AN EDUCATIONAL DEVICE FOR RELATING ELECTROMAGNETIC TORQUE TO VARIATIONS IN SELF-INDUCTANCE

by

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Chapter 1

1.1 STATEMENT OF THE PROBLEM

In recent years, there has been increased emphasis placed on the more sophisticated mathematical approaches to circuit and machine theory. Such tendencies have made it increasingly difficult for the student to grasp the fundamental physical concepts upon which the theory is based. This is especially true concerning the relationship between electromagnetic torque and the variation of the inductance parameter in rotating machines. By using the principles of energy storage and conversion, the problem is reduced to the development of a comparatively simple device which can be used to physically demonstrate the interrelations of these phenomena, relating them to their mathematical representations.

1.2 APPROACH TO THE PROBLEM

During the initial phases of this investigation, a rotary electromagnet, similar to the one shown in Fig. 1.1, was considered for demonstrating the variation of torque with inductance. The device was to be of laminated construction to avoid excessive eddy currents

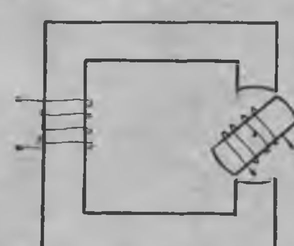

Fig. 1.1 Rotary electromagnet

and make a-c inductance measurements possible. However, subsequent consideration indicated that the mechanical difficulties which would be encountered in building the structure such as clearances, alignment, bearing, etc., warranted selection of a similar commercial device which could be suitably modified.

Of several motors available, a one-eighth horsepower d-c motor with laminated stator was chosen for modification to give suitable inductance and torque vs displacement curves. It was provided with a laminated rotor having two salient poles on which was placed a winding whose ends terminated on slip-rings. The modified motor is shown in Fig. 1.2. Exploring coils for fluxmeter readings were placed on the rotor and stator members.

Inductance measurements were made utilizing stator and rotor windings in different combinations for a complete cycle of variation--one-half a revolution. From 60 cycle a-c measurements with conventional indicating voltmeters, ammeters, and wattmeters, the self-inductances were calculated on a reactive power basis from which the torque vs angular displacement characteristics were obtained. Similar inductance curves were obtained for d-c measurements using a fluxmeter.

The effects on torque of the variation in inductance

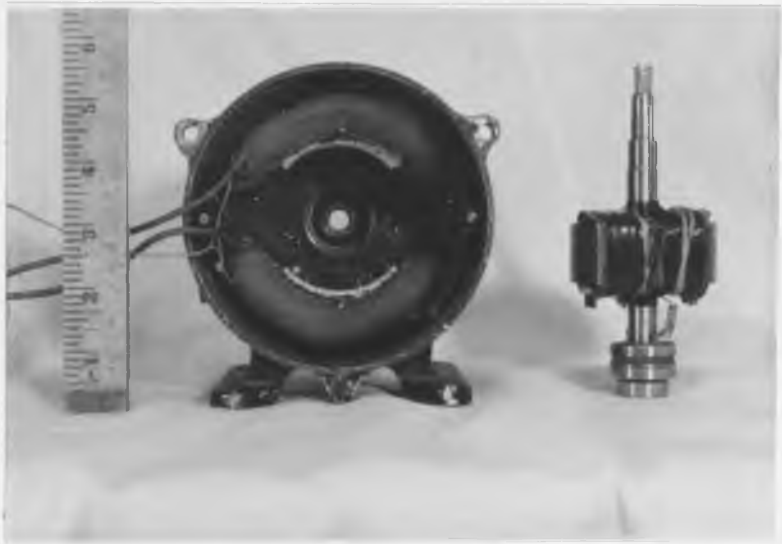

Fig. 1.2 The modified one-eighth horsepower d-c motor showing windings and slip-rings.

with rotor displacement due to the variation of stored energy in the magnetic circuit were calculated and compared with measured values. This was done: (1) by using incremental changes of inductance with displacement; and (2) by actually differentiating the analytical expression obtained for the inductance curve using Fourier analysis and numerical integration.

This thesis is appended with a glossary of the symbols--with their corresponding dimensions--used in the text. (See page 33).

Chapter 2

THE PRINCIPLES OF ENERGY STORAGE AND CONVERSION

2.1 ELECTROMECHANICAL SYSTEMS

Two fundamental principles were considered in this approach to the study of electromechanical energy conversion devices. One is the law of conservation of energy which states that energy is neither created nor destroyed but merely changed in its form. The other is Faraday's law of electromagnetic induction.

In the electromechanical system, electrical energy may be transformed into heat (i^2r losses), losses due to eddy current and hysteresis phenomena in the physical structure, or converted into mechanical motion. Mathematically, the foregoing leads to the basic differential equation,

$$d\, W_{elect} = d\, W_{fld} + d\, W_{mech} \qquad (2.1)$$

where $d\, W_{elect}$ is the net electrical input after i^2r losses; $d\, W_{fld}$ accounts for energy absorbed by the field; and $d\, W_{mech}$ is the net internal energy realized as mechanical energy including the mechanical losses. The equation holds for both motor and generator action although convention has established positive values of electrical and mechanical quantities for the former.

Faraday's law of magnetic induction,

$$e = d\lambda / d\, t \qquad (2.2)$$

relates an emf to a circuit in which the flux linking it is time-varying. In this thesis, the rationalized MKS system of units will be used wherein λ is expressed in weber turns.

2.2 THE SINGLY EXCITED SYSTEM

Consider a singly excited magnetic system, i.e. one which has but one series winding, such as that shown in the following simplified diagram.

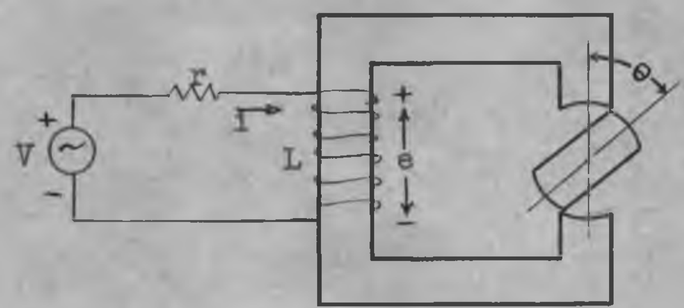

Fig. 2.1 A singly excited magnetic system.

Applying Kirchoff's law to the electrical circuit,

$$v = ir + d\lambda / dt \qquad (2.3)$$

where $d\lambda/dt$ is the induced emf in the winding due to the flux linking it. But since $\lambda = Li$, where L is in henries,

$$v = ir + d(Li)/dt$$

$$= ir + L\,di/dt + i\,dL/dt \qquad (2.4)$$

The electrical power input to the circuit expressed in watts is

$$p = vi$$

$$= i^2 r + Li\,di/dt + i^2\,dL/dt \qquad (2.5)$$

After heat losses, the differential electrical energy available for further conversion is

$$d\ W_{elect} = p\ dt$$
$$= Li\ di \neq i^2\ dL \qquad (2.6)$$

The energy in joules, stored in the field is given by

$$W_{fld} = \tfrac{1}{2}\ Li^2 \qquad (2.7)$$

The power required to change the stored energy written in differential form is

$$d\ W_{fld} = Li\ di \neq \tfrac{1}{2}\ i^2\ dL \qquad (2.8)$$

From Eqs. 2.1, 2.6, and 2.9, the expression for d W_{mech} is

$$d\ W_{mech} = \tfrac{1}{2}\ i^2\ dL \qquad (2.9)$$

which is the differential of power converted to mechanical energy. This relation may be expressed in terms of linear motion recalling that

$$d\ W_{mech} = f\ dx \qquad (2.10)$$

where f is a developed force acting through a distance dx in the direction of X.

Further, from Eqs. 2.9 and 2.10,

$$f\ dx = \tfrac{1}{2}\ i^2\ dL \qquad (2.11)$$

or,

$$f = \frac{i^2}{2}\ dL/\ dx \qquad (2.12)$$

It follows from Eq. 2.12 that when the developed force, in newtons, is positive, dL/ dx is positive and the device gives up mechanical energy; however, when the converse is true, the force is negative and mechanical energy must be supplied to the device. Stated in terms of energy relations, the equation indicates that for a given current, developed force seeks to store the maximum energy in the field. Equation 2.12 is general and applies to any singly excited system.

The systems in this investigation were operated below the region of magnetic saturation in order that the variation of inductance be a function of angular displacement only. Generally, economical design practice dictates that commercial electromechanical devices operate around the knee of the magnetic saturation curve. In such cases, saturation effect can be accounted for by using the relation[1]

$$f = \tfrac{1}{2}\,(L/L_0)\, i^2\, \partial L_0 / \partial x \qquad (2.14)$$

where L_0 is the unsaturated value of inductance and L is the value at the point considered.

2.3 MAGNETIC CIRCUIT RELATIONSHIPS

The inductance, L, may be expressed in terms of other magnetic parameters. Ohm's law for the magnetic circuit states

$$\mathcal{F} = \mathcal{R}\phi$$
$$= \phi / \mathcal{P} \qquad (2.15)$$

where $\mathcal{F}$ = MMF = Ni = ampere turns

$\mathcal{R}$ = magnetic reluctance = ampere turns per weber

$\mathcal{P}$ = permeance, the reciprocal of reluctance

ϕ = flux in webers

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[1] R. E. Doherty and R. H. Park, "Mechanical Force Between Electric Circuits," Trans. A.I.E.E., Vol. 45, Feb. 1926, pp. 241-43.

Again,

$$L = \lambda / i$$

$$= N\phi / i \qquad (2.16)$$

where ϕ is the equivalent flux linking all N turns of a winding. The flux is also given from Eq. 2.15 by

$$\phi = Ni\mathcal{P} \qquad (2.17)$$

Substituting for ϕ in Eq. 2.16 yields

$$L = N^2 \mathcal{P}$$

$$= N^2 / \mathcal{R} \qquad (2.18)$$

Equation 2.18 may be substituted into Eq. 2.13 to give the following force relation:

$$f = \frac{1}{2} (Ni)^2 \, d\mathcal{P} / dx$$

$$= \frac{1}{2} \mathcal{F}^2 d\mathcal{P} / dx \qquad (2.19)$$

Or, in terms of reluctance,

$$f = -\frac{1}{2} \mathcal{F}^2 / \mathcal{R}^2 \quad d\mathcal{R} / dx$$

$$= -\frac{1}{2} \mathcal{F}^2 \mathcal{P}^2 \quad d\mathcal{R} / dx \qquad (2.20)$$

Also from Eq. 2.15,

$$f = -\frac{1}{2} \phi^2 \, d\mathcal{R} / dx \qquad (2.21)$$

For rotating devices, torque and angular displacement are analogous to force and linear displacement; hence

$$\mathcal{T} = -\frac{1}{2} \phi^2 \, d\mathcal{R} / d\theta \qquad (2.22)$$

It follows, then, that the torque developed in the electromechanical device under consideration depends only on the equivalent flux and the angular rate of change of the reluctance. For a given value of flux, the torque is such as to make the stored energy a minimum.

Chapter 3

MEASUREMENTS

3.1 PHYSICAL ARRANGEMENTS

A protractor was mounted on the housing of the rotary electromagnet to make possible the measurement of angular displacement as shown in Fig. 3.1. A pointer to indicate angular displacement was attached to a cylindrical collar on the rotor shaft. The arrangement of the protractor and pointer was such that when the magnetic axes of rotor and stator coincided at the point of minimum reluctance, the displacement was approximately zero. As Fig. 3.1 illustrates, an extendable clamp positioned against an auxiliary lever on the pointer collar maintained any desired point of angular displacement while inductance measurements were being made.

To make torque measurements, a lever arm was fastened to a second collar which turned on the indicator collar and could be locked to it at any point by means of a thumb set-screw. A two spring-scale arrangement was used to measure the force exerted on the lever arm by the developed torque. Figure 3.2 shows the torque-measuring apparatus in operation.

Fig. 3.1 Physical arrangements for making inductance measurements.

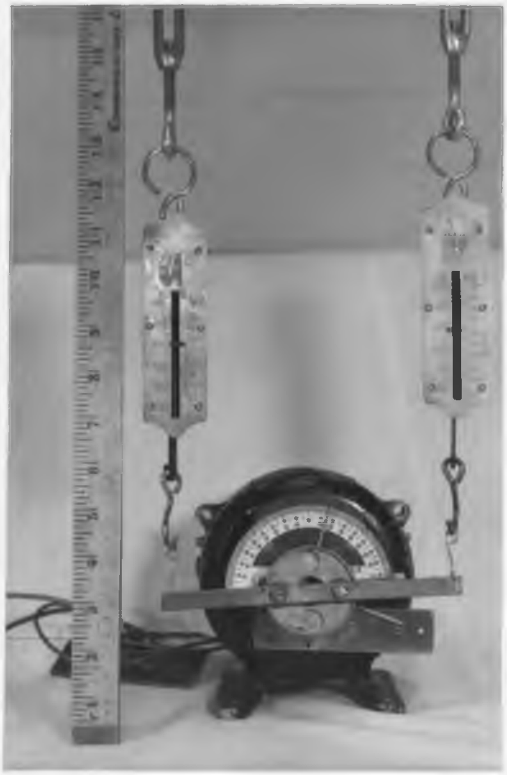


Fig. 3.2 Physical arrangements for measuring torque.

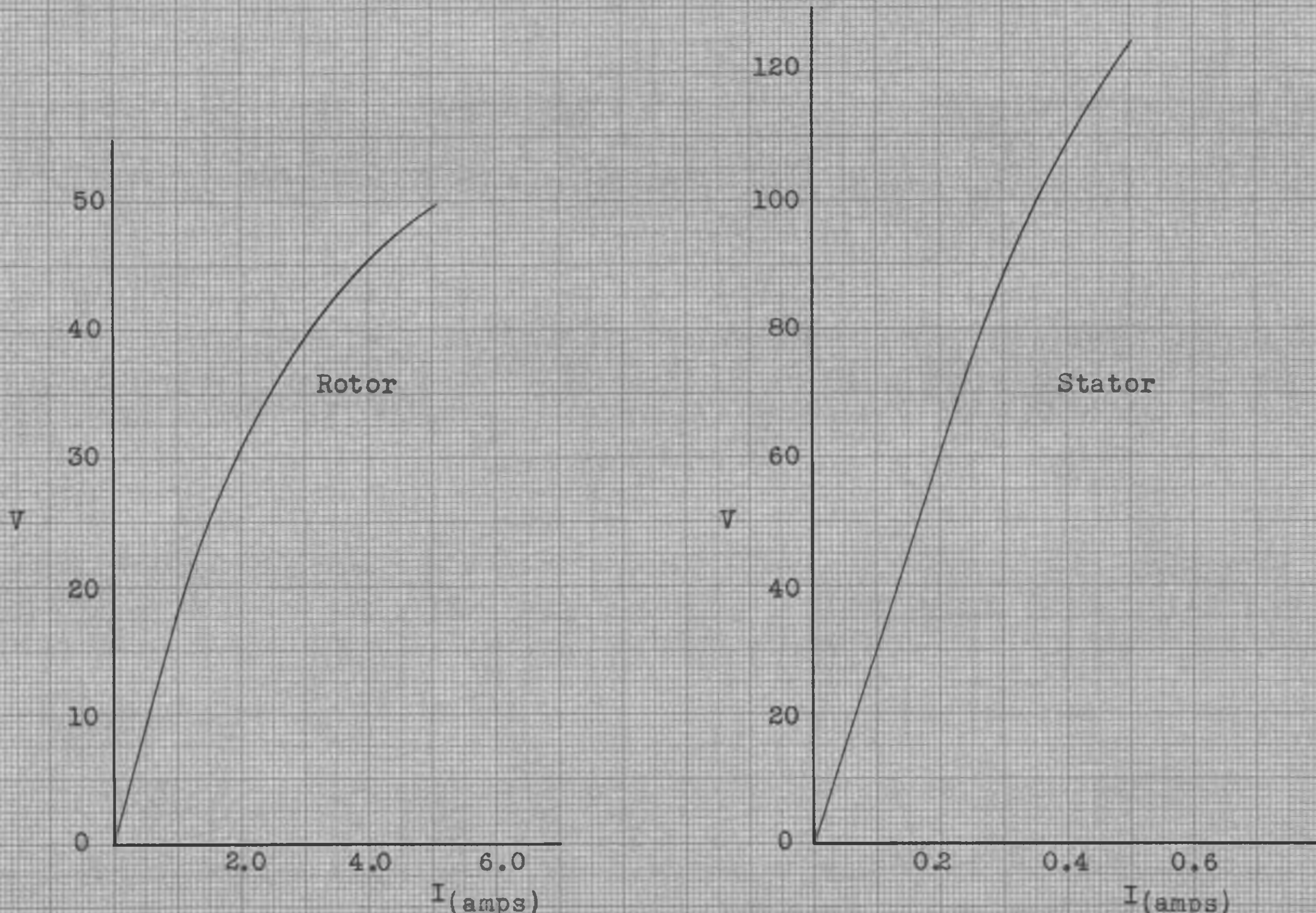
3.2 DETERMINATION OF INDUCTANCE

Prior to making measurements of inductance, the saturation characteristics of the rotor and stator were obtained to determine their limits of linear magnetization. The a-c characteristics of applied voltage vs exciting current are shown in Fig. 3.3. Inductance measurements could then be made based on operation over the linear portion of the magnetization curves.

Using a wattmeter, ammeter, and voltmeter arrangement, the reactive power inputs to the windings of the rotor, the stator, and the rotor and stator in series were measured for 10° increments over a cycle of variation, i.e. one-half a revolution. Inductance was then calculated from the relationships

$$L = X_1 / \omega$$
$$= \left[(V/I)^2 - (P/I^2)^2 \right]^{\frac{1}{2}} / \omega \qquad (3.1)$$

where V, I, and P are the quantities read from the voltmeter, ammeter and wattmeter respectively. This was done for each point of angular displacement and the results were plotted for three respective winding arrangements. The inductance vs angular displacement curves are shown in Fig. 3.4. The decrease in inductance during the second half cycle of the curve for the inductance of the rotor and stator windings in series, is due to the effects of magnetic saturation. At an angular displacement of 90°, the MMF of the windings



A-C Saturation Characteristics

Applied Voltage vs Exciting Current

Fig. 3.3

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Inductance Curves of Two-Pole-Rotor Electromagnet

A-C (60 cycle) Excitation

Fig. 3.4

changes from series aiding to series opposing, driving the operating points of the stator well into the saturated portion of its magnetization characteristic.

Efforts were made to measure flux with search coils and a fluxmeter when direct current was applied to the windings so that the inductance could be computed on a basis of flux linkages per ampere. The results of this effort are shown graphically in Fig. 3.5.

The stator inductance characteristics from Figs. 3.4 and 3.5 are compared in Fig. 3.6. The curves show that the values based on flux measurements are generally lower than the corresponding values for alternating-current excitation due to the search coils not sampling representative values of flux.

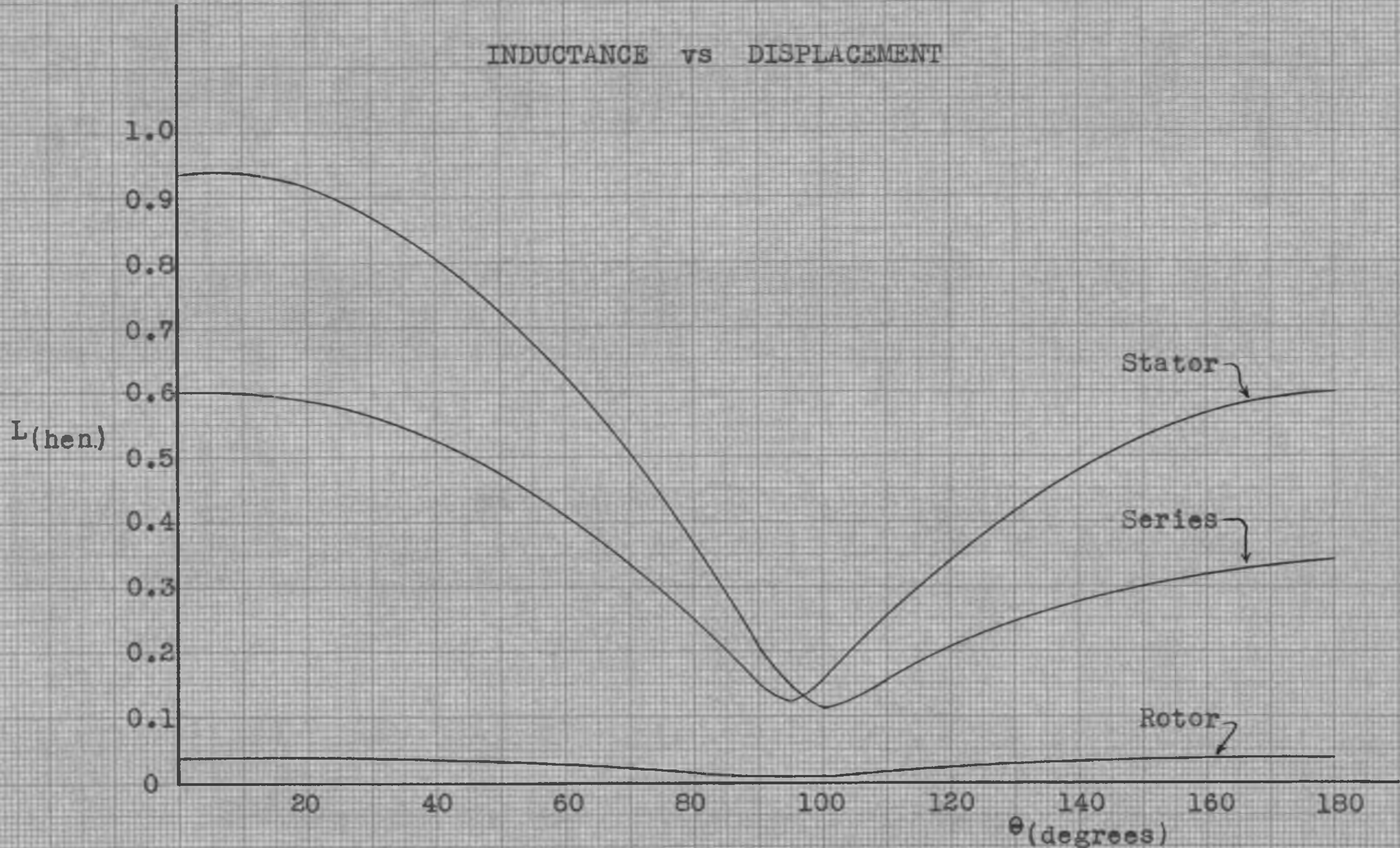
Mutual-inductance may be obtained using Fig. 3.4 and the relationships

$$L_s + L_r + 2\,M = L_{(\text{series aiding})} \qquad (3.2)$$

and

$$L_s + L_r - 2\,M = L_{(\text{series opposing})} \qquad (3.3)$$

where the values of L are taken from the respective stator, rotor or series (aiding or opposing) inductance curves. The variation of mutual-inductance with angular displacement is found by applying Eqs. 3.2 and 3.3 at each point of displacement of the curves.



Inductance Curves of Two-Pole-Rotor Electromagnet

DC Excitation

Fig. 3.5

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Stator Self-Inductance Curves
of
Two-Pole-Rotor Electromagnet
Fig. 3.6

3.3 MEASUREMENT OF TORQUE

Using the set-up shown in Fig. 3.2, d-c torque was measured for the three magnetic circuit arrangements described in section 3.2. The magnetomotive force, based on 200 rotor and 800 stator turns, was maintained constant throughout this portion of the experiment. All torque measurements were made with direct current obtained from a d-c power supply and passed through a reversing switch and ammeter. The torque was measured at 10° intervals over one-half a cycle of variation--one quarter revolution--except for the case in which the rotor winding was excited. The latter was taken through a full cycle of variation. At each point the current was reversed and an average reading taken to reduce errors caused by residual magnetism. The current was kept constant for each set of data.

The indicator and shaft were turned through the proper interval and the lever-arm collar locked to the indicator collar. Adjustment of the former was made until the desired displacement was achieved.

Force was measured by means of two spring-scales suspended on turnbuckles--see Fig. 3.2--which were adjusted to read half-scale at zero torque. The difference between the two readings yielded the net force exerted on the rotor at each point. The applied torque was the product of this force and the length of the lever arm. The results of these measurements are shown in Fig. 3.7 as applied torque vs displacement curves.

3.4 CALCULATION OF TORQUE

Two methods of calculating applied torque, both based on the curves of self-inductance shown in Fig. 3.4, were used in this development. The first involved using incremental values of inductance and displacement. The equation for applied torque, when written in terms of increments and angular displacement, is

$$\mathcal{T} = -\tfrac{1}{2}i^2 \Delta L/\Delta\theta \qquad (3.4)$$

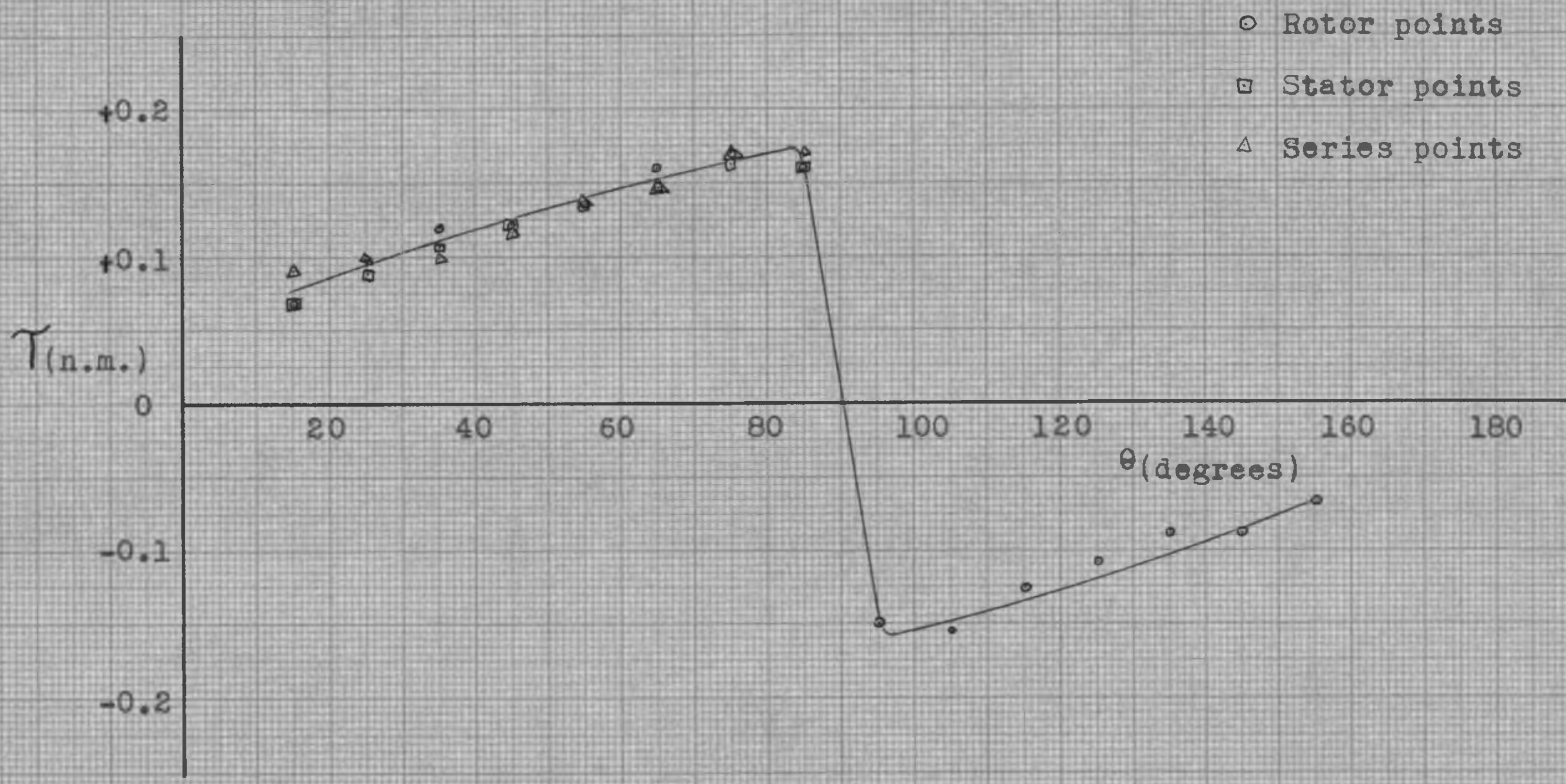
The increments, ΔL, were taken as the difference between the points of the stator inductance curve for $\Delta\theta = 10^\circ$. In order that the measured and calculated torques could be compared, the same value of direct current as was used to measure the stator torque was used to compute the applied torque characteristic shown in Fig. 3.8-A. Equation 3.4 was used for this purpose.

The other method used for calculating the torque, utilizing the equation

$$\mathcal{T} = -\tfrac{1}{2}\, i^2\, dL/d\theta \qquad (3.5)$$

required that an analytical expression be found for the stator inductance curve and differentiated with respect to angular displacement. Substituting these results--and the same values of direct current as previously used--into Eq. 3.5, yielded the curve shown in Fig. 3.8-B. Determination

TORQUE vs DISPLACEMENT



Measured Torque (DC Excitation)

Fig. 3.7

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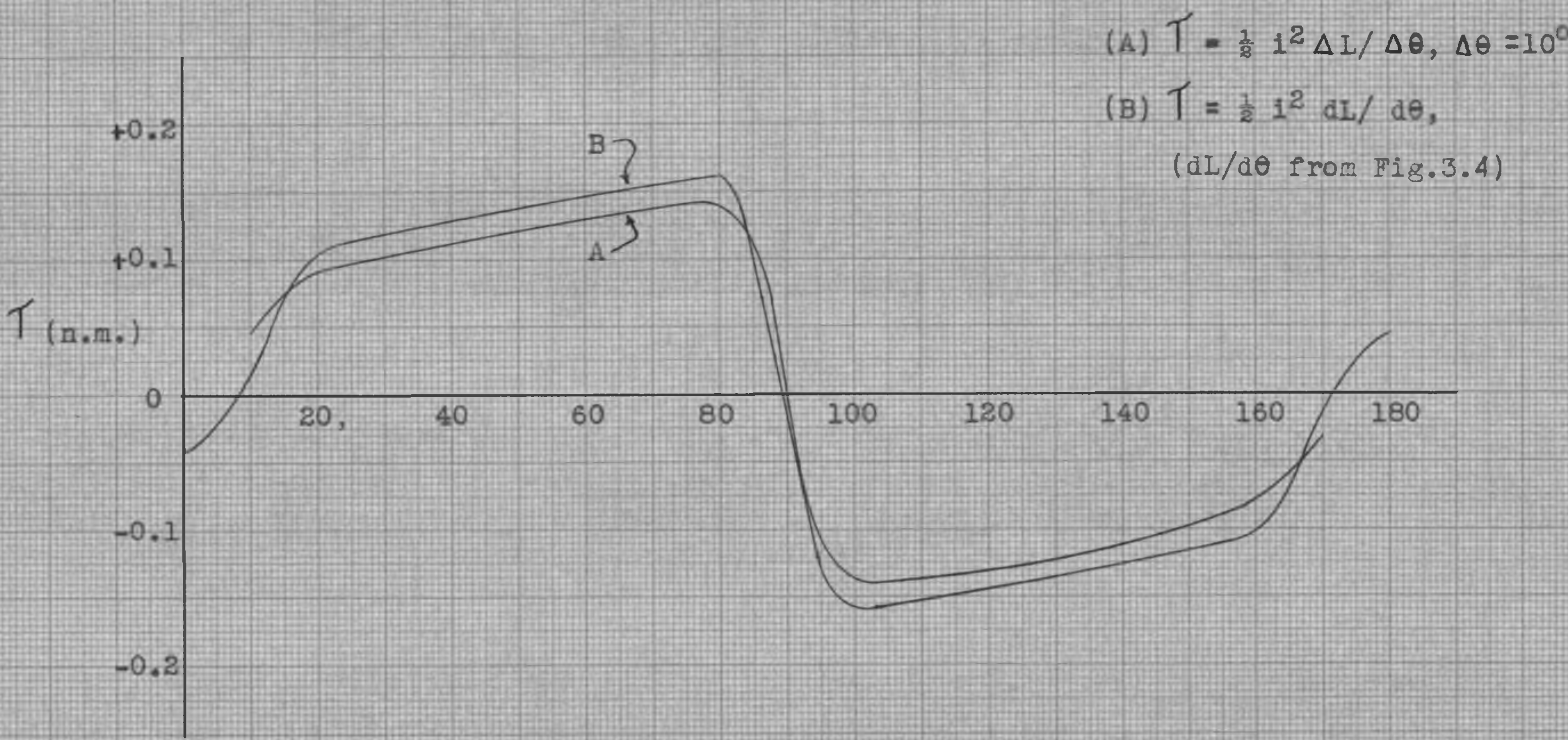
TORQUE vs DISPLACEMENT



Comparison of Calculated Torques for

Stator Winding With A-C Excitation

Fig. 3.8

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of the analytical expression for the curve is given in the appendix.

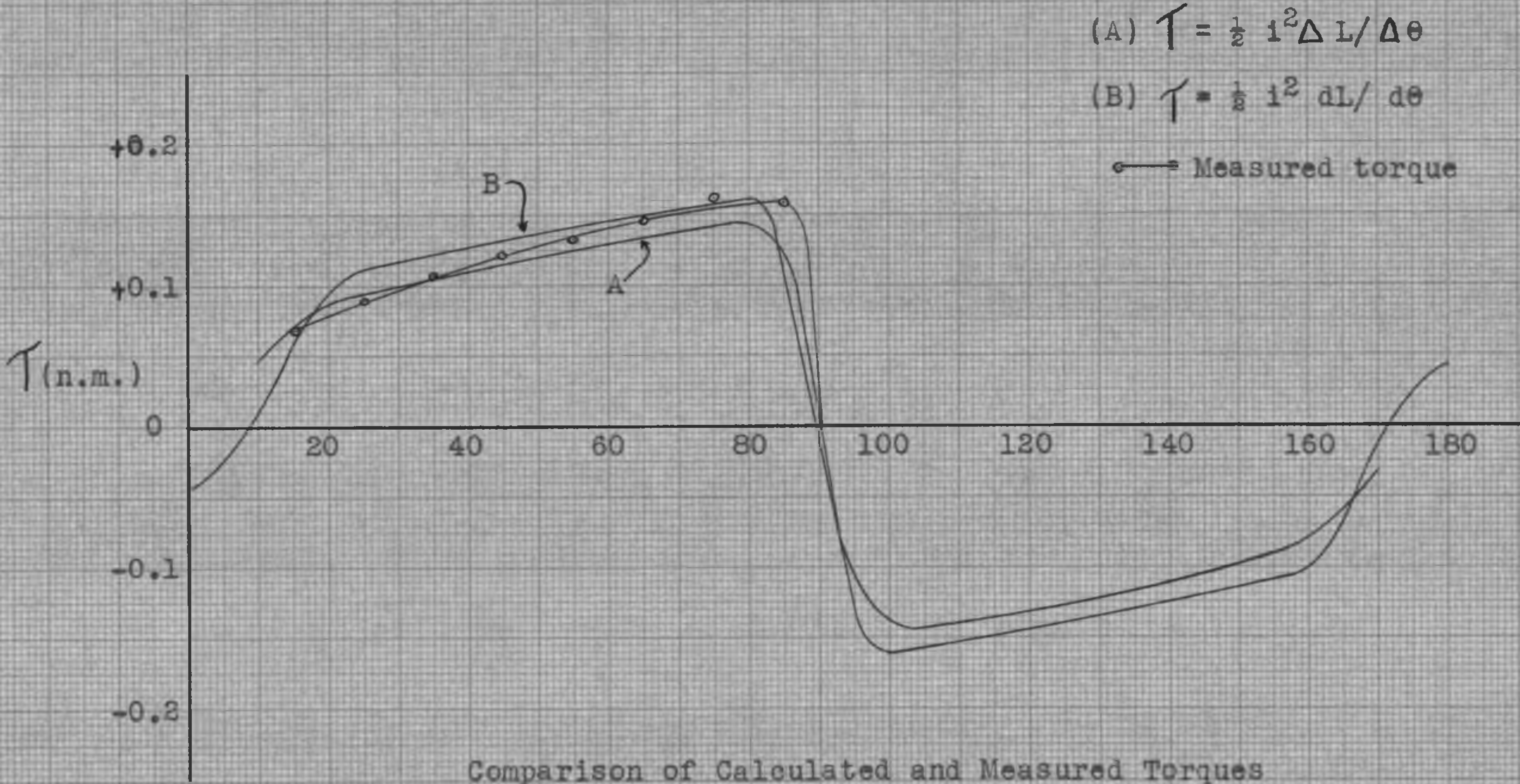
In Fig. 3.9, the measured and calculated values of applied stator torque are compared and seen to be in good agreement; hence, the incremental method would be the better one to use in the student laboratory since the determination of the analytical expression is tedious and time consuming.

3.5 TEST EQUIPMENT

The equipment used to determine the inductance based on a-c measurements at 60 cycles per second were conventional indicating instruments. The electrodynamometer type wattmeters had a nominal accuracy of one-quarter of one percent of full scale value. Ammeters and voltmeters were used whose nominal accuracy was one-half of one percent of their full scale values.

The fluxmeter used for the flux measurements was the D'Arsonval double pivot type. Its nominal accuracy was also one-half of one percent of full scale readings. However, there was some question as to the accuracy of the calibration of this particular instrument. Consequently, the results based on fluxmeter readings, though not conclusive, are used for comparative purposes only, to indicate the potential of such measurements.

APPLIED TORQUE vs DISPLACEMENT



Comparison of Calculated and Measured Torques

Stator Winding Excited

(Calculations based on Fig. 3.4)

Fig. 3.9

The accuracy of the spring-scales, based on checks made with known weights, was estimated to be three percent of the actual scale readings at mid-scale.

3.6 SOURCES OF ERROR

The largest instrument errors occurred in the measurement of the higher values of inductance because of the relatively small deflections of the instruments. In general, the maximum error, estimated to be approximately two percent, occurred at the lower readings. In the middle range of values and above, there were usually sufficient instruments and scales available to make readings well upscale, keeping the errors within the nominal accuracies of the individual instruments. As has already been mentioned, the error made in measuring torque with the spring-scales was approximately three percent of the scale readings.

The maximum error of observation made in reading the displacement indicator for inductance measurements was measured over the width of a protractor division and found to be approximately 0.001 henries. This would represent a maximum possible error of ten percent occuring where $dL/d\theta$ is the smallest.

The most probable cause of the discrepancies noted in the comparison of the curves of inductance obtained with a-c and d-c excitation--see Fig. 3.6--was due to the placement of the search coils on the structure of the rotary electromagnet. If the coils did not link representative values of flux, the resulting calculations for inductance would be low.

Another source of error was friction. The use of ball-bearings tended to reduce this error; however, in the cases when the rotor was excited, brush friction on the slip-rings increased it.

Chapter 4

4.1 CONCLUSIONS

The rotary electromagnet which was developed in this study, successfully demonstrated the relationships between electromagnetic torque and the variation of inductance. As an educational tool, it enables the student to physically visualize the basic principles underlying these relationships by performing experiments such as those described herein.

The similarity of results of self-inductance measurements made with alternating- and with direct-current is shown in Figs. 3.4 and 3.5. Although the results obtained using the fluxmeter and direct current were not altogether satisfactory, those measurements were more easily and quickly made. Analysis of the discrepancies, in light of the sources of error and instrument accuracies, tends to support the belief that with a better arrangement of exploring coils, fluxmeter measurements would be adequate to obtain the inductance characteristics of the rotary electromagnet in the student laboratory.

The variation of applied torque with changes in stored energy was computed using incremental changes of inductance with increments of angular displacement. The results were in good agreement with the measured torque and with the torque computed using the more rigorous analytical methods. Further refinements in measuring techniques would produce even closer results than those shown in Fig. 3.9.

Such a rotary electromagnet is economically feasible, also. Its size is such that it lends itself to the small laboratory group and can be easily manipulated, disassembled or examined. Since it is small, room for storage is not a problem either. The purchase and modification of such devices are relatively inexpensive, too, when compared with larger floor-mounted types.

4.2 RECOMMENDATIONS FOR FURTHER STUDY

It is the author's opinion that further work should be done with this and similar devices. An investigation should be made to determine the best location of exploring coils to obtain more reliable results with fluxmeter measurements.

In order to avoid the effects of saturation during measurements with the rotor and stator windings in series, a winding with a greater number of turns should be put on the rotor. Increasing the number of rotor turns to 400 or even 600 would lead to values of voltage, current and power which could be more readily measured with conventional instruments.

The device under study was of laminated construction to make a-c measurements of inductance possible. The feasibility of constructing a similar device with a solid core and rotor should also be explored for direct current excitation only. Such a device could also be used to demonstrate why laminations are necessary for a-c measurements.

Only single-phase operation was possible for the present rotary electromagnet. Further work should be done to modify suitable three-phase machines for similar experiments with polyphase excitation. Such a series of experiments would help the student correlate the theory and mathematical representations of three-phase devices with physical concepts and quantities he could actually measure.

Appendix

DETERMINATION OF THE FOURIER SERIES REPRESENTING THE STATOR INDUCTANCE CURVE[1]

The Fourier series, corresponding to the stator inductance curve, L (α), where $\alpha = 2\theta$, is written

$$L(\alpha) = \frac{1}{2} A_o + \sum_{n=1}^{\infty} (A_n \cos n\alpha + B_n \sin n\alpha) \qquad (A.1)$$

where the Fourier coefficients are given by

$$A_n = 2\left[1/2\pi \int_0^{2\pi} L(\alpha) \cos n\alpha d\alpha\right], \ 0 \leq \alpha \leq 2\pi \qquad (A.2)$$

$$B_n = 2\left[1/2\pi \int_0^{2\pi} L(\alpha) \sin n\alpha d\alpha\right], \ 0 \leq \alpha \leq 2\pi \qquad (A.3)$$

However, point-by-point numerical integration is used because the inductance curve is non-symmetrical and hence, difficult to integrate.

The period is first divided into m equal increments of width $\Delta\alpha$ such that $m\Delta\alpha = 2\pi$ radians or 360°. Hence,

$$\Delta\alpha = 2\pi/m \qquad (A.4)$$

Replacing $d\alpha$ by $\Delta\alpha$, substitution of Eq. A.4 into A.2 and A.3 yields

$$A_n \simeq 2/m \sum_{o}^{m} L(\alpha) \cos n\alpha \qquad (A.5)$$

$$B_n \simeq 2/m \sum_{o}^{m} L(\alpha) \sin n\alpha \qquad (A.6)$$

where $L(\alpha)$ is the ordinate taken at the mid-point of each

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[1]H. A. Thompson, Alternating-Current and Transient Circuit Analysis, McGraw-Hill Book Co., Inc., 1955, pp. 227-30.

increment, $\Delta\alpha$. The coefficients A_n and B_n are then found by carrying out the summations indicated in Eqs. A.5 and A.6 through n harmonics. Substitution of the coefficients into Eq. A.1 yields the solution.

$$
\begin{aligned}
L(\alpha) = {} & 0.482 + 0.00583 \sin\alpha + 0.261 \cos\alpha + 0.0202 \sin 2\alpha - \\
& 0.0335 \cos 2\alpha + 0.00416 \sin 3\alpha + 0.0266 \cos 3\alpha + \\
& 0.0019 \sin 4\alpha - 0.0102 \cos 4\alpha + 0.00171 \sin 5\alpha + \\
& 0.00582 \cos 5\alpha - 0.0041 \cos 6\alpha + 0.002 \cos 7\alpha + \\
& 0.0005 \sin 8 - 0.00202 \cos 8
\end{aligned}
\qquad (A.7)
$$

Performing the differentiation gives

$$
\begin{aligned}
dL/d\alpha = {} & 0.00583 \cos\alpha - 0.261 \sin\alpha + 0.0404 \cos 2\alpha + \\
& 0.067 \sin 2\alpha + 0.0125 \cos 3\alpha - 0.0798 \sin 3\alpha + \\
& 0.0076 \cos 4\alpha + 0.0408 \sin 4\alpha + 0.00855 \cos 5\alpha - \\
& 0.0291 \sin 5\alpha - 0.0246 \sin 6\alpha - 0.014 \sin 7\alpha + \\
& 0.004 \cos 8\alpha + 0.0162 \sin 8\alpha
\end{aligned}
\qquad (A.8)
$$

Since the inductance curve is periodic in π mechanical radians or one-half revolution, $\alpha = 2\theta$. The applied torque, Eq. 3.5, then becomes in terms of α,

$$\mathcal{T} = -2\left[\tfrac{1}{2}\, i^2\, dL/d\alpha\right]$$

$$\mathcal{T} = -i^2\, dL/d\alpha \qquad (A.9)$$

Evaluating Eqs. A.7 and A.8 for one-half revolution and Eq. A.9 for I = 0.75 amps, DC, gives the results tabulated in Table I and the curve shown in Fig. 3.8-B.

TABLE I.

Data for the Analytically Computed Stator Inductance Curve

θ	α	L	dL/ dα	T
0	0	0.732	-0.079	-0.0444
5	10	0.742	-0.0385	-0.0217
10	20	0.733	-0.022	0.0124
15	30	0.713	0.126	0.071
20	40	0.692	0.198	0.110
25	50	0.667	0.190	0.107
30	60	0.637	0.208	0.117
35	70	0.595	0.226	0.127
40	80	0.547	0.230	0.129
45	90	0.501	0.239	0.134
50	100	0.467	0.237	0.133
55	110	0.431	0.262	0.147
60	120	0.389	0.270	0.152
65	130	0.339	0.252	0.142
70	140	0.297	0.268	0.151
75	150	0.237	0.281	0.158
80	160	0.191	0.288	0.162
85	170	0.156	0.179	0.101
90	180	0.151	-0.025	-0.014
95	190	0.164	-0.236	-0.133

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GLOSSARY

$\alpha = 2\theta$ = angular displacement in degrees

e = induced emf in volts

f = force in newtons

$\mathcal{F}$ = Ni = MMF in ampere-turns

ϕ = equivalent flux in webers

i = current in amperes

L = self-inductance in henries

λ = flux linkages in weber-turns

M = mutual-inductance

N = number of turns of a given winding

ω = radian frequency, $2\pi f$

P = power in watts

$\mathcal{P}$ = magnetic permeance

r = resistance in ohms

$\mathcal{R}$ = magnetic reluctance in ampere turns per weber

Υ = torque in newton-meters

θ = angular displacement in degrees

w = applied emf in volts

$W_{elect, mech, fld}$ = energy in joules

x = displacement in meters

X_1 = inductive reactance in ohms