A COMPARISON OF MEASURED AND PREDICTED PHOTOGRAPHIC NOISE POWER SPECTRUM

by

David Alan Honey

A Thesis Submitted to the Faculty of the COMMITTEE ON OPTICAL SCIENCES (GRADUATE) In Partial Fulfillment of the Requirements For the Degree of MASTER OF SCIENCE In the Graduate College THE UNIVERSITY OF ARIZONA

1978
STATEMENT BY AUTHOR

This thesis has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: David Glenn Honey

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

R. R. SHANNON
Professor of Optical Sciences

27 Nov 78
Date
ACKNOWLEDGMENTS

I wish to thank the following people for their help in the preparation of this thesis. Professor Robert R. Shannon, my thesis advisor, supplied valuable insight and guidance to this project. Dr. Wayne Ferris of the Department of Cell and Developmental Biology, prepared all of the electron micrographs that were used. Mr. Don Hilliard operated the microdensitometer for us. I would also like to thank Mrs. Norma Emptage for her help in the preparation of the final copy of this thesis and for her completing the administrative work which my absence prevented me from doing.
## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>x</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. RENEWAL THEORY</td>
<td>4</td>
</tr>
<tr>
<td>Introduction</td>
<td>4</td>
</tr>
<tr>
<td>Fundamentals</td>
<td>4</td>
</tr>
<tr>
<td>Special Processes</td>
<td>7</td>
</tr>
<tr>
<td>The Equilibrium Renewal Process</td>
<td>8</td>
</tr>
<tr>
<td>The Poisson Process</td>
<td>8</td>
</tr>
<tr>
<td>The Type I. Counter</td>
<td>8</td>
</tr>
<tr>
<td>Asymptotic Distribution of N_t</td>
<td>9</td>
</tr>
<tr>
<td>Alternating Renewal Theory</td>
<td>10</td>
</tr>
<tr>
<td>3. THE TRABKA GRAIN MODEL</td>
<td>12</td>
</tr>
<tr>
<td>Introduction</td>
<td>12</td>
</tr>
<tr>
<td>Monolayer Monodisperse Emulsions</td>
<td>13</td>
</tr>
<tr>
<td>Introduction</td>
<td>13</td>
</tr>
<tr>
<td>Important Parameters</td>
<td>13</td>
</tr>
<tr>
<td>Application of Renewal Theory</td>
<td>14</td>
</tr>
<tr>
<td>Alternating Renewal Model</td>
<td>20</td>
</tr>
<tr>
<td>Multilayer Emulsions</td>
<td>22</td>
</tr>
<tr>
<td>Introduction</td>
<td>22</td>
</tr>
<tr>
<td>Transmittance</td>
<td>22</td>
</tr>
<tr>
<td>4. COMPUTER SIMULATION OF AN EMULSION</td>
<td>27</td>
</tr>
<tr>
<td>Introduction</td>
<td>27</td>
</tr>
<tr>
<td>Sampling from Any Known Distribution</td>
<td>27</td>
</tr>
<tr>
<td>Theory</td>
<td>27</td>
</tr>
<tr>
<td>Application</td>
<td>30</td>
</tr>
<tr>
<td>Emulsion Simulation</td>
<td>33</td>
</tr>
<tr>
<td>TABLE OF CONTENTS—Continued</td>
<td></td>
</tr>
<tr>
<td>----------------------------</td>
<td></td>
</tr>
</tbody>
</table>

5. MEASUREMENT OF EMULSION PARAMETERS. ........................................... 37
   Parameters Measured ............................................................... 37
   Methods of Measurement ......................................................... 37

6. COMPARISON OF MEASURED AND PREDICTED GRANULARITY SPECTRA. .......... 42
   Principles of Digital Spectral Analysis ...................................... 42
      Introduction. ........................................................................... 42
      Estimation of the Power Spectrum. ............................................ 46
   Principles of Coherent Spectral Analysis ..................................... 52
      Introduction. ........................................................................... 52
      Practical Techniques. ............................................................... 52
   The Relationship between Incoherent and Coherent Spectra. ............. 56
   Results ....................................................................................... 60
      Introduction. ........................................................................... 60
      Liquid Gated and Non-Liquid Gated Wiener Spectra. ....................... 61
      Measured Liquid Gated and Simulated Wiener Spectra. ..................... 65
      Measured Non-Liquid Gated and Simulated Wiener Spectra. ............... 65
      Measured and Predicted Optical Power Spectra. ............................. 72
      Liquid Gated Wiener and Optical Power Spectra ............................ 76
      Non-Liquid Gated Wiener and Optical Power Spectra ..................... 80

7. DISCUSSION AND CONCLUSIONS. ..................................................... 84

APPENDIX A: COMPUTER PROGRAMS. ...................................................... 90
APPENDIX B: EXPERIMENTAL DATA. ..................................................... 114
APPENDIX C: ELECTRON MICROGRAPH PHOTOS ................................. 115
REFERENCES .................................................................................... 122
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Simulation of random variables.</td>
<td>29</td>
</tr>
<tr>
<td>2.</td>
<td>Monte Carlo simulation and analytic solution of the Wiener spectrum of a monolayer monodisperse emulsion.</td>
<td>35</td>
</tr>
<tr>
<td>3.</td>
<td>Normalized grain frequency histogram for Tri-X shoulder sample</td>
<td>40</td>
</tr>
<tr>
<td>4.</td>
<td>Normalized gap frequency histogram for Tri-X shoulder sample</td>
<td>41</td>
</tr>
<tr>
<td>5.</td>
<td>Coherent spectrum analyzer</td>
<td>53</td>
</tr>
<tr>
<td>6.</td>
<td>Tri-X shoulder liquid gated and non-liquid Wiener spectra</td>
<td>62</td>
</tr>
<tr>
<td>7.</td>
<td>Tri-X mid-range liquid gated and non-liquid gated Wiener spectra</td>
<td>62</td>
</tr>
<tr>
<td>8.</td>
<td>Tri-X toe liquid gated and non-liquid gated Wiener spectra</td>
<td>63</td>
</tr>
<tr>
<td>9.</td>
<td>3414 shoulder liquid gated and non-liquid gated Wiener spectra</td>
<td>63</td>
</tr>
<tr>
<td>10.</td>
<td>3414 mid-range liquid gated and non-liquid gated Wiener spectra</td>
<td>64</td>
</tr>
<tr>
<td>11.</td>
<td>3414 toe liquid gated and non-liquid gated Wiener spectra</td>
<td>64</td>
</tr>
<tr>
<td>12.</td>
<td>Tri-X shoulder liquid gated Wiener spectrum and Monte Carlo simulation.</td>
<td>66</td>
</tr>
<tr>
<td>13.</td>
<td>Tri-X mid-range liquid gated Wiener spectrum and Monte Carlo simulation.</td>
<td>66</td>
</tr>
<tr>
<td>14.</td>
<td>Tri-X toe liquid gated Wiener spectrum and Monte Carlo simulation.</td>
<td>67</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>15.</td>
<td>3414 shoulder liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>67</td>
</tr>
<tr>
<td>16.</td>
<td>3414 mid-range liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>68</td>
</tr>
<tr>
<td>17.</td>
<td>3414 toe liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>68</td>
</tr>
<tr>
<td>18.</td>
<td>Tri-X shoulder non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>69</td>
</tr>
<tr>
<td>19.</td>
<td>Tri-X mid-range non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>69</td>
</tr>
<tr>
<td>20.</td>
<td>Tri-X toe non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>70</td>
</tr>
<tr>
<td>21.</td>
<td>3414 shoulder non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>70</td>
</tr>
<tr>
<td>22.</td>
<td>3414 mid-range non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>71</td>
</tr>
<tr>
<td>23.</td>
<td>3414 toe non-liquid gated Wiener spectrum and Monte Carlo simulation</td>
<td>71</td>
</tr>
<tr>
<td>24.</td>
<td>Tri-X shoulder optical power spectrum and Monte Carlo simulation</td>
<td>73</td>
</tr>
<tr>
<td>25.</td>
<td>Tri-X mid-range optical power spectrum and Monte Carlo simulation</td>
<td>73</td>
</tr>
<tr>
<td>26.</td>
<td>Tri-X toe optical power spectrum and Monte Carlo simulation</td>
<td>74</td>
</tr>
<tr>
<td>27.</td>
<td>3414 shoulder optical power spectrum and Monte Carlo simulation</td>
<td>74</td>
</tr>
<tr>
<td>28.</td>
<td>3414 mid-range optical power spectrum and Monte Carlo simulation</td>
<td>75</td>
</tr>
<tr>
<td>29.</td>
<td>3414 toe optical power spectrum and Monte Carlo simulation</td>
<td>75</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>30.</td>
<td>Tri-X shoulder liquid gated Wiener spectrum and OPS.</td>
<td>77</td>
</tr>
<tr>
<td>31.</td>
<td>Tri-X mid-range liquid gated Wiener spectrum and OPS.</td>
<td>77</td>
</tr>
<tr>
<td>32.</td>
<td>Tri-X toe liquid gated Wiener spectrum and OPS.</td>
<td>78</td>
</tr>
<tr>
<td>33.</td>
<td>3414 shoulder liquid gated Wiener spectrum and OPS.</td>
<td>78</td>
</tr>
<tr>
<td>34.</td>
<td>3414 toe liquid gated Wiener spectrum and OPS.</td>
<td>79</td>
</tr>
<tr>
<td>35.</td>
<td>3414 mid-range liquid gated Wiener spectrum and OPS.</td>
<td>79</td>
</tr>
<tr>
<td>36.</td>
<td>Tri-shoulder non-liquid gated Wiener spectrum and OPS.</td>
<td>80</td>
</tr>
<tr>
<td>37.</td>
<td>Tri-X mid-range non-liquid gated Wiener spectrum and OPS.</td>
<td>81</td>
</tr>
<tr>
<td>38.</td>
<td>Tri-X toe non-liquid gated Wiener spectrum and OPS.</td>
<td>81</td>
</tr>
<tr>
<td>39.</td>
<td>3414 shoulder non-liquid gated Wiener spectrum and OPS.</td>
<td>82</td>
</tr>
<tr>
<td>40.</td>
<td>3414 mid-range non-liquid gated Wiener spectrum and OPS.</td>
<td>82</td>
</tr>
<tr>
<td>41.</td>
<td>3414 toe non-liquid gated Wiener spectrum and OPS.</td>
<td>83</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Probability density and cumulative distribution for a sample population.</td>
<td>30</td>
</tr>
<tr>
<td>2.</td>
<td>Flow chart of program to simulate the occurrence of random variables with an arbitrary distribution.</td>
<td>31</td>
</tr>
<tr>
<td>3.</td>
<td>Theoretical vs. measured probability density values for a simulation of 1000 random variables.</td>
<td>32</td>
</tr>
</tbody>
</table>
This thesis presents an attempt to predict incoherent and coherent power spectra of uniformly exposed and processed photographic films from an emulsion model. The model was developed by Eugene Trabka and is based upon renewal theory. Comparisons are made between measured and predicted spectra and between different methods of obtaining power spectra. The conclusion is reached that the present model lacks sufficient parameters for accurate prediction of the power spectrum. Improvements to the model which might yield more accurate predictions are discussed.
CHAPTER 1

INTRODUCTION

One of the major areas of study in photographic sciences deals with mathematical modelling of the micro-structure in silver halide film. This micro-structure is comprised of the basic discrete elements of image formation known as grains. In 1913 Nutting\textsuperscript{1} presented a model which predicted the macro transmittance from a knowledge of the emulsion thickness and the number and area of developed grains. This model assumes all grains have the same size, that the emulsion is divided into one grain thick layers, and the transmittance for an aperture "A" is determined by the projected area of the grains from all layers within the aperture. It is also assumed that no scattering occurs within the emulsion.

The next model to arise came from Silberstein and Trivelli\textsuperscript{2} in 1938. This approach ignored emulsion thickness and considered opaque flat disks distributed at random. These disks were meant to represent the projected area of the grains from the entire emulsion. This model later became known as the random dot model.

Later improvements to modelling introduced such concepts as random grain diameters, random grain sensitivities, grain size amplification factors (in going from the undeveloped to the developed state), and variable exposure distributions within the emulsion. Despite these
new sophistications, there was still much room for improvement. This became especially apparent when attempts were made to predict noise characteristics. R. Clark Jones\(^3\) introduced the concept of describing photographic noise by Fourier analysis. In this paper, Jones also presented results showing a large discrepancy between measured and predicted noise characteristics. Jones attributed this discrepancy to the lack of "clumping of grains" in the emulsion predicted by the random dot model.

The early models discussed above dealt primarily with aperture transmittances. In 1955, B. Picinbono\(^4,5\) introduced the concept of the stochastic point process to the two dimensional random dot model. Picinbono was able to rigorously derive the mean, covariance, and power spectrum of point transmittance fluctuations. Jones had already described the effect of the aperture in relating the measured and true noise spectrum. Thus, the point process was the next logical step. Various refinements of this approach were carried out by Savelli\(^6\), Marchant and Dillon\(^7\), and Benton and Kronauer\(^8\) with regards to emulsion parameters.

Even with these improvements, it was noted by Bayer\(^9\) that these models predict a rapid increase in granularity at high densities which has not been observed experimentally. Trabka\(^10\) has undertaken to develop a stochastic point transmittance model based on renewal theory to account for the experimentally observed high density behavior. The first presentation of this model dealt solely with monodisperse emulsions of one layer. Later publications\(^11,12\) further refined this model to include various grain parameters and multilayer emulsions. It is the purpose of this thesis to examine this model and its experimental application.
It has been shown\textsuperscript{13} that the Fourier transform of the autocorrelation of transmittance or density fluctuations, gives a statistically complete description of photographic noise. The Fourier representation of the noise is especially useful from a linear systems point of view since it can be cascaded\textsuperscript{13}. Furthermore, it has been shown\textsuperscript{13} that there is a relationship between the noise spectrum of a photographic material and its detective quantum efficiency, which in turn has a direct influence upon the information carrying capacity of the film. Thus we see that the power spectrum of a photographic material is a very useful function to know. In this thesis we will examine how well Trabka's model predicts the measured noise spectrum of photographic film.

It is necessary to establish here certain conventions in terminology which are used throughout the remainder of this thesis. Two methods for measuring the noise spectrum of uniformly exposed and processed film are used. One method uses the light intensity distribution as measured in the Fourier transform plane of a coherent optical processor. The results of such a device will be referred to as the Optical Power Spectrum (OPS). The other method uses an incoherent measuring instrument such as a microdensitometer. The results from this device will be referred to as the Weiner spectrum. When referring to the general case, the term "noise spectrum" or just "spectrum" will be used. The mathematical distinction between OPS and the Weiner spectrum will be discussed later.
CHAPTER 2

RENEWAL THEORY

Introduction

As mentioned in Chapter 1, the basis for Trabka's model lies in renewal theory. This area of mathematics began as the study of the probability of failure and replacement of components. For a given interval in time, renewal theory can be used to describe the statistics of the age at which failure of a component will occur. An important modification of the renewal process is the alternating renewal process. Here, when a component fails, it is replaced with a component of opposite type. Thus, if we were concerned with light bulb replacement, we might alternate between two brands of light bulb. In the next chapter this idea will be extended to alternating between grains and gaps in order to simulate an emulsion. For the remainder of the present chapter the basic tenets of renewal theory will be described.

Fundamentals

Initially, we will be concerned with the distribution of "event occurrence" time. In the time domain this might be the length of failure time for a particular component type or in the spatial domain the distance from one silver halide grain to the next. We will let X be a random variable associated with the event occurrence time (or length) having the probability distribution function f(x), which is zero...
for negative x. We further define the cumulative distribution

\[ F(x) = \int_0^x f(u) \, du \tag{2.1} \]

and the survivor function

\[ R(x) = 1 - F(x). \tag{2.2} \]

The time specific occurrence rate

\[ (x) = \lim_{\Delta x \to 0} \frac{\text{Prob}(x < X < x + \Delta x \mid X < Y)}{\Delta x} \tag{2.3} \]

gives the probability of almost immediate occurrence of an event such as a component failure when the component is known to be of age x. It can be shown\textsuperscript{14} P. \textsuperscript{5} that

\[ f(x) = \phi(x) \exp(-\int_0^x \phi(u) \, du) \tag{2.4} \]

which means that \( \phi(x) \) uniquely determines the p.d.f., \( f(x) \).

The Laplace transform is of prime importance in renewal thoery. We will define this operation by

\[ L\{f(x); s\} = f^*(s) = \int_0^\infty e^{-sx} f(x) \, dx. \tag{2.5} \]

For the case where \( f(x) \) is a p.d.f.,

\[ f^*(s) = E(e^{-sx}), \tag{2.6} \]

where "E" denotes expected value. It can be further shown\textsuperscript{14} P. \textsuperscript{63} that for \( f(x) \) being a p.d.f

\[ F^*(s) = f^*(s)/s \tag{2.7} \]
and
\[ \mathcal{F}^*(s) = \{1 - f^*(s)\}/s. \quad (2.8) \]

Laplace transforms are valuable in renewal theory in connection with sums of independent random variables. If we have a series of non-negative random variables \(X_1, \ldots, X_n\) which are independently distributed with p.d.f.'s \(f_1(x), \ldots, f_n(x)\), the Laplace transform of \(X_1 + \ldots + X_n\) is then

\[
E(\exp(-s(X_1 + \ldots + X_n))) = E(e^{-sX_1} \ldots e^{-sX_n}) \\
= E(e^{-sX_1}) \ldots E(e^{-sX_n}) \\
= f_1^*(s) \ldots f_n^*(s). \quad (2.9)
\]

The inverse transform of this yields an explicit formula for the p.d.f. of \(X_1 + \ldots + X_n\) which is

\[
\int_{-\infty}^{\infty} \ldots f_1(u_1) f_2(u_2 - u_1) \ldots f_n(X - u_n - 1) du_1 \ldots du_{n-1} \quad (2.12)
\]

where all \(f(x)\) are zero for negative \(x\). This is called the convolution of \(f_1(x), \ldots, f_2(x)\). For the case where all of the random variables are identically distributed, the p.d.f. of \(X_1 + \ldots + X_n\) is denoted \(f_{(n)}(x)\) and is called the \(n\)-fold convolution of \(f(x)\). This brings about

\[
f_{(n)}^*(s) = (f^*(s))^n, \quad (2.13)
\]

which is a very useful result.

In any renewal process a random variable of great interest is the number of renewals occurring in an interval \((0, t)\), which we denote \(N_t\).
We further define

\[ N_{t_1,2} = N_{t_2} - N_{t_1} \quad \text{(2.14)} \]

We call the mean value of \( N_t \) the renewal function and denote it

\[ H(t) = E(N_t) \quad \text{(2.15)} \]

where

\[ E(N_{t_1,2}) = H(t_2) - H(t_1) \quad \text{(2.16)} \]

Another concept which will later be used is that of the forward recurrence time. This is the random variable which describes the time from any point "t" to the next renewal. This tells us how much life a particular component has left.

**Special Processes**

Let us suppose that we have a renewal process starting at some arbitrary point in time and we begin with a new component at time zero. Also, let us associate with each component a continuous random variable \( X_i \) with p.d.f. \( f(x) \). As the process employing the component continues and a component fails, it is immediately replaced with a new component. Thus, the failure time of the \( r \)th component used in the process will be \( X_r \). If we define \( S_r \) as the time at which the \( r \)th failure occurs,

\[ S_r = X_1 + \ldots + X_r \quad \text{(2.17)} \]

If all \( X_i \) are independent and identically distributed with p.d.f. \( f(x) \) we have an ordinary renewal process.
The Equilibrium Renewal Process

If we have an ordinary renewal process which has been running for a long time before it is first observed, we can no longer expect the first component to have the same p.d.f. as the rest of the components. It can be shown that the first component or event will have a p.d.f. \( \frac{F(x)}{\mu} \), where \( F(x) \) is the previously described survivor function and \( \mu \) is the mean. We call such a process an equilibrium renewal process.

The Poisson Process

For a renewal process where the p.d.f. is given by

\[
f(x) = pe^{-px},
\]

where \( p \) is a constant, we have a Poisson process. This has the special property that the time specific occurrence rate \( \lambda(x) = p \), its mean and standard deviation are both equal to \( 1/p \), and its coefficient of variation is equal to one. The quantity \( p \) is called the Poisson rate parameter.

The Type I. Counter

We will now consider a special case of the Poisson renewal process known as the Type I. counter. Suppose that we have a stream of particles whose arrival time is Poisson distributed with rate \( p \). Now let us also suppose that we employ a particle counter which works according to a special set of rules in which the counter may be blocked and unable to count, or open and able to count. The random variable \( X_i \) will be associated with the time the counter is blocked, and we will assume
that all $X''_i$'s are independently and identically distributed. The random variable $X''_i$ is the time it takes for a particle to arrive and be counted after $X'_i$ time has passed and the counter assumes an open state. The p.d.f. for the $X'_i$ we will call $f'_1(x)$, and as already stated, the $X''_i$ have p.d.f. $pe^{-px}$. Initially, let us start at $t = 0$ with the counter blocked. If we regard the counting of a particle as a "component failure" or an event occurrence, then we have an ordinary renewal process. The p.d.f. of event occurrence time will be the convolution of $f'_1(x)$ with $pe^{-px}$. This is known as a Type I. counter system. The counter could assume the open state initially, and we would still have a Type I. counter, but not an ordinary renewal process\cite{14, 31}. When considering this type of counter or similar system, we are generally interested in correcting the observed count for the blocked time.

**Asymptotic Distribution of $N_t$**

An important simplification when applicable is the case of the distribution of $N_t$ as $t \to \infty$. Cox\cite{14, 40} has shown that from the Central Limit Theorem that $N_t$ is asymptotically normally distributed with mean $t/\mu$ and variance $\sigma^2t/\mu^3$. Here, both $\mu$ and $\sigma$ refer to the distribution for the random variable $X$, as discussed earlier.

This is easily examined for the Poisson process of rate $p$ for which $\sigma = \mu = 1/p$. The distribution of $N_t$ for $t \to \infty$ will have both mean and variance equal to $pt$. 
Alternating Renewal Theory

We will now describe certain special properties of the alternating renewal process, a description of which was given earlier. Instead of one type of event occurring, we now consider two possible types which alternate in occurrence. For the life time of the Type I. event, we will use the random variable $X_1$, while using $X_2$ for the type II event. Respectively, we will use the p.d.f.'s $f_1(x)$ and $f_2(x)$. The event occurrence time distribution for the overall process will be the convolution of $f_1(x)$ and $f_2(x)$. For a Type I. process, we have a modified renewal process in which the first event occurrence has the p.d.f. $f_1(x)$ and all remaining events have as the p.d.f. the convolution of $f_1(x)$ with $f_2(x)$. Cox\textsuperscript{14} p. 40 has shown that for such a process the Laplace transform of the expected number of Type I. event occurrences in the interval $(0, t)$ is

$$H_1^*(s) = \frac{f_1^*(s)}{s(1-f_1^*(s)f_2^*(s))}.$$  \hspace{1cm} (2.19)

For the Type II. event occurrence, the corresponding analog to Eq. 2.19 is

$$H_2^*(s) = \frac{f_1^*(s)f_2^*(s)}{s(1-f_1^*(s)f_2^*(s))}.$$ \hspace{1cm} (2.20)

When using an alternating renewal model for some process, we may be interested in knowing the probability that a Type I. or II event is occurring at some arbitrary time "t." This probability, which we will denote $R(t)$, is the sum of the probability that the initial event life
time is greater than \( t \) and the probability that a Type II failure has occurred in \((u, u+du) < t\), for a Type I occurrence. The second condition also requires that the Type I component introduced when a Type II failure occurs has a lifetime greater than \( t-u \). Cox has shown that this leads to

\[
R_1(t) = H_2(t) - H_1(t) + 1 \quad \text{(2.21)}
\]

Of course, the probability that a Type II component is in use is simply

\[
R_2(t) = 1 - R_1(t) \quad \text{(2.22)}
\]

The preceding discussion gives a brief overview of the principles of renewal theory. In the next chapter we will go through Trabaka's approach to emulsion modelling using renewal theory. If desired, a more detailed treatment of renewal theory is given by Cox p.83.
CHAPTER 3

THE TRABKA GRAIN MODEL

Introduction

Historically, the random dot grain model has received most of the attention of photographic scientists. The reason for this emphasis was that this model lent itself to many interesting mathematical treatments. However, as noted by R. Clark Jones\textsuperscript{3}, this model did not accurately predict second order properties of granularity. In fact, most of the development in grain modelling has dealt with deriving mathematically tractable approaches to incorporating one or more physical properties of an emulsion in the model. Not much has been done toward getting better agreement between model and real emulsion performance. In the following sections we will examine the Trabka et al. approach to incorporating certain physical properties in an emulsion model, with the ultimate goal of relating this model to real commercial emulsions.

In analyzing the predictions of his model, Trabka examined both closed form and Monte Carlo solutions. In the following chapter we will place emphasis on the Monte Carlo approach, and are now concerned with developing those second order properties which will be needed. Less emphasis will be placed on the closed form solution of the predicted power spectrum, except where special insight is gained.
Monolayer Monodisperse Emulsions

Introduction

The "crowded" monolayer forms a basic element in the Trabka model. As will be shown later, if we know the second order properties of a single layer within an emulsion, then we know the properties of the entire multilayer sandwich. The term "crowded" refers to the principle of the impenetrability of matter (i.e., photographic grains), which is accounted for in the Trabka model via renewal theory. It is this factor which Trabka and Jones have hypothesized accounts for the discrepancy between the measured and predicted granularity spectra for the random dot model. This discrepancy has been especially large at high densities. The crowding of grains would tend to reduce the number of grains in an aperture, and thus reduce the granularity fluctuations. We will consider a crowded emulsion to be one in which the grains are so densely populated that they have significantly constrained locations. Traditionally, grain locations were considered to be distributed completely at random with a Poisson distribution. This will of course have to be modified in order to account for crowding.

Important Parameters

With regard to the emulsion under consideration, certain physical parameters will be discussed which we will now define. The first parameter is $t_{\min}$. This is the minimum average transmittance possible if all the grains in a layer were to receive a sufficient number of photons and develop. In this thesis we will not attempt to relate
the pre-development emulsion to the developed emulsion, since grain shapes may change in a manner that is too difficult to treat here. Therefore, for our purposes, $t_{\text{min}}$ will simply be thought of as the average transmittance when the model is applied to real commercial emulsions. However, in order to maintain a general approach, our development will use both $t_{\text{min}}$ and $\mu_t$ which is the average transmittance when the probability $p$ of a grain receiving sufficient exposure to develop is not equal to one.

As mentioned above, the grain locations will be constrained such that their distribution is not strictly Poisson. The amount by which this departure occurs will be denoted by $\varepsilon$, the Poisson excess. This quality will be determined by the coefficient of variation $C$ of the gaps between events (grains of length $\ell$).

The probability density function of the gaps will be for non-negative gap lengths with a probability $p_0$ of a gap length of zero. This $p_0$ is called the sticking parameter since it describes the probability that two grains will be in physical contact. We will also use $\lambda$ to denote the Poisson rate parameter described earlier.

Application of Renewal Theory

We will now describe the monolayer monodisperse emulsion as predicted by renewal theory. The renewal model relates the statistics of the number of events (or grains) in an interval to the statistics of the gaps between them. Thus, for every grain we encounter we will have a finite probability of encountering another grain after travelling a given
distance. As mentioned above, we will also have a finite probability $p_0$ of having an immediate occurrence of an event (or grain) following a previous occurrence.

If we are considering a large interval across an emulsion, the asymptotic approximations of renewal theory\(^1\) state that:

$$\mu_h \approx \frac{1}{\mu x}L$$  \hspace{1cm} (3.1)

and

$$\sigma^2_h = \frac{\sigma^2 x}{\mu^3 x}L$$  \hspace{1cm} (3.2)

where $h$ is the random variable associated with the number of grains in an interval $L$ and $x$ the random variable for the gaps between successive grains. This gives us

$$\frac{\sigma^2 x}{\mu^2 x} = C^2 = \frac{\sigma^2_h}{\mu^2 h}$$  \hspace{1cm} (3.3)

which for a Poisson process is equal to one. This means that the Poisson excess

$$\varepsilon = C^2 - 1$$  \hspace{1cm} (3.4)

Let the probability density function for the gap distribution be $f(x)$. If the grain locations were completely random, $f(x)$ would be exponential. Trabka\(^1\) has proposed a modification of this to incorporate crowding and sticking. This yields:

$$f(x) = p_0 \delta(x-\ell) + (1-p_0)U(x-\ell)\lambda e^{-\lambda(x-\ell)}$$  \hspace{1cm} (3.5)
U(x) is a step function which guarantees that the gap length between successive leading edges of a pair of grains is not less than the grain length \( l \). We consider this to be the crowding effect and arises from the finite size of the grains. \( \delta(x) \) is a delta function of strength 0 \( \leq p_0 \leq 1 \) when \( x = l \). If we take \( x \) as the gap length between successive leading edges for a grain pair, then this delta function is capable of providing a nonzero probability that \( x = l \). At \( x = l \) successive grains will be sticking together, thus \( p_0 \) is our sticking parameter.

For this type of distribution, Trabka has gone on to show that

\[
\mu_x = (1+\lambda l- p_0)/\lambda \quad \text{(3.6)}
\]
\[
\sigma_x^2 = (1-p_0^2)/\lambda \quad \text{(3.7)}
\]

and

\[
C^2 = (1-p_0^2)/(1+\lambda l- p_0)^2 . \quad \text{(3.8)}
\]

If we define the average transmittance as

\[
\mu_t = 1-p\bar{n}l \quad \text{(3.9)}
\]

where \( \bar{n} = \mu_h/L \) and \( p \) is the probability of a silver halide crystal absorbing sufficient photons to be made developable, then it can be shown\(^{10} \) that

\[
\mu_t = (1+(1-p)\lambda l- p_0)/(1+\lambda l- p_0) . \quad \text{(3.10)}
\]

When \( p = 1 \), we have

\[
t_{\text{min}} = (1-p_0)/(1+\lambda l- p_0) \quad \text{(3.11)}
\]
which yields

$$C^2 = \frac{(1+p_0)}{(1-p_0)}(t_{\text{min}})^2 \quad . (3.12)$$

Since we will not seek in later sections to relate pre-development to post development statistics, our main concern will be with those equations where $p = 1$.

We now have expressions for the mean, variance, and distributions of grain locations and their effect upon mean aperture transmittance.

We will now cover the main points of Trabka's derivation of a closed form solution for the single sided Wiener spectrum of the point transmittance fluctuations.

Let us define the single sided Wiener spectrum as

$$P(\omega) = 2\int_{-\infty}^{\infty} c(u)e^{-i\omega u} du \quad \text{for } 0 \leq \omega \leq \infty \quad . (3.13)$$

where $c(u)$ is the autocovariance function of a random process. Next, we will define a shot noise process given by Parzen as a stochastic process

$$a(x) = \sum_{n=-\infty}^{\infty} A(x-S_n; y_n) \quad . (3.14)$$

This equation represents a process as the superposition of impulses occurring at random locations ..., $S_{-1}, S_0, S_1, ...$. The shape of the impulse at a given location is determined by the random variable $y_n$.

If a stochastic process can be modelled by a modification of Eq. 3.14 such that $a(x)$ vanishes for $x<0$ and

$$a(x) = \sum_{n=1}^{\infty} A(x-S_n; y_n) \quad , (3.15)$$
then its power spectrum will be given by

\[ P(\omega) = \left( \frac{2}{\gamma_1} \right) \left( \gamma_2(\omega, -\omega) + 2\Re \rho^*(j\omega) \right) \] (3.16)

as has been shown by Smith\textsuperscript{16}. In this equation, \( \gamma_1 \) and \( \gamma_2 \) are the Fourier transforms

\[ \gamma_1(\omega) = \mathcal{F}[a_1(x)] = \int_{-\infty}^{\infty} a_1(x)e^{-i\omega x} dx \] (3.17)

\[ \gamma_2(\xi, \eta) = \mathcal{F}[a_2(u,v)] = \int_{-\infty}^{\infty} a_2(u,v)e^{-i(\xi u + \eta v)} du dv \] (3.18)

where

\[ a_1(x) = \hat{E}(A(x)) \] (3.19)

\[ a_2(u,v) = E(A(u,y_n)A(v,y_n)) \] (3.20)

and \( E \) denotes expectation with respect to the random process \( y_n \). The term \( \Re (\rho^*(i\omega)) \) is the real part of the Laplace transform

\[ \rho^*(s) = \int_0^{\infty} e^{-su} \rho(u) du \] (3.21)

of the renewal density function \( \rho(u) \) discussed earlier. This function gives the probability of one or more events occurring in a given interval, and its Laplace transform is

\[ \rho^*(s) = \frac{[f^*(s)]}{[1-f^*(s)]} \] (3.23)

where \( f^*(s) \) is the Laplace transform of the distribution for the underlying process. The mean of the process is \( \mu_x \), and the process in this case is the gaps between grains.
In order to apply these results to the crowded monolayer, Trabka first introduces the sequence of independent and identically distributed non-negative random variables $x_i$ with partial sums

$$S_n = \sum_{i=1}^{n} x_i .$$

(3.23)

The variable $x_i$ denotes the gaps between the left hand edges of grains, and the $S_i$ their linear positions. The $y_i$ mentioned above will denote the absorptance profile of the grain. In choosing to use the left hand edge of each grain for the origin of $A(u)$, it is apparent that for a given grain $A(u) = 0$ for $u \leq 0$. This precludes the overlapping of pulses or grains. Thus, we are really only superpositioning a single pulse in the monolayer at a given point. We now have the absorptance profile of a monolayer expressed in such a way that we can apply Eq. 3.15.

The next step in Trabka's development is to apply this result to the gap distribution in Eq. 3.5, along with the grain absorptance profile

$$A(u) = A_0 \quad \text{for } (0 \leq u \leq \ell)$$

$$= 0 \quad \text{for } (u < 0, u > \ell) .$$

(3.24)

$A_0$ is a random variable which has values of 1 and 0 with a probability $P$ for a value of 1. Trabka shows this now yields a power spectrum of the form

$$P(\omega) = \frac{2}{\pi \mu_{X}} |Y_1(\omega)|^2 [1 + 2 \rho \Re \left( i\omega \right)] .$$

(3.25)

Here, the term $|Y_1(\omega)|^2 = (p^2 \ell^2 \sin^2 X)/X^2$ in which $X = \omega \ell/2$, is the power spectrum of the average absorptance profile of the grains. He
further shows that when \( p = 1 \) (which is true for all cases we will consider), the remaining terms are due to the spectrum of the point process which governs the location of the leading left hand edges of the grains. This result shows us that the spectrum of a monolayer mono-disperse emulsion is a result of multiplying the spectrum of the average grain absorptance profile by the spectrum of the point process associated with grain locations. Eq. 3.25 will provide useful insight in developing a reliable Monte Carlo modelling technique in the next chapter.

**Alternating Renewal Model**

In the previous section we considered a distribution of gaps, the length of which could not be less than \( \ell \), since a grain of length \( \ell \) resided in each gap. We will now examine a model which considers the gaps and the grains both as random quantities, and no longer requires a minimum gap length of \( \ell \). Our approach here, as before, is for the one dimensional case.

Let \( a_i \) be random variables for the lengths of silver grains. We assume that these random variables are non-negative, independently and identically distributed. The cumulative distribution will be

\[
F_a(x) = \text{Prob}(a_i < x).
\]  

Let \( b_i \) be random variables for the lengths of the gaps between grains. These are assumed to be non-negative, independent of each other and the \( a_i \)'s, and identically distributed. The cumulative distribution function will be \( F_b(x) \). \( F_b(0) \) is the probability that successive grains have zero
gap length, or that they stick. The alternating sequence $a_1, b_1, a_{i+1}, b_{i+1}, \ldots$ is an alternating renewal process. When a Monte Carlo approach is used to generate such a monolayer, the functions $F_a(x)$ and $F_b(x)$ are of primary importance.

We will define $\mu_a$ as the mean grain length where

$$\mu_a = \int_0^{\infty} x dF_a(x),$$

and $\mu_b$ as the mean gap length where

$$\mu_b = \int_0^{\infty} x dF_b(x).$$

Castro et al.\textsuperscript{12} have shown that the single sided power spectrum for such a process to be

$$P(\omega) = \frac{2}{\mu \omega^2} \text{Re} \left( \frac{(i-f_a(i\omega))(i-f_b(i\omega))}{i-f_a(i\omega)f_b(i\omega)} \right)$$

where $c = a + b$, and

$$f_a(i\omega) = \int_0^{\infty} e^{-i\omega x} dF_a(x)$$

$$f_b(i\omega) = \int_0^{\infty} e^{-i\omega x} dF_b(x)$$

and Laplace-Stieltjes transforms. While it is too difficult to repeat the derivation here, the final result shows that the distributions $F_a(x)$ and $F_b(x)$ have equal weight in the power spectrum. Also, we see that the means of these distributions must be finite.
Multilayer Emulsions

Introduction

Up till now we have only considered the granularity properties of single layers. Now that we know what happens for a single layer, we will derive how to go to a multilayer sandwich. Our basic goal here is to show how granularity depends on emulsion thickness. This will be accomplished by examining how the granularity propagates through the layers of the emulsion. As before, we will explore only the one dimensional case.

Transmittance

We will define the random variable $t(p)$, where $0 \leq t(p) \leq 1$, as the transmittance at a single point of a monolayer. The aperture transmittance will be given by

$$T(p) = \frac{1}{|A|} \int_{A(p)} t(Q) dQ \; ,$$

which is the average transmittance over a region $A$ centered at $p$. If we assume that a photographic emulsion is an ergodic process, the macro-transmittance is

$$T_\infty = \lim_{|A| \to \infty} T(p) \; .$$

If the random process $t(p)$ is ergodic, homogeneous, and isotropic, then

$$\nu_T = \nu_t = T_\infty \; .$$
Since a point transmittance is the fraction of light transmitted at a point, point transmittances are multiplicative. Thus, the point transmittance of an $m$-layer sandwich will be

$$s(p) = \frac{m}{i=1} t_i(p) \quad (3.34)$$

However, we cannot substitute Eq. 3.31 into Eq. 3.34 because we cannot assume that the integral of the product is equal to the product of the integrals. Therefore,

$$s(p) \neq \frac{m}{i=1} T_i(p) ~, \quad (3.35)$$

meaning that we cannot assume that aperture transmittances are multiplicative.

If we let $T_\infty$ denote macrotransmittance for a single layer and $S_\infty$ the macrotransmittance for an $m$-layer sandwich, then we see from Eq. 3.33 that

$$S_\infty = E\left( \frac{m}{i=1} t_i(p) \right) = \frac{m}{i=1} E(t_i(p)) = T_\infty^m \quad (3.36)$$

We now see that while point and macrotransmittances can be multiplied, the same is not true for aperture transmittances.

Second Order Properties

Let $c_s(r)$ denote the autocovariance of $s(p)$ and $c_t$ the autocovariance of $t(p)$. By definition

$$c_s(r) = E(s(p)s(p+q)) - E(s(p))E(s(p+q)) \quad (3.37)$$
which, because of isotropy, is only a function of \( r = |q| \). Using Eqs. 3.33 and 3.36 and some algebraic manipulation, we can relate the covariance of the sandwich to that of the individual layers by

\[
c_s(r) = (c_t(r)+T_\infty^2)^m - T_\infty^{2m} . \tag{3.38}
\]

This assumes that all layers are generated by the same random process, and are independent. This is an assumption that we too shall make. Real emulsions do not, however, have this property. How much they deviate from this simplification is not known, and will not be explored here. If in future work it is decided that this simplification cannot be made, it will be necessary to determine a transmittance profile for each individual layer, and then for the resulting sandwich.

The single sided space power spectrum is then given by

\[
P_s(\omega) = 2 \int c_s(r)e^{-i\omega r}dr \tag{3.38}
\]

for the entire emulsion. Thus if we know the autocovariance function for a single layer of a multilayer sandwich and the mean level of the single layer, then we know the power spectrum for the sandwich as a whole via Eq. 3.38.

Using the binomial expansion we can express Eq. 3.38 as

\[
c_s(r) = \sum_{j=1}^{m} \binom{m}{j} [c_t(r)]^j T_\infty^{2(m-j)} \tag{3.39}
\]

which gives a power spectrum

\[
P_s(\omega) = 2 \sum_{j=1}^{m} \binom{m}{j} T_\infty^{2(m-j)} F_j(\omega) . \tag{3.40}
\]
In the above equation, the term
\[ F_j(\omega) = \int_{-\infty}^{\infty} [c_t(r)]^j e^{-i\omega r} dr \]  
(3.41)

or
\[ F_j(\omega) = (P_t(\omega))^{*j} \]  
(3.42)
denotes the \( j \)-fold convolution of \( P_t(\omega) \) with itself, since the Fourier transform of the product is equal to the convolution of the Fourier transform of the individual functions. We can therefore rewrite Eq. 3.40 such that
\[ \frac{P_s(\omega)}{S_\infty^2} = \sum_{j=1}^{m} \binom{m}{j} \frac{P_t(\omega)^{*j}}{T_\infty^{2j}} \]  
(3.43)

We now have an explicit relationship between the spectrum of the sandwich and the spectrum of the individual layers. It should be noted that this relationship does involve the mean macrotransmittance level of the individual layers.

To date, the Trabka model is only one dimensional in each layer. Extending the alternating renewal model to include two dimensional distributions of grains in a crowded layer is not a trivial task. We must tackle the question of how to extend to two dimensions the one dimensional gap. Furthermore, the extension would have to be done so that grains do not overlap in a single layer. This type of ordering would not be easy. This limitation represents a serious drawback to the Trabka model, since there may be some directional properties in photographic film which may effect the grains and gaps in their size distributions. We will not attempt to develop a more general point...
process to solve this problem here. Instead it will be assumed that the grains and gaps are homogeneous and isotropic in two dimensions. This means that a one dimensional treatment of the noise can then be used. However, if the Trabka model is to be used for future work in greater detail, an extension to two dimensions will have to be made.
CHAPTER 4

COMPUTER SIMULATION OF AN EMULSION

Introduction

It has been shown in the previous chapter than an analytical solution of the power spectrum for the multilayer alternating renewal model of an emulsion is available. This solution can be implemented through the use of a digital computer, but only with much difficulty. Also, this closed form solution does not give us any information with regards to the variability we should expect from measured power spectra of real emulsions. This poses a serious difficulty in comparing measured and predicted spectra. A method for solving this problem is to artificially generate realizations of the random process under study on a computer, and perform the pertinent statistical analysis on the simulations to obtain the desired information. In this manner we can obtain an estimate of the power spectrum and its associated variability based upon the previously derived emulsion models.

Sampling from Any Known Distribution

Theory

The basic problem faced in simulated sampling is that of selecting values of random variables with known distributions. In this manner we simulate their occurrences as real physical random processes.
The method we will use to accomplish this is known as Monte Carlo sampling. This technique relies upon using one set of independent observations (called the basic sequence) of a random variable with a known distribution to generate a set of independent observations of another random variable whose distribution is known. The basic sequence is usually a set of random numbers from a uniform distribution.

Let $x$ be a random variable with density function $f(x)$ and cumulative distribution function $F(x)$. Let $u$ be a uniformly distributed random variable on the interval $(0,1)$, with cumulative distribution $G(u)$. Since both $u$ and $F(x)$ are defined on the unit interval, we can write

\[ u = F(x) \quad . \]  \hspace{1cm} (4.1)

The inverse of this equation is

\[ F^{-1}(u) = x. \]  \hspace{1cm} (4.2)

Letting $x_1$ and $u_1$ be values such that

\[ u_1 = F(x_1) \]  \hspace{1cm} (4.3)

then

\[ \text{prob} \ (x \leq x_1) = F(x_1) \]  \hspace{1cm} (4.4)

\[ \text{prob} \ (u \leq u_1) = G(u_1) = u_1 \]  \hspace{1cm} (4.5)

This means that we can make random sampling from a uniform distribution over the interval $(0,1)$ equivalent to random sampling from any known
distribution. Since most computers have the ability to supply a list of numbers whose distribution is uniform over the interval (0,1), this type of sampling procedure is easily implemented. Once we have a sample from the uniform distribution, we use the inverse relationship of Eq. 4.2 to determine the value of the variable x. This is shown in Fig. 1.

Fig. 1. Simulation of random variables.

The variable to be simulated is on the horizontal axis. The cumulative distribution function and the input uniformly distributed random numbers are on the vertical axes.
Application

We will now demonstrate by example the application of Monte Carlo simulated sampling. Let us suppose that we wish to generate a sample of some population, with 1000 observations in the sample. Let the population consist of 21 discrete values running from 6 to 26. We will also let the probability density function be as shown in Table 1.

Table 1. Probability density and cumulative distribution for a sample population.

<table>
<thead>
<tr>
<th>x</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>0.0002</td>
<td>0.0008</td>
<td>0.0024</td>
<td>0.0065</td>
<td>0.0150</td>
<td>0.0300</td>
<td>0.0526</td>
<td>0.0809</td>
<td>0.1098</td>
</tr>
<tr>
<td>F(x)</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0034</td>
<td>0.0099</td>
<td>0.0249</td>
<td>0.0549</td>
<td>0.1075</td>
<td>0.1884</td>
<td>0.2982</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>0.1317</td>
<td>0.1399</td>
<td>0.1317</td>
<td>0.1098</td>
<td>0.0809</td>
<td>0.0809</td>
<td>0.0526</td>
<td>0.0150</td>
<td>0.0065</td>
</tr>
<tr>
<td></td>
<td>0.4299</td>
<td>0.5698</td>
<td>0.7015</td>
<td>0.8133</td>
<td>0.8922</td>
<td>0.9848</td>
<td>0.9748</td>
<td>0.9898</td>
<td>0.9963</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>25</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0024</td>
<td>0.0008</td>
<td>0.0002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9987</td>
<td>0.9995</td>
<td>0.9997</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The flow chart for the computer program to handle this is shown in Table 2, with the actual program listing given in Appendix A.
Table 2. Flow chart of program to simulate the occurrence of random variables with an arbitrary distribution.

Table 3 gives the theoretical probability density vs. the measured density of the 1000 simulated observations. A visual inspection of this table suggests that the simulated sample belongs to the parent population. The chi-square goodness-of-fit test\textsuperscript{17} gives us a determination of how well the Monte Carlo technique provides us with the desired sample population. In this test we will compare the observed frequencies for each subinterval in the sample to that of the theoretical parent population.
Table 3. Theoretical vs. measured probability density values for a simulation of 1000 random variables.

<table>
<thead>
<tr>
<th>x</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>measured f(x)</td>
<td>0.0000</td>
<td>0.0010</td>
<td>0.0020</td>
<td>0.0080</td>
<td>0.0130</td>
<td>0.0350</td>
<td>0.0350</td>
<td>0.0750</td>
<td>0.1150</td>
</tr>
<tr>
<td>theoretical f(x)</td>
<td>0.0002</td>
<td>0.0008</td>
<td>0.0024</td>
<td>0.0065</td>
<td>0.0150</td>
<td>0.0300</td>
<td>0.0526</td>
<td>0.0809</td>
<td>0.1098</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>0.1340</td>
<td>0.1540</td>
<td>0.1320</td>
<td>0.1090</td>
<td>0.0700</td>
<td>0.0470</td>
<td>0.0400</td>
<td>0.0180</td>
<td>0.0080</td>
<td></td>
</tr>
<tr>
<td>0.1317</td>
<td>0.1399</td>
<td>0.1317</td>
<td>0.1098</td>
<td>0.0809</td>
<td>0.0526</td>
<td>0.0300</td>
<td>0.0150</td>
<td>0.0065</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>25</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0030</td>
<td>0.0000</td>
<td>0.0010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0024</td>
<td>0.0008</td>
<td>0.0002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The statistic for the chi-square test is

\[ X = \sum_{i=1}^{K} \frac{(P_{oi} - P_{ei})}{P_{ei}} \]  

(4.6)

where

\( N \) = total number of observations

\( K \) = number of subintervals

\( P_{oi} \) = observed probability for the \( i^{th} \) subinterval

\( P_{ei} \) = expected probability for the \( i^{th} \) subinterval

The hypothesis being tested is that the sample observations have the same distribution as the parent population against the alternative that they do not have this distribution. We will choose an alpha risk (probability of rejecting the hypothesis when it is true) of 0.05. The number of degrees of freedom is equal to \( K-1 \), or 20 in this case. The test statistic computed according to Eq. 4.6 is \( X^2 = 20.290 \). From the chi-square test table we get \( X^2 = 34.170 \). Since 20.290 < 34.170, we accept the hypothesis that the sample has the desired distribution, as there is insufficient evidence to reject it. We can now conclude that we are capable of generating a series of random variables with a desired distribution to a certain level of confidence.

**Emulsion Simulation**

Emulsion simulations fall into one of two basic categories, depending upon whether we are using a simple renewal model (monodisperse emulsions) or the more sophisticated alternating renewal model (variable...
gaps and grains). For the simple renewal model in which the gap density follows Eq. 3.5 we can integrate to obtain the cumulative distribution function such that

\[ F(x) = 1 - (1 - p_0)e^{-\lambda(x-\ell)} \]  \hspace{1cm} (4.7)

Now we could evaluate this function at certain points, enter those values into the F and x arrays of the program in Table 2 and simulate the emulsion. However, we can use the inversion formula of Eq. 4.2 directly to get

\[ x = \ell \quad \text{for } (v<0) \]

\[ x = \ell - \frac{1}{\lambda} \ln((1-v)/(1-o_0)) \quad \text{for } (v\geq 0) \]  \hspace{1cm} (4.8)

where \( v \) is a uniformly distributed random variable on the interval \((0,1)\).

As explained before, this model uses the concept of each gap including a grain of length \( \ell \) whose leading left hand edge coincides with the origin of the gap. This means that no gap can be less than \( \ell \) in length. An example of such a simulation's power spectrum is shown in Fig. 2. In Fig. 2 we can see good agreement between the Monte Carlo calculation and the analytical calculation. Trabka's solution for this special case enabled us to check our results with those of an analytical solution. This was very important as it gave us a good indication of how to proceed in performing the digital spectrum calculation. The details of this operation will be discussed in a later chapter. In simulating the emulsion, wherever a grain occurs we have a transmittance equal to zero,
Fig. 2. Monte Carlo simulation and analytic solution of the Wiener spectrum of a monolayer monodisperse emulsion.
and a transmittance equal to one for empty spaces. Thus we end up with a one dimensional array of binary transmittance levels or profiles with which we calculate the transmittance power spectrum.

If we are dealing with an alternating renewal model, it may be possible to fit a function to each of the cumulative probability functions which can in turn be analytically inverted. If not, we need only turn to the program in Table 2 to come up with a series of random variables associated with grain and gap sizes. In this model we examine the gaps and grains separately. We alternate in generating grains and gaps, and associate a transmittance of zero with the former and one with the latter. For the case where we have a multilayer emulsion, we determine the properties of a single layer and then use Eq.3.40 to determine the properties of the entire sandwich. The program listings that deal with this problem are given in Appendix A.
MEASUREMENT OF EMULSION PARAMETERS

Parameters Measured

There are three inputs to the computer simulation which are obtained from the film type of interest. These are the grain size distribution, the gap size distribution, and the number of layers in the emulsion. The first two sets of data are used to form frequency histograms and input to the simulation routine. The third parameter is used to predict the effects of granularity propagation through the layers.

Methods of Measurement

In order to obtain the required data, we made electron micrographs of cross sections of the exposed and developed film samples. The technique for preparing the cross sections was supplied to us by Mr. C. Powell of the Polaroid Corporation Research Laboratory. Mr. Powell's method consisted of placing the film sample between two cellulose acetate butyrate (CAB) supports, and then sectioning on a microtome. What is of importance here, is that the CAB material is curved. The film sample is placed between the two CAB supports so that when placed in the jaws of the microtome vise, the CAB will press against the film sample. This pressing action gives the film some extra rigidity that permits sectioning.
We prepared our cross sections on a Sorvol MT-2 microtome, and used a glass blade. Diamond blades are also used in this type of work, but they are very expensive and easily damaged. Initially, a good glass blade can produce a section as thin as a diamond blade, but it wears down and becomes dull much quicker. The sectioning and electron microscopy were done in the laboratory of Dr. Wayne Ferris of the Department of Cell and Developmental Biology of the University of Arizona. Dr. Ferris, after talking with Mr. Powell, came up with a method of dry sectioning, which he then taught to us. All of the subsequent sectioning was done by us. Dr. Ferris did all of the electron microscope operation and processing of the film.

In preparing the cross sections, it is necessary to try for a thickness of around one micron. The main problem here is getting good penetration of the section with the electron beam. The thicker a sample is, the more power the beam must have to get good penetration. However, some of the beam's energy is absorbed by the film sample and converted to heat. If the beam's intensity is too great, the sample will melt. Unfortunately, the sections which were easily cut were too thick. The sections which were thin enough were very hard to handle. Once a section has been cut, it is removed from the blade and placed on a copper mesh using a hair attached to the end of a small stick. The microtome has a stereo microscope which makes all of this easy to see. Because it is stereo, you get a good feel for depth, which is very important in this operation. The sections were difficult to handle as they were very susceptible to the presence of static electricity. Much patience was required.
In order to alleviate the problem of sample heating from the electron beam, Dr. Ferris coated some grids with plastic and some with carbon. We found that if the sections made good thermal contact with the coated grid, the grid would act as a heat sink. This helped reduce sample heating.

The electron microscope used by Dr. Ferris is a Phillips EM-200. The recording film was Eastman-Kodak type 4489 Electron Microscope film. Development was in D-19, in accordance with the instructions accompanying the film. At one time we considered using a scanning electron microscope. However, early experiments with such a device proved unsuccessful. When the electron beam did not destroy the sample, the resolution we obtained was too poor for our purposes.

After obtaining the electron micrographs, the grain and gap information was digitized on a Tektronix 4956 Graphics Tablet connected to a Tektronix 4051 terminal. The graphics tablet has a four button cursor with cross hairs, whose position on the tablet was automatically recorded when any one of the four buttons was pushed. Depending on the button pushed, it was also recorded whether a grain or gap was being measured. The data, once recorded, were shipped from the Tektronix terminal to a Data General Eclipse computer for storage and analysis. The programs used for taking the data and shipping it to the Eclipse are in Appendix A.

Given that the tablet had a resolution of 0.127 mm., and that the electron microscope magnification was 6847X, the maximum resolution of
the measurements was about 0.019 microns. No histogram was given a cell interval smaller than this. Generally, the cell interval used in compiling the histograms was about 1/10 the standard deviation. The number of cells was adjusted so that not many cells had fewer than five members. An example of a grain and gap histogram is shown in Figures 3 and 4. The large number of small length gaps is due to the occurrence of sticking grains. This particular data is for Tri-X exposed in the shoulder region of the characteristic curve, where we would expect to find this behavior.

Fig. 3. Normalized grain frequency histogram for Tri-X shoulder sample.
Fig. 4. Normalized gap frequency histogram for Tri-X shoulder sample.
CHAPTER 6

COMPARISON OF MEASURED AND PREDICTED GRANULARITY SPECTRA

Principles of Digital Spectral Analysis

Introduction

A stochastic process is an ordered set of random variables (r.v.'s). From this set we get a probabilistic description of some phenomenon occurring in a given space such as distance or time. The set of functions which can be defined on the same space is called the ensemble. We will examine here techniques for obtaining probabilistic information about stochastic processes by examination of relatively few realizations from the ensemble. These techniques are used for their efficiency in computer time and their reliability.

Earlier, we gave the univariate moments of a stochastic process of a single point in space, \( x \), to be of the form

\[
E((T(x))^k) = \int_{-\infty}^{\infty} t^k f_{T(x)}(t) dt \tag{6.1}
\]

where \( f_{T(x)} \) is the pdf of the rv \( T(x) \). From this we can compute the mean function \( \mu(x) \) and the variance function \( \sigma^2(x) \). The bivariate moments are

\[
E((T(x_1))^{k}(T(x_2))^{\ell}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} t_{1}^{k} t_{2}^{\ell} f_{T(x_1)T(x_2)}(t_{1}, t_{2}) dt_{1} dt_{2} \tag{6.2}
\]
where \( f_{T(x_1)T(x_2)} \) is the joint pdf of the rv's \( T(x_1) \) and \( T(x_2) \) at distances \( x_1 \) and \( x_2 \). The bivariate moment with which we are concerned is the autocovariance function (acvf)

\[
\gamma_{TT}(x_1,x_2) = \mathbb{E}((T(x_1) - \mu(x_1))(T(x_2) - \mu(x_2)))
\]

(6.3)

where \( \gamma_{TT}(x_1,x_1) = \sigma^2(x_1) \). We define the autocorrelation function (acf) to be

\[
\rho_{TT}(x_1,x_2) = \frac{\gamma_{TT}(x_1,x_2)}{\sigma(x_1)\sigma(x_2)}
\]

(6.4)

which is a unitless normalized version of the acvf.

A simplification of the above occurs when we can apply the concept of stationarity. When a stochastic process is considered to be stationary, its pdf is not position dependent. Thus the process will have a constant mean \( \mu \) and variance \( \sigma^2 \). Also, the joint pdf will depend only on the difference \( u = x_2 - x_1 \). The acf will then reduce to

\[
\rho_{TT}(u) = \frac{\gamma_{TT}(u)}{\gamma_{TT}(0)} = \frac{1}{\sigma^2_T} \operatorname{cov}(T(x),T(x+u))
\]

(6.5)

where \( u \) is called the lag.

Another interesting concept is ergodicity. This means that the statistical properties averaged over one realization as the length of that realization tends to infinity will be equal to the ensemble average. As has been shown by Jenkins and Watts, the covariance function has this property. However, since record lengths are finite, this property is of questionable value.
Generally speaking, we consider a signal as either deterministic or stochastic. The difference between these two types of signals becomes obvious as we let the record length tend to infinity. For a zero mean record \( t(x) \) tending towards infinite length the variance can be shown\(^{18}\) to be

\[
\sigma^2 = \lim_{X \to \infty} \int_{-X/2}^{X/2} t^2(x) dx = \lim_{X \to \infty} \int_{-X/2}^{X/2} \lim_{X \to \infty} C_{tt}(f) df = C_{tt}(0) \quad (6.6)
\]

where

\[
C_{tt}(f) = \frac{1}{T} \int_{-X/2}^{X/2} t(x)e^{-i2\pi fx} dx \quad (6.7)
\]

is the sample power spectrum and \( C_{tt}(u) \) is the sample acvf. It can also be shown that the sample spectrum and acvf are Fourier transform pairs. Eq. 6.7 is satisfactory when dealing with a deterministic signal, but not for realizations of a stochastic process. The reason for this is that as we increase the record length, \( C_{tt}(f) \) will tend to converge to the true spectrum \( \Gamma(f) \) smoothly only for the deterministic case. For the stochastic case, all estimates of \( \Gamma(f) \) will be erratic, regardless of record length. In order to get a good sample estimate \( C_{tt}(f) \) of the true spectrum \( \Gamma(f) \), we need an approach other than just increasing the sample record length.

Based on the above discussion, we see that for a stochastic process, \( C_{tt}(u) \) and \( C_{tt}(f) \) are realizations of the rv's \( C_{TT}(u) \) and \( C_{TT}(f) \) respectively. If we can derive the distribution or moments of \( C_{TT}(f) \) then the erratic behavior of the realizations \( C_{tt}(f) \) can be
explained. Thus, we will take the power spectrum $\Gamma_{TT}(f)$ of a stochastic process to be the Fourier transform of the acvf of the random process $T(x)$ where\(^{18}\)

$$\Gamma_{TT}(f) = \lim_{X \to \infty} E(C_{TT}(f)) = \int_{-\infty}^{\infty} \gamma_{TT}(u) e^{-i2\pi fu} du$$  \hspace{1cm} (6.8)

for the continuous case and

$$\Gamma_{TT}(f) = \lim_{X \to \infty} E(C_{TT}(f)) = \Delta \sum_{k=-\infty}^{\infty} \gamma_{TT}(k)e^{-i2\pi kf\Delta}, -\frac{1}{2\Delta} \leq f < \frac{1}{2\Delta}$$  \hspace{1cm} (6.9)

for the discrete case where $\Delta$ is the sampling interval.

Estimation of the Autocovariance Function

The previously stated formula for the autocovariance of a stochastic process is not useful when dealing with real data. Instead, Jenkins and Watts\(^{18}\) have developed an estimator

$$c_{tt}(u) = \frac{1}{X} \int_{0}^{X-|u|} (t(x) - \bar{t})dx, -X \leq u \leq X$$  \hspace{1cm} (6.10)

or

$$c_{tt}(u) = \frac{1}{N} \sum_{x=1}^{N-j} (t_x - \bar{t})(t_{x+k} - \bar{t}) , k = 0,1,\ldots, N-1$$  \hspace{1cm} (6.11)

where $\bar{t}$ is the sample mean. A program computing the autocovariance of a discrete sample was written and tested. The program can be found in Appendix A.
Estimation of the Power Spectrum

Since we are computing power spectra from realizations of a stochastic process, we can expect some variability to be present in the resulting estimates. This variability can be reduced by dividing the data record into smaller sub-series, computing the spectrum for each sub-series, and averaging the individual estimates to obtain a smoother result. Bartlett has shown that subdividing a record of length $X$ into $\ell$ sections of length $M = X/\ell$, computing each subdivision's spectrum and averaging is equivalent to smoothing the original sample by convolving it with the function

$$W(f) = M \left( \frac{\sin \pi f M}{\pi f M} \right)^2 .$$  \hfill (6.12)

This is equivalent to multiplying the autocovariance function by

$$w(u) = \begin{cases} \frac{1}{M} & |u| \leq M \\ 0 & , u > M \end{cases} .$$  \hfill (6.13)

The function 6.12 is called a spectral window, and the function 6.13 is called a lag window.

We now have a smoothed spectral estimator of the form

$$\tilde{c}_{tt}(f) = \int_{-\infty}^{\infty} w(u) c_{tt} e^{-i2\pi fu} du = \int_{-\infty}^{\infty} c_{tt}(u) e^{-i2\pi fu} du$$  \hfill (6.14)

which will give a less erratic variance than the spectrum estimator $C_{tt}(f)$. An equivalent form of Eq. 6.14 is

$$\tilde{c}_{tt}(f) = \int_{-\infty}^{\infty} W(g) C_{tt}(f-g) dg .$$  \hfill (6.15)
Several spectral windows have been proposed, none of which is the best for all cases. The main trade off between windows is their smoothing ability vs. the amount of positive bias they introduce into the spectral estimate. The wider the spectral window is, the smoother but more positively biased the resulting estimate will be. With a narrow spectral window we will get a smaller positive bias, but larger variance. In choosing a particular window, it is helpful if one knows what to expect in the spectral estimate. In the simpler emulsion simulation, we found as did Trabka\textsuperscript{10}, that Monte Carlo derived estimates were best smoothed with a Parzen window and autocovariance function with 60 lags. This criteria was used for all digital spectrum estimates. For the discrete case, Eq. 6.14 becomes

\[
\bar{C}_{tt}(f) = \Delta \sum_{k=-(L-1)}^{(L-1)} w(k) c_{tt}(k) e^{-12\pi fk\Delta} \frac{-1}{2\Delta} \leq f \leq \frac{-1}{2\Delta} \quad (6.16)
\]

where: \( \Delta \) is the sampling interval

\[ L = M/\Delta \]

\( M = \) truncation point of the lag window.

Since the acvf and spectrum are both even functions, we need only compute one side of each function. Eq. 6.16 then reduces to

\[
\bar{C}_{tt}(f) = 2\Delta(c_{tt}(0) + 2 \sum_{k=1}^{L-1} c_{tt}(k)w(k)\cos2\pi fk\Delta) \quad 0 \leq f \leq \frac{1}{2\Delta} \quad (6.17)
\]

A program which uses this algorithm is given in Appendix A.

Before taking data, we must determine how many data points are needed, and at what interval sampling should occur. We must first...
decide at what interval (δf) and up to what frequency (f_{max}) the spectrum needs to be known. From the sampling theorem, we get

\[
\delta f = \frac{1}{X}
\]  

(6.18)

and

\[
f_{\text{max}} = \frac{1}{2\Delta}
\]  

(6.19)

where: \(X\) = length of the data record

\(\Delta\) = sampling interval.

The number of data points \(N\) is given by

\[N = \frac{X}{\Delta}.
\]  

(6.20)

Using these equations we can determine the parameters needed for data collection.

Once we have an estimate of the spectrum, we may wish to compare it with some other estimate. Since we are dealing with a stochastic process, an interval estimate such as the confidence interval about the mean should be used. Jenkins and Watts\textsuperscript{18} have shown that the distribution for smoothed spectral estimators is best approximated by the chi-squared functions. They have shown the confidence interval to be given by

\[
\frac{\bar{\sigma}_{tt}(f)}{X_{\nu}(1-\alpha/2)}, \frac{\bar{\sigma}_{tt}(f)}{X_{\nu}(\alpha/2)}
\]

(6.21)

where: \(\nu = \frac{2C}{\int_{-\infty}^{\infty} w^2(u)du}
\)
In measurement of the spectrum, the measuring equipment will introduce some effect. Let us define $C'_{tt}(f)$ to be the measured sample spectrum, $C_{tt}(f)$ the true sample power spectrum, $T'(x)$ the measured data record, $T(x)$ the true data record, and $h(u)$ the impulse response to the measuring equipment. We then have

$$
C'_{tt}(f) = \frac{1}{X} \left| \int_{-X/2}^{X/2} T'(x) e^{-i2\pi fx} dx \right|^2
$$

$$
= \frac{1}{X} \left| \int_{-X/2}^{X/2} h(u)T(x-u) e^{-i2\pi fx} dx \right|^2
$$

$$
= |H(f)|^2 C_{tt}(f) \quad . \quad (6.2)
$$

We see that the measured sample spectrum will be the result of the multiplication of the square of the measuring system's modulation transfer function with the true sample spectrum. If the measuring equipment is an image scanning microdensitometer, the scanning aperture and pick up objective determine to what frequency spectral estimates can be made. This is shown in

$$
T_M(u,v) = T_O(u,v)T_A(u,v) \quad (6.23)
$$

where: $T_M$ = the microdensitometer transfer function

$T_O$ = the pick up objective transfer function

$$
= \frac{2}{\pi} \cos \frac{1}{\omega} - \frac{\omega}{2\omega_0} (1-\omega_0^2)^{1/2} \quad \text{for diffraction limited optics}
$$

$$
\omega = (u^2 + v^2)^{1/2}
$$
NA₀ = numerical aperture of the pick up objective

λ = wavelength

\[ T_A = \frac{\sin \pi W}{\pi W} \] is the aperture transfer function

W = the slit width

In photographic Wiener spectrum analysis, the microdensitometer we use to gather the data is capable of scanning with either a rectangular slit or a circular spot. Since we can safely assume the two-dimensional Wiener spectrum is given by

\[ W(u, v) = \lim_{X, Y \to \infty} < \frac{1}{2X} \frac{1}{2Y} \int_{-X}^{X} \int_{-Y}^{Y} \Delta T(x, y)e^{-i2\pi(ux + vy)} \, dx \, dy \, ^2 > \quad (6.24) \]

where: \( \Delta T \) indicates fluctuations of transmittance about some mean level

\(< > \) indicates ensemble average

is rotationally symmetric for photographic materials which have been uniformly exposed and processed, we may restrict our attention to a one-dimensional cross section of Eq. 6.24. Since our microdensitometer scans with a two-dimensional aperture, we obtain some form of the two-dimensional spectrum. We wish to get from this the one-dimensional cross section.

We know that the two-dimensional measured autocovariance function \( C'(\xi, \eta) \) is related to the true spectrum by
\[ C'(\xi, \eta) = \int_{-\infty}^{\infty} W(u,v) |H(u,v)|^2 e^{i2\pi(u\xi + v\eta)} \, du \, dv. \] (6.25)

For a microdensitometer scan in one direction we can set \( \eta = 0 \) and get

\[ C'(\xi) = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} W(u,v) |H(u,v)|^2 \, dv \right) e^{i2\pi \xi u} \, du. \] (6.26)

Since

\[ C'(\xi) = \int_{-\infty}^{\infty} W'(u) e^{i2\pi \xi u} \, du \] (6.27)

we get

\[ W'(u) = \int_{-\infty}^{\infty} W(u,v) |H(u,v)|^2 \, dv \] (6.28)

which is the relationship between the one and two-dimensional spectra.

If we were to scan with a long narrow slit of length \( L \) and width \( A \)

\[ H(u,v) = \text{sinc}(Au) \text{sinc}(Lv). \] (6.29)

Thus,

\[ W'(u) = \text{sinc}^2(Au) \int_{-\infty}^{\infty} W(u,v) \text{sinc}^2(Lv) \, dv \] (6.30)

\[ = \text{sinc}^2(Au) W(u,0) \int_{-\infty}^{\infty} \text{sinc}^2(Lv) \, dv \] (6.31)

\[ = \frac{\text{sinc}^2(Au)}{L} W(u,0) \] (6.32)

where the assumption is made in the approximation that \( W(u,v) \) is constant throughout the spatial frequency range where \( \text{sinc}^2(Lv) \) is appreciably non-zero. This is a reasonable assumption for a long slit. If we scan with a circular aperture of radius \( r \) in the microdensitometer, we get

\[ W'(u) = \int_{-\infty}^{\infty} W(u,v) \left( \frac{2J_1(2\pi r (u^2 + v^2)^{1/2})}{2\pi r (u^2 + v^2)^{1/2}} \right)^2 \, dv \] (6.33)
which is a more complicated relationship. Based on this, we will scan only with long narrow slits, and modify the aperture transfer function accordingly.

**Principles of Coherent Spectral Analysis**

**Introduction**

The coherent spectrum analyzer we use is shown in Fig. 5. The mathematical description of the complex amplitude of the wave as it propagates through the system has been rigorously derived elsewhere. The field intensity $I_f$ in the back focal plane of the transform lens is given by

$$I_f(u,v) = \frac{AI_i}{\lambda f} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(\alpha,\beta) H_D(\alpha,\beta) e^{-i2\pi(u\alpha + v\beta)} d\alpha d\beta$$

(6.34)

where:

- $I_i$ = input plane wave intensity
- $A$ = aperture area
- $\lambda$ = wavelength of illumination
- $f$ = focal length of the transform lens
- $R(\alpha,\beta)$ = autocovariance function of the transmittance function $t(x,y)$
- $H_D(\alpha,\beta)$ = optical transfer function for the above system when diffraction limited.

$$u = \frac{x_f}{\lambda f}$$

$$v = \frac{y_f}{\lambda f}$$

$$(x_f,y_f) = \text{focal plane coordinates}$$
We call this focal plane intensity distribution the optical power spectrum (OPS). Since we are assuming that the statistical properties of the transmittance fluctuations of a uniformly exposed and processed piece of film are stationary and isotropic, \( R(\alpha, \beta) \) depends only on \( r = (\alpha^2 + \beta^2)^{1/2} \). According to Goodman\(^2\), this yields an OTF such that

\[
H_D(\alpha, \beta) = H_D(r) = \begin{cases} 
\frac{\pi}{2} \left( \cos^{-1} \left( \frac{r}{2b} \right) - \left( \frac{r}{2b} \right) \left( 1 - \left( \frac{r}{2b} \right)^2 \right) \right), & r \leq 2b \\
0, & \text{otherwise}
\end{cases} \quad (6.35)
\]

where: \( b = \text{aperture radius} \).
We can rewrite Eq. 6.35 such that

\[ I_f = \frac{\Delta I}{\lambda 2 f^2} W(u, v) \ast 4A \left( \frac{1}{2\pi b (u^2 + v^2)^{1/2}} \right)^2 \]  

(6.36)

We can now see the first major difference between the incoherent and coherent spectrum. With the former, the true spectrum was multiplied by the square of the MTF, while the latter is convolved with the MTF.

If the aperture radius of the coherent analyzer is large, the MTF can be assumed to approximate a delta function, and have no appreciable effect. Equation 6.36 assumes diffraction limited optics are used in the analyzer. In our work, we will assume diffraction limited optics are in use. Also, we will use the largest aperture possible and assume that the resulting MTF introduces no significant effects.

With coherent analysis, the samples we are measuring are still realizations of a stochastic process, and the resulting spectra in need of some type of smoothing. Since the autocovariance function is not directly available to us, we will smooth by averaging the spectra from several samples. A minimum bias smoothing window for use with coherent spectral estimates has been developed by Stark and Dimitriadis. However, their technique requires a spot scanner rather than the stationary scanner our system uses. Our detector averages over a certain frequency band for each measurement it takes, which helps to reduce the variability. The exact nature of this type of smoothing has not been developed as have other smoothing techniques, so that we will have to rely on the sample population to provide us with confidence intervals.
Practical Techniques

The optical layout for the coherent processor used to measure the optical power spectrum is shown in Fig. 5. Data acquisition is accomplished by a Recognition Systems, Inc. (RSI) Electronic Diffraction Pattern Sampling Unit (ESPSU) along with certain controlling electronics and data recording devices. This particular unit, along with some of its associated software is fully discussed by Cheatham. The detector used by the EDPSU is a silicon wafer with 64 photoactive elements. The detector is divided into 32 semi-annular rings and 32 wedges. The rings serve to measure over a certain frequency band of the power spectrum. The wedges measure a certain percentage of the total pattern over all frequencies present on the detector. Using the above relationships and the known physical dimensions of the detector, the bandwidth covered by each of the rings can be computed. The data supplied by the EDPSU is the output voltage of an amplifier which receives an input signal from any desired address. According to Cheatham, this output voltage is directly proportional and linear with respect to the irradiance falling on the detector.

The mathematics describing the measurements are

\[
\text{Ring Measurement} = \int_0^{\theta_2} \int_{r_1}^{r_2} \text{PS}(\rho, \theta) d\rho d\theta \\
\text{Wedge Measurement} = \int_{\theta_1}^{\theta_2} \int_0^{r_{\text{max}}} \text{PS}(\rho, \theta) d\rho d\theta
\]

(6.37)  
(6.38)

These equations describe the actual quantities measured. The parameter \(r_1\) denotes a bounding radius of a ring, and \(\theta\) the bounding angle of a wedge.
In order to equate the predicted one-dimensional spectrum with the measured two-dimensional spectrum we use

\[
\text{Predicted Power Spectrum } = \text{PS}_p = \int_{r_1}^{r_2} 2\pi \rho S(\rho) d\rho
\]

(6.39)

The function \(S(\rho)\) is the spectrum predicted by the computer model.

Once we have the power spectrum curve from the analyzer, we normalize by the total irradiance detected in all the rings (with no transparency in place). Data from different analyzers can then be compared, as the data are no longer laser power dependent. This is readily seen from Eq. 6.34.

If the surface of photographic film were to be observed on a microscopic scale, it would be seen that the surface is very irregular. These irregularities result from a tanning effect during development and from uneven application and drying of the emulsion during manufacture. We treat these two effects as noise components. These components are presented as phase effects which can be detected by a coherent analyzer. In order to eliminate this source of noise, every transparency must be liquid gated with an index matching fluid. There is another source of phase noise, which is a random fluctuation of the internal refractive index of the emulsion. Altman\(^{25}\) has shown this to be negligible for high resolution emulsions, and we will assume it to be true for all films examined here.

The Relationship between Incoherent and Coherent Spectra

The difference between the analyzers used to measure the incoherent and coherent spectra is their domain of linearity. The
incoherent analyzer is linear in intensity, while the coherent analyzer is linear in complex amplitude. Let us consider an illustrative example given by Armstrong to show what this means. Assume we have an amplitude transmittance given by

\[ T_a = 1 + \cos(\omega_0 x) \]  

This yields an intensity transmittance of

\[ T_i = (1 + \cos(\omega_0 x))^2 = 1 + 2\cos(\omega_0 x) + \cos^2(\omega_0 x) \]

Fourier transforming these functions gives

\[ \mathcal{F}(T_a) = \delta(\omega) + \frac{1}{2} \delta(\omega + \omega_0) + \frac{1}{2} \delta(\omega - \omega_0) \]  

\[ \mathcal{F}(T_i) = \frac{3}{2} \delta(\omega) + \delta(\omega + \omega_0) + \delta(\omega - \omega_0) + \frac{1}{4} \delta(\omega + 2\omega_0) + \frac{1}{4} \delta(\omega - 2\omega_0) \]

Now we square these transformed quantities to get

\[ (\mathcal{F}(T_a))^2 = \delta(\omega) + \frac{1}{4} \delta(\omega + \omega_0) + \frac{1}{4} \delta(\omega - \omega_0) \]  

\[ (\mathcal{F}(T_i))^2 = \frac{9}{4} \delta(\omega) + \delta(\omega + \omega_0) + \delta(\omega - \omega_0) + \frac{1}{18} \delta(\omega + 2\omega_0) + \frac{1}{16} \delta(\omega - 2\omega_0) \]

We see that the resulting power spectra differ in both amplitude and frequency components. A case where the two spectra would be the same is when the image is made up of square pulses of unit amplitude. For
the emulsion model we are using, all developed grain will have a transmittance of zero. Future work may reveal that this is an improper assumption to make about photographic grains. Should this be the case, we see that it will not be possible to assume that the optical power spectrum gives us the same information about the intensity transmittance function of a transparency as the Wiener spectrum. If the two measured spectra are different after accounting for system effects, the above relationships will be the most likely cause of the difference.

Another difference between the two spectral analysis techniques has to do with the fact that the transmittance of the film samples is not a zero mean process. For the incoherent method, our computer program calculates the mean transmittance of each microdensitometer scan. This quantity is then subtracted from all of the transmittance values before calculating the spectrum. This procedure is called pre-whitening, and it removes low frequency trends that obscure the true spectrum. We have verified this by tests using pre-whitened and non-pre-whitened emulsion simulations. Pre-whitening cannot be done with coherent analysis. To see the effect that this has on the optical power spectrum, consider the following. Let us suppose we have a transmittance distribution given by \( t(x) \) and a pupil function given by \( a(x) \). Let us define the corresponding Fourier transform pairs as

\[
\mathcal{F}(t(x)) = T(\xi) \tag{6.47}
\]

and

\[
\mathcal{F}(a(x)) = A(\xi) \tag{6.48}
\]
We know that the intensity distribution measured in the coherent analyzer will be of the form

\[ I(\xi) = |T(\xi) * A(\xi)|^2 \quad (6.49) \]

where * denotes convolution. Let us define the amplitude transmittance function as being the sum of two components, a mean \( \bar{t} \) and a series of oscillations \( t_1(x) \) about zero. Thus we have

\[ t(x) = \bar{t} + t_1(x) \quad (6.50) \]

If we Fourier transform both sides of Eq. 6.50 and use the result of Eq. 6.49 we get

\[ I(\xi) = |\bar{t}A(\xi) + T_1(\xi) * A(\xi)|^2 \quad (6.51) \]

Since we will liquid gate our samples, we will assume \( t(x) \) to be a real function. If we further assume \( A(\xi) \) to be real, then Eq. 6.51 becomes

\[ I(\xi) = |\bar{t}|^2|A(\xi)|^2 + |T_1(\xi) * A(\xi)|^2 + 2|\bar{t}|A(\xi)T_1(\xi) * A(\xi) \quad (6.52) \]

We now see that the optical power spectrum is composed of three components. The first term is due to the clear aperture term and the non-zero mean of the process. It accounts for the strong central order or low frequency bias noted in OPS measurements. The second term contains the true spectrum smoothed by the transform of the aperture function. The third component is a cross term. Although it contains information about the true spectrum, it is quickly attenuated by the transform of
the clear aperture. Thus, the cross term also contributes to the large low frequency bias.

Results

Introduction

In this section we will compare the several sets of spectra which were obtained. These are Figures 6 through 41. Figures 6 through 11 compare liquid gated and non-liquid gated samples measured on the microdensitometer. A significant difference between spectra would indicate the presence of strong partial coherence in the microdensitometer. This would make us question the linearity of the instrument. No correction for the microdensitometer transfer function is made.

Figures 12 through 29 are Monte Carlo simulations compared to actual measurements. Figures 30 through 41 are microdensitometer derived Wiener spectra converted to optical power spectra for comparison to measured optical power spectra. The simulated spectra have been filtered by the microdensitometer transfer function. The Wiener spectra were inverse filtered to remove microdensitometer effects before conversion by Eq. 6.39.

In these following graphs we have plotted the 95% confidence intervals about the mean for each spectrum. The degree of overlap of these intervals tells us how much any two spectra appear to belong to the same population.
Liquid Gated and Non-Liquid Gated Wiener Spectra

In each of these curves (Fig. 6. throu 11) we see that the non-liquid gated spectrum has more power than the liquid gated spectrum. This is most likely caused by random fluctuations across the surface of the film samples. While liquid gating eliminates some of this phase noise, it does not make the microdensitometer linear in intensity or complex amplitude. We used matched numerical apertures for the illuminating and pick up objectives, since this is all that is currently available from the manufacturer. The operators of the microdensitometer are seeking to remedy this problem with an improved optical system. Our results indicate a need for this. In the following sections where microdensitometer data is used, we will present both liquid gated and non-liquid gated data.
Fig. 6. Tri-X shoulder liquid gated and non-liquid Wiener spectra.

Fig. 7. Tri-X mid-range liquid gated and non-liquid gated Wiener spectra.
Fig. 8. Tri-X toe liquid gated and non-liquid gated Wiener spectra.

Fig. 9. 3414 shoulder liquid gated and non-liquid gated Wiener spectra.
Fig. 10. 3414 mid-range liquid gated and non-liquid gated Wiener spectra.

Fig. 11. 3414 toe liquid gated and non-liquid gated Wiener spectra.
Measured Liquid Gated and Simulated Wiener Spectra

The Tri-X mid-range and toe curves are very close to the predicted curves, but there is no significant overlap. The shoulder sample shows about twice the discrepancy present in the other two samples. The 3414 curves display a more constant amount of difference than the Tri-X curves. The 3414 curves also fail to show significant overlap. There is an average of 1 unit of log power difference for the 3414 curves, 1.4 units of log power difference for the Tri-X shoulder, and 0.5 and 0.7 for the Tri-X mid-range and toe samples respectively. See Figures 12 through 17.

Measured Non-Liquid Gated and Simulated Wiener Spectra

As with the liquid gated sample, the Tri-X mid-range and toe give a better fit to the predicted spectrum than the shoulder. We see an average 2 unit of log power difference between the measured and predicted spectra for the shoulder sample. There is a 0.7 difference in the mid-range and complete agreement in the toe. The shoulder and mid-range 3414 samples have a 1.8 unit of log power difference between measurements and predictions, while the toe has an average of 0.3 difference. None of the 3414 curves has significant overlap. See Figures 18 through 23.
Fig. 12. Tri-X shoulder liquid gated Wiener spectrum and Monte Carlo simulation.

Fig. 13. Tri-X mid-range liquid gated Wiener spectrum and Monte Carlo simulation.

Please note the change in meaning between dashed and solid curves for this section.
Fig. 14. Tri-X toe liquid gated Wiener spectrum and Monte Carlo simulation.

Fig. 15. 3414 shoulder liquid gated Wiener spectrum and Monte Carlo simulation.
Fig. 16. 3414 mid-range liquid dated Wiener spectrum and Monte Carlo simulation.

Fig. 17. 3414 toe liquid gated Wiener spectrum and Monte Carlo simulation.
Fig. 18. Tri-X shoulder non-liquid gated Wiener spectrum and Monte Carlo simulation.

Fig. 19. Tri-X mid-range non-liquid gated Wiener spectrum and Monte Carlo simulation.
Fig. 20. Tri-X toe non-liquid gated Wiener spectrum and Monte Carlo simulation.

Fig. 21. 3414 shoulder non-liquid gated Wiener spectrum and Monte Carlo simulation.
Fig. 22. 3414 mid-range non-liquid gated Wiener spectrum and Monte Carlo simulation.

Fig. 23. 3414 toe non-liquid gated Wiener spectrum and Monte Carlo simulation.
Measured and Predicted Optical Power Spectra

At a first glance we see a sharp contrast between the Wiener spectra and the optical power spectra (Figs. 24 through 29). We see that the coherent spectra increase with spatial frequency instead of decreasing, as does the incoherent spectra. This difference is due to the integration over different bandwidths by the coherent analyzer detector. As spatial frequency increases, so does the bandwidth over which our detector integrates. Thus, each address of the detector receives progressively more total energy as we move out from the central order. Our simulations take this integration into account, as can be seen from the plots. In some of the measured optical power spectra it may be noticed that the interval estimate is very noisy at about 10 cy./mm. This is due to the small amount of total power measured in the addresses in this region. This is due to the fact that the bandwidth of integration is so narrow in that area. Of course, the central order shows a large amount of power as was predicted earlier.
Please note the change in meaning for the dashed and solid curves.
Fig. 26. Tri-X toe optical power spectrum and Monte Carlo simulation.

Fig. 27. 3414 shoulder optical power spectrum and Monte Carlo simulation.
Fig. 28. 3414 mid-range optical power spectrum and Monte Carlo simulation.

Fig. 29. 3414 toe optical power spectrum and Monte Carlo simulation.
Liquid Gated Wiener and Optical Power Spectra

In this series of plots (Figs. 30 through 35) we have converted Wiener spectra to optical power spectra by the same method used for the Monte Carlo simulations. The Tri-X shoulder and mid-range samples show good agreement. We see a difference of about 0.1 to 0.3 units of log power. The Tri-X toe samples show much greater disagreement between the two methods of measurement. In this sample we see a difference of 2 units of log power. For all three samples of the 3414 film we have very poor agreement between the two methods. It is interesting to note that when the Wiener spectra are converted to optical power spectra, the estimates become much smoother. This is because we are integrating and thus smoothing even further.
Fig. 30. Tri-X shoulder liquid gated Wiener spectrum and OPS.

Fig. 31. Tri-X mid-range liquid gated Wiener spectrum and OPS.
Fig. 32. Tri-X toe liquid gated Wiener spectrum and OPS.

Fig. 33. 3414 shoulder liquid gated Wiener spectrum and OPS.
Fig. 34. 3414 toe liquid gated Wiener spectrum and OPS.

Fig. 35. 3414 mid-range liquid gated Wiener spectrum and OPS.
Non-Liquid Gated Wiener and Optical Power Spectra

In this data set (Figs. 36 through 41) we have converted micro densitometer derived power spectra which were not liquid gated to optical power spectra. The results of this series are almost identical to those for the liquid gated case. While we get good agreement between the shoulder and mid-range samples of Tri-X, there is very poor agreement in the toe sample. All of the 3414 samples give poor agreement between the two methods of measurement.

Fig. 36. Tri-shoulder non-liquid gated Wiener spectrum and OPS.

Please note the change in meaning for dashed and solid curves for this section.
Fig. 37. Tri-X mid-range non-liquid gated Wiener spectrum and OPS.

Fig. 38. Tri-X toe non-liquid gated Wiener spectrum and OPS.
Fig. 39. 3414 shoulder non-liquid gated Wiener spectrum and OPS.

Fig. 40. 3414 mid-range non-liquid gated Wiener spectrum and OPS.
Fig. 41. 3414 toe non-liquid gated Wiener spectrum and OPS.
CHAPTER 7

DISCUSSION AND CONCLUSIONS

Consider the curves for the liquid gated and non-liquid gated microdensitometer samples in Chapter 6, Results (Liquid Gates and Non-Liquid Gated Wiener Spectra). We see that the non-liquid gated spectra always have more power than their liquid dated counter parts. The reason for the difference is that a significant amount of partial coherence is present in the microdensitometer. With the Tri-X, the greatest difference between spectra is in the toe samples. The most likely reason for this is that with fewer grains present, phase fluctuations are more predominant. Just the opposite situation is present in the 3414 samples. Here, the shoulder shows the widest difference and the toe the least difference. It is very interesting to note how much larger the difference between the 3414 shoulder spectra is than any other set of curves. We believe that this is accounted for by the pellloid backing of the 3414 film. The shoulder region is close to the maximum saturation possible of grains. When this occurs, transmittance fluctuations will decrease, which causes a noise power spectrum decrease. However, the pellloid backing still contributes to the noise power. This accounts for the large difference in spectra for the 3414 shoulder sample. This factor would also explain the trend of the two methods of measurement to come closer together as the density level increases in 3414. As
we go from high to low density, we get farther from the grain saturation level, and the noise power level becomes less dominated by the pellloid noise. The maximum noise level varies with different films. With some, it occurs at the mid-range or the shoulder. With others it can be below the mid-range. It depends upon at what density level grain saturation occurs.

We turn now to the predicted and measured Wiener spectra of the Results (Measured Liquid Gated and Simulated Wiener Spectra, and Measured Non-Liquid Gated and Simulated Wiener Spectra) section. There is a similar trend in both the measured and predicted spectra. In going from shoulder to mid-range to toe the power spectrum increases. Care must be taken in noting that the scale on the vertical axis of both Figs. 12 and 18 are different from the other plots. The greatest difference between measured and predicted spectra is in the shoulder for both the liquid and non-liquid gated methods. The closest agreement is in the toe, with good agreement also in the mid-range. It is most likely we overestimated the amount of sticking grains when taking the Tri-X shoulder data. A lot of sticking would mean a near saturation grain level, which in turn would lower the power spectrum. In the liquid gated toe the predicted spectrum is higher than the measured spectrum, the opposite of which is true for all other Tri-X samples, except the non-liquid gated toe, which agrees completely with the prediction. These differences may be due to grain sizes which are actually larger and packed closer together than we measured.
The 3414 shows on the average much more disagreement between measured and predicted spectra with the non-liquid gated than the liquid gated method. The presence of the pelloid backing is again causing more noise in the non-liquid gated results. As with the Tri-X, the predicted 3414 spectra show an increase with decreasing density, but not the measured spectra. For the non-liquid gate samples, the mid-range is a little higher in power but nearly the same general level as both shoulder and toe. The only close agreement is in the toe. The non-liquid gated sample increases in power from the shoulder to the mid-range, and then drops some at the toe. The predicted curves increase steadily with decreasing density. The difference is attributable to improper estimation of grain saturation. The predicted spectra are less than the measured shoulder and mid-range for the non-liquid gated, but higher than the measured toe. In the liquid gated samples we see the predicted spectrum is less than the measured spectrum for only the mid-range. The pelloid backing makes predictions too difficult to make in the non-liquid gated case. The liquid gated predicted shoulder does not show the same drop off as the measured spectrum. This could be due to improper estimation of grain/gap size distributions. The level of agreement is nearly the same for both the Tri-X and 3414 liquid gated samples.

We now examine the predicted and measured OPS of Results (Measured and Predicted Optical Power Spectra). One observation is very apparent. Except for the Tri-X shoulder, all of the predicted curves are higher than the measured curves. This very likely indicates that our assumption of grain and gap transmittances being 0 and 1 respectively,
is incorrect. Gap transmittances are probably near 1, but grain transmittances are probably greater than 0. This means that the predicted point fluctuations are less in magnitude than predicted. This in turn could cause us to observe a lower noise power than predicted. There is very close agreement in the Tri-X shoulder and mid-range, but not in the toe. The poor agreement in the Tri-X toe and all of the 3414 is probably due to the above mentioned grain transmittances.

In the last two sets of curves we can compare the non-liquid and liquid gated spectra with measured OPS. In both instances, the Tri-X shoulder and mid-range give very close agreement. This would indicate a low level of phase effects in the film at those densities, and grain and gap transmittances of nearly 0 and 1. None of the 3414 curves gives as good a match as the shoulder and mid-range Tri-X. Of the 3414 curves only the liquid gated toe shows the OPS with more power than the converted Wiener spectrum curves. On the whole, it does not appear that either the liquid gated or non-liquid gated gives better agreement with the coherent data. This might be indicative of a measurement error or a conversion error. Scattering and diffraction effects are also likely sources of error.

We see from the results obtained here that we were unable to accurately predict the level of the power spectrum. We were, however, able to get good agreement between the shapes of measured and predicted spectra. In many cases we were also able to predict trends in spectra. This indicates that while the model may show promise as a predictor of power spectra, much work needs to be done.
One possible improvement of the model has to do with scattering. As of now, no emulsion model treats this topic. All models assume that the light incident on the emulsion is collimated and will not be scattered by the silver grains. A possible method for dealing with this problem would be the physical simulation of radiative transfer through an emulsion by computer. Such a technique has been used by DePalma and Gasper\textsuperscript{26} to predict the MTF of photographic films. Their paper outlines techniques for determining how supposed photons would be scattered in an emulsion and their probability of absorption when encountering a grain.

A second improvement of the model has to do with individual grain transmittance. As seen from the curves, there is a difference between incoherent and coherent power spectra. This may have to do with the transmittance of the grains themselves. As mentioned previously, we assumed unit amplitude pulse transmittances for grains. This is the only condition where we can expect the incoherent and coherent power spectra to be equal. Our results indicate that for medium to high densities of Tri-X the unit amplitude transmittance is all right. However, at low density Tri-X and for any density of 3414 the approximation is not valid. If the grain transmission problem is solved, there is still the possibility that there will still be some difference between predicted and measured optical power spectra. This difference may arise from diffraction effects as the coherent wave propagates through the emulsion. The Trabka granularity propagation relationships
do not account for this type of phenomenon. If this problem does arise, a much more complicated analysis which takes diffraction into account will be needed.

A third area of further investigation should be the methods used in emulsion parameter measurement. Obtaining the data involved tedious and time consuming efforts. It is possible that the number of experimental observations we have for constructing our grain and gap size distribution should be larger. At the low density end of the curve, there are so few grains in each electron micrograph, that many are needed to get the same number of observations available in just one high density micrograph. Time restricted us to the number of cross sections we were able to prepare. Generally, our low density observations numbered around 100, and our high density observations around 500. Also, in the future it may be better to get the size information by digitizing the electron micrographs on a microdensitometer instead of the Tektronix Graphics Tablet. This would be a much easier and more reliable operation.

As a final observation, we reiterate that this is only a one dimensional model. In order to be more general, it will have to be extended to two dimensions. Also, more study is needed to better characterize the statistics of each individual layer. Due to the fact that the developer must diffuse through several layers to reach the deeper grains, we would expect these deep grains to be fewer and smaller. Therefore, our assumption that all layers are independent and generated by the same random process is very likely to be in error.
APPENDIX A

COMPUTER PROGRAMS

DATA

This is the Tektronix program for taking grain and gap data. It is written in BASIC. The program allows for the inclusion of a header to aid in sample identification. A sample is taped to the graphics tablet, and the cursor moved from point to point. At each gap or grain ending, the appropriate button is pushed. The program then calculates and records the distance traversed from the last cursor position. When changing from one layer to another, pushing button four will inform the program of this, and it will take appropriate steps to avoid accidentally recording an erroneous data point. It should be noted that the character in quotes in lines 380 is a control G.

SEND

Once data has been recorded via program DATA, it must be sent to the Eclipse for further analysis. This is done by program SEND. In order to do this, the Tektronix must be attached to the Eclipse. A receiving program is created and put in the append mode. Control of the Tektronix keyboard is then returned to the Tektronix internal computer. This will have no effect upon the Eclipse. Program SEND is then executed. It will request the file number it is to send. It will find the file, convert the binary data to ASCII format, and ship it out the I/O port to
which the Eclipse is attached. At the end of data transfer, you return
control of the keyboard to the Eclipse and close the file. Control of
the keyboard is transferred to the Eclipse by typing in CALL "TERMIN,"
and to the internal Tektronix computer by pushing the return to basic
key on the keyboard. It should be noted that in line 210, the character
in quotes is a control J.

TEKREAD

When data files are shipped to the Eclipse from the Tektronix
4051, the grain and gap information is intermixed. This program will
separate the grain and gap size information and store it in two separate
files. The program also converts the size information as recorded by the
Tektronix to millimeters, and takes the microscope magnification into
account. When the data is originally taken on the Tektronix, it is taken
in blocks of 100. Each block constitutes one record in this program.
After you begin execution of this program, it will first ask for the
name of the file to process, and then ask how many records are in the
file. After this, the program will then ask you in which file you wish
to place the grain data, and in which the gap data. It will then place
in that file the name of the file from which the data was taken, the
number of observations, the mean, the standard deviation, and all of
the observations.

HISTO

Once TEKREAD has been executed, we need to convert that data to
a normalized frequency histogram for input to the simulation program.
When executed, the program will first ask for the name of the file to examine. It will then report the mean and standard deviation of the file data, and ask how many cells it should use and what cell interval is desired. Once the program has this information, it will calculate and display the frequencies of both the unnormalized and the normalized histogram. If it is decided that improper values were given for either or both the cell number and interval, execution can be terminated by typing in control C control B. If it is decided that the data are acceptable, the program will ask what output file is to be used. The program will then record the input file name, the number of observations, the number of cells, the individual cell intervals, and the normalized cell frequencies for each cell.

**HISTOP**

This program is executed just like HISTO, however, rather than specifying an output file, a plotting device is specified. When the program asks for the output value, 1 will cause the normalized frequency histogram to be plotted on the terminal (only when using a Tektronix), a value of 2 will cause the output to be plotted on the Verstaek. No data is output to a file.

**MONTE**

This program provides the transmittance values that would be obtained from a simulated emulsion, when given as input any normalized grain and gap frequency histograms. When executed, the program will first ask for the name of the input grain file and then the gap file.
SIMP

This program takes data which has been generated by the simulation routine MONTE and computes its Wiener spectrum. The lag and smoothing are the same as in POWER. In addition, certain microdensitometer parameters are used to modify the Wiener spectrum in the same way as the measured spectrum. During execution, the program will request the name of the input data file, how many data records the file contains, the sampling interval, how many layers the simulated emulsion is to have, the numerical aperture of the microdensitometer pick-up objective, and the microdensitometer slit width. A slit length of 100 microns is assumed. While executing, the program will print on the console what record number has just been processed. The log mean power spectra are then stored in any specified file.

COPOW

In predicting the optical power spectrum, we need estimates at certain particular frequencies. Furthermore, we need to integrate over certain bandwidths in order to get the same effect that the coherent analyzer detector introduces. This program performs such a calculation. The information needed during execution is the same as that in POWER. The log mean spectrum values are output to any specified file.

SCO

This program performs the same operation as COPOW, only it uses microdensitometer data instead of simulated data.
Once spectral estimates have been obtained from SIMP and POWER, we can plot them on the Versatek with this program. It will plot the first file with dashed lines and the second with solid lines. We will actually get the 95% confidence intervals about the mean plotted. During execution, the only inputs needed are the names of the files to be plotted.

This is a plot program similar to SMP, only it is for coherent spectra, and data which have been converted to coherent spectra.

These are data storage files. They are used by the DEC-10 data analysis programs for input, output, and temporary storage. File 20 is used to store raw data from the EDPSU unit only.

This program reads data from file 20 and computes the normalized and unnormalized power spectrum. The power spectrum values are then displayed on the DEC terminal.

Same as OPSN1.F4, except the power spectrum values are not displayed on the DEC terminal. This is convenient to use when large amounts of data have to be processed. Once the data has been processed
by the program, it can be sent to the line printer, card punch, or to a permanent storage file if one is created for that purpose.

OPSN2A.F4

This is the same as OPSN2.F4, except the normalized power spectrum values are expressed in $\log_{10}$.

OPSN2B.F4

This is the same as OPSN2.F4, except both the normalized and the unnormalized power spectrum values are expressed $\log_{10}$.

SAGAN1.CDC

Causes a card deck to be punched of any of the data storage files.

SAGAN1.S12

Causes any specified data storage file to be printed on the line printer.

SAGAN2.CDC

Causes any specified data storage file to be punched and printed.

CLEAN.DAT

Causes all data storage files to be emptied and condensed so as to minimize storage space. Since a charge is assessed to the account for the amount of storage space occupied, this program should be executed when data in the storage files is not longer needed.
PLOT.F4

Produced a plot on the DEC terminal of the unnormalized power spectrum values stored in file 23 after execution of any of the OPSN programs.

PLOT2.F4

When PLOT.F4 is executed, only one set of data can be handled. PLOT2.F4 is to be used when several sets of data are in file 23. There is a disadvantage to using PLOT.F4. Generally, two plots of each data set in file 23 are necessary. This is because of the wide dynamic range of the power spectrum values. Thus, two plots, each with a different maximum value along the power spectrum axis are made. This is not convenient with PLOT2.F4, as you must cycle back to the beginning of file 23 to repeat a particular plot.

SWITCH.INI

When .LOGIN is executed to enter on to the DEC system, SWITCH.INI is automatically executed. This program defines for the DEC computer the necessary parameters to enable use of the Texas Instruments Silent 700 terminal. Without this program, it would be necessary for the operator to manually inform the DEC of these parameters every time the operator logged into the DEC system. These parameters include such items as the number of characters in a line, whether or not a tape input is available, the length of time it takes for a carriage return, if messages are to be accepted from other terminals, etc. This program is not to be executed by the operator of the Silent 700 terminal, since it is automatically executed by the computer.
The method by which the above programs are executed is covered in Cheatham's dissertation$^{20}$. Below is an example of how one would send data elsewhere to be output on cards or paper.

```
.TOCDC SAGAN2.CDC, FOR24.DAT
```

This would cause file 24 to be sent to the CDC 6400 computer for printing and punching. The same format is used for any SAGAN program with any data file.

Multiple Spectra Recodings

There is a mistake in the operator's manual in Cheatham's dissertation$^{20}$ with regards to recording more than one spectrum file. The error in step is in step 26. Before returning to step 8 and typing the next data file header as directed, a line feed should be typed. It is then permissible to return to step 8 and proceed with the new data header typed. If the last piece of data has been recorded, step 26 is executed as normal.
PROGRAM SEND

100 INIT
110 PRINT @32,26:2
120 PRINT "THIS PROGRAM READS A DATA FILE WHICH WAS GENERATED BY THE"
130 PRINT "GRAIN-GAP LENGTH DATA PROGRAM AND SENDS IT TO THE ECLIPSE."
140 PRINT "MAKE SURE THAT YOU ARE PROPERLY ATTACHED TO THE ECLIPSE BEFORE"
150 PRINT "PROCEEDING FURTHER."
160 PRINT ""
170 PRINT "WHICH FILE DO YOU WISH TO TRANSMIT?"
180 INPUT L
190 FIND L
200 READ @33:W$  
210 PRINT @40:W$,"\"J"
220 PRINT W$
230 B$=""  
240 REM
250 ON EOF (0) THEN 350
260 READ @33:X  
270 X$=STR(X)
280 B$=B$&X$
290 I=LEN(B$)
300 IF I>50 THEN 320
310 GO TO 240
320 PRINT @40:B$  
330 PRINT B$
340 GO TO 230  
350 PRINT @40:B$  
360 PRINT B$
370 PRINT "ALL DONE, NOW CALL TERMIN I"  
380 END
THE NAME OF THIS PROGRAM IS TEKREAD
THIS PROGRAM READS THE GRAIN/GAP DATA FILES WHICH
ARE ALREADY ON DISK FROM THE TEKTRONIX 4051. IT
PREPARES THE DATA FOR INPUT TO THE HISTOGRAM
PROGRAM BY SEPARATING THE TFK DATA INTO DIFFERENT
FILES FOR GAP AND GRAIN. THIS PROGRAM ALSO
COMPUTES THE MEAN AND STANDARD DEVIATION OF THE DATA
AND STORES IT WITH THE APPROPRIATE DATA IN THE OUTPUT FILE
THIS PROGRAM ALSO ACCOUNTS FOR THE ELECTRON MICROSCOPE
MAGNIFICATION, WHICH = 6842.77X.

DIMENSION DATA(20),DATA1(20),RGP(1000),RGN(1000)
DIMENSION X(1000)

TYPE "WHICH FILE DO YOU WISH TO PROCESS ?"
READ (1,1)DATAM(1)

1 FORMAT (55)
CALL OPEN(1,DATAM(1),2,IER)
IF (IER.NE.1) TYPE "DATA FILE OPEN ERROR."
TYPE "HOW MANY RECORDS ARE IN THE ENTIRE FILE ?"
READ FREE(11,1)
READ (1,3)DATAK(1)

3 FORMAT (55)
READ FREE(1,END=124)

124 X1GAP=5.5
X1GRN=5.5
XGAP=5.5
XGRN=5.5
GAP=5
GRN=5

DO 4 I=1,11
DO 98 J=(I*200)-199,(I*200)-180
IF (X(J+180).EQ.8.0) GO TO 6
IF (X(J+180).EQ.4.0) GO TO 7
IF (X(J+180).EQ.1.0) GO TO 7
6 RG P (GAP+1)=X(J)*1.127*(1/6842.77)
GAP=GAP+1
XGAP=XGAP+RGP(GAP)
X1GAP=X1GAP+(RGP(GAP)**2)
GO TO 98
7 RG R (GRN+1)=X(J)*1.127*(1/6842.77)
GRN=GRN+1
XGRN=XGRN+RGR(GRN)
X1GRN=X1GRN+(RGR(GRN)**2)

98 CONTINUE

4 CONTINUE

SGAP=((GAP*X1GAP-(XGAP**2))/(GAP*(GAP-1)))*5.5
XGAP=XGAP/GAP
SGRN=((GRN*X1GRN-(XGRN**2))/(GRN*(GRN-1)))*5.5
XGRN=XGRN/GRN

TYPE "IN WHICH FILE DO YOU WISH THE GAP DATA PLACED ?"
READ(11,3)DATAK(10)
CALL OPEN(2,DATAK(10),2,IER)
IF (IER.NE.1) TYPE "GAP OUTPUT OPEN ERROR."
WRITE (2,3)DATAK(1)
WRITE FREE(2,GAP,XGAP,SGAP)
WRITE FREE(2,GAP,1,1,GAP)

TYPE "IN WHICH FILE DO YOU WISH THE GRAIN DATA PLACED ?"
READ(11,3)DATAK(10)
CALL OPEN(3,DATAK(10),2,IER)
IF (IER.NE.1) TYPE "GRAIN OUTPUT OPEN ERROR."

WRITE(3,3)DATAK(1)
WRITE FREE(3,GRN,XGRN,SGRN)
WRITE FREE(3,GRN,1,1,GRN)
TYPE "ALL DONE"
END
THE NAME OF THIS PROGRAM IS HISTO
THIS PROGRAM TAKES FILES WHICH HAVE BEEN PROCESSED BY
THE PROGRAM CALLED TEKREAD, GENERATES A FREQUENCY
HISTOGRAM, REPORTS THE MEAN AND ST. DEV., AND STORES
THE HISTOGRAM IN ANY SPECIFIED FILE.
DIMENSION X(1000),CELL(50),DATA(20),OUTPUT(10),CO(50)

TYPE "WHICH FILE DO YOU WISH TO EXAMINE?"
READ(11,11)DATA(1)
1 FORMAT(S15)
CALL OPEN(6,DATA(1),2,IER)
IF(IER.NE.1)TYPE "OPEN INPUT ERROR."
READ(6,1)DATA(10)
WRITE(10,99)DATA(10)
99 FORMAT('"THIS DATA FILE IS: ",S15)
READ FREE(7)NUM,XBAR,XSTD
WRITE FREE(6,END=180)*X
CONTINUE
TYPE "THE MEAN IS: " ,XBAR
TYPE "THE STD IS: ",XSTD
TYPE "HOW MANY HISTOGRAM CELLS ARE REQUIRED?"
READ(11,3)I1
3 FORMAT(I2)
DO 8 I=1,50
CELL(I)=0.5
8 CONTINUE
TYPE "WHAT CELL INTERVAL SHOULD BE USED?"
READ FREE(11)CINT
DO 10 J=1,I1
C0(J)=I-CINT
10 CONTINUE
DO 4 I=1,NUM
DO 5 J=1,I1-1
IF(X(I).GT.(CINT*I1)-CINT) GO TO 6
IF(X(I).LE.(J*CINT)) GO TO 7
5 CONTINUE
6 CELL(I)=(CELL(I))+1
GO TO 4
7 CELL(J)=(CELL(J))+1
4 CONTINUE
TYPE "CELL VALUES ARE:"
WRITE FREE(18)CELL(I)
37 CONTINUE
TYPE "NORMALIZED CELL VALUES ARE:"
DO 38 I=1,I1
CELL(I)=CELL(I)/NUM
WRITE FREE(20)CELL(I)
38 CONTINUE
TYPE "WHAT FILE DO YOU WISH TO USE FOR OUTPUT?"
READ(11,2)OUTPUT(1)
2 FORMAT(S15)
CALL OPEN(1,OUTPUT(1),2,IER)
IF(IER.NE.1)TYPE "OPEN OUTPUT ERROR."
WRITE(1,15)DATA(1)
15 FORMAT('"THIS DATA IS FOR FILE: ",S15)
WRITE(1,14)NUM
WRITE(1,39)
WRITE(1,40)I1
WRITE(1,9)
39 FORMAT('"THE NUMBER OF DATA OBSERVATIONS IS: ",I3)
WRITE(1,39)
WRITE(1,40)I1
WRITE(1,9)
40 FORMAT('"THE NUMBER OF CELL INTERVALS IS: ")
9 FORMAT('"THE CELL INTERVALS ARE: ")
WRITE(1,11)(CO(I),I=1,I1)
11 FORMAT(10E12.4)
WRITE(1,12)
12 FORMAT('"THE NORMALIZED CELL FREQUENCIES ARE: ")
WRITE(1,11)(CEL(I),I=1,I1)
WRITE(1,12)
TYPE "ALL DONE"
END
THE NAME OF THIS PROGRAM IS HISTOP

THIS PROGRAM TAKES FILES WHICH HAVE BEEN PROCESSED BY
THE PROGRAM CALLED TEKREAD, GENERATES A FREQUENCY
HISTOGRAM, REPORTS THE MEAN AND ST. DEV., AND PLOTS
THE HISTOGRAM.

DIMENSION X(1000),CELL(50),DATA(28)
INCLUDE"FPLTRM"

DATA X_WIDTH/5.5/,XI/1.,XJ/1.,BAUDRT/1200/
DATA LM_WIDTH/3./,VERCTL/1001./,LABELX(1)="SIZE"
DATA LABELY(1)="FREQ",LTSZ/.1/

TYPE"WHICH FILE DO YOU WISH TO EXAMINE ?"
READ(11,1)DATA(1)
1 FORMAT(5S)

CALL OPEN(6,DATA(1),2,IER)
IF(IER.NE.1)TYPE"OPEN INPUT ERROR."
READ(6,1)DATA(10)
WRITE(10,99)DATA(10)
99 FORMAT(1X,"THIS DATA FILE IS: ",S15)
READ FREE(6)NUM,XBAR,XSTD
READ FREE(6,END=180)X

180 CONTINUE
TYPE"THE MEAN IS: ",XBAR
TYPE"THE STD IS: ",XSTD
TYPE"HOW MANY HISTOGRAM CELLS ARE REQUIRED ?"
READ(11,1)I1
3 FORMAT(12)
DO 8 I=1,5F
CELL(I)=.0
8 CONTINUE

TYPE"WHAT CELL INTERVAL SHOULD BE USED ?"
READ FREE(11)CINT
DO 4 I=1,NUM
DO 5 J=1,I1-1
IF(X(I).GT.(CINT*I1)-CINT) GO TO 6
IF(X(I).LE.(J*CINT))GO TO 7
5 CONTINUE
6 CELL(I1)=CELL(I1)+1
DO 4
7 CONTINUE
CELL(J)=(CELL(J)+1)

TYPE"CELL VALUES ARE:

DO 37 I=1,I1
WRITE FREE(18)CELL(I)
37 CONTINUE

TYPE"NORMALIZED CELL VALUES ARE:

DO 38 I=1,I1
CELL(I)=CELL(I)/NUM
WRITE FREE(18)CELL(I)
38 CONTINUE

XMAX=I1*CINT
XMIN=CINT
ACCEPT"YMAX = ",YMAX
YMIN=.0
ICODE=2
EX=54
KY=49

IF(YMAX.GT.XMAX)CALL TEKREAD
IF(YMIN.LT.XMIN)CALL TEKREAD

CALL PLOTXY("SIZE","FREQ")
DO 9 I=1,I1
XP=I*CINT
9 CONTINUE

CONTINUE
IF(OUTPUT.EQ.1)CALL TSEND
IF(OUTPUT.EQ.2)CALL PLETEND
CALL PLETEND
END
THE NAME OF THIS PROGRAM IS MONT

This program takes as input any specified grain and gap
files. From them it simulates an emulsion and outputs the
resulting transmittances. These can then be called by any
of the spectral analysis programs.

DIMENSION CF1(50), CI1(50), CF2(50), CI2(50), DATA(4000), D(15)

Type: 'What is the input grain file name?'
READ(11,1) D(1)

FORMAT (S10)
CALL OPEN(1,D(1),2,IER)
IF (IER.NE.1) TYPE 'OPEN INPUT GRAIN FILE ERROR.'
READ(1,1) D(1)
READ(1,1) D(1)
READ D FREE (1) ICEL1
READ(1,1) D(1)
READ D FREE (1)(CI(1),I=1,ICE1L1)
READ(1,1) D(1)
READ D FREE (1)(CI(1),I=1,ICE1L1)
DO 3 I=2,ICE1L1

CI1(I)=CI1(I)+CI1(I-1)
CONTINUE

Type: 'What is the input gap file?'
READ(11,1) D(6)
CALL OPEN(2,D(6),2,IER)
IF (IER.NE.1) TYPE 'OPEN INPUT GAP FILE ERROR.'
READ(11,1) D(11)

CALL OPEN(3,D(11),2,IER)
IF (IER.NE.1) TYPE 'OPEN OUTPUT FILE ERROR.'

DO 4 I=2,ICE1L2

CF2(I)=CF2(I)+CF2(I-1)
CONTINUE

DO 17 I=1,5
DO 18 I=1,4000
DATA(I)=0.5
CONTINUE

CALL RNDF(X)
SUM=1.5
DO 5 I=1,ICE1L1

IF(X.GT.CI1(ICE1L1)) X=CI1(ICE1L1)
IF(X.GT.CI1(I))) GO TO 5
R1=CI1(I)/.002
GO TO 6
CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

SUM=SUM+R1
DO 11 J=1,J1
DATA(J)=.5
CONTINUE

10 CALL RNDF(X)
DO 9 I=1,ICE1L2

IF(X.GT.CI2(ICE1L2)) X=CI2(ICE1L2)
IF(X.GT.CI2(I))) GO TO 9
R1=CI2(I)/.002
GO TO 9
CONTINUE

SUM=SUM+R1
DO 12 J=1,J1
DATA(J)=.5
CONTINUE
11
10
IF(J1.GT.4000)GO TO 16
CALL RNDF(X)
DO 12 I1=1,ICELL1
IF(X.GT.CFI(ICELL1))X=CFI(ICELL1)
IF(X.GT.CFI(I1))GO TO 12
R1=CFI(I1)/.882
GO TO 13
12 CONTINUE
13 SUM=SUM+R1
IF(SUM.GT.J1+1)GO TO 14
J=J1+1
J1=INT(SUM)
IF(J1.GE.4000)J1=4000
DO 15 I1=J,J1
DATA(I1)=0.0
CONTINUE
15 CONTINUE
IF(J1.GE.4000)GO TO 16
GO TO 14
16 WRITE(3,40)(DATA(I1),I1=1,4000)
40 FORMAT(17F4.1)
TYPE"PASS NUMBER",I,"HAS BEEN MADE."
CON TINUE
TYPE"ALL DONE."
END
THE NAME OF THIS PROGRAM IS POWER.

THIS PROGRAM COMPUTES THE POWER SPECTRUM OF
MICRODENSITOMETER DATA. IT PARZEN SMOOTHES WITH A LAG
WINDOW OF 60.

DIMENSION INTE(4096)
DIMENSION X(4000),PSX(2001),COV(60),DATAA(20),OUTPUT(20)
DIMENSION AVG(50)
DO 30 I=1,50
AVG(I)=0.0
30 CONTINUE

ACCEPT "THE NUMBER OF FILES IS: ",J10
ACCEPT "THE SAMPLING INTERVAL IN MJ IS: ",DELTA
RINT=1/(2*DELTA*50)
DO 40 J5=1,J10
DO 50 I=1,4000
X(I)=0.0
INTE(I)=0
50 CONTINUE
DO 41 I=1,2001
PSX(I)=0.0
41 CONTINUE
DO 42 I=1,60
COV(I)=0.0
42 CONTINUE
DO 43 I=1,20
DATAA(I)=0.0
43 CONTINUE
OUTP(T)=0.0
44 CONTINUE

TYPE "WHICH FILE IS NUMBER",J5," ?"
READ(11,3)DATAA(1)
FORMAT(S20)
CALL OPEN(1,DATAA(1),1,IER)
IF(IER.NE.1)TYPE"OPEN ERROR 1"
READ BINARY(1)INTE
CLOSE(1)
L8=60
XAV=0.0
DO 21 I=1,4000
XAV=XAV+INTE(I)
21 CONTINUE
XAV=XAV/4000.0
DO 22 I=1,4000
X(I)=(INTE(I)-XAV)/500.0
22 CONTINUE
DO 24 I=1,L8
COV(I)=0.0
J=4000-(I-1)
DO 25 K=1,J
COV(I)=COV(I)+(X(K)*X(K+I-1))/4000
25 CONTINUE
24 CONTINUE
R=0.0
DO 41 I=1,L8
R=R+1.0
IF(I.GT.L8/2) GO TO 36
COV(I)=COV(I)*(1-R/L8)**2+6*(R/L8)**3
GO TO 41
36 COV(I)=COV(I)*(2*(1-R/L8)**3)
41 CONTINUE
L8=L8-1
DO 27 I=1,50
PSX(I)=0.0
27 CONTINUE
DO 28 K=1,110
PSX(I)=PSX(I)+COV(K+1)*COS(2*3.14159*I*RINT*K*DELTA)
28 CONTINUE
40 PSX(I)=PSX(I)+COS(COV(I)+2*PSX(I))
27 CONTINUE
DO 23 I=1,50
AVG(I)=AVG(I)+PSX(I)/J10
23 CONTINUE
DO 45 I=1,50
AVG(I)=ALOG10(AVG(I))
45 CONTINUE
TYPE"OUTPUT DATA FILE WILL BE?" READ(11,3)OUTPUT(1)
CALL OPEN(5,OUTPUT(I),2,IER)
IF (IER.NE.1) TYPE"OPEN ERROR IN OUTPUT FILE"
WRITE(5,46)
46 FORMAT('THE LOG MEAN POWER SPECTRUM VALUES ARE:'),I=1,50
6 FORMAT(10E12.4)
TYPE"" TYPE"
TYPE"ALL DONE"
END

R=ALOG10(X(I)+STD(I))
CALL PLTLIN(ICODE,XINC(I),R)
9 CONTINUE
ICODE=2
DO 9 I=1,32
IF(X(I).LE.STD(I)) GO TO 9
R=ALOG10(X(I)-STD(I))
CALL PLTLIN(ICODE,XINC(I),R)
ICODE=0
CONTINUE
IF(OUTPUT.EQ.1)CALL TSEND
IF(OUTPUT.EQ.2)CALL PLTEND
CALL PLTEQJ
END
THE NAME OF THIS PROGRAM IS SVSC.

THIS PROGRAM PLOTS ONE OPTICAL POWER SPECTRUM FILE
AND ONE INCOHERENT SPECTRUM FILE WHICH HAS BEEN
CONVERTED TO OPS DATA. THE OPS FILE MUST CONTAIN THE
MEAN VALUES ALONG WITH THE CONFIDENCE INTERVAL DATA.

INCLUDE "FILENAME"

DIMENSION X(32),DATA(10),DATA1(I),X(5),XINC(32),STD(32)
DATA XWIDTH/5.5/,WIDTH/5.5/,KJ/I/,K3/I/,BAUDRT/1200/
DATA LWIDTH/3/,VERTC1/1000/,LABELX(1)/"CY/MM,<0>/"
DATA LABELY(1)/"LOG POWER",LABELY(2)/"R,<0>/"
DATA (XINC(I),I=1,32)/0.1,1.2,2.1,2,9.3,6,4,5,5,6,2,7,2,
$ 8,3,9,5,10,8,12,3,13,9,15,0,17,9,20,3,23,0,26,1,29,5,33,4,
$ 37,8,42,7,48,2,54,3,61,1,68,6,76,9,96,8,96,1,107,2,119,3/

TYPE "THE INCOHERENT FILE TO PLOT IS ?"
READ(11,1)DATA(I)
1 FORMAT(S10)
CALL OPEN(5,DATA,2,IER)
IF(IER.NE.1)TYPE "OPEN INPUT ERROR"
TYPE "THE COHERENT FILE TO PLOT IS ?"
READ(11,1)DATA(6)
CALL OPEN(4,DATA(6),2,IER)
IF(IER.NE.1)TYPE "SECOND FILE OPEN ERROR."
READ(5,2)DATA(1)
2 FORMAT(S10)
READ FREE(5)X
TYPE "PS READ"
XMAX=1200.0
XMIN=-5.0
TYPE "X(1) = ",X(1)
TYPE "X(32) = ",X(32)
YMAX=1.0
YMIN=-6.0
KX=15
KY=39
ICODE=2
OUTPUT=2
LTS2=-1
A4=255.

CALL PLTAXS
CALL NODE(-15,A1,A2,A3)
ICODE=2
CALL NODE(15,A4,A2,A3)
DO 6 I=1,32
R=X(I)+0.0012
CALL PLTILM(ICODE,XINC(I),R)
ICODE=0
6 CONTINUE
ICODE=2
DO 7 I=1,32
R=X(I)-0.0072
CALL PLTILM(ICODE,XINC(I),R)
ICODE=0
7 CONTINUE
CALL NODE(15,A1,A2,A3)
ICODE=2
READ(4,2)DATA(6)
READ FREE(4)X
READ(4,2)DATA(6)
READ FREE(4)STD
TYPE "PS2 READ"
DO 8 I=1,32
IF(X(I).LE.STD(I))GO TO 8
CALL PLTILM(ICODE,XINC,R)
ICODE=0
8 CONTINUE
ICODE=2
DO 9 I=1,50
XINC=(I-1)*5.0
R=X(I)-0.0072
CALL PLTILM(ICODE,XINC,R)
ICODE=0
9 CONTINUE
IF(OUTPUT.EQ.1)CALL TSEND
IF(OUTPUT.EQ.2)CALL PTSEND
CALL PLTEOS
END
THE NAME OF THIS PROGRAM IS SPM

THIS PROGRAM PLOTS ANY TWO INCOHERENT POWER SPECTRUM CURVES. THE 95% CONFIDENCE INTERVALS ARE PLOTTED

ACCORDING TO PARZEN SMOOTHING. SPECIFYING AN OUTPUT OF
1 CAUSES THE PLOT TO BE DISPLAYED ON THE TEXT RONIX, 2
CAUSES THE PLOT TO BE SENT TO THE VERSATEC, ONCE VEPLOT
HAS BEEN EXECUTED.

INCLUDE "PLTprm"

DIMENSION X(50),DATA(10),DATA(10),Y(5)

DATA WIDTH/5.5/,YWIDTH/5.5/,X1/1,K1/1,BAUDRT/1200/
DATA LENGTH/3/,VDIRCL/1001,LABEL(1)/"CY./MH.<0>/"
DATA LABELY(1)/"LOG POME"/LABELY(2)/"<R>"/

TYPE"THE SPECTRUM FILE TO PLOT IS ?"
READ(1,1)DATA(1)

1 FORMAT(510)
CALL OPEN(5,DATA(10),2,IER)
IF(IER.NE.1)TYPE"OPEN INPUT ERROR"

TYPE"SECOND FILE IS ?"
READ(1,1)DATA(6)
CALL OPEN(4,DATA(6),2,IER)
IF(IER.NE.1)TYPE"SECOND FILE OPEN ERROR."
READ(5,2)DATA(1)

2 FORMAT(S18)
READ FREE(5)X
TYPE"PS READ"
XMIX=2500.B
XMIX=10.0
TYPE"X(1) read "
TYPE"X(50) read"
YMIN=-10.0

XK=19
KY=19
ICODE=2
OUTPUT=2
LXS=1
A4=255.

CALL PLTAXS
CALL MODE(-18,A1,A2,A3)
ICODE=2
CALL MODE(18,A4,A2,A3)
DO 6 I=1,15
XINC=(I-1)*5.0
R=X(I)+0.0812
CALL PLTAXSICODE,XINC,R)
ICODE=0

6 CONTINUE
ICODE=2
DO 7 I=1,15
XINC=(I-1)*5.0
R=X(I)+0.0812
CALL PLTAXSICODE,XINC,R)
ICODE=0

7 CONTINUE
CALL MODE(18,A1,A2,A3)
ICODE=2
READ(4,2)DATA(6)
READ FREE(4)X
TYPE"PS READ"
DO 8 I=1,5
XINC=(I-1)*5.0
R=X(I)+0.0812

PSX(I)=PSX(I)+COV(K+1)*COS(2*3.14159*FREQ(I)*DELTA)
CONTINUE

PSX(I)=PSX(I)+2*DELTA*(COV(I)+2*PSX(I))
CONTINUE

DO 23 I=1,32
PSX(I)=(FREQ(I)+1)-FREQ(I)*0.5*(PSX(I)+FREQ(I)*PSX(I)+1)
FREQ(I+1)=3.14159
AVG(I)=AVG(I)+(ALOG10(PSX(I)))/31#
CONTINUE

TYPE"PASS",J5,"HAS BEEN MADE."
CONTINUE

TYPE"OUTPUT DATA FILE WILL BE ?"
READ(11,3)OUTPUT(1)
CALL OPEN(5,OUTPUT(1),2,IER)
IF (IER.NE.1)TYPE"OPEN ERROR IN OUTPUT FILE"
WRITE(5,48)

FORMAT(1X,"THE LOG MEAN POWER SPECTRUM VALUES ARE:")
WRITE(5,62)(AVG(1),I=1,32)

FORMAT(10E12.4)
TYPE""
TYPE"ALL DONE"
END
THE NAME OF THIS PROGRAM IS SCO.

THIS PROGRAM CONVERTS MONTE CARLO SIMULATED DATA TO EQUIVALENT OPTICAL POWER SPECTRUM DATA.

DIMENSION X(4001), PSX(2001), COV(60), DATAA(20), OUTPUT(20)

DIMENSION AVG(50), FREQ(33)

DATA (FREQ(I), I=1,33)/0.0, 6.7, 1.7, 2.5, 3.2, 4.0, 4.9, 5.8, 6.7, 7.7,
8.9, 10.1, 11.5, 13.0, 14.8, 16.8, 18.9, 21.6, 24.5, 27.7, 31.4, 35.5,
40.1, 45.3, 51.1, 57.5, 64.6, 72.5, 81.2, 90.8, 101.4, 112.9, 125.6/

DO 39 I=1, MS

AVG(I)=0.0

CONTINUE

TYPE "WHAT IS THE FILE NAME?"

READ(11,3) DATAA(I)

IF (IER .NE. 1) TYPE "OPEN ERROR 1"

ACCEPT "HOW MANY DATA RECORDS ARE THERE TO EXAMINE?", J10

ACCEPT "THE SAMPLING INTERVAL IN MJ: IS: ", DELTA

ACCEPT "HOW MANY LAYERS ARE IN THIS EMULSION?", N

ACCEPT "THE NUMBER OF DATA POINTS IN EACH FILE IS: ", INT

DO 40 J5=1, J10

DO 42 I=1, 2001

PSX(I)=0.0

CONTINUE

DO 43 I=1, 60

COV(I)=0.0

CONTINUE

DO 44 I=1, 20

DATAA(I)=0.0

OUTPUT(I)=0.0

READ FREE(1) X

LA=60

XAV=0.0

DO 21 I=1, 4000

XAV=XAV+X(I)

DO 22 I=1, 4000

X(I)=(X(I)-XAV)

CONTINUE

DO 24 I=1, L8

COV(I)=0.0

J=4000-(I-1)

DO 25 K=1, J

COV(I)=COV(I)+(X(K)*X(K+I-1))/4000

CONTINUE

DO 26 I=1, L8

CONTINUE

DO 27 I=1, L8

CONTINUE

DO 28 I=1, L8

CONTINUE

DO 29 I=1, L8

CONTINUE

R=0.0

DO 31 I=1, L8

R=R+1.0

IF (I.GT.LS/2) GO TO 36

CONTINUE

CONTINUE

LS=LS-1

IF (I.GT.LS/2) GO TO 36

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1

CONTINUE

CONTINUE

LS=LS-1
DO 27 I=1,33
PSX(I)=0.0
DO 28 K=1,L10
PSX(I)=PSX(I)+COV(K)*COS(2*3.14159*FREQ(I)*K*DELTA)
CONTINUE
I=I-CT.1)GO TO 47
CT=1.0
GO TO 48

F=FREQ(I)/W0
CF1=(2/3.14159)*ACOS(F-((F/2)*SQR(1-(F**2))))
CF2=SIN(3.14159*W0*F*SW)/(3.14159*W0*F*SW)
CF=(CF1*CF2)**2
PSX(I)=2*DELTA*(COV(1)+2*PSX(I))/(CT**2)
CONTINUE
DO 23 I=1,32
PSX(I)=((FREQ(I+1)-FREQ(I))**0.5*(PSX(I)*FREQ(I)+PSX(I+1)-
FREQ(I+1))*3.14159
AVG(I)=AVG(I)+(ALOG10(PSX(I)))/J10
CONTINUE

TYPE"OUTPUT DATA FILE WILL BE ?" 
READ(11,3)OUTPUT(1)
CALL OPEN(5,OUTPUT(1),2,IER)
IF (IER.KE.1)TYPE"OPEN ERROR IN OUTPUT FILE"
WRITE(5,46)
WRITE(5,6)(AVG(I),I=1,32)
FORMAT(10E12.4)
TYPE" 
TYPE="ALLO DONE"
END
THE NAME OF THIS PROGRAM IS COPOW

THIS PROGRAM CONVERTS MICRODENSITOMETER DATA INTO EQUIVALENT OPTICAL POWER SPECTRUM DATA.

DIMENSION INTE(4600)

DIMENSION X(4600),PSX(2001),COV(60),DATAA(20),OUTPUT(20)

DIMENSION AVG(50),FREQ(31),X(1)-1,33)

DATA (FREQ(I),I-1,33)/0.0,0.7,1.7,2.5,3.2,4.0,4.9,5.8,6.7,7.7,
8.9,10.1,11.5,13.0,14.8,16.8,19.0,21.6,24.5,27.7,31.4,35.5,
40.1,45.3,51.1,57.5,64.6,72.5,81.2,90.8,101.4,112.9,125.6/

DO 23 I-1,50

AVERAGE(I)=0.0

CONTINUE

ACCEPT "THE NUMBER OF FILES IS:" ,J10

ACCEPT "THE SAMPLING INTERVAL IN MM. IS:" ,DELTA

RINT=1/(2*DELTA*50)

ACCEPT "THE NUMERICAL APERTURE OF THE MICRO-D PICKUP"

ACCEPT "OBJECTIVE IS:" ,RNA

ACCEPT "THE MICRO-D SLIT WIDTH IN MM. IS:" ,SW

W0=(2*RNA)/5000

DO 46 J5=1,510

DO 50 I=1,4600

X(I)=0.0

INTE(I)=0

CONTINUE

DO 42 I=1,2001

PSX(I)=0.0

CONTINUE

DO 43 I=1,60

CONTINUE

DO 44 I=1,2001

DATAA(I)=0.0

OUTPUT(I)=0.0

CONTINUE

CALL WHICH FILE IS NUMBER",J5," ?"

READ (11,3)DATAA(1)

3 FORMAT (62,0)

CALL OPEN,1,DATAA(1),1,IER

IF (IER.NE.1) TYPE "OPEN ERROR 1"

READ BINARY(11,INTE)

CLOSE(1)

L8 IS THE LAG VALUE.

L8=60

XAV=0.0

DO 21 I=1,4600

XAV=XAV+INTE(I)

CONTINUE

XAV=XAV/(4600.0)

DO 22 I=1,4600

X(I)=INTE(I)-XAV/5000.0

CONTINUE

DO 24 I=1,L8

COV(I)=0.0

J=4600-(I-1)

DO 25 K=1,J

COV(I)=COV(I)+(X(K)*X(K+I-1))/4600

CONTINUE

CONTINUE

R=0.0

R=0.0

IF (I.GT.L8/2) GO TO 36
L16-L8-1
DO 27 I=1,50
PSX(I)=X
DO 28 K=1,L16
PSX(I)=PSX(I)+COV(K+1)*COS(2*3.14159*I*RINT*K*DELTA)
26 CONTINUE
IF(I.GT.1) GO TO 47
CF=1.0
GO TO 48
27 CONTINUE
F=(I-1)*RINT/WI
CF1=(2/3.14159)*ACOS(F-(F/F)*Sin(1-(F**2)))
CF2=(2*3.14159*W*F*SW)/(3.14159*W*F*SW)
CF=(CF1*CF2)**2
47 CONTINUE
PSX(I)=2*DELTA*(COV(I)+2*PSX(I))*CP=.1
48 CONTINUE
DO 23 I=1,50
AVG(I)=AVG(I)+(PSX(I)/J10)
23 CONTINUE
"PASS",5,"HAS BEEN MADE."
40 CONTINUE
DO 45 I=1,50
AVG(I)=ALOG10(AVG(I))
45 CONTINUE
"OUTPUT DATA FILE WILL BE ?"
READ(11,3)OUTPUT(1)
CALL OPEN(5,OUTPUT(1),2,IER)
IF (IER.NE.1) TYPE "OPEN ERROR IN OUTPUT FILE"
WRITE(5,6)(AVG(I),I=1,50)
6 FORMAT(10E12.4)
TYPE "ALL DONE"
END
THE NAME OF THIS PROGRAM IS SIMP

THIS PROGRAM COMPUTES THE POWER SPECTRUM WE WOULD EXPECT TO GET WITH CERTAIN MICRODIFFRACTOMETERS CONFIGURATIONS, WHEN SIMULATED DATA IS INPUT. THE LAG IS 60 AND THE SMOOTHING IS PARZEN.

DIMENSION X(4800),PSX(2001),COV(60),DATAA(20),OUTPUT(20)
DIMENSION AVG(50)

DO 39 I=1,50
   AVG(I)=0.0
   CONTINUE

39 TYPE"WHAT IS THE FILE NAME ?"
READ(11,3)DATAA(1)
PFORM(22)
CALL OPEN(1,DATAA(1),1,IER)
IF(IER.NE.1)TYPE"OPEN ERROR 1"
ACCEPT"HOW MANY DATA RECORDS ARE THERE TO EXAMINE ?",J10
ACCEPT"THE SAMPLING INTERVAL IN NM. IS: ",DELTA
ACCEPT"HOW MANY LAYERS ARE IN THIS EMULSION ?",M
ACCEPT"THE NUMERICAL APERTURE OF THE MICRO-D PICKUP"
ACCEPT"OBJECTIVE IS: ",RNA
ACCEPT"THE MICRO-D SLIT WIDTH IN NM. IS: ",SW

W0=(2*RNA)/5000
ACCEPT"THE NUMBER OF DATA POINTS IN EACH FILE IS: ",INT
RINT=l/(2*DELTA*50)
DO 40 J5=1,J10
   DO 42 I=1,2001
      PSX(I)=0.0
      CONTINUE
   DO 43 I=1,60
      COV(I)=0.0
      CONTINUE
   DO 44 I=1,20
      DATAA(I)=0.0
      OUTPUT(I)=0.0
      CONTINUE
   READ FREE(1)X
   LB=60
   XAV=0.0
   DO 21 I=1,4800
      XAV=XAV+X(I)/4800
      CONTINUE
   DO 22 I=1,4800
      X(I)=X(I)-XAV
      CONTINUE
   DO 24 I=1,LB
      COV(I)=0.0
      J=4800-(I-1)
      DO 25 K=1,J
         COVI=COVI+(X(K)*X(K+I-1))/4800
      CONTINUE
   CONTINUE
   DO 25 I=1,LB
      COVI=(COVI+XAV**2)**2-(XAV)**2
      CONTINUE
   R=0.0
   DO 41 I=1,LB
      R=R+1.0
   CONTINUE
51 IF(I.GT.LB/2) GO TO 36
   CONTINUE
   COVI=COVI*(1-6*(R/LB)**2+6*(R/LB)**3)
   GO TO 41
   CONTINUE
APPENDIX B

EXPERIMENTAL DATA

The films used in this experiment were Eastman-Kodak 35 mm. Tri-X and 70 mm 3414. The Tri-X was developed in D-76 which was diluted 1:1 at 68° F for 10 minutes. Tray development was used. The stop bath was Kodak Indicator Stop Bath, and the fixer was Kodak F5 fixer. The fixing bath was followed by Kodak Hypo Clearing Agent. The 3414 was developed in D-19 at 69°F for 8 minutes. The remainder of the development process was the same as for the Tri-X.

The shoulder, mid-range and toe densities for the Tri-X were 1.74, 1.01, and 0.34 while for the 3414 they were 2.17, 1.16 and 0.30.

The microdensitometer used was a Perkin-Elmer PDS Microdensitometer. The illuminating and pick-up objectives were both of numerical aperture 0.25. The sampling distance was 2 microns. The scan speed setting was 20 (which converts to 3.92 mm/sec.). The microdensitometer was operated in the transmittance mode with a transmittance range of 0-500. This transmittance range is accounted for in the analyses programs.

The coherent spectrum analyzer was used in accordance with the directions in Cheatham's dissertation (1976). The only setting available to the operator is the transform lens focal length, which was 217 mm.
APPENDIX C

ELECTRON MICROGRAPH PHOTOS

The following pictures are electron micrographs of films used in this project. In each case the sample has been uniformly exposed and processed in accordance with Appendix B. The final magnification of these micrographs is 6842X.
Fig. C-1. Electron micrograph of Tri-X toe sample.
Fig. C-2. Electron micrograph of Tri-X mid-range sample.
Fig. C-3. Electron micrograph of Tri-X shoulder sample.
Fig. C-4. Electron micrograph of 3414 toe sample.
Fig. C-5. Electron micrograph of 3414 mid-range sample.
Fig. C-6. Electron micrograph of 3414 shoulder sample.
REFERENCES


