,

$$r = \frac{1}{\int_o^{T_o} R(t)\,dt} = \frac{Q(T_o)}{\int_o^{T_o} R(t)\,dt} + \frac{R(T_o)}{\int_o^{T} R(t)\,dt} \qquad (5.3)$$

Over a long mission t, the average number of preventive replacements is

$$E_{PN}(t) = \frac{t\,R(T_o)}{\int_o^{T_o} R(t)\,dt} \qquad (5.4)$$

and the average number of corrective replacements (number of failures) is

$$E_{CN}(t) = \frac{t\,Q(T_o)}{\int_o^{T_o} R(t)\,dt} \qquad (5.5)$$

and the total number of spares required for a mission time t becomes

$$E_N(t) = E_{PN}(t) + E_{CN}(t) = \frac{t}{\int_o^{T_o} R(t)\,dt} \text{ (1, p. 18).} \qquad (5.6)$$

The number of spares to be carried on a long mission spacecraft flight can be calculated for any required percent confidence by applying X^2 confidence limits to equation 5.6. See Section 5.3 for X^2 evaluation.

5.2.2 Type 2

In type 2 preventive maintenance, the preventive replacements are made strictly every T_o operating hours regardless of whether or not the component has failed and been correctively replaced in the period since its last preventive replacement. In general, the type 2 replacement policy is used when maintenance schedules are based on machine operating time rather than on parts time. It is obvious that if there were 10,000 of the same type of component being used on a spacecraft that it would be next to impossible for an astronaut to log and keep track of individual type 1 preventive maintenance schedules for each component. As more and more components failed before the preventive maintenance schedule time T_o, and were replaced and allowed to operate T_o hours on their own schedule, the time between maintenance operations would approach $\frac{T_o}{10,000}$ if the mission time $t \gg T_o$.

If there is only one of a component used in a spacecraft, say a radar tube, type 2 preventive maintenance would not increase the reliability (if an exponential

failure rate is assumed for the period T_o) and would increase the number of spare components that would have to be taken on a mission. Thus, since replacements that are not needed are made, the availability of the radar tube would be less (assuming that the repair time $\tau \neq 0$).

With type 2 preventive maintenance, the preventive maintenance rate is:

$$r_P = \frac{1}{T_o} \tag{5.7}$$

and the corrective replacement rate, which equals the stabilized long term average failure rate is given by the range:

$$\frac{\int_o^{T_o} Q(t)\,dt}{T_o \int_o^{T_o} R(t)\,dt} \leqq \lambda_{AV}(T_o) < \frac{Q(T_o)}{\int_o^{T_o} R(t)\,dt} \tag{5.8}$$

Using the upper limit for $\lambda_{AV}(T_o)$ the calculation for spares supplies and optimization of the preventive maintenance period T_o can again be made:

$$E_N(t) = t\left[\frac{1}{T_o} + \frac{Q(T_o)}{\int_o^{T_o} R(t)\,dt}\right] \tag{5.9}$$

See Section 5.3 for an example of type 2 preventive maintenance.

5.3 Spare Parts Provisioning Philosophy for Manned Flights

For the component outlined in Section 2.3, let us assume that the estimated failure rate $\hat{\lambda}_C = \frac{1 \text{ failure}}{10^6 \text{ hours}}$ was taken from the following data: 10,000 components were tested for 1000 hours and 10 failures occurred yielding an estimated failure rate of 1 failure/10^6 hours for the component. All of the components were fully debugged and the 1000 hours of test time did not push any of the components into wearout. From this data, the number of spares to be carried on a mission, during which no new spare supplies are obtainable, can be calculated for any required percent confidence that sufficient spares will be available to restore equipment operation in case of parts failures. Using a 90% confidence level for a one-sided X^2 upper limit, the upper limit for the chance failure rate is given by the following equations:

$$\frac{1}{\lambda_C} \geq \frac{2\,T_x}{X^2_{\vartheta\,;2r+2}} \tag{5.10}$$

where $T_x = \frac{r}{\hat{\lambda}_C}$

r = number of failures

T_x = total logged test time

$2r+2$ = degrees of freedom

λ_C = random failure rate

$\hat{\lambda}_C$ = estimated random failure rate

$$\lambda_C \leq \frac{\chi^2_{\alpha;\,2r+2}}{2\,T_x} \leq \frac{30.8}{2 \times 10^7} = \frac{1.54}{10^6} \text{ failures hours}$$

It is now known that for the random failure rate data generated for the component in Section 2.3, 90% of the time λ_C will be less than 1.54 failures per 10^6 hours. From Table III, considering the exponential case only, a value of T_o of 2000 hours was chosen as the replacement time of the component. This corresponds to a desired reliability of 0.9980.

Using equations 5.6 and 5.9, the number of spares needed for each component used aboard the spacecraft can be calculated with 90% confidence for type 1 and type 2 preventive maintenance respectively. This is done in the next two sections.

5.3.1 Spare Parts Provisioning for Type 1 Preventive Maintenance

Using equation 5.6,

$$E_N(t) = \frac{t}{\int_o^{T_o} R(t)\,dt}$$

$$E_N(17{,}520) = \frac{17{,}520}{\int_o^{2000} e^{-\lambda_C t}\,dt} = \frac{17{,}520}{\frac{1}{\lambda_C}\left[1-e^{-\lambda_C 2000}\right]}$$

$$E_N(17{,}520) = \frac{17{,}520}{2000} = 8.76 \text{ spares needed for a 17,520 hour flight.}$$

Thus, if there were 10 of the components being operated continuously in the space vehicle with type 1 maintenance every T_o = 2000 hours, there should be 88 spares with 90% confidence that a spare will always be available.

It should be remembered that type 1 preventive maintenance requires the least number of spares for the same level of confidence of the two types of preventive maintenance covered in this thesis.

5.3.2 Spare Parts Provisioning for Type 2 Preventive Maintenance

Using equation 5.9 and the X^2 confidence limit for λ_C from Section 5.3, the number of spare parts per application can be calculated for a 17,520 mission using type 2 preventive maintenance with 90% confidence.

$$E_N(t) = t\left\{\frac{1}{T_o} + \frac{Q(T_o)}{\int_o^{T_o} R(t)\,dt}\right\}$$

from Section 5.3.1 for T_o = 2000 hours and λ_C = 1.54 failures/10^6 hours; $\int_o^{T_o} R(t)\,dt = 2000$

$$Q(T_o) = 1 - e^{-\lambda_C T_o} = 0.00308$$

and

$$E_N(17,520) = \frac{17,520}{2000} + \frac{17,520\ (0.00308)}{2000}$$

$$E_N(17,520) = 8.760 + 0.027$$

$E_N(17,520) = 8.787$ spares needed for a 17,520 hour flight.

Thus, if there were 10 of the components being operated continuously in the space vehicle with type 2 maintenance every $T_o = 2000$ hours, there should be 88 spares with 90% confidence that a spare will always be available.

CHAPTER 6

COMPARISON OF SYSTEM RELIABILITY FOR MANNED AND UNMANNED FLIGHTS

6.1 Without Wearout

For a component in an unmanned spacecraft to operate entirely in the chance or constant failure rate area of the bathtub curve, the mission time t must be equal to the mean of the normal wearout distribution minus 4 to 6 standard deviations of the wearout distribution. The number of standard deviations that the mission time must lie to the left of the wearout mean is determined by the risk, or probability of failure which can be tolerated from wearout. In a manned flight, preventive maintenance can be used at $T_o = M - 4$ to 6σ to prevent the component from entering wearout. For a component with a set wearout mean and standard deviation, preventive maintenance in a manned flight is the only method of extending the possible mission time while maintaining a high reliability and prohibiting the component from entering wearout.

The component reliability and the slight contribution to the over-all failure rate by wearout in the -4 to -6σ range can be calculated using equation 2.10.

The probability of a component operating for a given mission time t in the exponential life of the component can be increased by applying the sequential and functional redundancy techniques covered in Chapter 3. In a manned flight using preventive maintenance to extend the mission time from T_o to t, the probability of the component operating until the replacement time T_o can be increased by using redundancy. This method is described in Section 6.4.

6.2 With Wearout

In a manned flight with wearout, all of the techniques covered in this thesis for increasing the reliability over a given mission time or for extending the maximum mission time possible for a given reliability can be used in any assorted combinations. Allowing wearout, the components may be replaced as they fail, giving a constant wearout failure rate after the third or fourth wearout replacement.

In an unmanned flight, all of the redundancy techniques covered in Chapter 3 may be used for increasing component reliability and increasing mission time. It was found that if the sensing and switching reliability

for a sequentially redundant circuit can be considered perfect, the sequentially redundant circuit with one operating component and n-1 standby components will have a higher reliability than a functionally redundant circuit of n components for values of λt between 0.0001 and 1.0.

6.3 With Preventive Maintenance

A preventive maintenance policy is justified when the component exhibits an increasing failure rate (23, p. 188). If the component has been operating with a constant failure rate, or during the chance failure period of its life, then an increase in failure rate indicates that the component is beginning to enter wearout. When a component enters wearout, the failure rate increases quite rapidly, and naturally the probability of survival of the component decreases. Replacing a component that has an operating age X with a new one returns the failure rate to the initial value at time zero.

If a component which has been operating in its useful life (chance failure rate) and will continue to operate in its useful life for the duration of the mission is replaced by a new identical component, the failure rate remains the same and nothing is gained. Thus, preventive maintenance should be used only when it is known that a component will enter wearout before the end of a mission. A preventive maintenance policy is effective when the

component exhibits an increasing failure rate with time and when the replacement time of components which have not failed is less than the replacement time of failed components. Since it is desirable to minimize the down-time of a component, availability may be enhanced by substituting preventive maintenance time for corrective maintenance time.

6.4 With Redundancy

If with preventive maintenance the component reliability is too low, i.e., the chance failure rate is too high, the component reliability may be increased by using redundancy with inspections at time intervals T_1 and the replacement of all redundant components at preventive maintenance time T_o where $T_1 < T_o$. The preventive replacement at time T_o insures that the redundant components are not entering wearout, i.e., are behaving exponentially, and the inspections insure full redundancy and a maximum mean time between failure for the system. It can be seen from Fig. 13 that if the system is inspected at intervals of time T_1, and all of the failed redundant components are replaced, the failure rate for the component returns to zero and begins retracing the average failure rate curve for a system maintained every T_o hours. If the system is inspected at small intervals of time T_1, the maximum failure rate is kept very low and the average failure rate

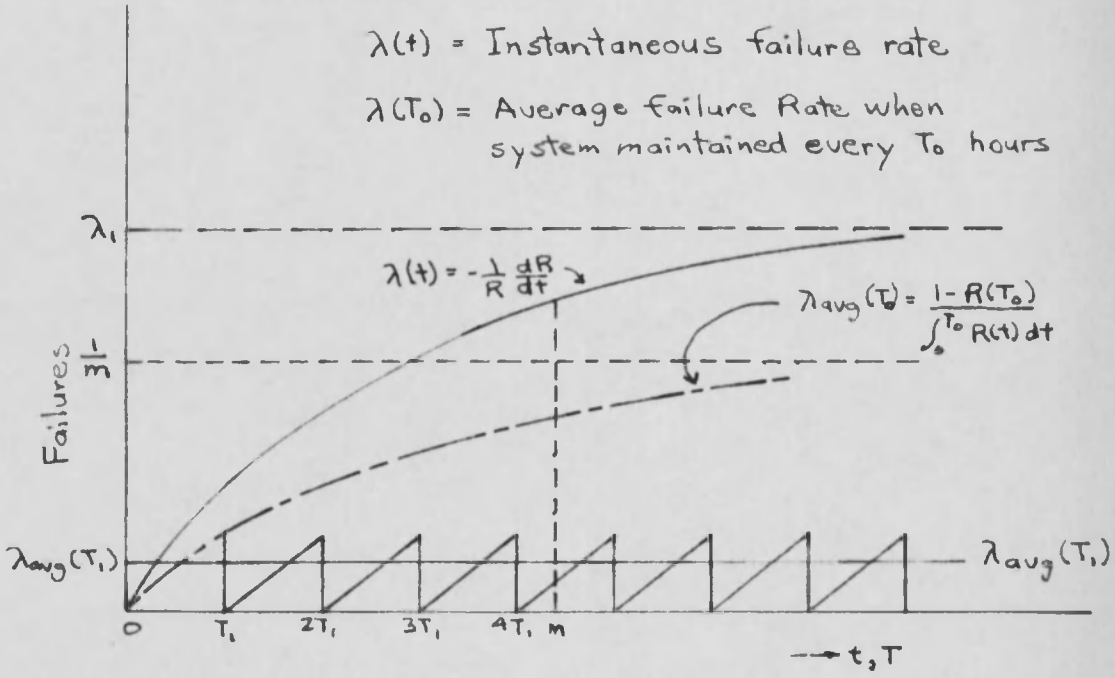

Figure 13. Failure Rate of Redundant System with Inspections at T_1 and preventive maintenance at time T_0.

for an inspection time T_1 ($\lambda_{AV}(T_1)$) is given by:

$$\lambda_{avg}(T_1) = \frac{1 - R(T)}{\int_o^{T_1} R(t)\, dt}$$

The application of redundancy to an already maintained system does not significantly add to time required to keep the system operating unless inspections are made at intervals T_1 to assure that the system has full redundancy. Frequent inspections of the preventively maintained redundant system give the maximum MTBF possible, thus insuring a maximum reliability between inspections times T_1.

CHAPTER 7

SUMMARY AND CONCLUSIONS

Considering the three failure areas on the failure rate versus time curve, a relationship for obtaining the component reliability from any time T to T + t was developed. Equations were then developed to determine the reliability of systems with sequentially redundant components and of systems with functionally redundant components in terms of the component reliabilities established by equation 2.10. It was found that when the sensing and switching reliability is close to unity, a sequentially redundant system will give higher reliabilities than a functionally redundant system for a given mission time and an equivalent number of identical components. The probabilistic method for calculating the reliability of sequentially redundant circuits was found to be an excellent method for handling all time dependent standby systems.

Preventive maintenance applied to redundant circuits in manned spacecraft flights was found to be an extremely useful tool in reducing the system failure rate.

REFERENCES

1. Reliability Mathematics as Applied to Shipboard Machinery, Igor Bazovsky, Raytheon Comp., February, 1963, 28 pp.

2. Analysis of Mariner R Flight Data for Ranger Application, Reliability Control Department, Northrop Space Laboratories, Hawthorne, Calif., July, 1963, 46 pp.

3. Research and Technological Problems of Manned Rotating Spacecraft, Langley Research Center Staff, Langley Station, Hampton, Va., August, 1962, 146 pp.

4. Silent Satellites Speak, Product Engineering, February 4, 1963, pp. 94-95.

5. Prelaunch Calculations of Space System Reliability, Vernon L. Gross, Northrop Ventura, January, 1963, 22 pp.

6. Human Engineering, Ernest J. McCormick, McGraw-Hill Book Comp., Inc., New York. 1st Ed. 1957, 467 pp.

7. A Reliability Model and Analysis for Project Mercury 3-Orbit Manned and Unmanned Mission, Wolman and Okano, National Aeronautics and Space Administration, Washington, D. C., December, 1962, 24 pp.

8. An Analysis of the Effects of Maintenance on Part Replacement, R. L. Madison, Proceedings 4th National Symposium on Reliability and Quality Control, pp. 19-29, January, 1958.

9. The Role of Reliability Testing on Satellite Reliability, J. H. Boechal, NASA X-320-62-41, May, 1962.

10. Wearout Failures Versus Random Failures, E. L. Eagle, The Bendix Corp., Ann Arbor, Michigan, 24 October 1960, 4 pp.

11. ARINC Report No. 2, Air Force Reliability Assurance Program, ARINC Research Corp., April 1, 1959.

REFERENCES
(Contd.)

12. Mathematical Foundation of Reliability Theory, E. Pieruschka, Redstone Arsenal, April, 1960.

13. Predicting Electronic Equipment Maintainability, James W. Thomas, Proceedings, 5th National Symposium on Reliability and Quality Control, pp. 265-269, January, 1959.

14. Predicting Systems Reliability, H. M. Gordy, International Science and Technology, July, 1963, pp. 72-79 (2 copies).

15. Principles of Reliability, Erich Pieruschka, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1st edition 1963, 365 pp.

16. Reliability Data Book, Paul H. Zorger, Martin Company, Baltimore 3, Maryland, June, 1962, 114 pp.

17. For Reliability ... Which Kind of Redundancy? R. H. Myers, Hughes Aircraft Co., Product Engineering, Vol. 32, No. 34, 21 August 1961, pp. 43-45.

18. Redundancy Considerations in Space and Satellite Systems, L. A. Aroian and R. H. Meyers, Proceedings 7th National Symposium on Reliability and Quality Control, pp. 292-301, January, 1961.

19. Reliability Theory and Practice, Igor Bazovsky, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1st edition, 1961, 292 pp.

20. On the Basic Concepts of Reliability Prediction, George T. Bird, 7th National Symposium on R. & Q.C., January, 1961, pp. 51-55.

21. Reliability Control in Aerospace Equipment Development, John de S. Coutinho and Lionel Levy, Society of Automotive Engineers, Inc., 485 Lexington Avenue, New York 17, New York.

22. Lecture Notes from Dr. Dimitri B. Kececioglu's Advanced Reliability Engineering Course taught spring semester, 1964, at The University of Arizona, Tucson, Arizona.

REFERENCES
(Contd)

23. System Reliability Engineering, Gerald H. Sandler, Prentice Hall, Inc., Englewood Cliffs, New Jersey, 1st edition, 1963, 221 pp.