SHEAR EFFECTS ON FLAME-FRONT STABILITY

by

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STATEMENT BY AUTHOR

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<th>Description</th>
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<tr>
<td>A/F</td>
<td>Air-fuel ratio</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Diameter</td>
<td>in.</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
<td>psi</td>
</tr>
<tr>
<td>f</td>
<td>Natural frequency of vibration of the flame-holder</td>
<td>cps</td>
</tr>
<tr>
<td>I</td>
<td>Moment of inertia</td>
<td>in.⁴</td>
</tr>
<tr>
<td>l</td>
<td>Length</td>
<td>in.</td>
</tr>
<tr>
<td>M</td>
<td>Photographic scale factor</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Mass per unit length</td>
<td>lb.-sec.²/in.²</td>
</tr>
<tr>
<td>N</td>
<td>Rotational frequency of the flame-holder</td>
<td>rev./sec.(rps)</td>
</tr>
<tr>
<td>r</td>
<td>Radial coordinate in polar coordinates</td>
<td>in.</td>
</tr>
<tr>
<td>R</td>
<td>Radius of the flame-holder</td>
<td>in.</td>
</tr>
<tr>
<td>V</td>
<td>Primary mixture speed</td>
<td>fps</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Angle between flame-front and vertical</td>
<td>degrees</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Circulation</td>
<td>in.²/sec.</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>Angular coordinate in polar coordinates</td>
<td>degrees</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
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<tr>
<td>--------</td>
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<tr>
<td>λ</td>
<td>Wavelength</td>
<td>in.</td>
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<tr>
<td>ψ</td>
<td>Stream function</td>
<td>in.$^2$/sec.</td>
</tr>
<tr>
<td>Ω</td>
<td>Wavelet frequency</td>
<td>cps</td>
</tr>
<tr>
<td>ω</td>
<td>Natural frequency of vibration of the flame-holder</td>
<td>rad./sec.</td>
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<tr>
<td>Z</td>
<td>Vorticity of steady motion</td>
<td>l/sec.</td>
</tr>
<tr>
<td>x, y</td>
<td>Cartesian coordinates</td>
<td>in.</td>
</tr>
<tr>
<td>U</td>
<td>Undisturbed velocity</td>
<td>fps</td>
</tr>
<tr>
<td>u, v</td>
<td>Perturbation velocities</td>
<td>fps</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Perturbation vorticity</td>
<td>l/sec.</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>sec.</td>
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<tr>
<td>k</td>
<td>Wave number</td>
<td>l/in.</td>
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<tr>
<td>n</td>
<td>Exponential growth factor</td>
<td>l/sec.</td>
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<tr>
<td>C</td>
<td>Constant</td>
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ABSTRACT

The results of an experimental investigation of the stability of a laminar "vee" flame-front are presented.

A rotating flame-holder device was constructed and the effects on flame-front structure were observed with a shadowgraph system. A limited number of measurements of shear-flow patterns in the vicinity of the flame-holder were taken with the aid of particle-track photographs.

The results of the investigation indicate that shear-flow instability is a predominant factor which influences the stability of a laminar "vee" flame-front.
1.1 Introductory Remarks

Recent advances in the field of chemical propulsion systems have brought about a need to investigate instabilities in the combustion process. For example, combustion instability in rocket motors has become a problem of increasing importance due to the enormous size of the combustion chambers of some of the boosters in use today.

Flames in such engines are extremely turbulent and very difficult to analyze on a mathematical basis. For this reason, a study of laminar flames has been approached in hopes that a more fundamental knowledge of laminar flames can be extended to turbulent flames.

1.2 Statement of the Problem

The behavior of laminar flames has been observed by various experimenters. Under certain operating conditions, instability occurs when the laminar flame-front breaks down into cells or wavelets which propagate along the flame with a speed approximately equal to the tangential component of the velocity of the approach flow.
This phenomenon has been observed in the burning of many hydrocarbon fuels in the presence of laminar approach flow.

Several theories on the stability of laminar flames have been presented, primarily the hydrodynamic theory of Landau, the hydrodynamical-chemical theory of Markstein, and the shear flow instability theory of Scurlock. None of these theories, however, takes into account the presence of the flame-holder.

Experimental evidence has shown a certain amount of agreement with theory (1) (2)*, but for very lean air-fuel ratios (20 to 1 or greater) the flame-fronts are unstable, in direct contrast with theory. It has been observed that flame-front stability is also dependent on the size and shape of the flame-holder. Furthermore, it has been postulated (3) that velocity gradients induced in the mixture flow field by the flame-holder produce shear effects which in turn affect flame-front stability. Since various mathematical attempts to account for the presence of the flame-holder have been unsuccessful, it has been proposed to investigate shear-flow effects by the technique of rotating the flame-holder so as to produce asymmetric shear-flow patterns. Although it is

* Numbers in parentheses refer to REFERENCES.
beyond the scope of this investigation, it is hoped that by using this type of approach, a phenomenological theory can be developed to account for the presence of the flame-holder.

1.3 The University of Arizona Combustion Tunnel

The equipment used in this investigation consisted of a combustion tunnel and its associated apparatus, a rotating flame-holder device, and equipment for measuring the rotational speed of the flame-holder.

The University of Arizona Combustion Tunnel (Fig. 1.1) consists of three main components, a vertical test section, a primary flow system, and a secondary flow system. The associated equipment includes a schlieren/shadowgraph photographic system for observation of flame-fronts, and a light source and chopper wheel used for qualitative and quantitative measurements of shear gradients in the vicinity of the flame-holder.

Air and propane are directed to a master control panel where they are metered, mixed, and directed to the primary flow system. Morris (4) has shown that the air-propane mixture emitted from the primary nozzle has a very flat velocity profile and a low turbulence level. The secondary air flow, which surrounds the primary flow system, is provided by a variable speed fan located on the roof of the laboratory. This system was modified by
Fig. 1.1. The University of Arizona Combustion Tunnel
Pyle (5) to produce a symmetric, low turbulence flow. The 18 inch by 18 inch test section is positioned vertically and connects the secondary flow plenum chamber to the roof of the laboratory. Two opposing walls of the tunnel contain glass so that flame-front photographs may be taken, and an additional window is provided for use in conjunction with the particle-track system. For a more detailed description of the combustion tunnel, the reader is referred to (4) and (5).

The rotating flame-holder apparatus consists of a cylindrical flame-holder, a driving mechanism, a variable speed A.C. motor, and a stroboscope for measuring the rotational speed. A more detailed description of the rotating holder apparatus is given in Chapter 2.
CHAPTER 2

EXPERIMENTAL APPARATUS

2.1 Flame-Front Optical System

In order to obtain a high-speed photographic record of the flame-front, a spark light source used in conjunction with a schlieren optical system has been used. A schematic of this system is illustrated in Fig. 2.1. A highly intense spark is passed through a 0.060 inch hole which is located in the focal plane of an 8-inch diameter parabolic mirror. The parallel rays of light pass through the test section to a second mirror which reflects the light rays to the camera. The camera is equipped with an adjustable knife-edge, a synchronized shutter system, and a device for mounting a 4 inch by 5 inch cut film holder and a Polaroid Land film holder.

2.2 Particle-Tracking System

This system is used to indicate the nature of the flow field in the vicinity of the flame-holder. By injecting magnesium oxide particles into the primary flow system and by stroboscopically illuminating the
Fig. 2.1. Flame Front Optical System
particles, measurements of shear gradients near the flame-holder can be made.

The system consists of a particle injector tank, a light source, and a device for chopping the light up to 10,000 times per second. Detailed descriptions of these parts are presented in the following paragraphs.

After air and propane have been metered and mixed, part of the mixture is directed to the cylindrical injector tank (Fig. 2.2). The flow is introduced tangentially into the injector tank so as to provide continuous agitation of the dust particles. However, at low flow rates, the swirling motion is not sufficient to introduce the dust particles into the flow so an agitator connected to a handle outside the tank has been added to the system.

The light source for the illuminating system consists of a General Electric BH-6 mercury vapor lamp. The light is focused to a narrow slit at the plane of the slotted wheel. The light is then refocused to form a narrow plane of light of approximately 4 inches in height at the center of the nozzle. Figure 2.3 is a schematic of this system.

By positioning a camera along the longitudinal axis of the flame-holder, the photographs obtained show a series of dashes, due to the intermittent illumination
Fig. 2.2. Particle Injector System
Fig. 2.3. Light Source and Chopper Wheel
of the magnesium oxide particles. Hence, by knowing the flashing rate and the scale factor of the photograph, and by measuring the distance between dashes, the velocity of the flow can be determined. The camera used in this test was a 35 mm Nikon Reflex model and is equipped with an f/2, 135 mm focal length lens system.

2.3 Rotating Flame-Holder Apparatus

In order to be able to study the nature of the flow in the vicinity of the flame-holder, the entire flow field surrounding the flame-holder must be unobstructed at at least one end of the flame-holder support. This, combined with the requirement that the rotating mechanism must be located outside the tunnel made it necessary to position the bearing supports in the windows of the tunnel. Because of these restrictions and because of the high rotational speed of the flame-holder (up to 15,000 R.P.M.), teflon was chosen as the bearing material. The bearings were machined from 3/8-inch teflon rod stock so as to provide a snug fit in a 1/4-inch hole in each of the two glass windows. A third bearing is mounted on the frame of the rotating mechanism to support the driving shaft of the flame-holder. The flame-holder consists of 3/8-inch drill rod machined at both ends. A schematic of the bearing supports and flame-holder is shown in Fig. 2.6.
Fig. 2.4. Motor and Drive Mechanism

Fig. 2.5. Rotating Flame-Holder Apparatus
Glass Walls

Fig. 2.6. Flame-Holder and Bearing Supports
A rotating mechanism has been constructed by McCabria (6). This design incorporates a friction drive mechanism in order to produce flame-holder rotation. However, difficulties with this device arose in that the rotational speed of the flame-holder fluctuated. Since this condition was considered undesirable, a gearing system has been incorporated into the basic design. The modified rotating mechanism (as shown in Fig. 2.7) consists of a driving gear on the shaft of a 1/7 H. P. Universal motor, an idler gear, and a gear on the extended shaft of the flame-holder. The driving and driven gears have 25 teeth and a pitch diameter of 0.391 inches, and the idler gear has 130 teeth and a pitch diameter of 2.031 inches. The variable speed A.C. motor, connected to a variable auto transformer (variac) provides for operation from 0 to 15,000 rpm. Both the motor and the frame of the drive mechanism are mounted on three-way positioning devices.

The rotational speed of the flame-holder is measured by means of a stroboscopic light source consisting of a Model 121 Strobex lamp and power supply, and a Hewlett Packard audio oscillator to provide the triggering impulses. The accuracy of the oscillator has been checked against line frequency and found to be satisfactory. Although the Strobex lamp has a maximum flashing rate
Fig. 2.7. Schematic of Motor and Drive Mechanism
of 100 flashes per second, and the shaft speeds reach 250 revolutions per second, the speed of the flame-holder can still be measured by operating the lamp at $1/2$, $1/3$, $1/4$, $1/5$, etc. the speed of the flame-holder.
CHAPTER 3

SCOPE OF THE INVESTIGATION

3.1 Flame-Holder Vibration

It has been observed by Petersen (1) that both horizontal and vertical vibration of the flame-holder cause distortions in the laminar flame-fronts. Since the purpose of this investigation was to study only the shear effects of the flame-holder, it was considered necessary to keep the vibration of the flame-holder at a minimum. Due to the wide range of shaft speeds needed, it was necessary to have a rough idea of the shaft's critical speeds. An accurate value of the critical speeds is difficult to obtain since the shaft is stepped at both ends, and also because the shaft is not simply supported at the driving end. Assuming the shaft to be simply supported at both ends, the equation for the \( n \)th critical speed is given by

\[
\omega_n = \frac{n^2 \pi^2}{\ell^2} \sqrt{\frac{EI}{m}}. \]
Using $E = 30 \times 10^6$ psi, $l = 18$ in., $m = 8.1 \times 10^{-5}$ lb-sec$^2$/in.$^2$, and for $d = 3/8$ in., $I = 9.7 \times 10^{-4}$ in.$^4$, the resulting values for the fundamental and first harmonic are as follows:

$$\omega_1 = 574 \text{ rad./sec.}$$

and $$\omega_2 = 2296 \text{ rad./sec.}$$

or $$f_1 = \frac{\omega_1}{2\pi} = 90.5 \text{ cps}$$

and $$f_2 = 362 \text{ cps}.$$  

Since the rotation of the rod was limited to 250 cps because of the available motor and gear combination, only the fundamental mode of vibration presented any problems. Test results showed that excessive vibration occurred in the range from 75 to 110 rps, hence, these speeds were avoided during testing.

The radial motion of the rod has been measured by placing a micrometer next to the flame-holder as illustrated in Fig. 3.1. By rotating the flame-holder by hand through 360° in 90° increments, the static double amplitude was found. In this manner the static center of rotation of the shaft was determined. The distance from the geometric center to the center of rotation was found to be 0.003 inches. A plot of amplitude versus frequency is given by Fig. 3.2.
Fig. 3.1. Apparatus for Measuring Flame-Holder Vibration
Fig. 3.2. Amplitude Versus Frequency for Rotating Flame-Holder
3.2 Test Program

The purpose of this investigation was to determine the influence of the flow in the vicinity of the flame-holder upon the stability of a laminar "vee" flame. This has been accomplished by comparing the flame-fronts with the rod stationary to the flame-fronts with varying degrees of rotation. Since previous investigations have shown the flame-front to become naturally distorted for various combinations of mixture speeds, $V$, and air-fuel ratio, $A/F$, a series of flame-front photographs have been obtained for particular combinations of $V$, $A/F$, and flame-holder rotational speed, $N$. $V$ was varied from 10 to 25 fps in increments of 5 fps. The upper limit was chosen because of the inability of the compressor to deliver enough air. Speeds below 10 fps were avoided because of the tendency of the flame to "jump back" from the rod to the primary nozzle. $A/F$ was varied from 16.5 to 22.5 in increments of 2. Flames leaner than 22.5 to 1 were not investigated because of the tendency of the lean flame to be "blown off" at high rod speeds. The lower limit of 16.5 to 1 was chosen because of the excessive amount of heat liberated by richer flames and also because of the danger of a "flash-back."

The rotational speed, $N$, was varied from 0 to 250 rps in increments of 25 rps. Due to the vibration near
the fundamental frequency of the rod, tests were not conducted at 75 and 100 rps. However, the vibration was noticeably smaller at 70 rps, so tests were conducted at this speed.

This combination of four mixture speeds, four air-fuel ratios, and ten flame-holder speeds gives a total of 160 test runs. The test procedure consisted of selecting a primary mixture speed and an air-fuel ratio, and then taking flame-front pictures at each of the ten rod speeds. With the same mixture speed, the air-fuel ratio was changed, and the series of ten pictures was taken. With the testing completed at the four air-fuel ratios, the mixture speed was then varied. This procedure was followed until all of the 160 flame-front photographs had been obtained.

The flow field in the vicinity of the flame-holder has been investigated by means of the particle-track system. The procedure for the particle-track data is essentially the same as that for taking flame-front photographs except that the holder speeds were limited to 0, 70, 150, 200, and 250 rps.

Due to the difficulty in controlling the density of the cloud of dust particles in the primary flow, a 35 mm roll film camera has been used. This allows for taking two or three pictures in rapid succession so as to increase the probability of obtaining at least one good particle-track picture.
4.1 Flame-Front Photographs

The flame-front data obtained have indicated a distinct influence of flame-holder rotation upon the stability of a laminar "vee" flame. Since it would be impractical to include all of the photographs taken, only a few taken at an air-fuel ratio of 20.5 to 1 and a mixture speed of 10 fps will be included for the purpose of illustration.

The pictures taken at rod speeds of 25 and 50 rps indicated a very small effect upon flame-front structure for all combinations of mixture speed and air-fuel ratio. However, at a speed of 70 rps, large cells were observed on the right flame wing, whereas, the left wing remained relatively undisturbed. It should be noted that the flame-holder rotation is clockwise in the photographs. Figures 4.1a and 4.1b show the flame structure with no rotation and with a rotational speed of 70 rps respectively. In Figs. 4.1c and 4.1d (rotational speeds of 175 and 225 rps) the wavelets are present in the left flame wing, whereas, the right wing is now relatively
Fig. 4.1. Flame-Front Photographs

(a) $N = 0 \text{ rps}$

(b) $N = 70 \text{ rps}$

(c) $N = 175 \text{ rps}$

(d) $N = 225 \text{ rps}$
smooth. The fact that the wavelets appear on the opposite flame wing with high flame-holder speeds is thought to be caused by the changes in the velocity gradient pattern near the flame-holder due to rotational effects.

The frequency of the wavelets has been calculated and found to be closely related to the rotational frequency of the flame-holder. This is accomplished by assuming that the velocity of propagation of the cells is equal to the tangential component of velocity of the approach flow (Fig. 4.2.). Since the scale factor of the photograph, M, is known, and the individual wavelengths, $\lambda$, can be measured, the frequency of the wavelets, $\Omega$, can be obtained as follows:

$$\Omega = \frac{V_{t} \times M}{n}$$

From Fig. 4.3 it is evident that

$$V_{t} = V \cos \alpha$$

where $\alpha$ is the angle between the flame-front and the vertical.

Hence,

$$\Omega = \frac{V \cos \alpha}{\lambda} \times M$$
Fig. 4.2. Schematic of Distorted Flame-Front (Idealized)
Fig. 4.3. Particle-Track Photographs (No Flame)
For the photographs illustrated in Figs. 4.1b and 4.1c and 4.1d, the frequencies measured are 63, 104, and 185 corresponding to the rotational frequencies of 70, 125, and 225 rps respectively.

The fact that one flame wing is stable and the other unstable under the influence of flame-holder rotation is thought to be due to the stability of the flow in the vicinity of the flame-holder. As stated by Lord Rayleigh (7), the stability of a fluid stream is largely dependent on the nature of the shear gradients present in the flow field. A brief discussion of Rayleigh's theory is given in Appendix B. The asymmetric flame-fronts obtained in this investigation indicate that the flame structure is dependent on the shear gradients produced by flame-holder rotation. It is planned in future work to obtain detailed measurements of the flow pattern (velocity profiles and shear gradients in the vicinity of the flame-holder.) A sample analysis of particle-track data is described in the following section.

4.2 Particle-Track Photographs

The particle-track data obtained thus far have not been considered completely satisfactory from the standpoint of defining the nature of the flow at all points in the vicinity of the flame-holder.
Figures 4.3a and 4.3b show the flow field in the absence of combustion and with rod speeds of zero and 250 rps. The flow patterns obtained from these photographs are compared with theoretical potential flow patterns in Appendix A. Both pictures were taken with a mixture speed of 10 fps. Close examination of these photographs indicates three undesirable features; one, the absence of detectable dashes close to the flame-holder; two, the shadow caused by the presence of the flame-holder; and three, the fluctuating intensity of the light source causes the dashes along a particular track to fade at some point and occasionally disappear.

These conditions can be improved by providing for a more intense D.C. light source and by providing for some means of directing the light into the shadowed portion of the flow field.

Figures 4.4a and 4.4b show the flow field under the same operating conditions as before, except that here the flame is present. As can be seen from these photographs, the flow field in the burned region of the flame is undefined either due to the lack of dust particles in this region or because the intensity of the light of the burned gas obscures the particle-tracks. This latter phenomenon is partially caused by the intensity of the light emitted by the flame itself, but even more so by
Fig. 4.4. Particle-Track Photographs (Flame Present)
the glowing of the magnesium oxide particles as they become heated in the burned region.

Attempts have been made to use photographic filters to selectively filter out the luminosity of the burned gases. So far this has met with little success.

A limited amount of particle-track data have been analyzed and a discussion of the data reduction procedure is given in the following paragraphs.

Enlargements of the 35 mm negatives (approximately three times the actual size of the apparatus) have been made and pin-holes punched in the center of each dash to facilitate measurements. The velocity at a given point along a particular track was obtained by averaging the distance between three dashes (the point in question being the middle of the three) and by multiplying this distance by the flashing rate and the scale factor. This procedure was followed for all points along the track primarily to reduce the amount of error involved when the velocity is computed for only one particular point, and also to obtain measurements of the acceleration of the flow along the track. Figure 4.5 shows a typical result of such an analysis. It is to be noted that the particle which was analyzed in Fig. 4.5 was increasing its speed as it moved along its path.
Distance From First Detectible Dash - $\frac{1}{64}$ inch (not to scale)

Fig. 4.5. Typical Velocity Variation Along a Particle-Track
By using the foregoing procedure, an analysis of a velocity profile of Fig. 4.4a has been obtained (Fig. 4.6). This particular profile was obtained along a line drawn through the center of the flame-holder. A considerable amount of scatter is evident from this plot. This might possibly be reduced by further enlarging the pictures so as to increase the distance between dashes.
Fig. 4.6. Velocity Profile Obtained From Particle-Track Photograph (No Flame-Holder Rotation)
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The asymmetric flame-fronts obtained under the influence of flame-holder rotation indicate that shear-flow instability is a predominant factor influencing the stability of a laminar "vee" flame-front.

The particle-track photographs obtained thus far have not been of such a nature as to completely define the flow near the flame-holder. After the proper revisions are made to the particle-tracking apparatus, and the flow field sufficiently defined, an analytical treatment of the dependence of flame-front stability on shear-flow instability can be attempted.

5.2 Recommendations

In order to correct some of the problems associated with the particle-track system, a number of modifications of the present system should be attempted:

1. The light source should be supplied with direct current so as to provide a constant source of light;

2. A system of mirrors should be used to eliminate the shadowed portion of the flow field; and
3. A more intense light source used in conjunction with photographic filters should be used in order to obtain particle-tracks in the burned region.

When reducing the particle-track data, it would be helpful to enlarge the photograph to approximately 40 times the size of the negative.

It is also recommended that the plate glass in the windows of the test section be replaced with heat-resistant glass of comparable quality. This will reduce the amount of time necessary to allow the glass to cool and reduce the chance of breakage.
APPENDIX A

COMPARISON OF ACTUAL AND THEORETICAL STREAMLINES

It might be interesting to compare the streamlines as given in Figs. 4.3a and 4.3b with those computed by considering potential flow theory. The flow was assumed to be two dimensional, inviscid, irrotational, and incompressible.

For the case of flow around a cylinder with no circulation, the relation for the stream function, $\psi$, in polar coordinates is given by:

$$\psi = \nabla r \sin \theta \left(1 - \frac{R^2}{r^2}\right) \quad (1)$$

For $V = 10$ fps and $R = 3/16$ in. the streamlines were plotted and shown as solid lines in Fig. A.1a. Also shown in this figure are a few streamlines as obtained from Fig. 4.3a. A discussion of Fig. A.1a is reserved until later.

For the case of flow around a cylinder with circulation (produced by flame-holder rotation), vorticity must be added to (1). The expression for $\psi$ then becomes

$$\psi = \nabla r \sin \theta \left(1 - \frac{R^2}{r^2}\right) + \frac{T}{2\pi} \ln \frac{r}{R} \quad (2)$$
Fig. A.1. Plot of Actual and Theoretical Streamlines
where the circulation \( \Gamma \) is assumed to be given by the expression,

\[
\Gamma = \text{rod circumference } \times \text{rim speed} = 2R \times V_r.
\]

But \( V_r = 2\pi RN \)

Hence \( \Gamma = 4\pi^2 R^2 N \) \( (3) \)

For the conditions of Fig. 4.3b, \( R = 3/16 \) in. and \( N = 250 \) rev./sec. and

\[
\Gamma = 347 \text{ in.}^2/\text{sec.}
\]

The actual and theoretical streamlines are shown in Fig. A.1b.

Inspection of Fig. A.1a indicates that the actual and theoretical streamlines agree very closely in the flow regime to the left of the flame-holder. However, the flow field to the right cannot be predicted by this method of analysis because of the separation which occurs near the centerline of the flame-holder.

From Fig. A.1b it is evident that the actual streamlines are not in close agreement with the predicted ones. There seem to be two main factors to which this may be attributed: one, the close proximity of the nozzle
inhibits the ability of the flow to turn before reaching the rod; and, two, the mass of the magnesium oxide particles reduces the tendency of these particles to follow the actual pattern in regions of high acceleration.
APPENDIX B

A BRIEF REVIEW OF RAYLEIGH'S THEORY
OF THE INSTABILITY OF JETS

As far back as 1877, Lord Rayleigh, in his classic treatise, "The Theory of Sound,"* analyzed the stability of the interface between two fluids moving at different velocities. The following is a brief summary of his method of analysis for the edification of the reader.

For the undisturbed flow between two walls (Fig. B.1) the velocity parallel to the x-axis, U, is assumed to be a function of y only. In the disturbed motion, assumed to be in two dimensions, the velocities are denoted by $U + u$, and $v$, and the vorticity by $Z + J$, where the vorticity of the steady motion is

$$Z = \frac{1}{2} \frac{\partial U}{\partial y}$$

and the perturbation vorticity, $J$, is

$$\frac{1}{2} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right)$$

It is also assumed that the fluid is incompressible and that the effects of viscosity are negligible.

\[ U = U(y) \]

Fig. B.1. Flow Between Parallel Walls

(a) Stable  
(b) Unstable

Fig. B.2. Flow Patterns Analyzed by Rayleigh
By using the theorem of Helmholtz, the general equation for the disturbed motion is

\[
\frac{\partial (Z+j)}{\partial t} + (U+u) \frac{\partial (Z+j)}{\partial x} + \nu \frac{\partial (Z+j)}{\partial y} = 0
\]

(1)
in which \(\frac{\partial Z}{\partial t} = 0\), and \(\frac{\partial Z}{\partial x} = 0\).

If the terms involving products of the perturbation velocities and the perturbation vorticity are neglected, (1) may be written

\[
\frac{\partial j}{\partial t} + U \frac{\partial j}{\partial x} + \nu \frac{\partial Z}{\partial y} = 0
\]

(2)
and the equation of continuity for an incompressible fluid gives

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(3)
If the values of \(Z\) and \(j\) in terms of the velocities are substituted in (2), then

\[
(-\frac{\partial}{\partial t} + U \frac{\partial}{\partial x})(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) + \nu \frac{\partial^2 U}{\partial y^2} = 0
\]

(4)
We now assume that the perturbation velocities \(u\) and \(v\) can be expressed as

\[
\begin{align*}
\mathbf{u} &= C_1 e^{i n t} e^{i k x} \\
\mathbf{v} &= C_2 e^{i n t} e^{i k x}
\end{align*}
\]
where \( n \) is the exponential growth factor and \( k \) is the wave number \( (k = 2\pi / \lambda, \; \lambda = \text{wavelength}) \).

Hence, from (3)

\[
i K u + \frac{d v}{d y} = 0
\]  

(5)

If this value of \( u \) is substituted in (4), we obtain

\[
\left( \frac{n}{k} + U \right) \left( \frac{d^2 v}{d y^2} - k^2 v \right) - v \frac{d^2 U}{d y^2} = 0
\]  

(6)

In (6), \( k \) may be regarded as real, and the object is to determine the sign of the imaginary part of \( n \) (if it is negative, the disturbance will grow exponentially).

Rayleigh points out that, in the case where \( \frac{d^2 U}{d y^2} \) is of the same sign so that the velocity profile is either wholly convex or wholly concave throughout the entire space between the two fixed walls, the imaginary part of \( n \) is zero and hence the disturbance is neutrally stable, i.e., neither growing nor decaying.

A class of problems admitting fairly simple solutions are obtained by supposing the vorticity \( \zeta \) to be constant throughout layers of finite thickness and to change its value only in passing a limited number of planes, for each of which \( y \) is constant. In such cases, the velocity profile is composed of portions of straight lines intersecting one another at finite angles.
Rayleigh has analyzed the stability of the flows given in Figs. B.2a and B.2b. For the velocity profile of Fig. B.2a, the flow is stable, whereas, the flow depicted in Fig. B.2b is unstable.

Other types of flow patterns have been investigated by Rayleigh; however, the procedure is essentially the same as that given previously.
REFERENCES


