

HAND DIGITALIZATION ERRORS IN TRANSIENT ANALYSIS

by

Burke Owen Buntz

A Thesis Submitted to the Faculty of the
DEPARTMENT OF ELECTRICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of
MASTER OF SCIENCE

In the Graduate College
THE UNIVERSITY OF ARIZONA

STATEMENT BY AUTHOR

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APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:



DONALD G. DUDLEY
Associate Professor of
Electrical Engineering

6/3/92

Date

To Ann, Douglas, and Heidi

ACKNOWLEDGMENTS

The author is most indebted to Professor Donald G. Dudley for his suggestion of the topic of this thesis and for his continued encouragement and guidance.

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ABSTRACT

A statistical analysis is presented to investigate the error which occurs during the hand digitalization of a waveform. This nondeterministic process is one of the latter sequences within the task of studying the electromagnetic pulse (EMP) effects in the frequency domain. Experimental details for data collection and the software used to reduce the data are given to provide a basis for the analysis.

CHAPTER 1

INTRODUCTION

Research is being conducted by the Department of the Army to determine the electromagnetic field response within various environments to a high intensity electromagnetic pulse (EMP transients). A method of analyzing these pulse effects has been to set up a test station with electric and magnetic field probes which sample the test volume. This sensor information is displayed on oscilloscopes with attached cameras. Synchronous with the pulse propagating through the test station, the cameras record the various time varying voltages, currents, or fields caused by the pulse. Two waveforms are recorded for each sensor. The oscilloscope sweeps are adjusted such that one oscilloscope records the entire time history of the pulse effects, while a second oscilloscope records, with a faster sweep, only the leading edge information. These two pictures are then tied together on one graph with the changing of sweep speeds accounted for by the proper mapping of the time axis. To obtain the spectral content in the frequency domain of these various time varying parameters, Fourier transforms of the recorded waveforms are taken.

To perform the Fourier transform process, the recorded waveform must be digitized. These digitized data are then stored by a means compatible to digital computer processing. A numerical Fourier transform technique is used to convert the time domain data into the frequency domain. Within the Army project, the analysis of the waveforms is performed by two means. At Fort Huachuca, an "in-house" investigation is being conducted using machine digitizers and the fast Fourier transform algorithm. Concurrently, at The University of Arizona, a large number of individuals hand digitize the waveforms and the transforms are achieved by using Filon's Numerical Integration Technique.

This thesis describes statistical errors in the hand digitalization method. The thesis is organized into various topics with Chapter 2 describing the nature of the digitalization effort, Chapter 3 contains the methods used to reduce the data into statistical displays and results. Chapter 4 is an analysis of the results from Chapter 3, and Chapter 5 offers the conclusions made during the analysis. The final portions of this study contain recommendations for future work, the appendices, and a list of references.

CHAPTER 2

EXPERIMENT DESCRIPTION

2.1 Initial Waveform Processing

A two picture set of one representative pulse (Figures 2.1 and 2.2) was chosen to illustrate the procedures that are required to analyze pulse waveforms recorded from oscilloscope traces at the project test site. Figure 2.1 recorded only the leading edge information of the pulse and its sweep was 10 times that used to record Figure 2.2. To facilitate and economize the reproduction of the waveform, tracing paper with a superimposed grid of 20 lines per inch was chosen to provide a trace that is intersected by 50 time lines. The amplitude reference was chosen to provide 200 quantization levels per inch along the amplitude axis. The exact waveform amplitude in volts is not required in this study. Therefore, the frequency domain levels will be relative quantities. The two original photographs were time tied at time interval 37 with intervals 1-36 having a 4 nsec duration, and intervals 37-50 having a 40 nsec duration. The two photographs were tied together so that the leading edge information has a moderate maximum slope for good

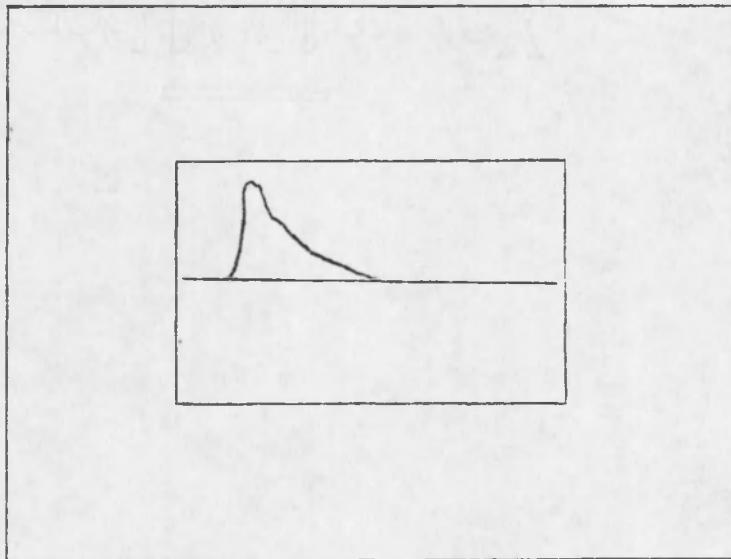


Figure 2.1. Oscilloscope Photo with Sweep ω

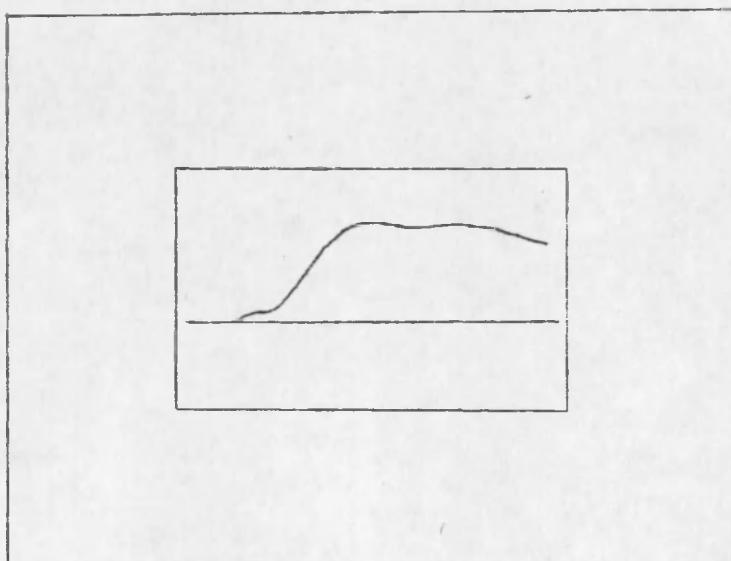


Figure 2.2. Oscilloscope Photo with Sweep 10ω

resolution and all of the history of the pulse could be analyzed on one waveform as shown in Figure 2.3.

2.2 Hand Digitalization Procedures

The final copy of the waveform (Figure 2.4) was reproduced and given to 204 undergraduate and graduate students at The University of Arizona. A one minute explanation of the task was given and then the students independently performed the digitalization task in groups of 10-40 individuals, at various times during the day, and at various psychological attitudes (i.e., during breaks, before and after formal lectures, and between classes). All of the individuals were told to perform the task as quickly as possible and the average time to complete the digitalization of the 50 time intervals was 5 minutes. After completing the task, the students' observations were transcribed onto data cards for storage and subsequent processing.

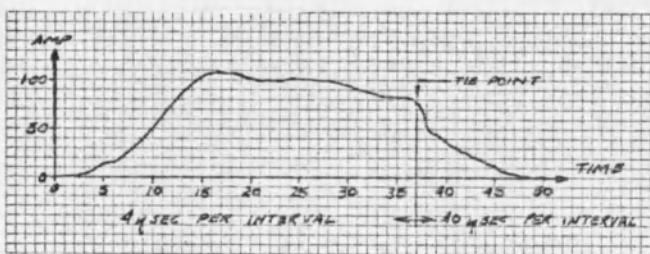
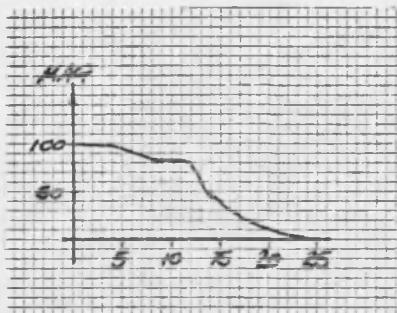
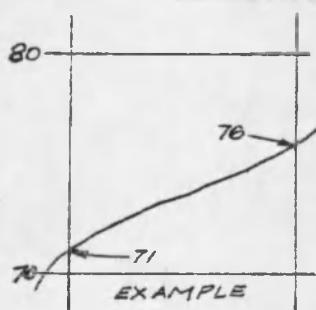
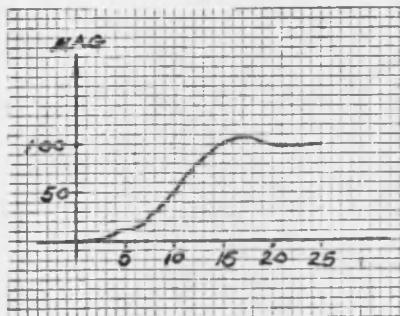


Figure 2.3. Time Tied Waveform Copy with Superimposed Reference Grid

Please indicate your estimate of the magnitude for each of the 25 points.



1-
2-
3-
4-
5-
6-
7-
8-
9-
10-
11-
12-
13-
14-
15-
16-
17-
18-
19-
20-
21-
22-
23-
24-
25-

1-
2-
3-
4-
5-
6-
7-
8-
9-
10-
11-
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23-
24-
25-

Figure 2.4. Digitalization Work Sheet

CHAPTER 3

DATA REDUCTION

3.1 Time Domain Reduction

A flow chart of the following narrative sequence is given in Figure 3.1. The first task in the sequence of analyzing the time domain data was to reject those data points that did not meet Chauvenet's Criterion (Section A.11 in Appendix A) using program TIMEA. The edited data were then processed to find their mean, standard deviation, skewness, and kurtosis with program TIMEB. The edited time domain data and moment statistics were then processed and displayed using the following programs:

1. A display of the mean and the respective high and low values for each of the 50 time intervals (program HILO) (see Figure 3.2).
2. A display of the PDF's and CDF's with a listing of the mean, standard deviation, skew, and kurtosis for each of the 50 time intervals (program CDFPDF) (see Figure 3.3 for the explanation of display notation, and Figures 3.4 through 3.53).
3. A display of the slope at each time interval versus its standard deviation (program SLVSD) (see Figure 3.54).

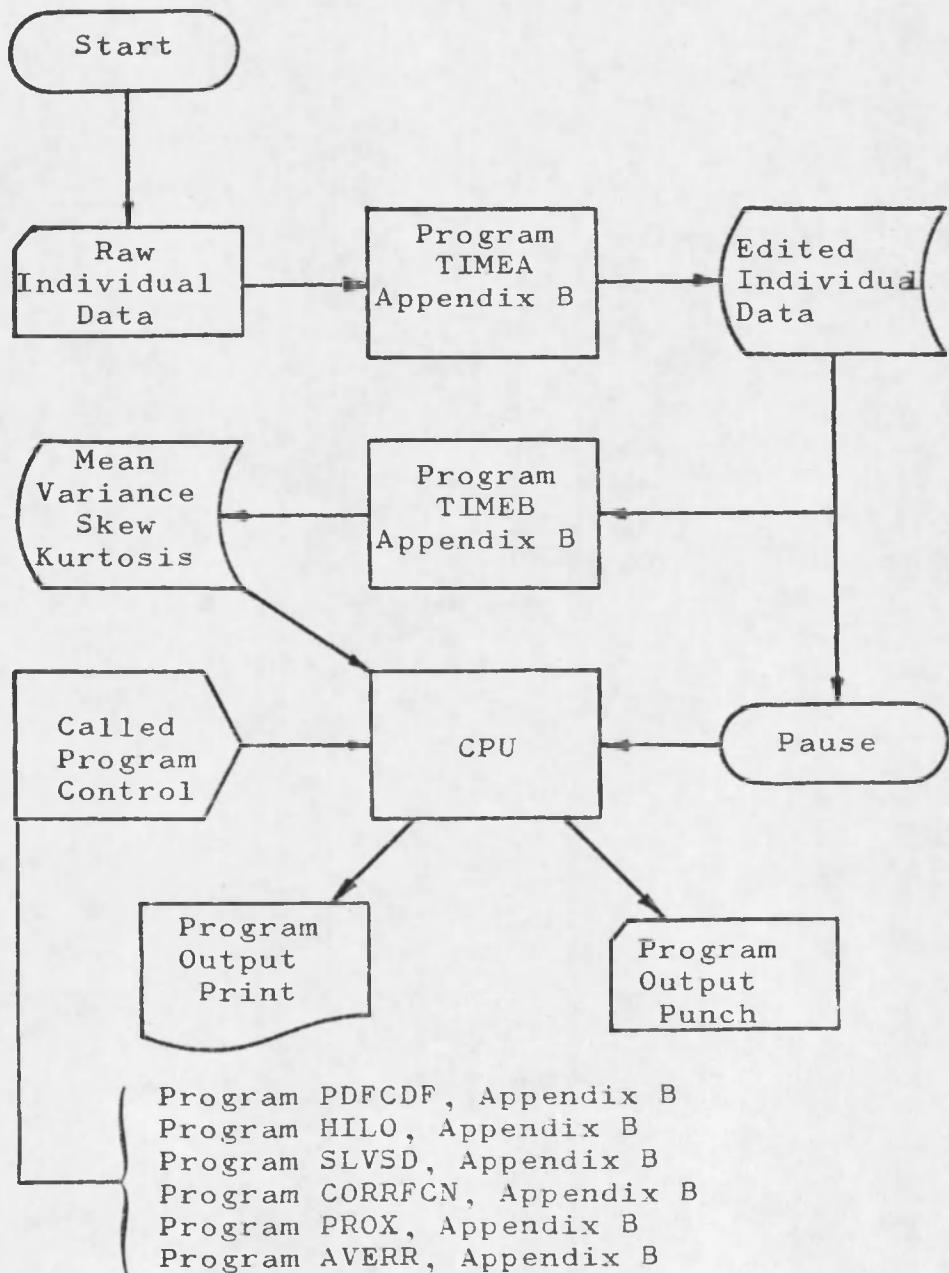


Figure 3.1. Time Domain Analysis Flow Chart

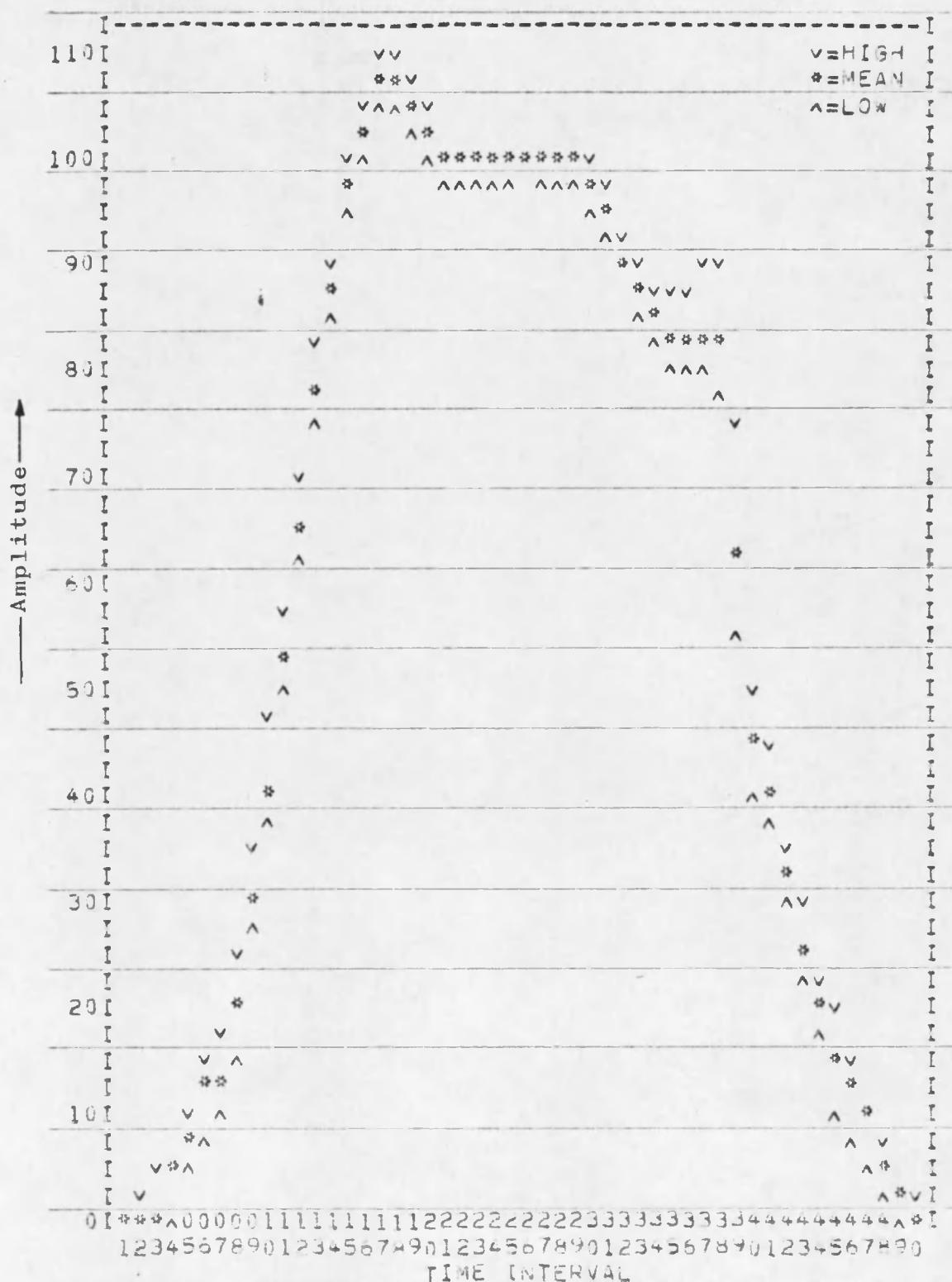


Figure 3.2. High-Mean-Low Time Domain Values

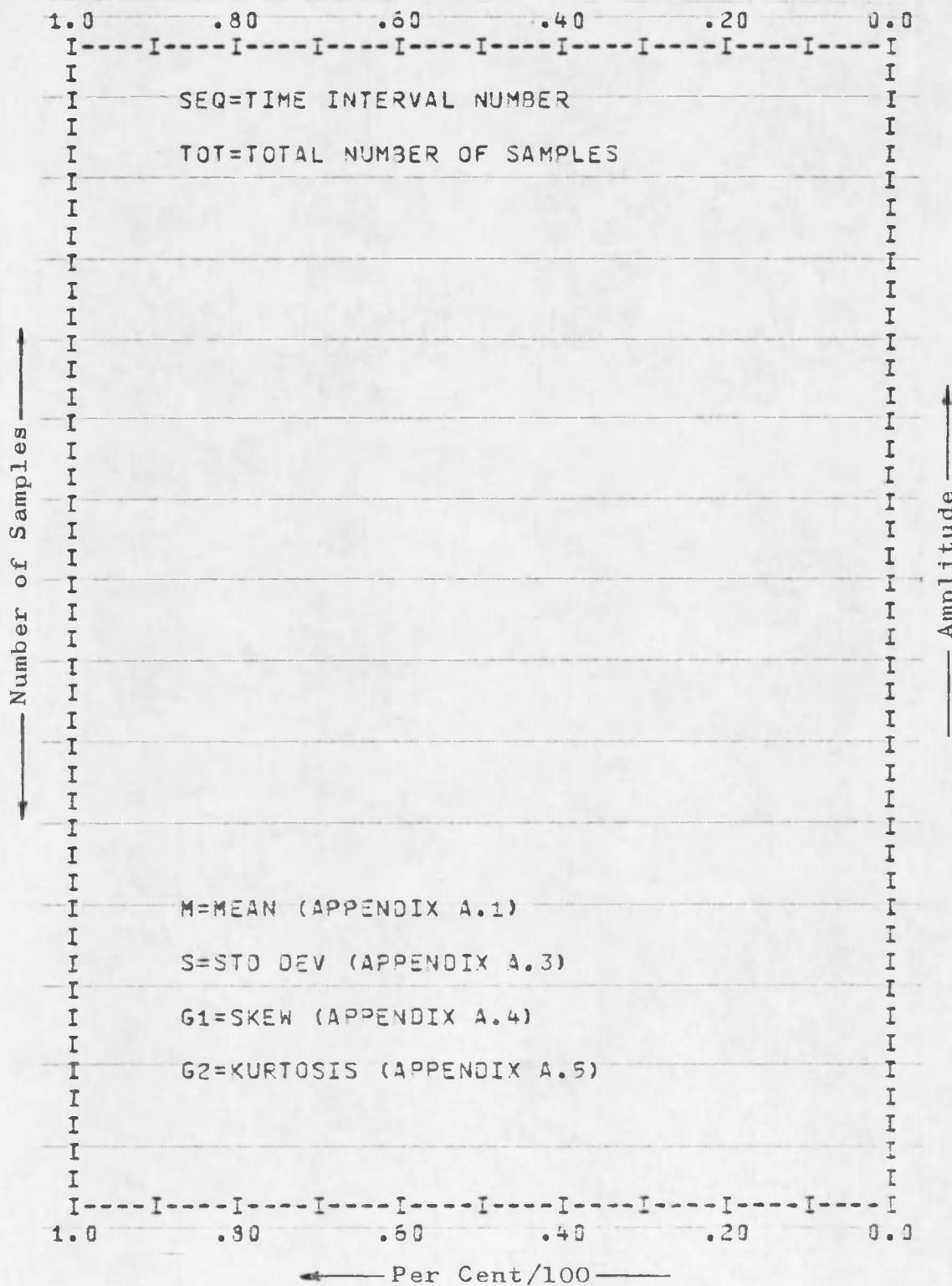


Figure 3.3. Explanation of the Display Nomenclature for Figures 3.4 through 3.53

The figure displays a distribution curve with the following parameters:

- Top x-axis values: .80, .60, .40, .20, 0.0
- Bottom x-axis values: .80, .60, .40, .20, 0.0
- Y-axis values: 1.0, .80, .60, .40, .20, 0.0 (increasing from top to bottom)
- Labels:
 - SET = 1
 - TOT = 188
 - M = 0.00 S = 0.00
 - G1 = 0.00 G2 = 0.00
 - + = PDF * = CDF
- Note: See Figure 3.3 for an explanation of the display nomenclature.

Figure 3.4. Statistics for Time Interval 1

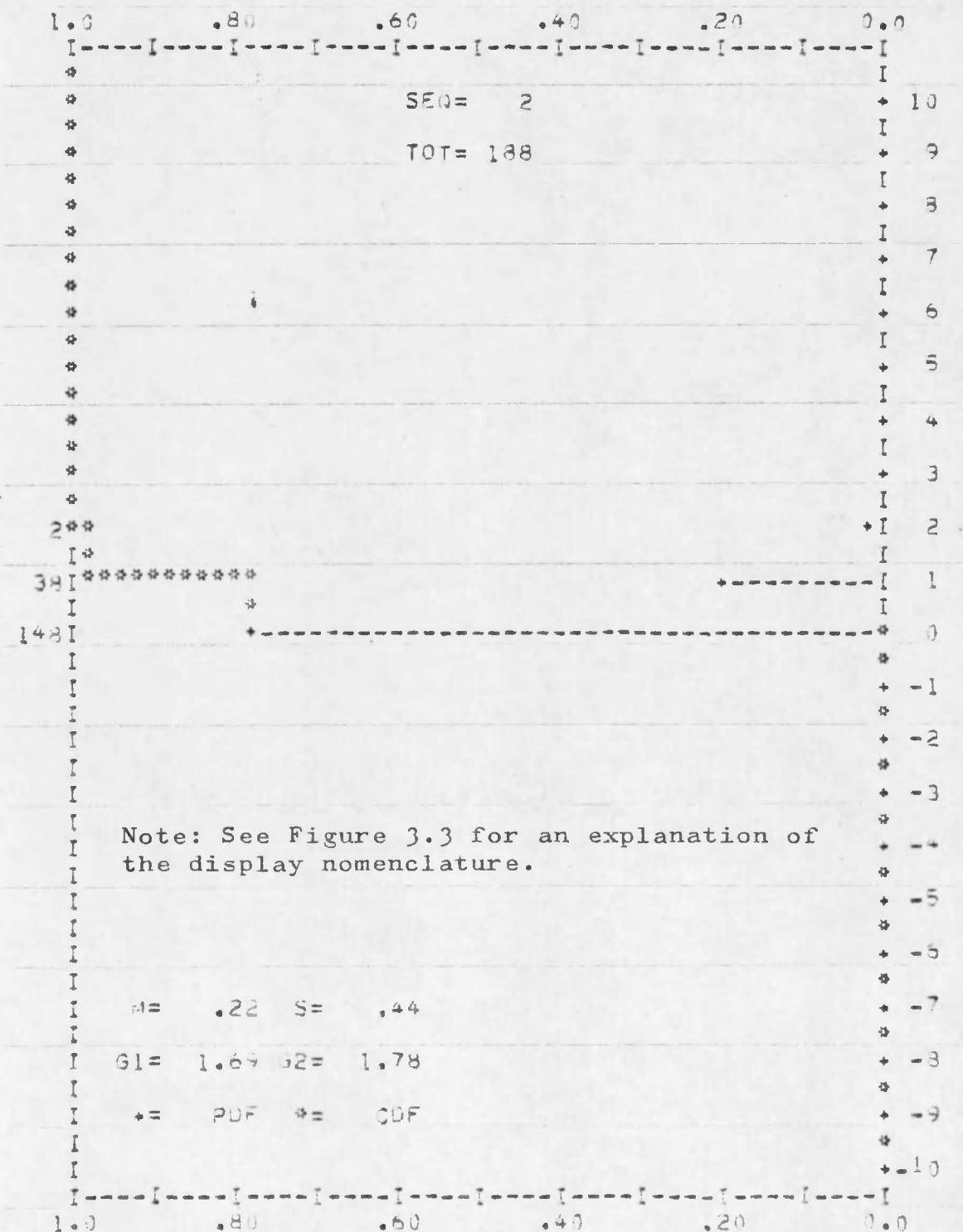


Figure 3.5. Statistics for Time Interval 2

Figure 3.6. Statistics for Time Interval 3

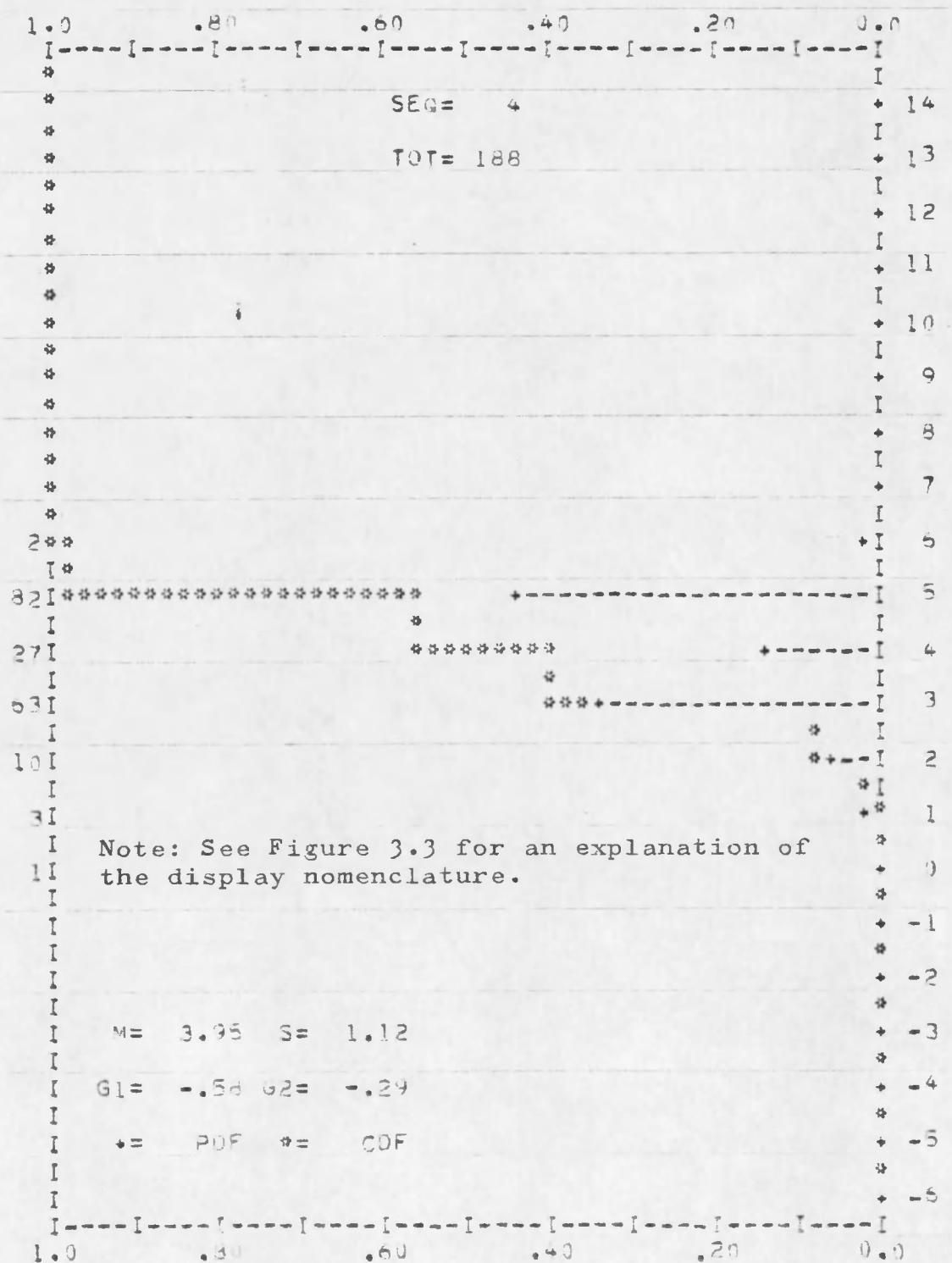


Figure 3.7. Statistics for Time Interval 4

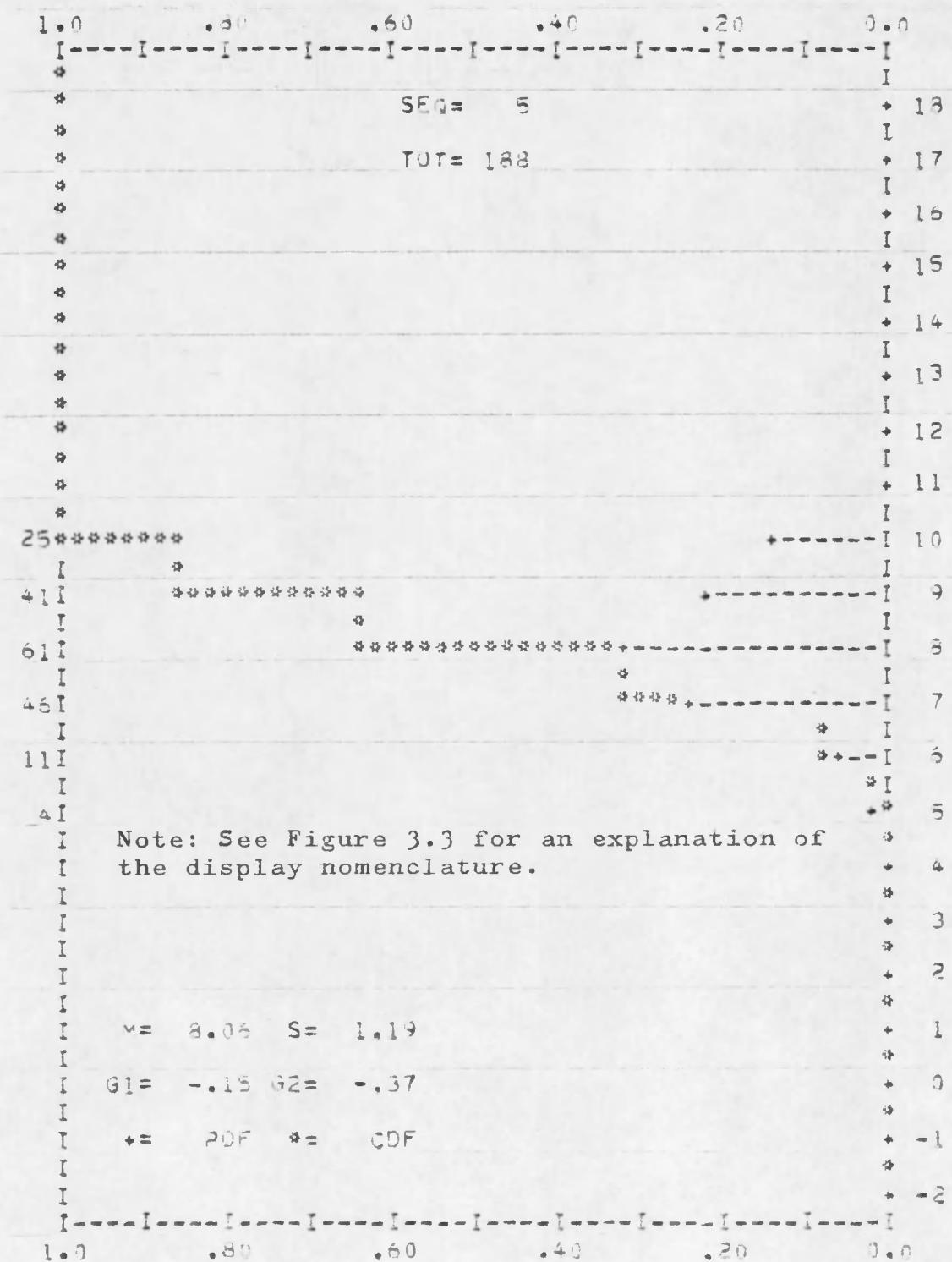


Figure 3.8. Statistics for Time Interval 5

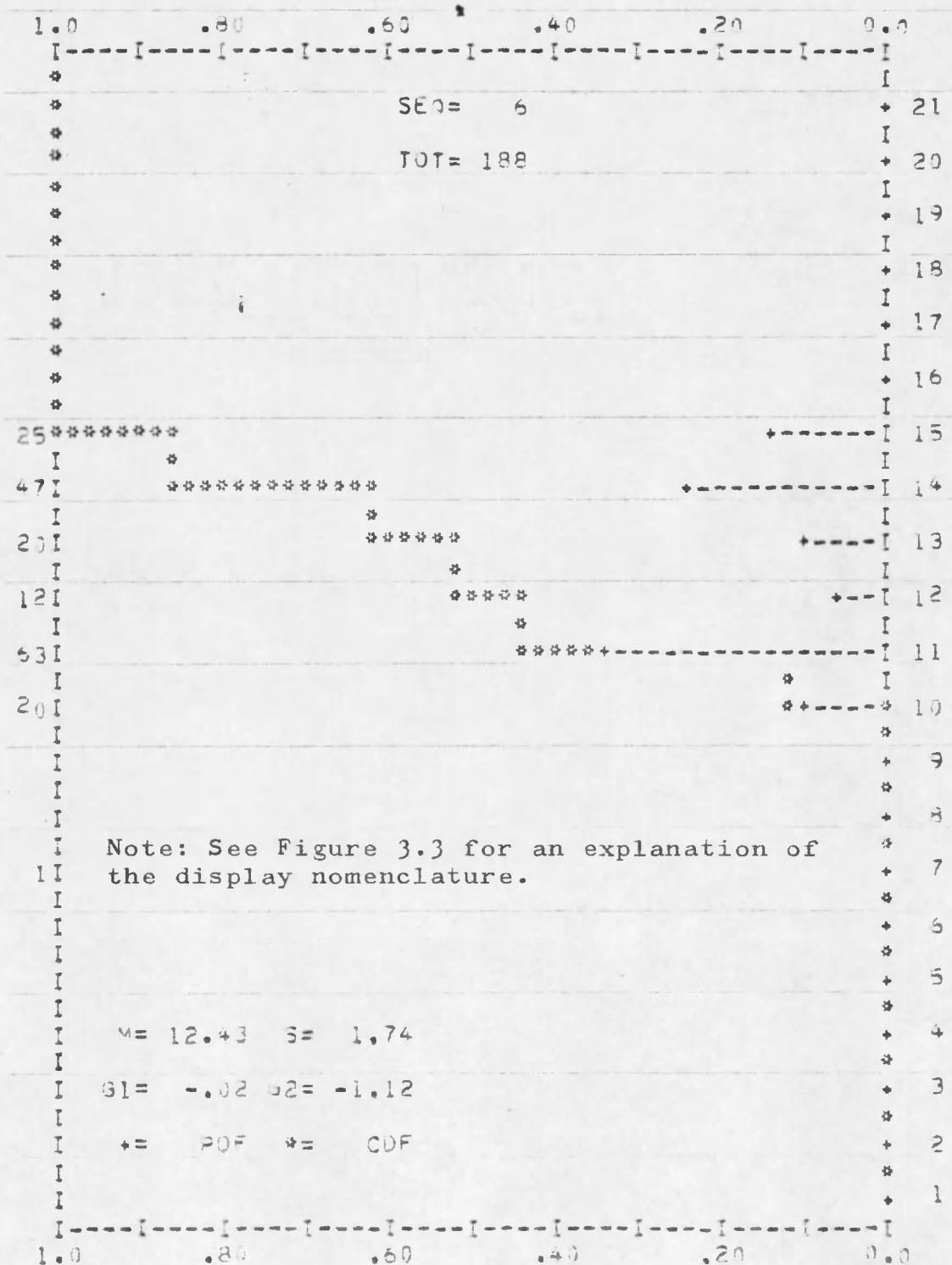


Figure 3.9. Statistics for Time Interval 6

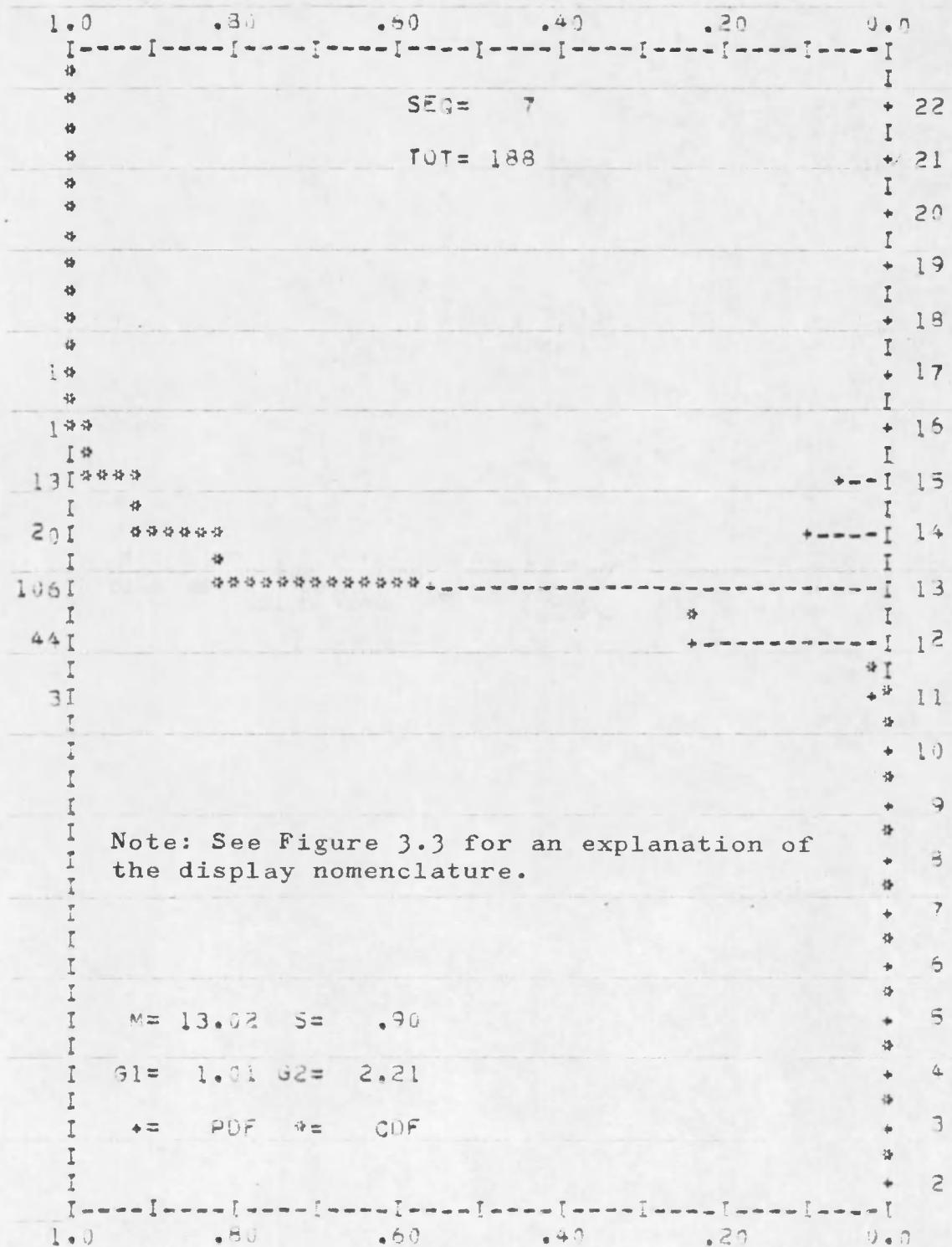


Figure 3.10. Statistics for Time Interval 7

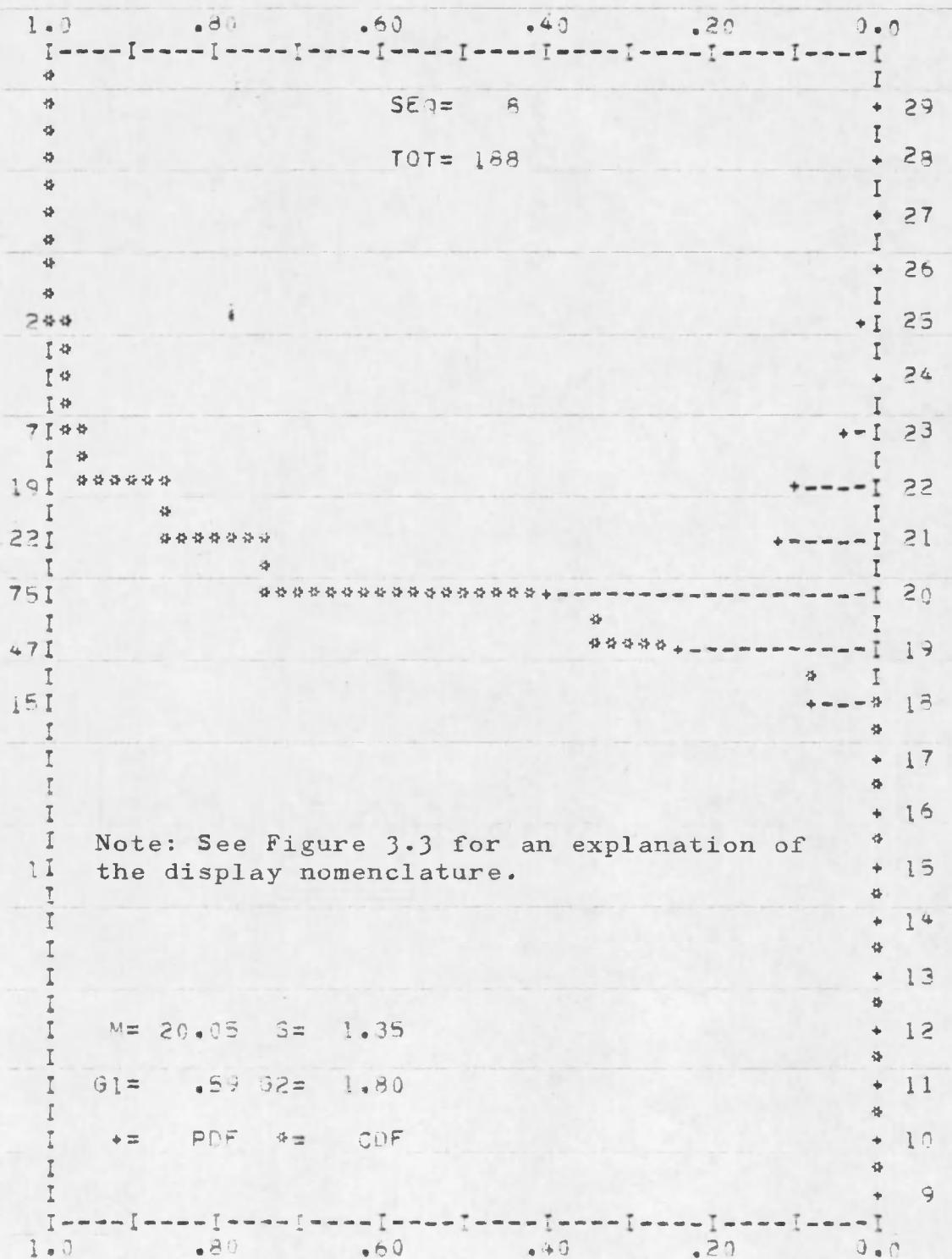


Figure 3.11. Statistics for Time Interval 8

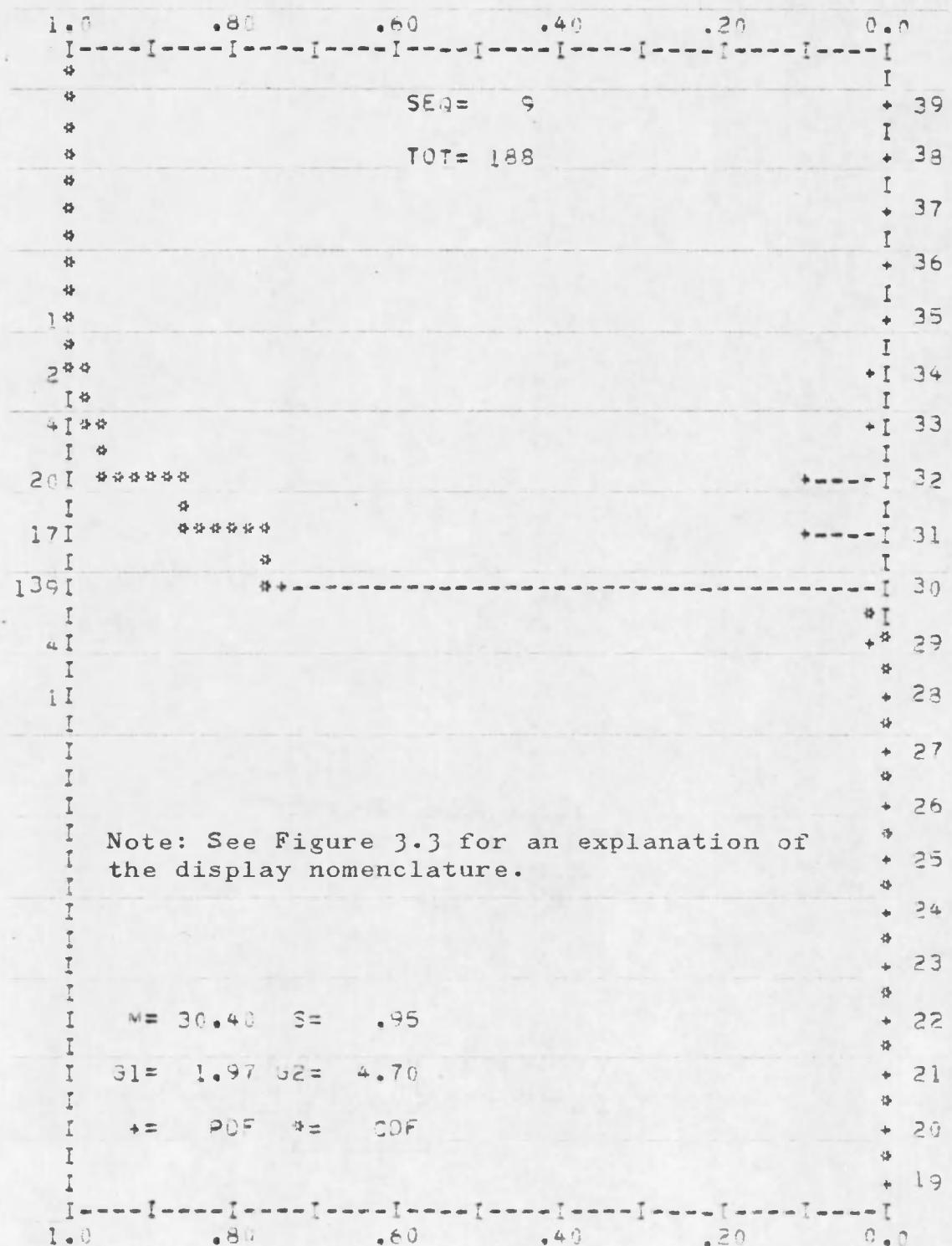


Figure 3.12. Statistics for Time Interval 9

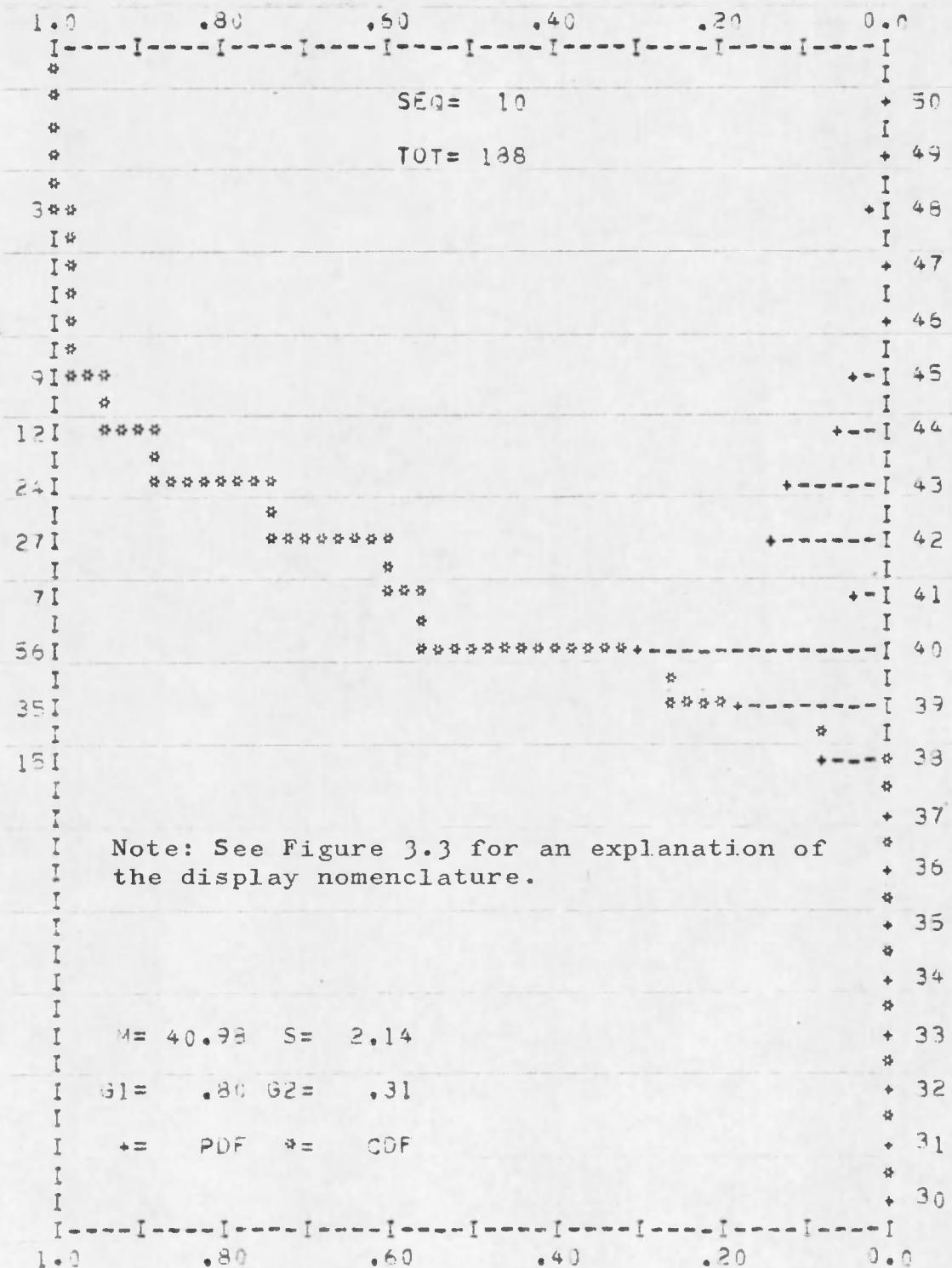


Figure 3.13. Statistics for Time Interval 10

1.0 .80 .60 .40 .20 0.0
 -----|-----|-----|-----|-----|-----|
 *
 * SEQ= 11 + 52
 * TOT= 188 + 61
 *
 *
 *
 *
 *
 *
 *
 4** + 60
 I + 59
 *
 6I*** + 58
 I + 57
 18I **** + 56
 I *
 52I ***** + 55
 I *
 7I *** + 54
 I *
 7I *** + 53
 I *
 32I ***** + 52
 I *
 9I *** + 51
 I *
 52I ***** + 50
 I *
 1I * 49
 Note: See Figure 3.3 for an explanation of * 48
 the display nomenclature. * 47
 *
 *
 *
 *
 M= 52.99 S= 2.45 + 46
 G1= .09 G2= -1.37 + 45
 *
 * PDF *= CDF + 44
 *
 *
 *
 *
 *
 *
 1.0 .80 .60 .40 .20 0.0

Figure 3.14. Statistics for Time Interval 11

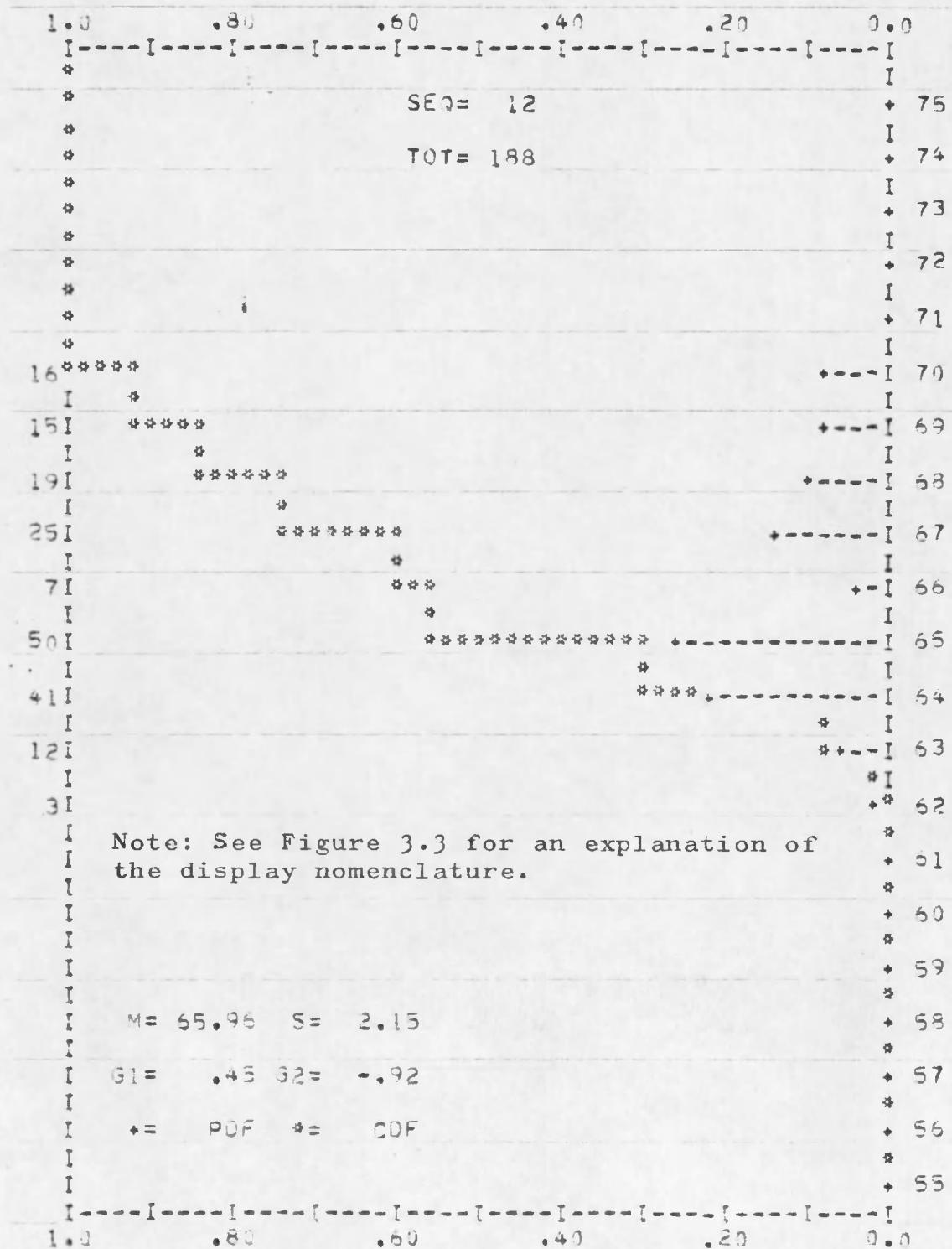


Figure 3.15. Statistics for Time Interval 12

Figure 3.16. Statistics for Time Interval 13

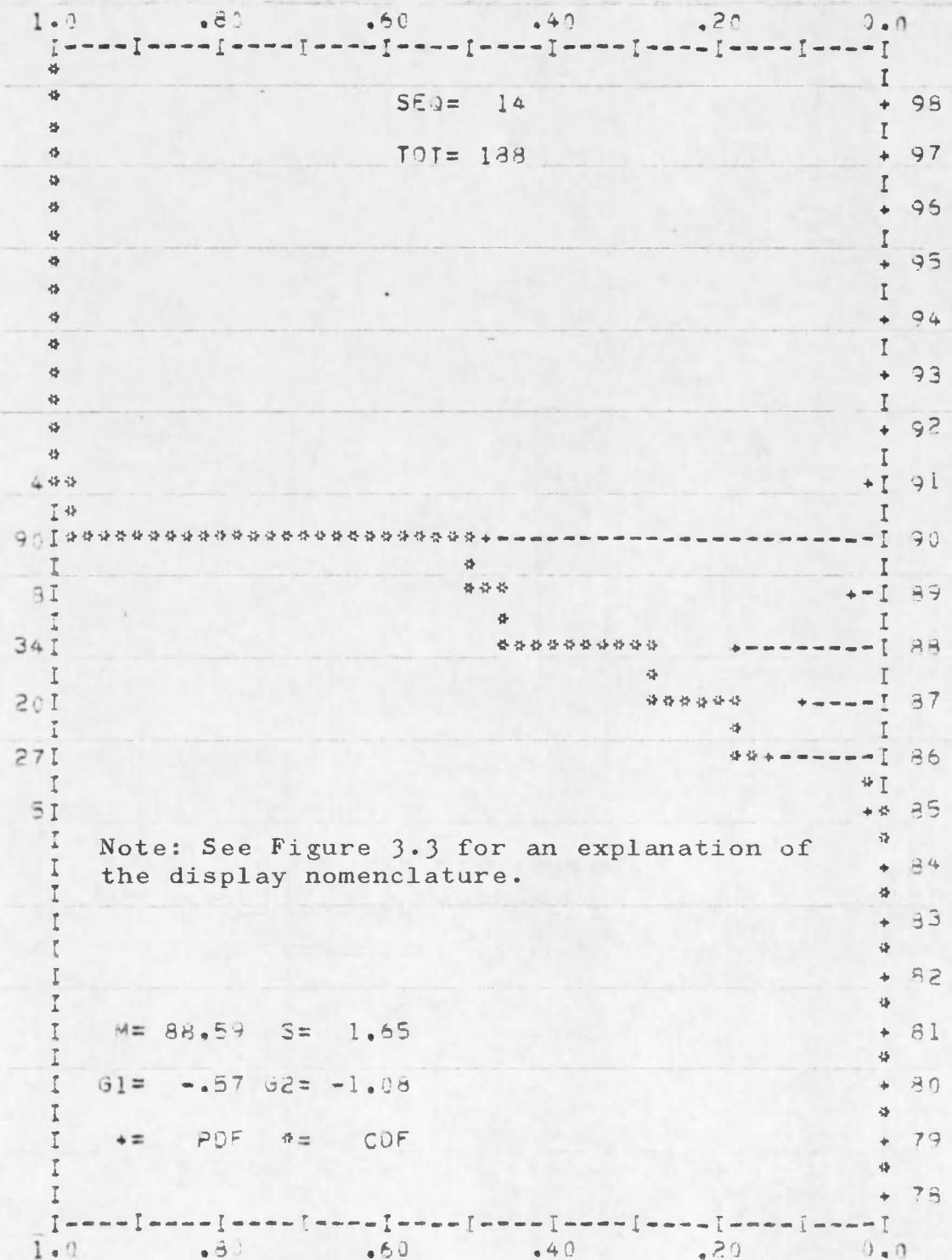


Figure 3.17. Statistics for Time Interval 14

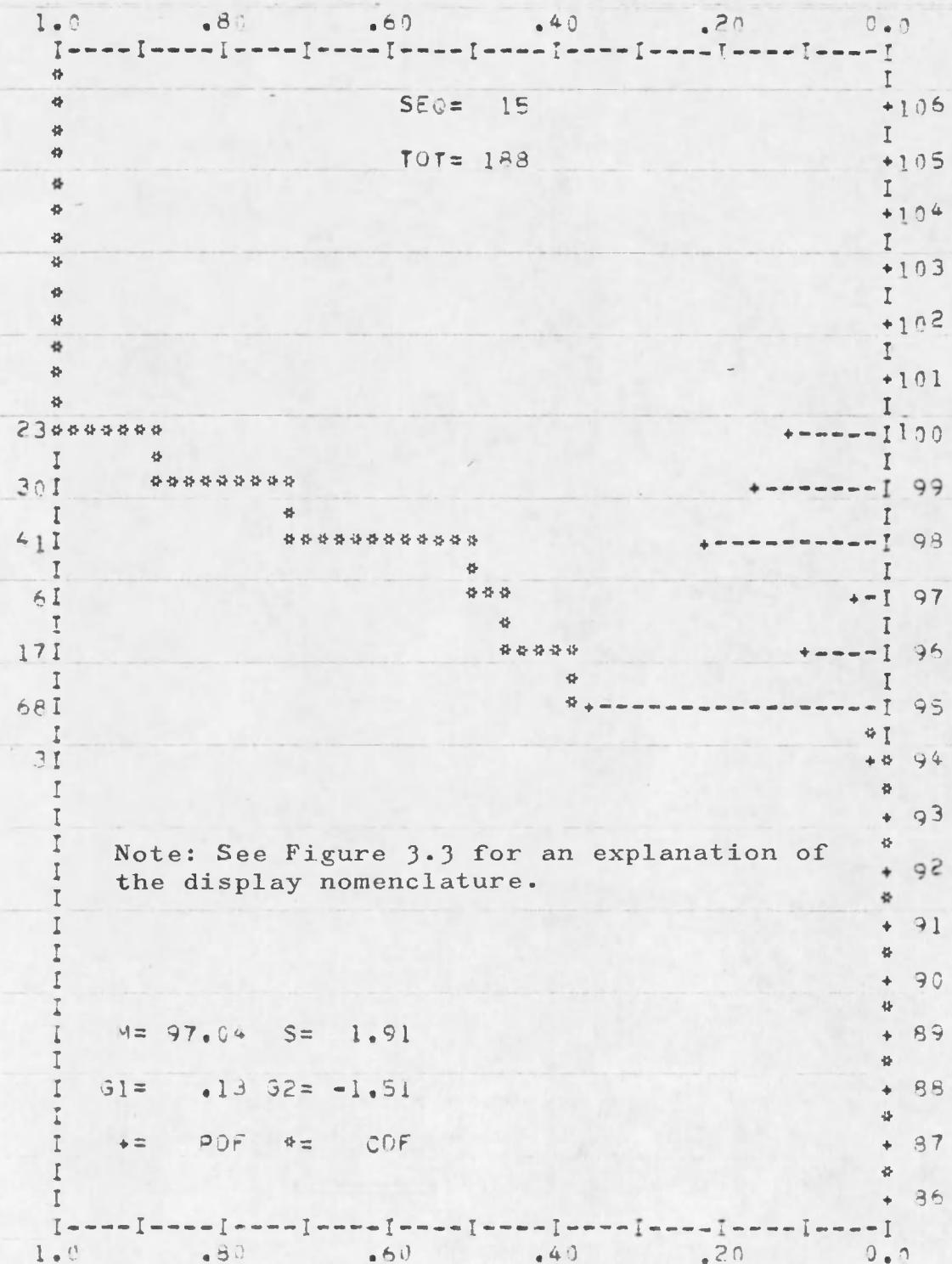


Figure 3.18. Statistics for Time Interval 15

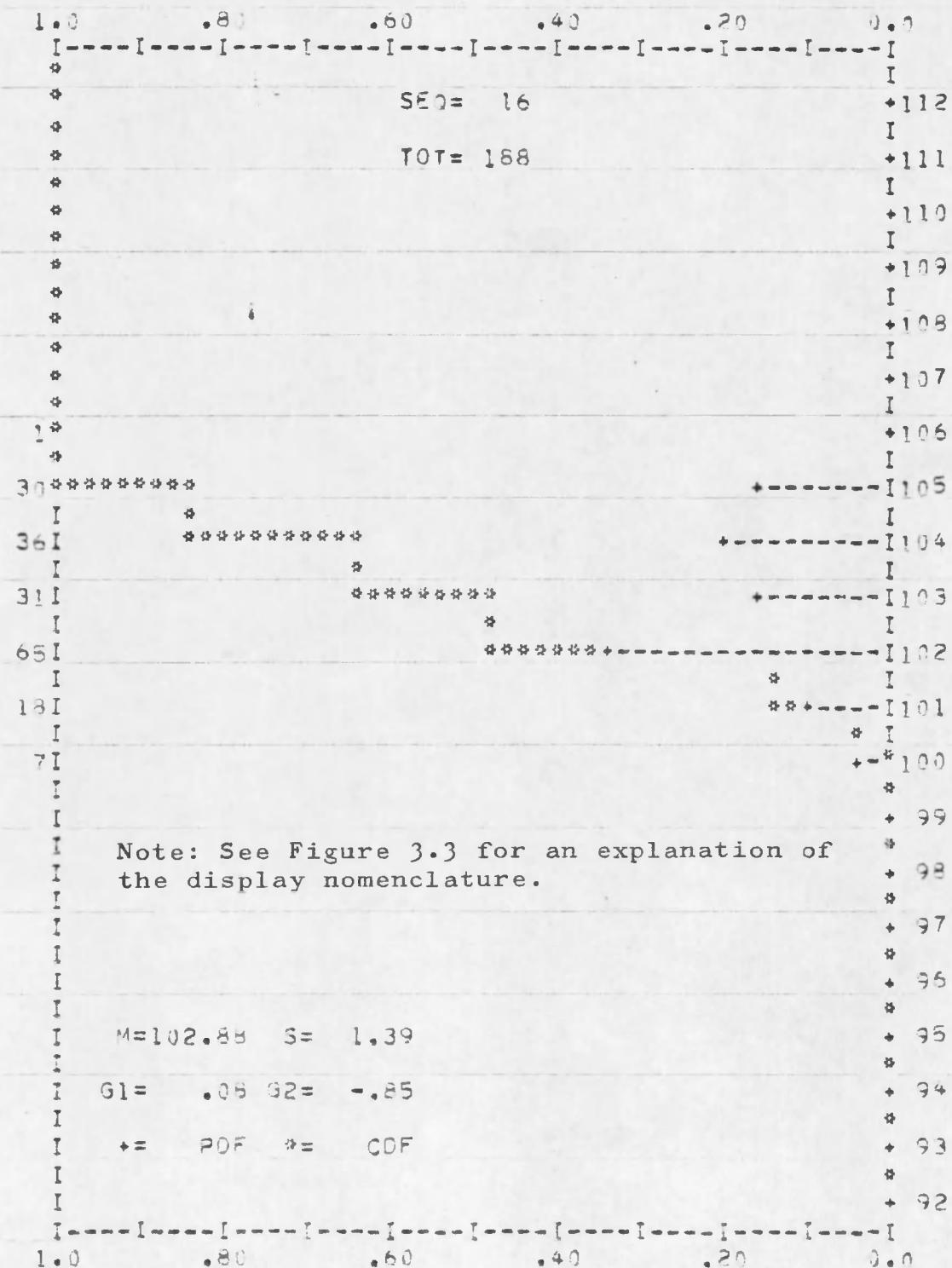


Figure 3.19. Statistics for Time Interval 16

Figure 3.20. Statistics for Time Interval 17

Figure 3.21. Statistics for Time Interval 18

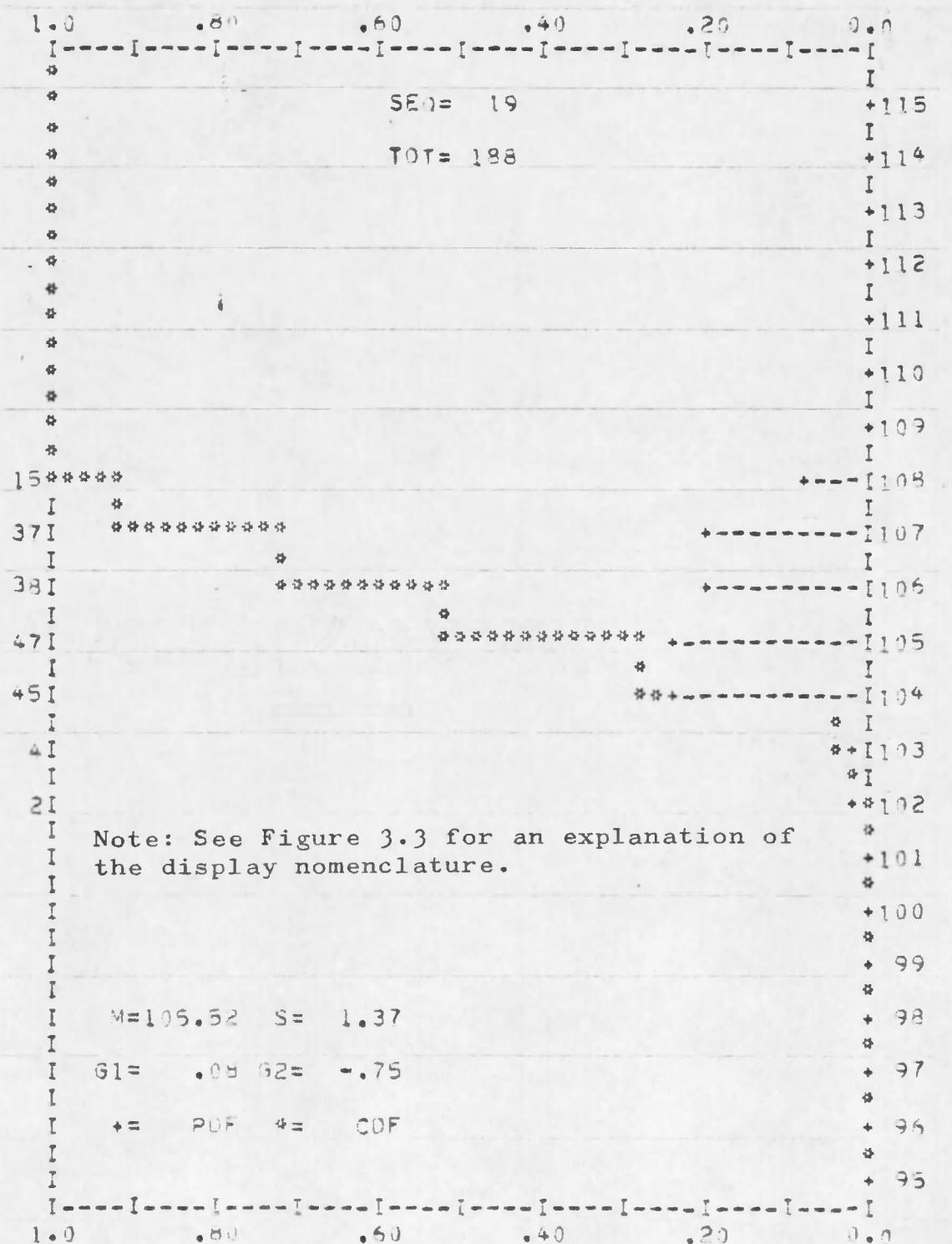


Figure 3.22. Statistics for Time Interval 19

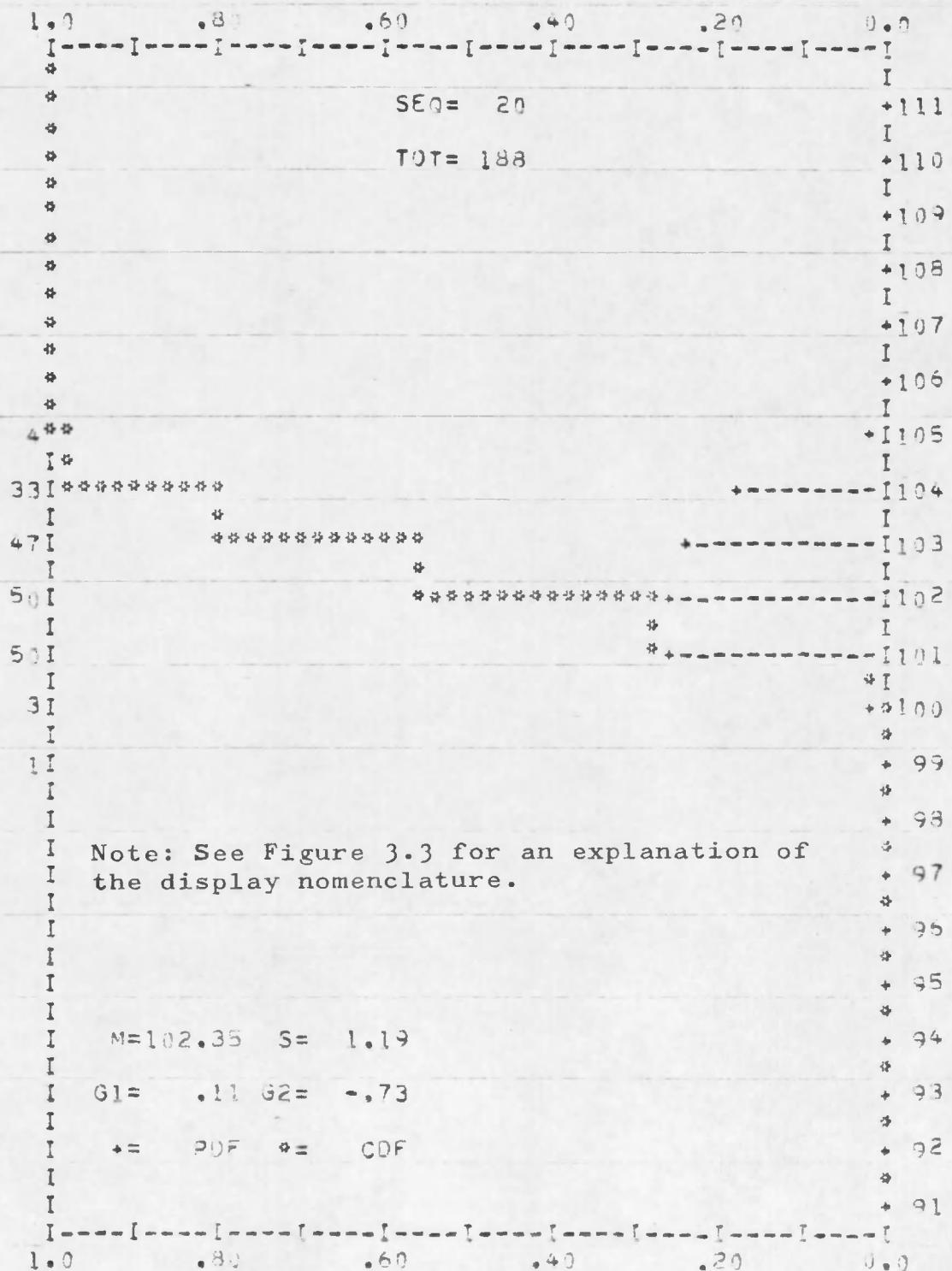


Figure 3.23. Statistics for Time Interval 20

Figure 3.24. Statistics for Time Interval 21

Figure 3.25. Statistics for Time Interval 22

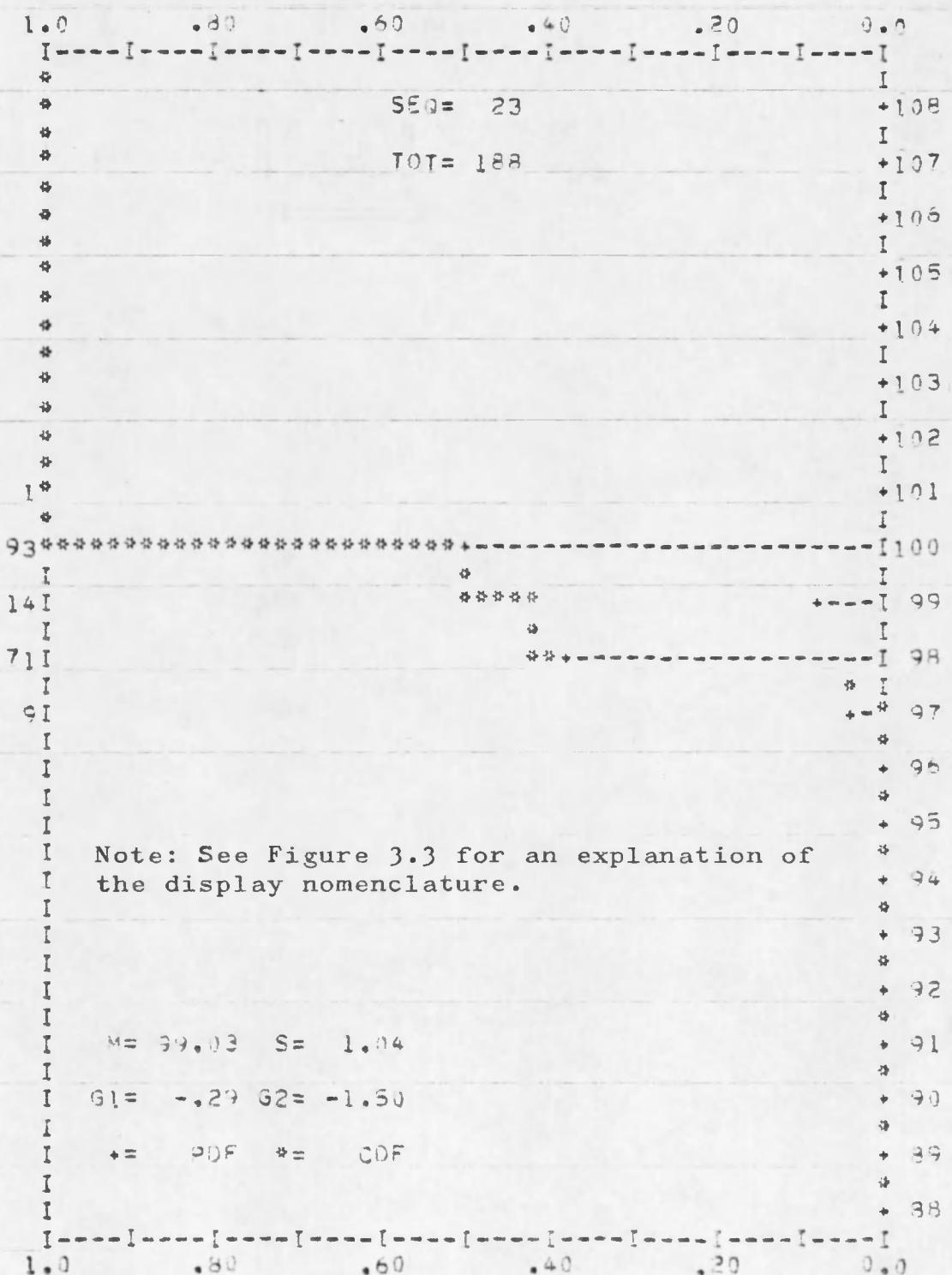


Figure 3.26. Statistics for Time Interval 23

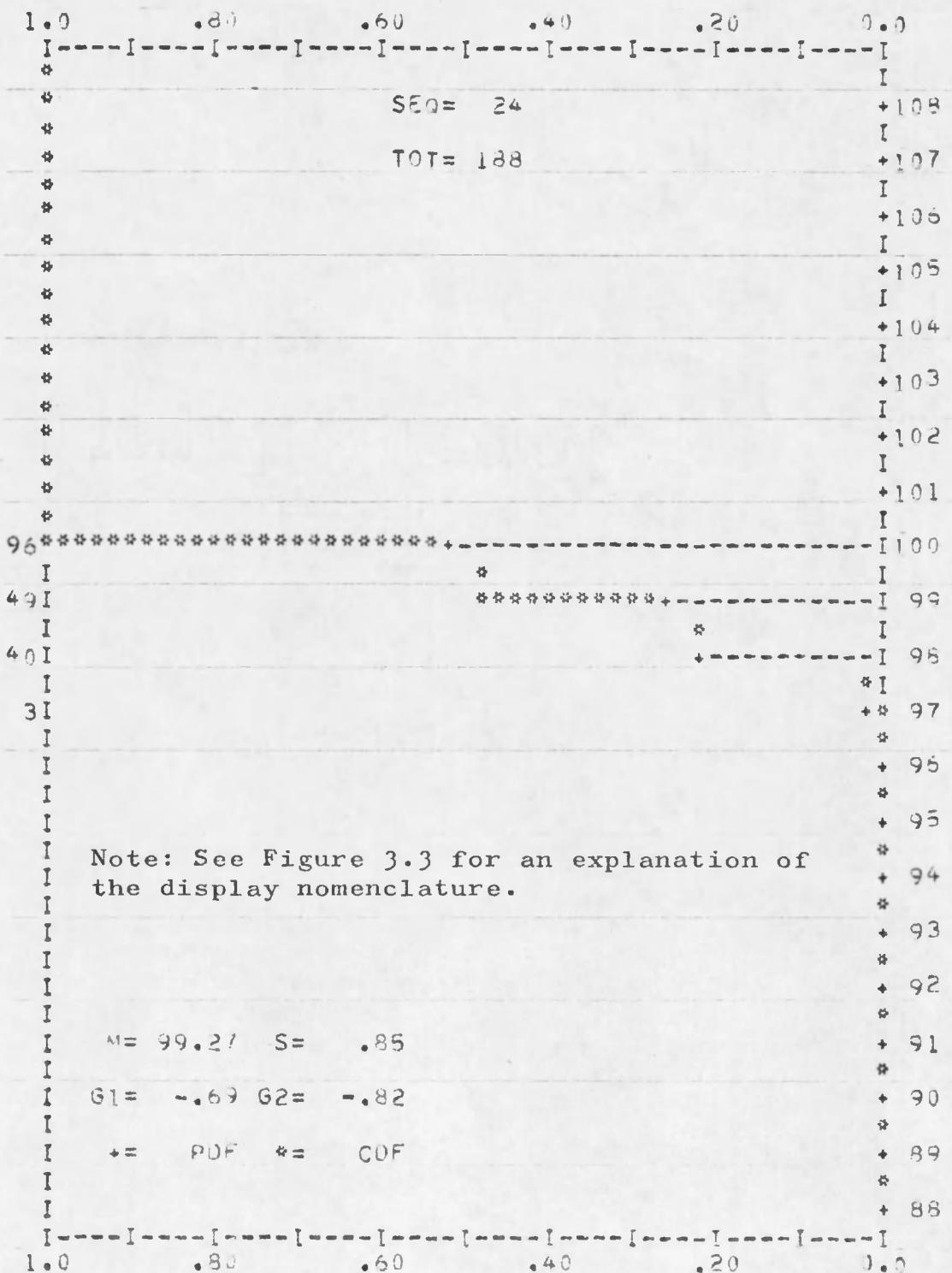


Figure 3.27. Statistics for Time Interval 24

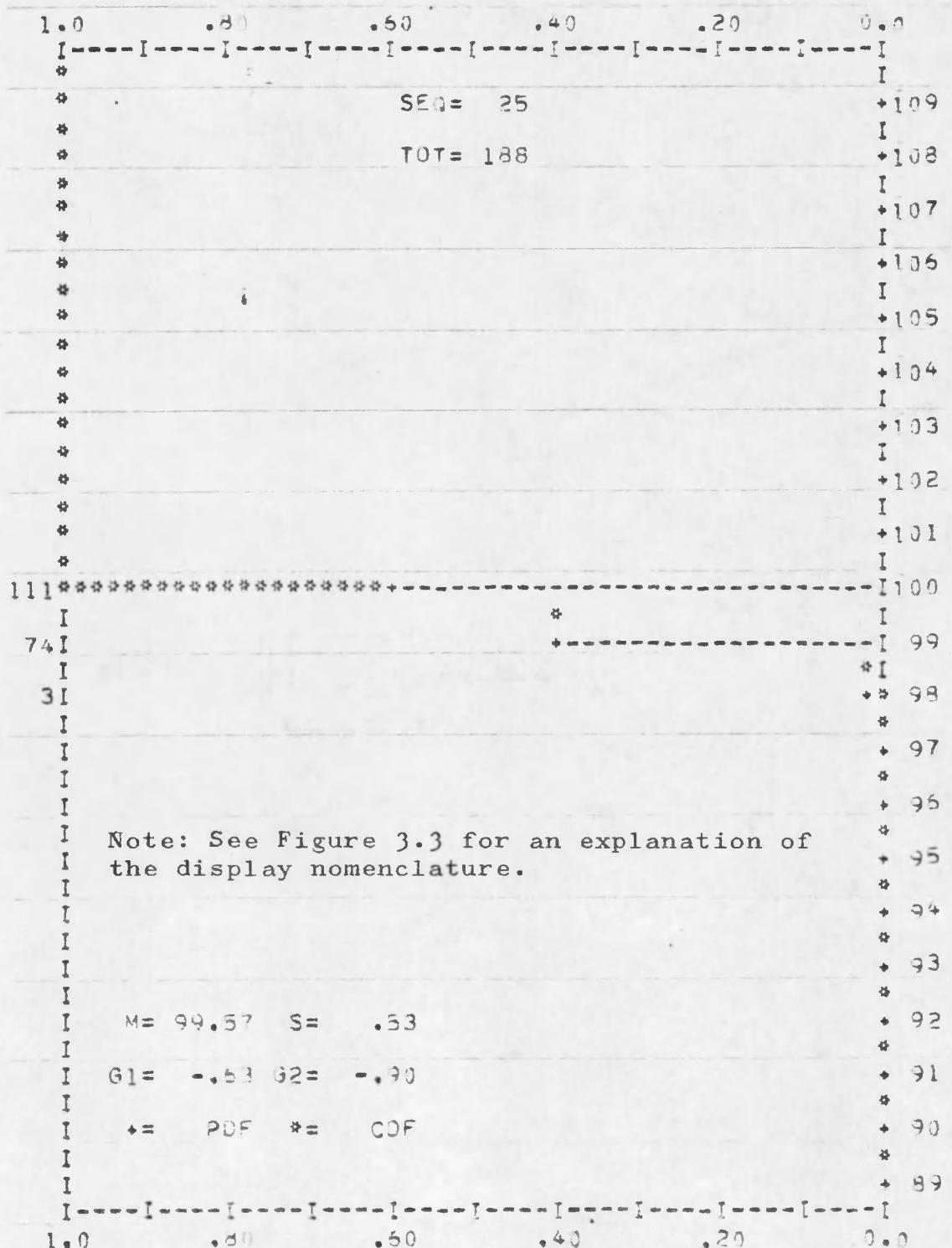


Figure 3.28. Statistics for Time Interval 25

Figure 3.29. Statistics for Time Interval 26

Figure 3.30. Statistics for Time Interval 27

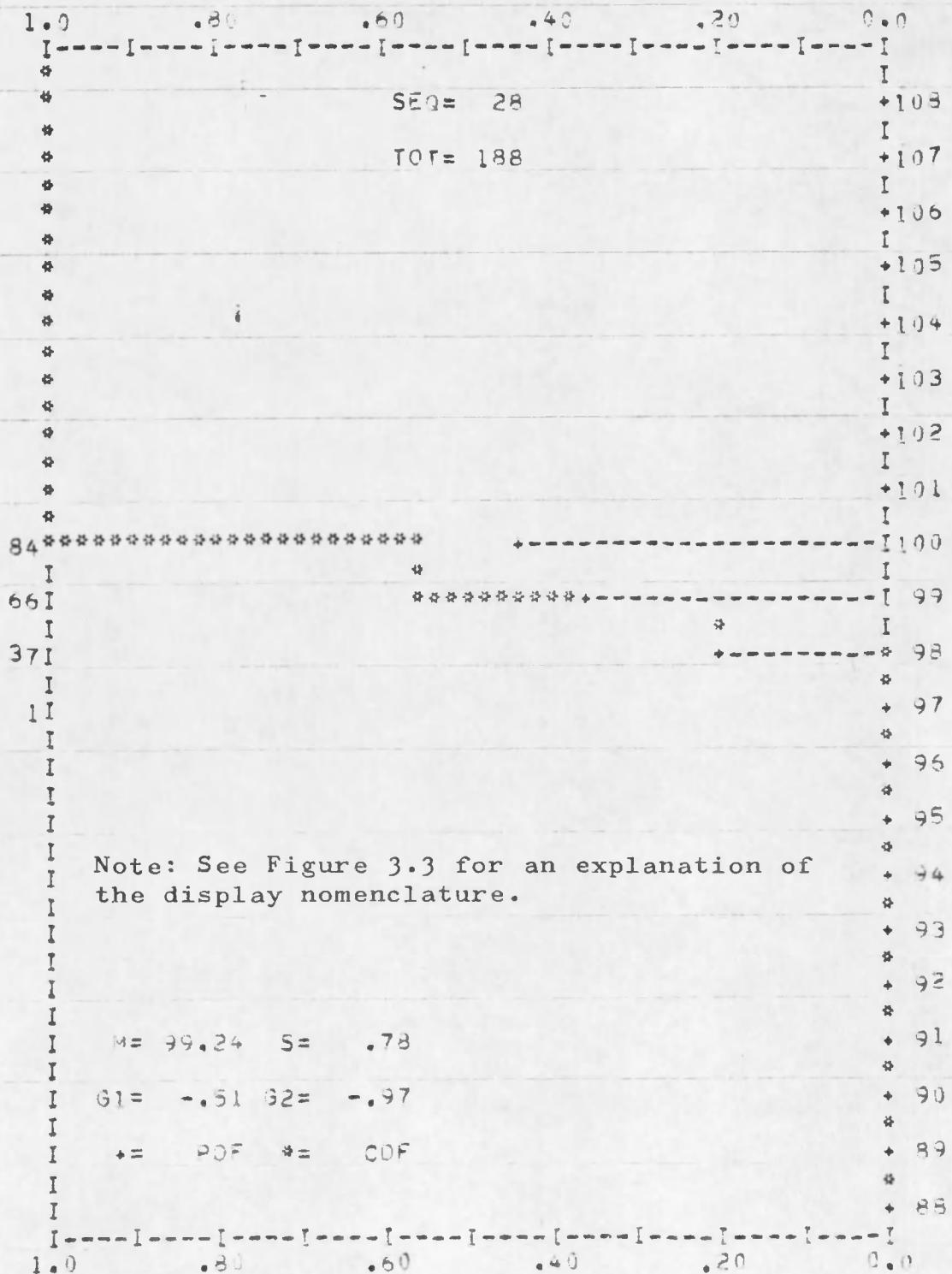


Figure 3.31. Statistics for Time Interval 28

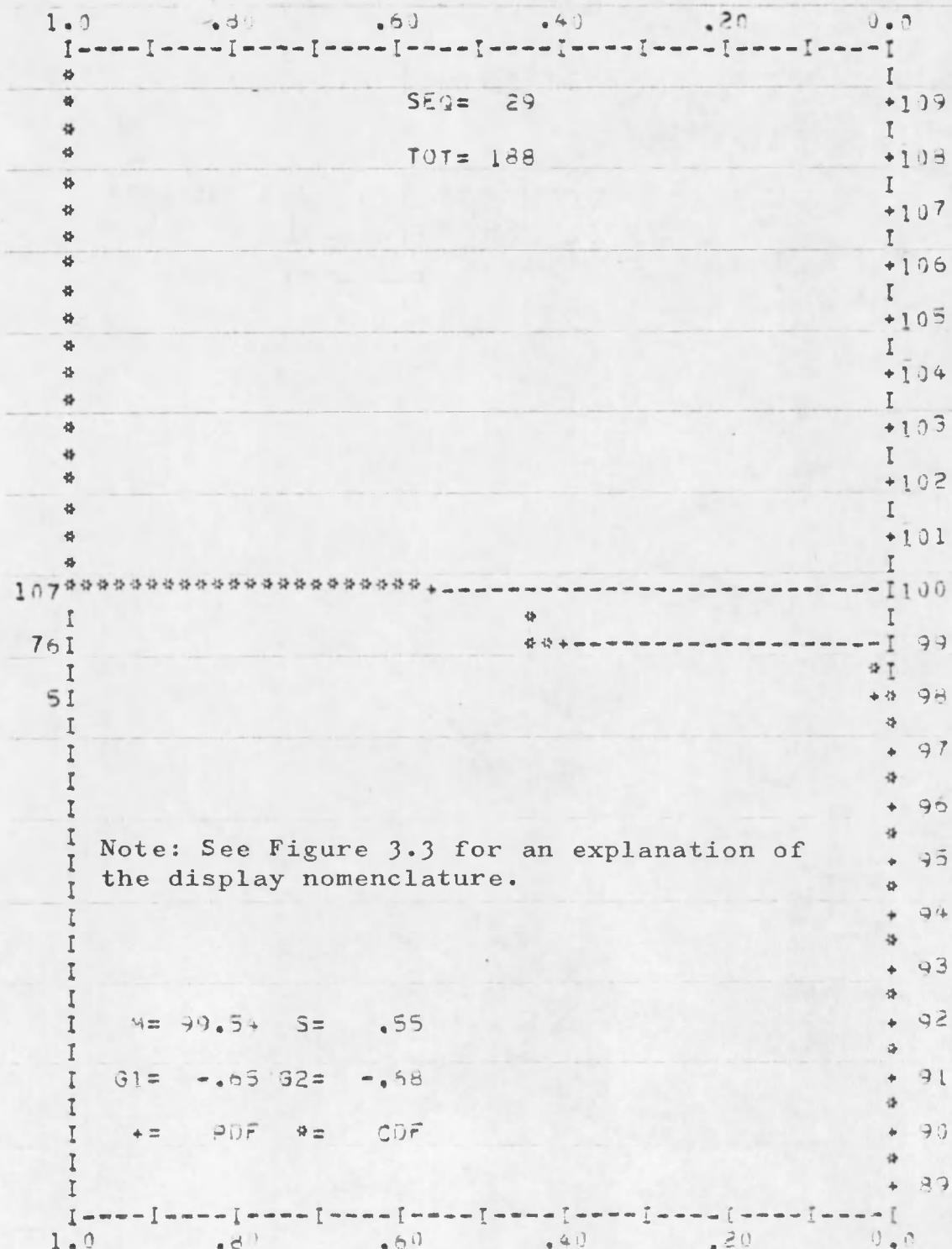


Figure 3.32. Statistics for Time Interval 29

Figure 3.33. Statistics for Time Interval 30

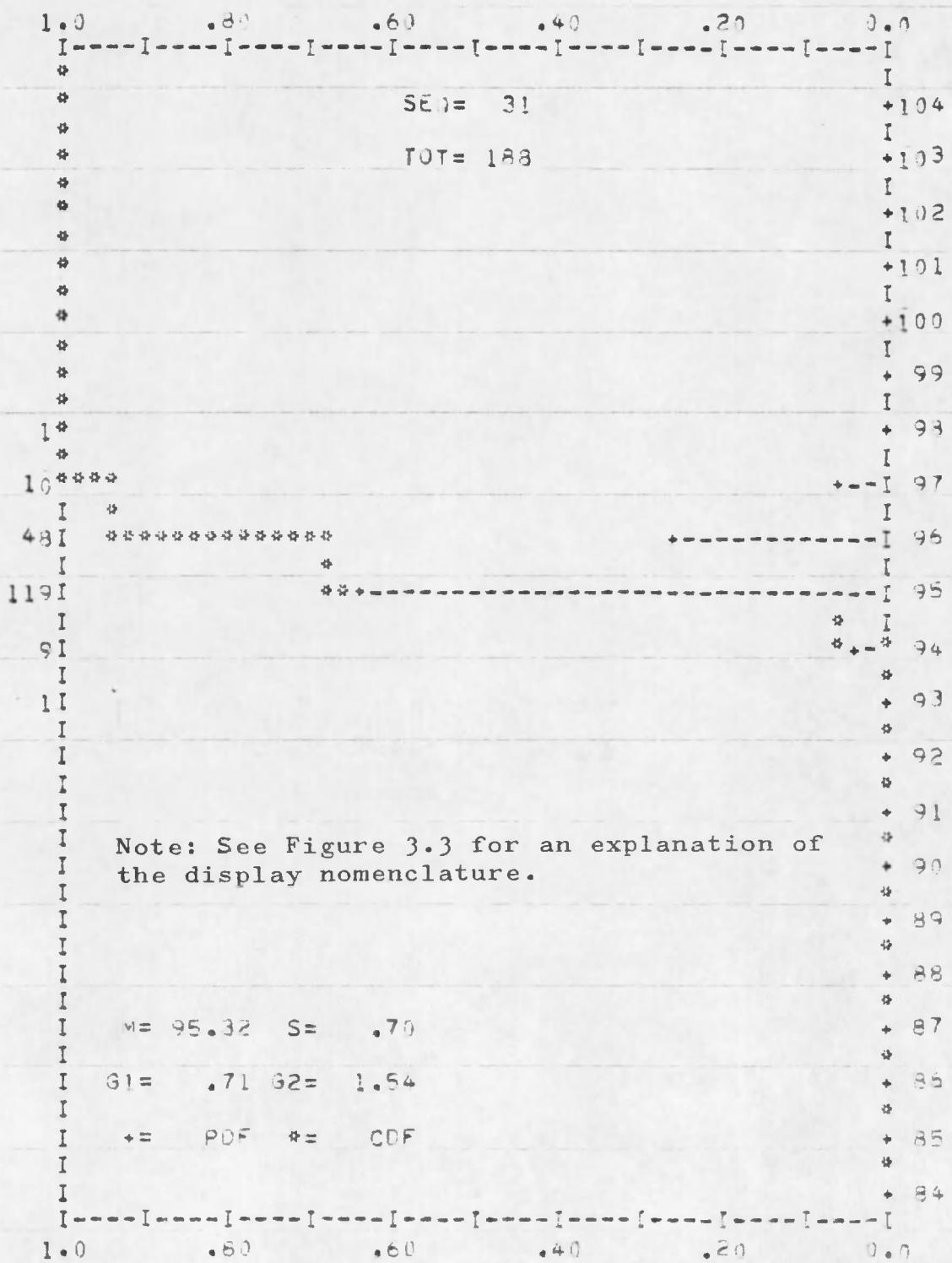


Figure 3.34. Statistics for Time Interval 31

Figure 3.35. Statistics for Time Interval 32

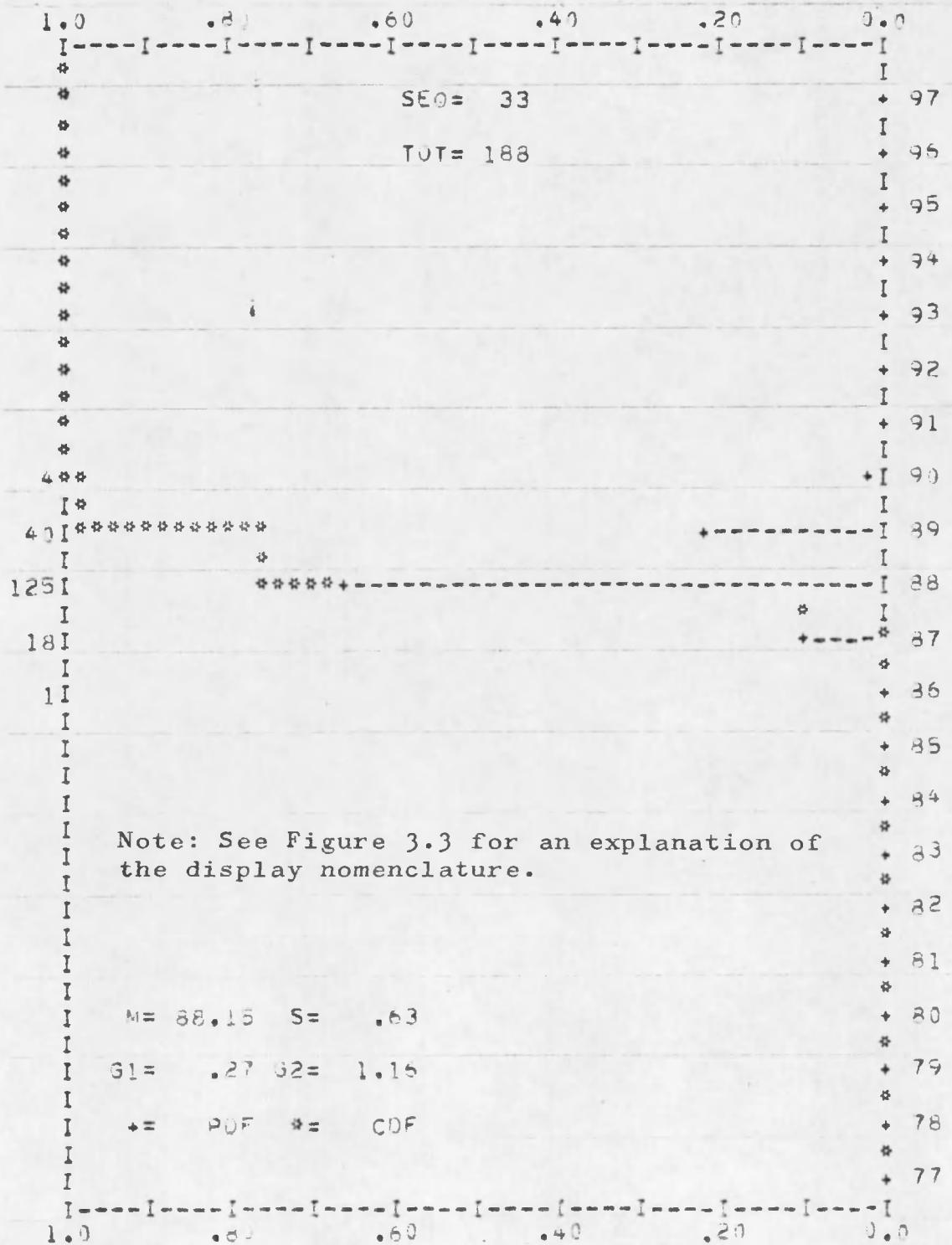


Figure 3.36. Statistics for Time Interval 33

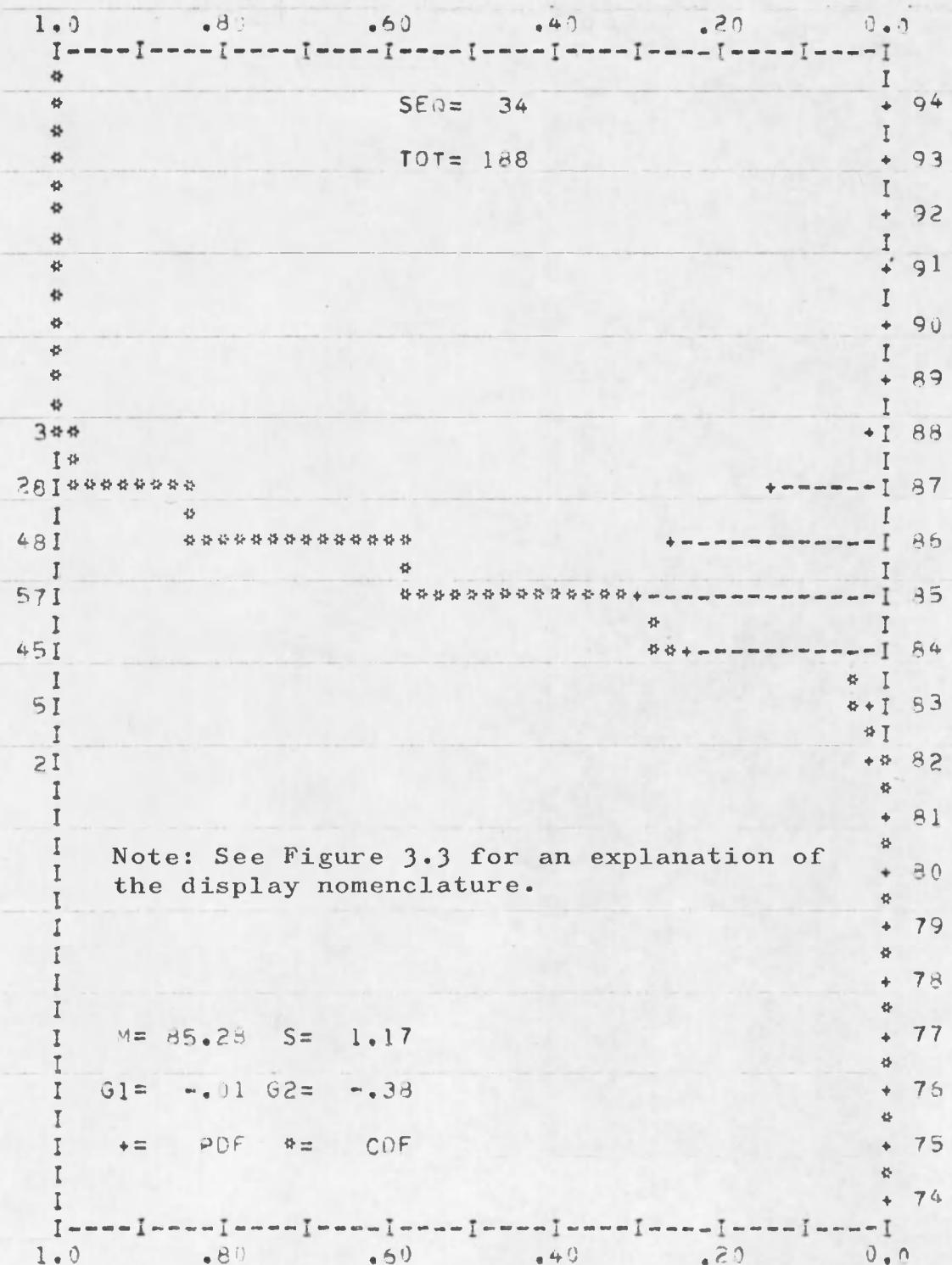


Figure 3.37. Statistics for Time Interval 34

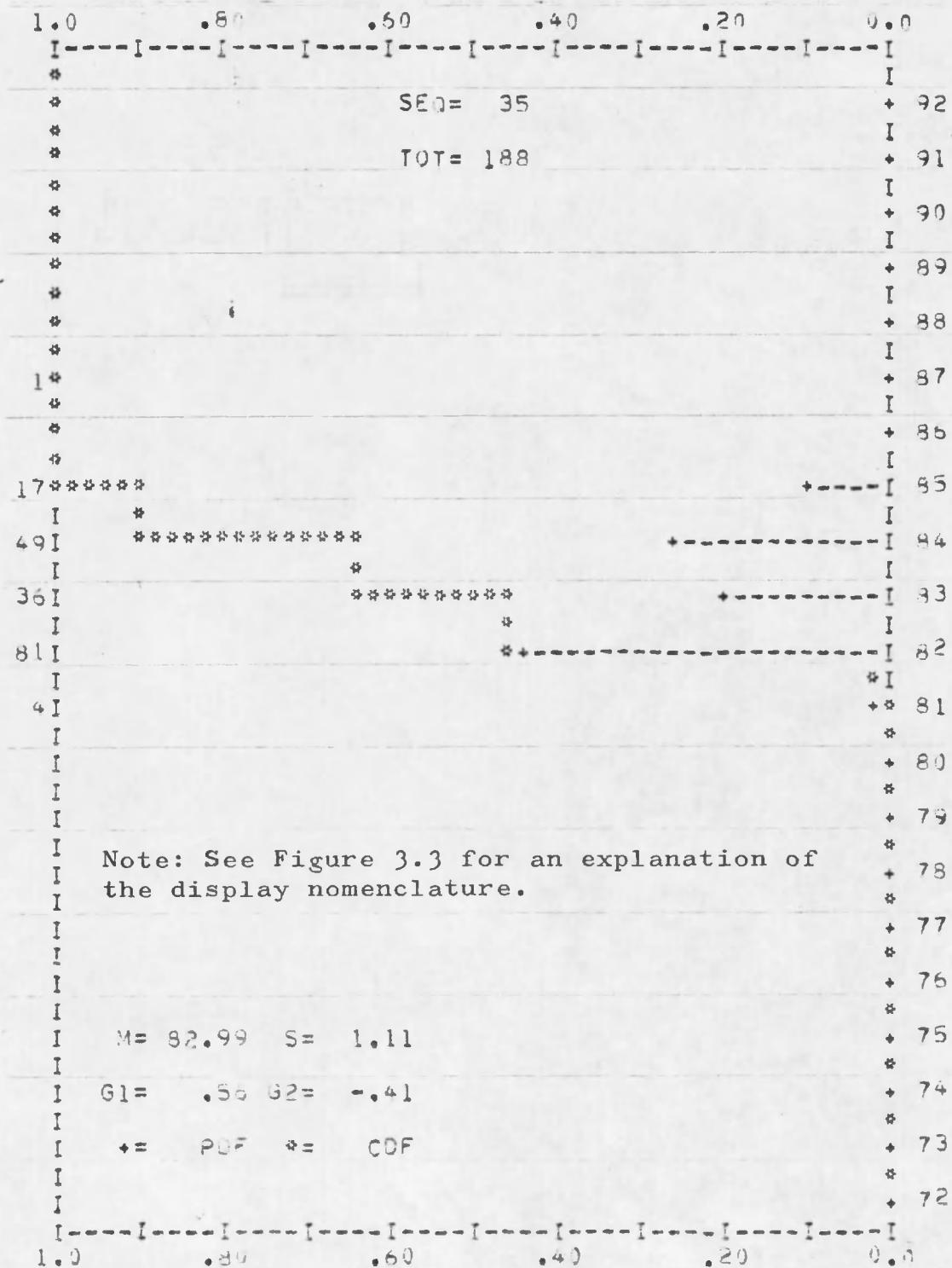


Figure 3.38. Statistics for Time Interval 35

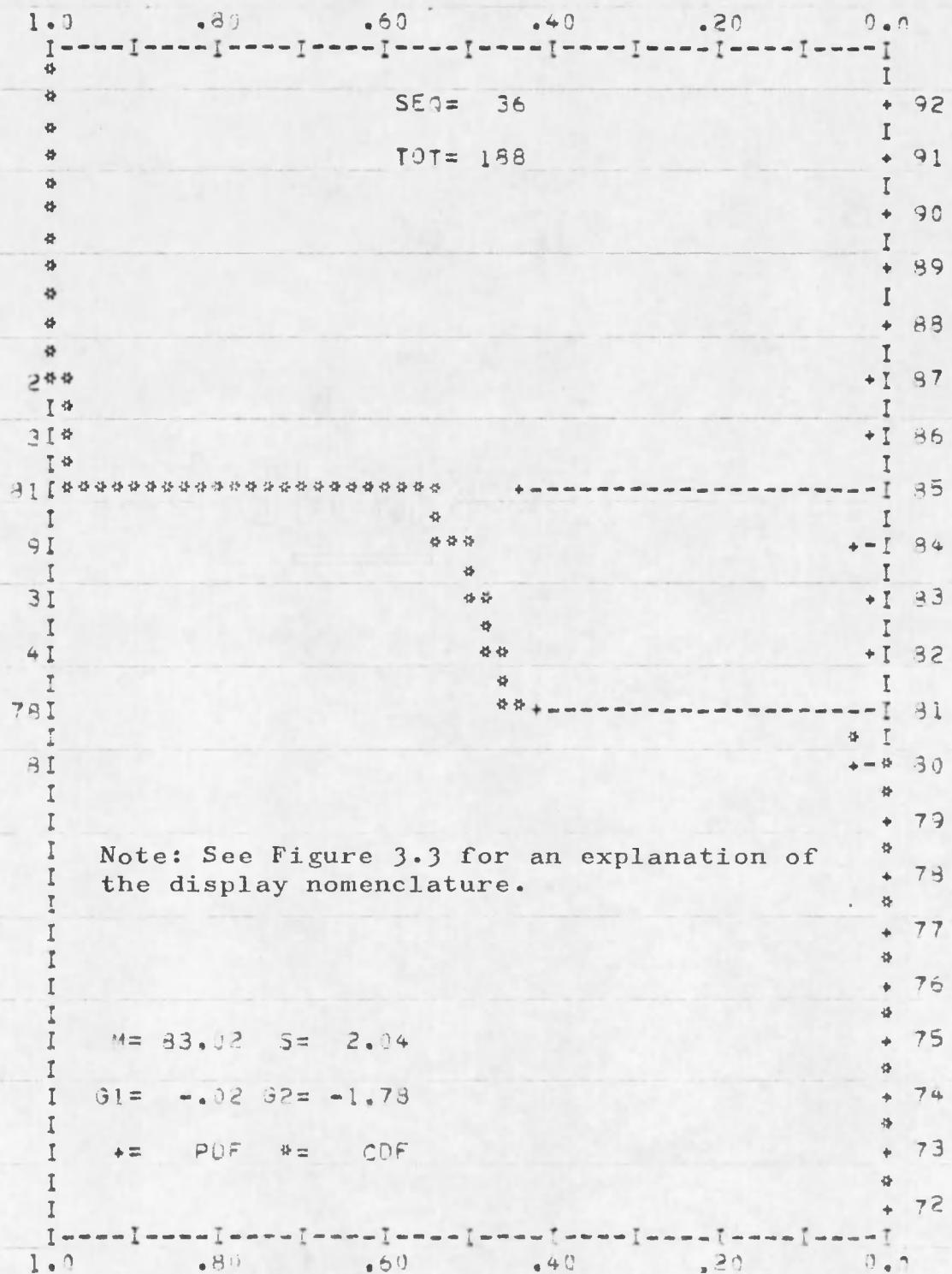


Figure 3.39. Statistics for Time Interval 36

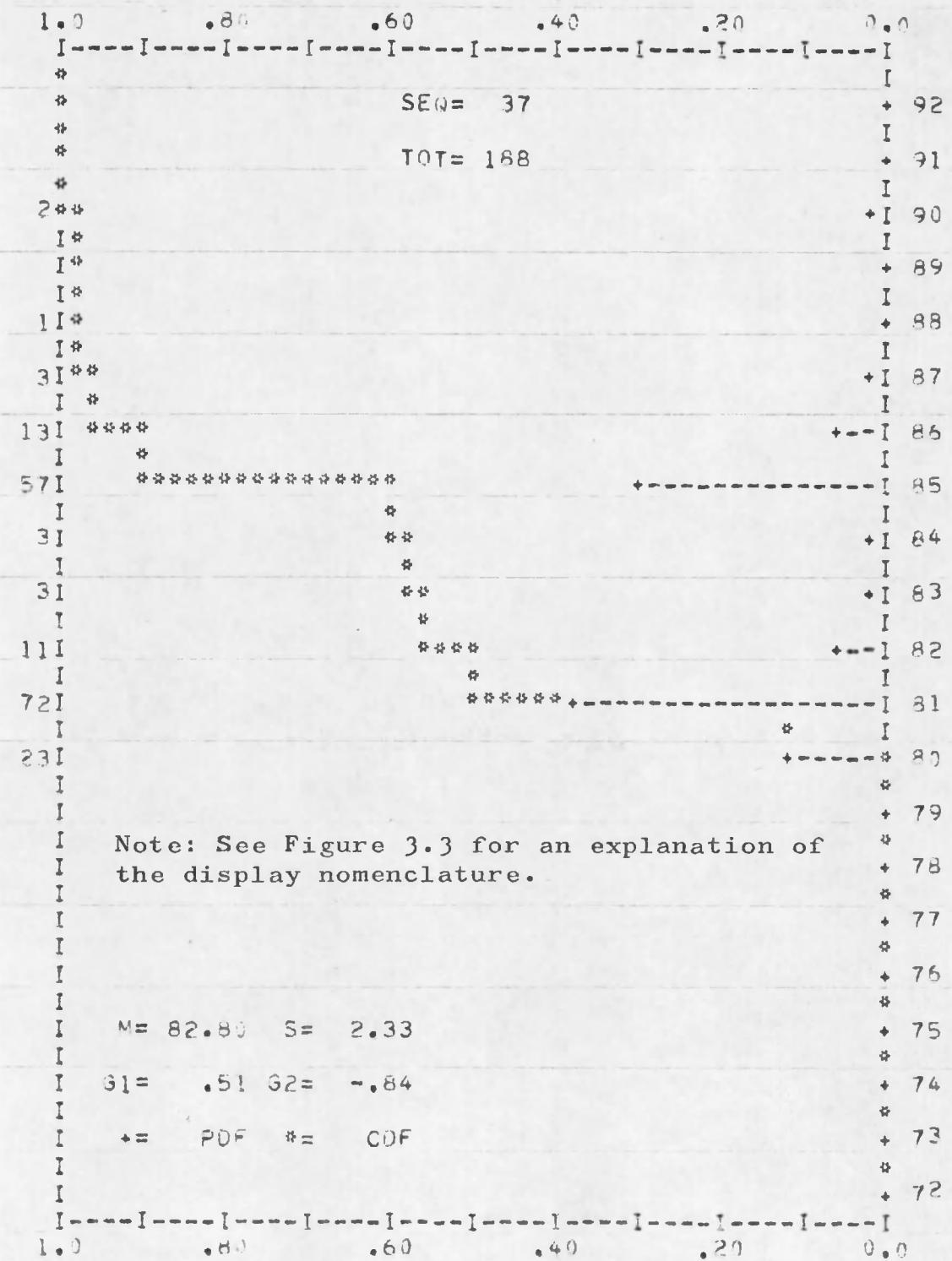


Figure 3.40. Statistics for Time Interval 37

1.0 .80 .60 .40 .20 0.0
 I-----I-----I-----I-----I-----I-----I-----I-----I
 *
 * SEQ= 38 + 91
 *
 * TOT= 188 + 90
 *
 *
 *
 *
 2** + I 88
 I* I
 1 I* + 87
 I* I
 18 I***** +---- I 86
 I * I
 64 I ***** +---- I 85
 I * I
 8 I *** + - I 84
 I * I
 1 I * + 83
 I * I
 I * + 82
 I * I
 3 I ** + I 81
 I * I
 35 I ***** +---- I 80
 I * I
 44 I ***+---- I 79
 I * I
 9 I * + I 78
 I Note: See Figure 3.3 for an explanation of + I
 ?I the display nomenclature. + * 77
 I * I
 I * 76
 I * 75
 I * 75
 I * 74
 I M= 82.29 S= 3.08 + 74
 I * I
 I G1= .03 G2= -1.58 + 73
 I * I
 I += PDF *= CDF + 72
 I * I
 I * 71
 I-----I-----I-----I-----I-----I-----I-----I
 1.0 .80 .60 .40 .20 0.0

Figure 3.41. Statistics for Time Interval 38

Figure 3.42. Statistics for Time Interval 39

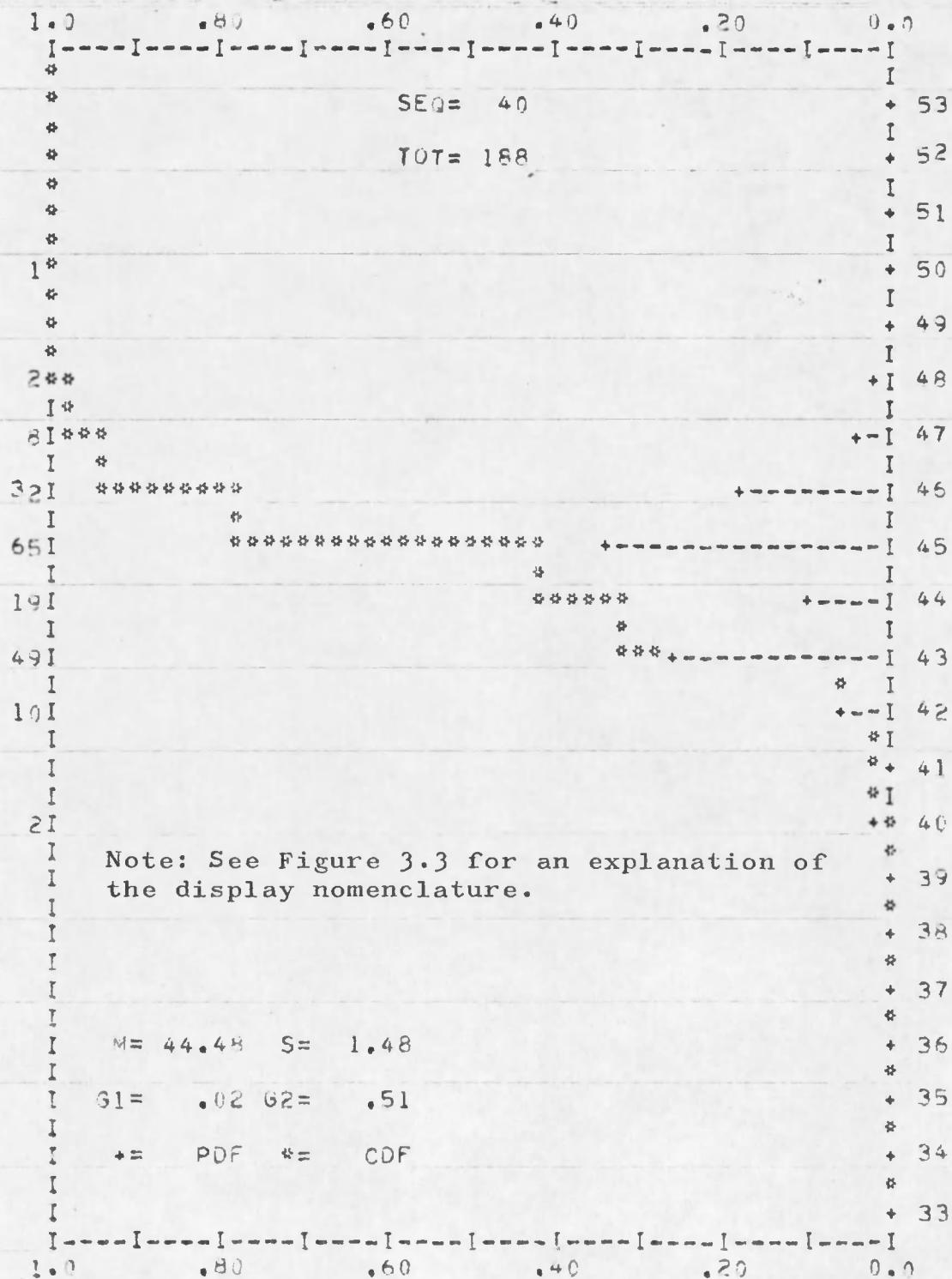


Figure 3.43. Statistics for Time Interval 40

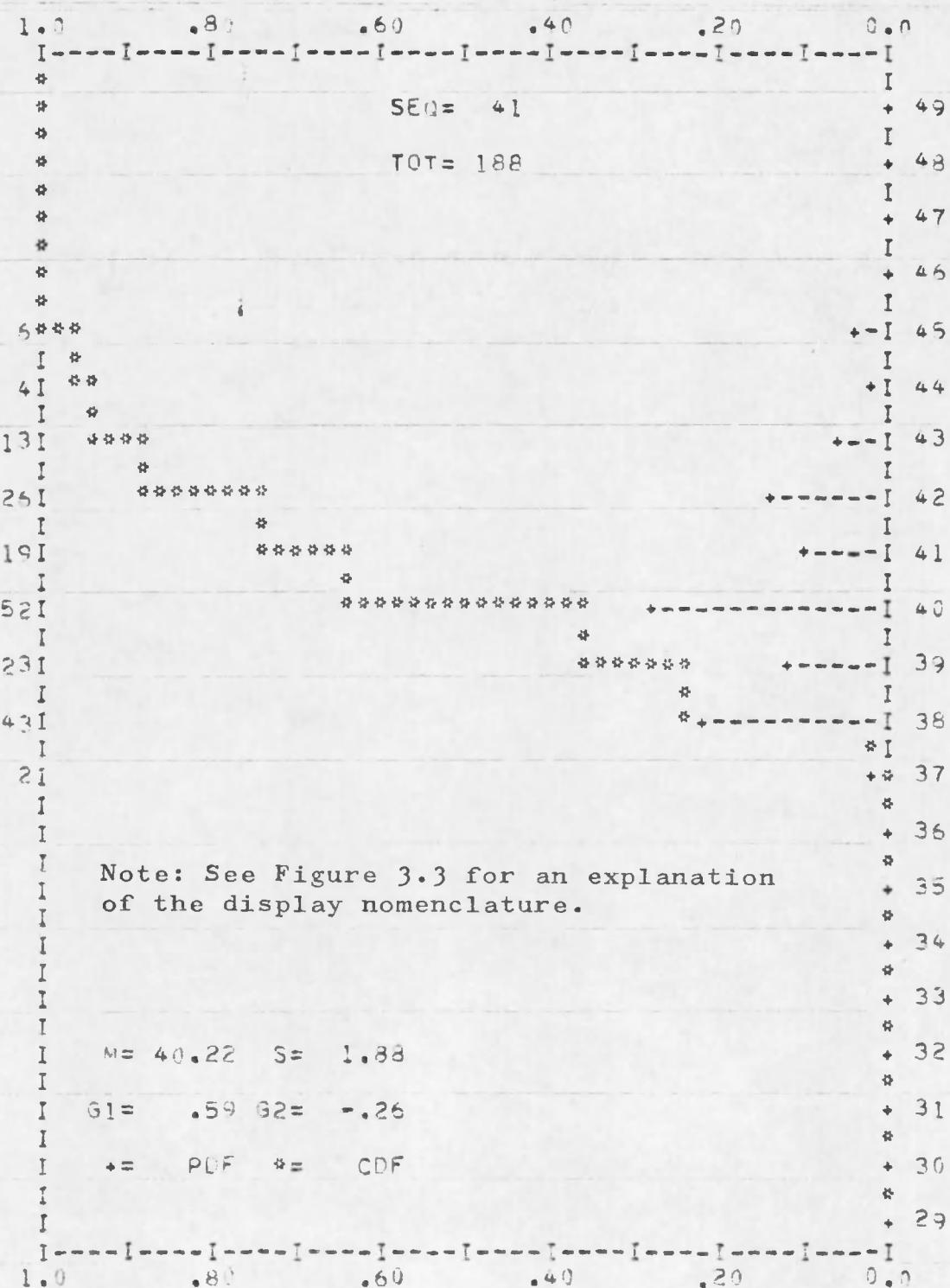


Figure 3.44. Statistics for Time Interval 41

Figure 3.45. Statistics for Time Interval 42

Figure 3.46. Statistics for Time Interval 43

Figure 3.47. Statistics for Time Interval 44

Figure 3.48. Statistics for Time Interval 45

Figure 3.49. Statistics for Time Interval 46

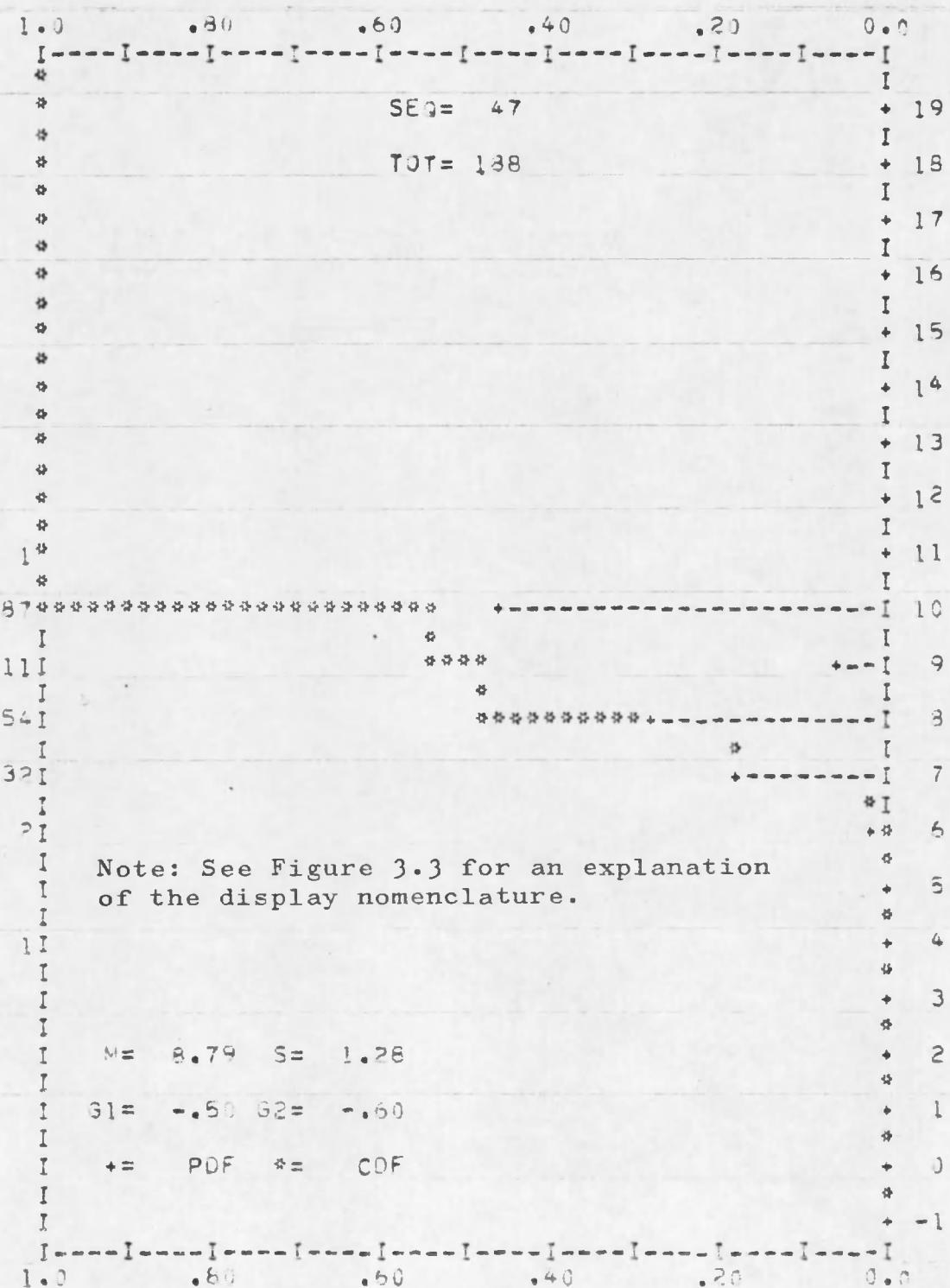


Figure 3.50. Statistics for Time Interval 47

Figure 3.51. Statistics for Time Interval 48

Figure 3.52. Statistics for Time Interval 49

Figure 3.53. Statistics for Time Interval 50

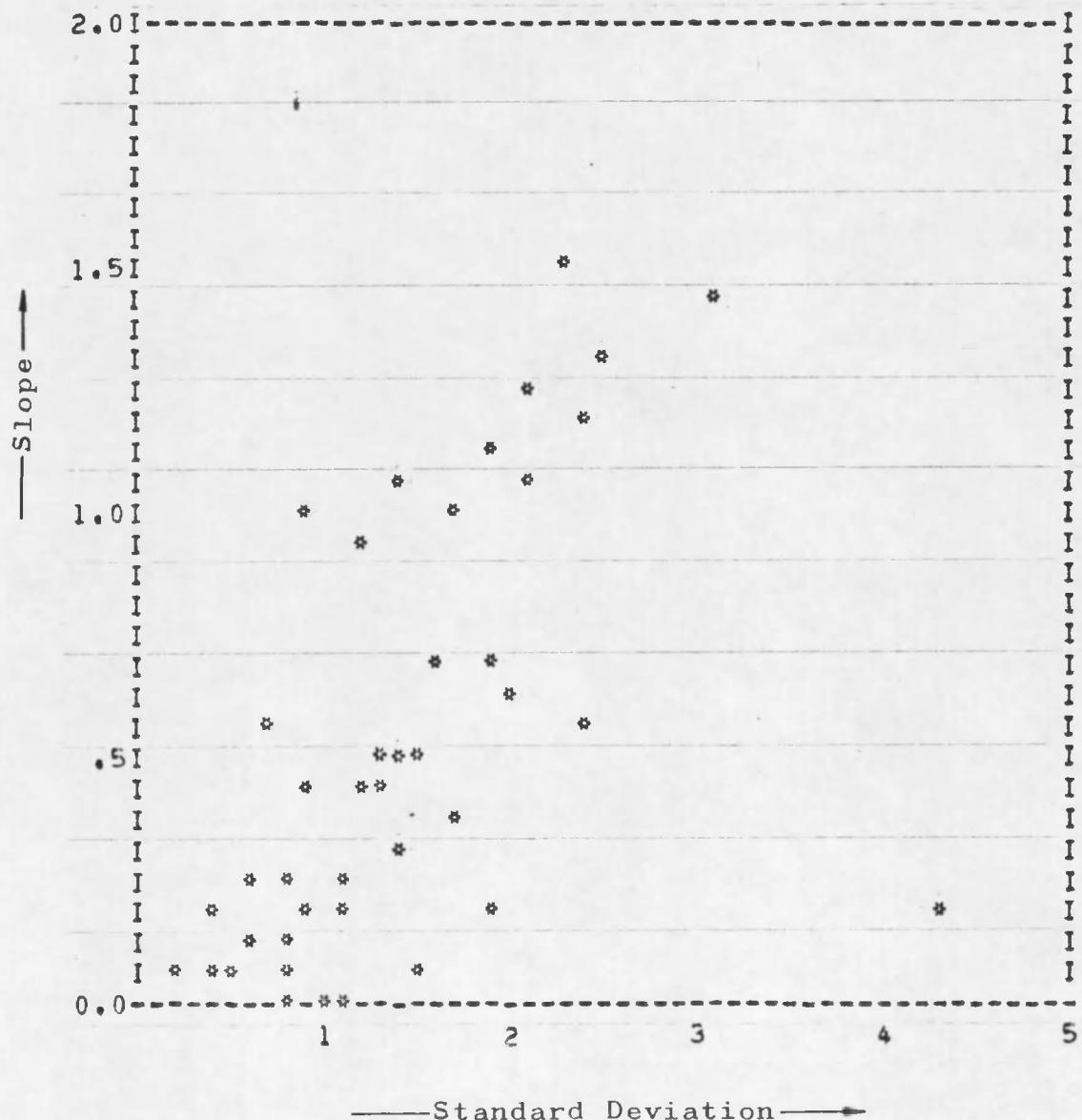


Figure 3.54. Slope Versus Standard Deviation

4. A display of the correlation function (Appendix A, Section A.7) for each time interval (program CORRFCN) (see Figure 3.55 and Figures 3.56 through 3.60).
5. A display of the standard deviation versus the proximity of the mean value to an amplitude grid reference line (program PROX) (see Figure 3.61).
6. A display of the average individual error per time interval (program AVERR) (see Figure 3.62).

3.2 Frequency Domain Reduction

A flow chart of the following narrative sequence is given in Figure 3.63.

Before the frequency analysis can begin, it is assumed that the frequencies (or band of frequencies) for the analysis are given. Previous to this experiment, it was determined that the lowest frequency transform required would be at .5 mhz and that frequencies higher than .5 mhz would be sampled until the magnitudes of the spectral levels fell off and stayed below -40 db of that level obtained for .5 mhz. Using the level obtained at .5 mhz as a zero db reference, the frequency domain statistics were found for frequencies in the range $.5 < F < 50$ mhz. Any quantity of frequencies within this band could be displayed, but the amount of computation time per frequency

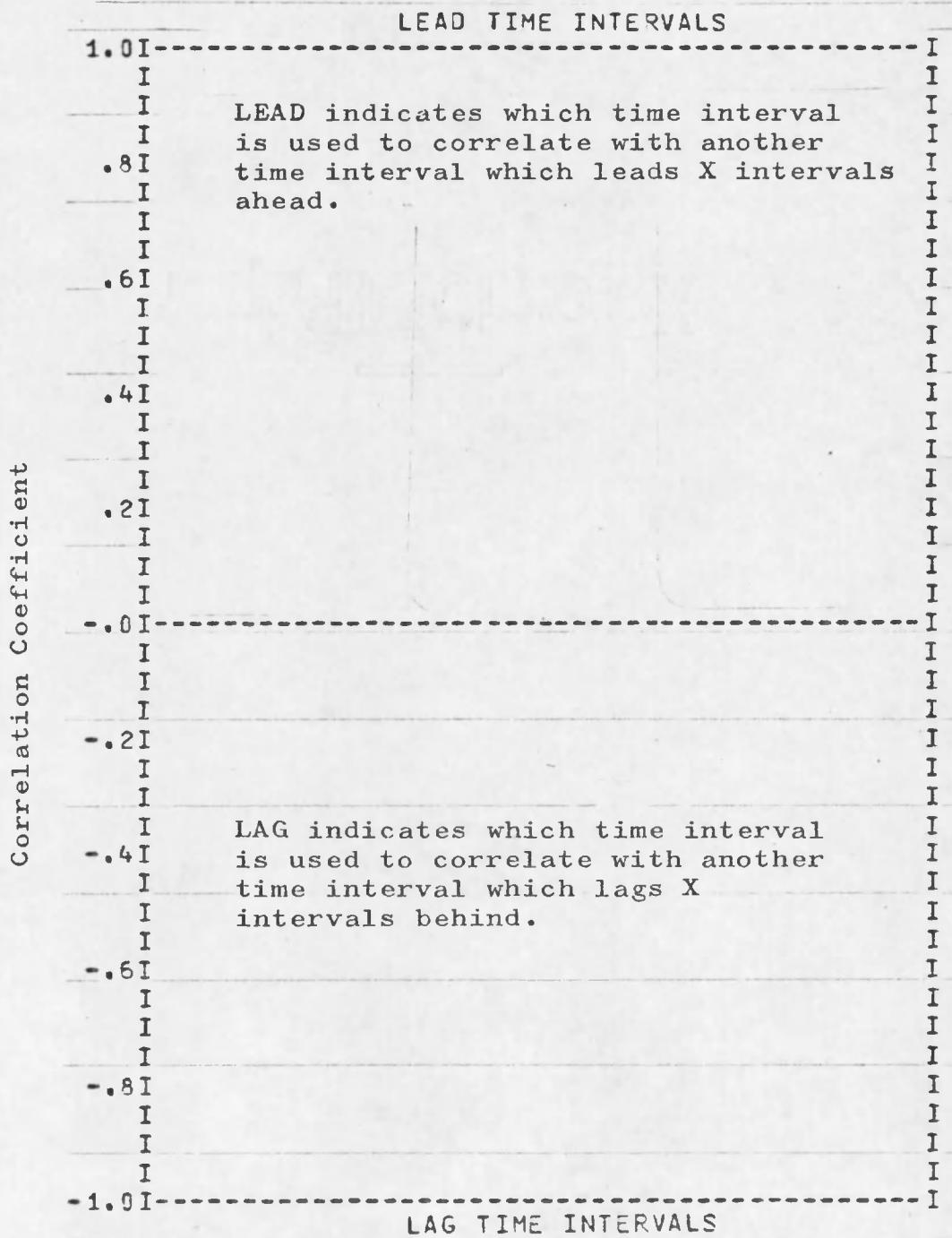


Figure 3.55. Explanation of the Display Nomenclature for Figures 3.56 through 3.60

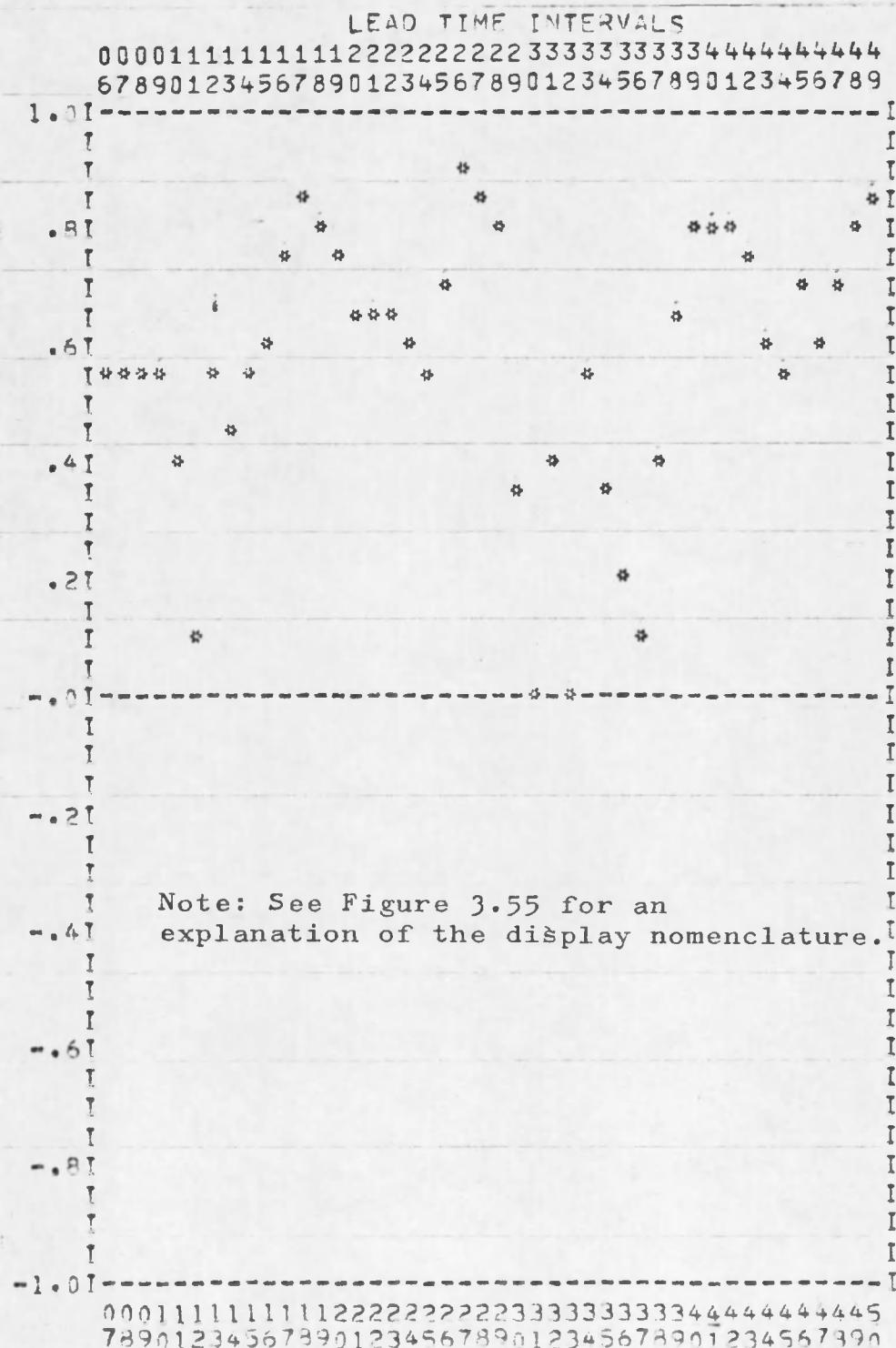


Figure 3.56. Correlation Diagram for LAG/LEAD = 1 Time Interval

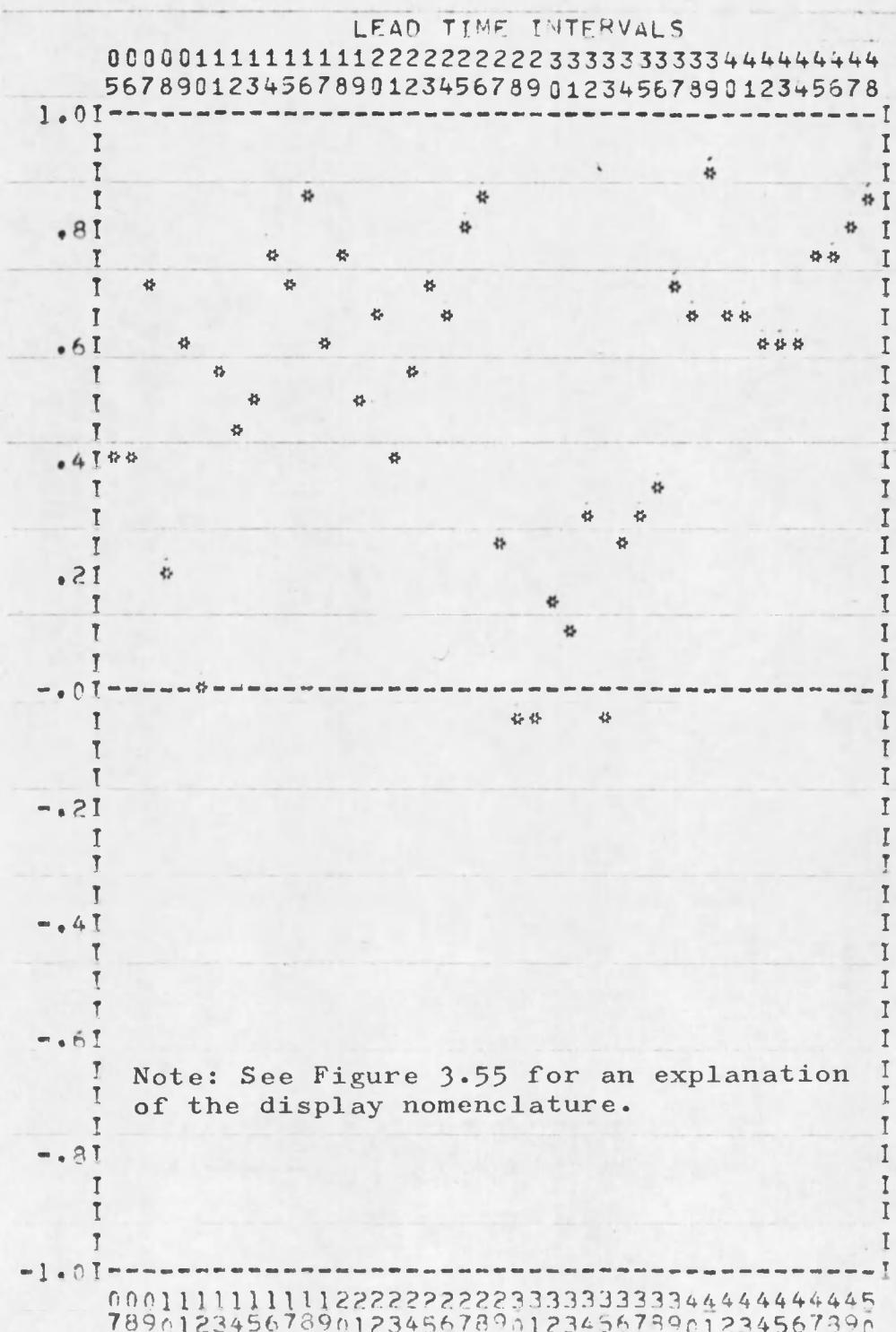


Figure 3.57. Correlation Diagram for LAG/LEAD = 2 Time Intervals

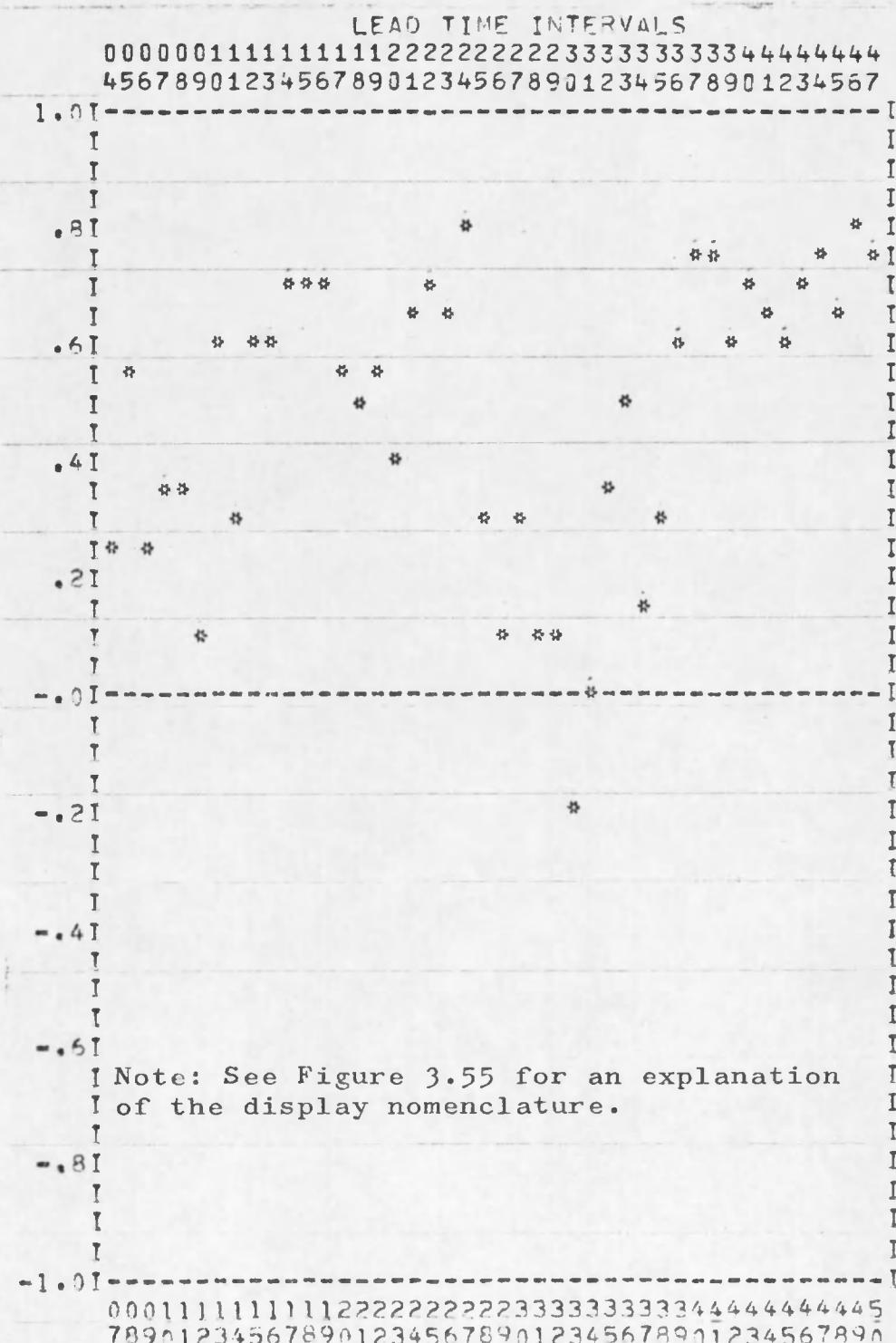


Figure 3.58. Correlation Diagram for LAG/LEAD = 3 Time Intervals

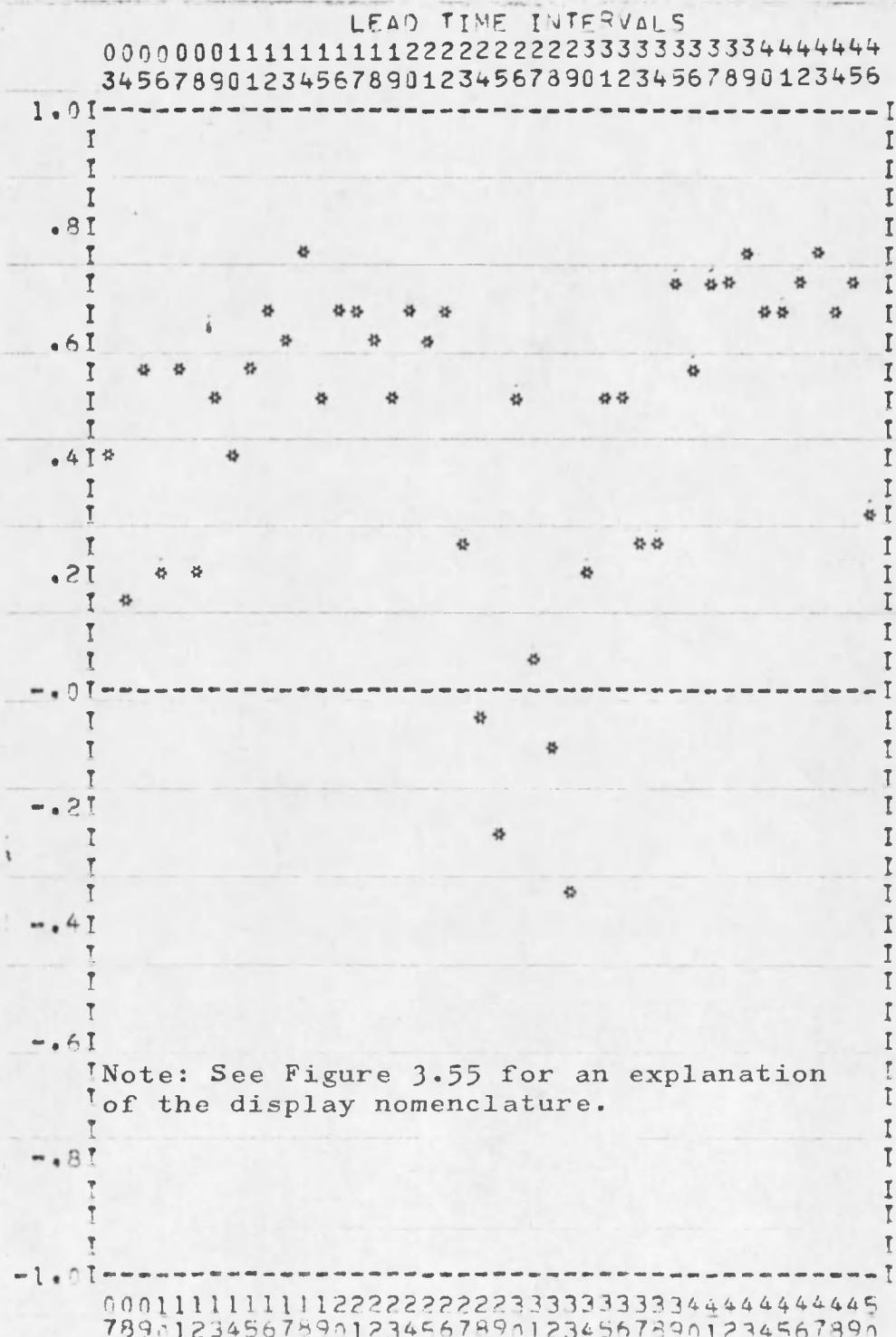


Figure 3.59. Correlation Diagram for LAG/LEAD = 4 Time Intervals

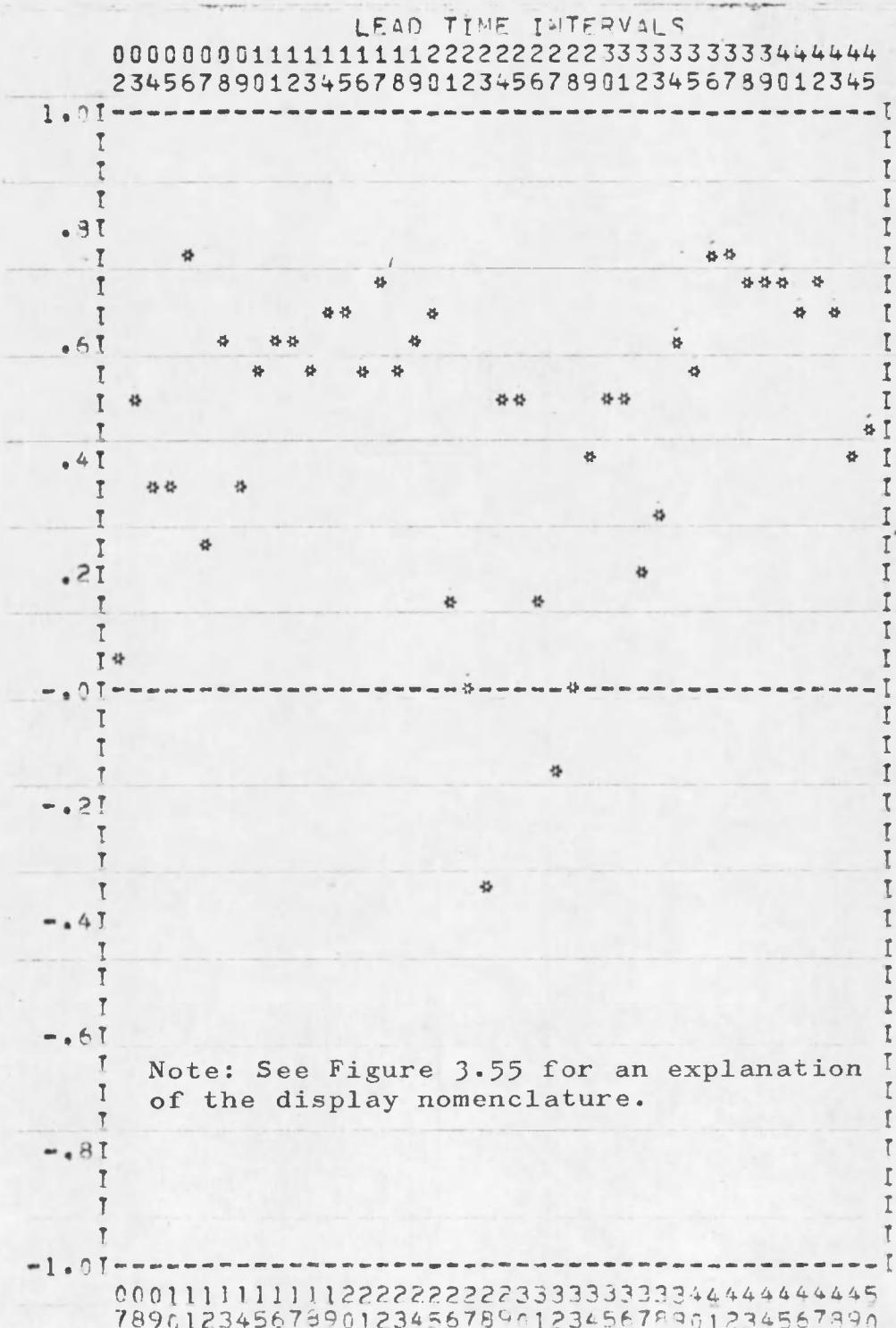


Figure 3.60. Correlation Diagram for $\text{LAG/LEAD} = 5$ Time Intervals

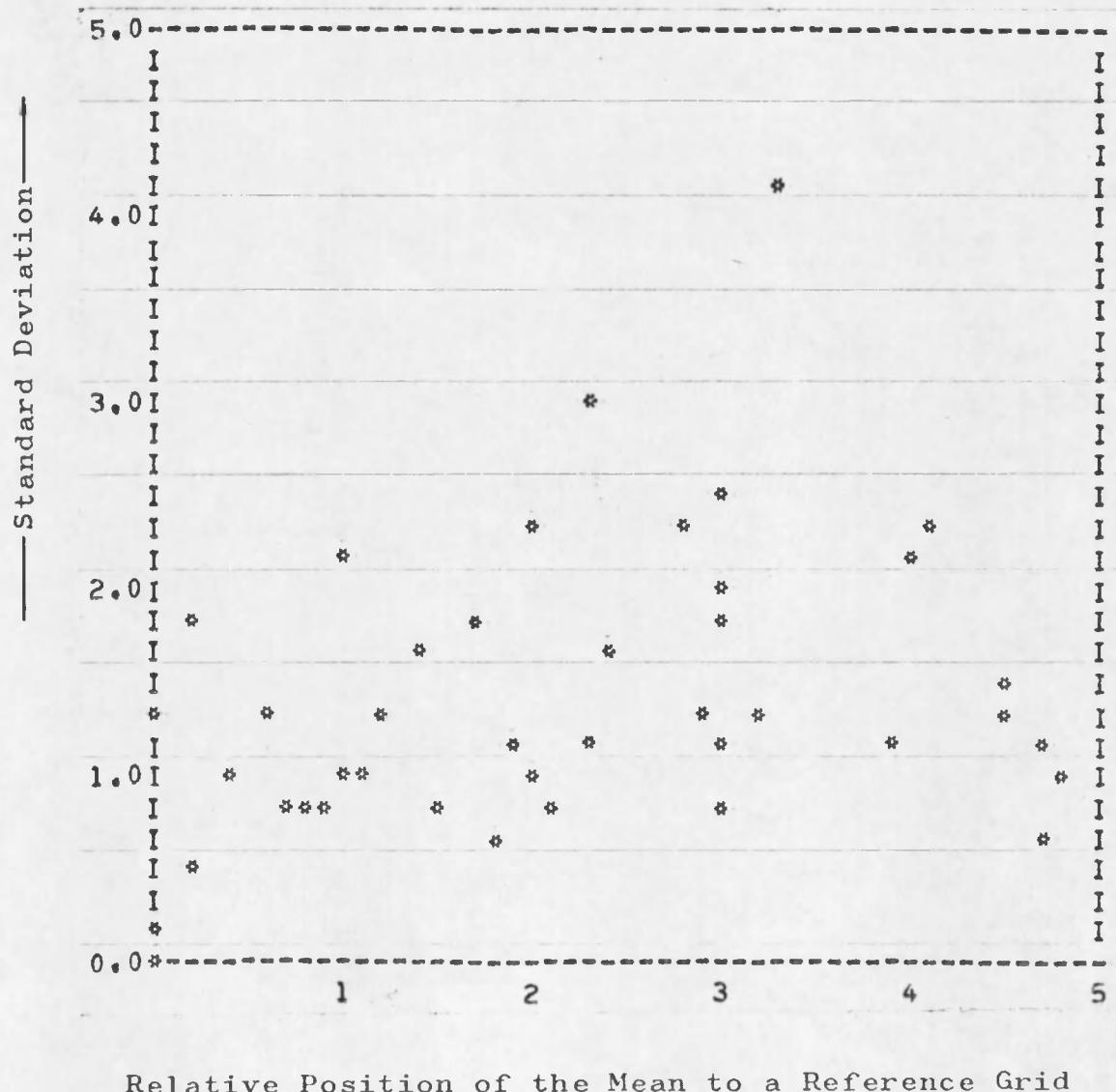


Figure 3.61. Standard Deviation Versus the Proximity of the Mean to a Grid

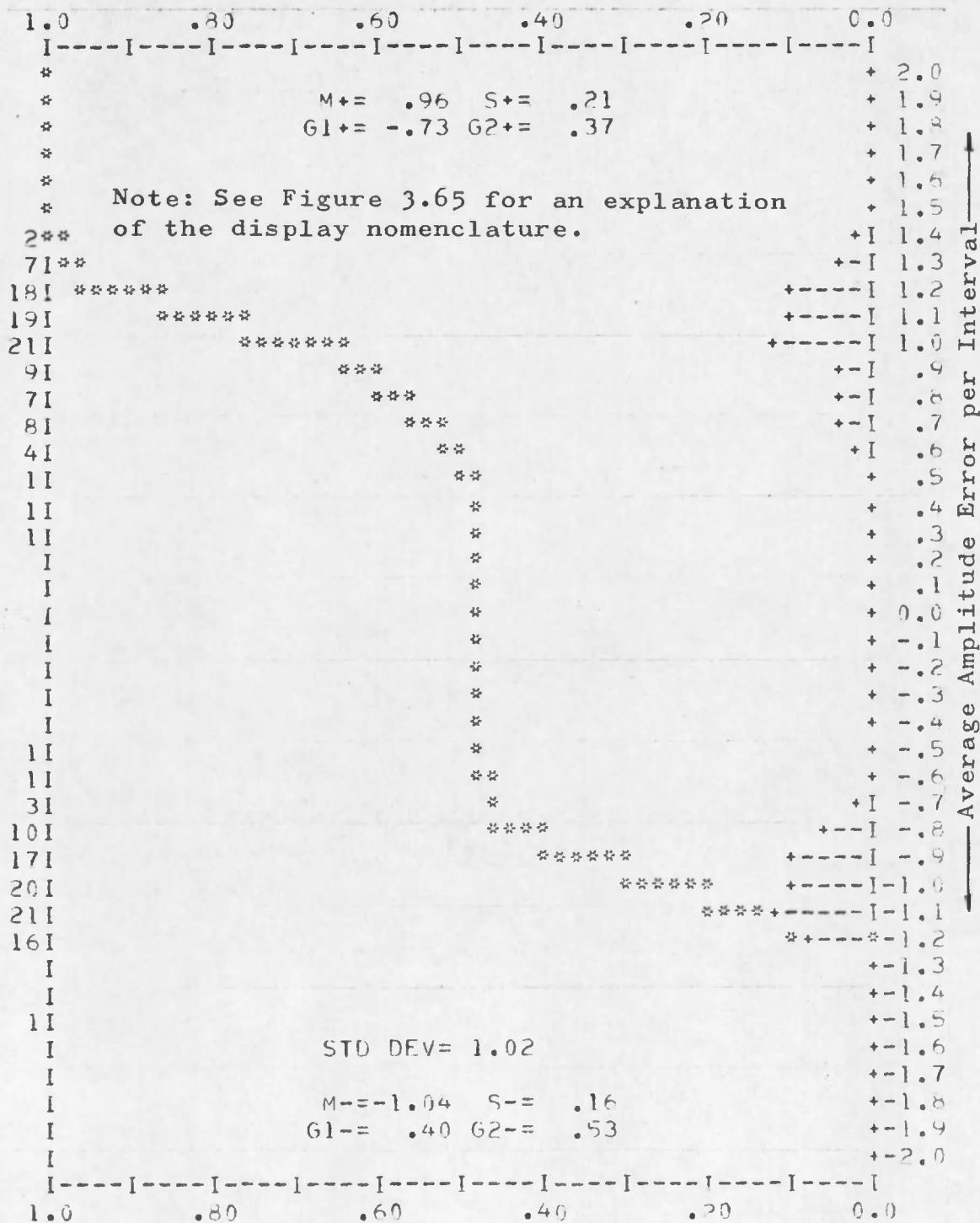


Figure 3.62. Average Individual Error Per Time Interval

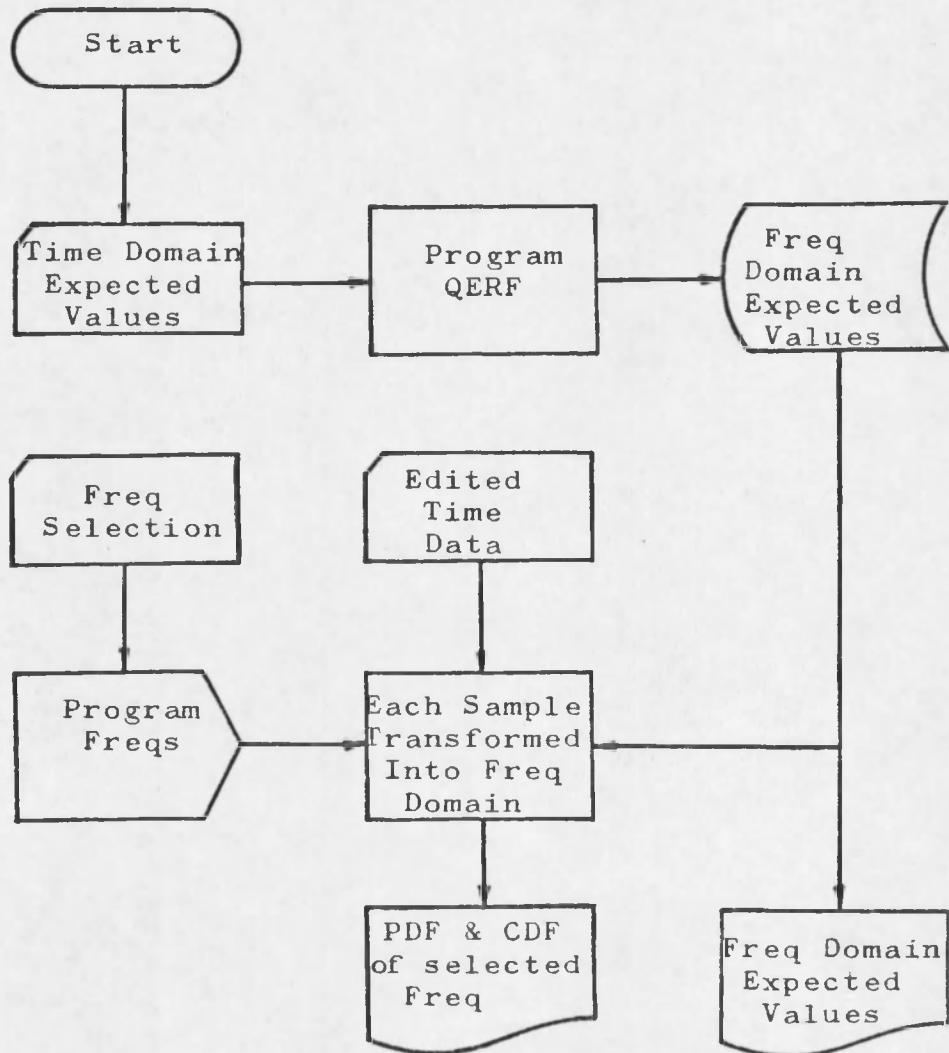


Figure 3.63. Frequency Domain Analysis Flow Chart

was substantial. Programs were compiled to display the following information:

1. A display of the expected values between .5 mhz and 50 mhz (program QERF) (see Figure 3.64).
2. A display of the frequency domain probability density functions and cumulative distribution functions with a listing of the mean, standard deviation, skewness, and kurtosis for selected frequencies (program FREQ) (see Figure 3.65 for an explanation of display notation, and Figures 3.66 through 3.75). Note: Each time domain sample is transformed into the frequency domain and the frequency statistics are formed from the ensemble of transforms.

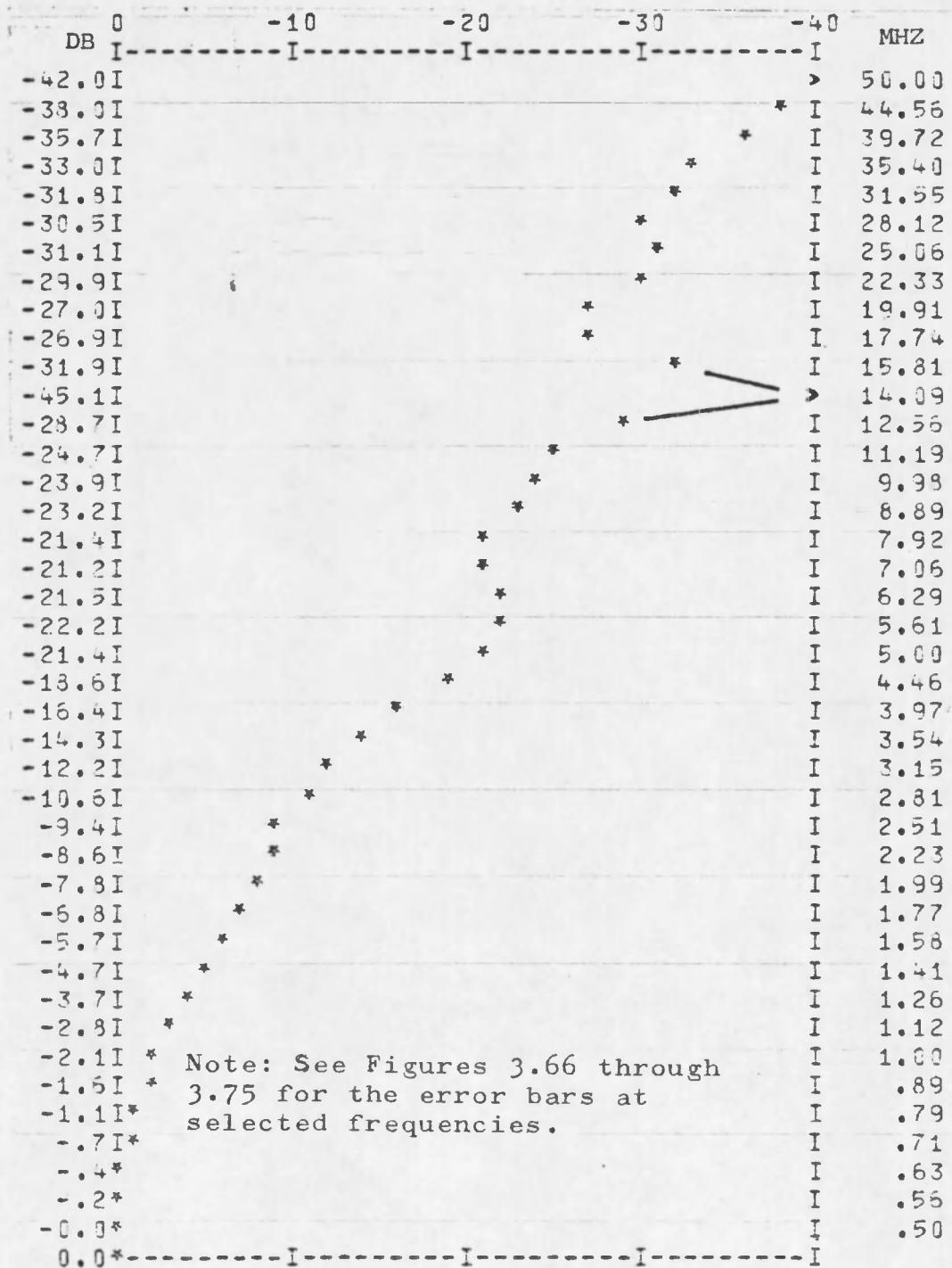


Figure 3.64. Frequency Domain Expected Values

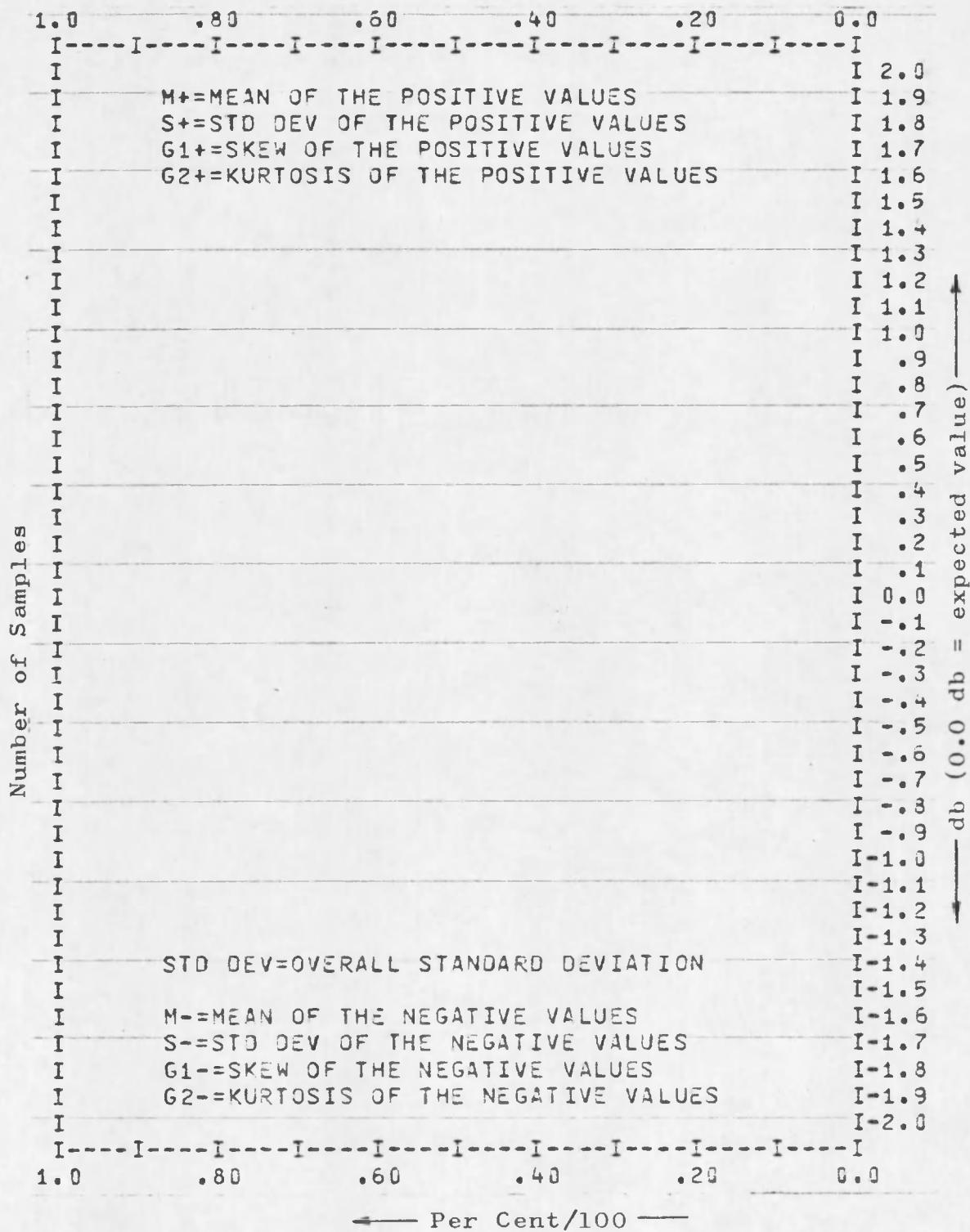


Figure 3.65. Explanation of the Display Nomenclature for Figures 3.66 through 3.75

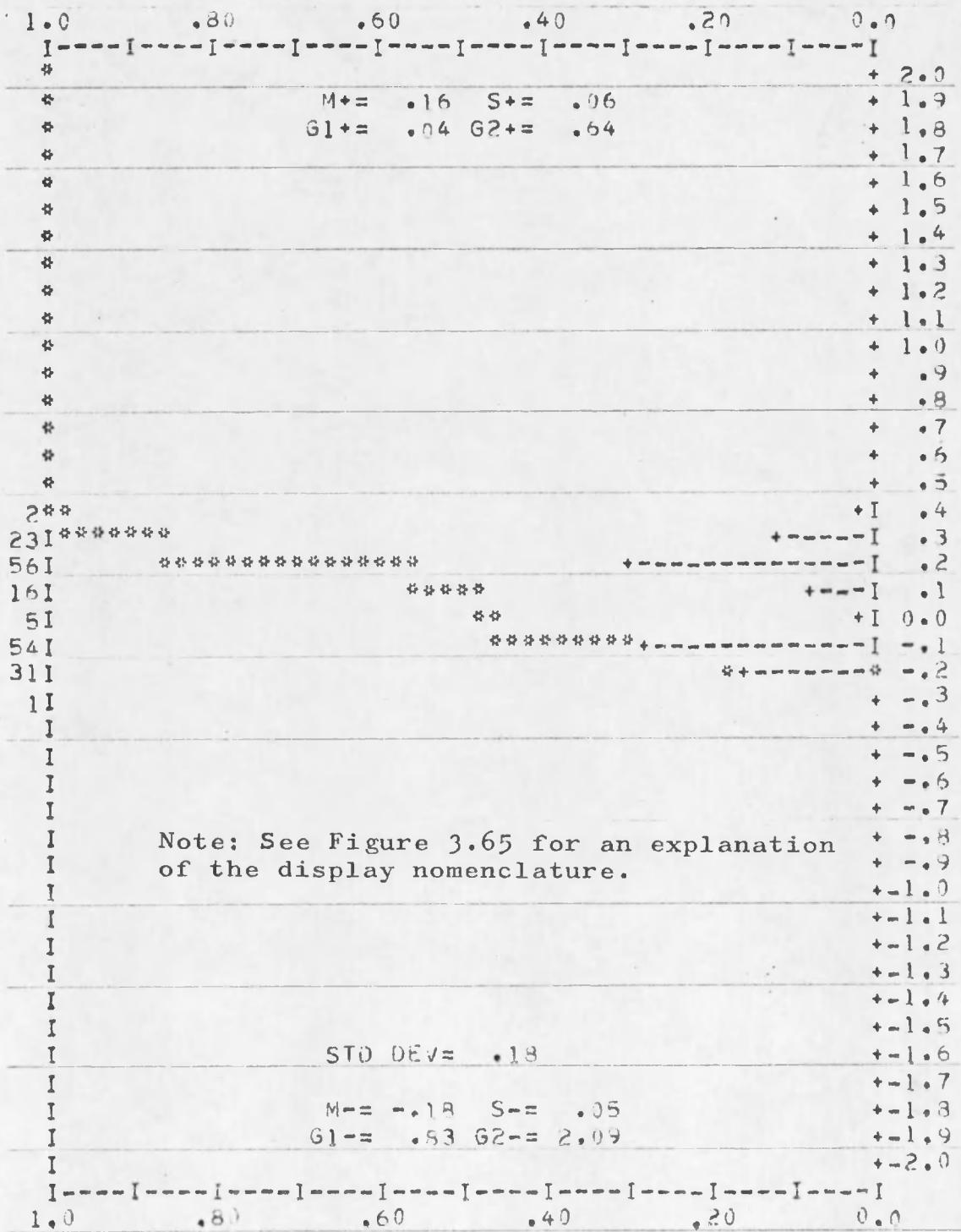


Figure 3.66. Frequency Domain Statistics for 1 mhz

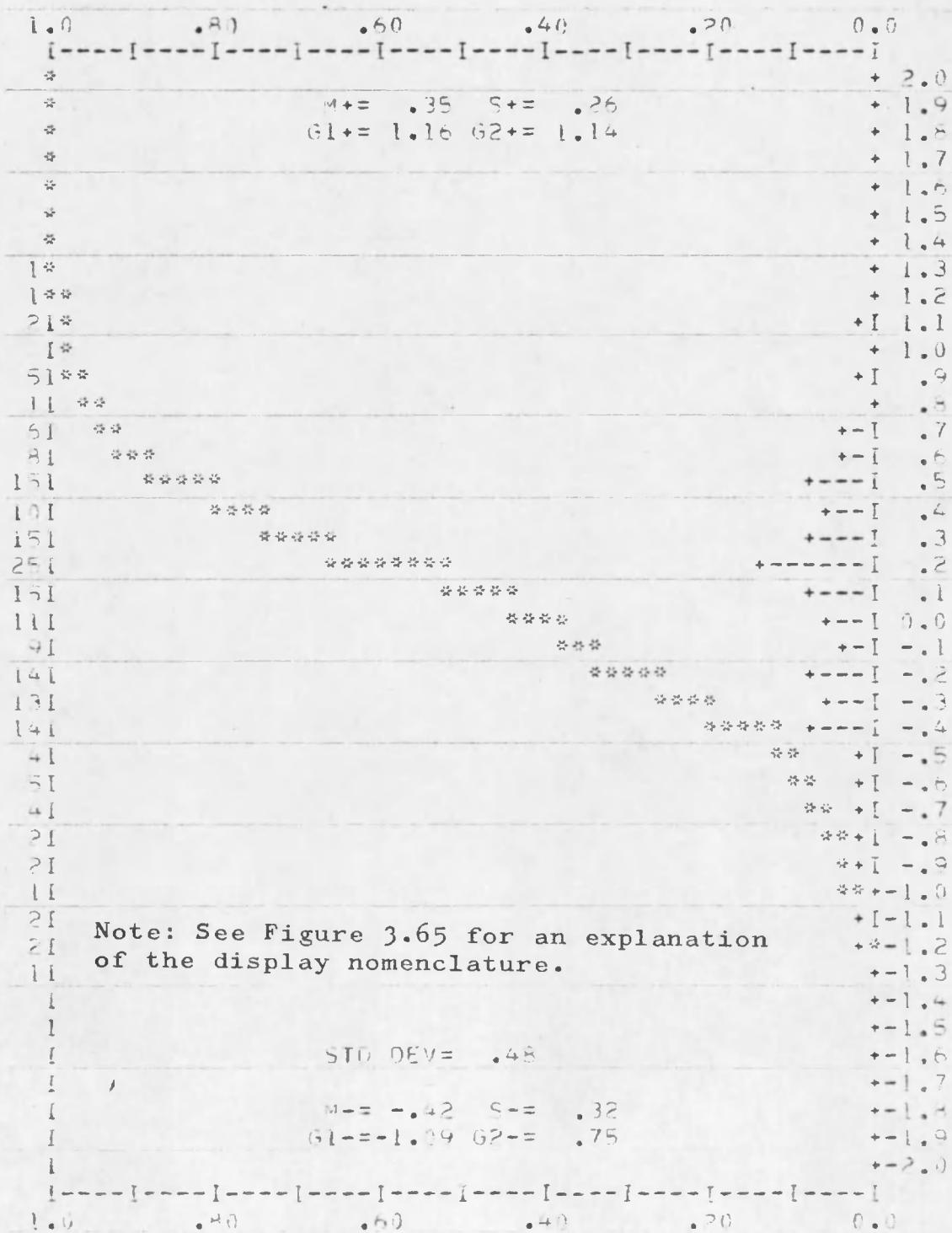


Figure 3.67. Frequency Domain Statistics for 5 mhz

1.0	.80	.60	.40	.20	0.0
I	I	I	I	I	I
*					♦ 2.0
*	M+= .46	S+= .26			♦ 1.9
*	G1+= .31	G2+= -.75			♦ 1.8
*					♦ 1.7
*					♦ 1.6
*					♦ 1.5
*					♦ 1.4
*					♦ 1.3
*					♦ 1.2
2**					♦ I 1.1
3I*					♦ I 1.0
8I***					+- I .9
6I ***					+- I .8
12I ****					+-+- I .7
8I ***					+- I .6
14I *****					+-+- I .5
15I *****					+-+- I .4
10I ****					+-+- I .3
12I ****					+-+- I .2
6I ***					+- I .1
6I **					+- I 0.0
3I **					+ I -.1
10I *****					+-+- I -.2
14I *****					+-+- I -.3
13I *****					+-+- I -.4
20I *****				+-+- I -.5	
10I *****				*****+- I -.6	
7I				**+- I -.7	
5I				*+ I -.8	
2I				+ I -.9	
1I				*+ I -1.0	
1I	Note: See Figure 3.65 for an explanation				+- I .1
I	of the display nomenclature.				+- I .2
I					+- I .3
I					+- I .4
I					+- I .5
I	STD DEV= .54				+- I .6
I					+- I .7
I	M-= -.49	S-= .24			+- I .8
I	G1-= -.17	G2-= -.06			+- I .9
I					+- 2.0
I	I	I	I	I	I
1.0	.80	.60	.40	.20	0.0

Figure 3.68. Frequency Domain Statistics for 7 mhz

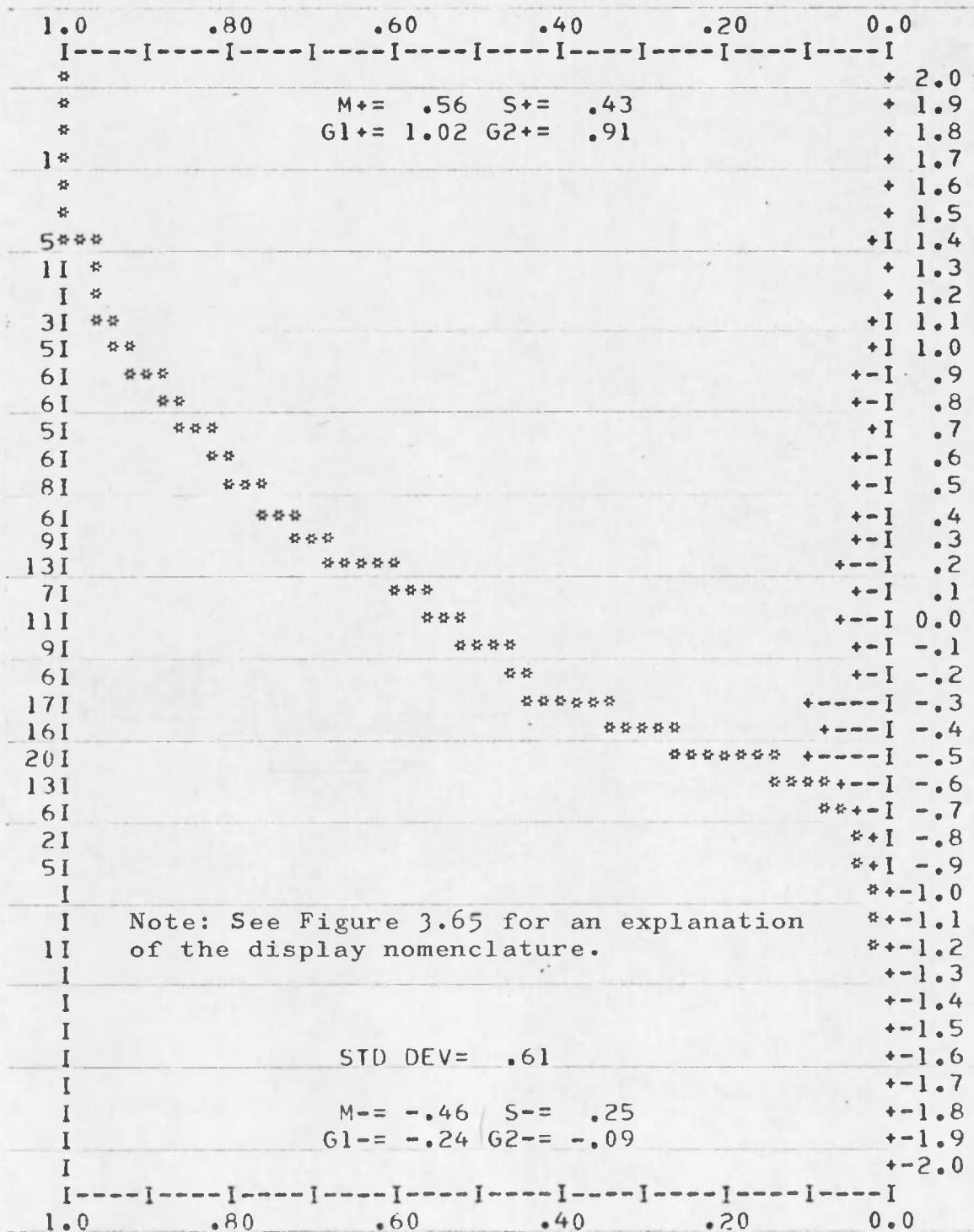


Figure 3.69. Frequency Domain Statistics for 12 mhz

Figure 3.70. Frequency Domain Statistics for 13 mhz

	1.0	.80	.60	.40	.20	0.0	
I-----I-----I-----I-----I-----I-----I-----I-----I-----I-----I							
3**						+ I 2.0	
1I*		M+= .92	S+= .58			+ I 1.9	
2I**		G1+= 1.02	G2+= 1.32			+ I 1.8	
1I *						+ I 1.7	
3I **						+ I 1.6	
I *						+ I 1.5	
3I *						+ I 1.4	
5I ***						+ I 1.3	
4I **						+ I 1.2	
4I **						+ I 1.1	
7I ***						+ - I 1.0	
12I ****						+--- I .9	
6I ***						+ - I .8	
5I **						+ I .7	
8I ***						+ - I .6	
4I **						+ I .5	
4I **						+ I .4	
6I ***						+ - I .3	
2I *						+ I .2	
4I **						+ I .1	
3I **						+ I 0.0	
10I ****						+--- I -.1	
7I ***						+ - I -.2	
10I ***						+--- I -.3	
12I *****						+--- I -.4	
8I ***						+ - I -.5	
6I **						+ - I -.6	
6I ***						+ - I -.7	
5I **						+ I -.8	
4I **						+ I -.9	
4I **						+ I -1.0	
3I Note: See Figure 3.65 for an				**		+ I -1.1	
4I explanation of the display nomenclature.				**		+ I -1.2	
6I				***		+ - I -1.3	
1I				*		+ - I .4	
1I				*		+ - I .5	
2I		STD DEV= 1.10				** + I -1.6	
I						*	+ - I .7
1I		M-= -.83	S-= .75			*	+ - I .8
I		G1=-2.20	G2= 6.08			*	+ - I .9
7I						* + - I -2.0	
I-----I-----I-----I-----I-----I-----I-----I-----I-----I-----I							
1.0	.80	.60	.40	.20		0.0	

Figure 3.71. Frequency Domain Statistics for 15 mhz

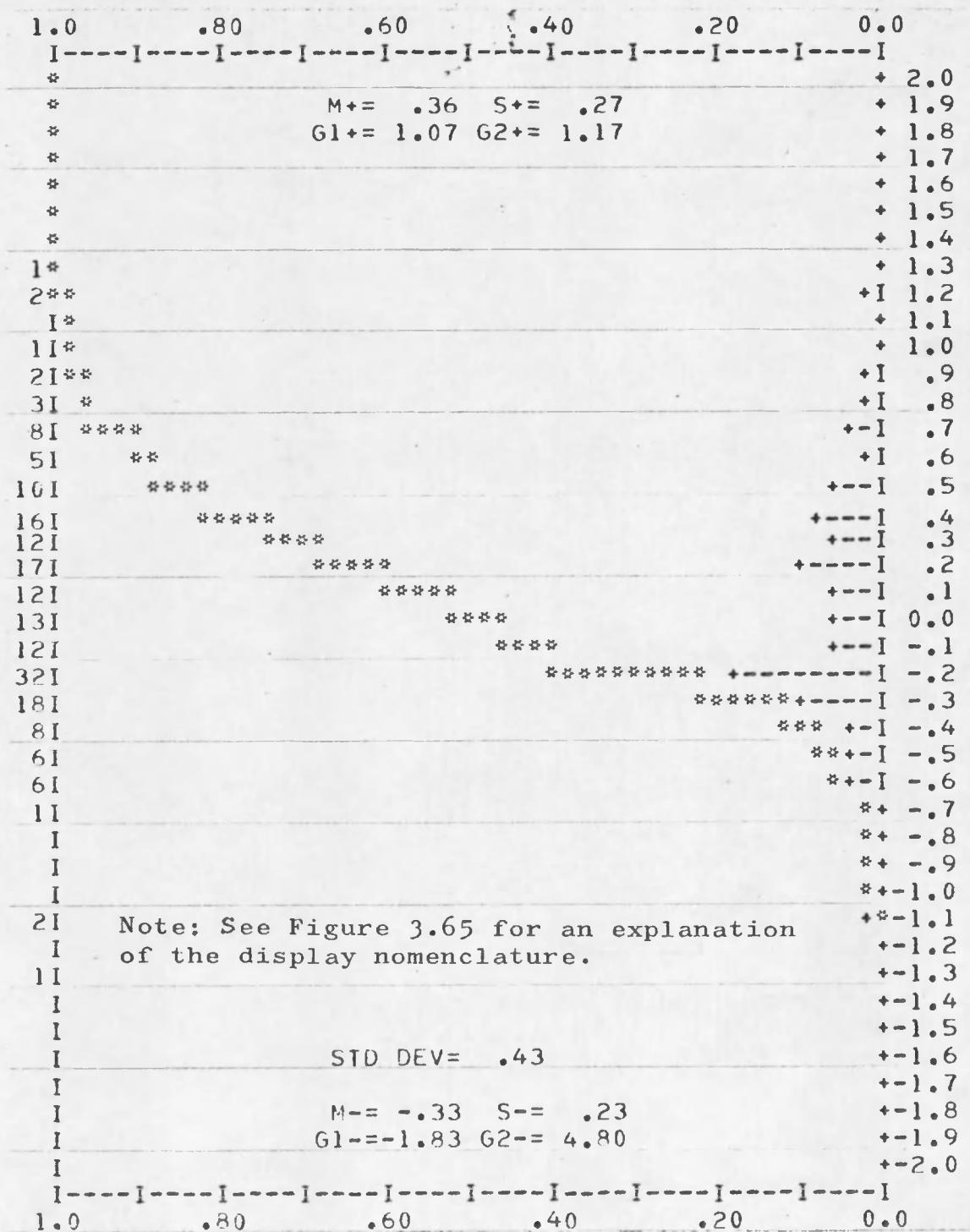


Figure 3.72. Frequency Domain Statistics for 16 mhz

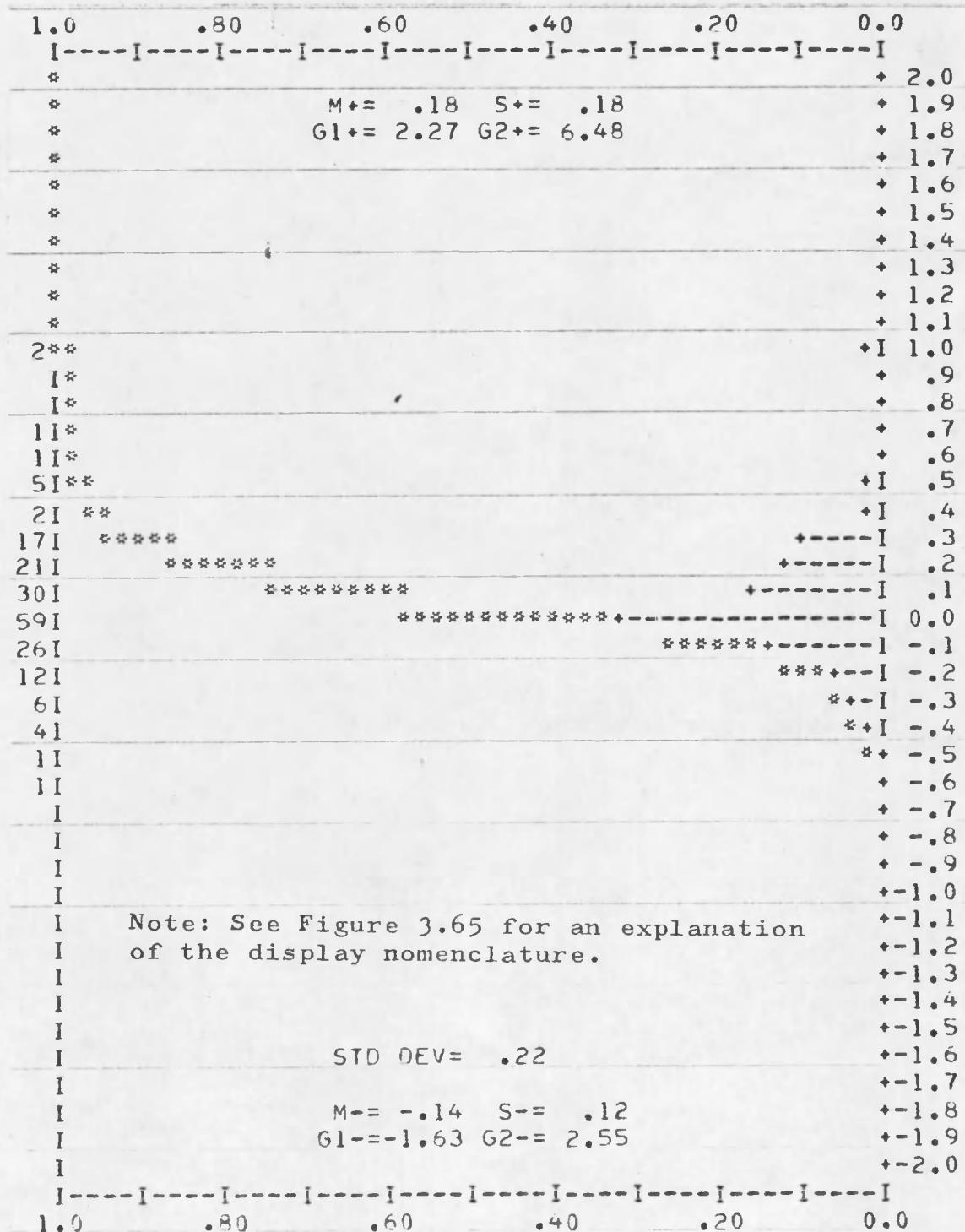


Figure 3.73. Frequency Domain Statistics for 20 mhz

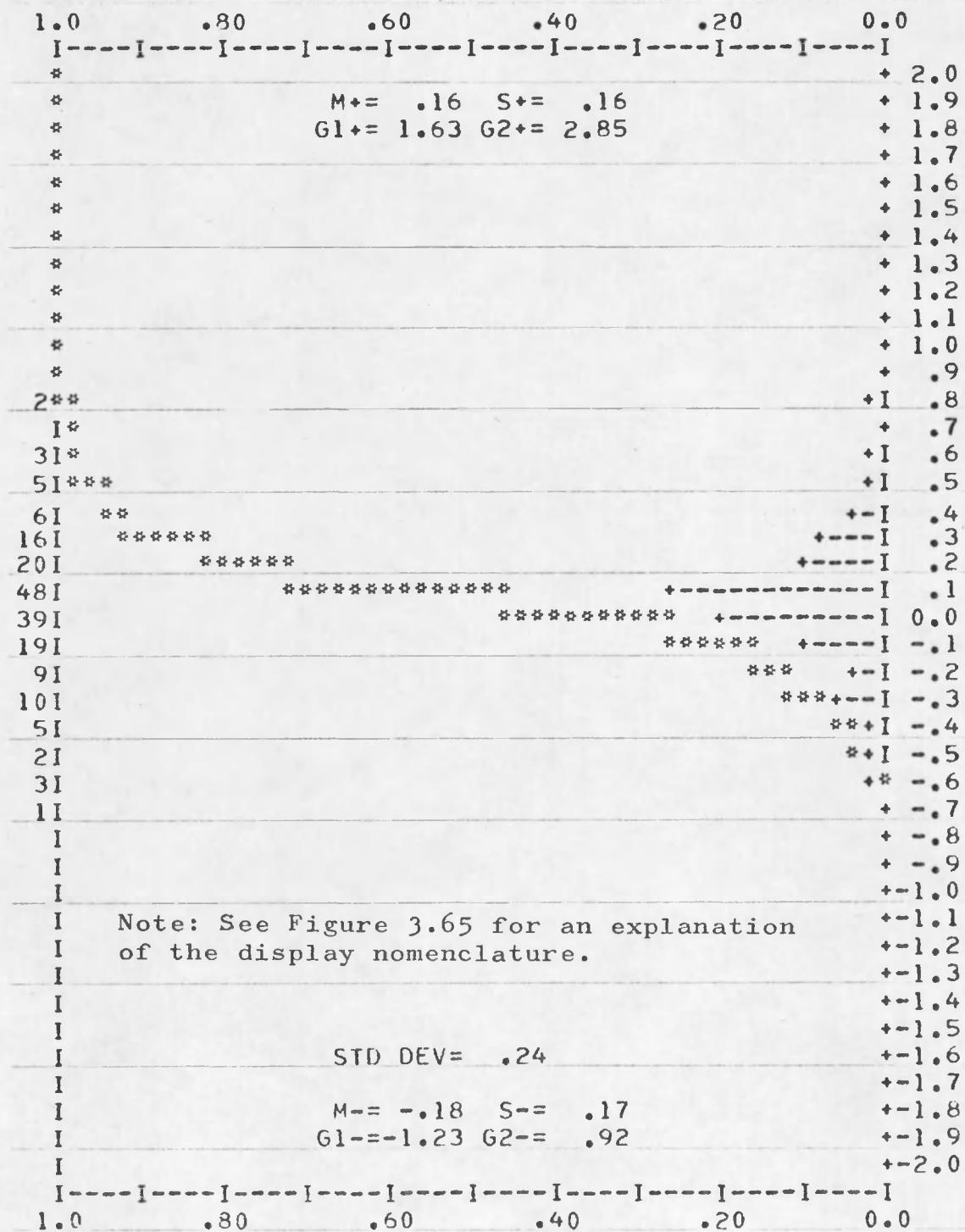


Figure 3.74. Frequency Domain Statistics for 30 mhz

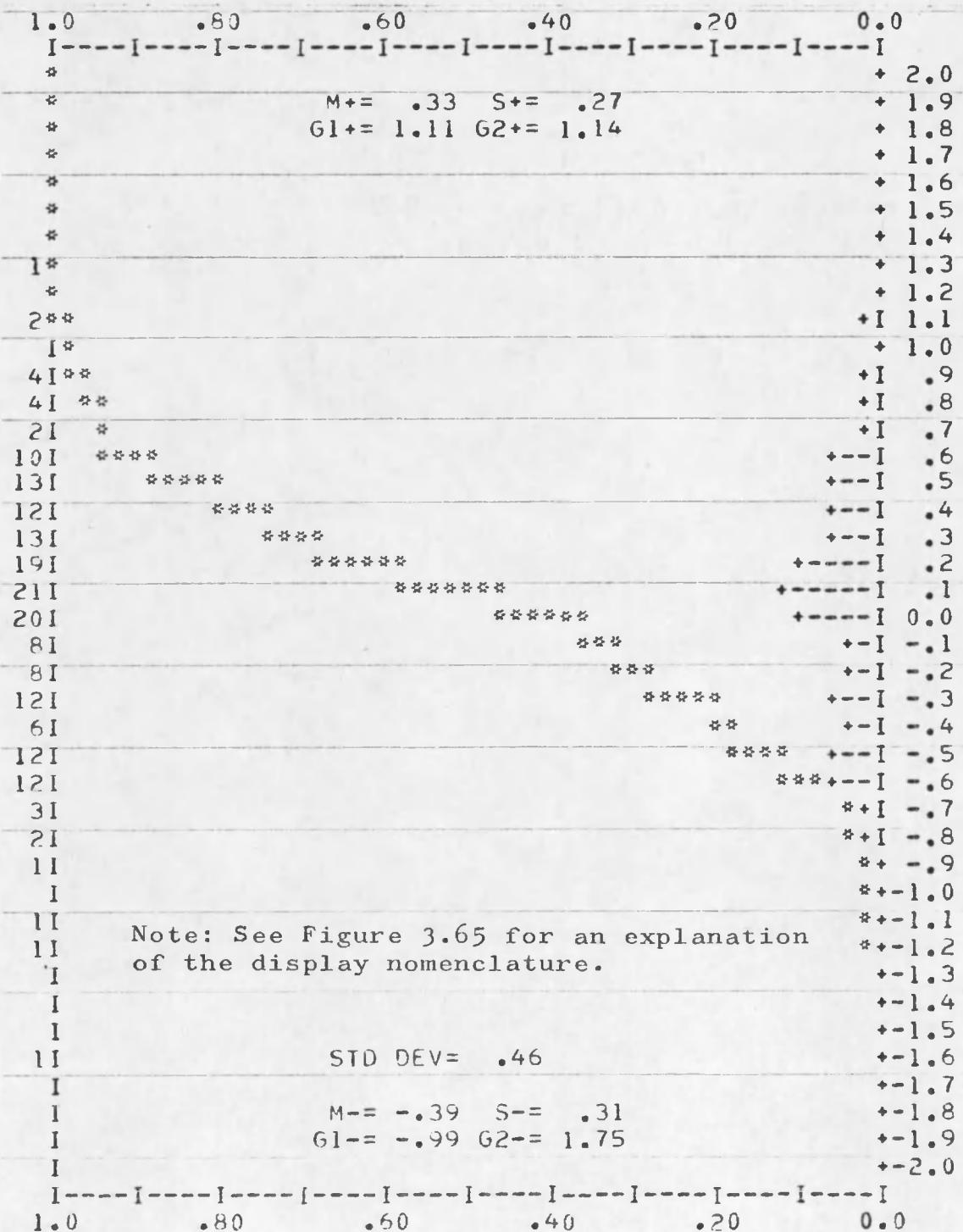


Figure 3.75. Frequency Domain Statistics for 45 mhz

CHAPTER 4

ANALYSIS OF THE EXPERIMENT

4.1 Scope of the Analysis

Many nondeterministic processes are contained in the overall task of studying EMP effects. A general sequence would include the process of sensing the pulse fields, photographing the oscilloscope screen, reproducing the photograph, quantizing the waveform, and numerically transforming the time domain data into the frequency domain. A variety of error sources are inherent in each of the sequences and an exhaustive list of these errors could be arbitrarily long. However, one of the dominant sources and the central purpose of this study is the error which occurs during the hand digitalization of the waveform. Therefore, the analysis will commence with an investigation of the time domain statistics of the digitalized waveform followed by the transformation of these statistics into the frequency domain.

4.2 Time Domain Analysis

4.2.1 Prelude

Each individual attempting to interpolate the waveform amplitude at each of the time intervals will bias his

observation with correlation of the amplitude to surrounding time intervals. Therefore, the observations can not be assumed to be independent to one another along the time axis. The set of observations for each time interval should be considered as a random variable $\tilde{X}(\xi)$. Therefore, we have a nonstationary stochastic process which correlated discrete time functions $x(t, \xi_i)$.

4.2.2 Analysis of the Mean Value Display With High and Low Values for Each Time Interval

The mean value display (Figure 3.1) provides a visual indication of the time domain error bars. The relative magnitude of the error bars provides an indication of ability of the quantizing personnel to agree on each time interval value, however, this display does not give an indication of the variance for each time interval. Therefore, this display should be used primarily as a reference of the expected values for the time domain intervals discussed in the analysis of the PDF's and CDF's.

4.2.3 Analysis of Probability Density Functions and Cumulative Distribution Functions

Figures 3.4 through 3.53 provide a display of the CDF and PDF for each of the 50 time intervals. The primary purpose of these displays is to analyze the time of distributions found in the time domain.

Only six time intervals, shown in Figures 3.8, 3.13, 3.14, 3.33, 3.43, and 3.51 are considered to display a

Gaussian distribution. Of these six, Figure 3.51 has the nearest skew and kurtosis statistics to those displayed by a Gaussian distribution.

Twelve additional intervals have distributions whose 4th order statistics, i.e., kurtosis, are much higher or lower than those for a Gaussian distribution. These intervals can be found in Figures 3.10, 3.11, 3.21, 3.34, 3.45, 3.49, and 3.50 which have a peaked distribution and Figures 3.15, 3.19, 3.22, 3.23, and 3.37 which display a flatter characteristic than associated with a Gaussian distribution.

The remaining 32 time intervals tend even less toward a Gaussian type distribution. For example, Figures 3.6, 3.9, 3.15, 3.38, and 3.48 do not display any of the common types of statistical distribution functions while Figures 3.5, 3.12, and 3.53 tend toward one value and Figure 3.4 could be called deterministic since it has zero variance for the present size of the sample.

Therefore, the study of Figures 3.4 through 3.53 indicates that the density functions in general are not Gaussian nor can we assume that the time domain densities are of any one type. To determine a confidence interval for these distributions, Wilks' tolerance theorem is employed since the distributions can not be categorized by any one type. The Wilks tolerance theorem (see Appendix A, Section A.10) provides that for a sample size of 188 (see

Table A.1) it is at least 97 per cent sure that 97 per cent of the future samples will fall between the established upper and lower limits of each time interval.

4.2.4 Analysis of Slope Versus Standard Deviation Display

This display provides an indication of slope versus standard deviation correlation. If a trend can be seen, assumptions concerning the standard deviation can be made so that future analysis of a certain type of waveform can be simplified. In Figure 3.54, the correlation between slope and standard deviation does show a linear trend after the slope values become greater than one; however, for slope values less than one, there is much less correlation between the two parameters. Since many of the waveforms to be studied can not be assumed to contain only slopes greater than one, then in general, the slope of the waveform should not be used to provide an indication of the standard deviation values.

4.2.5 Analysis of the Correlation Coefficient and t Statistic

The correlation coefficient display provides an indication of the correlation for different values of lag/lead time intervals. A study of Figures 3.56 through 3.60 indicates that the correlation function is in general positive and most of the values lie in the range between .2 and .8. From t statistic tables (Maisel, 1971, p. 276)

a 99 per cent confidence interval that the true correlation is not zero can be made for a sample size greater than 150 if the absolute value of the computed correlation function is greater than .208. Therefore, the majority of the time intervals can be assumed to be correlated.

4.2.6 Analysis of the Standard Deviation Versus the Proximity of the Mean Value to an Amplitude Grid Reference Line

The analysis of this display will indicate the correlation between the placement of the reference system and the standard deviation of the observations. Ideally, there should be no correlation between the readings and the placement of the grid but since it is easier to judge the value of the waveform as it approaches a reference line, then the standard deviation values could be influenced by the grid placement. Therefore, the location of the grid could bias the data. In general, Figure 3.61 indicates that the correlation between the grid placement and the standard deviation is not great except for the fact that the standard deviation is less than two as the amplitude mean approaches a grid line (0 value) or half way between grid lines (5 value).

4.2.7 Analysis of the Average Individual Error

Figure 3.62 indicates that the average errors were distributed both above and below the mean values in

generally the same manner. As shown, the extreme errors were within ± 1.5 of the mean for each time interval.

4.3 Frequency Domain Analysis

4.3.1 Displaced Time Origin

During the process of copying a waveform photograph onto the tracing paper, a grid line was chosen to represent the time origin ($t = 0$). Since the oscilloscope sensing makes no provision to indicate visually the time origin, this point in time is chosen arbitrarily. If the correct time origin was missed by an amount t_0 , then the resulting Fourier transform $F'(w)$ is given by (Papoulis, 1962, p. 14)

$$F'(w) = \int_{-\infty}^{\infty} f(t-t_0) e^{-j\omega t} dt = \int_{-\infty}^{\infty} f(x) e^{-j\omega(t_0+x)} dx \quad (4.1)$$

$$\Rightarrow F'(w) = F(w) e^{-j\omega t_0} \quad (4.2)$$

Therefore, the displaced time origin produces a phase shift of the correct Fourier transform, $F(w)$, and the absolute value $|F'(w)|$ equals the absolute value $|F(w)|$. Regardless of the amount of phase shift, the frequency domain absolute values will be the same.

4.3.2 Analysis of the Transform Expected Values

As shown in Appendix A, Section A.12, the transform of the expected values in the time domain will provide the

expected values in the frequency domain. The transform expected value display (Figure 3.64) indicates the spectral levels over the band of frequencies from .5 mhz to 50 mhz with a minimum level threshold of 40db down from the level at .5 mhz. This display should be used as a reference for the CDF's and PDF's of the frequencies selected in Figures 3.66 through 3.75.

4.3.3 Analysis of the Frequency Domain Probability Density Functions and Cumulative Distribution Functions

A study of Figures 3.66 through 3.75 indicates that as the frequency approaches a minimum (i.e., less than -40db relative) at approximately 14 mhz, the deviation of the frequency domain tends to get larger. In general, the standard deviations of the transformed samples remain in the range between .1 and .5 db except for those frequencies in the vicinity of the minimum point. Using Wilks' theorem in the same manner described in Section 4.2.3, the range of values in the frequency domain have a 97 per cent confidence of remaining within a 97 per cent tolerance interval.

CHAPTER 5

CONCLUSIONS

The most important statistic of the time domain data for this study is the expected value for each time interval. This statistic is transformed into the frequency domain to provide the expected values for the spectral levels at selected frequencies. The only method of obtaining higher order frequency domain statistics was to transform each time domain sample into the frequency domain then compiling the variance, skew, and kurtosis from the transformed samples.

The range of errors in the frequency domain remains less than 1 db except for those regions where the expected value of the energy levels approaches a minimum point (i.e., less than -40 db relative).

The size of the sample space should be determined within specified confidence intervals with a nonparametric (distribution free) test such as Wilks' theorem.

CHAPTER 6

RECOMMENDATIONS FOR FUTURE WORK

Future analysis of this subject should investigate a means of simplifying the analysis of EMP effects by finding an approximation of the actual transform,

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

with a discrete time function such as,

$$F(\omega) \approx \sum [a_n] e^{-int\omega}$$

then testing this approximation to find an analytic means of finding the frequency domain statistics from $[a_n]$ without transforming each time domain sample into the frequency domain.

To provide a faster more efficient method of recording each individual's observations onto data cards, the IBM 534 mark sensing system is suggested. An example scoring sheet is shown in Figure 6.1 which could be modified to properly punch data cards using the IBM 534 system thereby alleviating the card punch effort.

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APPENDIX A

STATISTICAL THEORY

The analysis tools required for quantization error analysis are basic statistical concepts found in most introductory texts on the subject. It is assumed that most readers of this thesis have a basic knowledge of statistics, therefore this appendix is intended to refresh these concepts.

A.1 Expected Value (M_k) of the set of Observations of the Waveform at a Given Time Interval "k" (Brunk, 1965, p. 136)

$$M_k = \frac{1}{N} \sum_{i=1}^N (\tilde{x}(t_k, \xi_i)) \quad (A.1)$$

A.2 Variance (s_k^2) of the set of Observations of the Waveform at a Given Time Interval "k" (Brunk, 1965, p. 139)

Let

$$U_k = \frac{1}{N} \sum_{i=1}^N (\tilde{x}(t_k, \xi_i) - M_k)^2 \quad (A.2)$$

then

$$s_k^2 = \left(\frac{N}{N-1} \right) U_k \quad (A.3)$$

A.3 Standard Deviation (S_k) of the set of Observations of the Waveform at a Given Time Interval "k" (Brunk, 1965, p. 53)

$$S_k = \sqrt{S_k^2} \quad (A.4)$$

A.4 Skew ($g_{1,k}$) of the set of Observations of the Waveform at a Given Time Interval "k" (Hays and Winkler, 1971, p. 162)

$$g_{1,k} = \frac{\sum_{i=1}^N (\tilde{x}(t_k, \xi_i) - M_k)^3}{N U_k^{3/2}} \quad (A.5)$$

Note, the skew value for a normal distribution function is zero (0). Plus values of skew indicate that the distribution mass tends (skews) toward the positive side of the mean value, while minus values indicate that the mass tends toward the negative side of the mean.

A.5 Kurtosis ($g_{2,k}$) of the set of Observations of the Waveform at a Given Time Interval "k" (Hays and Winkler, 1971, p. 162)

$$g_{2,k} = \frac{\sum_{i=1}^N (\tilde{x}(t_k, \xi_i) - M_k)^4}{N U_k^2} \quad (A.6)$$

Note, the kurtosis value for a normal distribution function is zero (0). Plus values of kurtosis indicate that the distribution function tends to be narrower (more peaked)

than a normal distribution, while negative values indicate a wider (flattened) distribution.

A.6 Pearson Product-Moment Correlation Coefficient (r_{kL}) Between Two Sets of

Observations of the Waveform at Two Given Time Intervals, k and L (Hays and Winkler, 1971, p. 602)

$$r_{kL} = \frac{\sum_{i=1}^N [(\tilde{x}(t_k, \xi_i) - M_k) (\tilde{x}(t_L, \xi_i) - M_L)]}{N S_k S_L} \quad (A.7)$$

Note, if the correlation coefficient is equal to +1, then the two time interval observations are said to be perfectly correlated; if it is -1, they are said to be perfectly negatively correlated; and if it is 0, they are said to be uncorrelated. For values other than +1, -1, and 0, a correlation coefficient closer to +1 or -1 implies a stronger relationship, and a value closer to zero implies a weaker relationship.

A.7 Student T Correlation Hypothesis Test (Hays and Winkler, 1971, p. 609)

To test the correlation hypothesis $H_0 : r_{kL} = 0$, a Gaussian distribution is assumed so that the T statistic can be used to test the hypothesis. The T statistic test is given as:

$$t = \frac{r_{kL} \sqrt{N-2}}{\sqrt{1-r_{kL}^2}} \quad (A.8)$$

Thus if the hypothesis is true, then this test will give the percentage of the time that the correlation coefficient r_{KL} is observed to be equal to or greater than its tested value.

A.8 Cumulative Distribution Function (CDF)
(Papoulis, 1965, p. 281)

$$F(x, t) = P\{\xi \in S \mid \tilde{x}(t, \xi) \leq x\} \quad (A.9)$$

A.9 Probability Density Function (PDF)
(Papoulis, 1965, p. 281)

$$f(x, t) = \frac{\partial}{\partial x} F(x, t) \quad (A.10)$$

A.10 Wilks' Tolerance Theorem
(Freeman, 1963, p. 196)

Let $x(1), \dots, x(N)$ be ordered elements of a sample from a population of random variables. If for a given α and β , N is the integer most nearly satisfying the equation:

$$N\beta^{N-1} - (N-1)\beta^N = 1 - \alpha \quad (A.11)$$

then $(x(1), x(N))$ is the two-sided β per cent statistical tolerance interval with confidence coefficient α (i.e., it is α per cent sure that β per cent of the future measurements will fall between the upper and lower bounds, see Table A.1). Note, the Wilks' tolerance theorem is a non-parametric (distribution free) test which does not require

Table A.1. Sample Size (N) for Wilks' Tolerance Theorem

α	β									
	.90	.91	.92	.93	.94	.95	.96	.97	.98	.99
.90	38	42	48	55	64	77	96	129	194	388
.91	39	44	49	56	66	79	100	133	200	401
.92	41	45	51	58	68	82	103	138	207	416
.93	42	47	53	61	71	85	107	143	215	432
.94	44	49	55	63	74	89	112	149	225	451
.95	46	51	58	66	78	93	117	157	236	473
.96	49	54	61	70	82	99	124	166	249	500
.97	52	58	65	75	88	105	132	177	266	534
.98	56	63	71	81	95	115	144	193	290	581
.99	64	71	81	92	108	130	164	219	330	662

α = Confidence coefficient.

β = Tolerance interval.

estimating parameters (mean and standard deviation) from the data. The more common χ^2 test requires a population mean and standard deviation which are not known in this case, therefore the Wilks' tolerance is used to provide a bound on the size of the required sample needed to give certain confidence limits.

A.11 Chauvenet's Criterion (Barford, 1967, p. 102)

A reading may be rejected if the probability of obtaining the particular deviation from the mean is less than $1/2N$ (i.e., for $N = 100$, any reading that falls outside of the range of $M \pm 2.84S$ may be rejected, for $N = 200$ the range is $M \pm 3.01S$). This study uses $N = 200$.

A.12 Expected Value of a Fourier Transform (Papoulis, 1965, p. 310)

$$\begin{aligned}
 E\{f(t, \xi)\} &= \int_{-\infty}^{\infty} \xi f(t, \xi) d\xi \\
 F(\omega, \xi_i) &= \int_{-\infty}^{\infty} f(t, \xi_i) e^{-j\omega t} dt \\
 \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \xi f(t, \xi) d\xi \right] e^{-j\omega t} dt &= \int_{-\infty}^{\infty} \xi \left[\int_{-\infty}^{\infty} f(t, \xi) e^{-j\omega t} dt \right] \\
 &= E\{F(\omega, \xi)\} \quad E\{F(\omega, \xi)\} = \int_{-\infty}^{\infty} E\{f(t, \xi)\} e^{-j\omega t} dt
 \end{aligned} \tag{A.12}$$

APPENDIX B

COMPUTER PROGRAMS

```
PROGRAM TIMEA ( INPUT , OUTPUT , TAPE5=INPUT , TAPE1 , TAPE2 ,
1 TAPE3 )
DIMENSION AA(50) , AB(50) , JA(50)
CALL READA ( M )
CALL COMPA ( AA , AB )
REWIND 3
REWIND 1$ II=0
DO 14 IA=1,M
READ(1)(JA(I),I=1,50)
IF (EOF,1) 88,89
89 DO 17 IB=1,50
IF (JA(IB)-AA(IB)) 34,34,16
16 IF (JA(IB)-AB(IB)) 17,34,34
17 CONTINUE
II=II+1
WRITE(3)(JA(I),I=1,50)
GO TO 14
34 PRINT 78,AA(IB),AB(IB),JA(IB),IB,(JA(I),I=1,50)
14 CONTINUE
88 PRINT 76,II
END FILE 3
STOP
76 FORMAT(I5)
78 FORMAT(2F10.3,2I10/2(25I5/))
END
```

Figure B.1. Program TIMEA

```
PROGRAM TIMEB (INPUT,OUTPUT,TAPE3,TAPE4,TAPE2)
DIMENSION JA(50),JB(50),CC(209),TX(209),CD(209)
REWIND 2 REWIND 4
PRINT 95
95 FORMAT(1H1////////)
DO 40 KA=1,50
REWIND 3
II=0
13 READ(3)(JA(I),I=1,50)
IF(EOP,3)20,14
14 II=II+1
41 CC(II)=FLOAT(JA(KA))
GO TO 13
20 PRINT75,KA
M=II
75 FORMAT(120)
CALL BASIC(CC,M,AM,AD,AS,AK)
CALL GUAGE(CC,M,IX,CD)
JS=IFIX(CD(1))
JT=IFIX(CD(M))
READ(2)DM,DD
EA=AM-DM$EB=AD-DD
PRINT 44,DM,AM,EA,DD,AD,EB
44 FORMAT(1M+,19X,3F8.1,5X,3F8.2)
WRITE(4)AM,AD,AS,AK,JA,JT
40 CONTINUE
END FILE 4
STOP
END
```

Figure B.2. Program TIMEB

```
PROGRAM HILO (INPUT,OUTPUT,TAPE4)
DIMENSION K(45,51)
DO 11 IA=1,50
READ(4)AM,AD,AS,AK,JS,JT
MA=JS*4/10$MB=JT*4/10$K(MA)=68B$K(MB)=67B
MC=IFIX(.4*AM)$K(MC)=47B
11 CONTINUE
DO 12 IB=1,45
PRINT 13,(K(IB,IC),IC=1,51)
13 FORMAT(15X,1HI,51R1,1HI)
12 CONTINUE
STOP
END
```

Figure B.3. Program HILO

```
PROGRAM CDFPDF (INPUT,OUTPUT,TAPE3,TAPE4)
DIMENSION JA(50),CC(250)
REWIND 3$REWIND4
DO 11 IA=1,50
  II=0
  REWIND 3
13  READ(3)(JA(I),I=1,50)
    IF(EOF,3)20,14
14  II=II+1
    CC(II)=FLOAT(JA(IA))
    GO TO 13
20  READ(4)AM,AD,AS,AK,JS,JT
    CALL AUDIT (CC,II,AM,AD,IA,JS,JT,1,AS,AK)
11  CONTINUE
    STOP
END
```

Figure B.4. Program CDFPDF

```
PROGRAM SLVSD(INPUT,OUTPUT,TAPE4)
DIMENSION SL(50),DD(50),AM(50),K(49,51)
REWIND 4
DO 30 IA=1,33$DO 30 IB=1,51
K(IA,IB)=55B
K(I,IB)=K(33,IB)=46B$K(IA,1)=K(IA,51)=11B
30 CONTINUE
DO 11 IA=1,50
READ(4)AM(IA),DD(IA),AS,AK,JS,JT
11 CONTINUE
CALL SLOPE(AM,50,SL)
DO 12 IA=2,46
M=IFIX(16.*ABS(SL(IA))+.499999)+1
L=IFIX(10.*DD(IA)+.499999)+1
12 K(M,L)=47B
PRINT 15
15 FORMAT(1H1/////)
G=2.5
DO 16 IA=1,33
IB=34-IA
IF(IA-IA/8*8-1)17,19,17
19 G=G-.5
PRINT 20,G,(K(IB,IC),IC=1,51)
20 FORMAT(25X,F3.1,51R1)
GO TO 16
17 PRINT 18,(K(IB,IC),IC=1,51)
18 FORMAT(2BX,51R1)
16 CONTINUE
PRINT 22,(IX,IX=1,5)
22 FORMAT(29X,5(9X,I1))
STOP
END
```

Figure B.5. Program SLVSD

```

PROGRAM CORRFCN(INPUT,OUTPUT,TAPE6,TAPE4)
DIMENSION CC(6,250),JA(50),DD(6),AA(5,50)
DIMENSION EA(5),EB(5),EE(5)
DIMENSION CD(50),AM(50),DY(6),DM(6),DX(6)
REWIND 4
DO 70 IA=1,50
70 READ(4)AM(IA),CD(IA),AS,AK,JS,JT
N=0
M=0
REWIND 6
READ(6)(CC(1,I),I=1,188)
DO 11 IA=2,6
N=M+1
READ(6)(CC(N,I),I=1,188)
22 DD(N)=CD(IA)
11 CONTINUE
DO 61 J8=7,50
IF(J8-J8/6*6-2)34,32,34
32 D=0
34 N=N+1
DO(N)=CD(JB)
READ(6)(CC(N,I),I=1,188)
39 M=M-M/6*6+1
DZ=DD(M)
DO 50 KB=1,5
L=M+KB
L=L-(L-1)/6*6
DY(KB)=DD(L)
EF(KB)=0.
DO 50 KA=1,188
50 EF(KB)=EF(KB)+CC(M,KA)*CC(L,KA)
DO 31 KB=1,5
AA(KH,JH)=EE(KB)/(FLOAT(II)*DZ*DY(KB))
IF(AA(KB,JB)-1.)41,41,40
40 AA(KB,JB)=1.$GO TO 31
41 IF(AA(KB,JB)+1.)42,31,31
42 AA(KB,JB)=-1.
31 CONTINUE
PRINT 201,(AA(I,JB),I=1,5)
201 FORMAT(5F20.5)
61 CONTINUE
CALL USHER(AA)
STOP
END

```

Figure B.6. Program CORRFCN

```
PROGRAM PROX (INPUT,OUTPUT,TAPE4)
DIMENSION K(49,51)
REWIND 4
DO 11 IA=1,50
READ(4)AM,AD,AS,AK,JS,JT
MA=IFIX(AM-AM/5*5)$MB=IFIX(10.*AD)
11 K(MA,MB)=47B
DO 12 IB=1,49
PRINT 13, (K(IB,IC),IC=1,51)
13 FORMAT(15X,1HI,51RI,1HI)
12 CONTINUE
STOP
END
```

Figure B.7. Program PROX

```
PROGRAM AVERR(INPUT,OUTPUT,TAPE3,TAPE4)
DIMENSION AM(50),AD(50)
REWIND 3$REWIND 4
DO 11 IA=1,50
11 READ(4)AM(IA),AD(IA),AS,AK,JS,JT
DO 12 IB=1,188
READ(3)(J(I),I=1,50)
DO 14 ID=1,50
14 Z(IB)=Z(IB)+(FLOAT(J(ID)-AM(ID))/50.
12 CONTINUE
CALL ADDER(Z,188,AM,AD)
STOP
END
```

Figure B.8. Program AVERR

```

PROGRAM QERF : (INPUT,OUTPUT,TAPE4,TAPE5)
DIMENSION K(41),LN(10),FR(5),DATA(51),X(51),DX(10),
1XM(5),T(5),S1(50),S3(50)
COMMON/ 999 /INDCATE,TWOP1,INDT,INDF
INDCATE=7HFOFTIME
TWOP1=2.*ACOS(-1.)
INDT=51
REWIND 5
READ(5)(X(I),I=1,51)
READ(5)(DX(I),I=1,3)
READ(5)(LN(I),I=1,4)
NF=1
NB=3
INDF=NF
REWIND 4
DO 59 IA=1,50
READ(4)DATA(IA),AD,AS,AK,JS,JT
59 CONTINUE
DO 34 KL=1,5
34 XM(KL)=0.
DO 37 KP=1,41
PK=FLOAT(KP-1)/20.+1.
PA=.5E+05*10.*PK
FR(1)=PA
CALL ORBIT (DATA,XM,T,X,LN,DX,NB,FR,1)
PRINT 101,T(1)
101 FORMAT(E20.10)
S1(KP)=T(1) $ S3(KP)=PA/1.E+06
37 CONTINUE
GO=S1(1)
G3=GO-60.
DO 21 IC=1,41
21 K(IC)=55B
PRINT 22
DO 11 IA=1,41
Z=S3(42-IA)
G=S1(42-IA)
IF(G-G3 )13,14,14
13 G=G3
14 A=GO-G
K(1)=K(41)=11B
J=IFIX(A+.499999)+1
A=-A
IF(J-41)16,16,17
16 K(J)=47B $ GO TO 18
17 J=41 $ K(J)=73B
18 PRINT 15,A,(K(L),L=L,41),Z

```

Figure B.9. Program QERF

```
K(J)=55B
11 CONTINUE
PRINT 41
STOP
15 FORMAT(15X,F5.1,41R1,F7.2)
22 FORMAT(1H1/////////20X*0*8X*-10*7X*-20*7X*-30*7X *-40*/
116X*DB I*4(*-----I*)* MHZ*)
41 FORMAT(17X,4HO.0*,4(*-----I*))
END
```

Figure B.9.--Continued

```

PROGRAM FREQ(INPUT,OUTPUT,TAPE5,TAPE3,TAPE4)
INTEGER LN(10),JA(50),IX(209)
REAL FR(5),VAL(51),DATA(51),X(51),DX(10),XM(5),T(5)
REAL S1(209),S2(209),S3(209),S4(209),S5(209)
COMMON/999/INDCATE,TWOPI,INDT,INDF
INDCATE=7HFOFTIME
TWOPI=2.*ACOS(-1.)
INDT=51
REWIND 5
READ(5)(X(I),I=1,51)
READ(5)(DX(I),I=1,3)
READ(5)(LN(I),I=1,4)
NF=1$NB=3
FR(1)=1.3E7
INDF=NF
REWIND 4
DO 101 IA=1,50
READ(4)DATA(IA),AD,AS,AK,JS,JT
101 CONTINUE
DO 34 KL=1,NF
34 XM(KL)=0.
CALL ORBIT(DATA,XM,T,X,LN,DX,NB,FR,NF)
DO 35 KJ=1,NF
35 XM(KJ)=T(KJ)
II=0
REWIND 3
13 READ(3)(JA(I),I=1,50)
IF.EOF,3)20,14
14 II=II+1
DO 15 JB=1,50
15 DATA(JB)=FLOAT(JA(JB))
CALL ORBIT(DATA,XM,T,X,LN,DX,NB,FR,NF)
JC=II
S1(JC)=T(1)$S2(JC)=T(2)$S3(JC)=T(3)$S4(JC)=T(4)
GO TO 13
20 ICA=II
CALL BASIC(S1,ICA,AM1,AD1,AS1,AK1)
CALL ADDER(S1,II,AM1,AD1)
STOP
END

```

Figure B.10. Program FREQ

```

SUBROUTINE ADDER (Z,I,AM,AD)
DIMENSION Z(I),IC(50),AKA(50),ADC(250),IXXX(250),
IADS(250)
CALL GUAGE(Z,I,IXXX,ADC)
II=0
11 II=II+1$IF(II-I)70,71,71
71 IJ=II$GO TO 12
70 IF(ADC(II))11,12,12
12 IJ=II-1
IF(IJ)66,66,67
67 CALL BASIC(ADC,IJ,CM,CD,CS,CK)
GO TO 68
66 CM=CD=CD=CK=0.
68 IF(II-I)69,75,75
69 KB=0
DO 13 KA=II,I
KB=KB+1
13 ADS(KB)=ADC(KA)
CALL BASIC(ADS,KB,DM,DD,DS,DK)
GO TO 76
75 DM=DD=DS=DK=0.
76 ID=1
AGB=2.0$GA=-2.0$AJB=.1$AXX=-2.1
DO 1 IA=1,41$IC(IA)=0$AXX=AXX+AJB$AKA(IA)=0.
DO 2 IB=ID,I
8 IF(ADC(IB)-AXX)3,3,4
3 IC(IA)=IC(IA)+1
2 CONTINUE
AKA(IA)=50.*FLOAT(IC(IA))/FLOAT(I)
IF(AXX-AGB)5,7,7
4 ID=IB$AKA(IA)=50.*FLOAT(IC(IA))/FLOAT(I)
1 CONTINUE
5 IG=IA+1
DO 6 IY=IG,41
AKA(IY)=0.
6 IC(IY)=0
7 CALL FRONT(AKA, IC,AJB,AM,AD,CM,CD,CS,CK,DM,
1 DD,DS,DK)
RETURN
END

```

Figure B.11. Subroutine ADDER

```
SUBROUTINE AUDIT (ADX,IACA,AAM,AAD,IASQ,IAJS,IAJT,  
IAMA,AS,AK)  
DIMENSION ADC(250),IANC(50),AKA(50),IAXZ(250),ADX(1)  
IACD=1$IAGA=IFIX(AAM-9.5000001)  
AXX=FLOAT(IAGA-1)$AGB=AXX+21.$CALL GUAGE(ADX,IACA,  
IAXZ,ADC)  
IAJS=IFIX(ADC(1))$IAJT=IFIX(ADC(IACA))  
DO 1 IAAA=1,21$IANC(IAAA)=0$AAX=AAX+1.$AKA(IAAA)=0.  
DO 2 IAAB=IACD,IACA  
8 IF (ADC(IAAB)-AXX)3,3,4  
3 IANC(IAAA)=IANC(IAAA)+1  
2 CONTINUE  
 AKA(IAAA)=50.*FLOAT(IANC(IAAA))/FLOAT(IACA)  
 IF (AXX-AGB)5,7,7  
4 IACD=IAAB  
 AKA(IAAA)=50.*FLOAT(IANC(IAAA))/FLOAT(IACA)  
1 CONTINUE  
5 TABA=IAAA+1  
DO 6 IAYA=TABA,21  
AKA(IAYA)=0.  
6 IANC(IAYA)=0  
7 CALL FACET(IAGA,IACA,AKA,IANC,IASQ,AAM,AAD,IAMA,AS,AK)  
RETURN  
END
```

Figure B.12. Subroutine AUDIT

```
SUBROUTINE BASIC(BAD,IBAN,BMN,BSD,BSK,BKT)
DIMENSION BAD(1)
BMN=BMA=BMB=BMC=0.
BAN=FLOAT(IBAN) $ BKA=0.
DO 1 IBAA=1,IBAN
3 BMN=BMN+BAD(IBAA)
1 CONTINUE
BMN=BMN/(BAN-BKA)
DO 2 IBAB=1,IBAN
4 BCA=BAD(IBAB)-BMN $ BCB=BCA*BCA $ BCC=BCB*BCE $ BCE=
1BCC*BCE
BMA=BMA+BCB $ BMB=BMB+BCC
BMC=BMC+BCE
2 CONTINUE
IF(BMA)5,5,6
5 BSD=BSK=BKT=0. $ GO TO 7
6 BAN=BAN-BKA
BSD=SQRT(SMA/(BAN-1.))
BMA=BMA/BAN $ BMB=BMB/BAN $ BMC=BMC/BAN
BSK=BMB/(BMA*SORT(BMA))
BKT=-3.+BMC/(BMA**2)
7 RETURN
END
```

Figure B.13. Subroutine BASIC

```
SUBROUTINE CAPER (BAD,IBAN,BMN,BSD)
DIMENSION BAD(1)
BMN=BMA=BMB=BMC=0.
BAN=FLOAT(IBAN) $ BKA=0.
DO 1 IBAA=1,IBAN
3 BMN=BMN+BAD(IBAA)
1 CONTINUE
BMN=BMN/(BAN-BKA)
DO 2 IBAB=1,IBAN
4 BCA=BAD(IBAB)-BMN $ BCB=BCA*BCA
BMA=BMA+BCB
2 CONTINUE
IF(BMA)5,5,6
5 BSD=BSK=BKT=0. $ GO TO 7
6 BAN=BAN-BKA
BSD=SQRT(BMA/(BAN-1.))
7 RETURN
END
```

Figure B.14. Subroutine CAPER

```
SUBROUTINE COMPA(AA,AB)
DIMENSION JA(50),CA(209),AA(50),AB(50)
REWIND 2.
DO 11 IA=1,50
REWIND 1
II=0
13 READ(1)(JA(I),I=1,50)
IF(EDF,1)20,12
12 II=II+1
CA(II)=FLOAT(JA(IA))
GO TO 13
20 CALL CAPER(CA,II,AM,AD)
WRITE(2)AM,AD
IF(AD-1.)22,23,23
22 AD=1.
23 AA(IA)=AM-3.03*AD-1.
11 AB(IA)=AM+3.03*AD+1.
END FILE 2
RETURN
END
```

Figure B.15. Subroutine COMPA

```

SUBROUTINE FACET(IFGA,IFTT,FKA,IFNC,IFSQ,FAM,FAD,IFMA,
1 LAS,AK)
DIMENSION JFF(51),FKA(50),IFNC(50),IFJJ(1)
IFRD=0$PRINT 24
IFKB=50.$IFKB=1
DO 2 IFAA=1,21$DO 1 IFAB=1,51
1 JFF(IFAB)=55B$IFAZ=22-IFAA$IFKB=IFKA(IFAZ)$IFKC=IFKB
IFKA=51-IFIX(FKA(IFAZ)+.499999)
IFKB=51-IFIX(IFKB+.499999)
IFAD=IFGA-IFAA+21$JFF(51)=11B$JFF(1)=11B$JFF(IFKC)=47B
1$PRINT 4,JFF
DO 5 IFTA=IFKC,IFKB
5 JFF(IFTA)=47B$JFF(IFKA)=45B$IFQB=IFKA+1$IF(IFQB-51)31,
139,39
31 DO 32 IFQA=IFQB,50
32 JFF(IFQA)=46B
39 IF(IFNC(IFAZ))6,6,7
5 PRINT 8,JFF,IFAD$GO TO 91
7 PRINT 3,IFNC(IFAZ),JFF,IFAD
91 IF(IFAA-2)13,12,11
12 PRINT 22,IFTT$GO TO 2
13 PRINT 23,IFSQ$GO TO 2
11 IFAX=IFAA-17$IF(IFAX)2,2,14
14 GO TO(15,16,17,2),IFAX
15 PRINT 25,FAM,FAD$GO TO 2
16 PRINT 26,AS,AK$GO TO 2
17 PRINT 27$GO TO 2
2 CONTINUE
PRINT 64
RETURN
3 FORMAT(22XI3,51R1,I3)
4 FORMAT(25X,51R1)
8 FORMAT(25X,51R1,I3)
22 FORMAT(1H+,45X,*TOT=*,I4)
23 FORMAT(1H+,45X,*SEQ=*,I4)
24 FORMAT(1H|||||24X*1.0*7X*.80*7X*.60*7X*.40*7X*,20*
172*0.0*/25X*I*,10(5H---I))
25 FORMAT(1H+,27X,*M=*F6.2,*S=F6.2)
26 FORMAT(1H+,25X,*G1=*F6.2,*G2=F6.2)
27 FORMAT(1H+,27X,15H+=PDF * =CDF)
64 FORMAT(25X*I*,10(5H---I)/24X*1.0*7X*.80*7X*.60*7X*.40*
17X*.20*7X**0.0*)
83 FORMAT(1H@,100X,I4)
END

```

Figure B.16. Subroutine FACET

```

SUBROUTINE FOURIER(X,Y,LN,DX,NB,U,V,CL,P)
DIMENSION X(1),Y(1),LN(1),DX(1),U(1),V(1),CL(1),P(1)
COMMON/ 999 /INDICATE,TWOPPI,INDT,INDF
KB=0$KP=INDT
IF(INDICATE.EQ.7HF0FFREQ) GO TO 25
KB=1$KB=INDF
25 DO 90 K=1,KP
AC=0.$BC=0.$TK=TWOPPI*O(K)
DO 80 L=1,NB
A1=0.$B1=0.$A2=0.$B2=0.$MO=LN(L)$MN=LN(L+1)$MO2=2*
1MO$MN2=2*MN
M1=MO+2$M2=MN-2$DL=DX(L)$CA=COS(TK*X(M1))$SA=SIN(TK*X
1(M1))
CB=COS(TK*2.*DL)$SB=SIN(TK*2.DL)$CZ=CA$SZ=SA
DO 30 M=M1,M2,2
A1=A1+Y(2*M-1)*CZ$B1=B1+Y(2*M-KB)*SZ$CA=CZ$SA=SZ$CZ=
1CZ=CA*CB-SA*SB
30 SZ=SA*CH+CASB
M1=M1-1$CA=COS(TK*X(M1))$SA=SIN(TK*X(M1))$CZ=CA$XZ=SA
DO 40 M=M1,MN,2
A2=A2+Y(2*M-1)*CZ$B2=B2+Y(2*M-KB)*SZ$CA=CZ$SA=SZ$CZ-
1CA*CB-SA*SB
40 SZ=SA*CB+CA*SB
EO=TK*X(MO)$EN=YK*X(MN)$CZO=COS(EO)$CZN=COS(EN)$SZO=
1SIN(EO)
SZN=SIN(EN)$A1=A1+.5*(Y(MO2-1)*CZO+Y(MN2-1)*CZN)
B1=B1+.5*(Y(MO2-KB)*SZO+Y(MN2-KB)*SZN)
AS=Y(MN2-1)*SZN-Y(MO2-1)*SZO$BS=Y(MN2-KB)*CZN-Y(MO2-KB)
1*CZO
TH=TK*DL$TH2=TH*TH
IF(ABS(TH).LT..1) GO TO 60
STH=SIN(TH)/TH$CTH=COS(TH)
A=(1.+SIH*CTH-2.*STH*STH)/TH$B=2.*((1.+CTH*CTH-2.*STH*
1CTH)/TH2
G=4.*(STH-CTH)/TH2$GO TO 75
60 TH4=TH2*TH2$A=2.*TH2*TH/45.*((1.-TH2/7.+TH4/105.))
B=2./3.+2.*TH2*((1./15.-2.*TH2/105.+TH4/567.))
G=4./3.+TH2/15.*((-2.+TH2/14.-TH4/756.))
75 AC=AC+DL*(A*AS+B*A1+G*A2)
80 BC=BC+DL*(-A*BS+B*B1+G*B2)
IF(INDICATE.EQ.7HF0FTIME) GO TO 85
CL(K)=4.*AC$P(K)=4.*BC $V(K)=2.*((AC+BC)$GO TO 90
85 V(2*K-1)=AC$V(2*K)=BC
90 CONTINUE
RETURN
END

```

Figure B.17. Subroutine FOURIER

```

SUBROUTINE FRONT(FKA,IFNC,FJB,AM,AD,CM,CD,CS,CK,DM,DD,
1DS,DK)
DIMENSION JFF(51),FKA(50),IFNC(50)
IERD=0$PRINT 24
FKH=50.$IFKB=1
DO 2 IFAA=1,41$DO 1 IFAB=1,51
1 JFF(IFAB)=558$IFAZ=42-IFAA$FKB=FKB-FKA(IFAZ)$IFKC=IFKB
IFKA=51-IFIX(FKA(IFAZ)+.499999)
IFKB=51-IFIX(FKB+.499999)
FAD=12.+FLOAT(41-IFAA)/10.
JFF(51)=JFF(1)=11B
DO 5 IFTA=IFKC,IFKB
5 JFF(IFTA)=47B$JFF(IFKA)=45B$IFOB=IFKA+1$IF(IFQB)-51)31,
139,39
31 DO 32 IFQA=IFQB,50
32 JFF(IFQA)=46B
39 IF(IFNC(IFAZ))6,6,7
6 PRINT 8,JFF, FAD $GO TO 91
7 PRINT 3,IFNC(IFAZ),JFF, FAD
91 IF(IFAA-3)92,92,11
92 GO TO(2,13,12),IFAA
13 PRINT 22,DM,DD$GO TO 2
22 FORMAT(1H+,41X,3HM+=,F5,2,5H S+=,F5.2)
12 PRINT 23,DS,DK$GO TO 2
23 FORMAT(1H+,4OX,4HG1+=,F5.2,5H G2+=,F5.2)
11 IFAX=IFAA-36$IF(IFAX)2,2,14
14 GO TO(15,2,16,17,2),IFAX
15 PRINT 25,AD$GO TO 2
25 FORMAT(1H+41X,*STD DEV=*F5.2)
16 PRINT 26,CM,CD$GO TO 2
26 FORMAT(1H+,41X,3HM-=,F5.2,5H S-=,F5.2)
17 PRINT 27,CS,CK$GO TO 2
27 FORMAT(1H+,4OX,4HG1-=,F5.2,5H G2-=,F5.2)
2 CONTINUE
PRINT 64
RETURN
3 FORMAT(22X,13,51R1,F4.1)
4 FORMAT(25X,51R1)
8 FORMAT(25X,51R1,F4.1)
24 FORMAT(1H1/////////24X*1.0*7X*.80*7X*.60*7X*.40*7X*.20*7X*
1*0.0*/25X*I*,10(5H----I))
64 FORMAT(25X*I*,10(5H----I)/24X*1.0*7X*.80*7X*.60*7X*.40
1*7X*.20*7X*0.0*)
END

```

Figure B.18. Subroutine FRONT

```
SUBROUTINE GUAGE (GSS,IGCA,IGTT,GTT)
DIMENSION GSS(IGCA),IGTT(IGCA),GTT(IGCA)
DO 3 IGAA=1,IGCA
1   GTT(IGAA)=1.E20
DO 4 IGAB=1,IGCA
IF(GSS(IGAA)-GTT(IGAB))5,5,4
4   CONTINUE
5   IGAC=IGAA-IGAB+1
IF(IGAA-IGAC)8,7,7
7   IGAE=2 $ GO TO 9
8   IGAE=1 $ GO TO 9
9   DO 6 IGAD=IGAE,IGAC
      IGTT(IGAA-IGAD+2)=IGTT(IGAA-IGAD+1)
6   GTT(IGAA-IGAD+2)= GTT(IGAA-IGAD+1)
      IGTT(IGAB)=IGAA
3   GTT(IGAB)=GSS(IGAA)
2   RETURN
END
```

Figure B.19. Subroutine GUAGE

```
SUBROUTINE ORBIT(D,Y,T,X,L,E,N,F,K)
DIMENSION F(1),X(1),E(1),L(1),D(1),T(1),Y(1),B(100),
1P(100)
DIMENSION FR(2)
COMPLEX AL(51),V(100),VV(1)
COMMON/999/INDCATE,TWOPi,INDT,INDF
DO 11 IA=1,35
11 AL(IA)=D(IA)
DO 12 IA=37,50
12 AL(IA)=D(IA)
AL(36)=.4*D(37)+.6*D(36)$AL(51)=0.
CALL FOURIER(X,AL,L,E,N,F,V,B,P)
INDE=1
FR(1)=5.E5
CALL FOURIER(X,AL,L,E,N,FR,VV,B,P)
H=CABS(VV(1))$W=H*1.E-03
DO 6 I=1,K
G=CABS(V(I))
IF(H-G*1.E+3)7,7,8
8 G=W
7 T(I)=20.* ALOG10(G)-Y(I)
6 CONTINUE
RETURN
END
```

Figure B.20. Subroutine ORBIT

```

SUBROUTINE USHER (AA)
DIMENSION AA(1,1),K(41,45)
DO 33 JJ=1,5
DO 11 IA=1,41$DO 11 IB=1,45
11 K(IA,IB)=55B
DO 12 IA=1,45
K(25,IA)=K(17,IA)=24B
K(1,IA)=K(41,IA)=46B
12 K(21,IA)=46B
DO 14 J=7,50
M=IFIX(20.*AA(JJ,J)+.499999)+21
14 K(M,J-6)=47B
PRINT 15
15 FORMAT(1H1////////)
G=1.2
DO 16 IA=1,41
IB=42-IA
IF(IA-IA/4*4-1)17,19,17
19 G=G-.2
PRINT 20,G,(K(IB,IC),IC=1,44)
20 FORMAT(25X,F4.1,1HI,44R1,1HI)
GO TO 16
17 PRINT 18,(K(IB,IC),IC=1,44)
18 FORMAT(29X,1HI,44R1,1HI)
16 CONTINUE
DO 21 IA=7,50$K(1,IA)=IA-IA/10*10
21 K(2,IA)=IA/10
PRINT 22,(K(2,II),II=1,44)
PRINT 22,(K(1,II),II=1,44)
22 FORMAT(29X,50I1)
22 CONTINUE
RETURN
END

```

Figure B.21. Subroutine USHER

LIST OF REFERENCES

- Barford, N. C. Experimental Measurements: Precision, Error, and Truth. Reading, Mass.: Addison-Wesley Publishing Co., 1967.
- Brunk, H. D. An Introduction to Mathematical Statistics. Waltham, Mass.: Blaisdell Publishing Co., 1965.
- Freeman, Harold. Introduction to Statistical Inference. Reading, Mass.: Addison-Wesley Publishing Co., 1963.
- Hays, William L., and Robert L. Winkler. Statistics: Probability, Inference, and Decision. New York: Holt, Rinehart and Winston, Inc., 1971.
- Maisel, Louis. Probability, Statistics, and Random Processes. New York: Simon and Schuster, Inc., 1971.
- Papoulis, Athanasios. The Fourier Integral and its Applications. New York: McGraw-Hill Book Co., 1962.
- Papoulis, Athanasios. Probability, Random Variables, and Stochastic Processes. New York: McGraw-Hill Book Co., 1965.

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