ANALYSIS OF THE GENERALIZED MACHINE AS A SYNCHRONOUS GENERATOR

by

Maurice W. Collins

A Thesis Submitted to the Faculty of the

DEPARTMENT OF ELECTRICAL ENGINEERING

In Partial Fulfillment of the Requirements
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ANALYSIS OF THE CENTRAL-DE MACHINES
AS A DYNAMICALLY CONSTRAINED

by

Marcia A. De Lima

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Maurice W. Collins

APPROVAL BY THESIS DIRECTOR

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Chapter 1

PROBLEM AND APPROACH

The staff at the Massachusetts Institute of Technology developed a generalized machine whereby the theoretical concepts of electromechanical energy conversion and the study of conventional ac and dc rotating machinery could be represented by a single unit. Westinghouse Electric Corporation produced the generalized machine laboratory set as an educational device in 1958. A generalized machine laboratory set was donated to the University of Arizona by Westinghouse in the spring of 1959.

In this investigation the generalized machine is operated as a two-phase synchronous generator. The transient and steady-state performance of the machine is to be analyzed for balanced and unbalanced conditions.

The only available performance data concerning the synchronous operation of the generalized machine are saturation curves for its operation as a single-phase alternator. The three-phase synchronous generator has been analyzed quite extensively but relatively little has been done concerning the analysis of a two-phase synchronous generator.

The machine is treated as a group of three inductively coupled electric circuits. The expressions for the flux linkages of the

field and armature windings are developed and from these the steady-state voltage and power equations are determined. Constants are defined which describe the steady-state performance and an equivalent circuit is developed for the generalized machine operating as a synchronous generator.

Equations describing the performance for a balanced two-phase short-circuit are developed from the expressions for the flux linkages. These equations are developed for the instant of short-circuit by the law of constant flux linkages. The transient behavior of the machine is then analyzed by including the effect of resistance. From the resulting expressions, constants are defined which describe the transient behavior of the machine.

The transient and steady-state behavior of the machine is similarly analyzed, and machine constants found, for an unbalanced single-phase short-circuit.

All the constants which describe the steady-state and transient performance of the machine are calculated from measured values of machine resistance and inductance.

The open-circuit, short-circuit, and zero power-factor characteristics are obtained by test for the machine. The steady-state machine constants are determined from these characteristics. Oscillograph recordings are made of the transient and steady-state behavior of the machine for a balanced two-phase short-circuit and an unbalanced single-phase short-circuit. These oscillograph recordings are used to verify the general validity of the machine constants.

Chapter 2

THE GENERALIZED MACHINE

2.1 DESCRIPTION

A visual description of the generalized machine and its associated equipment is given in Figures 2.1 through 2.4.

The dimensions of the generalized machine laboratory set shown in Figure (2.1) are: Length - 7 feet, Width - 3 feet, Height - 3 feet. The generalized machine, Figure (2.3), has an overall length of 30 inches.

The generalized machine and its associated equipment can be operated as:

1) Alternating-Current Operation

   Synchronous motor and generator - single-phase or two-phase

   Induction motor - single-phase or polyphase

2) Direct-Current Operation

   Shunt or series motor

   Separately-excited generator or self-excited shunt generator

   Amplidyne

The performance of the generalized machine as any one particular type of machine is sacrificed for flexibility. It is

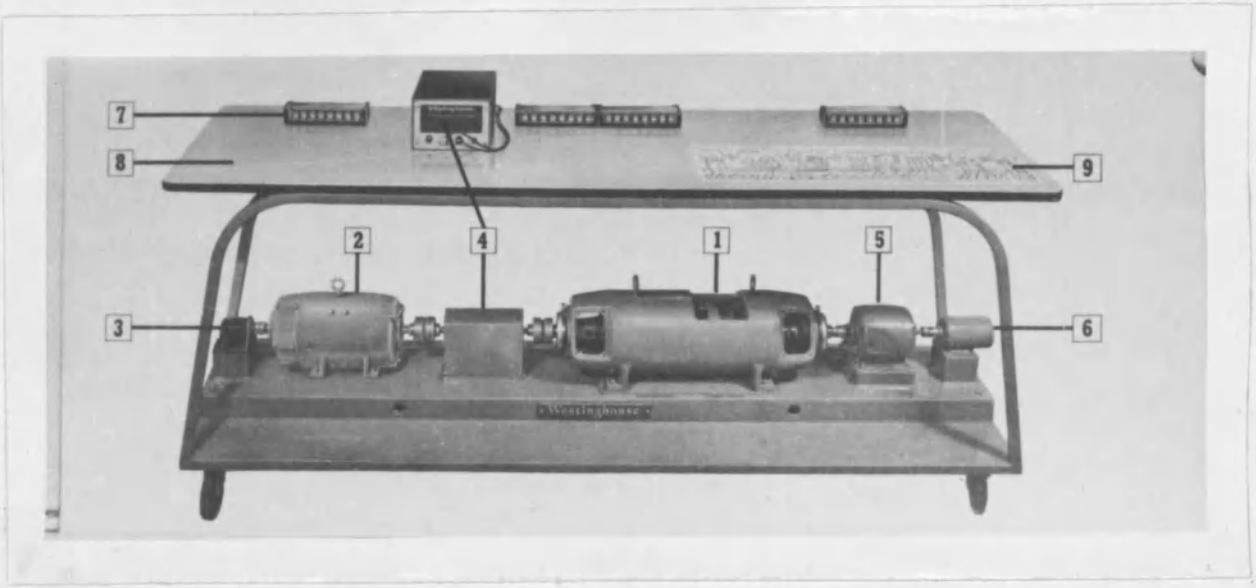

Figure 2.1 - Generalized Machine Laboratory Set

1- Generalized Machine

2- Rotor Drive Motor

3- Rotor Tachometer

4- Torque-meter

5- Brush Carriage Drive Motor

6- Brush Carriage Tachometer

7- Terminal Blocks

8- Table

9- Schematic Diagram

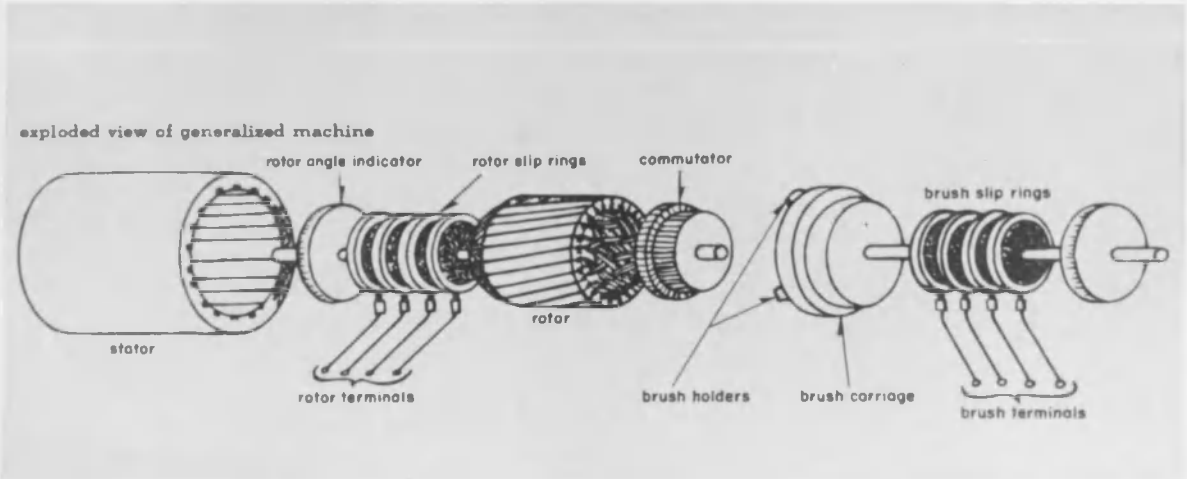

Figure 2.2 - Exploded View of Generalized Machine

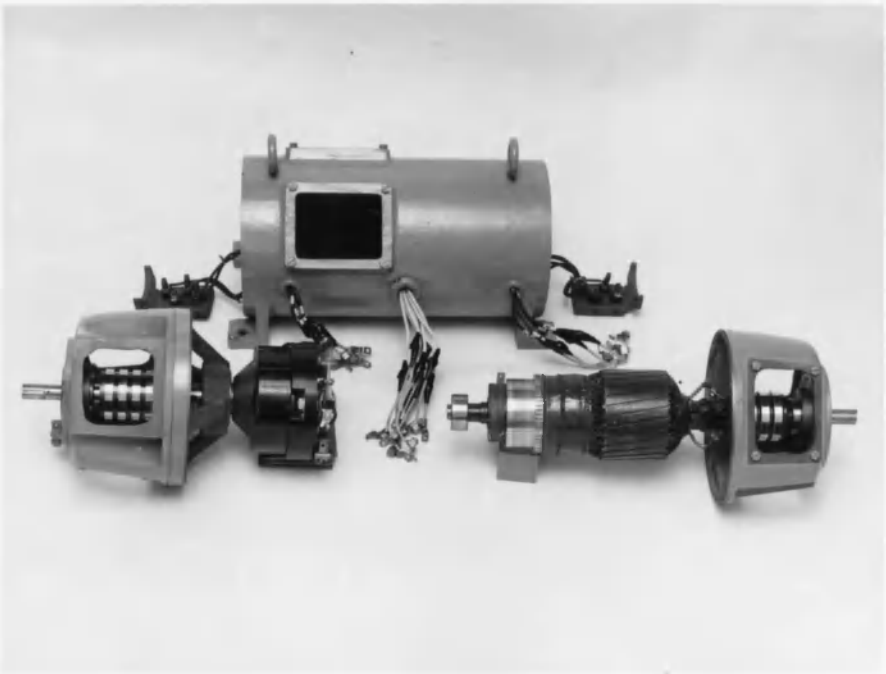

Figure 2.3 - Disassembled Generalized Machine

Figure 2.4 - Stator of Generalized Machine

impossible to incorporate the optimum design features of all the different types of machines. When the generalized machine is operated as a two-phase synchronous generator, the rotor, the rotor slip rings, and the stator, Figure (2.2), are utilized. The rotor drive motor and rotor tachometer, Figure (2.1), are also used.

2.2 ROTOR SPECIFICATIONS[1]

The mechanical specifications of the rotor are:

Diameter - 4.585 inches

Length - 3.0 inches

Moment of Inertia - 0.26 Lb. Ft.^{2}

Number of Rotor Slots - 28

Slot Skew - One slot pitch

The rotor has a two-pole, full-pitch, continuous lap winding. There are 560, 2 strand, number 20 inductors in the 28 slots or 20 inductors per slot. Each coil consists of 5 turns to give 4 coil sides per slot.

Direct current is supplied by means of slip rings mounted on the rotor shaft. The slip rings are connected to two diametrically opposite parts of the winding so as to give two parallel paths for the field current.

The rotor winding has a maximum rating of 230 volts and 8 amperes. The two parallel paths of the field winding have a combined

---

[1] Operating Manual for Generalized Machine, Westinghouse Electric Corporation, pp 2 and 3

resistance of 0.46 ohms and a self-inductance of 0.0662 henries.

2.3 STATOR SPECIFICATIONS[2]

The mechanical specifications of the stator are:

Inner Diameter - 4.631 inches

Air-gap - 0.023 inches

Width of Stator Iron - 3.0 inches

Number of Stator Slots - 36

The stator has a two-pole, two-phase, 11/18 pitch, distributed winding. There are two winding groups in each phase. Each winding group has 9 coils with 18 turns per coil. A turn consists of one number 19 wire in parallel with one number 20 wire.

For the experimental investigation, the windings in each phase are connected in series. For this connection the stator winding has a maximum rating of 230 volts and 3.6 amperes. The windings in series have a resistance of 2.8 ohms and a self-inductance of 0.459 henries. The maximum value of the mutual inductance between the field winding and the stator winding is 0.172 henries. The above values are for either phase.

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[2] Operating Manual for Generalized Machine, Westinghouse Electric Corporation, pp 2 and 3

Chapter 3

EXPERIMENTAL ARRANGEMENT

3.1 LABORATORY SETUP

The facilities of the Electrical Engineering Department's power laboratory were used for the experimental work. Two sources of variable dc voltage and a three-phase, 215-volt, 60-cycle bus were available.

The rotor was driven at synchronous speed by a 230 volt, dc shunt motor. Speed was monitored with a dc voltmeter from the dc tachometer generator. The voltmeter was calibrated for synchronous speed by a strobotac. The field of the generalized machine was excited from a variable dc source.

Ammeters, voltmeters, and wattmeters were used whose nominal accuracy was one-half of one percent of their full scale values. Sufficient instruments and scales were available to make most readings well up-scale, keeping the errors within the nominal accuracies of the individual instruments. Simultaneous readings were taken to minimize errors due to fluctuations. Quantities were recorded for both phases of the machine and averaged.

3.2 SCOTT TRANSFORMATION

The Scott connection is an arrangement of transformers for two-phase to three-phase transformation. It consists of a main transformer M and a teaser transformer T. The secondary of M has

a center-tap to which one side of the secondary of T is attached.

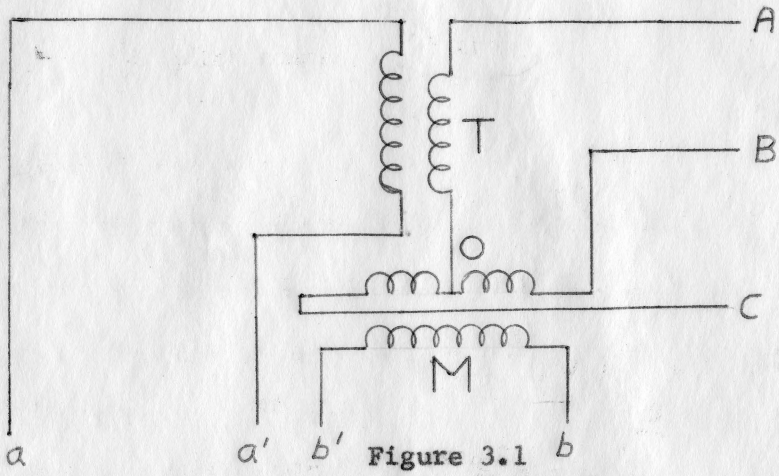

Figure 3.1

Scott Connection

M has a turns ratio such that when a two-phase voltage is applied to its primary, the corresponding line-to-line, three-phase voltage appears across its secondary. T has a turns ratio such that when the remaining two-phase voltage is applied to its primary, the vector sum of the voltage developed in its secondary and either half of the voltage developed in the secondary of M, equals the line-to-line, three-phase voltage.

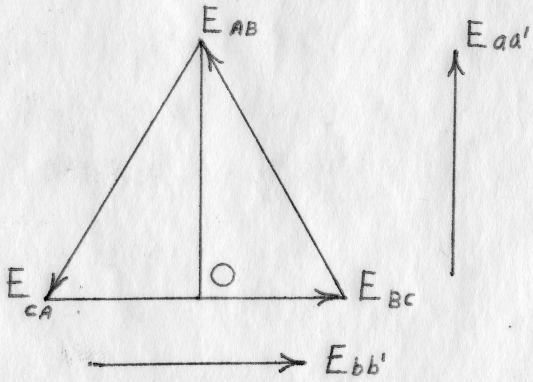

Figure 3.2

Vector Diagram

A Scott connection was used in conjunction with two auto-transformers to connect the two-phase, 230 volt generalized machine to the three-phase, 215 volt bus. The auto-transformers were used to obtain the proper voltage ratio and to insure good balance between the machine and the bus.

No-load measurements were made at the armature terminals of the generalized machine. Three-phase measurements could not be made as the transformers used in the Scott connection drew large exciting currents which distorted the voltage waveforms.

The nonsinusoidal exciting current of the transformer causes a large nonsinusoidal voltage drop in the machine impedance. Since the internal voltage of the machine is sinusoidal, transformer induced voltages must have harmonics equal and opposite to those in the voltage drop of the machine impedance. Both the two-phase and three-phase waveforms are badly distorted as the machine excitation is increased.

This difficulty disappears when the three-phase bus is connected to the system as the bus has very low impedance. The sinusoidal waveform of the bus voltage forces the transformer induced voltages to be sinusoidal.

Chapter 4

STEADY-STATE PERFORMANCE

4.1 MACHINE RATING

Synchronous machine constants are usually expressed in per-unit as they then fall within a relatively narrow range for all machines of similar design, although the size may vary over a wide range. The machine's own KVA rating is taken as the volt-ampere base and rated voltage is used as the voltage base. Base current and base impedance are then determined from these.

To find the KVA rating of the generalized machine it was connected to the three-phase, 215 volt bus by the Scott connection. The bus supplied or absorbed power as determined by the mechanical input to the machine. The terminal voltage of the machine was fixed at its rated value of 230 volts by the bus and the internal voltage kept constant by holding the field excitation at its rated value of 8 amperes. As the mechanical power input to the machine was increased the power output and hence the armature current increased. The reactive power decreased.

The real power output and armature current were measured and the power factor (cosine of the angle between armature terminal voltage and armature current) was computed for different mechanical inputs. The armature current and power factor are plotted as a function of the real power output in Fig. 4.1. Since the KVA output

is directly proportional to the armature current, the KVA for any lagging power factor may be read from the graph of Fig. 4.1.

The KVA rating of a synchronous generator is commonly specified for a power factor of 0.85 lagging. Synchronous generators are operated at lagging power factor to supply the lagging reactive power needed for inductive loads. When the generalized machine is operating as a two-phase synchronous generator, its KVA rating is 169 volt-amperes. The bases of the per-unit quantities are then per phase:

$$\begin{aligned} \text{Power} &= 169 \text{ volt-amperes} \\ \text{Voltage} &= 230 \text{ volts} \\ \text{Current} &= 0.735 \text{ amperes} \\ \text{Impedance} &= 313 \text{ ohms} \end{aligned} \tag{4.1}$$

Generalized Machine operating as a synchronous generator delivering power to an infinite bus.

Armature current and power factor (pf) vs real power output of generator.

Armature voltage = 230 volts
Field current = 8 amperes constant

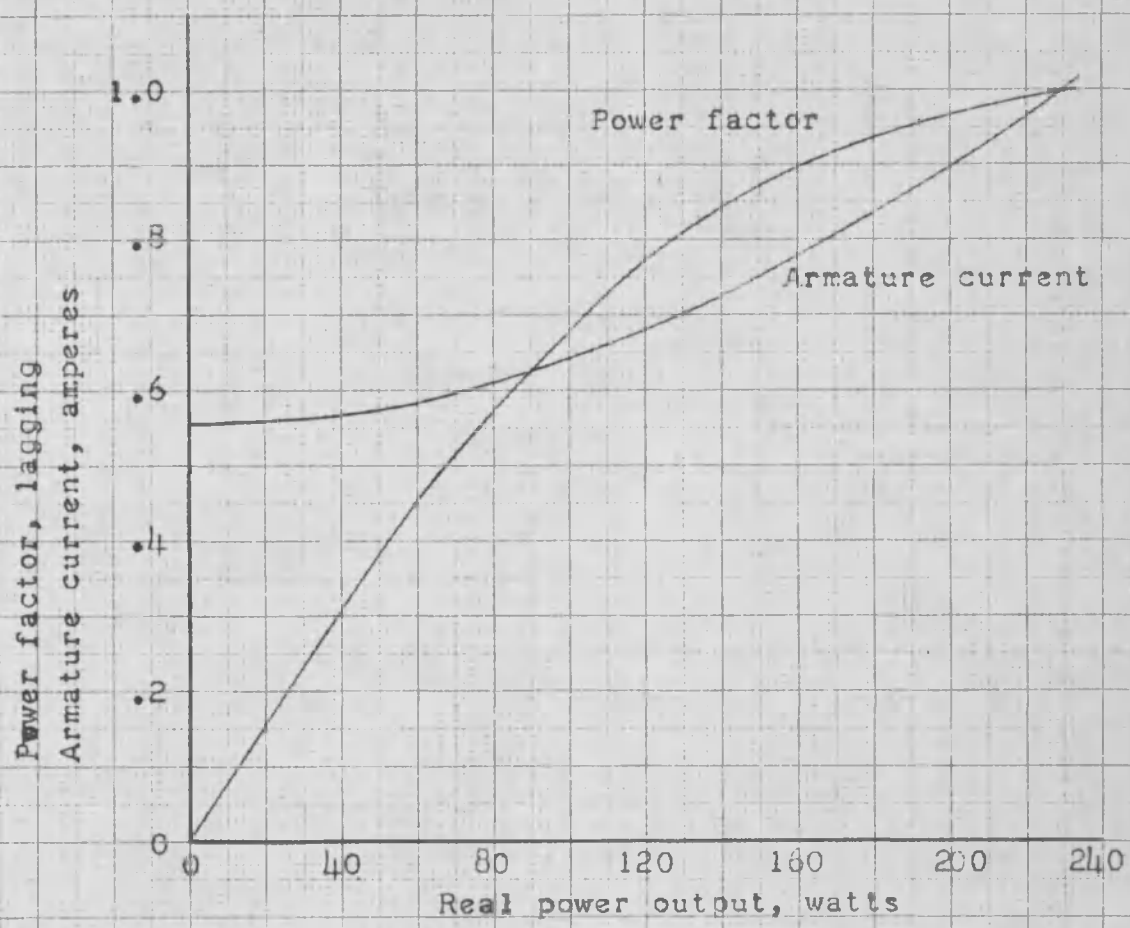

Figure 4.1

4.2 STEADY-STATE EQUATIONS

A synchronous machine may be thought of as a group of inductively coupled electric circuits. The flux linkage expressions for the field winding and for each phase winding can be developed and from these the voltage and power equations determined.

The generalized machine is represented as an idealized two-phase generator in Figure 4.2.

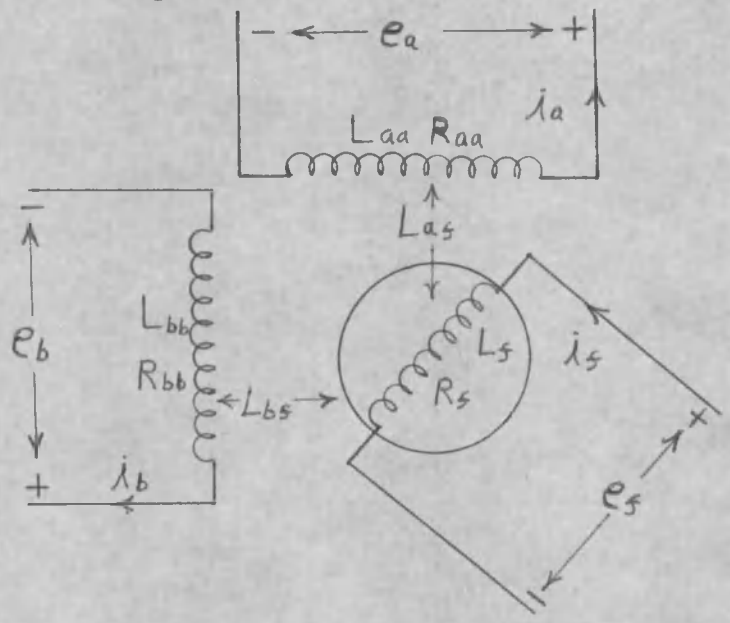

Figure 4.2 - Idealized Two-phase Generator

The phase windings are 90 electrical degrees apart so there is no mutual effect between them. Since the generalized machine has two poles, the phase windings are also 90 mechanical degrees apart. The flux linkages for each phase winding and the field winding are

$$\lambda_a = L_{aa}\, i_a + L_{af}\, I_f$$

$$\lambda_b = L_{bb}\, i_b + L_{bf}\, I_f$$

$$\lambda_f = L_f\, I_f + L_{af}\, i_a + L_{bf}\, i_b \qquad (4.2)$$

where

$L_d = L_{aa} = L_{bb}$ = Self-inductance of each phase winding

$R_a = R_b$ = Resistance of each phase winding

L_f = Self-inductance of the field winding

R_f = Resistance of the field winding

L_{af} = Mutual inductance between phase a and field

L_{bf} = Mutual inductance between phase b and field

The generalized machine has a uniform air-gap and the rotor slots are evenly distributed so the self-inductances of the phase windings and the field winding are constant. The mutual inductance between either phase winding and the field winding is developed in Appendix B. From Equation (B.5)

$$L_{af} = L_m \left[\cos(\omega t + \sigma_o) - 0.0078 \cos 3(\omega t + \sigma_o) - 0.0028 \cos 5(\omega t + \sigma_o) + \ldots\ldots\ldots\ldots\ldots\right] \tag{4.3}$$

All harmonics can be neglected giving

$$L_{af} = L_m \cos(\omega t + \sigma_o)$$

$$L_{bf} = L_m \cos(\omega t + \sigma_o - 90^o) = L_m \sin(\omega t + \sigma_o) \tag{4.4}$$

Where

L_m = Maximum value of mutual inductance between field and either phase winding

ω = Angular velocity in radians per sec.

σ_o = Initial angular displacement in radians with respect to phase a internal voltage. This is the angle between the magnetic axis of phase a and the magnetic axis of the field winding.

Substitution of Equation (4.4) in Equation (4.2) gives

$$\lambda_a = L_d\, i_a + L_m\, I_f \cos(\omega t + \sigma_o)$$

$$\lambda_b = L_d\, i_b + L_m\, I_f \sin(\omega t + \sigma_o) \qquad (4.5)$$

$$\lambda_f = L_f\, I_f + L_m\, i_a \cos(\omega t + \sigma_o) + L_m\, i_b \sin(\omega t + \sigma_o)$$

For balanced steady-state operation the armature terminal voltages are defined as

$$e_a = \sqrt{2}\,|E_a| \sin(\omega t + \alpha_o)$$

$$e_b = \sqrt{2}\,|E_b| \sin(\omega t + \alpha_o - 90^\circ) \qquad (4.6)$$

$$= -\sqrt{2}\,|E_b| \cos(\omega t + \alpha_o)$$

Then the armature currents are

$$i_a = \sqrt{2}\,|I_a| \sin(\omega t + \alpha_o + \beta_p)$$

$$i_b = -\sqrt{2}\,|I_b| \cos(\omega t + \alpha_o + \beta_p) \qquad (4.7)$$

Where

$|E_a| = |E_b|$ = RMS value of armature terminal voltage

$|I_a| = |I_b|$ = RMS values of armature current

β_p = Angle between armature terminal voltage and armature current

α_o = Initial angular displacement with respect to phase a armature terminal voltage

Substitution of Equation (4.7) in Equation (4.5) gives

$$\lambda_a = \sqrt{2}\ L_d\ |I_a|\ \sin\ (\omega t + \alpha_o + \beta_p) + L_m\ I_f\ \cos\ (\omega t + \sigma_o)$$

$$\lambda_b = -\sqrt{2}\ L_d\ |I_a|\ \cos\ (\omega t + \alpha_o + \beta_p) + L_m\ I_f\ \sin\ (\omega t + \sigma_o)$$

$$\lambda_b = L_f\ I_f + \sqrt{2}\ L_m\ |I_a|\ \sin\ (\omega t + \alpha_o + \beta_p)\ \cos\ (\omega t + \sigma_o) - \sqrt{2}\ L_m\ |I_a|\ \cos\ (\omega t + \alpha_o + \beta_p)\ \sin\ (\omega t + \sigma_o) \quad (4.8)$$

When simplified, the field flux linkages are seen to be independent of rotation

$$\lambda_f = L_f\ I_f + \sqrt{2}\ L_m\ |I_a|\ \sin\ (\beta_p + \alpha_o - \sigma_o) \quad (4.9)$$

The phase and field voltages are written for the circuits of Fig. 4.2. The phase windings are sources and the field winding is a load.

$$e_a = - R_a i_a - \frac{d\lambda_a}{dt}$$

$$e_b = - R_b i_b - \frac{d\lambda_b}{dt} \qquad (4.10)$$

$$e_f = + R_f I_f + \frac{d\lambda_f}{dt}$$

Substitution of Equation (4.8) in Equation (4.10) gives

$$e_a = - \sqrt{2}\ R_a \left|I_a\right| \sin(\omega t + \alpha_o + \beta_p)$$

$$- \sqrt{2}\ \omega L_d \left|I_a\right| \cos(\omega t + \alpha_o + \beta_p)$$

$$+ \omega L_m I_f \sin(\omega t + \sigma_o)$$

$$e_b = \sqrt{2}\ R_a \left|I_a\right| \cos(\omega t + \alpha_o + \beta_p)$$

$$-\sqrt{2}\ \omega L_d \left|I_a\right| \sin(\omega t + \alpha_o + \beta_p)$$

$$- \omega L_m I_f \cos(\omega t + \sigma_o) \qquad (4.11)$$

$$e_f = R_f I_f$$

The armature terminal voltages may be transformed from trigometric form to vector form. Writing Equation (4.11) in exponential notation

$$e_a = \text{Real}\ \sqrt{2}\ \left|I_a\right| \left[j R_a - \omega L_d\right] \epsilon^{j(\omega t + \alpha_o + \beta_p)}$$

$$- \text{Real}\ j \omega L_m I_f\ \epsilon^{j(\omega t + \sigma_o)}$$

$$e_b = \text{Real}\ \sqrt{2}\ \left|I_a\right| \left[R_a + j \omega L_d\right] \epsilon^{j(\omega t + \alpha_o + \beta_p)}$$

$$- \text{Real}\ \omega L_m I_f\ \epsilon^{j(\omega t + \sigma_o)} \qquad (4.12)$$

From Equation (4.5)

$$\text{Real} - j\sqrt{2}\,|E_a|\,\epsilon^{\,j(\omega t + \alpha_o)}$$

$$= \text{Real}\,\sqrt{2}\,|I_a|\left[j\,R_a - \omega\,L_d\right]\epsilon^{\,j(\omega t + \alpha_o + \beta_p)}$$

$$- \text{Real}\; j\,\omega\,L_m\,I_f\;\epsilon^{\,j(\omega t + \sigma_o)}$$

$$\text{Real} - \sqrt{2}\,|E_b|\,\epsilon^{\,j(\omega t + \alpha_o)}$$

$$= \text{Real}\,\sqrt{2}\,|I_a|\left[R_a + j\,\omega\,L_d\right]\epsilon^{\,j(\omega t + \alpha_o + \beta_p)}$$

$$- \text{Real}\;\;\omega\,L_m\,I_f\;\;\epsilon^{\,j(\omega t + \sigma_o)} \qquad (4.13)$$

The Real operator and the factor $\epsilon^{\,j\,\omega\,t}$ are common to each term and may be cancelled leaving

$$-j\sqrt{2}\,|E_a|\,\epsilon^{\,j\,\alpha_o} = \sqrt{2}\,|I_a|\left[j\,R_a - \omega\,L_d\right]\epsilon^{\,j(\alpha_o + \beta_p)}$$

$$- j\;\omega\,L_m\,I_f\;\epsilon^{\,j\,\sigma_o}$$

$$-\sqrt{2}\,|E_b|\,\epsilon^{\,j\,\alpha_o} = \sqrt{2}\,|I_a|\left[R_a + j\,\omega\,L_d\right]\epsilon^{\,j(\alpha_o + \beta_p)}$$

$$- \omega\,L_m\;I_f\;\epsilon^{\,j\,\sigma_o} \qquad (4.14)$$

Define

$$E_a = |E_a|\;\epsilon^{\,j\,\alpha_o}$$

$$E_b = |E_b|\;\epsilon^{\,j\,\alpha_o}$$

$$I_a = |I_a|\;\epsilon^{\,j(\alpha_o + \beta_p)}$$

$$E_i = |E_i|\;\epsilon^{\,j\,\sigma_o} = \left|\frac{\omega\,L_m I_f}{\sqrt{2}}\right|\;\epsilon^{\,j\,\sigma_o}$$

$\Delta = \sigma_o - \alpha_o$ = Angle by which E_i leads E_a

$\beta_i = \beta_p - \Delta$ = Angle by which I_a leads E_i

The vector form of phase a of the armature terminal voltage is

$$E_a = (- R_a - j \omega L_d) I_a + E_i \tag{4.15}$$

The mechanical power input to a group of inductively coupled electric circuits is written as

$$\begin{aligned} P_m = - \frac{1}{2} \Bigg[& i_1^2 \frac{dL_{11}}{dt} + i_1 i_2 \frac{dL_{12}}{dt} + \cdots i_1 i_n \frac{dL_{1n}}{dt} \\ & + i_2 i_1 \frac{dL_{12}}{dt} + i_2^2 \frac{dL_{22}}{dt} + \cdots i_2 i_n \frac{dL_{2n}}{dt} \\ & + i_n i_1 \frac{dL_{in}}{dt} + i_n i_2 \frac{dL_{2n}}{dt} + \cdots i_n^2 \frac{dL_{nn}}{dt} \Bigg] \end{aligned} \tag{4.16}$$

The mechanical power input to the generalized machine operating as a two-phase synchronous generator may be found using this equation.

$$\begin{aligned} P_m = - \frac{1}{2} \Bigg[& i_a^2 \frac{dL_{aa}}{dt} + i_b^2 \frac{dL_{bb}}{dt} \\ & + i_a I_f \frac{dL_{af}}{dt} + i_b I_f \frac{dL_{bf}}{dt} \Bigg] \end{aligned} \tag{4.17}$$

Only mutual terms will be present as the generalized machine has a uniform air-gap making all self-inductances constant. As developed in Equation (4.4), the mutual inductance terms are

$$L_{af} = L_m \cos(\omega t + \sigma_o)$$

$$L_{bf} = L_m \sin(\omega t + \sigma_o) \tag{4.18}$$

From Equation (4.7), the currents are

$$i_a = \sqrt{2}\,|I_a| \sin(\omega t + \alpha_o + \beta_p)$$

$$i_b = -\sqrt{2}\,|I_a| \cos(\omega t + \alpha_o + \beta_p) \tag{4.19}$$

Substitution of Equations (4.18) and (4.19) in Equation (4.17) gives

$$P_m = -\frac{1}{2}\Big[-\sqrt{2}\,\omega L_m I_f |I_a| \sin(\omega t + \alpha_o + \beta_p) \sin(\omega t + \sigma_o)$$

$$- \sqrt{2}\,\omega L_m I_f |I_a| \cos(\omega t + \alpha_o + \beta_p) \cos(\omega t + \sigma_o)\Big]$$

$$P_m = \frac{\sqrt{2}}{2}\,\omega L_m I_f |I_a| \cos(\alpha_o - \sigma_o + \beta_p)$$

$$P_m = \frac{\sqrt{2}}{2}\,\omega L_m I_f |I_a| \cos(\beta_i) \tag{4.20}$$

Equation (4.20) represents the mechanical power converted to electric power per phase by the generator.

4.3 EQUIVALENT CIRCUIT

The balanced steady-state performance of a synchronous machine may be analyzed from the phasor equation for phase voltage. For either phase,

$$E_a = E_i - (R_a + j\omega L_d)\, I_a \tag{4.21}$$

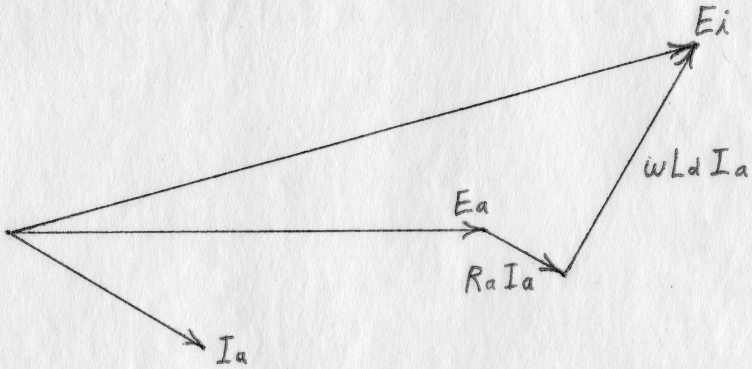

Fig. 4.3

Phasor Diagram

E_i is defined as the synchronous internal voltage. It is the voltage generated from flux produced by the field current. E_i is related to the field current as developed in section 4.2.

$$E_i = \omega L_m I_f/\sqrt{2} \tag{4.22}$$

Physically E_i can be realized as the open-circuit terminal voltage. A curve of the open-circuit terminal voltage as a function of the field excitation for the generalized machine operating at synchronous speed, is shown by the curve labeled occ in Fig. 4.4.

Open-circuit characteristic, short-circuit characteristic, and Potier triangle of the Generalized Machine as a 2-phase synchronous generator.

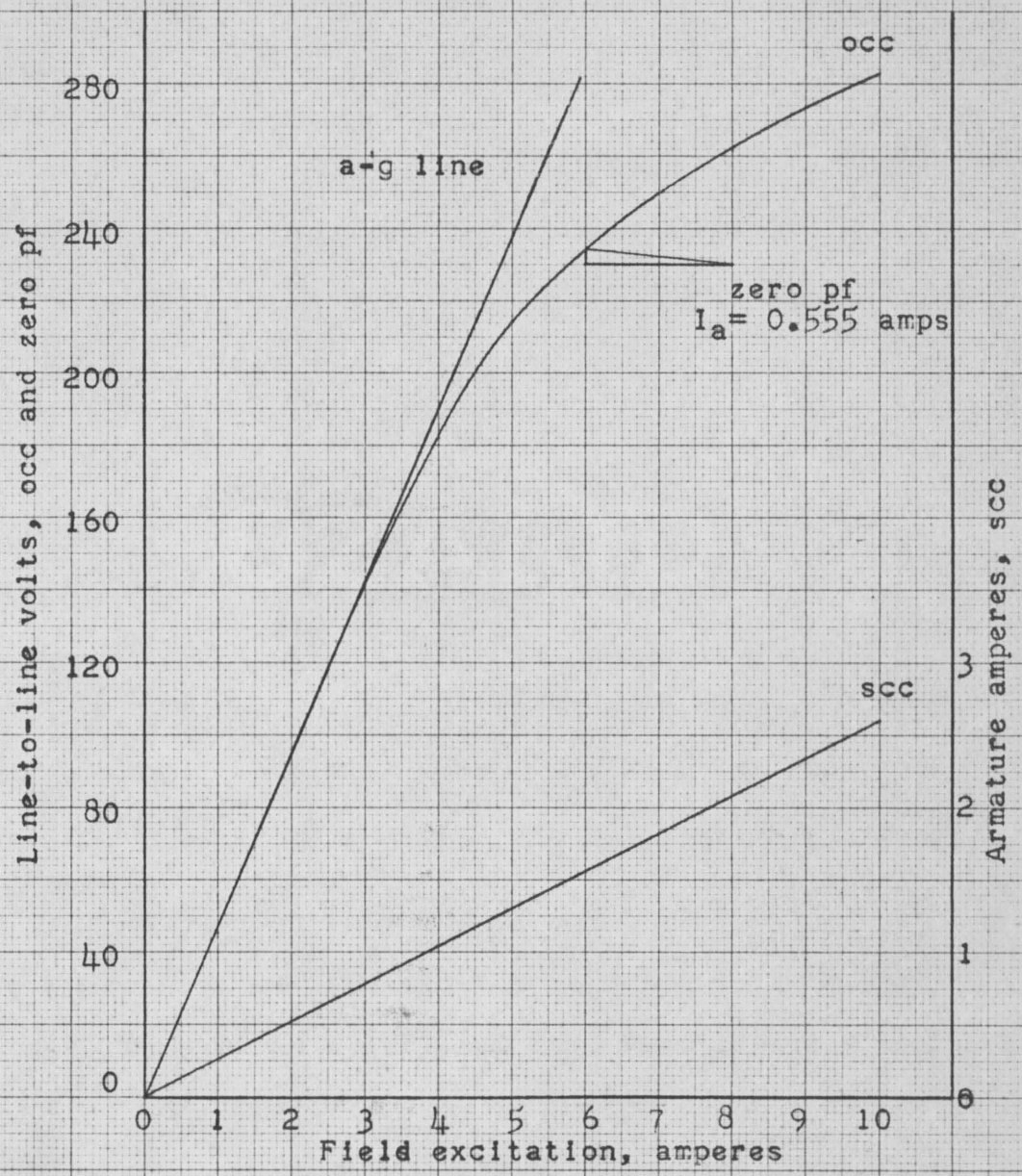

Figure 4.4

This curve is commonly called the open-circuit characteristic. It is seen from the occ that the relationship between the internal voltage and the field excitation is linear for low values of excitation but as the field current increases, the magnetic circuit becomes saturated and the curve becomes nonlinear. Although synchronous machines are generally operated in a slightly saturated condition the effect of saturation will be neglected for the present and the results modified later.

Equation (4.21) suggests that an equivalent circuit composed of a resistance and an inductance may be used to represent the steady-state behavior of the generalized machine. The term ωL_a is

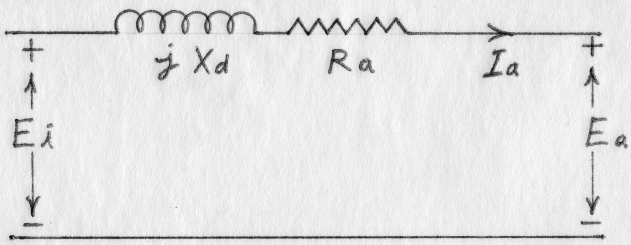

Fig. 4.5

Equivalent Circuit

defined as the synchronous reactance X_d. Together with the armature resistance R_a, it is used to account for the difference between the internal voltage that is developed by the field current and the voltage actually appearing at the armature terminals.

The armature resistance accounts for the voltage drop caused by the armature current and the resistance of the armature winding. The

significance of the synchronous reactance is best explained by determining why the internal and terminal voltage should differ and then using the synchronous reactance to represent these effects.

The resultant air-gap flux can be considered as the sum of component fluxes produced by the field current and by the armature current. The effect of the armature current on the field flux can be represented by a part of the synchronous reactance called the reactance of armature reaction, X_a.

The remainder of the synchronous reactance is used to account for the slot-leakage flux, coil-end-leakage flux, and harmonic fluxes.[1] It is called the leakage reactance X_1.

The equivalent circuit may now be drawn as,

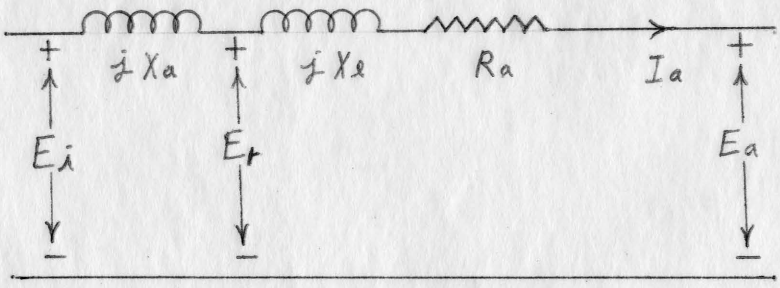

Fig. 4.6

Equivalent Circuit

The short-circuit characteristic of the machine was obtained by

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[1] A. E. Fitzgerald and C. Kingsley, Jr., Electric Machinery, McGraw-Hill Book Co., 1952, p. 202

short circuiting the armature terminals and measuring the armature current for different values of field excitation. The short-circuit characteristic is linear since the machine operates in an unsaturated condition for well above rated armature current. See Figure 4.4.

The synchronous reactance can be calculated from $X_d = \omega L_d$. This relationship was developed in section 4.2 in the flux-linkage derivation. Using the given value of 0.46 henries for L_d, the synchronous reactance is calculated to be 172 ohms or 0.552 per-unit. All per-unit values are computed using 169 VA and 230 volt bases.

The unsaturated synchronous reactance can be found from the open-circuit and short-circuit characteristics.
For the short-circuit condition,

$$E_i = I_a (R_a + j X_d) \tag{4.23}$$

Since R_a has a value of 2.8 ohms or 0.009 per-unit, it can be neglected and

$$X_d = E_i / I_a \tag{4.24}$$

Using the same value of field current, I_a is read from the short-circuit characteristic and E_i from the open-circuit characteristic. The unsaturated synchronous reactance of the generalized machine is computed to be 172 ohms or 0.55 per-unit.

The leakage-reactance can be approximated using the Potier triangle method. The Potier triangle for the generalized machine

is drawn in Fig. 4.4 using the method described by March and Crary.[2]

If the open-circuit characteristic were an exact relation between the air-gap voltage and the resultant mmf under load, the vertical leg of the triangle, ab, would be the leakage-reactance voltage drop and the horizontal leg bc would equal the armature-reaction mmf. For a nonsalient-pole machine this relation is a good approximation and the Potier reactance equals the leakage reactance.

The leakage reactance of the generalized machine is calculated to be 7.5 ohms or 0.024 per-unit. The unsaturated reactance of armature-reaction is then 164.5 ohms or 0.526 per-unit.

The unsaturated synchronous reactance may be modified to take into account the effect of saturation. Following the method developed by Kingsley,[3] a saturation factor, K = 1.144, is calculated for the generalized machine. The paths of the leakage fluxes are mainly in air and are thus assumed independent of saturation. The saturation factor is therefore applied only to the reactance of armature-reaction. The modified reactance of armature-reaction is 143.7 ohms or 0.459 per-unit. Adding the leakage reactance, the synchronous reactance is 150.2 ohms or 0.480 per-unit.

---

[2] L. A. March and S. B. Crary, "Armature Leakage Reactance of Synchronous Machines", AIEE Trans., Vol. 54, April 1935, pp 378-381

[3] C. Kingsley, Jr., "Saturated Synchronous Reactance", AIEE Trans., Vol. 54, March 1935, pp 300-304

Chapter 5

TRANSIENT PERFORMANCE

The transient state of a machine may be defined as the transition from one steady state to another. For a synchronous machine the transient state will occur whenever the terminal conditions or the speed suddenly change. The transient behavior of the generalized machine will be analyzed for balanced faults. Unbalanced faults will be studied in Chapter 6.

5.1 MACHINE EQUATIONS

Equations describing the performance of the generalized machine operating as a synchronous generator will be written for a balanced two-phase short circuit. These equations will be developed by neglecting the effect of resistance and applying the law of constant flux linkages.

The theorem of constant flux linkages states: "In an inductive circuit of negligible resistance which is closed on itself, with no external voltages in the circuit, the total flux linkages must remain constant".[1]

For the generalized machine operating under balanced steady-state conditions, the flux linkages prior to short-circuit are

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[1] R. E. Doherty "A Simplified Method of Analyzing Short-Circuit Problems", Trans. AIEE, Vol. 42, 1923, p 849

$$\lambda_{ao} = L_d\, i_{ao} + L_m\, i_{fo} \cos \sigma_o$$

$$\lambda_{bo} = L_d\, i_{bo} + L_m\, i_{fo} \sin \sigma_o$$

$$\lambda_{fo} = L_f\, i_{fo} + L_m\, i_{ao} \cos \sigma_o + L_m\, i_{bo} \sin \sigma_o \tag{5.1}$$

where

i_{ao} = Phase a armature current before short-circuit

i_{bo} = Phase b armature current before short-circuit

i_{fo} = Field current before short-circuit

Applying the law of constant flux linkages to the field and armature circuits

$$L_d\, i_a + L_m\, i_f \cos (\omega t + \sigma_o) = \lambda_{ao}$$

$$L_d\, i_b + L_m\, i_f \sin (\omega t + \sigma_o) = \lambda_{bo} \tag{5.2}$$

$$L_f\, i_f + L_m\, i_a \cos (\omega t + \sigma_o) + L_m\, i_b \sin (\omega t + \sigma_o) = \lambda_{fo}$$

The armature and field currents are solved from Equations (5.1) by determinants

$$i_a = \frac{D_a}{\Delta}$$

$$i_b = \frac{D_b}{\Delta}$$

$$i_f = \frac{D_c}{\Delta} \tag{5.3}$$

where

$$\Delta = L_d\, (L_d L_f - L_m^2)$$

$$D_a = \lambda_{ao}\, (L_d L_f - \frac{1}{2} L_m^2) + \lambda_{fo}\, L_d L_m \cos (\omega t + \sigma_o)$$

$$+ \frac{1}{2}\, \lambda_{bo}\, L_m^2 \sin 2\, (\omega t + \sigma_o) + \frac{1}{2} \lambda_{ao}\, L_m^2 \cos 2\, (\omega t + \sigma_o)$$

$$D_b = \lambda_{bo} \left(L_d L_f - \frac{1}{2} L_m^2\right) - \lambda_{fo} L_d L_m \sin(\omega t + \sigma_o)$$

$$+ \frac{1}{2} \lambda_{ao} L_m^2 \sin 2(\omega t + \sigma_o) - \frac{1}{2} \lambda_{bo} L_m^2 \cos 2(\omega t + \sigma_o)$$

$$D_f = \lambda_{fo} L_d^2 - \lambda_{ao} L_d L_m \cos(\omega t + \sigma_o)$$

$$- \lambda_{bo} L_d L_m \sin(\omega t + \sigma_o) \qquad (5.4)$$

The transient inductance L_d', transient reactance X_d', and transient internal voltage E_i', are defined as

$$L_d' = L_d - \frac{L_m^2}{L_f}$$

$$X_d' = X_d - \frac{\omega L_m^2}{L_f}$$

$$E_i' = \frac{\omega L_m \lambda_{fo}}{\sqrt{2} L_f} \qquad (5.5)$$

The physical significance of these equations will be discussed in section (5.2).

Substitution of Equations (5.2), (5.4), and (5.5) in Equation (5.3) gives

$$i_a = \frac{\sqrt{2}}{2} E_i \frac{X_d' + X_d}{X_d' X_d} \cos \sigma_o +$$

$$i_{ao} \frac{(X_d' + X_d)}{2 X_d'} - \frac{\sqrt{2} E_i'}{X_d'} \cos(\omega t + \sigma_o)$$

$$- \frac{\sqrt{2}}{2} E_i \frac{(X_d' - X_d)}{X_d' X_d} \cos(2 \omega t + \sigma_o)$$

$$- \frac{(X_d' - X_d)}{2 X_d'} \left[i_{ao} \cos 2(\omega t + \sigma_o) + i_{bo} \sin 2(\omega t + \sigma_o) \right]$$

$$i_b = \frac{\sqrt{2}}{2} E_i \frac{X_d' + X_d}{X_d' X_d} \sin \sigma_o + i_{bo} \frac{(X_d' + X_d)}{2 X_d'}$$

$$- \sqrt{2} \frac{E_i'}{X_d'} \sin (\omega t + \sigma_o) - \frac{\sqrt{2}}{2} E_i \frac{(X_d' - X_d)}{X_d' X_d} \sin (2\omega t + \sigma_o)$$

$$- \frac{(X_d' - X_d)}{2 X_d'} \left[i_{ao} \sin 2 (\omega t + \sigma_o) - i_{bo} \cos 2 (\omega t + \sigma_o) \right]$$

$$i_f = \frac{X_d}{X_d'} i_{fo} + \frac{X_d X_m}{X_d' X_f} \left[i_{ao} \cos \sigma_o + i_{bo} \sin \sigma_o \right]$$

$$+ \left[\frac{X_d' - X_d}{X_d'} \left[i_{fo} \cos \sigma_o - \frac{X_d X_m}{X_d' X_f} i_{ao} \right] \cos (\omega t + \sigma_o) \right.$$

$$+ \left[\frac{X_d' - X_d}{X_d'} i_{fo} \sin \sigma_o - \frac{X_d X_m}{X_d' X_f} \right] i_{bo} \left. \right] \sin (\omega t + \sigma_o) \qquad (5.6)$$

When the short-circuit occurs from no load, i_{ao} and i_{bo} are zero and the current equations reduce to

$$i_a = \frac{\sqrt{2}}{2} E_i \frac{X_d' + X_d}{X_d' X_d} \cos \sigma_o - \sqrt{2} \frac{E_i}{X_d'} \cos (\omega t + \sigma_o)$$

$$- \frac{\sqrt{2}}{2} E_i \frac{X_d' - X_d}{X_d' X_d} \cos (2\omega t + \sigma_o)$$

$$i_b = \frac{\sqrt{2}}{2} E_i \frac{X_d' + X_d}{X_d' X_d} \sin \sigma_o - \sqrt{2} \frac{E_i}{X_d'} \sin (\omega t + \sigma_o)$$

$$- \frac{\sqrt{2}}{2} E_i \frac{X_d' - X_d}{X_d' X_d} \sin (2\omega t + \sigma_o) \qquad (5.7)$$

$$i_f = \frac{X_d}{X_d'} i_{fo} + \frac{X_d' - X_d}{X_d} i_{fo} \cos \omega t$$

When the resistance of the windings is included, the flux linkages of the armature and field will not remain constant. The rate of change of these flux linkages is given by Equation (4.10), modified for the short-circuit conditions.

$$\frac{d\,\lambda_a}{dt} = -R_a\, i_a$$

$$\frac{d\,\lambda_b}{dt} = -R_b\, i_b$$

$$\frac{d\,\lambda_f}{dt} = +e_f - R_f\, i_f \tag{5.8}$$

The currents of Equation (5.6) and (5.7) are still good approximations for the currents at the instant of short-circuit since the flux linkages of any closed inductive circuit cannot change instantaneously. Equation (5.8) may be written as

$$\lambda_{a2} - \lambda_{a1} = \int_{t_1}^{t_2} -R_a\, i_a\, dt$$

$$\lambda_{b2} - \lambda_{b1} = \int_{t_1}^{t_2} -R_b\, i_b\, dt$$

$$\lambda_{f2} - \lambda_{f1} = \int_{t_1}^{t_2} (e_f - R_f\, i_f)\, dt \tag{5.9}$$

From Equation (5.9), the change in flux linkages must approach zero as the time interval $t_2 - t_1$ approaches zero.

The currents resulting from a two-phase short-circuit will be transient, having an initial value given by Equations (5.6) and (5.7) and a steady-state value which can be found from Equation (4.11).

Development of transient equations for the generalized machine is facilitated if the armature currents are expressed in terms of their direct-axis and quadrature-axis components. The direct-axis current i_d is the component of armature current that leads the internal voltage E_i by 90 degrees. The quadrature-axis current i_q is the component of armature current in phase with the internal voltage E_i. For the generalized machine they are defined as

$$\begin{aligned} i_d &= K_d\,(i_a \cos\sigma + i_b \sin\sigma) \\ i_q &= K_q\,(i_a \sin\sigma - i_b \cos\sigma) \end{aligned} \tag{5.10}$$

The direct-axis, quadrature-axis, and field flux linkages of the generalized machine may be written as

$$\begin{aligned} \lambda_d &= L_d\, i_d + K_d\, L_m\, i_f \\ \lambda_q &= L_d\, i_q \\ \lambda_f &= L_f\, i_f + \frac{1}{K_d}\, L_m\, i_d \end{aligned} \tag{5.11}$$

The mutual inductance between the direct-axis and the field winding will be reciprocal, $K_d L_m = \frac{1}{K_d} L_m$, if K_d is equal to one. For simplicity, K_q will also be given the value one.

The armature currents and armature flux linkages may be written in terms of the direct-axis and quadrature-axis components.

$$
\begin{aligned}
i_a &= i_d \cos\sigma + i_q \sin\sigma \\
i_b &= i_d \sin\sigma - i_q \cos\sigma \qquad (5.12) \\
\lambda_a &= L_d i_d \cos\sigma + L_d i_q \sin\sigma + L_m i_f \cos\sigma \\
\lambda_b &= L_d i_d \sin\sigma - L_d i_q \cos\sigma + L_m i_f \sin\sigma \\
\lambda_f &= L_f i_f + L_m i_d
\end{aligned}
$$

For a balanced two-phase short-circuit from no-load, substitution of Equation (5.7) in Equation (5.10) gives

$$
\begin{aligned}
i_d &= \sqrt{2}\,\frac{E_i}{X_d''}\,(\cos\omega t - 1) \\
i_q &= -\sqrt{2}\,\frac{E_i}{X_d''}\,\sin\omega t \qquad (5.13) \\
i_f &= \frac{X_d}{X_d''}\,i_{fo} + \frac{X_d'' - X_d}{X_d}\,i_{fo}\cos\omega t
\end{aligned}
$$

The change in the flux linkages of the windings in the time interval $t_1 - t_2$, is given by Equation (5.9). If the interval chosen is an integer number of cycles, the ac components of the

currents will have no net effect on the flux linkages as the integral of a periodic function over one cycle is zero. If the time interval contains a fraction of a cycle, the ac component can still be neglected as its effect cannot increase progressively and would remain small. With this approximation, the ac components of the flux linkages must be zero at the instant of short-circuit and remain so. The flux linkages of the windings then consist only of a dc term which decreases at a rate directly proportional to the dc component of current and the resistance of the winding. The $I^2 R$ losses produced by the alternating components affect the flux linkages but will be neglected.

For a balanced two-phase short-circuit the transient equations for the generalized machine can be written as

$$
\begin{aligned}
i_f &= i_f\,(dc) + i_f\,(c) \cos \omega t \\
i_d &= i_d\,(dc) + i_d\,(c) \cos \omega t \qquad (5.14) \\
i_q &= i_q\,(s) \sin \omega t
\end{aligned}
$$

If the machine were operating at no load prior to the short-circuit, the initial values of the components are found from Equation (5.13).

$$i_f\ (dc) = \frac{X_d}{X_d'}\ i_{fo}$$

$$i_f\ (c) = \frac{X_d' - X_d}{X_d}\ i_{fo}$$

$$i_d\ (dc) = \sqrt{2}\ \frac{E_i}{X_d'}$$

$$i_d\ (c) = \sqrt{2}\ \frac{E_i}{X_d'}$$

$$i_q\ (s) = \sqrt{2}\ \frac{E_i}{X_d'} \tag{5.15}$$

The transient currents, Equation (5.14), are substituted in the expressions for the flux linkages of the field and armature windings, Equation (5.12), giving

$$\lambda_f = L_f\ i_f\ (dc) + L_m\ i_d\ (dc) + \left[L_f\ i_f\ (c) + L_m\ i_d\ (c)\right] \cos \omega t \tag{5.16}$$

$$\lambda_a = \frac{1}{2}\left[L_d\ i_d\ (dc) + L_d\ i_q\ (s) + L_m\ i_f\ (c)\right] \cos \sigma_o$$

$$+ \left[L_d\ i_d\ (dc) + L_m\ i_f\ (dc)\right] \cos\ (\omega t + \sigma_o)$$

$$+ \frac{1}{2}\left[L_d\ i_d\ (c) - L_d\ i_q\ (s) + L_m\ i_f\ (c)\right] \cos\ (2\ \omega t + \sigma_o)$$

Using the approximate procedure developed, the dc components of the flux linkages are substituted in Equation (5.8) to account for the change in flux linkages, and the ac components are simply equated to zero. The resulting relationships are

$$L_f \frac{d\, i_f\,(dc)}{dt} + L_m \frac{d\, i_d\,(dc)}{dt} = e_f - R_f\, i_f\,(dc)$$

$$L_d \frac{d\, i_d\,(dc)}{dt} + L_d \frac{d\, i_q\,(s)}{dt} + L_m \frac{d\, i_f\,(c)}{dt}$$

$$= - R_a\,(i_d\,(c) + i_q\,(c))$$

$$L_f\, i_f\,(c) + L_m\, i_d\,(c) = 0$$

$$L_d\, i_d\,(dc) + L_m\, i_f\,(dc) = 0$$

$$L_d\, i_d\,(c) - L_d\, i_q\,(s) + L_m\, i_f\,(c) = 0 \qquad (5.17)$$

Solving for the transient current components from Equation (5.17) and using the initial conditions of Equation (5.15)

$$i_f\,(dc) = i_{fo}\left[1 - \frac{X_d' - X_d}{X_d'}\, \epsilon^{-\left[\frac{X_d\, R_f}{X_d'\, L_f}\, t\right]}\right]$$

$$i_d\,(dc) = -\sqrt{2}\, E_i \left[\frac{1}{X_d} - \frac{X_d' - X_d}{X_d'\, X_d}\, \epsilon^{-\left[\frac{X_d\, R_f}{X_d'\, L_f}\right] t}\right]$$

$$i_d\,(c) = \frac{\sqrt{2}\, E_i}{X_d'}\, \epsilon^{-\left[\frac{\omega R_a\,(X_d' + X_d)}{2\, X_d'\, X_d}\right] t}$$

$$i_f\,(c) = i_{fo}\, \frac{X_d' - X_d}{X_d'}\, \epsilon^{-\left[\frac{\omega R_a\,(X_d' + X_d)}{2 X_d'\, X_d}\right] t}$$

$$i_q\,(s) = \frac{\sqrt{2}\, E_i}{X_d}\, \epsilon^{-\left[\frac{\omega R_a\,(X_d' + X_d)}{2 X_d'\, X_d}\right] t} \qquad (5.18)$$

The direct-axis transient time constant T_d' and the armature time constant T_a are defined as

$$T_d' = \frac{X_d' \, L_f}{X_d \, R_f}$$

$$T_a = \frac{2X_d' \, X_d}{\omega R_a \, (X_d' + X_d)} \tag{5.19}$$

The transient currents of the armature windings and the transient current of the field winding are found by substitution of Equations (5.18) and (5.19) in Equations (5.14) and (5.12)

$$i_a = -\sqrt{2}\, E_i \left[\frac{1}{X_d} - \left[\frac{X_d' - X_d}{X_d' \, X_d} \right] \epsilon^{-\left[\frac{t}{T_d'}\right]} \right] \cos(\omega t + \sigma_o)$$
$$+ \frac{\sqrt{2}}{2} E_i \left[\frac{X_d' + X_d}{X_d' \, X_d} \, \epsilon^{-\left[\frac{t}{T_a}\right]} \right] \cos \sigma_o$$
$$- \frac{\sqrt{2}}{2} E_i \left[\frac{X_d' - X_d}{X_d' \, X_d} \, \epsilon^{-\left[\frac{t}{T_a}\right]} \right] \cos(2\omega t + \sigma_o)$$

$$i_b = -\sqrt{2}\, E_i \left[\frac{1}{X_d} - \left[\frac{X_d' - X_d}{X_d' \, X_d} \right] \epsilon^{-\left[\frac{t}{T_a'}\right]} \right] \sin(\omega t + \sigma_o)$$
$$+ \frac{\sqrt{2}}{2} E_i \left[\frac{X_d' + X_d}{X_d' \, X_d} \, \epsilon^{-\left[\frac{t}{T_a}\right]} \right] \sin \sigma_o$$
$$- \frac{\sqrt{2}}{2} E_i \left[\frac{X_d' - X_d}{X_d' \, X_d} \, \epsilon^{-\left[\frac{t}{T_a}\right]} \right] \sin(2\,\omega t + \sigma_o)$$

$$i_f = i_{fo} \left[1 - \frac{X_d' - X_d}{X_d'} \left(\epsilon^{-\left[\frac{t}{T_d'}\right]} - \epsilon^{-\left[\frac{t}{T_a}\right]} \cos \omega t \right) \right] \tag{5.20}$$

The above equations describe the transient performance of the generalized machine operating as a synchronous generator when subjected to a balanced two-phase fault from no load. Similar equations

could be derived for a short-circuit when the machine has a load. In the preceding derivation it has been assumed that the speed and the voltage applied to the field circuit remained constant.

For steady-state conditions, Equation (5.20) reduces to

$$i_a = -\sqrt{2}\,\frac{E_i}{X_d}\cos\,(\omega t + \sigma_o)$$

$$i_b = -\sqrt{2}\,\frac{E_i}{X_d}\sin\,(\omega t + \sigma_o)$$

$$i_f = i_{fo} \qquad (5.21)$$

5.2 PHYSICAL RELATIONSHIPS

The currents of Equation (5.20) mathematically describe the transient performance of the generalized machine for a balanced two-phase short-circuit from no load. These equations become more significant if their components are explained physically. The effect of resistance is neglected and the law of constant flux linkages applied.

At the instant of short-circuit the armature mmf attempts to drive a demagnetizing flux into the field circuit. To keep the field flux linkages constant the field current must increase. The fundamental armature current also increases since there is always a constant of proportionality between the fundamental armature current and the unidirectional field current.

Before the short-circuit occurs, the field flux linkages are constant but the armature flux linkages vary as the field rotates. At the instant of short-circuit the flux linkages trapped in each armature winding are dependent upon the angular position of the rotor. In order to keep the armature flux linkages constant the initial armature current of each phase must have a unidirectional component whose magnitude depends on the armature flux linkages of that phase. The unidirectional current in the armature will produce a fundamental current in the field winding, which in turn produces a second-harmonic in the armature winding. Each

armature winding produces a third-harmonic in the field winding but they are of opposite phase and cancel.

5.3 TRANSIENT CONSTANTS

The transient performance of the unloaded generalized machine for a balanced two-phase short-circuit may be characterized by two reactances - the synchronous reactance X_d and the transient reactance X_d' - and by two time constants - the direct-axis transient time constant T_d' and the armature time constant T_a.

The measured parameters of the generalized machine are

R_a = 2.8 ohms = Resistance of armature winding

R_f = 0.46 ohms = Resistance of field winding

L_d = 0.459 henries = Self-inductance of armature winding

L_f = 0.0662 henries = Self-inductance of field winding

L_m = 0.172 henries = Mutual inductance between armature winding and field winding

Using these parameters the reactances and time constants of the generalized machine may be calculated from Equations (5.5) and (5.19). Base quantities for the per unit values are 169 volt-amperes and 230 volts.

$$X_d = \omega L_d = 172 \text{ ohms} = 0.552 \text{ pu}$$

$$X_d' = \omega\left(L_d - \frac{L_m^2}{L_f}\right) = 4.9 \text{ ohms} = 0.016 \text{ pu}$$

$$T_d' = \frac{L_d' \, L_f}{L_d \, R_f} = 0.0041 \text{ sec.}$$

$$T_a = \frac{2 \, L_d' \, L_d}{R_a \, (L_d' + L_d)} = 0.009 \text{ sec.}$$

The synchronous reactance X_d determines the steady-state short-circuit current while the transient reactance X_d' is so defined that it determines the initial value of fundamental armature current. The initial value of this component therefore differs from its steady-state value by the ratio X_d' / X_d. The initial unidirectional component in the field winding will also differ from its steady-state value by this ratio. The two components will decrease from their initial values to their steady-state values by an exponential curve having the time constant T_d'.

The remaining components of the field and armature winding will have initial components that are a function of both X_d' and X_d, and that decrease to zero by an exponential curve having the time constant T_a.

5.4 EXPERIMENTAL STUDY

A Hathaway Type S 14-E Oscillograph was used to record the transient behavior of the generalized machine. The field current, a-phase armature current, b-phase armature current, and b-phase armature voltage were recorded for a balanced two-phase short circuit from no load. See Figure (5.1).

The general validity of the transient reactance X_d', the direct-axis transient time constant T_d', and the armature time constant T_a may be verified from this recording.[1] It is difficult to accurately determine these constants from the recording due to the high harmonic content. It would be necessary to isolate the unidirectional and fundamental components of the field and armature currents.

The steady-state, short-circuit armature current contains a fifth-harmonic component. This fifth-harmonic is not visible in the steady-state, open-circuit voltage so the short-circuit impedance of the machine for the fifth-harmonic component of current is probably very low. A fourth-harmonic component of field current is associated with the fifth-harmonic component of armature current.

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[1] S. H. Wright, "Determination of Machine Constants by Test", AIEE Trans., Vol. 50, December, 1931, pp 1331-1351.

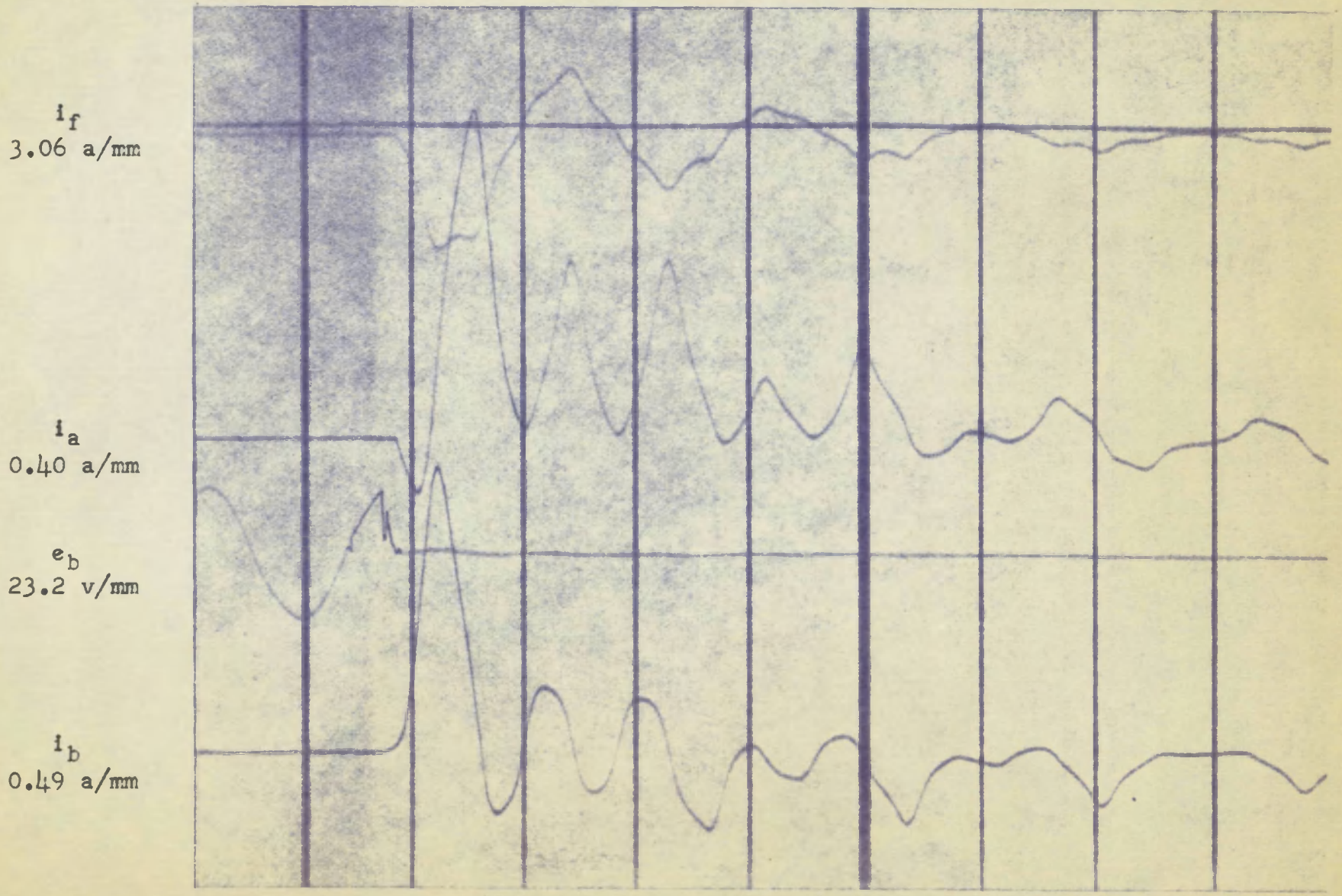

Fig. 5.1 - Two-phase Short-circuit

Chapter 6

UNBALANCED CONDITIONS

6.1 MACHINE EQUATIONS

The transient and steady-state behavior of the generalized machine is analyzed for an unbalanced single-phase fault from no load. Equations describing the performance of the machine are based on the procedure developed in chapter 5. Initially all resistance is neglected and the law of constant flux linkages is applied to obtain the currents at the instant of short circuit. The effect of resistance is then included and the transient behavior of the currents found.

Applying the law of constant flux linkages when phase a is short-circuited

$$L_d\, i_a + L_m\, i_f \cos\sigma = L_m\, i_{fo} \cos\sigma_o$$

$$L_f\, i_f + L_m\, i_a \cos\sigma = L_f\, i_{fo} \tag{6.1}$$

Solving for the armature current i_a from Equation (6.1)

$$i_a = \frac{L_m\, i_{fo}\, (\cos\sigma_o - \cos\sigma)}{\left(L_d - \frac{L_m^2}{L_f} \cos^2\sigma\right)} \tag{6.2}$$

Using the definitions of transient reactance X_d and internal voltage E_i

$$i_a = \frac{2\sqrt{2}\, E_i\, (\cos\sigma_o - \cos\sigma)}{(X_d' + X_d) + (X_d' - X_d)\cos 2\sigma} \tag{6.3}$$

Define the negative sequence reactance X_2 and the constant b as

$$X_2 = \sqrt{X_d' \; X_d} = \sqrt{(173)\,(4.9)} = 29.1 \text{ ohms} = .093 \text{ pu}$$

$$b = \frac{X_d - X_2}{X_d + X_2} = 0.712 \qquad (6.4)$$

This definition is commonly used for the impedance offered by a synchronous machine to the flow of negative sequence current during an unbalanced fault.

Expanding Equation (6.3) as a Fourier series using the definitions of Equation (6.4)

$$i_a = -\frac{2\sqrt{2}\;E_i}{X_d' + X_2}\left[\cos\sigma + \sum_{n=1}^{n=\infty} b^n \cos(2n+1)\,\sigma\right]$$

$$+ \frac{\sqrt{2}\;E_i \cos\sigma_o}{X_2}\left[1 + 2\sum_{n=1}^{n=\infty} b^n \cos 2n\,\sigma\right] \qquad (6.5)$$

From Equations (6.1) and (6.5) the field current may be written as

$$i_f = i_{fo} + \frac{(X_d - X_d')\,i_{fo}}{X_d' + X_2}\left[\left[1 + \frac{1+b}{b}\sum_{n=1}^{\infty} b^n \cos 2n\,\sigma\right] - 2\cos\sigma\left[\cos\sigma + \sum_{n=1}^{n=\infty} b^n \cos(2n+1)\right]\right] \qquad (6.6)$$

Equations (6.5) and (6.6) give the phase a armature current and field current at the instant of short circuit. The phase b armature current will of course be zero.

As in section (4.1), the effect of resistance is taken into account by assuming that the unidirectional component of flux linkages of each winding decreases at a rate determined by the unidirectional component of current and the winding resistance. The alternating component of flux linkages is assumed zero. With these assumptions all the harmonic terms of the same series have the same time constant.

The armature current and field current are found to be

$$i_a = - \left[\frac{2\sqrt{2}\ E_i}{X_d + X_2} + \left[\frac{2\sqrt{2}\ E_i}{X_d' + X_2} - \frac{2\sqrt{2}\ E_i}{X_d + X_2} \right] \epsilon^{\frac{t}{T_d'}} \right]$$

$$\times \left[\cos\sigma + \sum_{n=1}^{n=\infty} b^n \cos(2n+1)\sigma \right]$$

$$+ \frac{\sqrt{2}\ E_i \cos\sigma_o}{X_2} \epsilon^{\frac{t}{T_a}} \left[1 + 2 \sum_{n=1}^{n=\infty} b^n \cos 2n\sigma \right]$$

$$i_f = i_{fo} + (X_d - X_d') \, i_{fo} \left[\frac{\epsilon^{-\frac{t}{T_d'}}}{X_d' + X_2} + \right.$$

$$\left[\frac{1}{X_d + X_2} + \left[\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right] \epsilon^{-\frac{t}{T_d'}} \right] \left[\frac{1 + b}{b} \right.$$

$$\left. \sum_{n=1}^{n=\infty} b^n \cos 2n\sigma \right] - \frac{2 \cos \sigma_o}{X_d' + X_2} \epsilon^{-\frac{t}{T_a}} \left[\cos \sigma \right.$$

$$\left. \left. + \sum_{n=1}^{n=\infty} b^n \cos (2n + 1) \sigma \right] \right] \tag{6.7}$$

where the direct-axis transient time constant T_d' and the armature time constant T_a are given by

$$T_a = \frac{X_2}{\omega R_a} = 0.028 \text{ sec.}$$

$$T_d' = \frac{(X_d' + X_2) \, L_f}{(X_d + X_2) \, R_f} = 0.024 \text{ sec.} \tag{6.8}$$

For steady-state the currents reduce to

$$i_a = - \frac{2\sqrt{2}\ E_i}{X_d + X_2} \left[\cos\sigma + \sum_{n=1}^{n=\infty} b^n \cos(2n+1)\sigma \right]$$

$$i_f = i_{fo} + \frac{X_d - X_d'}{X_d + X_2}\ i_{fo}\ \frac{1+b}{b} \sum_{n=1}^{n=\infty} b^n \cos 2n\sigma \qquad (6.9)$$

6.2 PHYSICAL RELATIONSHIPS

At the instant of short-circuit both the armature and field currents contain a unidirectional component, a fundamental component, and both even and odd harmonics. The odd harmonics in the armature, and the unidirectional component and even harmonics in the field decrease exponentially with the time constant T_d'. For both the armature and field currents, the ratio of the initial values to the steady-state values is given by $(X_d + X_2) / (X_d' + X_2)$. The even harmonics in the armature and the odd harmonics in the field decrease to zero exponentially with the time constant T_a.

The relative magnitude of the harmonics in the steady-state currents of Equation (6.9) are

$$i_a = - \frac{2\sqrt{2}\ E_i}{X_d + X_2} \quad (\cos \omega t + .712 \cos 3\omega t + .507 \cos 5\omega t + \ldots\ldots\ldots)$$

$$i_f = i_{fo}\ (1 + 1.358 \cos 2\omega t + .996 \cos 4\omega t + \ldots.)$$

Figure (6.1) shows an oscillograph recording made for a single-phase short-circuit on a-phase. The field current, armature currents and b-phase armature voltage are recorded.

From Figure (6.1) the ratio of the first peak to the steady-state peak is measured as 5.2. This gives a low approximation for the ratio between the initial current and the steady-state current. This ratio may be calculated from the expression $(X_d + X_2) / (X_d' + X_2)$ using the measured values of inductance. The ratio is computed to be 5.9, which shows that the reactances are of the proper order of magnitude.

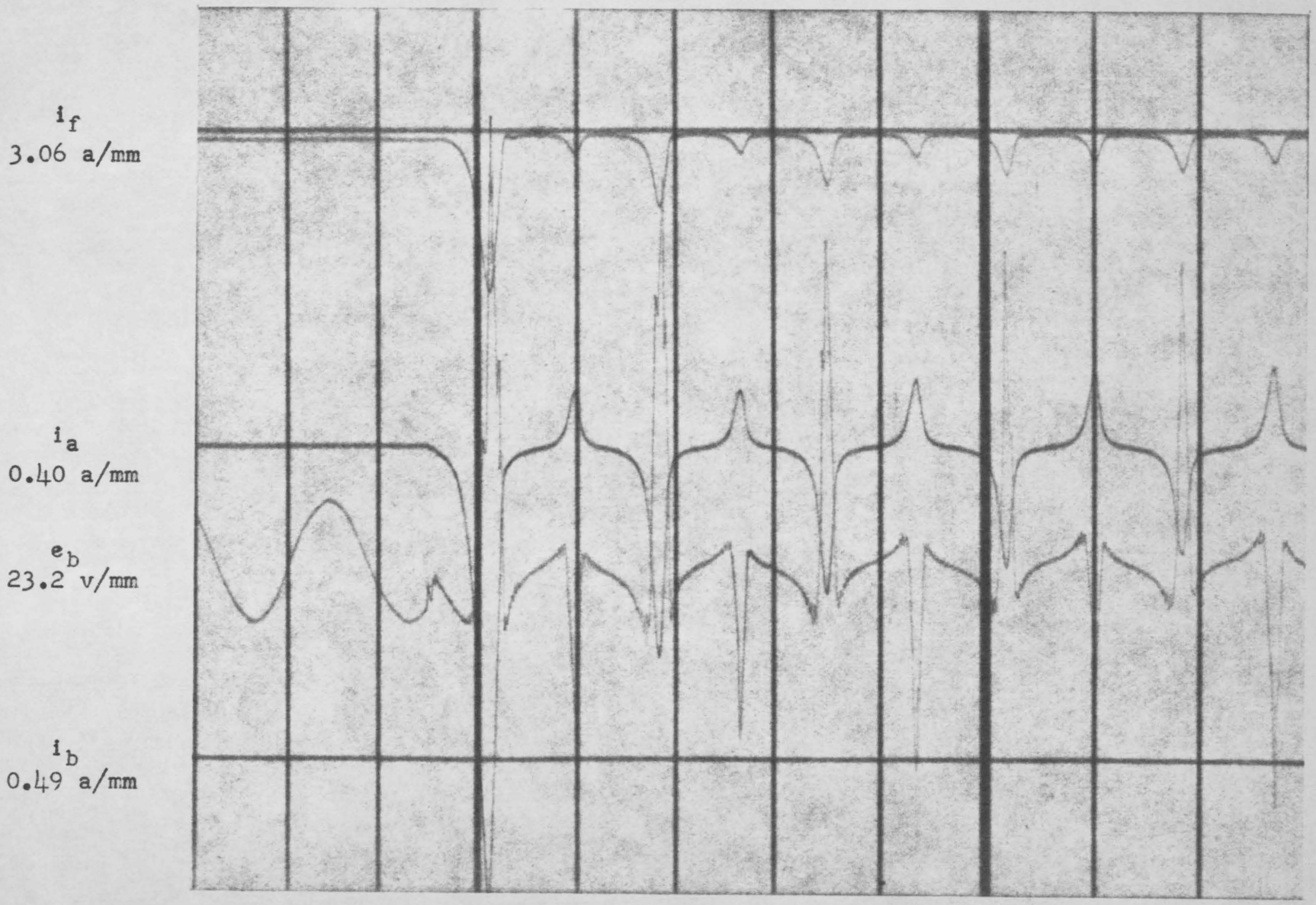

Fig. 6.1 - Single-phase Short-circuit

Chapter 7

SUMMARY OF RESULTS AND CONCLUSIONS

The constants which describe the performance of the generalized machine are given in Table I. These constants were calculated from measured values of inductance and resistance. Included for comparison purposes are the constants of a conventional synchronous generator.[1] Reactances are given in per unit values which are based on their corresponding machine.

Table I - Machine Constants

	Generalized Machine	Conventional Machine
X_d	0.552	0.95 - 1.45
X_d'	0.016	0.12 - 0.21
X_2	0.093	0.07 - 0.14
T_d'	0.004	0.6
T_a	0.009	0.04 - 0.24

[1] Transmission and Distribution Reference Book, Westinghouse Electric Corporation, 4th Ed., 1950, p 189

The armature of the generalized machine is rated at 230 volts and 3.6 amperes or 828 volt-amperes. The maximum current rating of the field winding, 8 amperes, prevents the machine from obtaining this VA rating. When the internal voltage E_i and the field current I_f are set at their maximum rated values of 230 volts and 8 amperes respectively, the armature current is 1.0 ampere (unity power-factor) as compared with its rated value of 3.6 amperes. At 0.85 power-factor lagging, (the VA rating of synchronous generators are commonly expressed at 0.85 p.f. lagging) the armature current is 0.735 amperes which gives a VA rating of 169 volt-amperes.

The low per unit value of synchronous reactance results from the utilization of only a fraction of the current rating of the armature. The synchronous reactance could be raised to 1.0 p.u. if the maximum rating of the field current could be increased.

The low per unit value of transient reactance, in relation to the synchronous reactance, is a consequence of the high coefficient of coupling between the field and armature circuits. The coefficient of coupling K for the generalized machine is

$$K = \frac{L_m}{\sqrt{L_d \, L_f}} = 0.98 \qquad (7.1)$$

Expressing the transient reactance X_d' in terms of the synchronous reactance X_d and the coefficient of coupling K

$$X_d' = X_d \, (1 - K) \qquad (7.2)$$

As the coefficient of coupling approaches unity the transient reactance and the leakage reactance converge and approach zero.[2] The value of the leakage reactance is computed in section (4.3) to be 0.024 p. u. as compared with a value of 0.016 p. u. for the transient reactance.

The transient reactance, as computed from the inductance measurements, is not precise since the difference of two nearly equal quantities is taken. An error in the third significant figure of the inductance would produce a corresponding error in the second significant figure of the transient reactance. From the oscillograph recordings and the value of leakage reactance, it may be concluded that while not precise, the stated value of transient reactance is of the proper order of magnitude.

The direct-axis time constant T_d' is usually several times larger than the armature time constant T_a for conventional synchronous generators. The fundamental component of field current thus decays more rapidly than the unidirectional component and no reversal of the field current results. For the generalized machine under investigation, the direct-axis time constant is less than half of the armature time constant. Equation (5.20) indicates that a reversal of the field current will occur for time constants of this magnitude. The value at the end of one cycle is computed to be

$$i_f = i_{fo}\left[1 + \frac{X_d - X_d'}{X_d'}(1 - \cos \omega t)\right] = -3.5\ i_{fo} \qquad (7.3)$$

[2] Transmission and Distribution Reference Book, Westinghouse Electric Corporation, 4th Ed., 1950, p 154

This is verified by the oscillograph recording of Figure (5.1) which shows a value at the end of one cycle of

$$i_f = -4.5\, i_{fo} \tag{7.4}$$

From Equation (A.12) it is seen that the flux produced by the field mmf is nearly sinusoidal. The oscillograph recordings show that the resulting open-circuit armature voltage is also practically sinusoidal. The steady-state armature current for a two-phase short-circuit contains a fifth-harmonic component which indicates that the impedance of the machine to the fifth-harmonic component of current is low.

In conclusion, the transient and steady-state performance of the generalized machine operating as a synchronous generator under balanced and unbalanced conditions, may be effectively analyzed from the constants given in Table I. These constants are not typical of those for a conventional synchronous generator.

Appendix A

FIELD FLUX PER POLE

The flux density produced by the field circuit of the generalized machine may be represented as a function of angular displacement by the Fourier series[1]

$$B_{fr} = \frac{4\, B_{fm} \left(\sin \frac{\pi r}{4}\right)^2}{q_p\, K_r\, \pi r \sin \frac{\pi r}{4\, q_p}} \cos r\,(\omega t + \sigma_o) \qquad (A.1)$$

where

B_{fm} = Maximum flux density at the center of each pole

r = Order of the harmonic (only odd harmonics present due to half-wave symmetry)

q_p = Number of field coils per pole

K_r = Correction factor for rotor slots

It is assumed that the machine is operating in an unsaturated condition so that the iron has infinite permeability and all the mmf is consumed in forcing the flux through the air-gap. The maximum flux density is then given by

$$B_{fm} = \frac{\mu_o\, M_{fm}}{g'} \qquad (A.2)$$

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[1] W. A. Lewis, The Principles of Synchronous Machines, 2nd. Ed., Edward Brothers, Inc., 1954, p 5.16

where

μ_o = Permeability of air

M_{fm} = Maximum mmf produced by field

g' = Effective length of air-gap

The maximum mmf produced by the field is[2]

$$M_{fm} = \frac{Z\ I_f}{2\ pa} \tag{A.3}$$

where

Z = Number of rotor inductors

p = Number of poles

a = Number of parallel paths in field circuit

I_f = Field current

At rated field current, I_f = 8 amperes, M_{fm} = 560 ampere-turns per pole.

The flux density is affected by armature curvature, stator slots, rotor slots, and end fringing. These effects may be taken into account by modifying the air-gap length.

1- Correction for armature curvature[3]

$$K = \frac{d}{2g}\ \ln \frac{1}{1 - \frac{2g}{d}} \tag{A.4}$$

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[2] A. E. Fitzgerald and C. Kingsley, Jr., Electric Machinery, McGraw-Hill Book Co., Inc., 1952, p 178

[3] W. A. Lewis, The Principles of Synchronous Machines, 2nd Ed., Edward Brothers, Inc., 1954, p 5.8

where

d = Diameter of armature surface

g = Actual air-gap length

For the generalized machine $K_c = 1.01$.

2- Correction for stator slots[4]

$$K_s = \frac{1 + \frac{s_s}{t_s}}{1 + C_s \frac{s_s}{t_s}} \qquad (A.5)$$

where

s = Width of stator slot

t = Width of stator tooth

C_s = Slot correction factor

For the generalized machine $K_s = 1.13$.

3- Correction for rotor slots[5]

$$K_r = \frac{1 + \frac{s_r}{t_r}}{1 + C_r \frac{s_r}{t_r}} \qquad (A.6)$$

where

s = Width of rotor slot

t = Width of rotor tooth

C_r = Slot correction factor

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[4] W. A. Lewis, The Principles of Synchronous Machines, 2nd. Ed., Edward Brothers, Inc., 1954, p 5.10

[5] W. A. Lewis, The Principles of Synchronous Machines, 2nd. Ed., Edward Brothers, Inc., 1954, p 5.11

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For the generalized machine $K_r = 1.15$.

4- Correction for end fringing[6]

$$K_f = \frac{f}{f + g} \tag{A.7}$$

where

f = Length of rotor

g = Actual air-gap length

For the generalized machine $K_f = 0.992$.

The actual air-gap of 0.023 inch is multiplied by these correction factors to give an effective length of $g' = 0.03$ inch.

The maximum flux density is then calculated to be 5.96×10^{-4} webers per square inch.

From Equation (A.1), the flux density for the generalized machine is

$$B_{fr} = 4.2 \times 10^{-4} \left[\cos(\omega t + \sigma_o) + 0.113 \cos 3(\omega t + \sigma_o) - 0.041 \cos 5(\omega t + \sigma_o) - 0.002 \cos 7(\omega t + \sigma_o) + \ldots\ldots \right] \tag{A.8}$$

The flux per pole is found by integrating Equation (A.8), with respect to the area, over one pole. The differential area in terms

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[6] W. A. Lewis, The Principles of Synchronous Machines, 2nd. Ed., Edward Brothers, Inc., 1954, p 5.12

of the differential angular displacement is

$$dA = \frac{D\ell}{p} \, d\sigma \tag{A.9}$$

where

ℓ = Rotor length

p = Number of poles

D = Armature diameter

Then

$$\phi_{fr} = \frac{D\ell}{p} \int_{\sigma - \frac{\pi}{2}}^{\sigma + \frac{\pi}{2}} \left[4.2 \times 10^{-4} \cos \sigma + \ldots\ldots \right] d\sigma \tag{A.10}$$

Integrating

$$\phi_{fr} = 2.92 \times 10^{-3} \left[\sin \sigma + \ldots\ldots \right]_{\sigma - \frac{\pi}{2}}^{\sigma + \frac{\pi}{2}} \tag{A.11}$$

Substitution of the limits gives

$$\phi_{fr} = 5.84 \times 10^{-3} \left[\cos (\omega t + \sigma_o) - 0.037 \cos 3 (\omega t + \sigma_o) - 0.008 \cos 5 (\omega t + \sigma_o) + \ldots\ldots \right] \tag{A.12}$$

Appendix B

MUTUAL INDUCTANCE

An analytical expression can be developed for the mutual inductance between the field winding and either phase winding of the armature. The mutual inductance equals the linkages of the phase winding with the field flux, divided by the field current.

$$L_{af} = \frac{\lambda_a}{I_f} \tag{B.1}$$

where

λ_a = Armature flux linkages per phase

I_f = Field current

The armature has a fractional-pitch, distributed winding which reduces the effective number of turns. The effective number of turns is related to the actual turns by the pitch factor K_p and the distribution factor K_b. These factors may be calculated using the relationships.[1]

$$K_{pr} = \sin\left(r\, p_t \frac{\pi}{2}\right) \tag{B.2}$$

$$K_{br} = \frac{\sin\left(r \frac{\pi}{\sigma}\right)}{q_a \sin\left(\frac{r}{q_a}\frac{\pi}{\sigma}\right)} \tag{B.3}$$

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[1] W. A. Lewis, The Principles of Synchronous Machines, 2nd. Ed., Edward Brothers, Inc., 1954, pp 6.5 and 6.9

where

r = Order of the harmonic

p_t = Coil pitch

q_a = Coils per phase

The computed results for the first seven harmonics are

K_{p1} =	0.820	K_{b1} =	0.955
K_{p3} =	0.258	K_{b3} =	0.638
K_{p5} =	- 0.995	K_{b5} =	- 0.280
K_{p7} =	0.423	K_{b7} =	- 0.031

The mutual inductance is now written

$$L_{af} = \frac{N_a}{I_f} \sum_{r=1}^{r=\infty} K_{pr} K_{br} \phi_{fr} \tag{B.4}$$

where

N_a = Armature turns per phase

ϕ_{fr} = Field flux

Substitution of Equation (A.12) results in

$$L_{af} = 0.185 \left[\cos(\omega t + \sigma_o) - 0.0078 \cos 3(\omega t + \sigma_o) - 0.0028 \cos(\omega t + \sigma_o) + \ldots\ldots \right] \tag{B.5}$$

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