A MODIFIED MILNE-EDDINGTON CURVE
OF GROWTH METHOD

by

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ABSTRACT

The Milne-Eddington curve of growth method is modified in two ways. The assumption that the ratio of line to continuous absorption is constant with optical depth throughout the atmosphere is modified by assuming that the atmosphere is divided into two layers. It is assumed that line formation occurs in only one of the layers.

The assumption that the lines of an element in a particular ionization state and particular excitation interval are formed at one optical depth is modified by defining a mean depth of formation for each line of an element.

The abundances of carbon I, nitrogen I, oxygen I, and sulphur I in the sun are calculated on the basis of these two modifications. The abundances from this paper are compared with the abundances determined by Goldberg, Müller, and Aller (1960). It is concluded that the method presented is acceptable for abundance determinations.

I. INTRODUCTION

Abundance determination methods can be grouped into two broad categories, the fine analysis methods and the coarse analysis methods. The fine analysis methods take into account the structure of the atmosphere while the coarse analysis methods are based on the approximation that a unique temperature and pressure can be specified as the conditions under which a spectral line is formed.

This paper is concerned with the modification of the Milne-Eddington curve of growth method. This coarse analysis method assumes that η, the ratio of line to continuous absorption, is constant throughout the atmosphere. The modification of this method assumes that the atmosphere can be divided into two types of layers, no line absorption occurring in one of the layers and η = constant $\neq 0$ in the other layer. All comments about line formation in this paper are to be interpreted in the light of the above assumption, unless otherwise stated. The purpose of this modification is to combine the two types of analysis so that much of the accuracy of the fine analysis method and simplicity of the coarse analysis can be retained.

τ_1 is defined as the optical depth above which no line absorption occurs and τ_2 is defined as the optical depth below which no line absorption occurs. τ_1 and τ_2 are measured in terms of the continuum optical depth. In order to simplify matters, it is assumed that either $\tau_1 = 0$ or $\tau_2 = \infty$ for each line, although this simplification is not necessary for the analysis; therefore, the atmosphere is assumed to be composed of two layers. If $\tau_1 = 0$, the line forming layer (η = constant) is the upper layer; if $\tau_2 = \infty$, the line forming layer is the lower layer. Once τ_1 or τ_2 is known, the mean depth of formation of a line τ_M can be found. Knowledge of τ_M enables all depth dependent quantities required by the analysis to be obtained from an available model atmosphere. Each line will have its own τ_1 or τ_2; each line will have its own τ_M. This means each line will have its own temperature, electron pressure, etc. This is to be contrasted with the unmodified Milne-Eddington method, where the lines have the same temperature, etc.

Although it has been assumed that local thermodynamic equilibrium holds, it is not a necessary feature of this analysis. Since LTE is assumed, the source function in the line and continuum is the Planck function.

II. DEFINITIONS

1- a = the damping constant

2- α_ν = the atomic absorption coefficient

3- B_A = the partition function

4- C = the velocity of light

5- $\Delta\nu_D$ = the Doppler half-width

6- e = the charge on an electron

7- η = the depth dependent ratio of line to continuous absorption

8- $\bar{\eta}$ = the constant value of η used to replace the actual varying one

9- η_o = the value of η at the line center

10- f = the oscillator strength

11- g_1 = the statistical weight of the lower level

12- h = Planck's constant

13- I_ν = the specific intensity in the line

14- I_c = the specific intensity in the continuum

15- IP = the ionization potential

16- k = the Boltzmann constant

17- $\bar{\varkappa}$ = the absorption coefficient upon which the independent depth parameter of the model atmosphere is based

18- $\varkappa_L$ = the monochromatic line absorption coefficient

19- $\varkappa_c$ = the monochromatic continuous absorption coefficient

20- $\overline{\tau}$ = the depth variable of the model atmosphere

21- τ_c = the optical depth in the continuum, at some wavelength

22- τ_L = the optical depth in a line

23- $\tau_\nu = \tau_L + \tau_c$

24- τ_M = the mean depth of formation of a line in terms of $\varkappa_c$

25- $\overline{\tau}_M$ = the $\overline{\tau}$ value of the mean depth of formation of a line

26- τ_1 = the continuum optical depth above which no line absorption occurs

27- τ_2 = the continuum optical depth below which no line absorption occurs

28- λ_o = the wavelength of the line center

29- ν_o = the frequency of the line center

30- m = the mass of the atom

31- $\mu = \cos\varphi$, where φ is the angle between the line of sight and the normal to the star's surface

32- N = the total number of atoms and ions per gram of material

33- N_A = the number of atoms in a given degree of ionization per gram of material

34- N_H = the total number of hydrogen atoms per gram of material

35- N_1 = the number of atoms per gram of material in the lower level of the line in question

36- S_ν = the source function

37- P_e = the electron pressure

38- T_o = the boundary temperature of a star

39- T = the temperature

40- θ = 5040/T

41- V_T = the microturbulent velocity

42- χ_1 = the excitation potential of the lower level

43- $W_\nu(\mu)$ = the equivalent width in frequency units

44- $W_\lambda(\mu)$ = the equivalent width in wavelength units

45- the subscript 1 refers to the lower level in the transition, and the subscript 2 refers to the higher level

III. THEORY

The purpose of this part of the paper is to develop the expressions for the quantities needed to construct the theoretical and observational curves of growth and to explain the fitting procedure. It is assumed that the source function can be expressed as a linear function of the continuum optical depth, i.e.,

$$S_\nu = B_0 + B_1 \tau_c \ . \tag{1}$$

The general solutions to the equation of transfer are:

$$I_\nu = \int_0^\infty S_\nu(\tau_\nu) e^{-\tau_\nu/\mu} \, d\tau_\nu/\mu \ ; \tag{2}$$

$$I_c = \int_0^\infty S_\nu(\tau_c) e^{-\tau_c/\mu} \, d\tau_c/\mu = B_0 + B_1 \mu \ . \tag{3}$$

The equivalent width of a line in frequency units is defined by

$$W_\nu(\mu) = \int_0^\infty \left(1 - \frac{I_\nu}{I_c}\right) d\nu \ . \tag{4}$$

It is seen from equation 4 that the expression for I_ν must be evaluated before the equivalent width can be determined. The relation between τ_ν and τ_c must be found in order to evaluate equation 2, since the source function is expressed in terms of τ_c . The desired relation can be obtained from the definition of τ_ν ,

$$d\tau_\nu = d(\tau_c + \tau_L) = d\tau_c(1 + \overline{\eta}) . \tag{5}$$

As mentioned in the introduction, the atmosphere is assumed to be divided into two layers. Stated mathematically:

$$\overline{\eta} = 0 \qquad \tau_c < \tau_1 , \tag{6a}$$

$$\overline{\eta} = \text{const} \neq 0 \qquad \tau_1 \leq \tau_c \leq \tau_2 , \tag{6b}$$

and

$$\overline{\eta} = 0 \qquad \tau_c > \tau_2 , \tag{6c}$$

where we remember that either $\tau_1 = 0$ or $\tau_2 = \infty$.

The desired relation between τ_ν and τ_c is obtained by combining equations 5 and 6:

$$\tau_\nu = \tau_c \qquad \tau_c < \tau_1 , \tag{7a}$$

$$\tau_\nu = (1 + \overline{\eta})\tau_c - \overline{\eta}\tau_1 \qquad \tau_1 \leq \tau_c \leq \tau_2 , \tag{7b}$$

and

$$\tau_\nu = \tau_c + \overline{\eta}(\tau_2 - \tau_1) \qquad \tau_c > \tau_2 , \tag{7c}$$

With the aid of equations 7, I_ν can be evaluated for the two different cases. When $\tau_2 = \infty$,

$$I_\nu = B_0 + B_1\mu(1 - \frac{\bar{\eta}}{1+\bar{\eta}} e^{-\tau_1/\mu}) \quad . \tag{8a}$$

When $\tau_1 = 0$,

$$I_\nu = B_0 + \frac{B_1\mu}{1+\bar{\eta}} (1 + \bar{\eta}e^{-(\frac{1+\bar{\eta}}{\mu})\tau_2}) \quad . \tag{8b}$$

For stars other than the Sun, where one must work with fluxes, the properties of the fluxes may be approximately obtained from the corresponding properties of the intensities evaluated at $\mu = 2/3$.

Combining equations 3 and 8a, one finds that

$$1 - \frac{I_\nu}{I_c} = \frac{\mu}{B_0/B_1+\mu} \frac{\bar{\eta}}{1+\bar{\eta}} e^{-\tau_1/\mu} \qquad \tau_2 = \infty \; ;$$

combining equations 3 and 8b, one finds that

$$1 - \frac{I_\nu}{I_c} = \frac{\mu}{B_0/B_1+\mu} \frac{\bar{\eta}}{1+\bar{\eta}} \left[1 - e^{-(\frac{1+\bar{\eta}}{\mu})\tau_2}\right] \qquad \tau_1 = 0 \; .$$

The expression (equation 4) for the equivalent width can now be written as

$$W_\nu(\mu) = \int_0^\infty \frac{\mu}{B_0/B_1+\mu} G(\bar{\eta},\tau_i)d\nu \quad , \tag{9}$$

where

$$G(\bar{\eta},\tau_i) = \begin{cases} \dfrac{\bar{\eta}}{1+\bar{\eta}}\, e^{-\tau_1/\mu} & \tau_2 = \infty\,, \quad (10a) \\[2ex] \dfrac{\bar{\eta}}{1+\bar{\eta}}\left[1 - e^{-\left(\frac{1+\bar{\eta}}{\mu}\right)\tau_2}\right] & \tau_1 = 0\,. \quad (10b) \end{cases}$$

Defining $u = \dfrac{\nu-\nu_o}{\Delta\nu_D}$, where $\Delta\nu_D = \dfrac{\nu_o}{C}\left[\dfrac{2kT}{m} + V_T^2\right]^{\frac{1}{2}}$, and dividing both sides of equation 9 by ν_o, the expression for equivalent width can be written finally as

$$\frac{W_\nu(\mu)}{\nu_o} = \frac{W_\lambda(\mu)}{\lambda_o} = \frac{\Delta\nu_D}{\nu_o}\left(\frac{\mu}{B_o/B_1+\mu}\right)\int_{-\infty}^{\infty} G(\bar{\eta},\tau_i)\,du\,, \quad (11)$$

where $G(\bar{\eta},\tau_i)$ is again given by equations 10a and 10b.

The lines are assumed to be characterized by the Voigt profile. Therefore,

$$\alpha_\nu = (1 - e^{-h\nu/kT})\frac{\sqrt{\pi}e^2}{mC}f\frac{1}{\Delta\nu_D}H(a,u) = \alpha_o H(a,u)\,, \quad (12)$$

where

$$H(a,u) = \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}\,dy}{(u-y)^2+a^2}\,.$$

Therefore, one can write

$$\eta = \frac{\kappa_L}{\kappa_c} = \frac{N_1\alpha_\nu}{\kappa_c} = \frac{N_1\alpha_o H(a,u)}{\kappa_c} = \eta_o H(a,u)\,.$$

From this equation it is seen that

$$\eta_o = \frac{N_1 \alpha_o}{\kappa_c} . \qquad (13)$$

Using Boltzmann's Law, equation 13 can be expressed as

$$\eta_o = \frac{N_A}{N_H} \frac{g_1}{B_A} \frac{\alpha_o}{\kappa_c'} 10^{-\theta\chi_1} . \qquad (14)$$

$N_H^{-1} \kappa_c'^{-1}$ is introduced by expressing the continuous absorption coefficient in terms of the abundance of hydrogen such that,

$$\kappa_c = \kappa_c' N_H . \qquad (15)$$

It should be noted that α_o and κ_c' both have to be corrected for induced emission, but since the correction factors are essentially the same, these factors may be left out.

It is now possible to discuss the observational and theoretical curves of growth and the fitting procedure. Writing equation 11 in logarithmic form, we define log F as

$$\log F = \log \frac{W_\lambda(\mu)}{\lambda_o} - \log \frac{\mu}{B_o/B_1+\mu} = \log \frac{\Delta\nu_D}{\nu_o} + \log D \qquad (16)$$

where $D = \int_{-\infty}^{\infty} G(\overline{\eta}, \tau_i) dU$.

Combining the expression for α_0 from equation 12 and

$$\log \frac{N_A}{N_H} = \log \frac{N}{N_H} + \log \frac{N_A}{N}$$

with equation 14, it is found that

$$\log \eta_0 = \log \frac{N}{N_H} - \log \frac{\Delta\nu_D}{\nu_0} + \log \frac{g_1 f}{B_A \nu_0} + \log \frac{N_A}{N}$$

$$- \log \kappa_c' - \theta\chi_1 - 1.82 .$$

The above equation can be written as

$$\log \eta_0 = \log z + \log \frac{N}{N_H} - \log \frac{\Delta\nu_D}{\nu_0} , \qquad (17)$$

where

$$\log z = -1.82 + \log \frac{N_A}{N} + \log \frac{g_1 f}{B_A \nu_0} - \log \kappa_c' - \theta\chi_1 . \qquad (18)$$

It is to be noted that practically all of the quantities that vary from line to line are included in the quantity log z. The knowledge of the value of τ_1 or τ_2, and from this τ_M, enables one to evaluate these quantities with the aid of a model atmosphere.

The theoretical curve of growth is constructed by plotting log D versus log η_0 ; the observational curve of growth is constructed by plotting log F versus log z. The observational curve of growth is so named because the quantities involved are either directly obtained from observations or from the model atmosphere once τ_M is known.

When the two curves are compared, it is found that the observational curve of growth is essentially the same shape as the theoretical curve of growth. This is because all the quantities that vary from line to line are included in log F and log z, so that the difference between the curves are essentially constant. However, the observational curve of growth is shifted both vertically and horizontally with respect to the theoretical curve of growth, i.e., there is a difference in zero points on the two curves. The vertical shift is due to the essentially constant factor $\log \frac{\Delta\nu_D}{\nu_0}$, while the horizontal shift is due to the essentially constant factor $\log \frac{\Delta\nu_D}{\nu_0} - \log \frac{N}{N_H}$. Once the vertical shift is known, $\log \frac{N}{N_H}$ can be found from the horizontal shift.

An important fact should be mentioned about the theoretical curve of growth. For the case $\tau_2 = \infty$, $\overline{\eta}$ is separable from τ_1, as seen in equation 10a. This means that only one theoretical curve of growth is necessary for different τ_1 values. In this case, the τ_1 factor may be included in the ordinate of the observational curve of growth plot. The observational curve of growth is constructed by plotting log F' versus log z, where

$$\log F' = \log \frac{W_\lambda(\mu)}{\lambda_0} - \log\left(\frac{\mu}{B_0/B_1+\mu}\right) + \log e^{\tau_1}$$

$$= \log D' + \log \frac{\Delta\nu_D}{\nu_0} \qquad (19)$$

where

$$\log D' = \log D + \log e^{\tau_1} \quad .$$

Since $\bar{\eta}$ is not separable from τ_2 for the case $\tau_1 = 0$, a different theoretical curve of growth is needed for each τ_2 value.

IV. PROCEDURE

The way η varies with depth determines whether $\tau_1 = 0$ (η decreases with depth) or $\tau_2 = \infty$ (η increases with depth). The most strongly depth dependent part of η can be written as

$$\eta = \frac{\kappa_L}{\kappa_c} = \frac{N_1 \alpha_0 H(a,u)}{\kappa_c} \propto \frac{N_1}{\kappa_c T^{\frac{1}{2}}} \qquad (20)$$

where the $T^{-\frac{1}{2}}$ factor comes from the $\frac{1}{\Delta\nu_D}$ factor in the expression for α_0 (equation 12).

It is seen from the excitation and ionization equations that

$$N_1 \approx N_I 10^{-\theta\chi_1} \;, \qquad (21a)$$

$$N_1 \approx N_{I-1} T^{5/2} P_e^{-1} 10^{-(IP_{I-1}+\chi_1)\theta} \;, \qquad (21b)$$

$$N_1 \approx N_{I+1} T^{-5/2} P_e 10^{+(IP_I-\chi_1)\theta} \qquad (21c)$$

where N_I is the number of atoms in the same stage of ionization as the line, N_{I-1} is the number in the next lower stage of ionization, and N_{I+1} is the number in the next higher stage of ionization. Usually, most of the element in question will be in one of the three consecutive stages of ionization, so that the corresponding N (N_{I+1} , N_I, or N_{I-1}) in the applicable equation (21a, 21b, or 21c) is approximately constant with depth.

Using equations 20 and 21, the depth dependence of η may be expressed as

$$\log \eta \cong \text{const} - \log \kappa_c + \begin{cases} \frac{1}{2}\log\theta - \theta\chi_1 & (22a) \\ -2\log\theta - \theta(IP_{I-1}+\chi_1) - \log P_e & (22b) \\ 3\log\theta + \theta(IP_I - \chi_1) + \log P_e & (22c) \end{cases}$$

The relative depth dependence of η can be determined from equations 22 with the aid of a model atmosphere.

The methods of choosing τ_1 and τ_2 are as follows: τ_1 can be approximately determined by noting that it is the optical depth where line absorption starts becoming important. This knowledge enables one to make a suitable choice for τ_1 , as will be seen in the next section of the paper. τ_2 can be chosen by noting that it should be the region where η is small compared to the surface value. The important fact to remember when choosing τ_2 (or τ_1) is that the precise value of τ_2 (or τ_1) is not as important as the knowledge of whether it is much larger than, about equal to, or much smaller than unity.

τ_M may be evaluated as the average τ value in the region of line formation weighted by attenuation factor $e^{-\tau_\nu}$.

Therefore,

$$\tau_{\nu_M} = \frac{\int_{\tau_{\nu_1}}^{\tau_{\nu_2}} \tau_\nu e^{-\tau_\nu}\, d\tau_\nu}{\int_{\tau_{\nu_1}}^{\tau_{\nu_2}} e^{-\tau_\nu}\, d\tau_\nu} ,$$

which readily reduces to

$$\tau_{\nu_M} = 1 + \tau_{\nu_1} \qquad \tau_2 = \infty \tag{23}$$

$$\tau_{\nu_M} = 1 - \frac{\tau_{\nu_2}}{e^{\tau_{\nu_2}}-1} \qquad \tau_1 = 0 . \tag{24}$$

Since $\tau_\nu = \tau_1$, because there is only continuous absorption in this layer, equation 7b can be written as

$$\tau_{\nu_M} - \tau_1 = (1 + \overline{\eta})(\tau_{c_M} - \tau_1) . \tag{25}$$

Therefore, equation 23 becomes

$$\tau_M = \tau_1 + \frac{1}{1+\overline{\eta}} \qquad \tau_2 = \infty \tag{26}$$

while equation 24 becomes

$$\tau_M = \frac{1}{1+\bar{\eta}}\left[1 - \frac{\tau_2(1+\bar{\eta})}{e^{\tau_2(1+\bar{\eta})} - 1}\right] \qquad \tau_1 = 0 \ , \qquad (27)$$

where τ_M is now in terms of the continuum optical depth.

It is seen that $\bar{\eta}$ must be known before τ_M can be determined. $\bar{\eta}$ must be estimated, which means an iterative scheme is to be employed. An accurate value of $\bar{\eta}$ is not needed, only the knowledge of whether $\bar{\eta}$ is large, small, or comparable to unity.

The procedure is then:

1. Estimate $\bar{\eta}$.

2. Determine τ_1 or τ_2 (from the considerations mentioned above).

3. Determine τ_M (from equation 26 or 27).

4. Determine $\bar{\tau}_M$ from

$$\bar{\tau}_M = \tau_M \div \frac{\varkappa_c}{\bar{\varkappa}}$$

where $\bar{\tau}_M$ is the optical depth at which all depth dependent quantities are evaluated in the model atmosphere.

5. Determine $\varkappa_c$ from

$$\varkappa_c = \bar{\varkappa} \times \frac{\varkappa_c}{\bar{\varkappa}}$$

where $\bar{\varkappa}$ is the value of the absorption coefficient from the model atmosphere at optical depth τ_M . Log κ_c' can then be determined from equation 15.

6. $\frac{N_A}{N} = \frac{1}{1+N_I/N_A}$, where N_I , the number of ions per gram, is determined by using the ionization equation.

7. Log z is determined from equation 18.

8. Log F' (or log F) is determined from equation 19 (or 16).

9. The observational curve of growth can be constructed, once log z and log F' (or log F) are known for each line. The observational curve is then fitted to the theoretical curve, and the abundance by mass of the element is found. This is done by determining the vertical and horizontal shifts in the zero point (vertical shift = $\log \frac{\Delta\nu_D}{\nu_0}$; horizontal shift = $\log \eta_0 - \log z$). Then apply equation 17.

10. The estimate of $\bar{\eta}$ (Step 1) should be compared with $0.7\ \eta_0$ to see whether the estimate was good or not. $0.7\ \eta_0$ is chosen because $\bar{\eta}$ is the average over the line.

The quantity $\kappa_c/\bar{\kappa}$ is needed a number of times, for instance in equation 28. In order to evaluate this quantity, $\bar{\kappa}$ must be chosen. $\bar{\kappa}$ is usually either some mean value of κ_c or a convenient monochromatic value. In either case $\kappa_c/\bar{\kappa}$ is not very sensitive to depth in the atmosphere, and so it may be assumed without much loss in accuracy that this ratio depends only upon wavelength. Table 1 shows the wavelength variation of $\varkappa_c/\bar{\varkappa}$ for the case in which $\bar{\varkappa}$ is the monochromatic absorption at λ 5050 and for moderate temperatures in which the main source of continuous absorption is negative hydrogen.

Once $\varkappa_c/\bar{\varkappa}$ is known, the quantity B_o/B_1 can be determined by using equation 8-94 Aller[1],

$$B_o/B_1 = \frac{8}{3}\,\frac{1-e^{-X_o}}{X_o}\,\frac{\varkappa_c}{\bar{\varkappa}}, \tag{30}$$

where $X_o = h\nu/kT_o$. The quantity B_o/B_1 is used in equations 16 and 19 in the form $\log \frac{\mu}{B_o/B_1+\mu}$. The present investigation is concerned with intensities at the center of the solar disc, so $\mu = 1$ has been used throughout the calculations. Therefore, the quantity $\log \frac{1}{B_o/B_1+1}$ must be known before equation 19 (or 16) can be used in the construction of the observational curve of growth. The way this quantity varies with wavelength is shown in Table 1.

TABLE 1

The Variation of κ_c/κ and log $\frac{1}{B_0/B_1+1}$ with Wavelength

λ(in Å)	$\frac{\kappa_c}{\kappa}$	log $\frac{1}{B_0/B_1+1}$
4000	.80	- .09
5000	1.00	- .17
6000	1.15	- .21
7000	1.28	- .25
8000	1.33	- .28
9000	1.32	- .30
10000	1.21	- .30
11000	1.05	- .29
12000	.90	- .27
13000	.74	- .25
14000	.57	- .24
15000		- .22
16000		- .20
17000		- .18
18000		- .17

V. CALCULATIONS AND RESULTS

This section of the paper will be concerned with the specific methods and simplifications used to carry out the calculations. The purpose of this paper was to reanalyze the data from the paper by Goldberg, Müller, and Aller[2] using the method described in this paper. Abundances were determined for carbon I, nitrogen I, oxygen I, and sulphur I. A number of elements from the Goldberg, Müller, Aller data were tested to see how η varied with depth. Of the elements investigated, a significant variation of η with depth was found only in the four above mentioned elements. In order to see whether or not this method is worthwhile to use, abundances were again determined for the four elements under the condition $\tau_1 = 0$ and $\tau_2 = \infty$. All of the steps and formulae used in the modified method are applicable to this "unmodified" method. The criteria employed for deciding whether or not the modified method is worthwhile are: the accuracy in the abundance determinations (these values will be compared to the values found in the Goldberg, Müller, Aller paper), the quality of the fit of the observational curve of growth to the theoretical curve of growth, and the difference in turbulent velocities found. Since the time required for both methods is about the same, the time element is not a factor in determining whether or not the modified method is worthwhile. It should be noted that the "unmodified" method itself is a more detailed method than is usually employed to determine abundances by the coarse analysis method.

The Goldberg, Müller, and Aller data were used to obtain the quantities log $W_\lambda(\mu)/\lambda_0$, log gf, χ_1, and log ν_0. $\chi_1 = IP - \Delta\chi$, was determined for carbon I and sulphur I with the aid of the Multiplet Tables[3], from which the values of the ionization potential for the two elements were obtained.

The value of the partition function for the element must be known in order to calculate log N_A/N (Step 6). Tables 8a and 8b from Aller[4] were used to determine the required values of the partition function.

The depth dependent quantities, such as θ and κ_c were obtained from the model atmosphere supplied by Dr. T. L. Swihart. The necessary calculations for the construction of the theoretical curve of growth were also obtained from Dr. Swihart. The two branches of the theoretical curve of growth correspond to log a = -1 for the upper branch and log a = -3 for the lower branch.

Making use of the model atmosphere is contingent upon being able to define τ_1 or τ_2 such that τ_M, and subsequently $\overline{\tau}_M$, can be found. τ_1 signifies the region of the atmosphere where line absorption starts becoming important. Since τ_1 is measured in the continuum, the quantity $\tau_1\overline{\eta}(\tau_1)$ is essentially the optical depth in the line. An optical depth of less than .1 means that line absorption is unimportant, while an optical depth of greater than .3 or .4 means that line absorption is more than just becoming important. The criterion used for determining τ_1 is $\tau_1\overline{\eta}(\tau_1) = .1$, where the value of .1 was chosen because it gave fairly consistent results. An upper limit of $\tau_1 = 1$, was imposed, as seen in the calculations in Appendix A.

In order to determine whether or not the modified method is worthwhile, a study of the results of the calculations for each element is beneficial.

Carbon I:

(a) Abundance: The abundance from the modified method is better than the abundance from the "unmodified" method as seen in Table 2.

(b) Quality of the fit: The fit using the modified method is essentially the same as the fit using the "unmodified" method, as seen in Figures 1 and 2.

(c) Remarks: The results in point (b) are not surprising once the calculations shown in Appendix A are examined. It is seen that the τ_1's are small for the modified method. Hence, the values of $\overline{\tau}_M$ when using the modified method are similar to $\overline{\tau}_M$ values calculated using the

TABLE 2

Abundances

Element	Modified	Unmodified	Goldberg, Müller, and Aller
CI	- 3.26 ± .20	- 3.02 ± .25	- 3.28
NI	- 4.14	- 4.12	- 4.02
OI (τ_1)	- 2.93 ± .08	- 2.79 ± .27	- 3.04
OI (τ_2)	- 3.20		- 3.04
SI	- 4.82 ± .15	- 4.85 ± .14	- 4.70

This Table gives the values of log N/N_H for each element.

"unmodified" method. Therefore, both observational curves of growth, and hence, the results derived from both curves, should differ only slightly.

Nitrogen I:

(a) Abundance: From Table 2, it is seen that the abundances are essentially the same.

(b) Quality of the fit: Since there are no strong lines present for nitrogen I, the fit for both methods can be accomplished anywhere on the 45° portion of the theoretical curve of growth (See Figures 3 and 4). The quality of the fit is the same for either method.

(c) Remarks: Upon inspection, it is found that Step 9 of the abundance calculations cannot be used. The calculations were made by noting that equations 17 and 19, which are used in Step 9, can be modified to fit the situation. The modified equations are:

$$\log \eta_o = \log z + \log \frac{N}{N_H} - \log \frac{\Delta\nu_D}{\nu_o} - K' \; ; \qquad (31)$$

$$\log F' = \log D' + \log \frac{\Delta\nu_D}{\nu_o} + K' \; , \qquad (32)$$

where K' is a constant. These equations say the vertical shift between the two curves og growth = $\log(\Delta\nu_D/\nu_o) + K'$, and the horizontal shift = $\log \eta_o - \log z + K'$; therefore, $\log N/N_H$, although not $\log \Delta\nu_D/\nu_o$, can be determined by using equations 31 and 32.

Since all the lines are weak lines, falling on the 45° part of the theoretical curve, it is to be expected that results will not differ significantly between the two methods.

Oxygen I:

(a) Abundance: The abundance from the modified method is better than the abundance from the "unmodified" method, as seen in Table 2.

(b) Quality of the fit: The fit for the modified method is clearly much better than the fit for the "unmodified" method, as is seen in Figures 5 and 6.

(c) Remarks: Only for oxygen I did the case $\tau_2 = \infty$ occur. The case occurred for lines 10-12 (See Appendix A). τ_2 should be the region of the atmosphere where η is small compared to the surface value. It was felt that $\tau_2 = 1$ was a good choice for these lines. Since the lines are weak, the same problem that occurred when using the nitrogen I lines, namely having to fit to the 45° portion of the theoretical curve, occurred for these three lines. The abundance from these three lines is somewhat better than using the "unmodified" method.

The important point in the oxygen I calculations is the effect of dividing the lines into τ_1 and τ_2 lines, instead of grouping all of the lines together. Comparing

Figures 5, 6, and 7, it is seen that the scatter of the points is cut down considerably by dividing the lines into τ_1 and τ_2 lines.

Sulphur I:

(a) Abundance: From Table 2 it is seen that there is no great improvement in the abundance determined using the modified method over the "unmodified" method.

(b) Quality of the fit: The fit using either method is of the same quality, as seen in Figures 8 and 9.

(c) Remarks: None.

The modified method is much better than the "unmodified" method in approximating the physical conditions. The modified method does take into account the fact that certain layers are more important than others in the formation of lines by including the definition of a τ_1 or τ_2 in the method, and using the τ_1 or τ_2 in the determination of τ_M.

The values of the turbulent velocity for the different lines of each element (where applicable) were found using the formula,

$$\log \frac{\Delta\nu_D}{\nu_0} = \log \left[\frac{1}{C} \left(\frac{2kT}{m} + V_T^2\right)^{\frac{1}{2}}\right] . \qquad (33)$$

Since $\log \Delta\nu_D/\nu_0$ was not obtainable at all for the lines of nitrogen I, or for lines 10-12 of oxygen I when using the modified method, V_T was not obtainable for these lines. In this case, T was obtained from the calculations and $\log \Delta\nu_D/\nu_0$ was then calculated from equation 33 using $\bar{V}_T = 0$. Then $\log \eta_0$ could be determined to see whether or not the estimate of $\bar{\eta}$ was good.

The values of the turbulent velocities calculated by the modified method differ appreciably from the values calculated by the "unmodified" method for both sulphur I and oxygen I (See Appendix A). There is no appreciable difference for carbon I, which is not surprising in the light of the remarks about carbon I (See Carbon I: Remarks).

It has been seen for sulphur I that the "quality of the fit" and the abundance are essentially the same whether the modified or "unmodified" method is used. Although sulphur I was chosen because η varies significantly with τ, most of the lines were formed at small τ_1's ; therefore, the shape of the observational curve of growth and hence the fit was not significantly changed when using the modified method. The value of $\log \Delta\nu_D/\nu_0$ is higher and value of $\log \eta_0 - \log z$ is lower when using the modified instead of the "unmodified" method. This means that the changes produced by the modified method tend to cancel each other in the determination of $\log N/N_H$ (See equation 17). This fact together with the fact that the observational curve of growth is not significantly altered explains why the derived abundance for sulphur I is essentially the same whether the modified or "unmodified" method is used.

It is seen that the modified method improves the abundance determination for carbon I, while it does not affect the "quality of the fit." The reason the "quality of the fit" is not affected is due to the fact that the lines of carbon I are formed at small τ_1's. The observational curve of growth, although not altered in shape, is shifted with respect to the theoretical curve of growth, when using the modified method as against using the "unmodified" method (See Figures 1 and 2). This explains why the determined abundance is different for the two methods.

For oxygen I, it has been seen that there is a considerable difference in the "quality of the fit" and the abundance determination when using the two methods. Lines 7 and 8 determine the fit of the observational curve to the theoretical curve of growth. These two lines are on the 45° portion of the curves of growth. Hence, although a number of lines have small τ_1's (i.e., lines 1-5), a considerable difference between the observational curve of growth for the modified method and "unmodified" method is to be expected; therefore, a considerable difference in the values of V_T calculated by the two methods is to be expected.

In conclusion, I feel that the modified method is worthwhile to use. I base this judgment on four facts:

(1) The modified method approximates physical reality much better than the "unmodified" method, since it takes into account the fact that certain layers of the atmosphere are more important in the formation of lines than other layers.

(2) There is a significant difference in the values of V_T for the two methods.

(3) There is a significant difference in the "quality of the fit" and the abundance determination for oxygen I and in the abundance determination for carbon I.

(4) The time required for each method is essentially the same.

Figure 1
Carbon I (Modified)

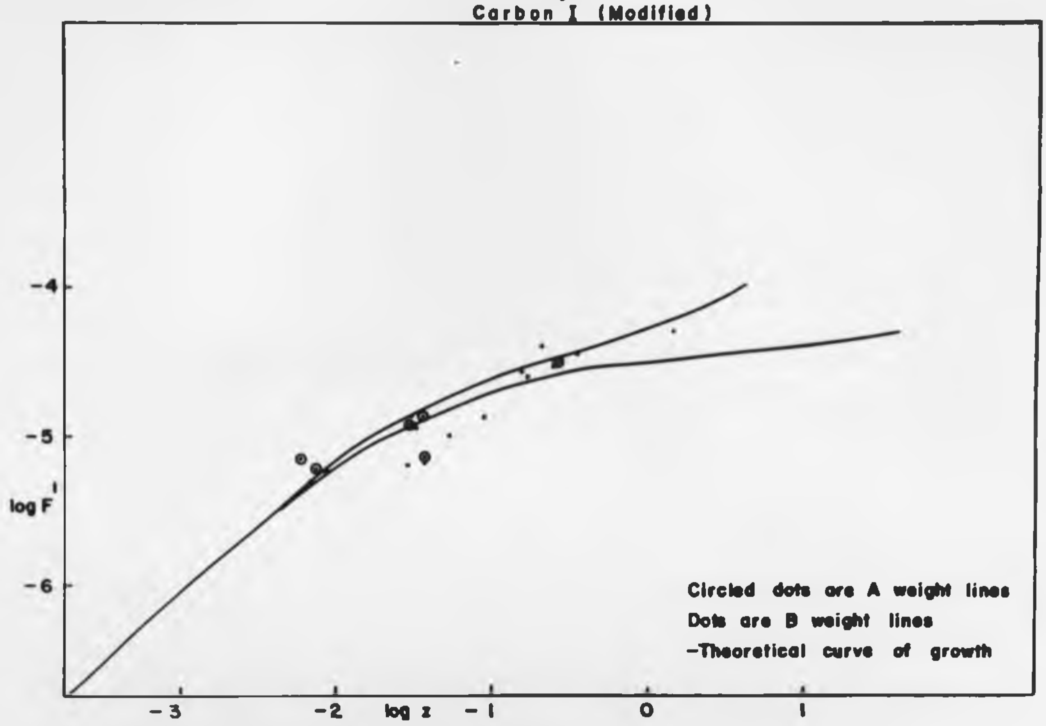


Figure 2
Carbon I (Unmodified)

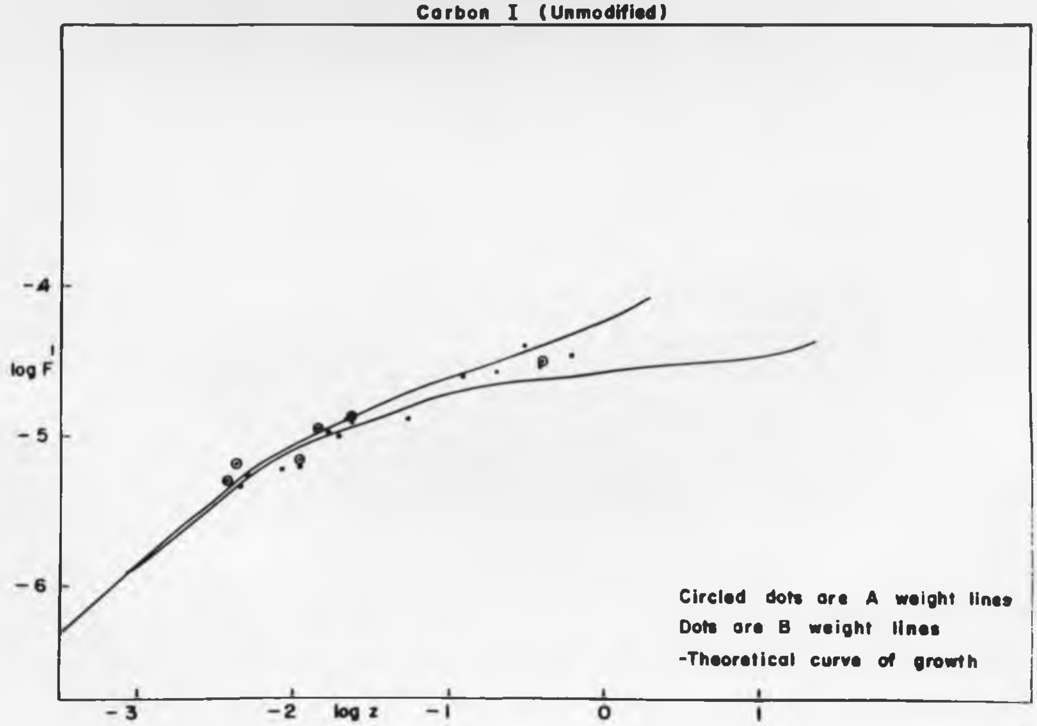


Figure 3
Nitrogen I (Modified)

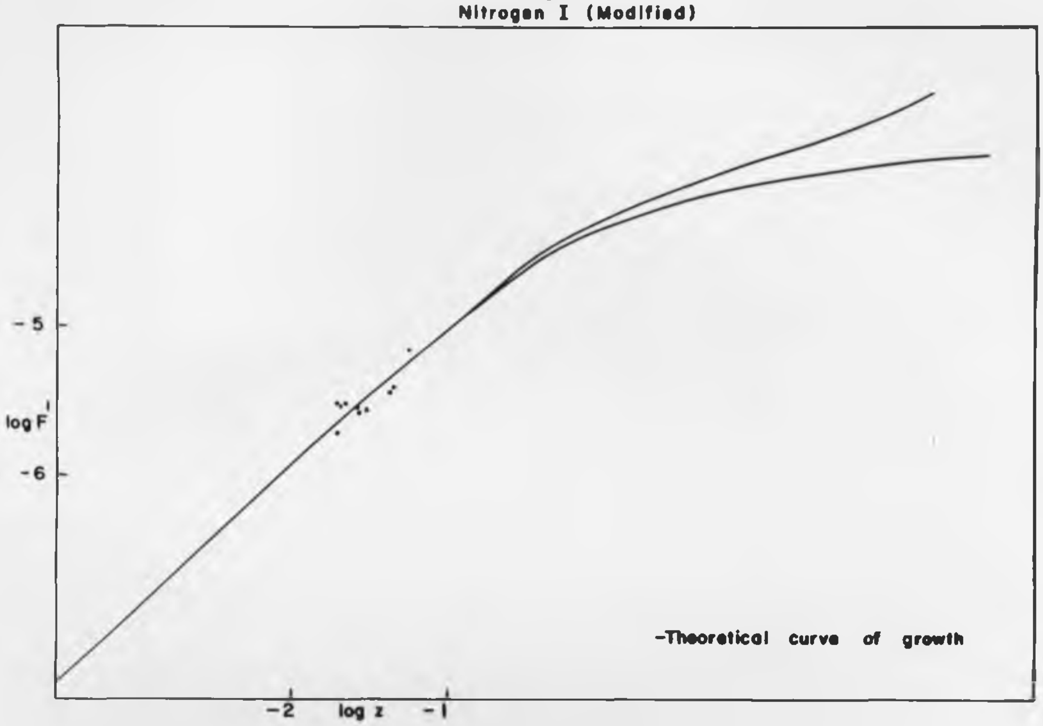


Figure 4
Nitrogen I (Unmodified)

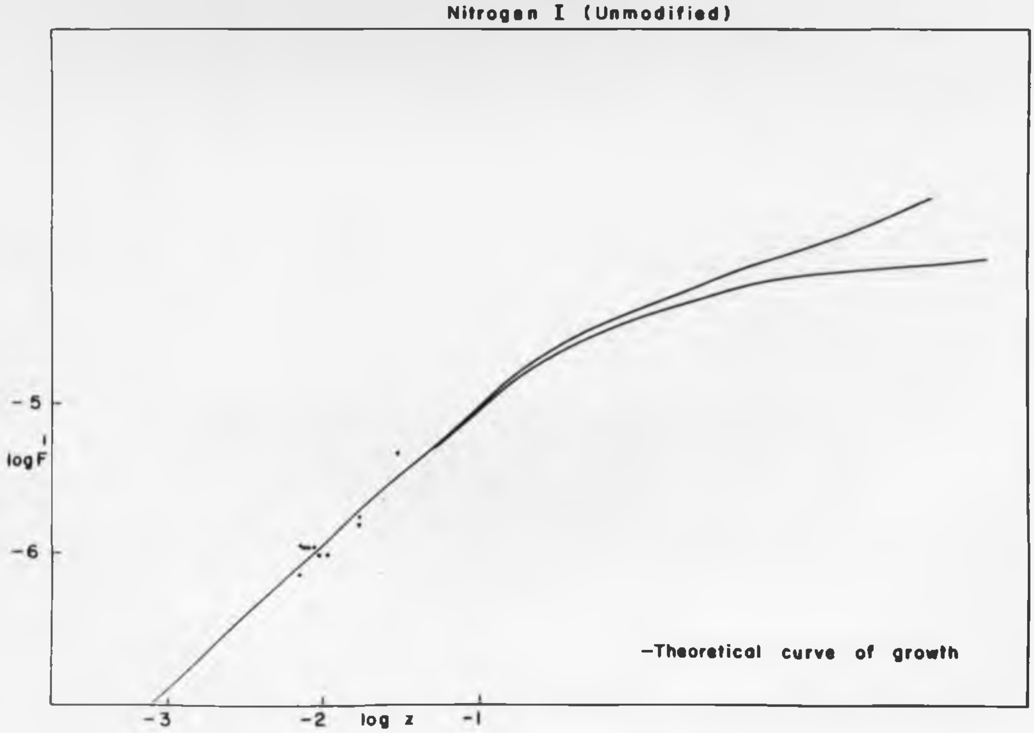


Figure 5
Oxygen I (Modified)(τ_1 lines)

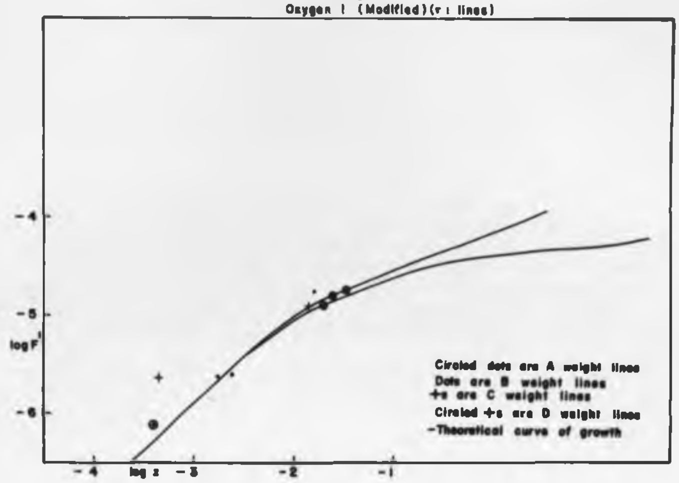


Figure 6
Oxygen I (Unmodified)

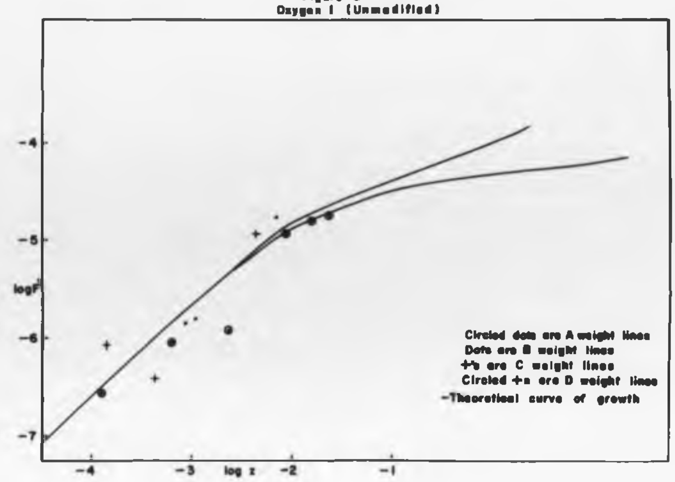


Figure 7
Oxygen I (Modified) (τ_2 lines)

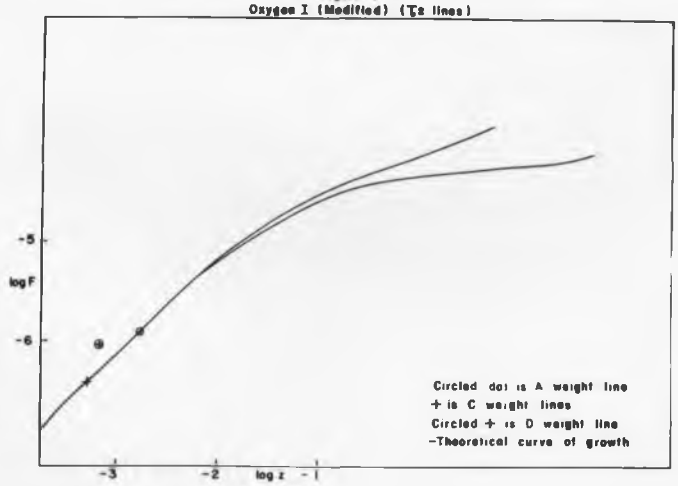


Figure 8
Sulphur I (Modified)

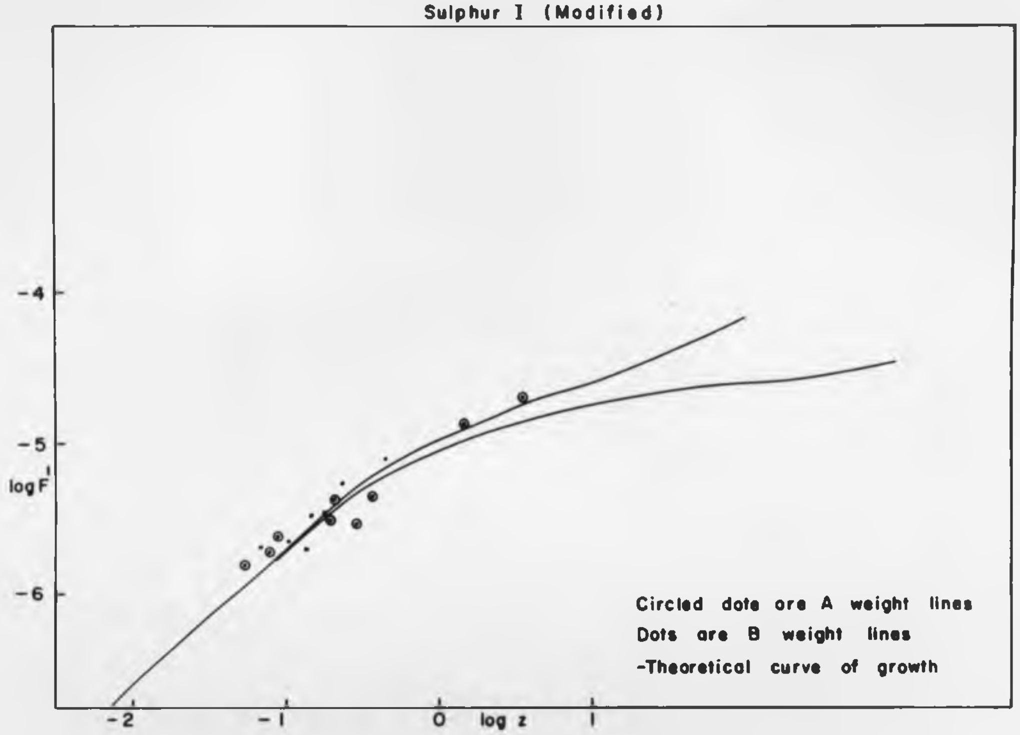
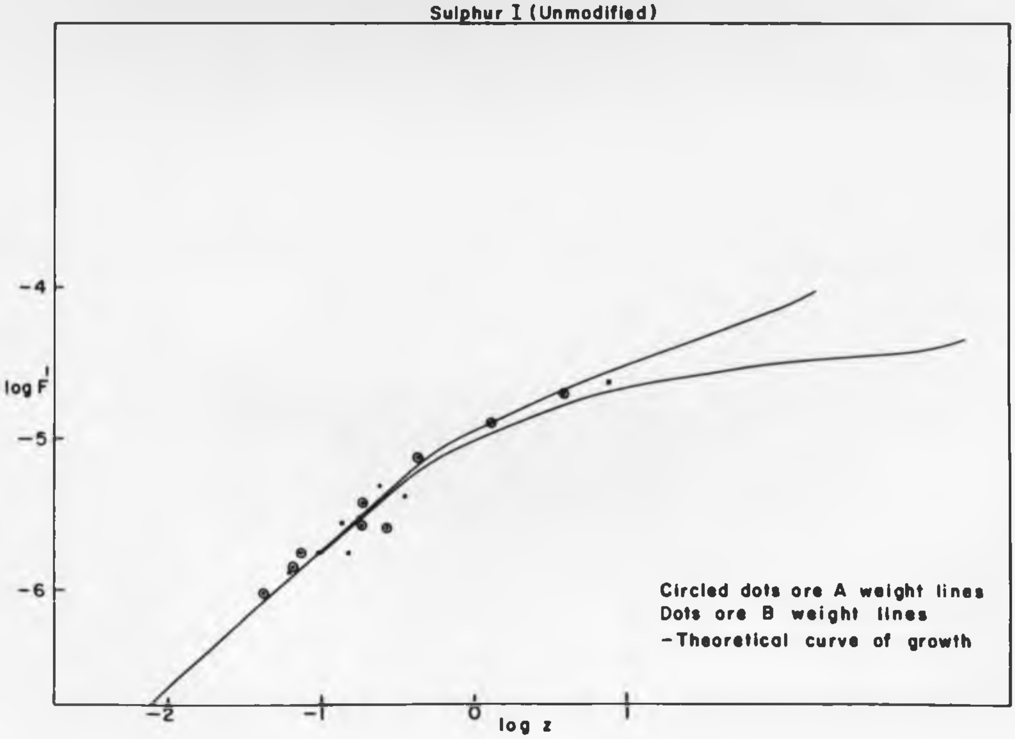


Figure 9
Sulphur I (Unmodified)



CARBON I (MODIFIED)

λ	Line No	χ_1	$\log \frac{W_\lambda}{\lambda_c}$	$\log \frac{1}{B_2/B_1+1}$	$\log \frac{gf}{B_\lambda \nu_c}$	$\bar{\eta}$	τ_1	$\bar{\tau}_m$	$\frac{k_c}{k}$	$\log \kappa'$	Θ	$\Theta\chi_1$	Log z	Log F'	η_0	V_T (km/sec)
10691.24	1	7.52	-4.58	-.29	-15.10	29.	.003	.0333	1.09	-24.92	1.045	7.86	.14	-4.29	77.	0
10685.36	2	7.51	-4.75	-.29	-15.70	14.	.007	.0676	1.09	-24.78	1.028	7.72	-.46	-4.46	19.	0
10729.59	3	7.52	-4.82	-.29	-15.83	9.	.01	.102	1.08	-24.70	1.014	7.63	-.58	-4.53	14	0
10707.36	4	7.51	-4.80	-.29	-15.83	9.	.01	.101	1.09	-24.70	1.014	7.62	-.57	-4.51	14	0
10754.02	5	7.52	-5.27	-.29	-17.00	2.4	.04	.309	1.08	-24.44	.944	7.10	-1.48	-4.96	1.8	0
9111.88	6	7.52	-4.70	-.30	-15.84	7.	.01	.103	1.31	-24.61	1.014	7.63	-.68	-4.40	11.	0
4770.00	7	7.52	-5.50	-.16	-17.85	1.5	.07	.480	.98	-24.33	.905	6.81	-2.15	-5.31	.38	0
4775.89	8	7.51	-5.42	-.16	-17.76	1.5	.07	.480	.98	-24.33	.905	6.80	-2.05	-5.23	.47	0
8335.15	9	7.53	-4.86	-.29	-15.96	7.	.01	.102	1.33	-24.61	1.014	7.64	-.81	-4.57	8.	0
5380.32	10	7.53	-5.37	-.18	-17.86	1.5	.07	.448	1.05	-24.33	.912	6.87	-2.22	-5.16	.31	0
5052.16	11	7.53	-5.07	-.17	-16.92	5.	.02	.187	1.00	-24.60	.979	7.37	-1.51	-4.89	1.6	0
6587.64	12	8.56	-5.54	-.24	-16.84	.8	.125	.553	1.23	-24.17	.891	7.63	-2.12	-5.25	.4	0
11748.28	13	8.66	-4.89	-.28	-15.08	6.	.02	.175	.93	-24.65	.983	8.51	-.76	-4.60	8.5	0
11801.10	14	8.67	-5.15	-.27	-15.80	5.	.02	.203	.92	-24.62	.974	8.44	-1.44	-4.87	1.9	0
11630.50	15	8.67	-5.29	-.28	-15.81	2.5	.04	.343	.95	-24.47	.935	8.11	-1.27	-4.99	2.8	0
12581.68	16	8.81	-5.21	-.26	-15.88	5.	.02	.239	.78	-24.65	.963	8.48	-1.53	-4.94	1.6	0
12549.50	17	8.81	-5.42	-.26	-15.98	2.5	.04	.407	.80	-24.48	.920	8.11	-1.43	-5.14	2.0	0
12562.21	18	8.81	-5.46	-.26	-15.98	2.5	.04	.407	.80	-24.48	.920	8.11	-1.43	-5.18	2.0	0
12569.11	19	8.81	-5.48	-.26	-16.10	2.3	.04	.429	.80	-24.46	.916	8.07	-1.53	-5.20	1.6	0
12614.14	20	8.81	-5.15	-.26	-15.40	5.	.02	.239	.78	-24.65	.963	8.48	-1.05	-4.88	4.7	0

CARBON I (UNMODIFIED)

λ	Line No	χ_1	$\log \frac{W_\lambda}{\lambda_c}$	$\log \frac{1}{B_{c/\beta_1}+1}$	$\log \frac{g_1 f}{B_A \nu_c}$	$\bar{\eta}$	$\bar{\tau}_M$	$\frac{\kappa_c}{\bar{\kappa}}$	$\log \kappa_c'$	θ	$\theta\chi_j$	Log z	Log F'	η_0	V_T (km/sec)
10691.24	1	7.52	-4.58	-.29	-15.10	543	.00169	1.09	-25.51	1.058	7.96	.63	-4.29	490.	0
10685.36	2	7.51	-4.75	-.29	-15.70	143	.00637	1.09	-25.24	1.056	7.93	-.21	-4.46	70.	0
10729.59	3	7.52	-4.82	-.29	-15.83	100	.00917	1.08	-25.17	1.055	7.93	-.41	-4.53	44.	0
10707.36	4	7.51	-4.80	-.29	-15.83	106	.00857	1.09	-25.18	1.055	7.92	-.39	-4.51	46.	0
10754.02	5	7.52	-5.27	-.29	-17.00	18	.0487	1.08	-24.85	1.037	7.80	-1.77	-4.98	1.9	0
9111.88	6	7.52	-4.70	-.30	-15.84	79	.00954	1.31	-25.08	1.055	7.93	-.51	-4.40	35.	0
4770.00	7	7.52	-5.50	-.16	-17.85	3	.255	.98	-24.54	.959	7.21	-2.34	-5.34	.53	0
4775.89	8	7.51	-5.42	-.16	-17.76	4	.204	.98	-24.59	.974	7.31	-2.30	-5.26	.58	0
8335.15	9	7.53	-4.86	-.29	-15.96	60	.0123	1.33	-25.02	1.053	7.93	-.69	-4.57	23.	0
5380.32	10	7.53	-5.37	-.18	-17.86	2.5	.272	1.05	-24.49	.954	7.18	-2.37	-5.19	.47	0
5052.16	11	7.53	-5.07	-.17	-16.92	25	.0385	1.00	-24.93	1.041	7.84	-1.65	-4.90	2.5	0
6587.64	12	8.56	-5.54	-.24	-16.84	2.2	.254	1.23	-24.44	.959	8.21	-2.43	-5.30	.42	0
11748.28	13	8.66	-4.89	-.28	-15.08	50	.0211	.93	-25.07	1.049	9.08	-.91	-4.61	14.	0
11801.10	14	8.67	-5.15	-.27	-15.80	9	.109	.92	-24.76	1.011	8.77	-1.63	-4.88	2.7	0
11630.50	15	8.67	-5.29	-.28	-15.81	32	.0319	.95	-24.99	1.045	9.06	-1.70	-5.01	2.3	0
12581.68	16	8.81	-5.21	-.26	-15.88	31	.0401	.78	-25.03	1.041	9.17	-1.84	-4.95	1.6	0
12549.50	17	8.81	-5.42	-.26	-15.98	24	.0500	.80	-24.97	1.036	9.13	-1.96	-5.16	1.3	0
12562.21	18	8.81	-5.46	-.26	-15.98	24	.0500	.80	-24.97	1.036	9.13	-1.96	-5.20	1.3	0
12569.11	19	8.81	-5.48	-.26	-16.10	18	.0658	.80	-24.92	1.029	9.07	-2.07	-5.22	.95	0
12614.14	20	8.81	-5.15	-.26	-15.40	9	.128	.78	-24.80	1.003	8.84	-1.26	-4.89	6.3	0

NITROGEN I (MODIFIED)

λ	Line No	χ_1	Log $\frac{W_\lambda}{\lambda_0}$	Log $\frac{1}{B_0/B_1+1}$	Log $\frac{g_1 f}{B_A \nu_c}$	$\bar{\eta}$	τ_1	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	θ	$\theta\chi_1$	Log z	Log F'	$.1\eta_0$
8683.38	1	10.29	-6.04	-.29	-15.05	.2	.83	1.308	1.32	-23.65	.790	8.13	-1.35	-5.39	.36
8680.10	2	10.29	-5.66	-.29	-14.77	.20	.50	1.010	1.32	-23.82	.823	8.47	-1.24	-5.15	.49
8718.76	3	10.29	-6.24	-.29	-15.42	.06	1.00	1.472	1.32	-23.57	.775	7.97	-1.64	-5.52	.18
8711.67	4	10.29	-6.29	-.29	-15.34	.07	1.00	1.466	1.32	-23.57	.775	7.97	-1.56	-5.57	.22
8703.15	5	10.28	-6.24	-.29	-15.45	.06	1.00	1.472	1.32	-23.57	.775	7.97	-1.67	-5.52	.17
8216.30	6	10.29	-6.07	-.28	-15.05	.12	.83	1.298	1.33	-23.65	.791	8.14	-1.36	-5.43	.36
8222.88	7	10.29	-6.22	-.28	-15.45	.06	1.00	1.461	1.33	-23.57	.776	7.99	-1.62	-5.51	.16
7468.27	8	10.29	-6.27	-.27	-15.30	.08	1.00	1.482	1.30	-23.57	.774	7.96	-1.51	-5.57	.25
7442.23	9	10.29	-6.40	-.27	-15.49	.06	1.00	1.495	1.30	-23.56	.773	7.95	-1.70	-5.70	.16
8629.16	10	10.64	-6.24	-.29	-15.08	.07	1.00	1.466	1.32	-23.57	.775	8.25	-1.58	-5.52	.21

NITROGEN I (UNMODIFIED)

λ	Line No	χ_l	Log $\frac{W_\lambda}{\lambda_0}$	Log $\frac{1}{B_u/B_l+1}$	Log $\frac{g_l f}{B_l \nu_c}$	$\bar{\eta}$	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	θ	$\theta\chi_l$	Log z	Log F'	η_0
8683.38	1	10.29	-6.04	-.29	-15.05	.11	.682	1.32	-24.04	.869	8.94	-1.77	-5.75	.10
8680.10	2	10.29	-5.66	-.29	-14.77	.21	.626	1.32	-24.09	.878	9.03	-1.53	-5.37	.18
8718.76	3	10.29	-6.24	-.29	-15.42	.05	.722	1.32	-24.01	.862	8.87	-2.10	-5.95	.05
8711.67	4	10.29	-6.29	-.29	-15.34	.06	.715	1.32	-24.02	.864	8.89	-2.03	-6.00	.06
8703.15	5	10.28	-6.24	-.29	-15.45	.05	.722	1.32	-24.01	.862	8.86	-2.12	-5.95	.05
8216.30	6	10.29	-6.07	-.28	-15.05	.11	.677	1.33	-24.05	.870	8.95	-1.77	-5.79	.10
8222.88	7	10.29	-6.22	-.28	-15.45	.05	.717	1.33	-24.01	.863	8.88	-2.14	-5.94	.04
7468.27	8	10.29	-6.27	-.27	-15.30	.06	.726	1.30	-24.02	.862	8.87	-1.97	-6.00	.07
7442.23	9	10.29	-6.40	-.27	-15.49	.04	.740	1.30	-24.01	.860	8.85	-2.15	-6.13	.04
8629.16	10	10.64	-6.24	-.29	-15.08	.05	.722	1.32	-24.01	.862	9.17	-2.06	-5.95	.05

OXYGEN I (MODIFIED)

λ	Line No	χ_1	Log $\frac{W_\lambda}{\lambda_0}$	Log $\frac{1}{\beta_1/\beta_1+1}$	Log $\frac{g_i f}{\beta_A \nu_0}$	$\bar{\eta}$	τ_1	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	θ	$\theta\chi_1$	Log z	Log F'	η_0	V_T (km/sec)
7771.95	1	9.11	-5.02	-.27	-15.21	4.	.03	.174	1.32	-24.50	.983	8.96	-1.49	-4.74	2.1	1.39
7774.18	2	9.11	-5.07	-.27	-15.36	3.5	.03	.191	1.32	-24.48	.978	8.91	-1.61	-4.79	1.6	1.38
7775.39	3	9.11	-5.19	-.27	-15.58	2.3	.04	.267	1.32	-24.39	.955	8.70	-1.71	-4.90	1.3	1.34
8446.35	4	9.48	-5.06	-.29	-15.28	2.6	.04	.239	1.33	-24.42	.963	9.13	-1.81	-4.75	1.0	1.35
8446.73	5	9.48	-5.23	-.29	-15.52	1.4	.07	.366	1.33	-24.30	.930	8.82	-1.86	-4.91	.9	1.28
6453.61	6	10.69	-6.33	-.25	-16.95	.05	1.00	1.627	1.20	-23.54	.762	8.15	-3.38	-5.65	.03	.63
6158.18	7	10.69	-6.02	-.22	-15.94	.21	.48	1.107	1.18	-23.81	.811	8.67	-2.62	-5.59	.16	.91
6156.81	8	10.69	-6.06	-.22	-16.09	.20	.50	1.130	1.18	-23.80	.809	8.65	-2.76	-5.62	.12	.89
5436.85	9	10.69	-6.74	-.19	-17.14	.05	1.00	1.842	1.06	-23.50	.745	7.96	-3.42	-6.12	.03	.50

λ	Line No	χ_1	Log $\frac{W_\lambda}{\lambda_0}$	Log $\frac{1}{\beta_1/\beta_1+1}$	Log $\frac{g_i f}{\beta_A \nu_0}$	$\bar{\eta}$	τ_2	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	θ	$\theta\chi_1$	Log z	Log F	η_0
5577.35	10	1.96	-6.23	-.19	-23.90	.07	1	.378	1.09	-24.37	.927	1.82	-3.17	-6.04	.04
6300.33	11	0.00	-6.13	-.22	-25.31	.18	1	.339	1.19	-24.37	.936	0.00	-2.76	-5.91	.10
6363.88	12	0.02	-6.63	-.22	-25.80	.06	1	.344	1.20	-24.36	.935	0.02	-3.28	-6.41	.03

OXYGEN I (UNMODIFIED)

λ	Line No	χ_1	$\log \frac{W_\lambda}{\lambda_0}$	$\log \frac{1}{B_c(B_1+1)}$	$\log \frac{g_1 f}{B_A \nu_0}$	$\bar{\eta}$	$\bar{\tau}_m$	$\frac{K_c}{\bar{K}}$	$\log k_c'$	θ	$\theta\chi_1$	Log z	Log F'	η_0	V_T (km/sec)
7771.95	1	9.11	-5.02	-.27	-15.21	41.	.0180	1.32	-24.95	1.051	9.57	-1.65	-4.75	2.6	2.24
7774.18	2	9.11	-5.07	-.27	-15.36	31.	.0237	1.32	-24.90	1.048	9.55	-1.83	-4.80	1.7	2.24
7775.39	3	9.11	-5.19	-.27	-15.58	18	.0399	1.32	-24.80	1.041	9.48	-2.08	-4.92	1.0	2.24
8446.35	4	9.48	-5.06	-.29	-15.28	15	.0470	1.33	-24.76	1.038	9.84	-2.18	-4.77	.76	2.23
8446.73	5	9.48	-5.23	-.29	-15.52	9.0	.0752	1.33	-24.67	1.025	9.72	-2.39	-4.94	.47	2.22
6453.61	6	10.69	-6.33	-.25	-16.95	.03	.809	1.20	-23.99	.849	9.08	-3.86	-6.08	.02	1.96
6158.18	7	10.69	-6.02	-.22	-15.94	.26	.673	1.18	-24.10	.871	9.31	-2.97	-5.80	.12	2.00
6156.81	8	10.69	-6.06	-.22	-16.09	.18	.718	1.18	-24.07	.863	9.23	-3.07	-5.84	.10	1.99
5436.85	9	10.69	-6.74	-.19	-17.14	.02	.925	1.06	-23.97	.834	8.92	-3.91	-6.55	.01	1.94
5577.35	10	1.96	-6.23	-.19	-23.90	.9	.483	1.09	-24.28	.905	1.77	-3.21	-6.04	.07	2.06
6300.23	11	0.00	-6.13	-.22	-25.31	2.8	.221	1.19	-24.49	.968	0.00	-2.64	-5.91	.27	2.15
6363.88	12	0.02	-6.03	-.22	-25.80	.86	.448	1.20	-24.28	.912	0.02	-3.36	-6.41	.05	2.07

SULPHUR I (MODIFIED)

λ	Line No	χ_1	Log $\frac{W_\lambda}{\lambda_c}$	Log $\frac{1}{B_\lambda/B_1 + 1}$	Log $\frac{gf}{B_\lambda y_c}$	$\bar{\eta}$	τ_i	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	θ	$\theta\chi_1$	Log $\frac{N_A}{N}$	Log z	Log F'	η_0	V_T (Km/sec)
4694.13	1	6.50	-5.53	-.15	-17.07	1.3	.08	.531	.97	-24.29	.895	5.82	-.01	- .43	-5.35	.94	0
4695.45	2	6.50	-5.72	-.15	-17.22	1.	.1	.619	.97	-24.23	.879	5.71	-.01	- .53	-5.53	.75	0
4696.27	3	6.50	-5.67	-.15	-17.44	.7	.14	.751	.97	-24.13	.858	5.58	-.02	- .73	-5.46	.47	0
10455.46	4	6.83	-4.91	-.29	-15.14	42.	.002	.224	1.13	-24.51	.968	6.61	-.01	.93	-4.62	23.	.44
10459.44	5	6.83	-4.99	-.29	-15.36	13.	.01	.0721	1.13	-24.75	1.027	7.01	.00	.56	-4.70	9.4	.59
10456.75	6	6.83	-5.18	-.29	-15.83	5.6	.02	.152	1.13	-24.60	.993	6.78	.00	.17	-4.88	3.9	.51
5278.96	7	6.83	-6.05	-.18	-17.64	.23	.43	1.207	1.03	-23.81	.799	5.45	-.04	-1.14	-5.68	.18	0
8694.64	8	7.84	-5.41	-.29	-15.44	2.	.05	.290	1.32	-24.37	.949	7.44	-.01	- .34	-5.10	1.16	.37
8671.31	9	7.83	-6.04	-.29	-16.16	.8	.125	.516	1.32	-24.17	.898	7.03	-.01	- .85	-5.70	.36	0
8693.96	10	7.84	-5.71	-.29	-16.03	.7	.14	.552	1.32	-24.15	.891	6.99	-.01	- .70	-5.36	.54	0
8670.63	11	7.83	-5.86	-.29	-16.05	.7	.14	.552	1.32	-24.15	.891	6.98	-.01	- .71	-5.51	.51	0
8678.95	12	7.83	-6.04	-.29	-16.52	.3	.33	.833	1.32	-23.94	.846	6.62	-.02	-1.04	-5.61	.23	0
8670.20	13	7.83	-6.04	-.29	-16.40	.4	.25	.731	1.32	-24.01	.861	6.74	-.02	- .97	-5.64	.27	0
7686.13	14	7.83	-6.11	-.27	-16.57	.3	.33	.833	1.32	-23.94	.846	6.62	-.02	-1.09	-5.70	.20	0
7679.60	15	7.83	-6.28	-.27	-16.79	.2	.50	1.010	1.32	-23.82	.823	6.44	-.03	-1.26	-5.79	.14	0
6757.20	16	7.83	-5.55	-.25	-15.94	1.	.10	.480	1.25	-24.23	.905	7.09	-.01	- .63	-5.26	.60	.08
6052.66	17	7.83	-5.78	-.23	-16.30	.55	.18	.705	1.17	-24.08	.865	6.77	-.02	- .83	-5.47	.38	0

SULPHUR I (UNMODIFIED)

λ	Line No	χ_1	Log $\frac{W_\lambda}{\lambda_c}$	Log $\frac{1}{g_c/g_1+1}$	Log $\frac{g_1 f}{g_A \nu_c}$	$\bar{\eta}$	$\bar{\tau}_m$	$\frac{\kappa_c}{\bar{\kappa}}$	Log κ_c'	Θ	$\Theta\chi_1$	Log $\frac{N_A}{N}$	Log z	Log F'	η_0	V_T (km/sec)
4694.13	1	6.50	-5.53	-.15	-17.07	1.2	.469	.97	-24.35	.908	5.90	-.01	- .45	-5.38	.47	1.23
4695.45	2	6.50	-5.72	-.15	-17.22	.9	.543	.97	-24.29	.893	5.80	-.01	- .56	-5.57	.35	1.21
4696.27	3	6.50	-5.67	-.15	-17.44	.52	.678	.97	-24.18	.869	5.65	-.02	- .75	-5.52	.23	1.17
10455.46	4	6.83	-4.91	-.29	-15.14	41.	.0211	1.13	-24.99	1.049	7.16	0	.87	-4.62	10.	1.37
10459.44	5	6.83	-4.99	-.29	-15.36	24.	.0354	1.13	-24.89	1.043	7.12	0	.59	-4.70	5.2	1.37
10456.75	6	6.83	-5.18	-.29	-15.83	8.	.0983	1.13	-24.69	1.015	6.93	0	.11	-4.89	1.7	1.35
5278.96	7	6.83	-6.05	-.18	-17.64	.15	.844	1.03	-24.04	.844	5.76	-.02	-1.20	-5.87	.08	1.14
8694.64	8	7.84	-5.41	-.29	-15.44	1.8	.271	1.32	-24.39	.954	7.48	-.01	- .36	-5.12	.56	1.28
8671.31	9	7.83	-6.04	-.29	-16.16	.36	.557	1.32	-24.14	.890	6.97	-.01	- .82	-5.75	.20	1.20
8693.96	10	7.84	-5.71	-.29	-16.03	.47	.515	1.32	-24.17	.898	7.04	-.01	- .73	-5.42	.25	1.21
8670.63	11	7.83	-5.86	-.29	-16.05	.46	.519	1.32	-24.17	.897	7.02	-.01	- .73	-5.57	.25	1.21
8678.95	12	7.83	-6.04	-.29	-16.52	.16	.653	1.32	-24.07	.873	6.84	-.02	-1.13	-5.75	.10	1.18
8670.20	13	7.83	-6.04	-.29	-16.40	.21	.626	1.32	-24.09	.878	6.87	-.02	-1.02	-5.75	.12	1.19
7686.13	14	7.83	-6.11	-.27	-16.57	.14	.665	1.32	-24.06	.872	6.83	-.02	-1.18	-5.84	.09	1.18
7679.60	15	7.83	-6.28	-.27	-16.79	.08	.701	1.32	-24.03	.865	6.77	-.02	-1.37	-6.01	.06	1.17
6757.20	16	7.83	-5.55	-.25	-15.94	.65	.485	1.25	-24.22	.903	7.07	-.01	- .62	-5.30	.31	1.22
6052.66	17	7.83	-5.78	-.23	-16.30	.33	.643	1.17	-24.13	.875	6.85	-.02	- .86	-5.55	.18	1.18

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