DEVELOPMENT OF AN ANALOG COMPUTER NOISE GENERATOR

by

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ABSTRACT

A new random noise generator yields both random-telegraph-wave output with a mean count rate of 80 kc and Gaussian noise having a power spectrum flat between d-c and 20 kc. The noise generator consists of a radioactive source, pulse amplifier, Schmitt trigger, binary, limiter, filter amplifier, and a feedback circuit regulating the mean count rate. Accurately predictable characteristics and relatively wide bandwidth make the new noise generator useful for fast analog computation and, in particular, for repetitive computers.

The output waveform preceding the filter amplifier is an accurately limited random telegraph wave of ± 20 volts with a stable zero mean. Theoretical design considerations and performance data are presented.

ACKNOWLEDGMENT

The circuit was developed in the course of a repetitive analog computer project directed by G. A. Korn. Acknowledgment is due the Electrical Engineering Department of the University of Arizona and Dr. P. E. Russell for continuing support of this work.

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CHAPTER 1

INTRODUCTION

Analog-computer studies of random processes require the generation of randomly varying voltages with appropriate statistical properties to represent randomly varying variables, initial conditions, and/or parameters.[1]

An analog-computer noise generator should possess the following characteristics:[2]

1. A stationary random output voltage of known amplitude.
2. Adequate RMS output, to minimize further amplification requirements.
3. A stable zero mean. A mean other than zero can be obtained by addition of a d-c voltage.
4. Low correlation (less than 0.1% of mean square) for delays made above 1/bandwidth; in particular, the output must not contain periodic components.
5. A stable power spectral density, preferably flat from zero frequency to a frequency sufficiently large when compared to any noise spectrum to be simulated.

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[1] See, for instance, W. Feller, Probability Theory and Its Applications, John Wiley & Sons, Inc., New York, 1950

[2] H. Low, in Grabbe, Ramo, and Wooldridge, Handbook of Automation, Computation, and Control, Chapter 26, John Wiley & Sons, Inc., New York, 1959

A noise generator intended to provide random input signals to a repetitive analog computer is likely to require a flat power spectrum extending from d-c to several thousand cycles per second. The output power spectral density may be altered to meet a particular requirement by means of linear shaping filters; the output amplitude distribution can be shaped by means of diode function generators.

1.1 Random Noise Generators

Gaussian-noise generation by means of noisy resistors,[3] diodes,[4] phototubes,[5] and thyratrons in a magnetic field[6] suffers from the necessity of elaborate regulation and/or monitoring of RMS and d-c output levels, and also from non-uniformity of the power spectrum at low frequencies. Low-frequency spectral non-uniformities, including line-frequency components due to hum pickup, can be removed through sampling or demodulation and filtering of the noise generator output, which re-centers a flat portion of the spectrum about d-c;[7] but the resulting noise output spectra are flat only to 100-500 cps, and, hence, are not useful for high-speed repetitive computation.

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[3] "White Noise Test Set, Type OA. 1249A," Marconi Instruments Limited, England

[4] W.R. Bennett, "Equipment for Generating Noise," Electronics, April, 1956, p. 135

[5] Ibid., p. 135

[6] R.R. Bennett, D.E. Beecher, and H. Low, "Electronically Stabilized Noise Generation," Electronics, 27, July, 1954, pp. 163-165

[7] H. Low, op.cit., Chapter 26

1.2 An Improved Noise Generator Circuit

A flip-flop (binary) symmetrically triggered by a radiation detector actuated with a radioactive source yields a random-telegraph signal with Poisson-distributed zero crossings (Fig. 1.1).

If the levels $\pm$ E are accurately set by a precision limiter,

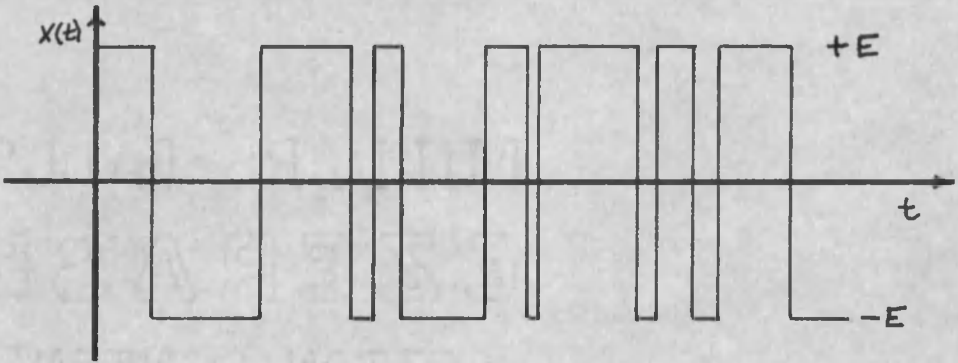

Fig. 1.1, Random-Telegraph-Wave

the mean output is zero, whereas the mean-square, autocorrelation, and power spectral density are accurately given by

$$\varphi(0) = E^2 \tag{1.1}$$

$$\varphi(\tau) = E^2 e^{-2\alpha|\tau|} \tag{1.2}$$

$$W(\omega) = \frac{E^2}{\alpha} \frac{1}{1 + \frac{\omega^2}{4\alpha^2}} \tag{1.3}$$

where α is the mean count rate of the radiation-detector output pulses (Appendix A). Low-pass filtering of the random telegraph signal yields essentially Gaussian noise of zero mean and spectral density precisely determined by the filter characteristics (Appendix A). The inherently flat-spectrum random-telegraph signal is, however, extremely useful in its own right. The binary nature permits its direct use for random switching (binary multiplication), e.g., in correlators.[8] It also serves as a source of randomly timed events (Poisson process) in simulations of equipment failures and queuing problems such as traffic control.[9]

An earlier noise generator of this type utilized a self-quenched Geiger-Mueller tube as the radiation detector;[10] but this device was limited to relatively low count rates (below 500 cps) by its deionization time. The new noise generator to be described replaces the Geiger-Mueller tube by a scintillation detector (Type 931A photomultiplier) and permits count rates well in excess of 75 kc for wideband operation.

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[8] G.W. Anderson, J.A. Aseltine, A.R. Mancini, C.W. Sartare, "A Self-Adjusting System for Optimum Dynamic Performance," IRE Nat'l Convention Record, March, 1958

[9] "Two Applications of Geda Computers to Statistical Problems of Operations Research," Goodyear, GER-6729, June, 1955, pp. 18-31

[10] "Random Noise Generator for Simulation Studies," Goodyear Aircraft Corporation, December, 1954

CHAPTER 2

DESCRIPTION OF NOISE GENERATOR

The block diagram of Fig. 2.1 illustrates the operation of the circuit described in this paper. The noise generator consists of a source of radioactive material mixed in a light-emitting phosphor and a photomultiplier detector whose output produces current pulses corresponding to the emission of light impulses in the scintillating phosphor. After amplification in a 5-stage pulse amplifier, pulses exceeding a certain threshold voltage trigger a cathode-coupled binary (Schmitt trigger) to produce short pulses of constant amplitude and random time occurrence. These pulses are inverted and applied to a binary (Eccles-Jordan trigger) to produce a square wave of constant amplitude with random zero crossings. The output square wave has a d-c component which is eliminated by a cathode follower followed by a symmetrical limiter. The cathode follower output is clipped accurately at equal positive and negative voltage levels by means of a diode bridge limiter. The result is a random-telegraph signal having accurate levels of $+E$ and $-E$ volts. The random-telegraph signal is amplified and/or filtered in a chopper-stabilized wide-band operational amplifier. Assuming a low-pass filter cutting off well below the mean count rate of the random-telegraph signal, the filter output will be approximately Gaussian.[11] The power

[11] Y.W. Lee, Statistical Theory of Communication, John Wiley & Sons, Inc., 1960, pp. 323-342

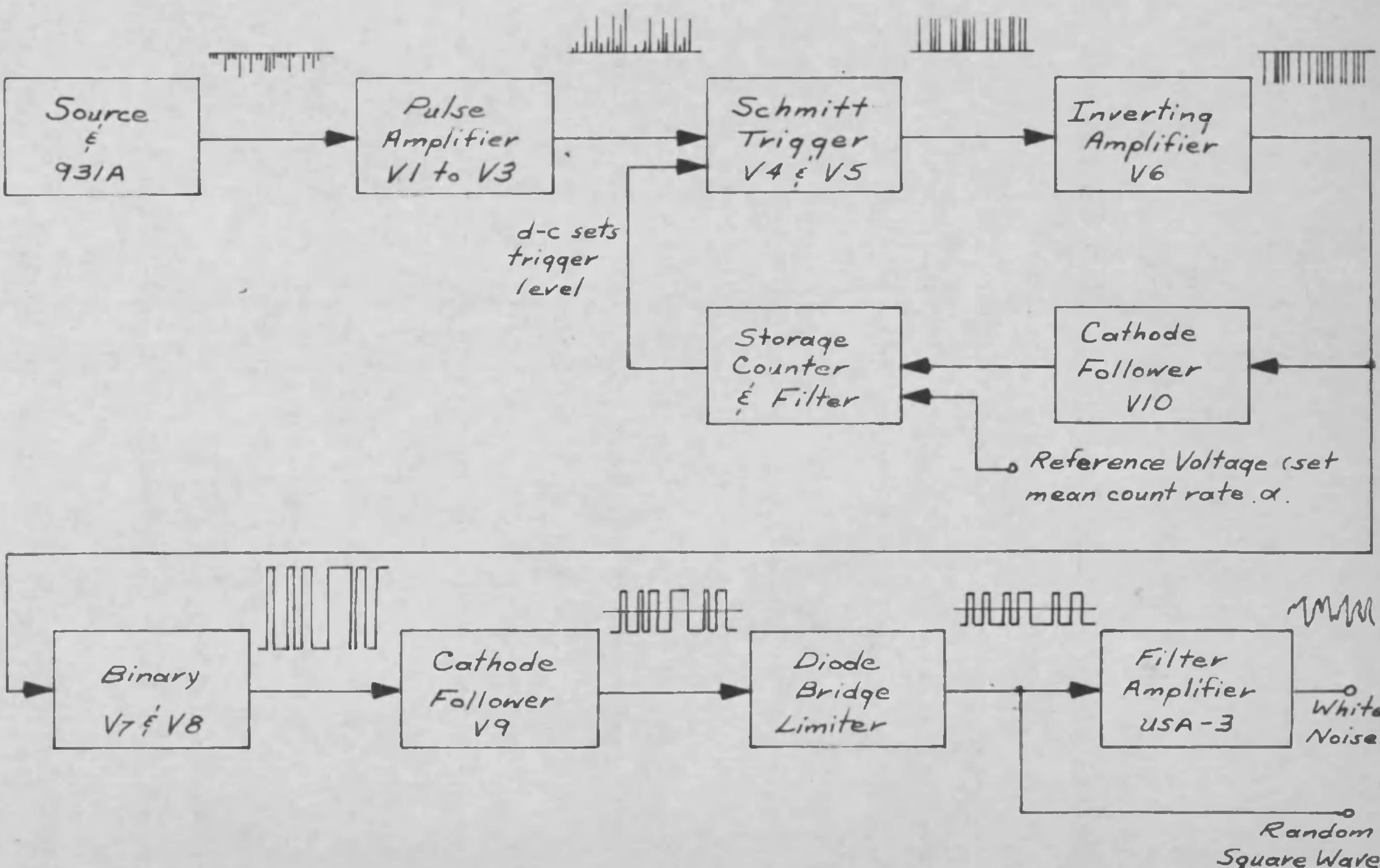

Fig. 2.1, Block Diagram of Noise Generator

spectral density will be essentially flat from zero frequency to near the filter cutoff frequency.

Since the mean count rate of the random-telegraph signal affects the spectral density of the noise output, the threshold level of the Schmitt trigger is controlled by means of feedback through a storage counter employing an operational amplifier integrator. This counting circuit produces a d-c voltage proportional to the mean repetition rate of the Schmitt trigger output pulses and compares it with a reference voltage. The resultant error voltage is filtered and fed back to regulate the Schmitt trigger threshold level.

2.1 Design Requirements

The rate of decay of the available radioactive source determines the mean count rate. The scintillator, excited by the radioactive isotope, emits light in the form of discrete packets of energy called photons. The persistency of the scintillating phosphor should be low so as to permit quick recovery or short flashes. If this condition is not satisfied, pulses will have long trailing edges and tend to overlap.

Since the pulses occur at random times and with random amplitudes, depending upon the energy of the photons, one requires a pulse amplifier which can recover quickly after a severe overload and can amplify small pulses faithfully occurring between very large and frequent overload pulses.

The random square wave generating circuitry requires a Schmitt trigger and binary with speeds comparable to that of the pulse amplifier. In the case of the binary, a large output is desired (100 volts or more).

A compromise between speed and output should result. To ensure a stationary output voltage with a stable zero mean, an accurate limiter is required.

With a constant potential across the phototube, the maximum available count rate is determined by the radioactive source. The count rate is also critically determined by the bias voltage at the input of the Schmitt trigger, which is used to stabilize the mean count rate through the aforementioned feedback-regulator circuit.

CHAPTER 3

THE SCINTILLATION DETECTOR

3.1 Radioactive Source and RCA 931A Photomultiplier

The radioactive source consists of a radioactive isotope mixed with a light emitting phosphor. It is the same substance that is used by jewelers for the dial of luminous watches. This jeweler's paint was applied to a small region of the tube envelope covering the cathode of the photomultiplier.

The 931A is a 9-stage photomultiplier with an S-4 response peaking at about 4,000 angstroms. This photomultiplier was selected because of its relatively low cost, and because it is of the vacuum type, which, unlike a gas phototube, suffers little or no fatigue (loss of sensitivity with use) when operated continuously near full plate current.

Sensitivity is dependent on the respective amplification of each dynode stage. The overall amplification is equal to the average amplification per stage raised to the nth power, where n is the number of dynode stages.

A supply of 1000 volts is applied to the phototube across a voltage divider providing 100 volts between successive dynode stages. A curve of count rate vs voltage across the phototube is shown in Fig. 3.1 for a Schmitt trigger threshold level set near maximum count rate. A base socket of high grade, low-leakage plastic prevents leakage of high voltage between the multiplier tube electrode terminals.

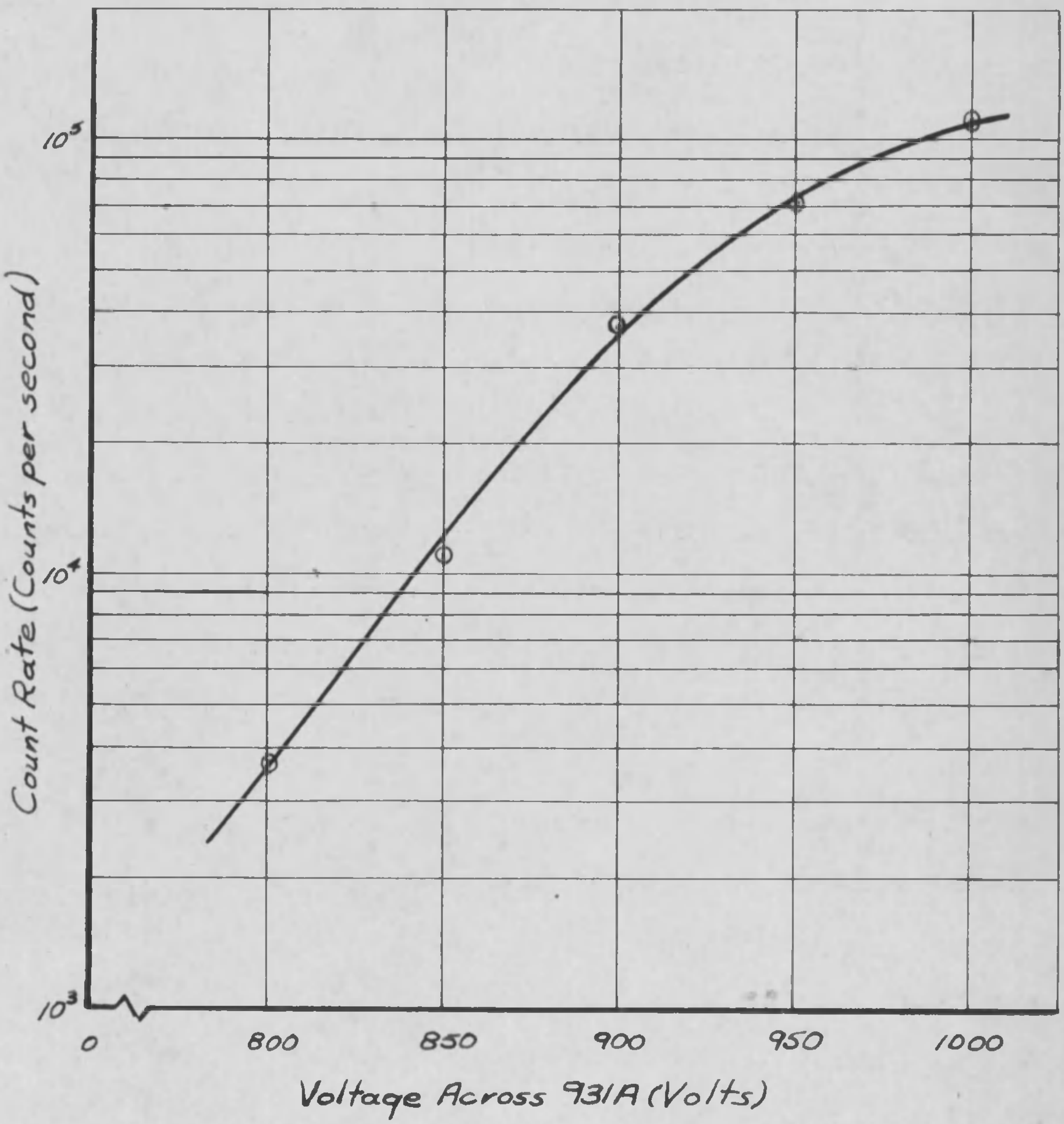

Fig. 3.1, Count Rate vs Voltage Across 931A.

As can be seen in Fig. 3.2, the last 3 dynode stages have been by-passed. If peak pulse outputs are continuously drawn from the tube, a momentary decrease in the bleeder current results and causes a corresponding decrease in the output voltage. This type of degenerative feedback may lead to nonlinear response from the tube. The value of the capacitors C_1, C_2, and C_3 is chosen so that the time constant determined by them and the bleeder resistance will be large with respect to the pulses to be detected.

The source and photomultiplier tube are completely enclosed by a Mu-metal shield mainly to prevent a-c fields from modulating the count rate; the earth's field might also cause a decrease in signal.[12]

3.2 Non-overloading Pulse Amplifier

The input signal to the pulse amplifier varies with the intensity of the incident light flashes; each photon in turn excites the cathode of the photomultiplier and yields photoelectrons which cause an output pulse amplified by secondary emission. The measured pulse output is of negative polarity and varies from near zero to ten or fifteen volts.

The count rate is determined by the number of these pulses which will exceed a certain threshold value. The pulses occur at random times and can be very close together. Hence, one requires a fast amplifier which can amplify the small pulses above the threshold value, and will not be overloaded by the larger pulses. The problem of fast response was not as difficult to solve as the problem of overloading. If the

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[12] "Dumont Multiplier Phototubes," Allen B. Dumont Laboratories, Inc., Clifton, N. J., 1959, p. 11

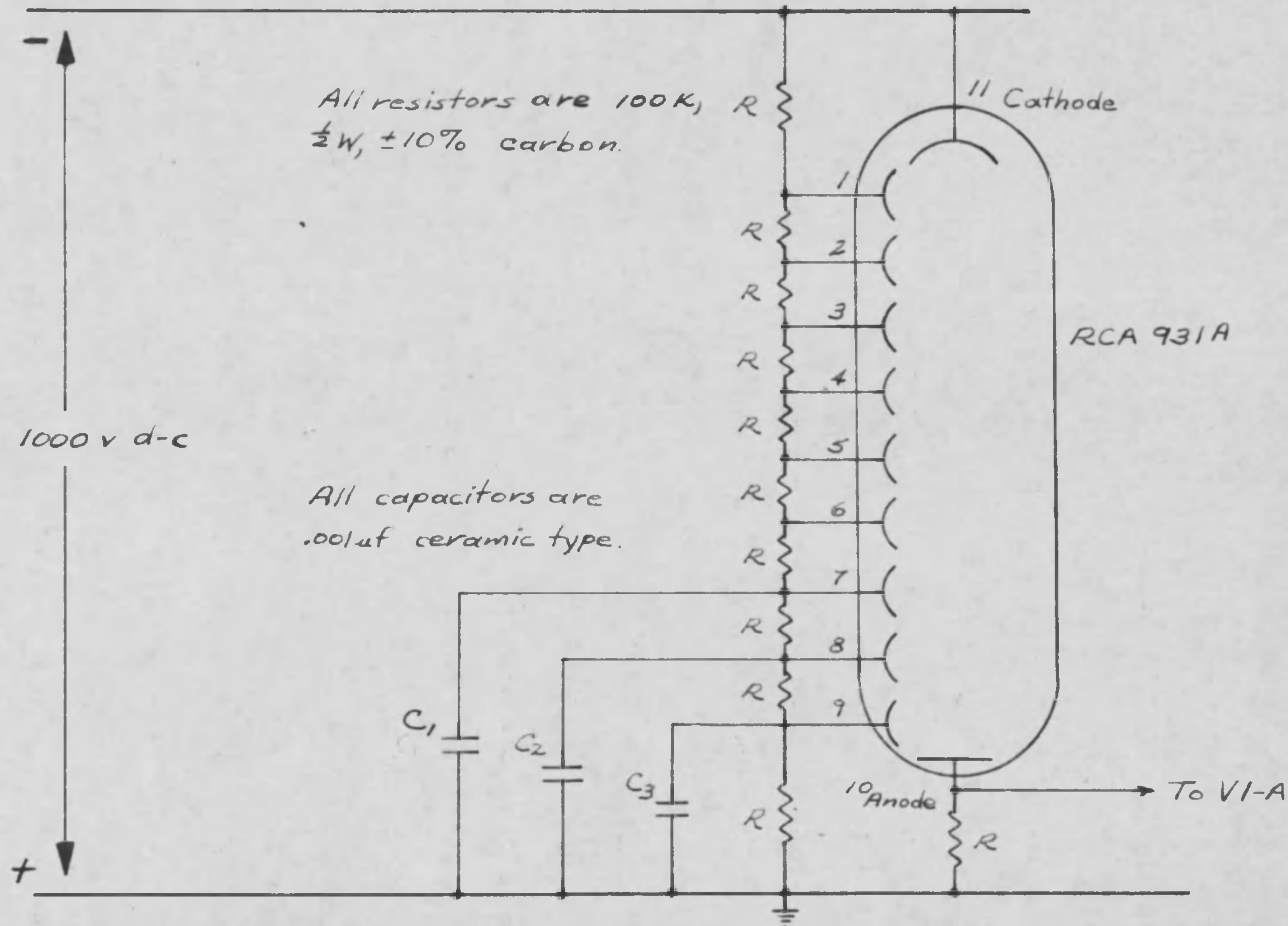

Fig 3.2, RCA 931A Photomultiplier.

control grid of one of the amplifier tubes is driven above cathode potential by a large pulse, the resistance of the coupling condenser charging circuit is greatly reduced. With the grid above cathode potential, a small grid-cathode resistance is placed in parallel with the large grid leakage resistor. The decreased time constant results in a large change in coupling-condenser voltage during the time of a strong pulse. This change causes the output voltage to overshoot with the long time constant in the output circuit. During the first part of the recovery period, the grid potential of the next stage may be near or below cutoff. The amplifier will then not pass small signals and is said to be blocked or overloaded. Succeeding pulses will not be passed and the count rate decreases.

One solution that was explored was to introduce a diode in the grid circuit of one of the stages to limit the size of the pulse; but a corresponding loss in speed was experienced due to the internal capacitance of the diode.

An ideal solution would be a fast amplifier that would amplify the small pulses a great deal and the large pulses only slightly. The circuit finally used is one commonly used in nuclear physics experimentation[13] and employs difference amplification. The first stage of the pulse amplifier is a linear amplifier and the two following stages comprise the difference amplifier. D.C. coupling between the first stage and the difference amplifier avoids the problem of charging coupling

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[13] "Model 215 Non-overloading Amplifier," Baird-Atomic, Inc., Massachusetts

capacitors.

The non-overloading pulse amplifier is shown in Fig. 3.3. The feedback loops and basic design are very similar to the "Oak Ridge-Fairstein" and "Brookhaven-Chase" circuits.[14] The negative input signal from the photomultiplier is amplified through V1-A and appears as a positive signal at the grid of V1-B. V1-B and V2-A are connected as a difference amplifier, with their grids coupled through a long time constant (220 kilohms and 0.1 microfarad). Under normal conditions, i.e., when the positive input signal at the grid of V1-B is small, the difference amplifier can operate with low effective cathode impedance (13 kilohms in parallel with the r_p of the other tube) since both V1-B and V2-A are conducting. When the signal is large, V2-A cuts off and all current flows through V1-B, the effective cathode impedance of which is now 13 kilohms. For large signals V1-B will draw grid current; but since there is no coupling capacitor in the grid circuit, it will return to its quiescent operating point as soon as the large signal ceases. V3 is an inverting amplifier whose output is the triggering signal for the Schmitt trigger.

Two feedback loops help to stabilize the gain characteristics of the pulse amplifier. High-frequency negative feedback is applied from the cathode of V2-B to the cathode of V1-A through a capacitor and resistor in shunt. The capacitor in shunt with the feedback resistor overcomes the capacitance from the cathode of V1-A to ground. There is

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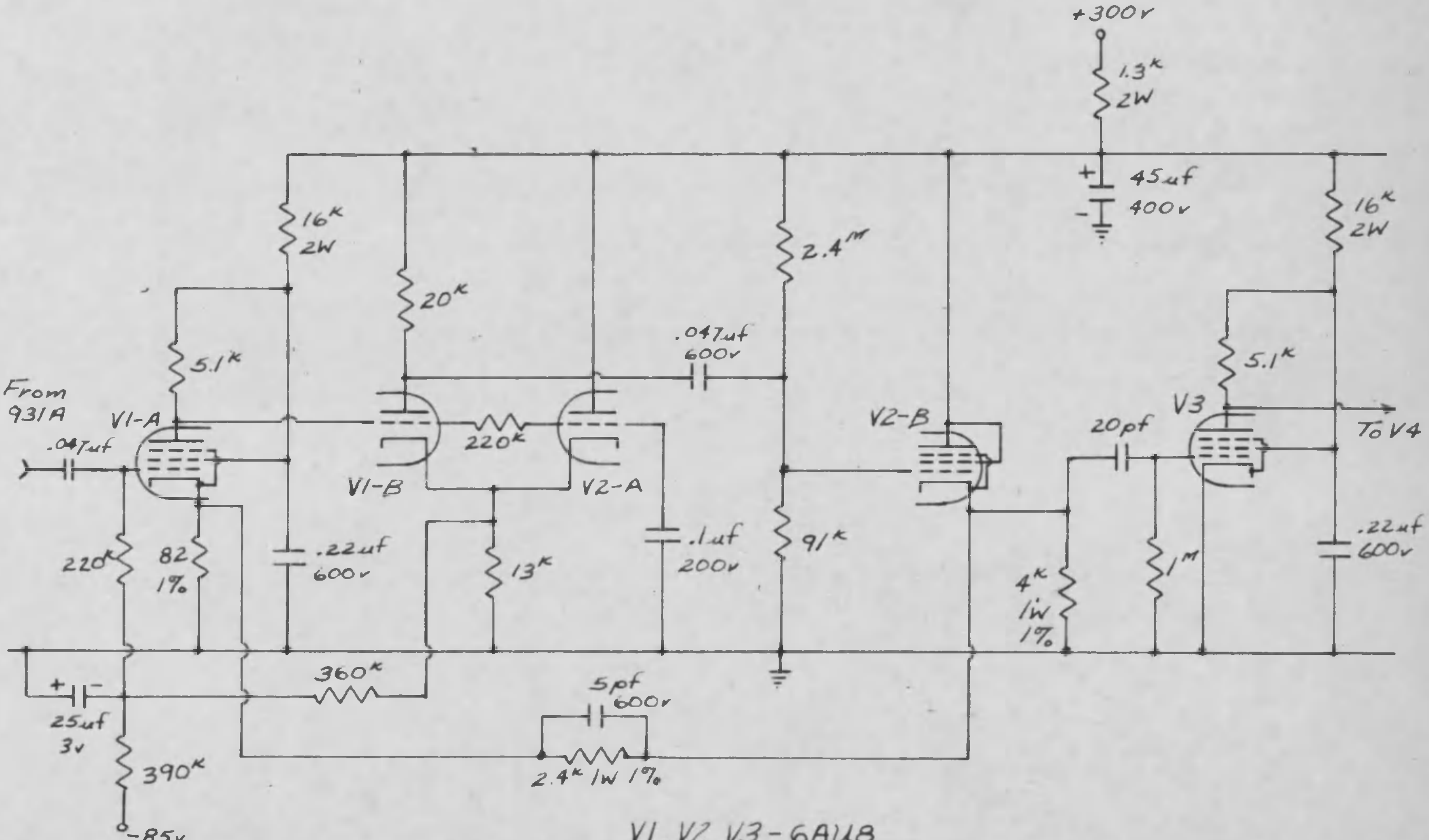
[14] Ibid.



V1, V2, V3 - 6AU8

All resistors are ½W, ±5% unless otherwise noted.

Fig. 3.3, Non-Overloading Pulse Amplifier.

also a second, low-frequency feedback connection from the cathode of V1-B and V2-A to the grid of V1-A to stabilize low-frequency fluctuations in voltage (caused for example by changes in tube characteristics). The long time constant of the 390 kilohm resistor and 25 microfarad capacitor confines the effectiveness of this feedback to the quiescent current conditions of V1-A and the difference amplifier.

3.3 Performance of the Non-overloading Amplifier

Tests made on the amplifier were made with an astable multivibrator at 2 mc. The rise time of the input pulse was 0.04 microseconds, and the recovery time was 0.16 microsecond. The amplifiers rise time is no greater than 0.22 microsecond; the recovery time 0.16 microsecond. The effective pulse gain varies with input amplitude, due to the non-linear difference amplification. The output voltage never exceeds 60 volts. This occurs for an input of 4 volts. Reliable amplification is obtained in excess of 2.5 mc.

CHAPTER 4

RANDOM SQUARE WAVE GENERATING CIRCUITS AND OUTPUT FILTER AMPLIFIER

4.1 Design of the Schmitt Trigger

The Schmitt trigger (V4 and V5) shown in Fig. 4.1 is of conventional design.[15] The output amplifier (V6) inverts the trigger to the proper polarity and prevents loading of the trigger by the binary.

A resistance is used in the cathode circuit of the first tube to reduce the loop gain to near unity and thus reduce hysteresis to approximately 1.8 volts. If the hysteresis were eliminated entirely, the loop gain would be equal to unity which is a relatively unstable condition. Drift (due to supply voltage changes, tube aging, etc.) could cause negative hysteresis (loop gain less than unity) and the circuit would no longer change states.[16] The + 1.8 volts hysteresis will ensure that the loop gain will remain greater than unity even if the circuit drifts somewhat.

4.2 Performance of the Schmitt Trigger

The performance of the Schmitt trigger was determined while it was operating in the circuit. The unit was fed by the pulse amplifier and loaded by the inverting amplifier. The rise time depends somewhat on the speed of the triggering voltage, but is in no case less than 0.16 microsecond. The output switches between + 250 and + 300 volts

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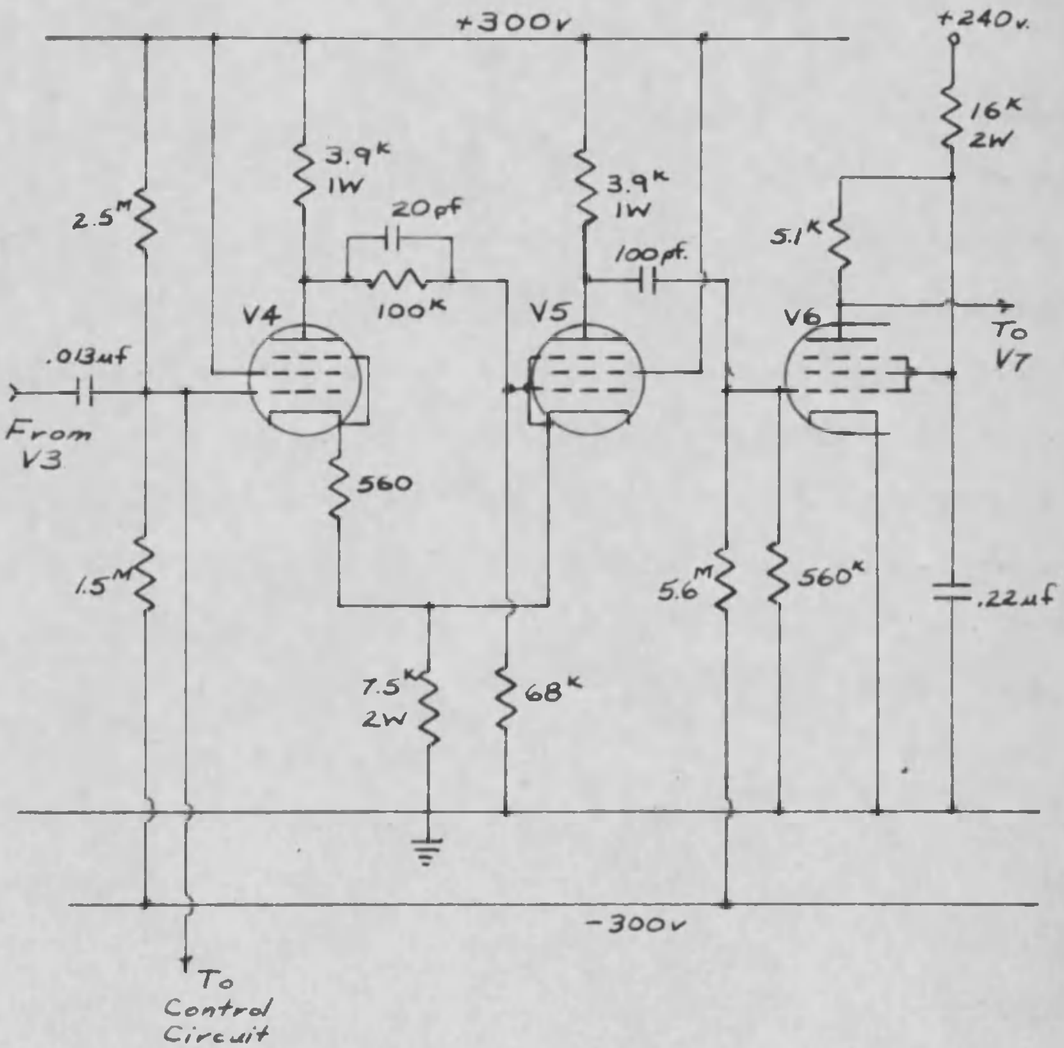
[15] J. Millman and H. Taub, Pulse and Digital Circuits, McGraw-Hill Book Co., Inc., 1956, pp. 164-172

[16] Ibid.



All resistors ± 10%, ½ watt unless otherwise noted.

V4 & V5 – 6CB6 , V6 – 6AU8

Fig. 4.1, Schmitt Trigger

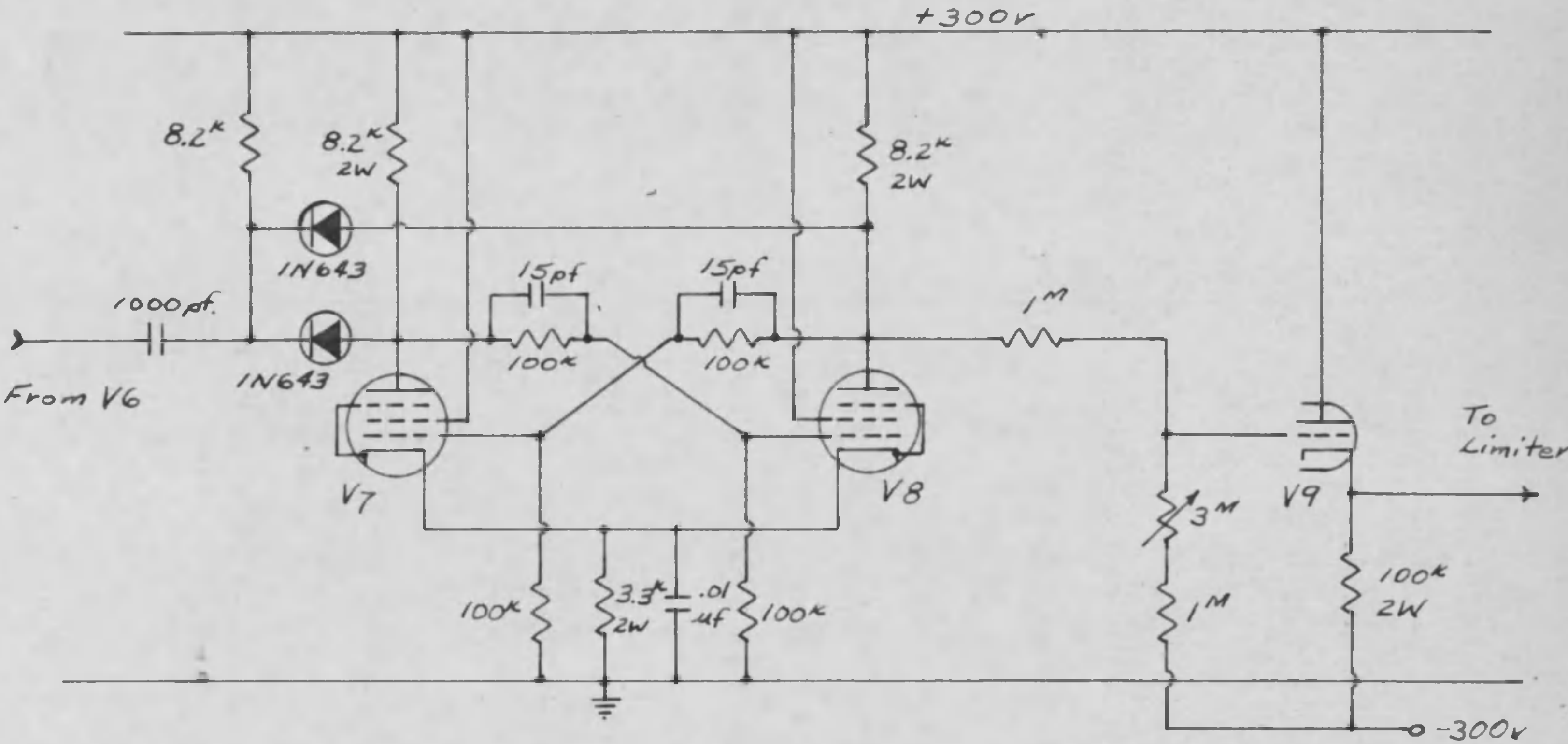
with a recovery time of 0.18 microsecond. The maximum reliable count rate obtained using the astable multivibrator connected to the input of the pulse amplifier, approximately 2.6 mc. The drift of the switching point is eliminated by the feedback control circuit.

4.3 Design of the Bistable Multivibrator

The binary (V7 and V8) shown in Fig. 4.2 is also of conventional design.[17] The output cathode follower (V9) reduces the output voltage to near the desired level, lowers the output impedance, and prevents loading the binary.

The shortest interval between triggering pulses for which the binary will operate reliably is called the resolving time and can be divided into the transition time and the recovery time. The transition time is the time it takes the circuit to switch states, while the recovery time is the time that must elapse before the circuit has established itself in its new state. As can be seen in the figure, the coupling attenuator is compensated. This additional capacitance in the circuit decreases the transition time of the binary but unfortunately also increases the recovery time. In this application, the signal that induces the transition between states is a spike-type voltage. If the spike is of a sufficiently short duration, the binary may be completely insensitive to it unless it is compensated. Thus, a short transition time is more important than a short recovery time in this application. The triggering pulse from the output of the inverting amplifier is negative. This is desirable, since a binary is ordinarily more

[17] Ibid., pp. 140-164



All resistors are ½W, ±10% unless otherwise noted.

V7 & V8 - 6CB6 V9 - 12AU7

Fig. 4.2, Bistable Multivibrator.

sensitive to a negative pulse than to a positive pulse.[18] Symmetrical triggering is desired as a transition must occur for every input pulse. Triggering was first attempted through an additional resistor connected directly in series with the supply voltage. It was found that to ensure reliable triggering the pulse must be large and the commutating capacitors had to be made larger than was consistent with a short recovery time. The problem was solved by triggering directly at the plates through diodes. This results in two definite advantages: 1. A decreased recovery time due to the use of smaller commutating capacitors. 2. The use of a trigger of smaller amplitude since it is no longer attenuated by the plate load resistor. Decreasing the load and attenuating resistors will improve the resolving time. The price that must be paid for this improvement in resolution time is increased power dissipation, and, unless it is possible to increase the tube current in proportion, as the load resistor is reduced, the plate swing will become smaller. Hence, not only will the useful output signal be reduced, but the total grid swing will become smaller, and it may become difficult to maintain d-c stability in the binary. When this occurs, it may become necessary to use precision components.

The output of the cathode follower (V9) is slightly greater than ± 30 volts, as the diode bridge will limit to ± 20 volts. This 20 volt margin ensures the elimination of any unwanted noise riding on top of the random square wave, and also results in sharper pulses. Since the

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[18] Ibid., pp. 156-157

random square wave is also attenuated in the resistance network preceding the cathode follower, the output of the binary must be fairly large. Note that d-c coupling is used all the way from the binary to the noise generator output to preserve the components of the random signal all the way down to d-c. The cathode follower also acts as an isolation network preventing the diode bridge from loading the binary and at the same time feeds the bridge through a low input impedance.

4.4 Performance of the Bistable Multivibrator

The output of the binary swings from + 300 to + 180 volts or an output of 120 volts. The rise time is 0.6 microsecond and the recovery time is 0.3 microsecond. The maximum number of reliable transitions is 0.7 million transitions per second. The output of the cathode follower is from + 30 to - 30 volts.

4.5 Design of a Diode Bridge Limiter

To obtain an output voltage of adequate root mean square with a stable zero mean, the output cathode follower is followed by the diode bridge limiter shown in Fig. 4.3.[19] To allow fast rise times, the impedance level should be as low as possible. Precision carbon deposited resistors provide for accurate limiting levels. The bridge-bias voltages must be well regulated and preferably should be obtained directly from the reference supplies of the computer in which the device will be used. The particular circuit shown here limits at ± 20 volts within 0.3 volts for both short and long pulses.

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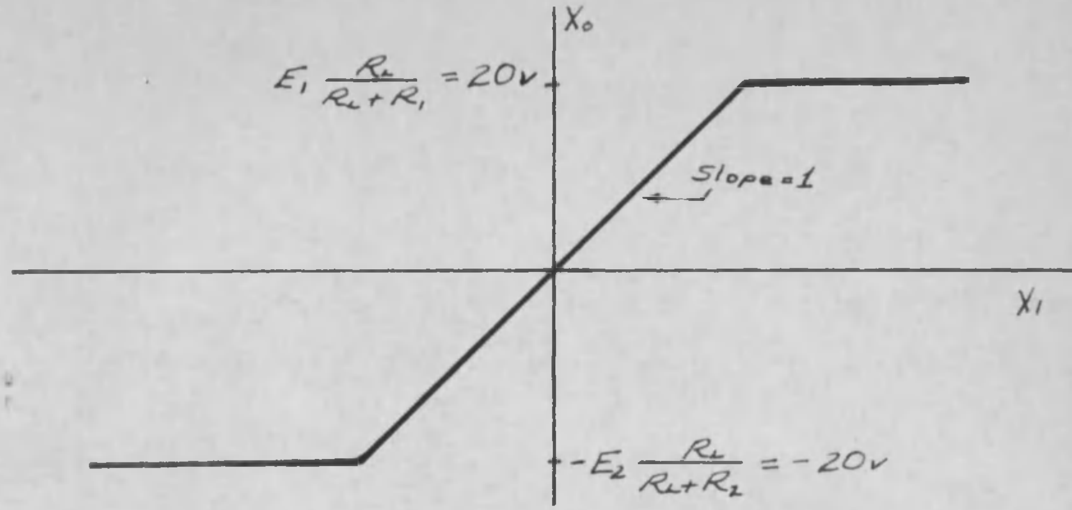
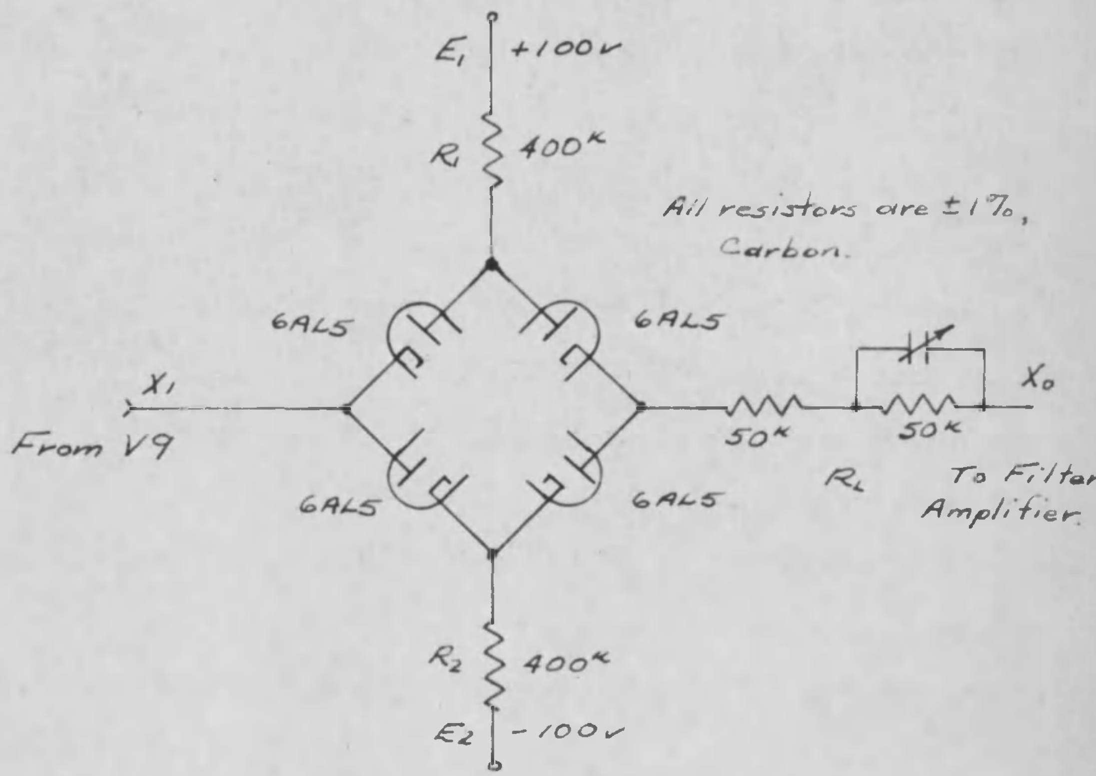
[19] G.A. Korn, T.M. Korn, Electronic Analog Computers, McGraw-Hill Book Co., New York, 1956, p. 294



Transfer Characteristic



Circuit

Fig. 4.4, Diode Bridge Limiter

4.6 Output Filter Amplifier

The output element of the complete random noise generator must be a wide-band, d-c amplifier capable of supplying the required gain and low output impedance. A Philbrick Type USA-3 chopper-stabilized amplifier is used. The output low-pass filter is placed in the feedback loop of this amplifier; plug-in terminals permit use of different feedback networks.

CHAPTER 5

MEAN-COUNT-RATE CONTROL CIRCUIT

The count rate can be set by adjustment of the threshold level at the Schmitt trigger input (Fig. 5.1). Note that two curves are plotted. The method used was to measure the count rate with a voltmeter at the grid of the Schmitt trigger; the voltmeter was then removed, and the count rate was remeasured. The voltmeter capacitance accounts for the difference in curves. Figure 5.2 shows the feedback circuit. The count rate was measured at the output of the inverting amplifier (V5) with the setup of Fig. 5.3.

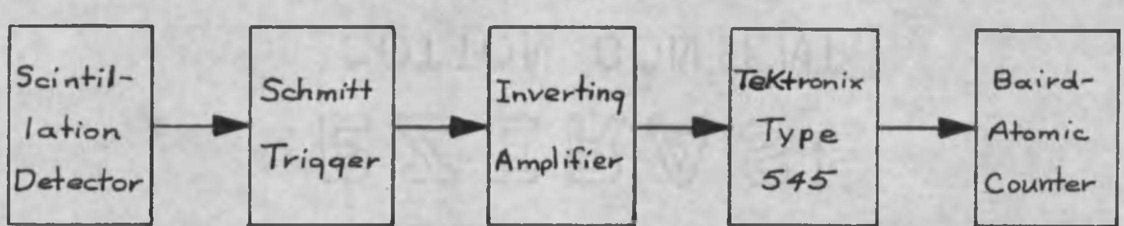


Fig. 5.3, Mean-count-rate Measuring Setup

Due to the low input impedance of the Baird-Atomic counter (1600 ohms), a Tektronix type 545 oscilloscope was used as an impedance isolating device preventing the counter from loading the inverting amplifier. The counter was triggered from the oscilloscope's vertical signal output terminal. Direct use of the cathode follower in the control circuit was not as satisfactory as the oscilloscope for

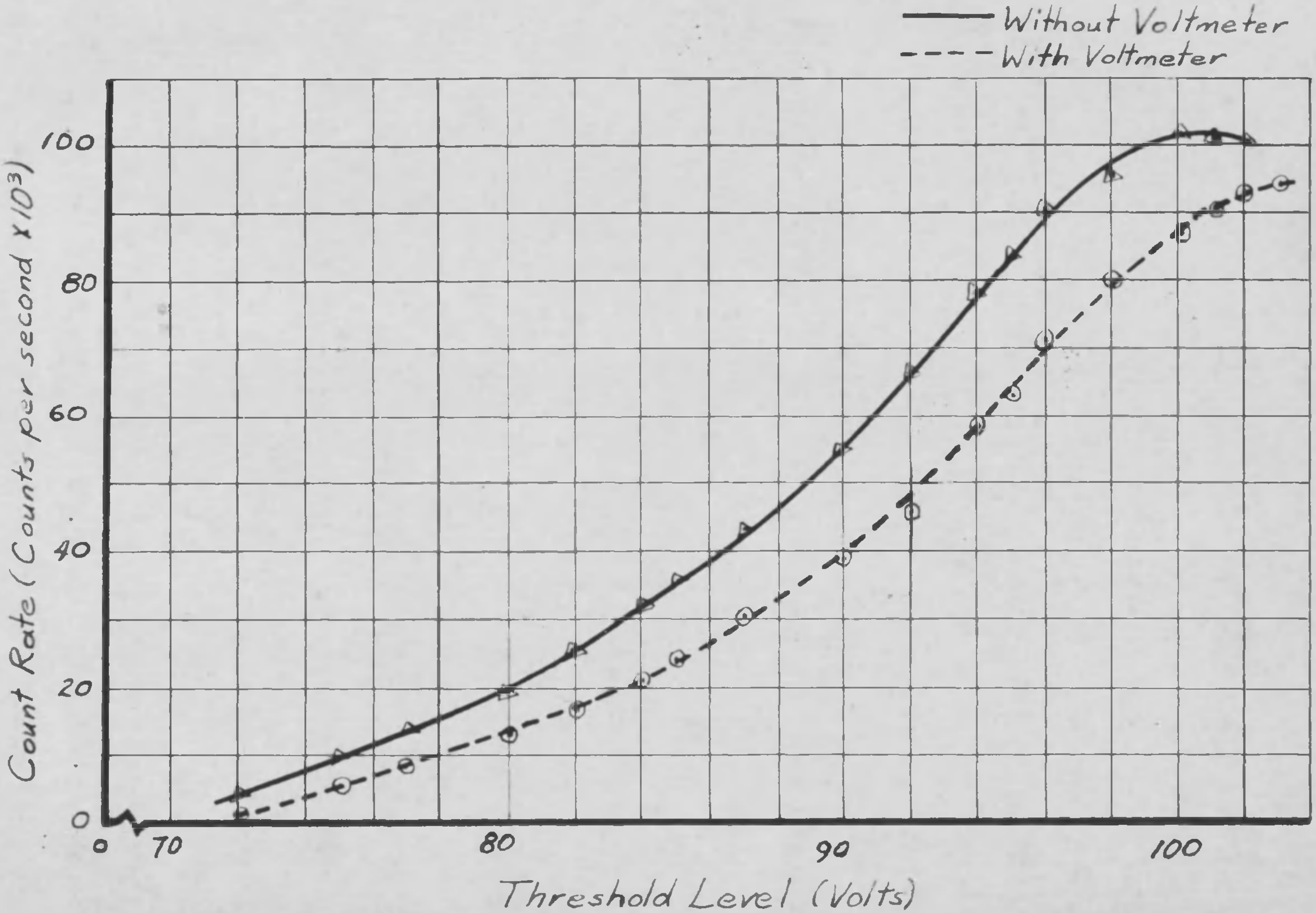
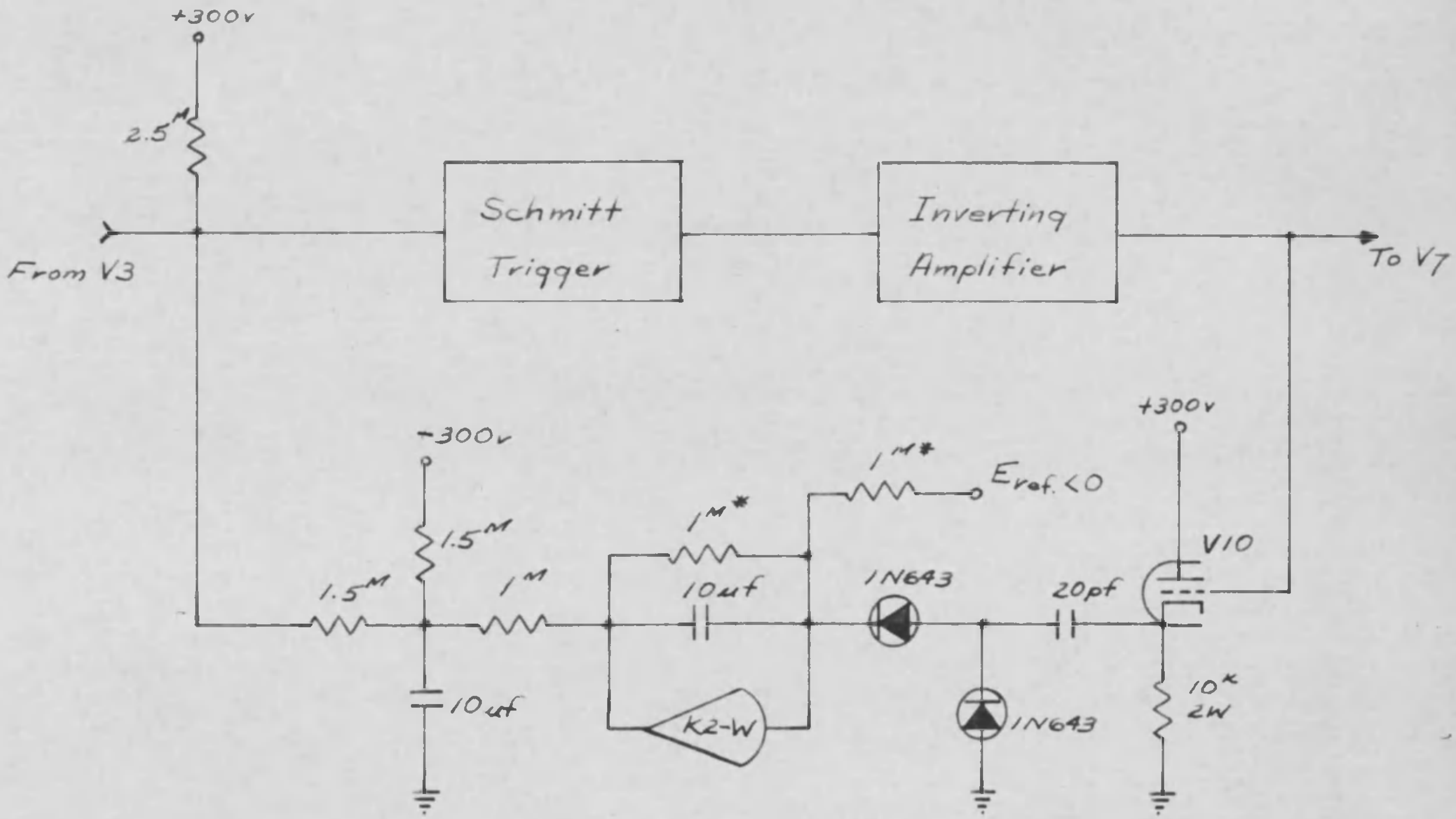


Fig. 5.1, Count Rate vs Threshold Level



All resistors are ½ W, ±10% unless otherwise noted.

Operational Amplifier is a Philbrick K2-W.

V10 - 12AU7

Fig. 5.2, Mean-Count-Rate Control Circuit measurement purposes.

Count-rate regulation could also have been accomplished through feedback to the power supply of the phototube. This would have the advantage of having more elements within the control loop. The reason this method is not used is because eventually other phototubes may use the same power supply.

5.1 Design of a Storage Counter With an Operational-Amplifier Circuit

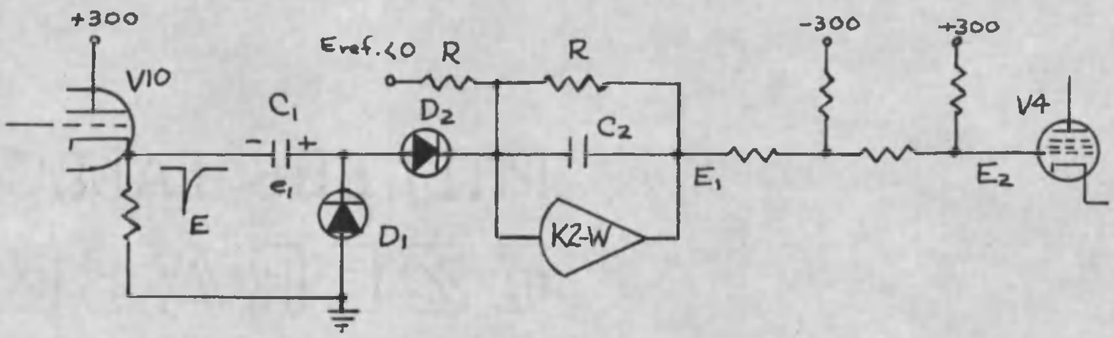


Fig. 5.4, Storage Counter With an Operational-Amplifier Circuit

Referring to Fig. 5.4, the cathode follower (V10) applies negative pulses through a low impedance to the storage counter. The negative pulses cause the capacitor (C_1) to charge through the diode (D_1). The time constant with which C_1 charges is the product of C_1 times the sum of the diode and cathode follower resistances. If this time constant is small in comparison with the duration of the pulse, then C_1 will charge fully to the value $e_1 = E$, with the polarity indicated. During the charging time of C_1, the diode D_2 does not conduct and no current flows into the operational amplifier. At the termination of the input pulse, the capacitor C_1 is left with the voltage $e_1 = E$, which now appears across D_1 and across the series combination of D_2 and the operational amplifier. The polarity of this voltage is such that D_1 will not conduct. The capacitor C_1 will, however, discharge through D_2 into the operational amplifier. The time constant with which this transfer of charge takes place must be quite small in comparison with the interval between pulses in order to allow equilibrium to be established between the capacitor voltages (C_1 and C_2). The capacitor C_2 is ordinarily quite large in comparison with C_1.[20] The average current flowing into the operational amplifier is $EC_1\alpha$. A node equation at the input of the operational amplifier is

$$EC_1\alpha = -\frac{E_1}{R} \tag{5.1}$$

at d-c. Solving for E_1

$$E_1 = -ERC_1\alpha \tag{5.2}$$

Equation 5.2 indicates the direct dependence of the operational amplifiers output voltage on the mean count rate. If the mean count rate increases, the output voltage decreases and lowers the threshold voltage at the input of the Schmitt trigger, causing the count rate to decrease, and thus regulating the noise generator. One can adjust the threshold level by varying the reference voltage at the operational-

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[20] Ibid., pp. 344-353

amplifier input.

5.2 Performance of the Control Circuit

The performance of the control circuit is tabulated in Table I.

Table I

E_1	E_2	ΔV of Phototube
-10	+ 96	0
+ 5	+100	-50
+22	+103	-100

The count rate was varied by decreasing the voltage across the photomultiplier. The large changes were for illustration purposes and would never be expected to occur in actual practice. The reference voltage for the above test was 100 volts.

CHAPTER 6

RESULTS AND DISCUSSION

The noise generator described in this thesis has been built and will be incorporated in a repetitive computer now being built in the computer laboratory.[21,22] It will be used primarily in connection with random-process studies.

The frequency capabilities of the generator circuitry meet the design requirements for this purpose and exceed the count rates obtainable from the source used. This permits one to use a source of higher frequency if one should become available; and provisions have been made for using two sources in parallel. If they are identical and independent, the mean count rate of the noise output should double.[23]

A primary consideration in random-noise-generator design is freedom from periodic components. The pulse circuitry generating the random telegraph signal inherently discriminates against line-frequency hum pickup, although stray 60 cps fields and currents can modulate the mean count rate both in the photomultiplier and at the Schmitt input. The resulting effects on second-order statistics, such as the power

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[21] G.A. Korn, "Repetitive Analog Computers at the University of Arizona," Instruments and Control Systems, Sept., 1960, pp. 1551-1553

[22] T. Brubaker and H. Eckes, "Digital Clock Delay Generators and Run Counter for a Repetitive Analog Computer," Western Joint Computer Conference Proceedings, May, 1961

[23] W.B. Davenport and W.I. Root, Random Signals and Noise, McGraw-Hill Book Co., Inc., 1958, p. 154

spectrum appear to be largely removed when the random telegraph signal is filtered for Gaussian output. It is believed that the main periodic interference in the desired random output is due to line-frequency components originating in the output amplifier.[24] This is due mainly to chopper ripple and spikes. This could be minimized by omission of the chopper stabilizer, and by d-c operation of the output amplifier filaments.

Small line-frequency components of this type are, unfortunately, rampant through most existing analog computers and measure about 3 to 5 millivolts RMS. They are completely negligible in ordinary computations over the usual ± 100 volt range, but might add up in statistical work. In the application of the present noise generator to the new repetitive analog computer, this has been taken into account; precision crystal-oscillator clock control[25] carefully prevents computer repetition rates and sampling frequencies harmonically related to the line frequency, so that line-frequency components are averaged out in all presently projected random-voltage measurements.

Photographs of the random-telegraph-wave and noise output are shown in Fig. 6.1.

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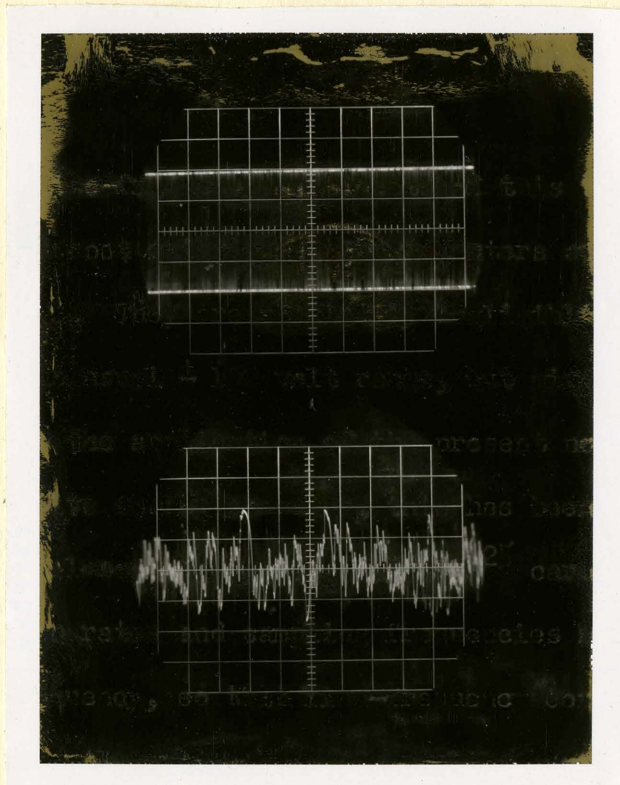
[24] G.A. Korn, Unpublished Communication

[25] T. Brubaker and H. Eckes, op. cit.



Scale

Amplitude - 10 volts/cm.
Time - 2 milliseconds/cm.

Fig. 6.1, Random-Telegraph-Wave and Noise Output

APPENDIX A

POWER SPECTRUM AND AMPLITUDE DISTRIBUTION OF THE NOISE OUTPUT[26]

The photomultiplier in conjunction with the pulse amplifier is basically a scintillation detector. The radioactive decay causes the scintillation detector to emit random voltage pulses so that the probability of having n pulses large enough to trigger the Schmitt trigger in τ seconds is that of the Poisson distribution:

$$P(n,\tau) = \frac{(\alpha\tau)^n e^{-\alpha\tau}}{n!} \qquad (A.1)$$

where α is the average rate of occurrence of the pulses in a unit time interval or the mean count rate.

For each random pulse large enough to trigger the Schmitt, a transition occurs in the binary. Each transition in the binary produces a random zero crossing of the telegraph wave at the output of the diode bridge limiter.

The power spectrum of the noise generator output can be computed from the autocorrelation function of the resultant square wave with random zero crossings. If 2E is the peak-to-peak amplitude of the square wave, the correlation between signals τ seconds apart is $+E^2$ for an even number of pulses and $-E^2$ for an odd number. For τ positive, the time autocorrelation function is defined by

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[26]Lee, <u>op. cit.</u>, pp. 331-334

$$\varphi(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T f(t) f(t + \tau)\, dt \qquad (A.2)$$

In the use of the random telegraph signal, the autocorrelation function is

$$\varphi(\tau) = E^2 \sum_{n=0}^{\infty} (-1)^n P(n, \tau)$$

$$= E^2 e^{-2\alpha|\tau|} \qquad (A.3)$$

A Fourier transformation of the autocorrelation function yields the power spectral density

$$W(\omega) = \int_{-\infty}^{+\infty} \varphi(\tau) e^{-j\omega\tau} d\tau \qquad (A.4)$$

$$= \frac{E^2}{\alpha} \frac{1}{1 + \frac{\omega^2}{4\alpha^2}} \qquad (A.5)$$

The average power is then

$$\varphi(0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} W(\omega) d\omega = E^2 \qquad (A.6)$$

If the noise generator output is passed through a sharp cutoff, low-pass filter, the resulting power spectrum will be essentially flat up to near the cutoff frequency, and the output signal will be approximately Gaussian.[27] The frequency at which the power spectrum of

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[27] J. A. McFadden, "The Probability Density of the Output of a Filter when the Input is a Random Telegraphic Signal," <u>IRE National Convention Record</u>, 1959

equation (A.5) is down 1 db is given by

$$\omega \approx 1.02\alpha \approx \alpha \tag{A.7}$$

The effect of a linear filter on the power spectrum is given by

$$W_2(\omega) = |T(j\omega)|^2 W_1(\omega) \tag{A.8}$$

where $W_2(\omega)$ is the filter output power spectrum in response to the filter frequency response $T(j\omega)$ and the input power spectrum $W_1(\omega)$. Consider a simple RC filter, whose frequency-response function is

$$T(j\omega) = \frac{1}{j\frac{\omega}{\omega_c} + 1}, \left(\omega_c = \frac{1}{RC}\right) \tag{A.9}$$

The output autocorrelation function is

$$\varphi_2(\tau) = \frac{E^2}{2\pi\alpha} \int_{\omega=-\infty}^{\omega=+\infty} \frac{e^{j\omega\tau}}{\left(1 + \frac{\omega^2}{4\alpha^2}\right)\left(1 + \frac{\omega^2}{\omega_c^2}\right)} d\omega \tag{A.10}$$

$$= E^2 \left[\frac{2\alpha\omega_c e^{-\omega_c\tau} - \omega_c^2 e^{-2\alpha\tau}}{(4\alpha^2 - \omega_c^2)} \right] \tag{A.11}$$

In particular, if $\omega_c \ll 2\alpha$,

$$\varphi_2(\omega) \approx E^2 \frac{\omega_c}{2\alpha} e^{-\omega_c\tau}; \tag{A.12}$$

and for $\omega_c = \alpha$,

$$\varphi_2(\tau) = E^2 \frac{2e^{-\omega_c\tau} - e^{-2\omega_c\tau}}{3} \qquad (A.13)$$

Finally, for $\omega_c \gg 2\alpha$,

$$\varphi_2(\tau) = E^2 e^{-2\alpha\tau} \qquad (A.14)$$

McFadden[28] has shown that for $K = \alpha\tau = \alpha RC = 4$, the probability density of the output of an RC low-pass filter for a Poisson square-wave input approximates the Gaussian process quite well.

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[28] Ibid.

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