THE USE OF A D-C ANALOG COMPUTER WITH RELAY LOGIC TO IMPLEMENT AN ULTRASTABLE FEEDBACK SYSTEM

by

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INTRODUCTION

Consider the following hypothetical application:
The design of a process control system requires that its four variables, \( X_i \), be set equal to the set point values, \( Q_i \), by a multi-loop feedback system which senses the error

\( (X_i - Q_i) \)

Each \( X_i \) shall be controlled by a "power amplifier" such as a regulated heat source, motor, valve or control surface, so that,

\[
\frac{dX_i}{dt} = \sum a_{ik}(X_k - Q_k) \quad \text{where} \quad \det \begin{bmatrix} a_{ik} \end{bmatrix} \neq 0 \quad (a)
\]

The coefficients \( a_{ik} \) are specified in part by fixed system dynamics and interactions; partially changeable due to changing environment; and partially available as design feedback-circuit parameters. The block diagram of such an application appears below.
Our adaptive system sets the feedback parameters so that Equation (a) describes a \textit{stable} system, for then, when a steady state is reached

\[ \frac{dx_i}{dt} = 0, \quad \text{and} \quad \sum a_{ik}(X_k - Q_k) = 0 \quad \text{det} [a_{ik}] = 0 \quad (b) \]

which yields the desired solution

\[ x_i = Q_i \quad (c) \]

Note that the values of the coefficients \( a_{ik} \) do not affect the \( x_i \) if the system is stable.

The ultrastable system readily adapts itself to the hypothetical application presented here. It functions in accordance with a principle that requires switching of some of the \( a_{ik} \), to be initiated when the \( x_i \) reach prescribed overload levels. Although limited by the fact it corrects only instability but not for example a lack in response speed, the ultrastable system should warrant considerations with regard to industrial control applications.
CHAPTER 1

ULTRASTABILITY

1.1 The Ultrastable System. Let us consider an autonomous system, i.e., one with no outside control, specified by two variables, \( y_1 \) and \( y_2 \). The simultaneous differential equations determining the behavior of this system can be written as:

\[
\frac{dy_1}{dt} = a_{11}(S)y_1 + a_{12}(S)y_2 \quad (1-1)
\]

\[
\frac{dy_2}{dt} = a_{21}(S)y_1 + a_{22}(S)y_2 \quad (1-2)
\]

The coefficients, \( a_{11}, a_{12}, a_{21} \) and \( a_{22} \), are defined as monotonic functions of a parameter, \( S \), to be chosen so that the resulting dynamical system becomes stable.

The phase-plane equation for the \( y_1y_2 \) plane, then, is:

\[
\frac{dy_1}{dy_2} = \frac{a_{11}(S)y_1 + a_{12}(S)y_2}{a_{21}(S)y_1 + a_{22}(S)y_2} \quad (1-3)
\]

The phase-plane representation gives some insight into the

---

solution of nonlinear equations. The autonomous system under consideration is nonlinear, of the second order. One way to solve a system of this type would be by the isocline method. Although the concept of ultrastability can best be explained utilizing phase-plane analysis, the isocline method does not readily adapt itself to the occasion.

The pattern of behavior of the autonomous system is determined by lines of the loci of the point \((y_1, y_2)\) as time increases, the lines starting from various initial points in the phase plane. For each value of parameter, \(\delta\), a new pattern is formed. There also will exist as many sets of the terms \(a_{11}(\delta)y_1, a_{12}(\delta)y_2, a_{21}(\delta)y_1\) and \(a_{22}(\delta)y_2\) as there are values of \(\delta\). Thus it can be stated that there will be as many definite patterns of behavior as there are sets of coefficients. The lines of the loci are known as lines of behavior.

Lines of behavior that converge upon a point in the phase plane, designated the stable equilibrium point, indicate a stable behavior pattern. An unstable pattern is one in which the lines tend to diverge from the equilibrium point. Therefore, to realize a stable autonomous system we merely reject the unstable patterns and retain the stable ones.

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Let us surround the equilibrium point with a closed boundary and investigate the action that takes place within as the value of parameter, $\delta$, is changed in discrete steps, as might result through the activation of a switching device.

Figures 1.1, 1.2, 1.3 and 1.4 illustrate this action. Suppose the first pattern of behavior is that of Figure 1.1. The system's behavior will correspond to the movement of a representative point along one line in the phase plane. In 1.1, the representative point is started at $P_1$. The line of behavior from $P_1$ is not stable in the immediate region. Thus the representative point follows the line to the boundary. At the instant it crosses at $P_2$, the switching device is activated and $\delta$ changes value. This leads to the unstable pattern of Figure 1.2. The representative point now moves out of the boundary at $P_3$. A new pattern arises as shown in Figure 1.3. Although a stable equilibrium point is contained within the region, the line of behavior is required to move further out of the region from $P_3$. Another switching action takes place. The value of $\delta$ resulting from this latest activation leads to the pattern of Figure 1.4. In this pattern, the line of behavior from $P_3$ is stable with regard to the region. The

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representative point moves to the equilibrium point and terminates its movement. The switching device will remain inactive as the system is no longer required to move out of the boundary. Stability has now been achieved.

Briefly reviewing the preceding paragraphs, we see that through the addition of a switching device and by prescribing a switching boundary in the phase plane, the system was able to reject unstable patterns of behavior and retain the stable one. In doing so, it automatically sought stability. A system that behaves in this manner is called ultrastable, after W. Ross Ashby.  

1.2 Ashby's Ultrastable System. In order to demonstrate the behavior of an ultrastable system, Ashby constructed a device known as the homeostat. 

Basically, the system comprises four identical units, one of which is shown in Figure 1.5. Each unit contains a magnet, free to rotate in an angular motion. Four coils are wrapped around each magnet. The amount of angular deflection is proportional to the summation of currents through the coils. The magnet also contains a wire that dips into a trough of water which heavily

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5 Ibid., pp. 93-96.
dampens its motion. Electrodes at either end provide a potential gradient. As the magnet rotates, the immersed wire, acting like the arm of a potentiometer, derives a potential that is sent to the grid of a vacuum tube. Referring to Figure 1.5, voltage, $E_1$, is greater than $E_2$. With the magnet's pointer centered, $R_1$ is adjusted so all plate current flows through it and none to the output circuit. Current $I_{R_1}$ is always a constant value. Deflection of the magnet, heavily damped by the movement of the wire in the water, varies the tube's bias, thus altering the amount of plate current. In order that $I_{R_1}$ remain constant, the current to the output circuit must change as shown in Figure 1.5. In addition to flowing through three coils, each of which is wrapped about the magnet of another unit, current in the output circuit passes through a coil ($K_4$) wrapped around a magnet located in its own unit. Currents through coils, $K_1$, $K_2$ and $K_3$ originate in the output circuits of the other units.

Let the angular deflections of the magnets be represented by the four variables, $\gamma_1$, $\gamma_2$, $\gamma_3$ and $\gamma_4$. The equations of motion are obtained by equating the turning torques produced by the coils to the damping torques. Thus,

---

Figure 1.5
\[
\frac{dy_1}{dt} = a_{11}y_1 + a_{12}y_2 + a_{13}y_3 + a_{14}y_4 \tag{1-4}
\]
\[
\frac{dy_2}{dt} = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + a_{24}y_4 \tag{1-5}
\]
\[
\frac{dy_3}{dt} = a_{31}y_1 + a_{32}y_2 + a_{33}y_3 + a_{34}y_4 \tag{1-6}
\]
\[
\frac{dy_4}{dt} = a_{41}y_1 + a_{42}y_2 + a_{43}y_3 + a_{44}y_4 \tag{1-7}
\]

The values of the coefficients \(a_{ik}\) - both magnitude and sign - are changed through the adjustment of potentiometers, \(R_3, R_4\) and \(R_5\), and by the action of the commutating devices, \(D_1, D_2, D_3\) and \(D_4\), respectively. The current feeding one coil on each magnet first passes through the contacts of a 25 position uniselector. The position of the uniselector jumps randomly whenever the angular deflection of the magnet reaches 45 degrees in either direction. For every setting of the 'a's, the four uniselectors give \(25^4 = 390,625\) combinations of the four coefficients they control. These 390,625 combinations represent a like number of behavior patterns for the homeostat. Automatically, the unstable patterns are rejected, while the stable ones are retained.
CHAPTER 2

ANALOG COMPUTER REPRESENTATION OF AN ULTRASTABLE SYSTEM

In the preceding chapter, the reader was acquainted with the principle of ultrastability as conceived by W. Ross Ashby. A brief description of his homeostat, a machine he designed to implement an ultrastable system was then presented. Chapter 2 is devoted to the development of a system, constructed in the University laboratory by this student, to behave in accordance with the same principle.

2.1 The Laboratory System in Brief. Four identically constructed stages, when properly interconnected, provide the basis for our n-th order ultrastable system (where n = 1, 2, 3 and 4). A block diagram of one stage is shown in Figure 2.1. Two operational amplifiers form the nuclei of each stage. One amplifier, with its associated feedback network, functions as an integrator, the other as a dead space limiter and, incidentally as a current amplifying device. The inputs to the integrator are derived from the outputs of three other stages in addition to the feedback voltage of the stage itself. Settings of
parameter units, denoted by $a_{11}$, $a_{12}$, $a_{13}$ and $a_{14}$, in Figure 2.1, determine the magnitude and polarity of the voltage appearing at the input to the integrating circuit. If the system is unstable for a given set of coefficients $a_{ik}$, an integrator output voltage will exceed the threshold of the dead-space limiter whose output will then operate a sensitive relay. This relay applies a pulsed d-c voltage to the coil of a stepping relay, which then changes one of the feedback coefficients $a_{ik}$ in a random search for stable configurations.

The system enforces the relations

$$\sum_{k} a_{ik} X_k = \frac{dX_i}{dt} \quad \text{det} \begin{bmatrix} a_{ik} \end{bmatrix} \neq 0 \quad (2-1)$$

If the system is stable,

$$\frac{dX_i}{dt} = 0 \quad \text{and we have} \quad (2-2)$$

$$\sum_{k} a_{ik} X_k = 0 \quad \text{where} \quad \text{det} \begin{bmatrix} a_{ik} \end{bmatrix} \neq 0 \quad (2-3)$$

Therefore, each variable $X_k$ tends to zero in the manners shown below.
Figure 2.1

Figure 2.2
2.2 D-C Analog Computer Components. Philbrick type \( K_2-W \) plug-in operational amplifiers, available in the University's Analog Computer Laboratory, were chosen to implement the system. These units served as building blocks for the feedback computing devices. For the sake of convenience, the input \( K_2-W \) amplifier will henceforth be referred to as \( K_2-W_1 \), and the output amplifier as \( K_2-W_2 \).

2.3 The Integrating Circuit. To implement the system's equations, amplifier \( K_2-W_1 \) was connected as an integrator whose output voltage, \( X_i \), is given by

\[
X_i = -\frac{1}{C_1P} \left( \frac{X_1}{R_1} + \frac{X_2}{R_2} + \frac{X_3}{R_3} + \frac{X_4}{R_4} \right)
\]

or, depending upon the position of the switches, \( a_{ik} \)

\[
X_i = -\frac{1}{C_1P} \left( \frac{X_1}{R_8} + \frac{X_2}{R_7} + \frac{X_3}{R_6} + \frac{X_4}{R_5} \right)
\]

To utilize the plus and minus inputs of \( K_2-W_1 \), the bias circuit shown in Figure 2.2 was employed. Potential \( E \) is composed of two 1.3 volt mercury cells wired in series. The bias adjusting control, \( R_9 \), necessary for balancing the amplifier, is a 100,000 ohm, 2 watt, ±1 percent wire-wound potentiometer shunted by a .05 microfarad paper condenser. The values of input summing resistors, \( R_1 \) through \( R_8 \), are discussed in paragraph 2.8.
Prior to operating the system, it is necessary to balance the Philbrick Operational Amplifiers. The inputs to the integrating stages are effectively grounded by placing manual selector components (See Figure 2.5) in the diagonal slots of the matrix, then turning the arms of the switches to their mid-settings. Balance potentiometer, Rg, is adjusted for a meter null indication at tip plug, TP-1.

2.4 The Dead-Space Limiter. The dead-space limiter circuit, Figure 2.3 (a), for each variable \( X_i \) performs a dual function:

1. It limits the absolute value of the integrator output voltage, \( X_i \), to about 9 volts and thus simulates an overload condition.

2. It actuates a stepper if, and only if, \( |X_i| \) reaches a threshold value of about 2.5 volts.

The ratio of the limiting (simulated overload) voltage to the threshold voltage must be chosen in relation to the stepper repetition rate. That is to say, the \( |X_i| \) should have time to return below the threshold values during one stepper period whenever the feedback configuration is reasonably stable. If this condition is not met, the stepping devices will continue to step even when the system is stable.
In the laboratory system, limiting voltages of about 9 volts and threshold voltages of about 2.5 volts have good results with a stepper repetition rate of approximately two steps per second.

The procedure for balancing the stage is accomplished in two steps. The first consists of grounding tip plug, TP-l, then adjusting the threshold level potentiometer, R₁, for a voltage reading at TP-3 equal to that at TP-4. The second involves adjusting bias potentiometer, R₈, to obtain a meter null indication at tip plug, TP-2.

2.5 Pulsed D-C Sources for Operating the Stepping Relays. A continuously applied, uninterrupted, voltage to the coil of the stepping relay will do no more than step the arm once. For correct operation, the system requires a pulsed d-c source. Although a single source for all steppers would be simpler and indeed preferable, for our system under consideration, an earlier building-block approach dictated promise of separate d-c sources and individual relays for each stepper. The multivibrator circuit shown in Figure 2.3 (b) was designed to provide this pulsing source. The four relays in the plate circuit of tube section B, each associated with a different stage, are actuated periodically at a frequency that
Figure 2.3

(a) TP-3 270K ohm +300 V.

(b) Sigma Type B454395-1 Relays 47K ohm

(c) 115 volts, 60cps. 200 ohms 10 watts

See Figure 2.5
determines the basic stepper repetition rate. The complete circuit is shown in Figure 2.3.

2.6 Fabrication of the Variable Coefficient Components. Sixteen parameter boxes were constructed, twelve containing single pole, seventeen position manual-selector switches and four housing twenty-five position stepping switches capable of operating continuously through 360 degrees of rotation.

Through the remainder of the thesis, the first twelve switches shall be referred to as manual selectors and the last four as autoselectors. The autoselectors, as the readers shall see, are synonymous to Ashby's uni-selectors.

Figure 2.4 (a) is a schematic diagram of the manual selector component. The seventeen position switch and the resistors are packaged in a 2" x 2 3/4" x 4" metal box. Terminals 1 through 6, on the schematic, represent the connections to a 6 pin, chassis mount, Jones plug (See 2.4 (b)).

Figure 2.5 is a schematic diagram of the autoselector component.

Unlike the resistor connections for the manual selector, here they are wired to the contacts in a random
Figure 2.5 - Autoselector Component

\[ R_1 - R_{25} = 100 \text{K ohm} \]

Figure 2.4 - Manual Selector Component

\[ R_1 - R_{17} = 100 \text{K ohm} \]

5 No Connection

6 No Connection
fashion. To achieve this, twenty-five pieces of paper, numbered 1 to 25 inclusive, were drawn from a hat four different times. Chart 2.1 tabulates the results. As an example, consider autoselector Unit 1. Contact 1 of the stepping switch is wired to point 24 or the junction of R_{23} and R_{24}; contact 2 to the junction of R_{16} and R_{17}, etc. The movement of the arm thus introduces a random quantity at the input of the integrating device each instant the autoselector coil is pulsed; hence a random search for stable feedback configurations is approximated.

The four silicon rectifiers, of Figure 2.3 (c), convert the incoming 115 volt, 60 cycles to approximately 100 volts, d-c. A 200 ohm, 10 watt resistor serves to limit the current passing through the contacts of the two sensitive relays, the devices that gate the incoming 60 cycle voltage. A neon glow tube wired in parallel with the autoselector coil enables the observer to tell when the arm is stepping to a new position.

All components, excluding relay K are packaged in a 2" x 5" x 7" chassis box. Connections to pins 1 to 6 inclusive, are brought out through a 6 pin Jones plug, similar to the one used on the manual selector. This feature permits the 16 parameter boxes to be interchanged in the Ultrastable System.
CHART 2.1 TABLE OF RANDOM VALUES FOR AUTO-SELECTOR CONTACTS.

<table>
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<tr>
<th>CONTACT</th>
<th>UNIT 1</th>
<th>UNIT 2</th>
<th>UNIT 3</th>
<th>UNIT 4</th>
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2.7 The Chassis Layout. The complete ultrastable system is constructed on two 3" x 14" x 17" chassis, bolted together to form one unit. A great deal of time was spent planning the layout of the various components prior to the actual construction. There were a number of design considerations involved, the primary one being that the finished product, in addition to functioning properly, must also look neat. It was important that the items comprising each of the four identical stages be grouped together. Placement of the Jones sockets, on the chassis, to enable rapid installation and removal of the parameter boxes, was of equal importance. Balance potentiometers for the Philbrick K₂₋₆ Amplifiers had to be located in accessible spots for easy adjustment. As it later turned out, this initial planning proved worthwhile.

Briefly, the layout is as follows. On the top side of the chassis, twelve octal tube sockets are mounted in a column. Eight hold the Philbrick plug-in Amplifiers, with four spares for possible use at a later date. Adjacent to these are four miniature tube sockets for the 6AL5 duodiodes. Occupying nearly three-fourths of the chassis are the sixteen 6 pin, chassis mount Jones sockets. A more detailed discussion of their layout follows shortly. Along the side of the chassis are mounted
the amplifiers' bias networks. The mercury cells supplying bias voltage to the K₂-W Amplifiers are all above ground potential. Their installation required a special insulated mounting plate. The shafts of the balance pots extend out the side, making them accessible for adjustment. Tip plugs for measuring the inputs and outputs of all K₂-W Amplifiers are installed on the top side, next to the octal sockets. The four sensitive relays, input summing networks for the integrating circuits and all resistors and condensers in the four stages, are located beneath the chassis. A power plug is mounted on one end of the chassis. Voltages necessary for the system's operation, namely, ±300 volts d-c regulated, and 115 volts, 60 cycles, are entered through this receptacle. The regulated voltages are supplied by a Philbrick type R-100 Power Unit, while 6.3 volts, 60 cycles filament voltage is obtained from two transformers, situated on the under side of the chassis.

2.8 Wiring the Sixteen Jones Sockets. Figure 2.6 is a partial wiring diagram of the sixteen Jones sockets. A close examination of the diagram will reveal how the manual selector and autoselector units, when inserted, form the input circuitry to the integrating components. The eight summing resistors associated with the diagonal
coefficients are 470,000 ohms in value as compared with the remaining twenty-four that are one megohm. This has the effect of reducing the strength of diagonal elements, producing more stable configurations. The authenticity of this last statement will be verified in Chapter 4.

Referring to Figure 2.6, the output from $K_2^{-W_1-1}$ feeds the number 3 terminals of the sockets in column 1, that from $K_2^{-W_1-2}$ the number 3 terminals of the sockets in column 2, etc. The summed voltages to the minus input of $K_2^{-W_1-1}$ are derived from the number 2 terminals of the sockets in row 1, those to the plus input from the number 1 terminals of the same sockets. Likewise, the summed voltages to the inputs of $K_2^{-W_1-2}$ are derived from the number 1 and number 2 terminals of the sockets in row 2. To introduce flexibility, i.e., enable the parameter boxes to be interchanged rapidly during system operation, pin 6 of each socket is connected to one side of the 115 volt, 60 cycle line. In addition, the number 5 terminals of all sockets in column 1 are tied to one contact of the sensitive relay energized by the output voltage of amplifier, $K_2^{-W_2-1}$. Thus, an a-c voltage to pulse the coil of the autoselector is available at all sockets.

Figure 2.7 represents the system in a matrix form. Note how the input to each stage is affected by
the outputs of the other three plus its own feedback. The 'a' coefficients symbolize the values of the manual selector and autoselector units. $a_{11}$, $a_{22}$, $a_{33}$ and $a_{44}$ are the diagonal terms of the matrix. Any possible combination, involving the sixteen units is physically realizable. However, as we shall see later in the thesis, some combinations may not yield stable system performance.
Notes: 1. All Number 6 pins are wired together.
2. All Number 4 pins are wired to ground.

Figure 2.6
Figure 2.7
CHAPTER 3

ANALYSIS OF SYSTEM STABILITY BY ROUTH'S METHOD

Before discussing the work involved in perfecting our ultrastable system, let us consider the requirements the system must satisfy before it can realize a condition of stability.

3.1 Conditions for Stability by the Routh Method. Most of us are already familiar with stability criteria for feedback systems and the general techniques of analysis. After considering the relative complexity of the ultrastable system it was decided to apply Routh's method for determining these requirements.

The laboratory system permitted the operator to implement \((1 \times 1)\), \((2 \times 2)\), \((3 \times 3)\) or \((4 \times 4)\) matrices at his discretion. With the exception of the \((1 \times 1)\) matrix, all configurations will be analyzed using Routh's method. The contents of Figure 2.7 provide the reader with reference data associated with the following paragraphs.

3.2 The \((1 \times 1)\) Matrix. Here, the system performs essentially as a single stage feedback device employing one autoselector unit. The unit is inserted in
any one of the four diagonal slots. The \((1 \times 1)\) matrix requires only that the term, \(a_{11}, a_{22}, a_{33}\) or \(a_{44}\) be less than zero. The phrase, "less than zero," refers to a particular setting of the selector component occupying the matrix slot. An examination of Figure 3.1, accompanied by a brief explanation, should clarify this last statement.

Consider the situation wherein the arm of the selector unit is at position B. Regardless of any output, at terminal \((6)\), from the operational amplifier, the voltage fed back to the input circuit will not produce a further change in this output since the arm is at ground potential. Let the arm now move to position C. Next assume a small positive output from the amplifier, resulting possibly from an unbalance within. Fed back, it will be felt at
the positive input terminal (1), leading to an increased output in the same direction. Eventually, the voltage will build up to a very high positive value, producing an unstable condition. Likewise, a small negative output fed back to the positive input terminal will result in a high negative output. Now let us assume the arm is resting at position A, and the amplifier output is initially a small value of positive voltage. Impressed upon the negative input terminal, (2), of the amplifier, the output will tend to rise in the negative direction, immediately cancelling out the positive voltage, producing a desired stable condition. Similarly, a negative voltage fed back to this same terminal, (2), will force the output to rise in the opposite polarity resulting in cancellation of the original signal.

Our less than zero term as applied to the sixteen coefficients of Figure 2.7 will imply that the arm of the autoselector or manual selector component is resting at a point along the resistive network between ground and the negative input terminal of the operational amplifier. Detailed drawings of these networks appear in Figures 2.4 and 2.5. Note that in the latter figure, points 14 through 25 do not necessarily correspond to autoselector contact positions 14 through 25. To determine the less than zero positions one must refer to Chart 2.1.
3.3 The (2 x 2) Matrix. Our ultrastable system will comprise two feedback stages implemented by a combination of four autoselector and manual selector units. The matrix slots they occupy will depend solely upon the two stages the operator chooses to utilize.

The characteristic equation describing system behavior is formulated from equations (3-1) and (3-2).

\[ a_{11}X_1 + a_{12}X_2 = PX_1 \]  \hspace{1cm} (3-1)

\[ a_{21}X_1 + a_{22}X_2 = PX_2 \]  \hspace{1cm} (3-2)

\[ (a_{11} - P)X_1 + a_{12}X_2 = 0 \]  \hspace{1cm} (3-3)

\[ a_{21}X_1 + (a_{22} - P)X_2 = 0 \]  \hspace{1cm} (3-4)

\[ (a_{11} - P)(a_{22} - P) - a_{12}a_{21} = 0 \]  \hspace{1cm} (3-5)

\[ a_{11}a_{22} - P(a_{11} + a_{22}) + P^2 - a_{12}a_{21} = 0 \]  \hspace{1cm} (3-6)

\[ P^2 - P(a_{11} + a_{22}) - (a_{12}a_{21} - a_{11}a_{22}) = 0 \]  \hspace{1cm} (3-7)

From the characteristic equation, (3-7), we can prepare a Table of Coefficients.

<table>
<thead>
<tr>
<th>Coefficient of</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^2 )</td>
<td>1 ( - (a_{12}a_{21} - a_{11}a_{22}) )</td>
</tr>
<tr>
<td>( p^1 )</td>
<td>( - (a_{11} + a_{22}) )</td>
</tr>
<tr>
<td>( p^0 )</td>
<td>( - (a_{12}a_{21} - a_{11}a_{22}) )</td>
</tr>
</tbody>
</table>

Table 3.1
The data included in Table 3.1 indicates there are two requirements that must be satisfied to stabilize a (2 x 2) matrix.

Number 1 \[ (a_{11} + a_{22}) < 0 \]
Number 2 \[ (a_{11}a_{22} + a_{12}a_{21}) > 0 \]

3.4 The (3 x 3) Matrix. For this configuration our ultrastable system will comprise three feedback stages. The matrix slots to be occupied by the combination of nine manual and autoselector components will again depend upon the (three) stages that are utilized.

Our system characteristic equation is now formulated from:

\[ a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = px_1 \]  \hspace{1cm} (3-8)
\[ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = px_2 \]  \hspace{1cm} (3-9)
\[ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = px_3 \]  \hspace{1cm} (3-10)

The resulting table of coefficients establishes requirements for system stability. The two most noteworthy terms contained in the table specify the following:

\[ \sum_{j=1}^{n} a_{ij} < 0 \]  \hspace{1cm} (1)
\[ \text{det} \left[ E_{ik} \right] < 0 \]  \hspace{1cm} (2)

3.5 The (4 x 4) Matrix. Our ultrastable system now comprises four feedback stages implemented by a
combination of all sixteen manual selector and autoselector units.

Development of the characteristic equation describing system behavior is indeed laborious. It begins with the following four equations.

\[
\begin{align*}
  a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= P_x_1 \quad (3-11) \\
  a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &= P_x_2 \quad (3-12) \\
  a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &= P_x_3 \quad (3-13) \\
  a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 &= P_x_4 \quad (3-14)
\end{align*}
\]

A rather cumbersome table of coefficients is derived from the resulting characteristic equation. Six terms specify overall requirements for system stability. Two of the more easily interpreted conditions are listed below.

\[
\begin{align*}
  (1) \quad \sum_{j=1}^{j=n} a_{ij} &< 0 \\
  (2) \quad \det [a_{ik}] &> 0
\end{align*}
\]
CHAPTER 4

SYSTEM PERFORMANCE RESULTS. REFINEMENT OF THE STABILITY ANALYSIS

In Chapter 2, the principle of ultrasatbility materialized through the perfection of a multi-loop feed-back system. Following this developmental phase, many hours were spent in the laboratory gathering test data. Recordings of output variations from each stage, as obtained on a four channel Sanborn instrument, appear later in the chapter. A chart, 4.1, is included to illustrate the matrix configurations associated with each recording.

Analyses on the test data revealed a discrepancy in the stability criteria derived in chapter 3. The discrepancy was particularly evident with second and third order systems. An immediate solution to the problem, one that substantiated the derivations, involved the development of an improved theory relative to the system characteristics affecting stability. This solution will be discussed later, in detail.
4.1 **Explanation of Symbols Used on Chart 4.1.** A sample portion of this chart appears below.

![Figure 4.1](image)

Figure 4.1

Referring to Figure 4.1, the symbols located in the matrix slots indicate the presence of an autoselector (numbered circle) or manual selector (arrow) component. The meaning behind each arrow's direction can best be explained by referring to Figure 3.1. The three arm positions, A, B and C, correspond to the similarly marked arrow directions on Figure 4.1. Note that is synonymous to a positive coefficient $a_{ik}$ and to a negative coefficient $a_{ik}$. These relationships will be used frequently throughout the remainder of this chapter.

4.2 **Ultrastability - The Simplest Case.** Our first example involves a single stage, whose output is fed back into its input through an autoselector component. For this situation, the component may be inserted in any one of the
four diagonal element slots, depending upon the stage to be used. Figure 4.2 is the recording associated with this example. The lower curve is the output of the integrating amplifier, while the upper curve indicates autoselector action.

Initially the system rests in a stable state. To demonstrate the principle of ultrastability, we must induce instability by manually triggering the stepping mechanism of the autoselector unit. Further movement then becomes automatic. Referring to Figure 4.2, we note that at each point, A, the output of the amplifier increases as a result of these manual disturbances. Upon reaching the switching boundary (designated by the dotted line) autoselector action takes place. For each step, a new behavior pattern is produced. If it is an unstable one, the autoselector will step to a new position. In this manner, unstable patterns are rejected. The stepping action will cease when the output decreases to a value below the switching boundary, an indication that the system has accepted a stable behavior pattern. To initiate additional action, we must repeat the manual triggering process. An examination of the upper curve reveals the varying number of random jumps required to produce stability. To satisfy the requirements specified
by the stability criteria, the arm of the autoselector component must settle each time as a position corresponding to a coefficient value that is less than zero.

4.3 The Ultrastable System, Second Order. Here, two feedback stages are required. The matrix composition is illustrated in Figures A, B and C of Chart 4.1. Note that manual selector units set to zero are inserted in the diagonal slots of the two idle stages to prevent the operational amplifiers from "taking off."

Figure 4.3 is a Sanborn recording associated with the second order system. Unlike the previous recording, Figure 4.3 does not include a curve that indicates stepping action by the autoselector component. Marks appearing on the upper trace in Figure 4.2 were reproduced from pips that inadvertently appeared on the original, lower curve. Failure to include facilities for monitoring this stepping action eliminates the possibility of analyzing the number of parameter changes taking place while the system stabilizes.

Initial settings of manual selector units, \(a_{11}\) and \(a_{22}\), Figure A, Chart 4.1, produced a sum \((a_{11} + a_{22})\) that was greater than zero. Referring to Figure 4.3, power was applied to the system at point, \(S_1\). Approximately two
Chart 4.1 (Continued)
seconds later outputs, $X_1$ and $X_2$, increased to their saturation levels of 8.5 to 9.0 volts. Outputs of this magnitude were sufficient to initiate stepping action by autoselector components, $a_{12}$ and $a_{21}$. Considering the dead space device's amplification factor, the saturation level, either positive or negative, greatly exceeds the value required to produce an actuation voltage of 6.5 at the coil. The exact level that is analogous to our switching boundary described in chapter 1 is undetermined.

As noted on Figure 4.3, outputs, $X_1$ and $X_2$, varied between their saturation levels. During this time, unstable behavior patterns were continually being rejected by the switching action. A reversal in the polarity of $X_1$ resulted from a change in parameter, $a_{12}$. Likewise, a reversal in $X_2$ was caused by a change in $a_{21}$. Knowing in advance that the existing settings of $a_{11}$ and $a_{22}$ would not yield a stable behavior pattern, selector unit $a_{11}$ was reset to zero (Figure B, Chart 4.1). Point $S_2$, Figure 4.3, denotes the occurrence of this adjustment. Action by both autoselector components continued for a number of seconds. Then, contrary to the stability criteria derived in chapter 3, outputs, $X_1$ and $X_2$, simultaneously broke into oscillations (Point $S_3$) that seconds
later dissipated completely. It was quite evident that a stable behavior pattern had been produced within the system. This discrepancy will be discussed in Paragraph 4.4.

Having achieved stability, the system was purposely made unstable by returning parameter unit $a_{11}$ to its greater than zero setting (Figure C, Chart 4.1). Within a few seconds, outputs $X_1$ and $X_2$, began increasing in amplitude until they broke into oscillations (Point $S_4$), increasing still further until they reached their saturation values. The system was allowed to operate in this unstable mode throughout the remainder of the recording.

4.4 The Improved Theory. The need for an improvement on the simple stability theory described in sections 3.3, 3.4 and 3.5, was apparent following the analysis of second order system operation. It will be necessary to take into account the fact that the integrator transfer function in our physical system is not actually $\frac{1}{p}$, but more accurately described by a transfer function of the form,

$$G(s) = \frac{A}{1 + AbS} \quad \text{where} \quad b = RC$$

The system equations become
The system described by equations, 4-3 and 4-4, will be stable if and only if all roots $S$ of its characteristic equation

$$\det\left\{a_{ik} - \frac{n+1}{G(S)} \delta_{ik}\right\} = 0 \quad \delta_{ik} = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{if } i \neq k \end{cases}$$

have negative real parts, i.e., if and only if all roots $S$ of the $n$ equations

$$1 - G(S) \frac{\alpha_j}{n+1} = 0 \quad (j = 1, 2, \ldots, n)$$

have negative real parts.\(^7\)

Let us now examine the transfer function of a high gain d-c amplifier with several feedback networks. Reference is made to the block diagram of Figure 4.4.

In terms of $S$, transfer function $G(S)$ equals

$$\frac{e_o(S)}{e_i(S)} = \frac{Z_f}{A(S)} = \frac{1}{\frac{Z_f}{A(S)}}$$  \hspace{1cm} (4-7)

Consider the schematic diagram illustrated in Figure 4.5. Here, $R = Z_i$ and $\frac{1}{SC} = Z_f$.

If the gain of the amplifier is sufficiently high our transfer function becomes

$$\frac{e_o(S)}{e_i(S)} = -\frac{Z_f}{Z_i}$$  \hspace{1cm} (4-8)

Substituting the above values for $Z_i$ and $Z_f$ in equation (4-8)

$$G(S) = -\frac{1}{RCS}$$  \hspace{1cm} (4-10)

Equation (4-10) describes a pure integrating device.

The transfer function of an imperfect (finite gain) integrator is obtained in the following manner.

$$G(S) = \frac{e_o(S)}{e_i(S)} = -\frac{1}{SC} \frac{1}{R(\frac{1}{A} + 1)} + \frac{1}{ASC}$$  \hspace{1cm} (4-11)

$$= -\frac{ASb}{Sb(1 + A) + 1}$$  \hspace{1cm} (4-12)

$$= -\frac{A}{Sb(1 + A) + 1}$$  \hspace{1cm} (4-13)

where $b = RC$  \hspace{1cm} (4-14)
At low frequencies

\[ A = A(S) > 0 \quad (4-15) \]

thus

\[ G(S) = -\frac{1}{bS + \frac{1}{A}} \quad (4-16) \]

Consider now the arrangement of Figure 4.6. Here,

\[ Z_f = \frac{R_2}{SC_1} \quad (4-17) \]

and

\[ Z_i = R_1 \quad (4-18) \]

Substituting the values of (4-17) and (4-18) in equation (4-7) yields a transfer function for the circuit of Figure 4.6. As with the perfect integrator, amplifier gain is assumed to be much greater than unity.

\[ G(S) = \frac{e_o}{e_1(S)} = -\frac{R_2}{R_1} \frac{1}{bS + 1} \quad (4-19) \]

where \( b = R_2C_1 \) \quad (4-20)

Information of particular interest to the reader is summarized in Table 4.1. Our improved theory can best be explained by analyzing the requirements each circuit imposes on the system towards achieving stability.
Figure 4.4

Figure 4.5

Figure 4.6
<table>
<thead>
<tr>
<th>Fig.</th>
<th>Circuit</th>
<th>Transfer Imped. Function</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| a.   | ![Circuit](image) | $-\frac{1}{RCS}$ | Pure Integrator.  
(A > 0) |
| b.   | ![Circuit](image) | $-\frac{1}{bS + \frac{1}{A}}$ | Imperfect (Finite Gain) Integrator.  
b = RC |
| c.   | ![Circuit](image) | $-\frac{R_2}{R_1} \frac{1}{bS + 1}$ | Parallel R-C Feedback Network.  
(A > 0)  
b = $R_2C_1$ |

Table 4.1
Let us initially consider the circuit in Figure a., namely, the pure integrator, whose transfer function is given as \(-\frac{1}{RCS}\). \(G(S)\) reduces to \(-\frac{1}{S}\) for a one second time constant \((R = \text{one megohm}, C = \text{one microfarad})\). The roots of the system's characteristic equation are determined by substituting \(G(S)\) in equation (4-6) and solving for \(S\).

\[
1 - \frac{\lambda_j}{3} = 0, \quad j = 1, 2 \quad (4-21)
\]

\[
S = \frac{\lambda_j}{3}
\]

For stability, the real parts of \(\lambda_j\) must be negative, or

\[
a_{11} + a_{22} < 0 \quad (4-23)
\]

\[
a_{11}a_{22} - a_{12}a_{21} > 0 \quad (4-24)
\]

since, the \(\lambda_j\) are the latent roots of

\[
\begin{vmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22} - \lambda
\end{vmatrix} = 0 \quad (4-25)
\]

or

\[
\lambda_{1,2} = \frac{a_{11} + a_{22} \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2} \quad (4-26)
\]

Consider next the circuit in Figure b., with a transfer impedance function equal to \(-\frac{1}{bS + \frac{1}{A}}\). Substituting in equation (4-6) and solving for \(S\) we obtain
\[ 1 - \frac{\lambda_j}{3\left(bS + \frac{1}{A}\right)} = 0 \]  
\[ 3bS + \frac{3}{A} = \lambda_j \]  
or \[ S = \frac{1}{b} \left( \frac{\lambda_j}{3} - \frac{1}{A} \right) \quad A > 0 \]  
j = 1, 2.

The integrator sign reversal is understood to be contained in \( a_{11}, a_{12}, a_{21}, \) and \( a_{22}. \) From (4-29) we see that \( S \) can stay negative even for slightly positive \( \lambda_j. \)

Our improved theory completely agrees with the results of laboratory tests on the ultrasifable system.

Second order system stability was tested with one megohm resistors placed in parallel with the feedback capacitators. The result was the system stabilized for all combinations of parameter settings. For an explanation regarding this situation we must apply the principles of our improved theory.

The transfer impedance function of the circuit is given in Figure c., Table 4.1. Substituting this term in equation (4-6) and solving for roots \( S, \) yields

\[ 1 - \frac{\frac{R_2}{R_1}}{3\left(bS + 1\right)} = 0. \quad j = 1, 2. \]  
\[ \frac{R_2}{R_1} = 1 \]  
\[ b = R_2C_1 = 1 \]
\[3(S + 1) = \lambda_j \quad (4-33)\]

\[S = \frac{\lambda_j}{3} - 1 \quad (4-34)\]

An examination of equation (4-34) reveals that \(\lambda_j\) can possibly assume large positive values and still maintain negative \(S\), thereby accounting for the omni-stable states.

4.5 The Ultrastable System, Third Order. Requirement (1), Paragraph 3.3, specified a summation of \((a_{11} + a_{22} + a_{33})\) less than zero for stability. Contrary to this, however, the system performed in accordance with our improved theory as depicted in Figures 4.6, 4.7 and 4.8.

Non-stable configuration.

\[(a_{11} + a_{22} + a_{33}) > 0.\]

Figure 4.6
Figure 4.7

Stable configuration \((a_{11} + a_{22} + a_{33}) > 0\).

Figure 4.8

Stable configuration \((a_{11} + a_{22} + a_{33}) \ll 0\).

The results of three recorded runs are shown in Figures 4.9, 4.10 and 4.11. During each run, fixed parameter values were altered at various times to study the effect on stability. The occurrence of parameter change \(n\) is denoted by symbol \(S_n + 1\) along the horizontal axis. Associated with 4.9, 4.10 and 4.11 are Figures D through I on Chart 4.1. The relationship between parameter change and symbol is shown in Tables 4.2, 4.3 and 4.4.
Run 1, Figure 4.9.

<table>
<thead>
<tr>
<th>Time</th>
<th>Parameter Change</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>None</td>
<td>Initial settings of Figure D.</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$a_{32} = 0$</td>
<td>System remains unstable.</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$a_{32} &lt; 0$</td>
<td>System stabilizes.</td>
</tr>
<tr>
<td>$S_4$</td>
<td>None</td>
<td>After stabilizing, outputs begin to increase.</td>
</tr>
<tr>
<td>$S_5$</td>
<td>None</td>
<td>System regains stability after five seconds. Final parameter settings shown in Figure E.</td>
</tr>
</tbody>
</table>

Table 4.2

Run 2, Figure 4.10.

<table>
<thead>
<tr>
<th>Time</th>
<th>Parameter Change</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>None</td>
<td>Initial settings of Figure F.</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$a_{33} = 0$</td>
<td>This change produced a stable pattern.</td>
</tr>
<tr>
<td>$S_3$</td>
<td>None</td>
<td>System stability. Final parameter settings shown in Figure G.</td>
</tr>
</tbody>
</table>

Table 4.3
Run - 3, Figure 4.11.

<table>
<thead>
<tr>
<th>Time</th>
<th>Parameter Change</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>None</td>
<td>Initial settings of Figure H.</td>
</tr>
<tr>
<td>S₂</td>
<td>( a_{22} &lt; 0. )</td>
<td>System remained unstable for approximately 80 seconds prior to this change.</td>
</tr>
<tr>
<td>S₃</td>
<td>None</td>
<td>System stability. Final parameter settings shown in Figure I.</td>
</tr>
</tbody>
</table>

Table 4.4

Additional matrix configurations are illustrated in Figures J, K and L of Chart 4.1. On each of the associated runs, a single parameter value was altered to attain system stability. The initial and final settings are lettered U and S, respectively.

4.6 The Ultrastable System, Fourth Order. One recorded run, Figure 4.12, is included to illustrate the performance of a fourth order system. Following the application of power, a condition of instability existed. One parameter change was required to produce a stable behavior pattern. Table 4.5 and Figures M and N on Chart 4.1, contain pertinent information with reference to this run.
Run - 1, Figure 4.12.

<table>
<thead>
<tr>
<th>Time</th>
<th>Parameter Change</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>None</td>
<td>Initial settings of Figure M. Application of power.</td>
</tr>
<tr>
<td>S2</td>
<td>$a_{44} = 0.$</td>
<td>Introduction of stable pattern.</td>
</tr>
<tr>
<td>S3</td>
<td>None</td>
<td>System stability. Final parameter settings shown in Figure N.</td>
</tr>
</tbody>
</table>

Table 4.5

The parameter settings shown in Figures O through R, Chart 4.1, produced stable behavior patterns.

A rigid analysis of fourth order system operation becomes difficult due to the complexity of the associated stability criteria.
CHAPTER 5

DISCUSSION

5.1 Adaptive Behavior. The development of adaptive behavior patterns by man and animal alike, throughout the ages, must certainly be considered as being the primary factor affecting their survival. Ashby defines a form of behavior as adaptive if it maintains the essential variables (for survival) within physiological limits. In relating adaptation to stability, he states, "adaptive behavior is equivalent to the behavior of a stable system, the region of the stability being the region of the phase space in which all the essential variables lie within their normal limits." Ashby further states that "stability is necessary for survival unless the environment is wholly inactive."  

Concepts associated with these patterns characterize a class of mechanical feedback systems we know today as adaptive control systems. The ability of our ultrastable

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9Ibid., pp. 64-65.
10Ibid.
device to seek a condition of stability by automatically rejecting unstable behavior patterns certainly places it in the aforementioned category.

5.2 Adaptive Control Systems. Aseltine, Mancini and Sarture in their article titled, A Survey of Adaptive Control Systems, list five classes that cover recent developments. Here, Ashby's device is placed in the System-Variable Adaptation category.

The ultrastable system is unique in that it is the only autonomous system. All others described require external inputs as a prelude to becoming adaptive, or self-adjusting. It seems apparent, from a practical viewpoint, that industrial requirements for an autonomous adaptive control system would be few, if any, in comparison to the demands for a non-autonomous device. In its present form, the system serves only to demonstrate the principle of ultrastability, as characterized by its intrinsic purposeful behavior. However, as pointed out in the introduction, it can be utilized in a process control system to establish desired set point values.

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Before discussing ultrastable system considerations, let us first examine some philosophies associated with the design of adaptive control devices.

5.3 Adaptive-Control Philosophies. Adaptive systems always compensate, automatically, for either of two basic types of disturbances: 12

1) Changes in the system input, i.e., the actual character of the driving signal, or a change in the relationship between several different input signals.

2) Changes in system parameters, such as those due to component aging, load fluctuations or environmental variations.

Design techniques for implementing system adaptation involve the consideration of several characteristics, namely, a) System function, b) What the system adapts itself to, c) The means of adaptation, and d) Criterion for adjustment. When applied to our ultrastable device, these four factors take on the following meanings:

a) Demonstrates purposeful behavior.

b) Changes in system parameters and structure.

c) System parameter alterations.

d) Stable equilibrium.

Perhaps the best approach towards familiarizing the reader with the philosophies of adaptive control systems would be by describing the characteristics of several representative types.

5.4 Model-Reference Type System. Here, the input command signal is also fed to a model unit whose performance is characterized by prescribed specifications. The block diagram of Figure 5.1 illustrates an application involving a reference model. As shown, it depicts adaptation for an aircraft, wherein the surface controls affecting yaw, pitch and roll movement are self adjusting through the action of the adaptive loop. Normally, system specifications are incorporated into the model. If performance functions for both the model and system are alike, their outputs will be the same. In most instances, however, they will not be identical and an error response will be developed. This error can then be used to generate a signal for actuating a control surface mechanism in an attempt to minimize the error function. This is accomplished by altering a controllable parameter (or in some cases the composition of the incoming signal). The
Figure 5.1
performance error, then, is an indication of the system's deviation from such specified characteristics as response time, damping and dynamic and static errors.

There are a number of advantageous features associated with the reference model type system that might be worth mentioning at this time.\(^\text{13}\)

1) Information required for making the system adaptive is obtained through normal operating inputs to the system.

2) Adaptive control equipment is making adjustments to a control system whose feedback control loops are closed independently of the adaptive controls rather than being closed through the adaptive equipment. If the latter equipment fails, the control system still remains as a closed loop system.

3) Adaptive control features techniques can be added to an existing control system without major alteration of the control signal paths.

4) There is the possibility of treating stabilization and transient response requirements separately in

a multi-loop control system, wherein separate models can be used for the various loops. In a multi-loop system, the adaptive features can often readjust necessary controllable parameters to result in satisfactory system performance, even though complete failure of the adaptive controls for one parameter occurs.

5) Noise on the input command signal is treated approximately the same by both the model and the system so that the presence of noise should not degrade desired performance.

6) It is theoretically possible to make the model "learn" by storing the solutions as they are obtained so as to make use of them if the same conditions occur again.

Other examples of model-reference type systems are contained in Wright Air Development Center Technical Note 58-330.14

5.5 Systems That Adapt Through Variable Sensing.

Flugge-Lotz and Taylor describe a linear second order system with added possible combinations of proportional

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and derivative feedback. A block diagram illustrating their principle appears in Figure 5.2. The combination of $R$ and $Y$ are controlled by binary switching logic that results from the sensing of two variables, $y$ and $e$ (output and error voltages respectively) in addition to their derivatives, $y'$ and $e'$. System response to an arbitrary input is improved by this method.

Another sensing type system, designed to improve aircraft response is shown in Figure 5.3. Here, the controlled variable is the aircraft's angular rate information. Frequency deviations from the desired closed loop frequency are felt by the lead and lag networks. The gain of the multiplier unit is varied accordingly, the end result being an increase, or decrease, in the system's response frequency in an adaptive fashion.

5.6 Ultrastable System Considerations. Although limited with regard to its practical applications the ultrastable system, characterized by unique behavior, certainly warrants a great deal of discussion in addition to further investigation.

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Figure 5.2

Figure 5.3
When discussing applications for the ultrastable system one must consider, a) the probability of stability, b) number of switching actions to reach stability, and c) the multistable system. As we shall soon see, the multistable system makes the principle of ultrastability practically realizable.

5.7 Probability of Stability. Given an autonomous system of n variables, one might readily ask the question, "What is the probability of its stability if all combinations of coefficients $a_{ik}$ are equally likely to occur?" Criteria derived in Chapter 3 and later modified by an improved theory, specified conditions necessary for achieving stability. Laboratory results further verified this criteria. It was pointed out that the real parts of the latent roots, $\lambda_j$, of the determinantal equation:

$$|a_{ij} - \delta_{ij}\lambda| = 0 \quad \delta_{ij} = 1 \text{ for } i = j$$

$$\delta_{ij} = 0 \text{ for } i \neq j$$

(5-1)

can be slightly greater than zero for a stable system. Thus, the probability of stability for such a system as described above is the probability that the latent roots of the matrix $a_{ij}$ associated with it will have real parts that are slightly greater than zero.
The following statement can be made regarding the probability of stability for a randomly assembled system with \( n \) variables: To increase the probability, choose favorable values for the diagonal elements of the matrix. As the number of variables increase, i.e., the system becomes larger, the probability will decrease accordingly.

5.8 Number of Switching Actions to Reach Stability.

For the \( n \)-th order autonomous system, there will be as many different patterns of behavior as there are different values of parameter, \( \delta \). The phase space pattern corresponding to any one setting is termed by Ashby as the field of lines of behavior. The field analogous to a stable behavior pattern, i.e., one existing after the last parameter change has resulted in system stability is called a terminal field. Naturally, the designer is interested in the number of switching actions required to reach the terminal field. The allowable time is governed primarily by system application. Performance of the ultrastable device in the laboratory indicated the number of switching actions can vary, due to the system's inherent random characteristics, for identical settings of the manual selector component. For example, consider a matrix configuration that produces system stability. Upon varying
the setting of one manual selector unit, the system reverts to a state of instability. Knowing the probability of stability is zero, the selector unit is returned to its stable setting. This procedure is repeated several times. Laboratory results have shown that the length of time to restabilize varies between successive runs. Furthermore, the number of switching actions increases as the number of variables increase. In other words, the search for the terminal field will be long for complex systems. Tsien specifies the average number of switching actions as follows:

\[
N = \sum_{m=1}^{\infty} \frac{mpq^{m-1}}{(1-q)^{-1}} = \frac{(1-q)^{-2}}{1-q} = \frac{1}{1-q} = \frac{1}{p}
\] (5-2)

where, \( p \) = the ultrastable system's probability of stability,

\( q = 1 - p \), the probability of not reaching the terminal field,

and, \( m \) = the \( m \)-th switching action.

It is entirely possible that an ultrastable system comprising a large number of variables might require a very long period of time for reaching a stable state. Ashby has approximated the probability of stability by:  

\[ p = \frac{1}{2^n} \]

where, \( n = \) number of variables.

Also, in the previous section we saw where the average number of switching actions involved was given as:

\[ N = \frac{1}{p} \]

In its present configuration, the laboratory system can switch coefficient values at the rate of approximately two per second. Assuming the system contains as many as 50 variables, then the time to achieve stability is:

\[ P = \frac{1}{2^{50}} \]

\[ N = \frac{1}{P} = 2^{50} = 11 \times 10^{14} \]

and,

\[ t \approx 5.5 \times 10^{14} \text{ seconds} \]

or

\[ t \approx 6.4 \times 10^9 \text{ days} \]

Obviously, a system taking this long to stabilize would be virtually useless for any application.

\[ ^{18} \text{W. Ross Ashby, op. cit., p. 141.} \]
5.9 Multistability. The necessity for increasing the probability of stability in the complicated, higher order system resulted in the development of the multistability concept. Ashby observed that only a relatively small number of variables, in such a system, are actually affected by any single disturbance or change in environmental conditions. Thus, if these variables could be isolated to form their own ultrastable system the probability would be increased. For example, consider our laboratory system with four variables. The following comparisons can be made.

\[ n = 4 \quad n = 50 \]
\[ P = \frac{1}{16} \quad \frac{1}{11 \times 10^{14}} \]
\[ N = 16 \quad 11 \times 10^{14} \]
\[ t = 8 \text{ Sec.}^* \quad 5.5 \times 10^{14} \text{ Sec.}^* \]

*Assume two switching actions per second.

The time now required for the disturbed variables to reach their terminal field is 16 seconds. For a system comprising 50 variables, an entirely new set of operating conditions are realized in 200 seconds (16 x 12.5). Two hundred seconds is a vast improvement in comparison to \(5.5 \times 10^{14}\).
The process of grouping the disturbed variables into separate sub-systems is called by Ashby, *the dispersion of behavior*. The multistable system is composed of ultrastable sub-systems, organized with the possibility of dispersion. Its relatively short time in achieving stability makes the multistable a more realistic proposition than the ultrastable system.

5.10 **Recommended Further Study.** Further study should involve the design, construction and investigation of a multistable feedback system. This would include a technique for regrouping the variables in a logical fashion. Upon completion, the system should provide a means for solving decision-making problems and studying criteria dealing with stability.


ABSTRACT

This thesis describes the performance of an adaptive feedback system designed to behave in accordance with W. Ross Ashby's principle of ultrastability. Included is a brief description of Ashby's homeostat, the machine he designed to demonstrate his principle. The construction of a laboratory system by the student, to demonstrate the same principle is covered in detail.

Analog computer components implement four randomly interconnected feedback stages, each comprised of an integrator and dead space device. Coefficient values, analogous to the settings of fixed and stepping switches, when properly chosen provide a stable dynamic system. An overload condition from any stage initiates a switching action by the stepping component(s) controlled by the stage. In this manner, unstable behavior patterns are continually rejected by automatically selecting new coefficient values. Likewise, desired stable patterns are retained.

Routh's method was employed to analyze system stability. Failure by the system to perform in accordance with the derived criteria required the development of an
improved theory. Characteristics of the imperfect (finite gain) integrator were permitting the system to stabilize for coefficient values that theoretically comprised unstable configurations.

Output variations as recorded on a four channel Sanborn instrument illustrate the performance of an ultrastable system while seeking stability.

Two factors, namely, the probability of stability and number of switching actions to reach stability reduce the usefulness of higher order systems. To make the principle of ultrastability practically realizable, one must then consider the multistable system.