RESPONSE OF THE CITRUS FRUIT-STEM SYSTEM TO FRUIT REMOVING ACTIONS

by

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ABSTRACT

The past decade has seen many advances in harvest mechanization of many of our agricultural crops. Recent years have seen a concentrated effort by many investigators to mechanize the harvest of citrus fruits. A complete harvest mechanization system for citrus does not exist at this time. A deterrent to the development of such a system has been the lack of basic data on methods of removing the citrus fruit from its stem.

The citrus fruit can be removed from its tree by application of forces on the fruit-stem system. These forces may be applied directly to the fruit itself by, for example, grasping and pulling on the fruit. Forces can be indirectly applied to the fruit when the fruit-stem system is vibrated and inertial forces are developed. The latter two means of applying forces to the fruit-stem system deal with forces applied to the fruit itself. These forces must result in one or a combination of tension, bending, or shearing stresses being developed across the button of the fruit of such a magnitude as to cause fruit removal. The second means of applying forces to the fruit-stem system is to apply forces to the stem. Cutting the stem is used to detach the fruit from the fruit-stem system. Here shearing forces induced by clippers result in severance of the stem and the fruit is removed.

In this study the removal of the citrus fruit from its stem by torsional, vibratory, and pulling actions has been considered. Removing forces were applied to the fruit itself. Removal of fruit by forces applied directly to the stem (as in cutting the stem) were not considered. The torsional and pull tests were conducted such that the individual and additive effects of tension, bending, and torsional stresses could be observed. Vibrational tests on the fruit-stem system were conducted to gather basic data and observe the response of the system to vibratory removal actions.

Results of pull tests have shown that trends of removal pull force throughout the growth and maturation of the citrus fruit are characteristic of the particular citrus variety. It was also shown that when pull forces are applied at an angle to the major axis of the fruit the magnitude of the removal pull force is substantially reduced.

Torsional removal tests have shown that removal torques are independent of maturity of fruit and independent of amount of tension (up to 10 pounds) that was placed on the stem at the same time the torque was being applied to the fruit. In other words, for a particular variety, the removal torque remained constant throughout the test season.

Vibratory removal tests have shown that Marsh grapefruit require a less severe shaking action to cause removal than do Valencia oranges.

An increase in the stroke (amplitude) of the shaking motion applied to the fruit-stem system has been shown to decrease the frequency necessary to remove the fruit from the stem.

The fruit-stem system responds in definite and predictable mannerisms to a sinusoidal shaking action. These particular responses, or regimes of vibration, were found to be useful in predicting how, why, and if fruit removal would occur. Three modes of separation of the fruit from its stem were encountered. These were, fruit removed with button intact on the fruit, fruit removed with no button on the fruit, and fruit removed with a portion of its stem intact. The modes of separation were found to be related to the regime in which the fruit was vibrating.

A theoretical analysis of steady-state forced vibration of the fruit-stem system was made. The analysis yielded a mathematical model of the fruit-stem system undergoing a sinusoidal shaking action. The analysis yields, among other things, deflection and stress across the button for a sinusoidal shaking action.

INTRODUCTION

CITRUS INDUSTRY IN THE UNITED STATES

Citrus was first grown in the United States in Florida during the mid 1500's. It was not until 1821, however, that groves of sufficient size were established to mark the presence of a citrus industry in Florida. By the early 1900's the boundaries of this country's citrus industry had been established. Concentrations of citrus were found in Arizona, California, Florida, and Texas. Today, these same four states produce nearly all of the commercially grown citrus in the United States. Small areas of production exist in Alabama, Georgia, Mississippi, and Louisiana.

Citrus fruit represent a major portion of all the fruit produced in the United States (1). In the 1964-1965 season some 7,699,060 tons of citrus were produced. The total fruit production for that year was 18,892,580 tons. The value of the citrus crop in 1964-1965 was 522,369,000 dollars. Production from all tree crops is a one and one-third billion dollar industry in the United States. Therefore, citrus accounts for in excess of one-third of the dollar value of tree crop production in the United States.

Florida is the leading citrus production state as it produced 71.6 per cent of the nation's citrus crop during the 1964-1965 crop season. This state has experienced a rapid increase in production over the past 15 years. The California industry ranks second to Florida having produced 23.9 per cent of the nation's crop in 1964-1965. Production in California has decreased over the past 20 years, primarily because of the large influxes of population which are displacing the industry. The citrus industry in Arizona amounts to 32,354 acres. This state produced 2.9 per cent of the nation's crop in 1964-1965. Recent plantings in Yuma County and in the Chandler Heights area of the Salt River Valley have greatly expanded Arizona's industry. Texas contributes 1.6 per cent of the total production of the citrus industry of the United States. Most of the Texas industry is concentrated in the lower Rio Grande Valley where a high quality grapefruit is grown.

HARVEST METHODS AND COSTS

Citrus fruits are harvested by hand picking. Hand labor is tedious, time consuming, and expensive. Two methods of fruit removal are used to separate the fruit from its stem. One method is the use of clippers to cut the fruit from the stem. The clippers have been designed to fit the palm of one hand and are secured there by a strap. With this one hand the picker grasps and cuts the stem. The other

hand is used to convey the clipped fruit to a receptacle. Clipping, as it is called, leaves the button (calyx) attached to the fruit. This is desirable in fruit which are marketed fresh. An intact button inhibits deterioration which might easily occur had the button been removed. Also, a fruit with a button is more easily marketed fresh because of its pleasing appearance. The second method of fruit removal is called snapping. This method is used primarily in Florida. Removal results from a combination of sharp movements of the hand. These movements consist in simultaneously twisting and jerking upon the fruit at an angle to its major axis. The movements are shown in Figure 1. Snapping removes the button from the fruit when it is picked. Improper snapping may result in plugging which is the removal of part of the peel with the stem. This is highly undesirable as the raw flesh of the fruit is left exposed and is susceptible to quick deterioration. The twisting motion of the hand in the snapping process tends to evenly distribute the force applied to the button and lessens the chance of plugging. The angular pull on the fruit tends to lessen plugging and also decreases the amount of force necessary to remove the fruit from the stem.

Harvest costs for hand-picked citrus represent a substantial cash output to the grower (2). Harvest costs for Navel and Valencia oranges in Arizona in 1962-1963 were

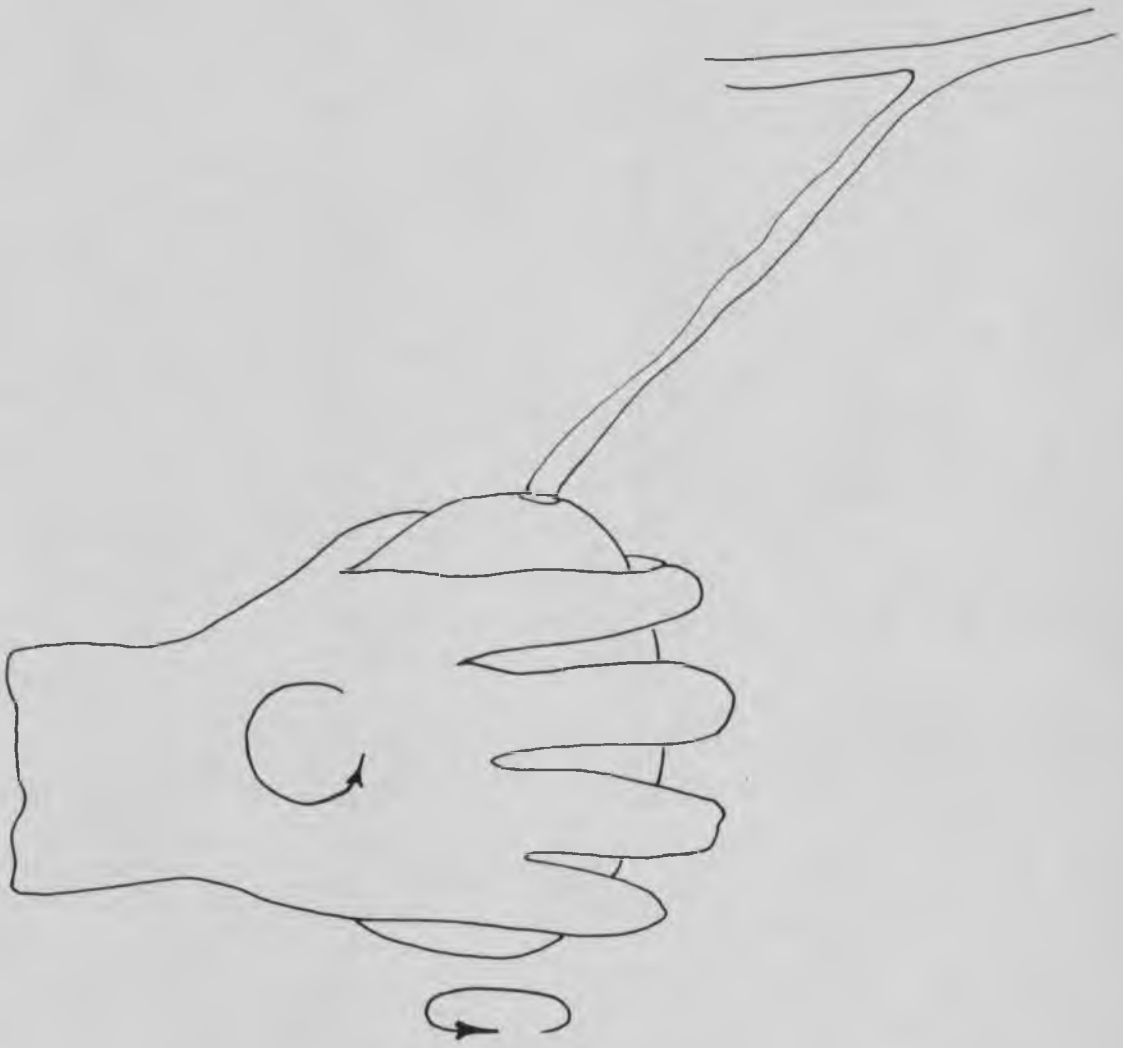

FIGURE 1: MOTION IN HAND SNAPPING OF CITRUS FRUIT

$0.25 per packed carton.[1] The harvest cost for lemons was $0.50 per carton and for grapefruit the figure was $0.175. By comparison, production costs were $0.97 per packed carton for Valencias, $1.55 per carton for Navels, and $0.71 per carton for grapefruit. These typical figures show that the harvest operation may be 25 per cent of the production costs.

HARVEST MECHANIZATION

Increasing labor costs coupled with an actual decrease in a dependable, proficient labor supply have pressed the need for some degree of mechanization of citrus harvest. Complete or 100 per cent mechanization of citrus would result in mass removal of the crop from the tree with a minimum of human effort or decision. Other mechanization programs are aimed at improving the efficiency of the picker by giving him the ability to position himself about the tree in a quick and easy manner.

Some work has been initiated on mass removal harvest systems for citrus. The "shake and catch" method has been used on some varities of citrus with encouraging results (3). Other harvest systems, still very much in the experimental stage, are being worked upon.

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[1]weight of packed carton:
Marsh grapefruit . . . . 32.0 pounds
Valencia orange . . . . 37.5 pounds
Navel orange . . . . 37.5 pounds

OBJECTIVES OF STUDY

Very little information is available on mechanically induced detachment of citrus fruits from their stem. In years past, citrus has been entirely hand picked and knowledge of the strength of the fruit-stem union and response of the fruit-stem system to various removing actions was not needed. Recent efforts to mechanize the citrus crop has brought a need to understand the "hows and whys" of mechanically induced detachment.

One method of mechanically removing a fruit from its stem is to create forces on the fruit which are greater than the bond between the fruit and its stem. The removing forces may be induced from direct physical contact with the fruit or may result from inertial forces as when the fruit is shaken. In either case, detachment results from one or a combination of a pull or a twist on the fruit.

A primary purpose of this thesis was to study and observe various methods of mechanically detaching citrus fruits from their stems. Pull, twisting, and shaking actions were used to cause detachment. Testing was done under controlled conditions so that the removal actions could be studied for a more complete understanding of the removal process.

Mathematical modeling can often shed light on understanding a system. With such a motive in mind, response of the fruit-stem system to vibratory removal actions were modeled.

It was expected that data would vary with maturity of the fruit. Hence tests were made throughout the growth season of the fruit.

REVIEW OF LITERATURE

PULL TESTS

Data pertaining to removal characteristics of citrus fruits is very limited. Coppock and associates (4) have reported on the results of pull tests on Valencia oranges in Florida. Tests were conducted by pulling on the stem at various angles with respect to the major axis of the fruit as shown in Figure 2. Results of this work show a decreasing removal force as the pull is directed away from the major axis of the fruit. Tests also have shown a decrease in removal force with an increase in maturity of the fruit.

Similar pull tests were conducted by Lamouria and Brewer (5) on California olives. The authors expressed results of their work in terms of a F/W ratio. F was the force necessary to remove the olive from the stem and W was the weight of the individual olive. Conclusions reached by the authors indicate the F/W ratio to be dependent upon the rate of force application. Results also show the F/W ratio to be influenced by fruit maturity, physiological changes within the fruit, and varietal and geographical differences.

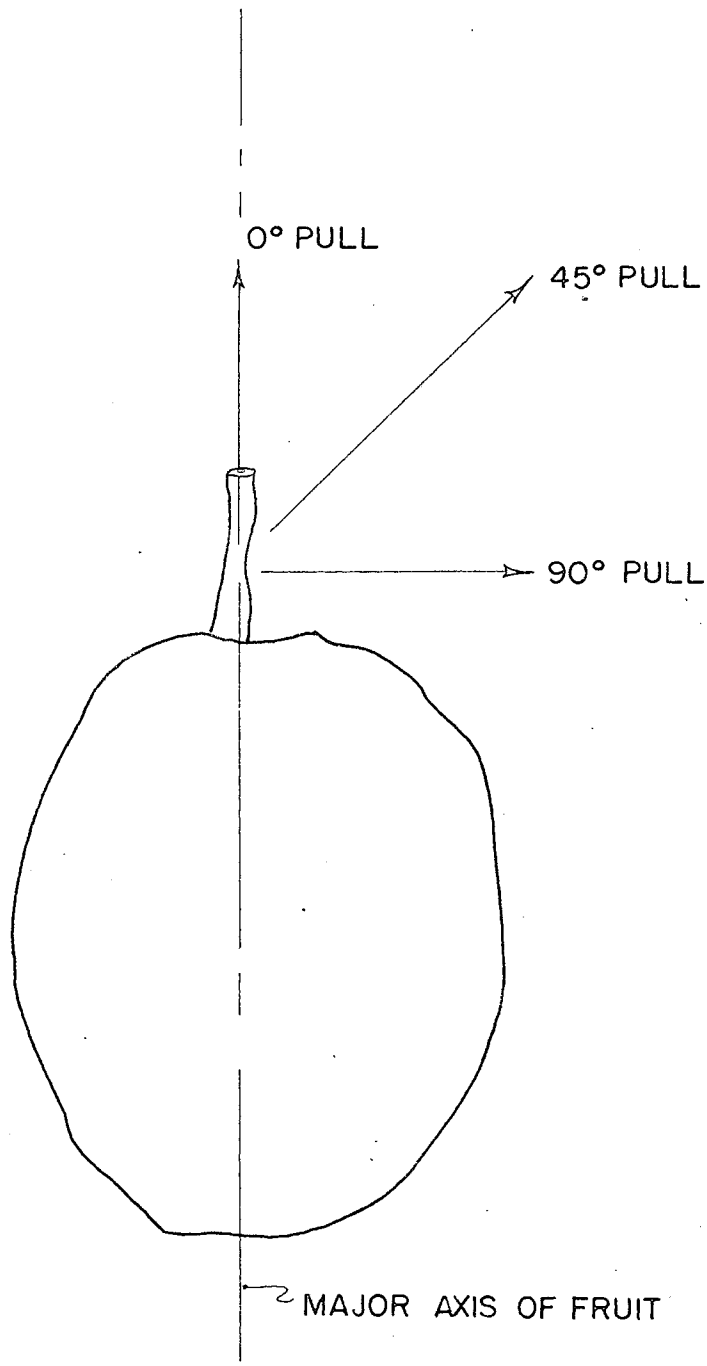

FIGURE 2: ANGLES OF PULL APPLIED TO CITRUS FRUIT STEMS

VIBRATORY REMOVAL TESTS

Several field studies have been made on vibratory removal of citrus. Coppock (4) and Lenker and Hedden (6) have reported on vibratory removal tests on citrus in Florida. Work by Coppock in the 1964-1965 harvest season proved the feasibility of the tree shaker harvest system for citrus. Work showed fruit removal by such a system to be dependent upon fruit maturity and fruit variety. Work by Lenker and Hedden was done to determine optimum shaking action. In their study, the effect of limb stroke, limb diameter, and shaking frequency on fruit removal were determined. A regression equation was determined from field data. This equation expressed fruit removal as an algebraic combination of a linear function of limb diameter and limb stroke, and a second order function of the product of shaking frequency and limb stroke. Other results showed different optimum shaking actions for different citrus varieties.

Field studies of vibratory removal of citrus have so far been limited to the work described above in Florida. Considerable work has been done in vibratory harvest of other tree crops (7, 8, 9). As yet, however, little work has been done to study the effect of shaking actions on the fruit-stem system. Lamouria and Brewer (5) have studied the revoval characteristics of the olive-stem system. Results

of this study showed removal effectiveness to be proportional to the product of applied stroke times frequency raised to a power. The power was theorized to be between one and two. A power of two would indicate removal effectiveness is proportional to the maximum acceleration imposed upon the fruit-stem system. A power of one indicates removal is dependent upon maximum velocity.

Coppock (9) has commented on two tree-shaking concepts which represent possible modes of vibration of the fruit-stem system. Figure 3 illustrates the two shaking concepts. Concept one is the result of applying the vibration normal to the major axis of the fruit. The magnitude of the removal force is proportional to the mass of the fruit, length of the stem, and the applied frequency and stroke. Concept number two has an excitation motion applied parallel to the major axis of the fruit. Therefore inertial and gravity forces always act in the same plane. For vertically imposed velocities of sufficient magnitude, the fruit will tend to rotate as shown. This rotation causes the resultant forces to be applied at an angle to the button, causing separation. In both removal concepts, forces induced in the stem which cause separation result from a combination of gravity and inertial forces.

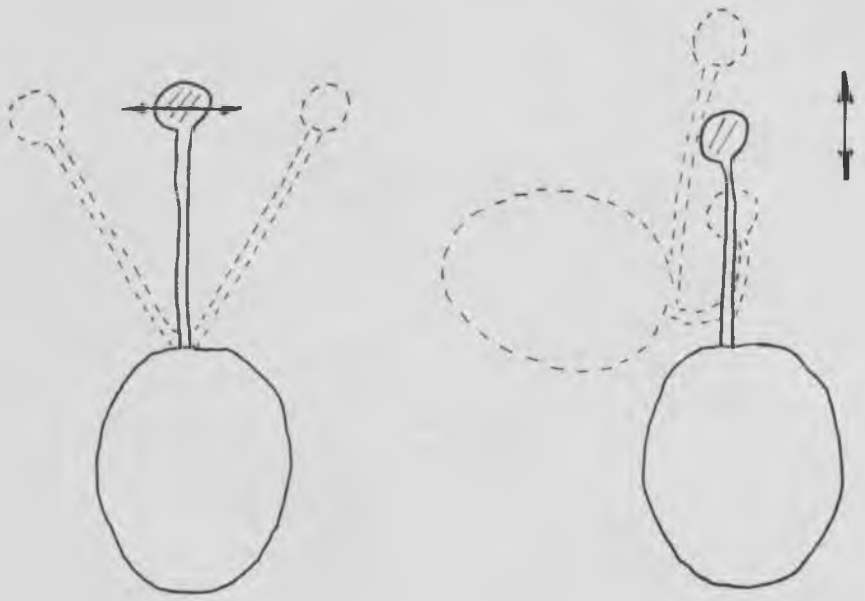

FIGURE 3: TWO SHAKING CONCEPTS FOR CITRUS FRUIT REMOVAL

TORSIONAL REMOVAL TESTS

A third mode of removal has been discussed by Coppock (9). This method consists in applying a torque in a plane normal to the major axis of the fruit. At the same time a tensile force is applied to the stem. Results of tests have shown that this method of fruit removal minimizes plugging.

INVESTIGATION

EXPERIMENTAL PROCEDURE AND APPARATUS

Mechanically induced detachment may result from one or a combination of straight or angled pulls and a twisting action. These forces may be applied directly to the fruit or may arise through inertial action. Equipment was constructed to perform three removal tests utilizing pulling, twisting, and vibratory removing actions.

PULL TESTS

The most basic removal test performed was a pull test. The apparatus consisted of a maximum reading force gauge and an alligator clamp. The clamp was attached to the stem of the fruit and provided a means for the force gauge to be attached to the stem. Removal force could be read directly from the gauge when the stem was pulled from the fruit.

Pull tests were conducted at three positions with respect to the major axis of the fruit. Figure 4 shows the pull test apparatus as connected to pull in line with the major axis of the fruit. This test was designated as the zero degree pull test. The test places the fruit stem system in direct tension and is therefore a measure of the

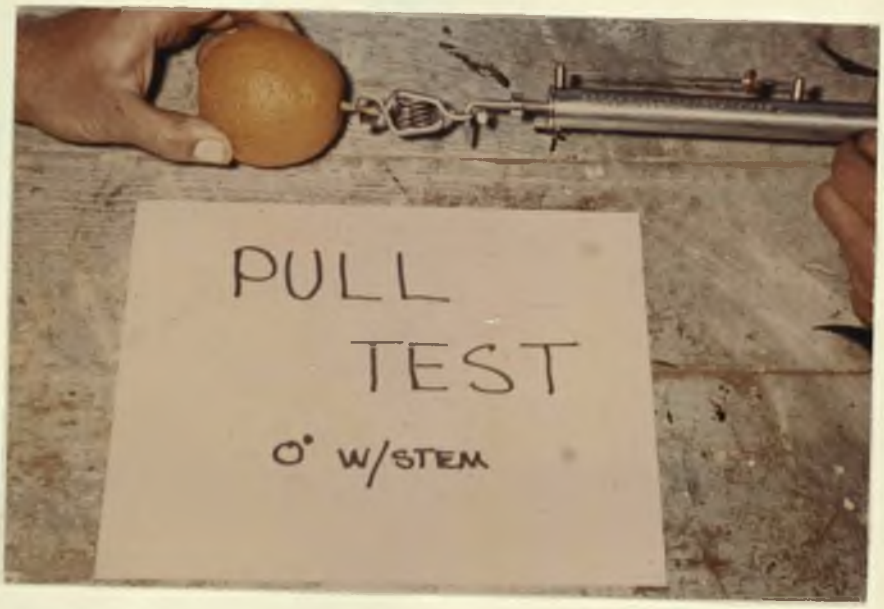

FIGURE 4: ZERO DEGREE PULL TEST

tensile strength of the fruit-stem union. Figure 5 shows the apparatus in position to perform an angled pull on the stem. This second pull test was conducted by pulling at a 45 degree angle with respect to the major axis of the fruit. The clamp was attached at one-half inch above the button of the fruit. The third pull test conducted is shown in Figure 6. Here the pull was exerted at 90 degrees with respect to the major axis.

All pull tests were conducted by firmly holding the fruit against a surface with one hand and pulling on the force gauge with the other. All attempts were made to exert a uniform, even rate of pull on the stem.

TORSIONAL REMOVAL TESTS

Torsional removal tests were performed with the apparatus pictured in Figure 7. The purpose of this type of removal device was to apply a torque normal to the major axis of the fruit while at the same time placing a constant tensile load upon the stem. Figure 8 shows the desired action.

Figure 9 shows the test specimen mounted in the torsional removal device. The spring loaded clamp was adjustable to compensate for size variation among the samples.

The torsional action was provided by the force gauge acting on a one-quarter-inch radius. The torque was

FIGURE 5: FORTY-FIVE DEGREE PULL TEST

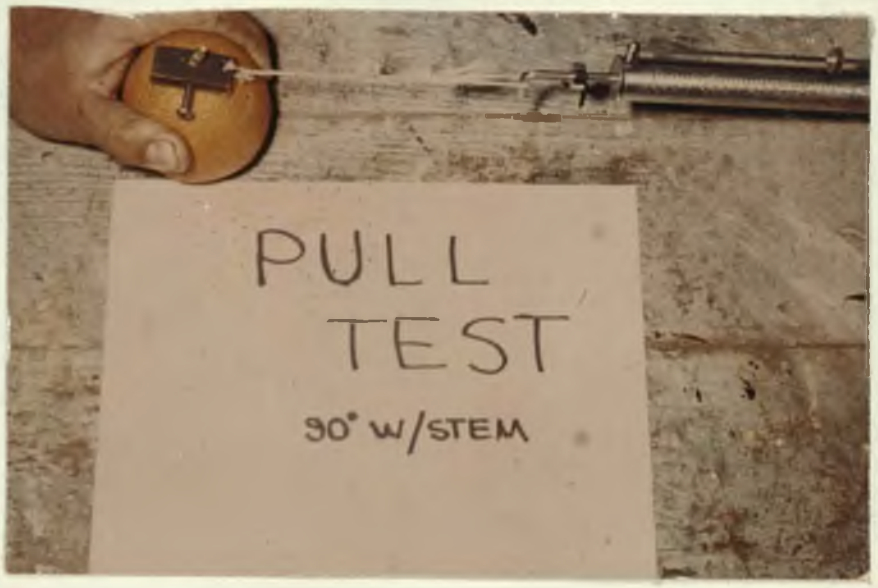

FIGURE 6: NINETY DEGREE PULL TEST

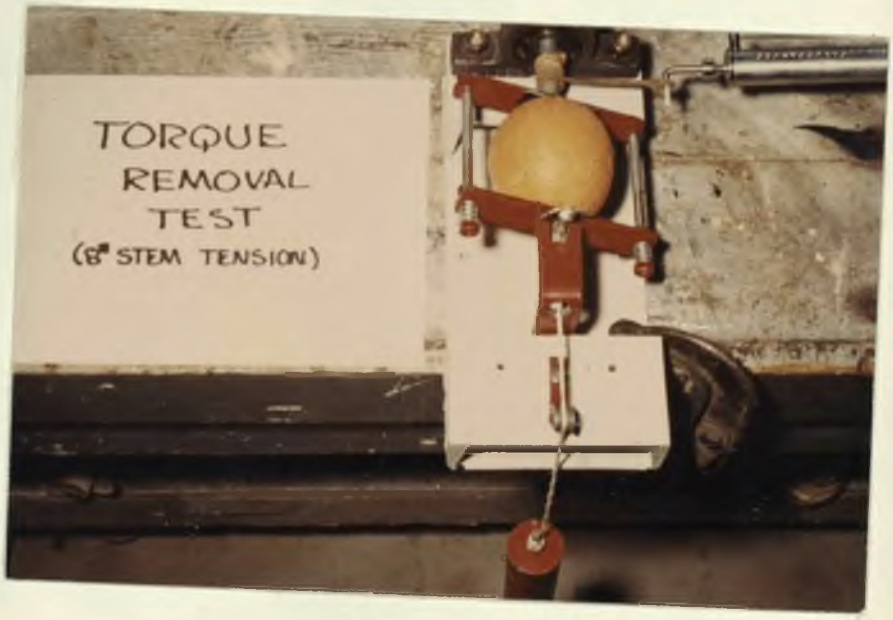

FIGURE 7: TORSIONAL REMOVAL DEVICE

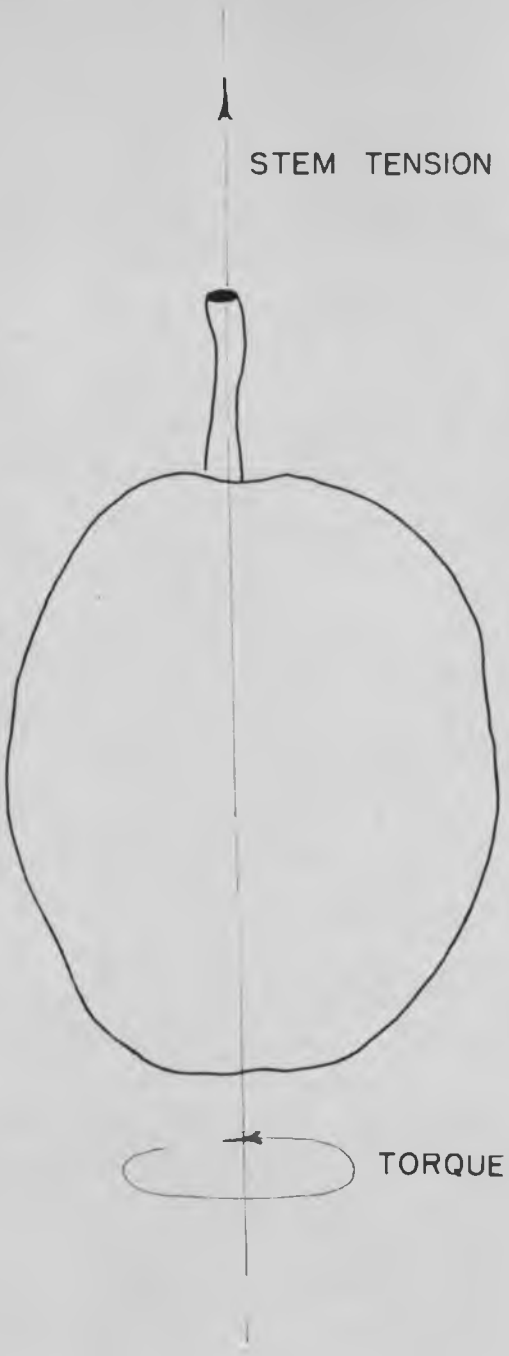

FIGURE 8: TORSIONAL REMOVAL ACTIONS

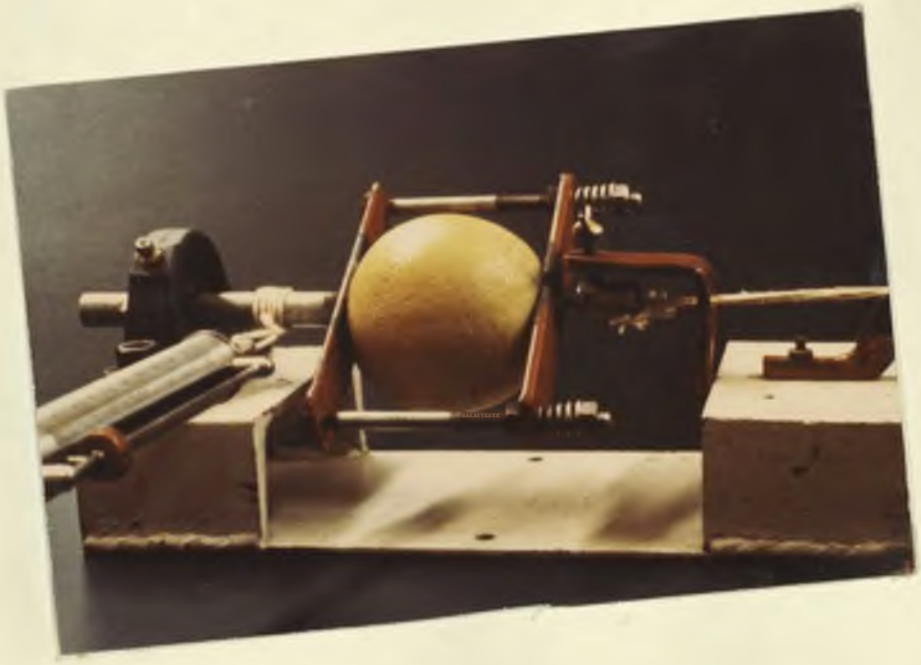

FIGURE 9: TEST SPECIMAN IN TORSIONAL RFMOVAL DEVICE

applied at a uniform rate. Torque application was increased until separation of the stem from the fruit occurred.

Tests were conducted at three different stem tensions. These were zero, eight, and ten pounds. The stem length was not varied, and was set at one-half inch.

VIBRATORY REMOVAL TESTS

A device was constructed which applied a sinusoidal exciting motion normal to the major axis of the fruit. The device is shown in Figure 10. A scotch yoke device supplied theoretically pure sinusoidal motion. The scotch yoke, Figure 11, was driven by a variable speed electric motor. The exciting motion could be varied continuously between zero and 720 rpm. The scotch yoke was constructed such that it could be set at either a one or two-inch stroke.

A screw down clamp was used to attach the fruit and its stem to the shaker arm. The stem was held between a metal and rubber surface. The rubber padding minimized injury to the stem as it was being clamped for the test.

Tests of fruit removal were made at different settings of the shaking device. Variables which could be changed on the shaker were frequency and stroke. Variables concerning the fruit were maturity and stem length.

A particular shaking test consisted in setting the frequency, stroke, and stem length at pre-determined values. The shaker was turned on and the test was timed until the

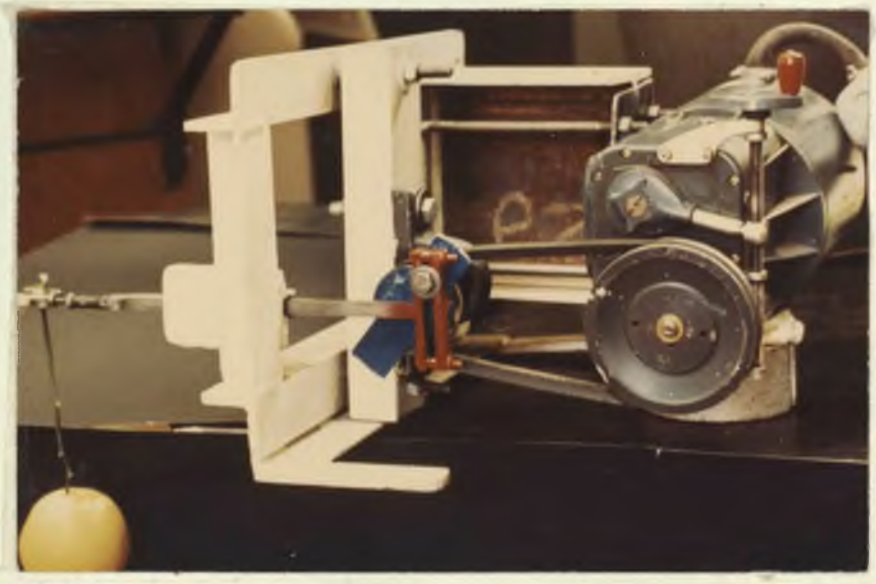

FIGURE 10: VIBRATORY REMOVAL DEVICE

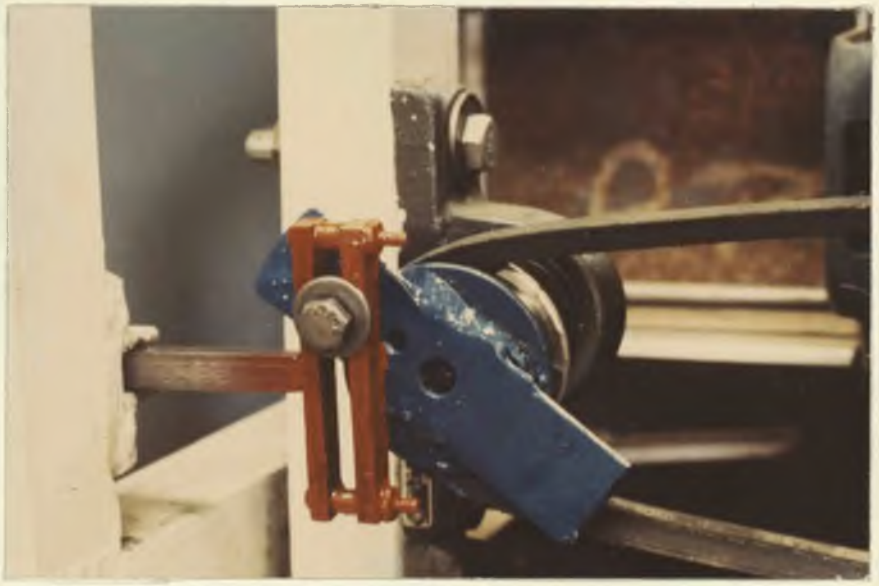

FIGURE 11: SCOTCH YOKE

fruit was shaken from its stem. If fruit removal had not occurred within one minute the fruit was considered not removable at those particular shaking conditions. Experimental design enabled the determination of dependence of fruit removal on frequency, stroke, and maturity.

STATISTICAL DESIGN

Citrus fruit were gathered and tested at the University of Arizona Salt River Valley Citrus Farm in Tempe, Arizona. Fruits of three citrus varieties were all gathered within a one-acre block on the farm. Tests were conducted upon Navel and Valencia oranges and Marsh grapefruit. The Navel, Valencia, and grapefruit trees were planted in 1932.

Samples were randomly clipped from the tree for testing. Each fruit to be tested was clipped with a three to four inch stem attached, and brought into the shop to be tested. All samples were tested within three hours after they had been cut. In all 3625 citrus fruit were picked and tested.

Twenty-five fruit were used as a sample size for the tests. For example, to determine the average zero degree pull force to separate the stem from the fruit, twenty-five fruit were tested. Likewise, twenty-five fruit determined the removal effectiveness of the shaker for a given stroke, frequency, and stem length.

Tests on Navel oranges were begun in mid-December and Valencia oranges and Marsh grapefruit were begun in February.

THEORY

SCOPE

A powerful tool of the engineer is his ability to represent physical systems with mathematical models. Such modeling usually enables him to study a system in greater detail than could be done by running actual tests on the system. In this study the fruit-stem system was modeled. Fundamental mode frequencies of the fruit-stem system were determined. Mathematical modeling was then extended to determine the steady-state forced vibration response of the system to a sinusoidal exciting motion. Results of the analysis yielded an equation which would give, among other things, deflection and stress at the button for the fruit-stem system undergoing a sinusoidal forcing motion.

SYMBOLS	USED IN THEORETICAL ANALYSIS	DIMENSIONS
a	cross sectional area of stem	L^2
a_o	amplitude of scotch yoke	L
E	modulus of elasticity of stem	M/LT^2
I	cross sectional moment of inertia of stem	L^4
k	radius of gyration of cross sectional moment of inertia of stem	L^2
l	length of stem	L
M	moment at button	ML^2/T^2

p	frequency of forcing vibration	$1/T$
p_n	fundamental frequency	$1/T$
R	radius of fruit	L
r	radius of stem	L
ρ	weight of stem per unit volume of stem	M/L^2T^2
t	time	T
U	amplitude of stem at any position x along the stem	L
W	weight of fruit	ML/T^2

NORMAL MODES OF VIBRATION

Normal modes of vibration are special cases of the infinitely many ways in which a system may vibrate. A beam is said to be undergoing a normal mode of oscillation when every portion of the beam is undergoing simple harmonic motion in the same period and phase. Each portion of the beam will have a different amplitude. In other words, period is independent of position along the beam, whereas amplitude is dependent upon position.

Normal mode response of the beam can be represented by the equation.

$$y = U \sin p_n t \qquad \ldots (1)$$

The general equation of motion for transverse oscillations of thin beams is given by,

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = -\frac{\rho a}{g} \frac{\partial^2 y}{\partial t^2}. \qquad \ldots (2)$$

Substitution of equation (1) into (2) enables the determination of the amplitude of the normal mode vibration, U. (See Appendix B for complete derivation of equations in the theory section).

$$U = A\cos mx + B\sin mx + C\cosh mx + D\sinh mx \qquad \ldots (3)$$

where $$m^4 = \frac{\rho P_n^2}{gEI^2}.$$

The four constants A, B, C, and D may be determined by four boundary conditions for the beam, or in this case, the fruit-stem system. The boundary conditions were derived by considering the fruit-stem system as a vertical cantilevered beam with a concentrated weight at one end. The fruit-stem system is shown in Figure 12.

The first boundary condition is that the amplitude of the beam is zero at the clamped end. In mathematical notation,

$$U = 0 \quad \text{AT} \quad X = 0. \qquad \ldots (4)$$

The second boundary condition is that the slope of the stem at the clamp end is zero. This is expressed as,

$$\frac{dU}{dX} = 0 \quad \text{AT} \quad X = 0. \qquad \ldots (5)$$

The third boundary condition is the value of the shear force at the button end of the stem (at x = 1). The shear force is equal to the inertia force developed from the concentrated mass during transverse motion. The shear

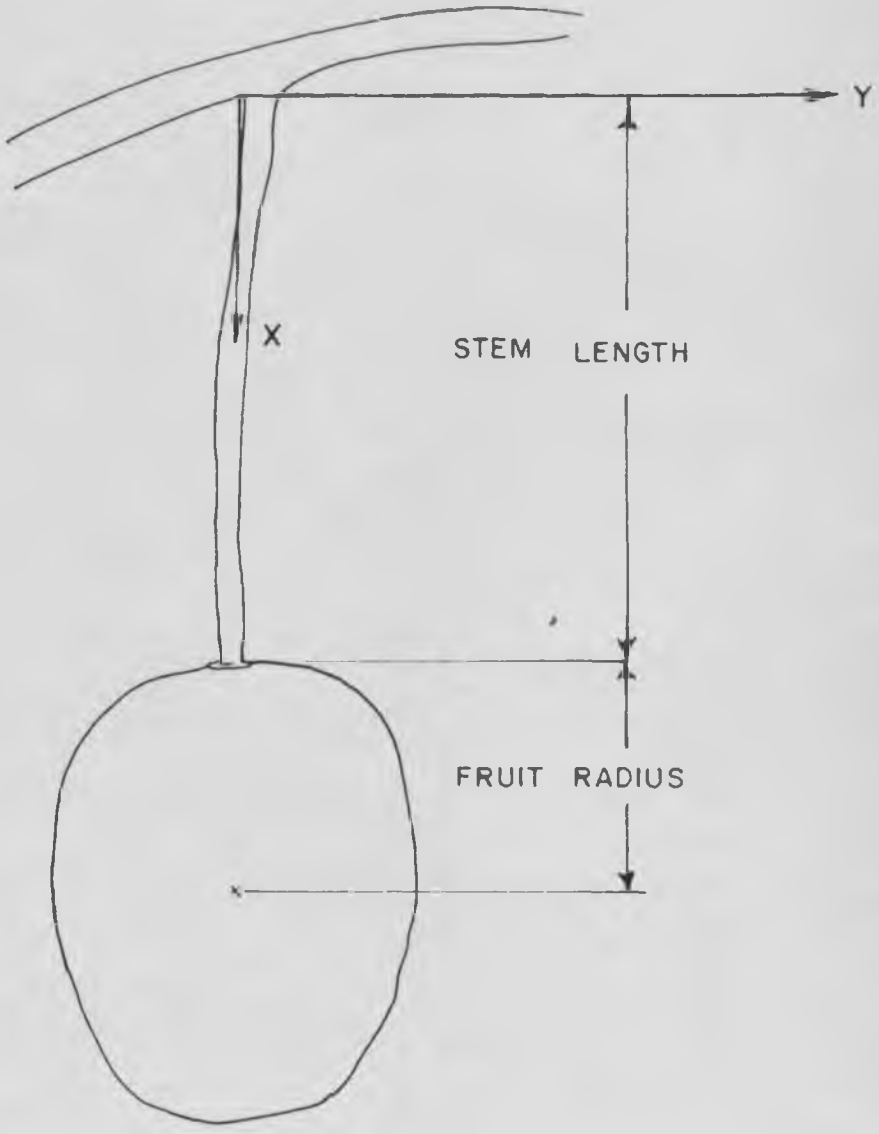

FIGURE 12: THE FRUIT-STEM SYSTEM

forces and bending moments developed from the inertial forces are shown in Figure 13. The shear developed at x = l is a negative shear and is,

$$F_s = -\frac{W}{g}\frac{d^2y}{dt^2} . \qquad \ldots (6)$$

From beam equations, shear is expressable by,

$$F_s = EI\frac{d^3y}{dx^3} .$$

But the co-ordinate system used in Figure 13 is not a right handed system as was used in the derivation of the latter equation. For this particular co-ordinate system, the shear equation becomes,

$$F_s = -EI\frac{d^3y}{dx^3} . \qquad \ldots (7)$$

Equating equations (6) and (7), we have,

$$-\frac{W}{g}\frac{d^2y}{dt^2} = -EI\frac{d^3y}{dx^3} .$$

But,

$$y = U \sin p_w t .$$

Therefore,

$$\frac{W}{g}p_w^2 \sin p_w t = -EI\frac{d^3U}{dx^3}\sin p_w t$$

From which,

$$\frac{d^3U}{dx^3} = -m^4\left(\frac{WU}{pa}\right) \quad \text{AT} \quad x = l . \qquad \ldots (8)$$

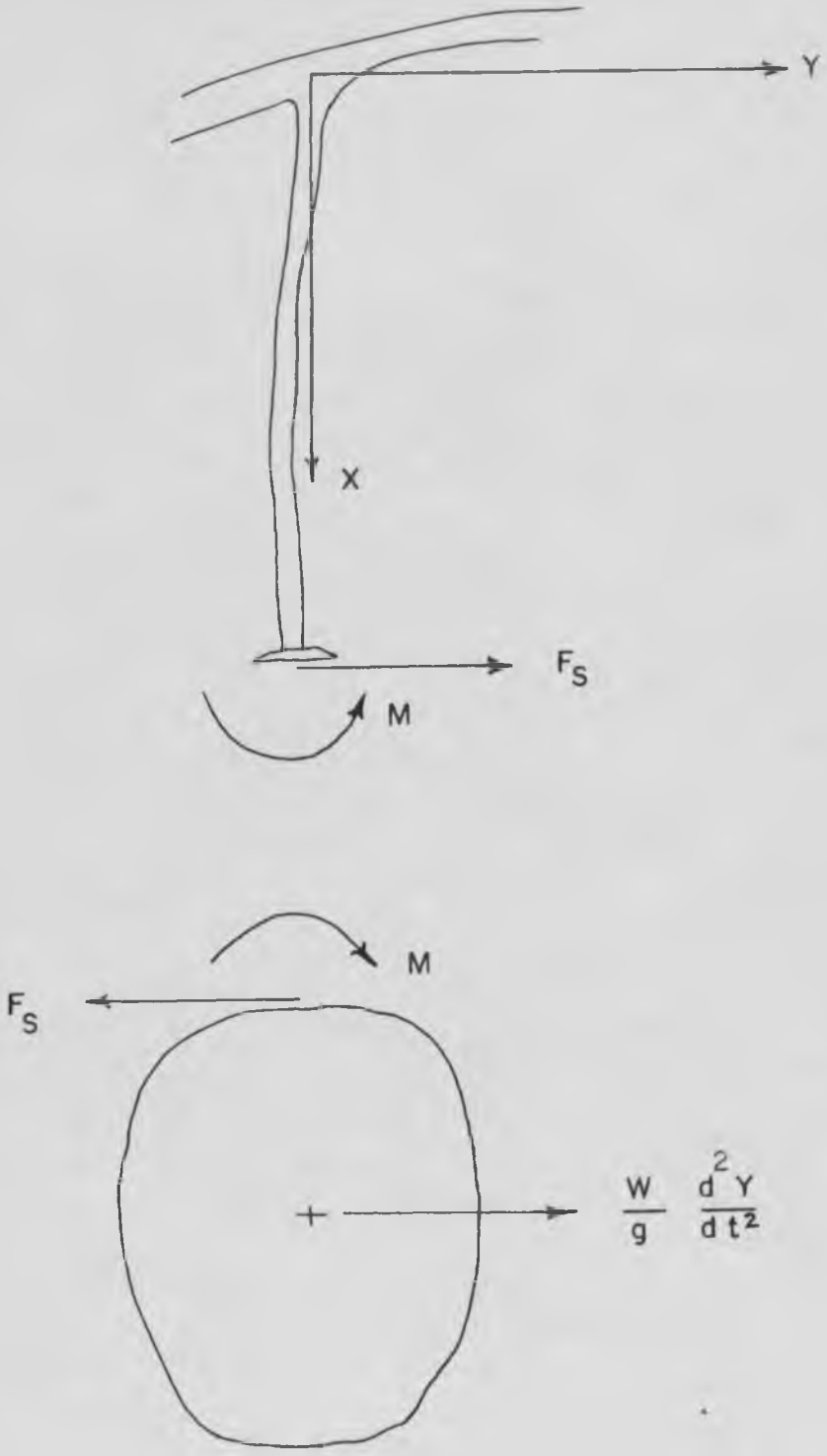

FIGURE 13: REACTIONS TO INERTIAL FORCES AT BUTTON

In several sources of literature reviewed by the author the fourth boundary condition was that the bending moment at x = l was zero. However, because of the large mass and large diameter of the citrus fruit, it was felt that this assumption was not good. Therefore, the fourth boundary was assumed to be the moment arising from the inertial force acting over the radius of the fruit. This condition can be expressed by,

$$M = \frac{W}{g} R \frac{d^2y}{dt^2} \qquad \text{AT } x = l. \qquad \ldots (9)$$

The beam equation for moments is,

$$M = -EI \frac{d^2y}{dx^2}. \qquad \ldots (10)$$

Equating equations (9) and (10) we have,

$$-\frac{W}{g} R P_N^2 U \sin P_N t = -EI \frac{d^2U}{dx^2} \sin P_N t \qquad \text{AT } x = l$$

$$\frac{W}{g} P_N^2 R U = EI \frac{d^2U}{dx^2}$$

and,

$$\frac{d^2U}{dx^2} = \frac{m^4 R W}{\rho a} U \qquad \text{AT } x = l \qquad \ldots (11)$$

where,

$$m^4 = \frac{\rho P_N^2}{g E k^2}. \qquad \ldots (12)$$

The four boundary equations are now expressed in equation 4, 5, 8, and 11. Substitution of the four conditions into equation (3) will eliminate the unknown constants

A, B, C, and D. This done, the resulting equation will be solved for values of m. The frequencies for the fundamental modes of vibration can then be determined from equation (12) which relates m to p_n, the fundamental mode frequency.

Substitution of boundary conditions from equations (4) and (5) into (3) yields,

$$U = A(\cos mx - \cosh mx) + B(\sin mx - \sinh mx).$$

Letting,

$$z = ml$$

$$c = w/\rho a l$$

and using the last two boundary conditions, we eliminate A and B and obtain,

$$[1 + \cosh z \cos z] + cz[\sinh z \cos z - \sin z \cosh z] - \frac{R}{l} cz^2 [\sinh z \sin z] = 0.$$

This becomes,

$$\frac{1 + \cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} = cz + \frac{R}{l} cz^2 \frac{\sin z \sinh z}{\cosh z \sin z - \sinh z \cos z} \quad \ldots \quad (12a)$$

This equation must be solved for z so that the fundamental mode frequencies may be determined. The equation was solved for z by an iterative process utilizing the digital computer. Values of z which were determined are presented in Table 1. Typical values of fruit radius and the weight of fruit to weight of stem ratio were used.

Once z has been determined, the fundamental mode frequencies can be determined in the following manner.

TABLE 1

VALUES OF "Z" TO BE USED IN FUNDAMENTAL FREQUENCY EQUATION

1. FIRST MODE FREQUENCY

Stem length (in.)	Fruit radius (in.)	C*	Z
2.0	1.5	406.0	0.24285
3.0	1.5	271.0	0.28205
4.0	1.5	203.0	0.31185
6.0	1.5	135.0	0.35645

2. SECOND MODE FREQUENCY

Stem length (in.)	Fruit radius (in.)	C*	Z
2.0	1.5	406.0	3.925
3.0	1.5	271.0	3.925
4.0	1.5	203.0	3.925
6.0	1.5	135.0	3.925

* Weight of fruit = 300 gms.

Weight of stem/in. = 0.37 gm/in.

From equation (12),

$$P_w = \left(\frac{gEk^2}{\rho}\right)^{1/2} m^2.$$

But,

$$m^2 = z^2/l^2.$$

Therefore,

$$P_w = \frac{z^2}{l^2}\left(\frac{gEk^2}{\rho}\right)^{1/2}. \qquad \ldots (13)$$

This equation shows that the normal mode frequencies are a function of six variables. Of these six, three are assumed constant for the fruit-stem system. These are g, the acceleration due to gravity, ρ, the density of the stem, and E, the modulus of elasticity of the stem material. Hence, we may express the above equation as,

$$P_w = K\frac{z^2}{l^2}k$$

where,

$$K = \left(\frac{gE}{\rho}\right)^{1/2} = \text{constant.}$$

Thus we see that the fundamental frequencies are inversely proportional to the square of the stem length and directly proportional to the radius of gyration of the stem. The z^2 term is influenced by the weight of the stem, weight of the fruit, diameter of the fruit, and length of the stem. It was assumed that there was little variation in the diameter

of stems among the three citrus varieties. Therefore, the k^2 term was combined with K and the resulting fundamental frequency equation becomes,

$$P_N = K_1 z^2/\ell^2$$

where,

$$K_1 = \left(\frac{gEk^2}{\rho}\right)^{1/2}.$$

The constant K_1 cannot be evaluated because the modulus of elasticity of the stem was not known. Therefore, the fundamental frequency curves can be plotted from the equation,

$$P_N/K_1 = z^2/\ell^2. \qquad \ldots (14)$$

A plot of P_N/K_1 vs. 1 for the first fundamental mode is shown in Figure 14. The natural frequency of the system is highly dependent upon stem lengths for lengths under approximately four inches.

In its final form, the equation for fundamental mode frequencies has been determined as,

$$P_N/K_1 = z^2/\ell^2.$$

Considering this equation relative to any given fruit-stem system let us discuss its inferences. The fundamental frequencies are a function of the mass of the stem, mass of the fruit, length of the stem, density of the stem, radius of gyration of the stem, and modulus of elasticity of the

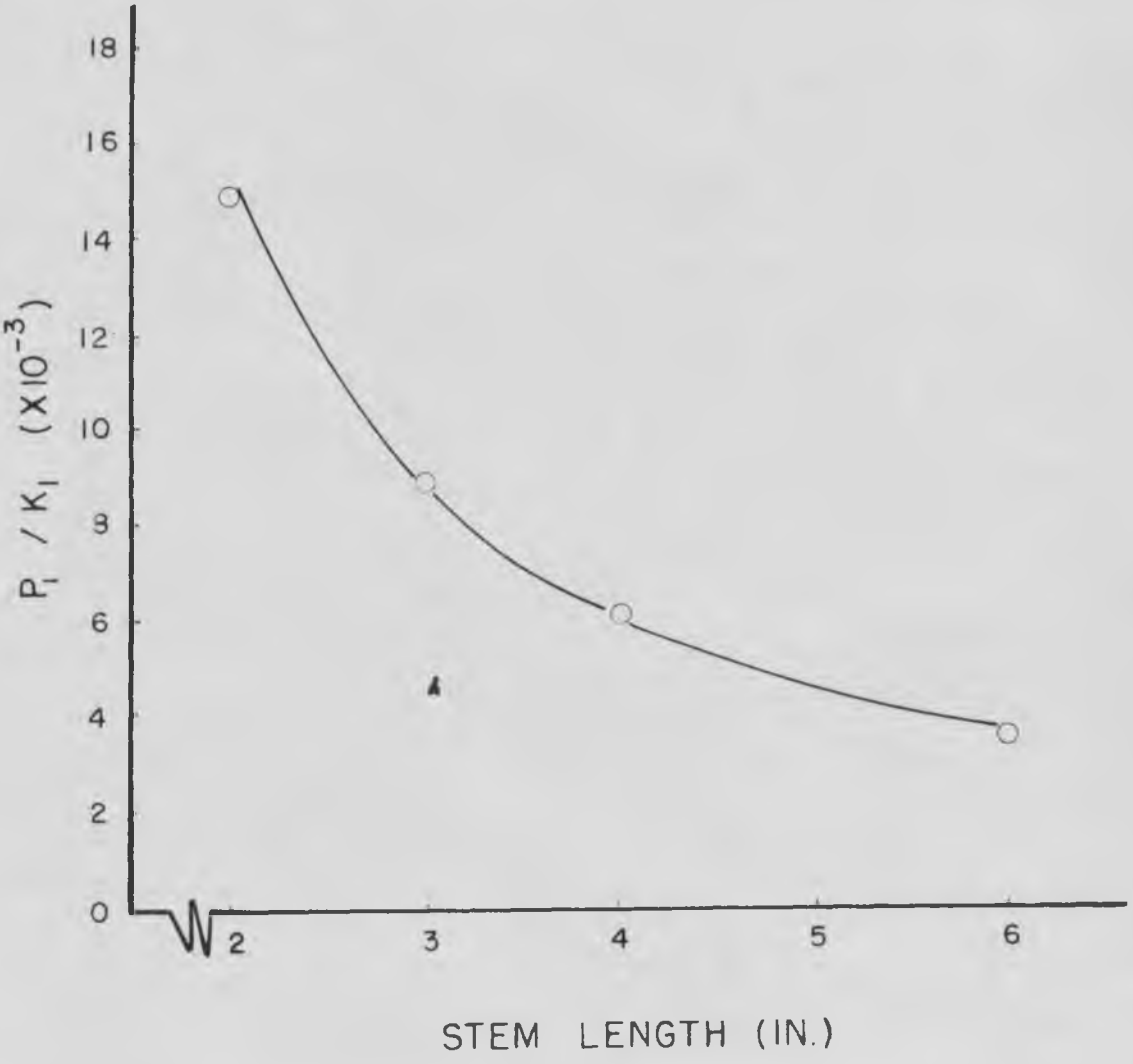

FIGURE 14: FIRST FUNDAMENTAL MODE FREQUENCY VS. STEM LENGTH

stem. One would expect a wide variation of these physical properties of the system within a given tree. It is impossible to say, without field data, which of the variables change more than others and hence determine the span of fundamental frequencies one would find in a tree.

In this analysis it has been assumed that the modulus of elasticity and radius of gyration are constant along the length of the stem. In fact, all variables contained in the constant K_1 are assumed to be independent of stem length. This assumption is valid when we view the stem as a uniform, homogeneous rod. In reality, however, the stem may change in section along its length. This means that the radius of gyration would not be constant with stem length. Similarly, the stem may not be homogeneous along its length. It might be expected that differences in stem composition could be found above and below nodes which are commonly found on the stem. Non-homogeneity means that the modulus of elasticity would vary along the length of the stem.

The analysis of a non-uniform, non-homogeneous fruit-stem system would be much more difficult. Because of uniformity and homogeneity, equation (2) could be written and solved as,

$$EI \frac{\partial^4 y}{\partial x^4} = -\frac{\rho a}{g} \frac{\partial^2 y}{\partial t^2}.$$

Equation (2) can only be written in this manner when the modulus of elasticity and moment of inertia are constant along the length of the stem. These assumptions were made and the analysis presented is only as good as the fruit-stem conforms to them.

STEADY-STATE FORCED VIBRATION ANALYSIS

The differential equation of motion for steady-state forced vibration is,

$$EI\frac{\partial^4 y}{\partial x^4} + \frac{\rho a}{g}\frac{\partial^2 y}{\partial t^2} = \left(\frac{\rho a}{g}\right) A_o p^2 \sin pt . \qquad \ldots (15)$$

The forcing vibration is applied at the fixed end (at $x = 0$ in Figure 12) of the fruit-stem system. The right-hand side of equation (15) is the additional load per unit length imposed upon the system. This load arises from the fact that the whole system is being accelerated by the external vibration. Hence the right-hand side is an inertial force.

Assuming the deflection of the system relative to the clamp as,

$$y = U \sin pt ,$$

equation (15) becomes,

$$EI\frac{d^4 U}{dx^4} - \left(\frac{\rho a}{g}\right) U p^2 = \left(\frac{\rho a}{g}\right) A_o p^2 . \qquad \ldots (16)$$

The particular solution to this differential equation is chosen as,

$$U_p = -a_0$$

The homogeneous solution is derived in a manner similar to the free vibration solution. Therefore the total solution is,

$$U = A\cosh mx + B\sinh mx + C\cos mx + D\sin mx - a_0 \quad \ldots (17)$$

The boundary conditions are the same as determined in the free vibration analysis.

$$U = \frac{dU}{dx} = 0 \qquad \text{AT } x = 0$$

$$\frac{d^2U}{dx^2} = \frac{m^4 R W}{\rho a} U \qquad \text{AT } x = l$$

$$\frac{d^3U}{dx^3} = -\frac{m^4 W}{\rho a} U \qquad \text{AT } x = l$$

where,

$$m^4 = \rho P_w^2 / g E k^2.$$

Using the first two boundary conditions we obtain,

$$U = A(\cosh mx - \cos mx) + B(\sinh mx - \sin mx) + a_0(\cos mx - 1).$$

Using the latter two boundary conditions and after considerable substitution and simplification, the total solution is,

$$U = \frac{A_1}{A_2 A_3}(\cosh mx - \cos mx) + \frac{B_1}{B_2 B_3}(\sinh mx - \sin mx) + a_0(\cos mx - 1) \quad \ldots (18)$$

where,

$$A_1 = a_0[1 + \cos z \cosh z + \sin z \sinh z] + a_0 C z [2\cos z \sinh z - \sin z - \sinh z] + a_0 C z^2 (R/l)[1 + \cos z \cosh z - \sin z \sinh z - \cos z - \cosh z]$$

$$A_2 = [(\cos z + \cosh z) - (R/l) C z^2 (\cosh z - \cos z)]$$

$$A_3 = \left[\frac{1 + \cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} - Cz - \left(\frac{R}{l}\right) C z^2 \frac{\sin z \sinh z}{\cosh z \sin z - \sinh z \cos z}\right]$$

$$B_1 = -a_0[\cosh z \sin z + \cos z \sinh z] + a_0 cz[-2\cosh z \cos z + \cos z + \cosh z] + a_0 (R/l) cz^2 [\sin z \cosh z - \cos z \sinh z + \sinh z - \sin z]$$

$$B_2 = [\cos z + \cosh z - (R/l) cz^2 (\cosh z - \cos z)]$$

$$B_3 = \left[\frac{1 + \cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} - cz - \left(\frac{R}{l}\right) cz^2 \frac{\sinh z \sin z}{\cosh z \sin z - \sinh z \cos z}\right].$$

Note that A_3 and B_3 are the composite equation for the fundamental frequencies of free vibration (see equation (12a). At the fundamental frequencies the right and left-hand sides of equation (13) are equal. Since the right-hand side is subtracted from the left-hand side in A_3 and B_3, A_3 and B_3 are zero at the fundamental frequencies. But A_3 and B_3 appear in the denominator of the forced vibration solution. Therefore, the amplitude U, of the forced vibration becomes infinite when the frequency of the forcing function becomes equal to one of the fundamental mode frequencies.

Equation (18) gives the amplitude of the fruit-stem system at steady-state conditions for a sinusoidal forcing function. A steady-state condition is achieved in a very short time after the vibration of the system is commenced. During this short time period a transient solution would have to be added to equation (18) to represent the total solution to the fruit-stem system. But as mentioned, the transient affect is short-lived. For this reason it was decided to deal with the steady-state response of the system.

Figure 15 shows the amplitude of the stem end (at the button) plotted against the ratio of the forcing frequency to the first fundamental mode frequency. Points were determined from evaluation of equation (18). The analysis was done for a 6-inch stem length, a weight of fruit of 300 grams, a fruit diameter of 3 inches, and a stem diameter of 1/8 inch. The weight and stem diameter correspond to typical Marsh grapefruit. It was not possible to express the ordinate of the graph in frequencies because properties of the fruit-stem system were not known (namely, the modulus of elasticity). Figure 15 shows the amplitude of the stem end to be a minimum at a forcing frequency of 3.5 times the natural frequency of the system. The amplitude increases as the forcing frequency is increased. Calculations not plotted on this graph show that the amplitude will level off, decrease, and finally become zero at the second fundamental mode frequency. Hence, at the second mode frequency the button end of the stem will remain stationary, i.e., it is a node position.

The maximum bending stress across the button is plotted versus the frequency ratio in Figure 16. The maximum bending stress can be derived from equations (9), (13), and by using,

$$\text{MAXIMUM BENDING STRESS} = \frac{M_{MAX}(J)}{I}$$

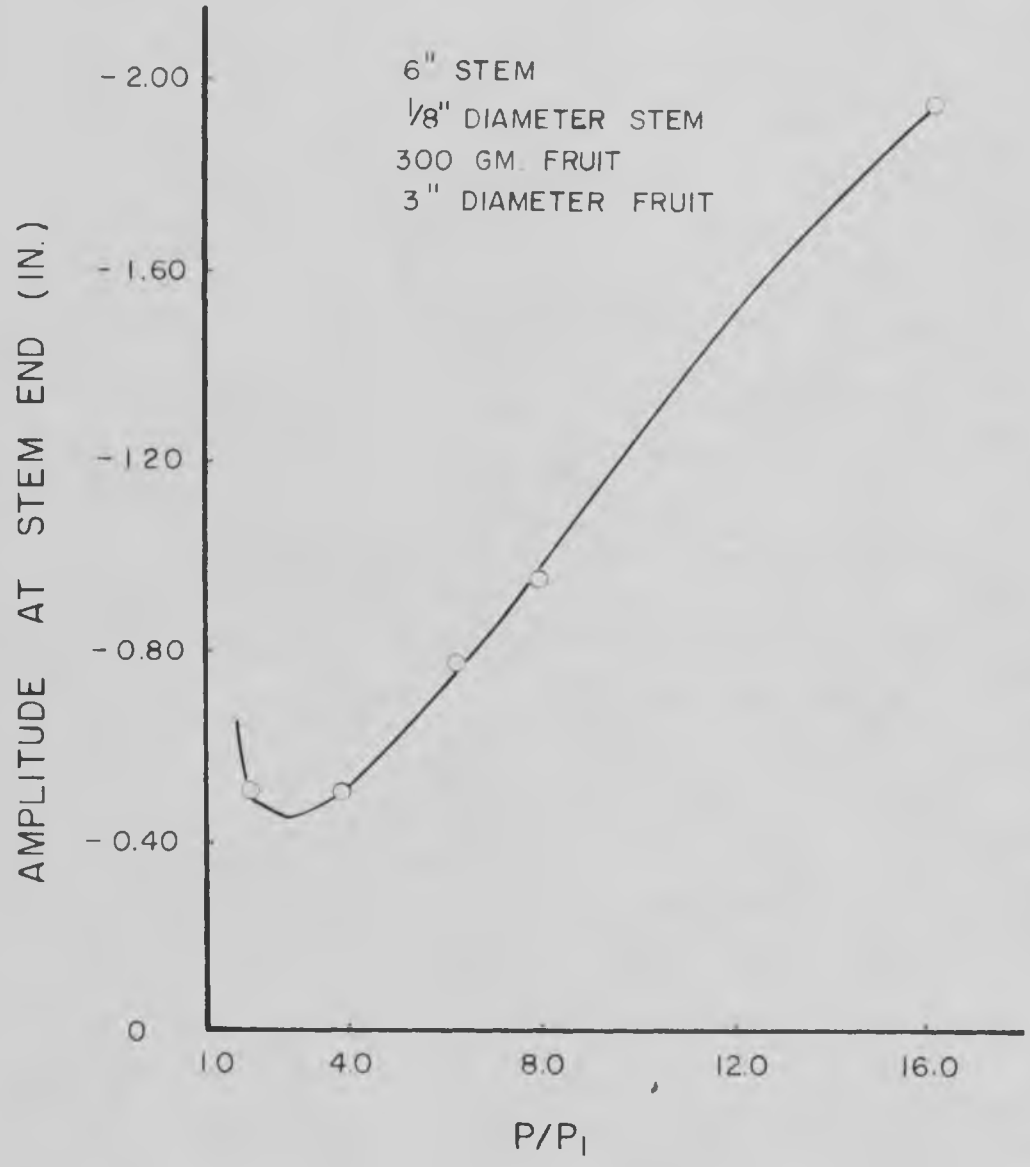

FIGURE 15: AMPLITUDE OF STEM END UNDER STEADY-STATE FORCED VIBRATION VS. RATIO OF FREQUENCY OF FORCING FUNCTION TO FIRST FUNDAMENTAL MODE FREQUENCY OF SYSTEM

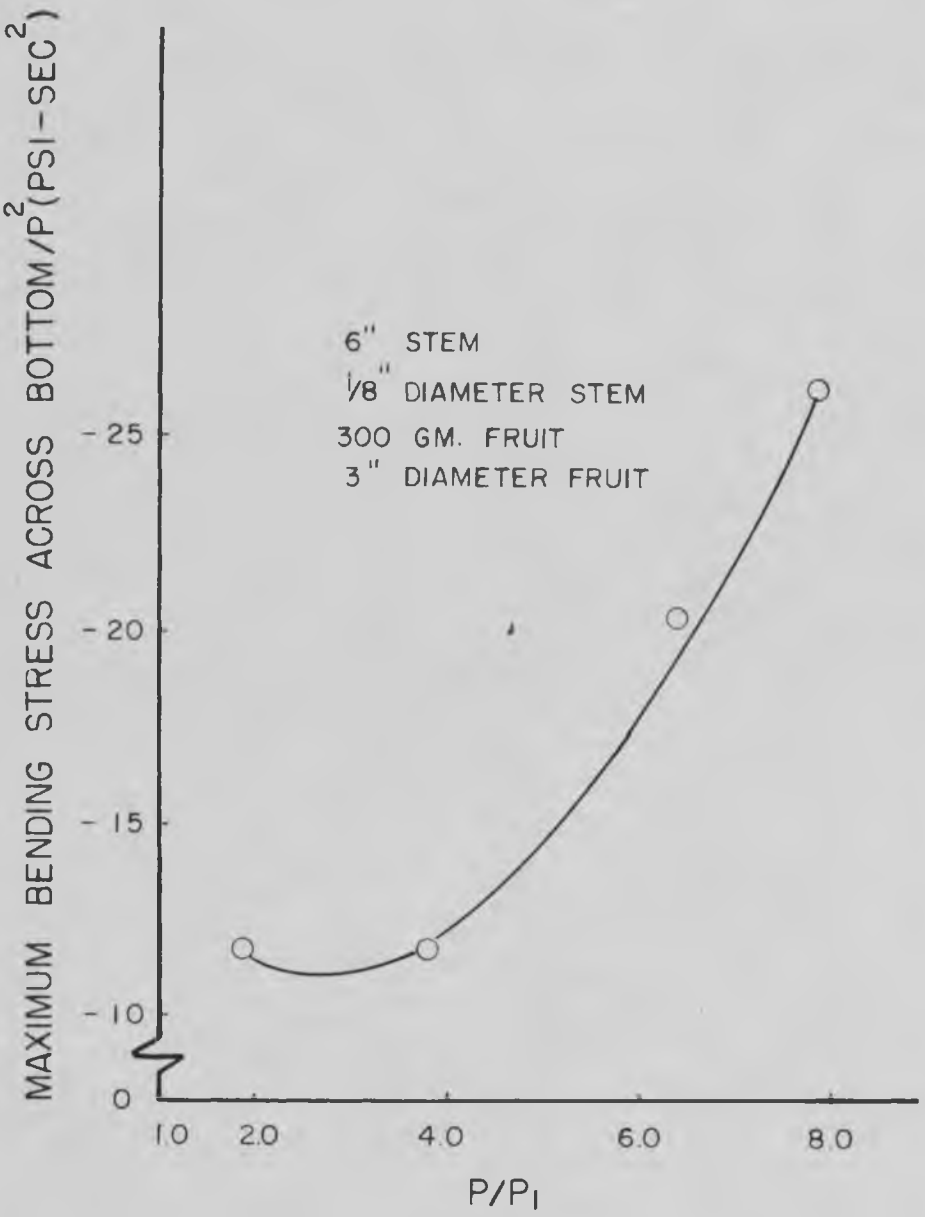

FIGURE 16: BENDING STRESS ACROSS BUTTON VS. RATIO OF FREQUENCY OF FORCING FUNCTION TO FIRST FUNDAMENTAL MODE FREQUENCY OF SYSTEM

In its final form, the maximum bending moment is,

$$M_{MAX} = (26.9)|U|\rho^2.$$

Therefore, at a given forcing frequency, the maximum bending moment is proportional to forcing frequency and to the amplitude of the stem at the point where the moment is to be calculated. Again, the ordinate of the plot in Figure 16 would have to be multiplied by the first fundamental mode frequency to give absolute values along the ordinate. Likewise, the abscissa would have to be multiplied by the square of the forcing frequency to give absolute values of the maximum bending stress. As in the deflection plot, the maximum bending stress will begin to decrease at speeds greater than those plotted and will become zero at the second mode frequency because the amplitude at this speed is zero.

Theoretical analysis has shown that properties of the fruit-stem system, namely the modulus of elasticity of the stem, will have to be determined before numerical values for deflection, bending stress, or shearing stress can be calculated. More such physical data concerning the fruit-stem system would open the door to a more complete study of the vibrational response of the fruit-stem system. Other empirical studies such as determination of first fundamental mode frequencies should be done so that theoretical calculations and field data can be compared for agreement. This would be necessary as a check on the validity of the mathematical model of the fruit-stem system.

RESULTS AND DISCUSSION

PULL TESTS

MEAN ZERO DEGREE REMOVAL FORCE VS. TIME OF SEASON

Tables 2 through 4 show the analysis of the means of zero degree pull tests of the three citrus varieties through the test season. Raw data with means and standard deviations of these and other tests are given in Appendix A. In Table 2, Section 1, means from the Valencia zero degree pull tests are arranged in descending magnitude. Section 2 of Table 2 shows the results of computations for Duncan's New Multiple Range Test that were performed on the data. Duncan's test is a method of making all possible comparisons among the means. The results of all possible comparisons are shown in Section 3 of Table 2. Means which are underscored are not significantly different from one another at the one per cent level. There is some question as to whether mean number 2 is with mean 1 or is with mean 3. But the magnitude of the difference between 2 and 1 is less than the difference between 2 and 3. Hence mean 2 was considered to be significantly different from mean 3 and not significantly different from mean 1. Thus data shows that no significant differences exist in mean zero degree

TABLE 2
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
VALENCIA ORANGE MEAN ZERO DEGREE REMOVAL FORCE VS. TIME OF SEASON

1. RANKING OF MEANS OF ZERO DEGREE PULL TEST

	Date	Mean of zero degree pull test (lbs.)
1.	March 17	18.95
2.	February 24	18.34
3.	April 23	16.60
4.	May 14	16.11
5.	June 7	15.61
6.	June 27	15.58

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
1-6	3.37	2.36	Significant
1-5	3.34	2.33	Significant
1-4	2.84	2.28	Significant
1-3	2.35	2.22	Significant
1-2	0.61	2.13	Not significant
2-6	2.76	2.33	Significant
2-5	2.73	2.28	Significant
2-4	2.23	2.22	Significant
2-3	1.74	2.13	Not significant
3-6	1.02	2.28	Not significant
3-5	0.99	2.22	Not significant
3-4	0.48	2.13	Not significant
4-6	0.53	2.22	Not significant
4-5	0.50	2.13	Not significant
5-6	0.03	2.13	Not significant

3. RESULTS

March	February	April	May	June	June
18.95	18.34	16.60	16.11	15.61	15.58

TABLE 3
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
MARSH GRAPEFRUIT MEAN ZERO DEGREE REMOVAL FORCE
VS. TIME OF SEASON

1. RANKING OF MEANS OF ZERO DEGREE PULL TEST

	Date	Mean of zero degree pull test (lbs.)
1.	February 24	15.78
2.	June 7	14.11
3.	February 2	13.80
4.	April 23	13.60
5.	May 14	13.14
6.	March 17	13.07
7.	June 27	11.84

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
1-7	3.94	2.13	Significant
1-6	2.71	2.10	Significant
1-5	2.64	2.06	Significant
1-4	2.18	2.04	Significant
1-3	1.98	1.97	Significant
1-2	1.67	1.89	Not significant
2-7	2.27	2.10	Significant
2-6	1.04	2.06	Not significant
2-5	0.97	2.04	Not significant
2-4	0.51	1.97	Not significant
2-3	0.31	1.89	Not significant
3-7	1.96	2.06	Not significant
3-6	0.73	2.04	Not significant
3-5	0.66	1.97	Not significant
3-4	0.20	1.89	Not significant
4-7	1.76	2.04	Not significant
4-6	0.53	1.97	Not significant
4-5	0.46	1.89	Not significant
5-7	1.30	1.97	Not significant
5-6	0.07	1.89	Not significant
6-7	1.23	1.89	Not significant

3. RESULTS

February	June	February	April	May	March	June
15.78	14.11	13.80	13.60	13.14	13.07	11.84

TABLE 4
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
NAVEL ORANGE MEAN ZERO DEGREE REMOVAL FORCE
VS. TIME OF SEASON

1. RANKING OF MEANS OF ZERO DEGREE PULL TEST

Date	Mean of zero degree pull test (lbs.)
1. December 21	21.09
2. December 29	20.37
3. January 31	18.27
4. February 24	12.71

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means Compared	Difference	SSR	Conclusion
1-4	3.38	3.12	Significant
1-3	2.82	3.02	Not significant
1-2	0.72	2.91	Not significant
2-4	7.66	3.02	Significant
2-3	2.10	2.91	Not significant
3-4	5.56	2.91	Significant

3. RESULTS

December	December	January	February
21.09	20.37	18.27	12.71

pull forces determined in February and March. Similarly, no significant differences occurred in April, May, and June. But there was a significant difference in data taken in March and April. Hence the data can be separated into two significantly different groups. The first group includes means from February and March, and the second group includes means from April, May and June.

The means of zero degree pull forces for Valencia orange data are plotted in Figure 17. The standard deviation for each mean is given near the plotted mean. A linear regression analysis of the data gave a value of R^2 to be 0.154. This low value indicates that a linear relation does not exist between time and pull force. The equation of the curve in Figure 17 was supplied by a computer program designed to give a quartic equation that best fits the given data. The derived equation is shown on the plot.

Table 3 shows the ranking of means of zero-degree pull tests which were performed on Marsh grapefruit. Duncan's New Multiple Range Test was performed on the data and the computations are shown in Section 2 of Table 3. Results of the test are shown in Section 3. Results show means can be grouped. The positions of means 2 and 7 are not clearly determined. It appears that mean 2 is more closely related to mean 3 than it is to mean 1. Also,

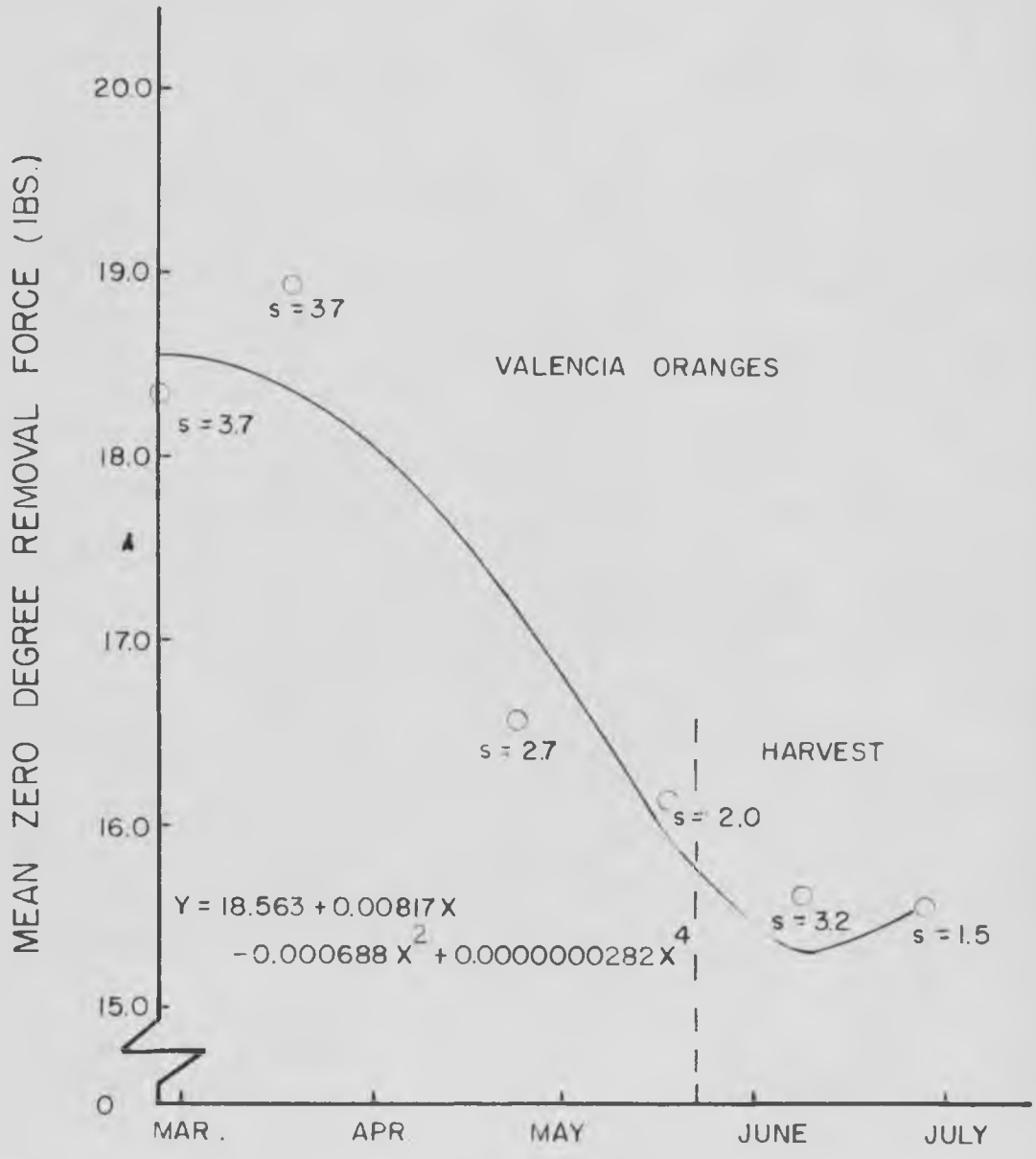

FIGURE 17: VALENCIA ORANGE MEAN ZERO DEGREE REMOVAL FORCE VS. TIME OF SEASON

mean 6 appears to be more closely related to mean 5 than it is to mean 7. Hence the means can be arranged into three groups. These are 1, 2 through 6, and 7. That is to say, means determined in late February and late June stand alone, significantly different from the rest of the means.

The zero degree pull force means for Marsh grapefruit are plotted in Figure 18. The R^2 value calculated was 0.054. Maximum and minimum removal forces occurred in mid-February and late June, respectively. The fruit was harvested on June 8 when the removal force was 14.1 pounds. The maximum mean removal force was 15.8 pounds and the minimum mean removal force was 11.8 pounds.

Means of the zero degree pull tests for Navel oranges are ranked in Section 1 of Table 4. Computations for Duncan's New Multiple Range Test are shown in Section 2. Results of the test are presented in Section 3 of Table 4. The test shows that no significant differences exist between data on December 21, 29, and January 31. There was significant difference between the mean determined on February 24 and the previous three means.

Figure 19 shows the plot of zero degree pull force means for Navel oranges. The R^2 value was 0.382. The tests were initiated in the latter part of December, only several weeks before harvest. Removal forces were high during the

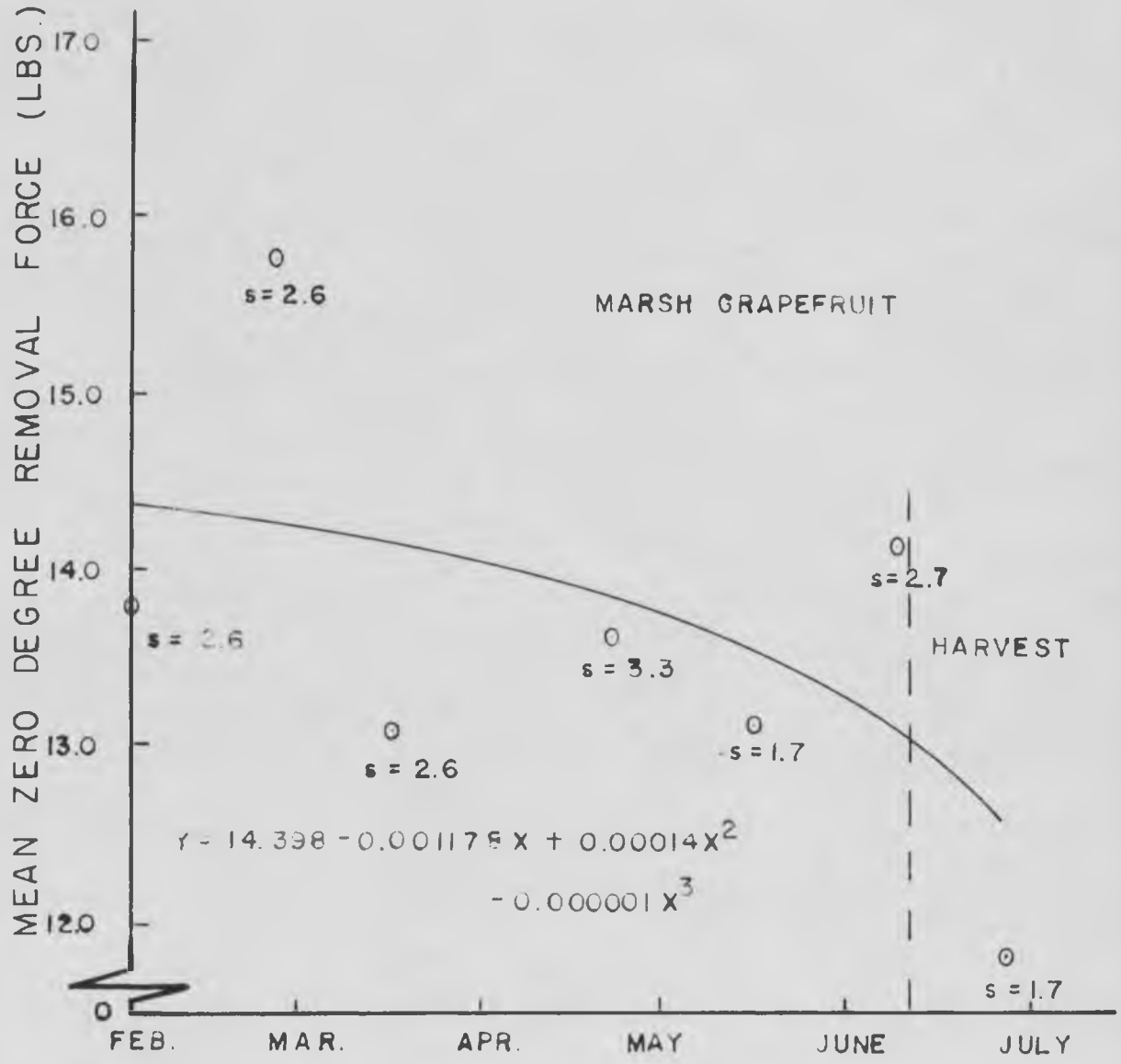

FIGURE 18: MARSH GRAPEFRUIT MEAN ZERO DEGREE REMOVAL FORCE VS. TIME OF SEASON

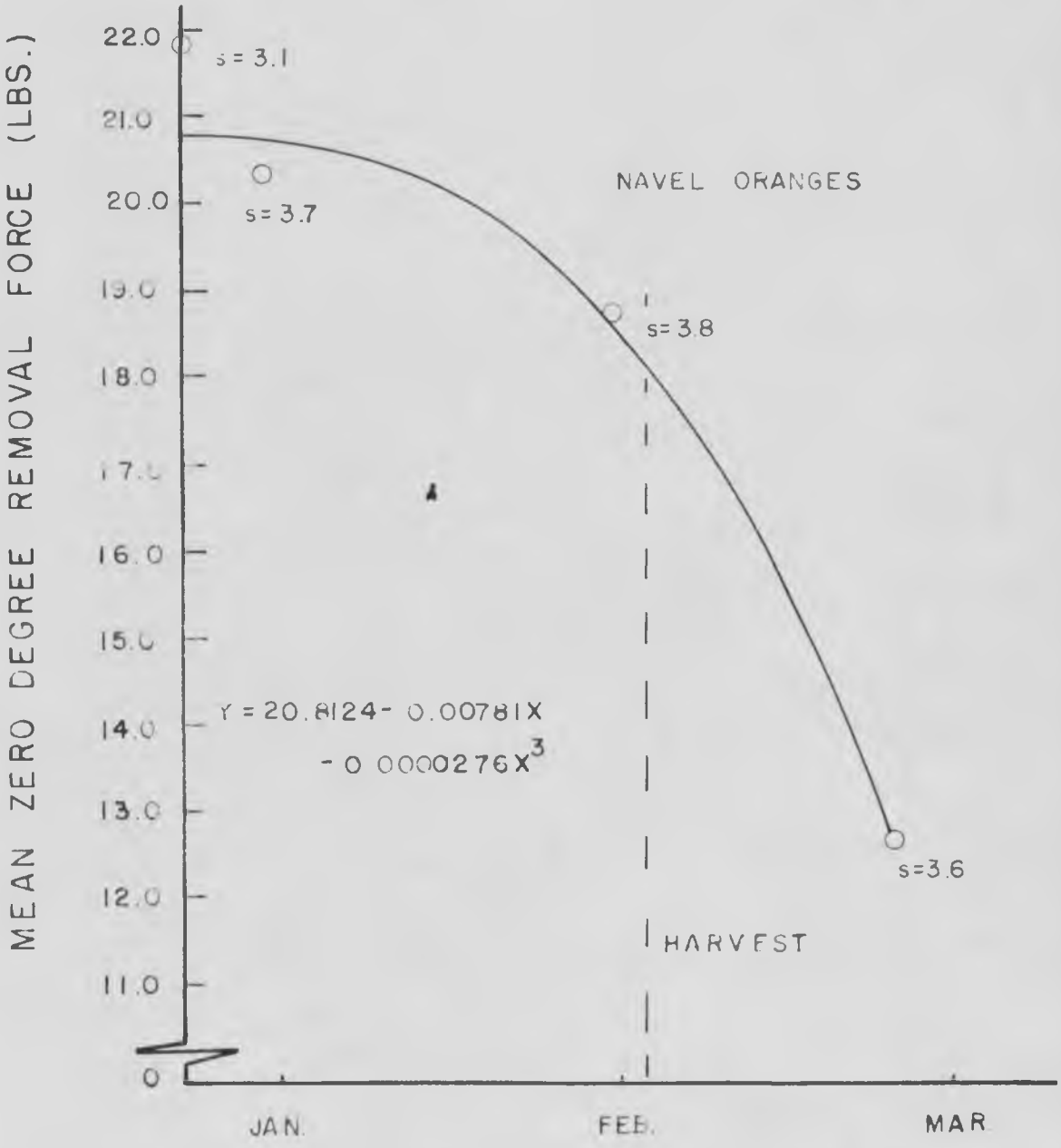

FIGURE 19: NAVEL ORANGE MEAN ZERO DEGREE REMOVAL FORCE VS. TIME OF SEASON

harvest period. Navel oranges from the test block were harvested on February 2. Interpolation from the curve shows the removal force to be 18.2 pounds at harvest. Tests were continued after the harvest season. Data show a very rapid decrease in removal force occurring between the end of January and the end of February.

It is interesting to compare the Navel and Valencia pull force data relative to their harvest dates. The most obvious difference is that the Navel data are near their peak at harvest whereas the Valencia removal forces are near their minima at harvest. The data show opposite trends if compared relative to the harvest period. Approaching the harvest period, the Valencia orange removal force is decreasing rapidly. Conversely, during this same relative time period, the Navels are slowly decreasing. At harvest, the Navel require near 18 pounds to be removed whereas the Valencias require 15.5 pounds. After harvest, the pull force data for Navels decreases rapidly. This is consistent with the fact that if not harvested, Navel oranges will drop of their own accord. The downward trend of the data is evidence that drastic changes in the strength of the fruit-stem union enables the fruit to fall easily. The leveling off of the Valencia data after harvest is also consistent with observed phenomena. It is well known that if Valencia fruit are not harvested, they will remain on the tree until the next harvest season. In other words, the strength

of the fruit-stem union is maintained. The Valencia data show this to be true. Through June, the strength of the union was leveling off near 15 pounds.

MEAN ZERO DEGREE REMOVAL FORCE VS. ANGLE OF PULL

Results of the 0, 45, and 90 degree pull tests are summarized in Section 1 of Table 5 through 7. Duncan's New Multiple Range Test was used to compare between the three types of pull tests. In all instances, there were significant differences between the means of 0 and 90, and 0 and 45 degree pull tests. Approximately half of the tests showed no significant differences between 45 and 90 degree pull tests. The results of Duncan's New Multiple Range Test are summarized in Section 2 of Tables 5 through 7.

Typical pull force vs. angle of pull data are plotted in Figures 20 and 21. Note the near 50 per cent reduction in removal force when attachment is made at a 45 degree angle with respect to the stem. It should be noted that attachment to the stem for the 45 degree pull was made at one-half inch above the button. Therefore, in addition to tension forces, bending and shearing forces were induced in the stem during the pull which served to reduce the force to cause fruit separation. The important fact gained from the angular pull tests is that the magnitude of the removal force is substantially reduced when the force is directed at an angle to the stem axis of the fruit.

TABLE 5
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
VALENCIA ORANGE MEAN REMOVAL FORCE VS. ANGLE OF PULL

1. RANKING OF MEANS (MEANS IN POUNDS)

Date	0 Pull	90 Pull	45 Pull
1. February 24	18.34	11.96	10.46
2. March 17	18.95	13.05	9.91
3. April 23	16.60	12.25	8.97

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means Compared	Difference	SSR	Conclusion
February 24			
0-45	7.88	2.29	Significant
0-90	6.38	2.19	Significant
90-45	1.50	2.19	Not significant
March 17			
0-45	9.04	2.36	Significant
0-90	5.90	2.26	Significant
90-45	3.14	2.26	Significant
April 23			
0-45	7.63	1.96	Significant
0-90	5.90	1.88	Significant
90-45	3.14	1.88	Significant

TABLE 6
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
GRAPEFRUIT MEAN REMOVAL FORCE VS. ANGLE OF PULL

1. RANKING OF MEANS (MEANS IN POUNDS)

Date	0 Pull	90 Pull	45 Pull
1. February 24	15.78	7.54	6.88
2. March 17	13.07	8.49	7.96
3. April 23	13.60	7.77	5.88

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means Compared	Difference	SSR	Conclusion
February 24			
0-45	8.90	1.80	Significant
0-90	8.24	1.73	Significant
90-45	0.66	1.73	Not Significant
March 17			
0-45	5.11	1.66	Significant
0-90	4.58	1.59	Significant
90-45	0.53	1.59	Not Significant
April 23			
0-45	7.72	1.78	Significant
0-90	5.83	1.70	Significant
90-45	1.89	1.70	Significant

TABLE 7
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
NAVEL ORANGE MEAN REMOVAL FORCE
VS. ANGLE OF PULL

1. RANKING OF MEANS (MEANS IN POUNDS)

Date	0 Pull	45 Pull	90 Pull
1. December 21	21.09	15.68	12.84
2. December 29	20.93	12.89	11.44
3. January 31	18.63	8.22	8.64

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
December 21			
0-90	8.25	3.35	Significant
0-45	5.41	3.22	Significant
45-90	2.84	3.22	Significant
December 29			
0-90	9.49	2.77	Significant
0-45	8.04	2.66	Significant
45-90	1.45	2.66	Not Significant
January 31			
0-90	9.99	2.59	Significant
0-45	10.41	2.48	Significant
45-90	0.42	2.48	Not significant

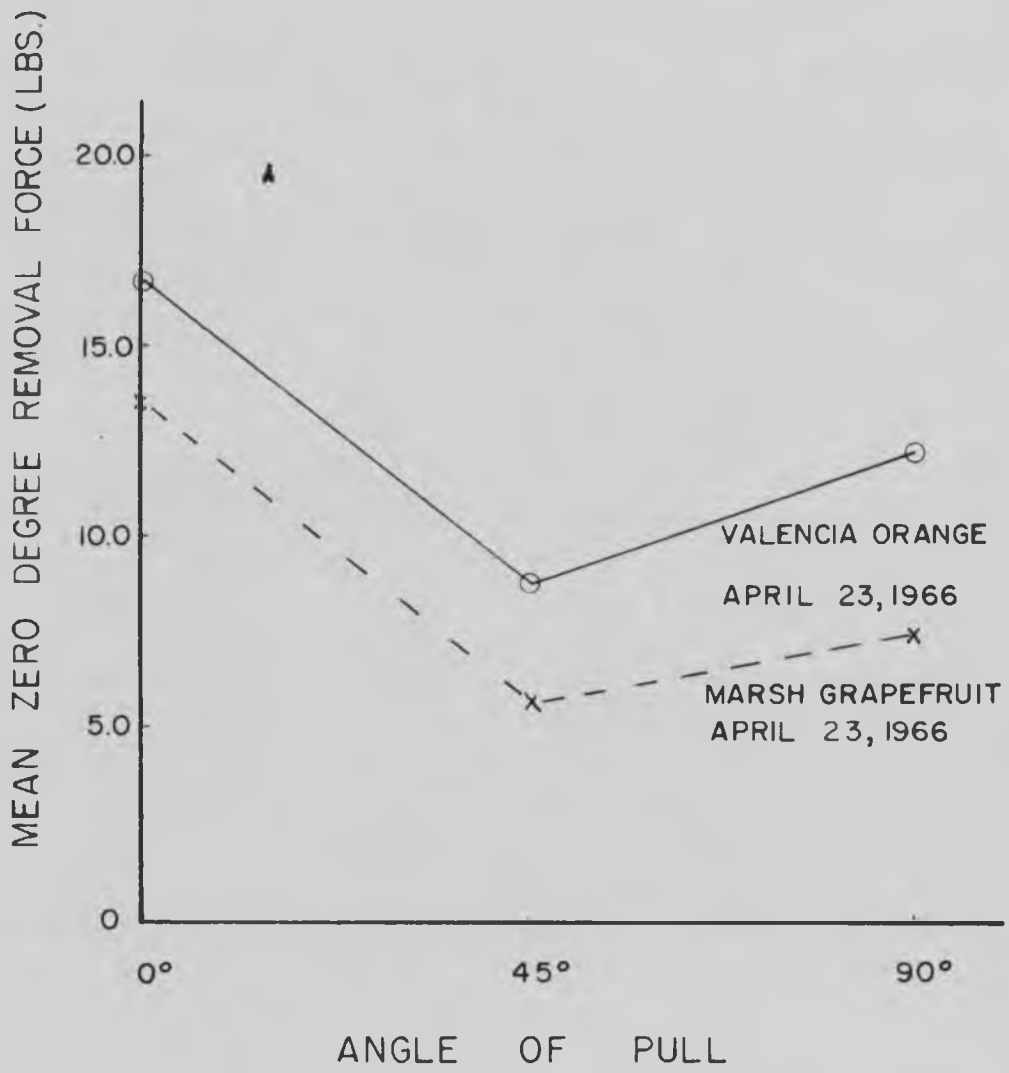

FIGURE 20: MEAN REMOVAL FORCE VS. ANGLE OF PULL

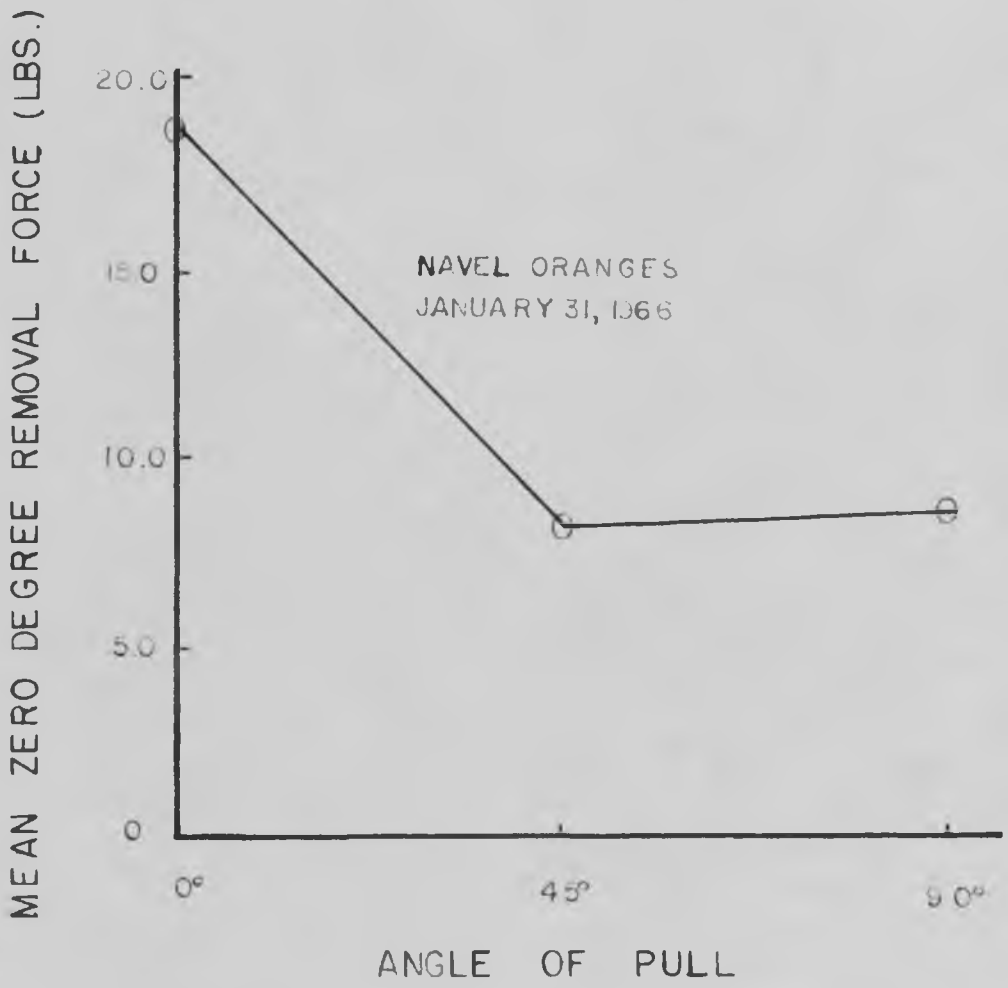

FIGURE 21: MEAN REMOVAL FORCE VS. ANGLE OF PULL

VIBRATORY REMOVAL TESTS

PER CENT REMOVAL VS. FREQUENCY OF VIBRATION

Figure 22 shows a removal curve for Marsh grapefruit on June 27, 1966. Per cent removal, i.e., the per cent of fruit removed from its fruit-stem system is plotted as the ordinate and the rpm of the exciting motion is plotted as the obscissa. For this particular curve, the stroke of the scotch yoke was set at two inches and the stem length of the oranges tested were two inches. The data shows a linear relationship between per cent removal and shaker rpm. Most plots of this type showed the linear relationship to exist. Some curves showed slight convexity and these curves were consistently those determined for 1 inch stem lengths. Figures 23 and 24 show Valencia orange removal curves under two different strokes and stem lengths.

It is interesting to note the slope of the removal curves. Figures 23 and 24 show that the span between zero and one-hundred per cent removal was only 80 rpm. The steep slope indicates a rather rapid change in the vibrational response of the system to its exciting motion. The slope of the curve was somewhat dependent upon time. The less mature fruit, especially young Valencias, give removal curves with lesser slopes than the mature fruit. Thus, removal curves shifted to the left along the abscissa and increased in slope as the season progressed and the fruit became easier to remove.

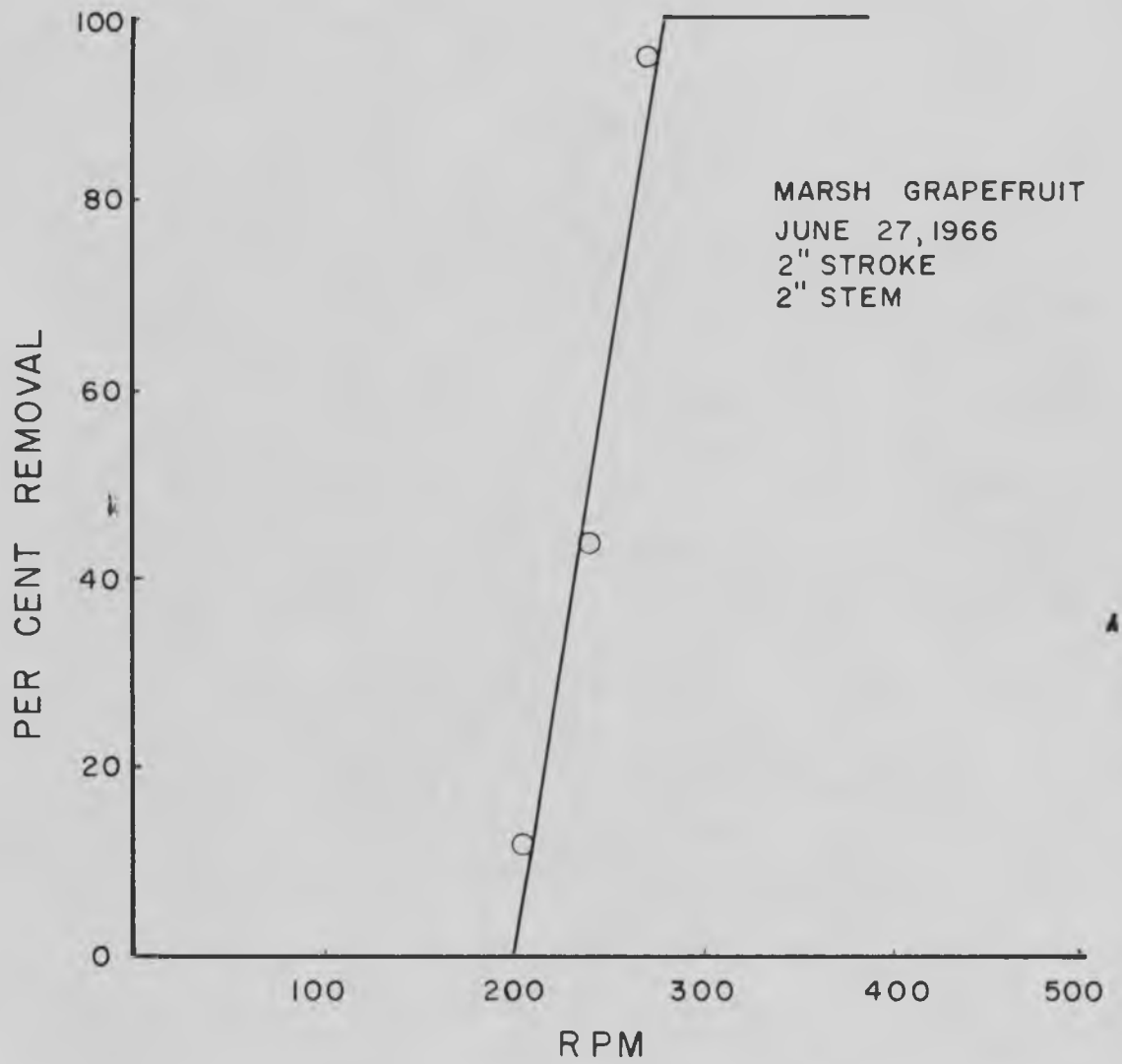

FIGURE 22: PER CENT REMOVAL VS. FREQUENCY OF VIBRATION

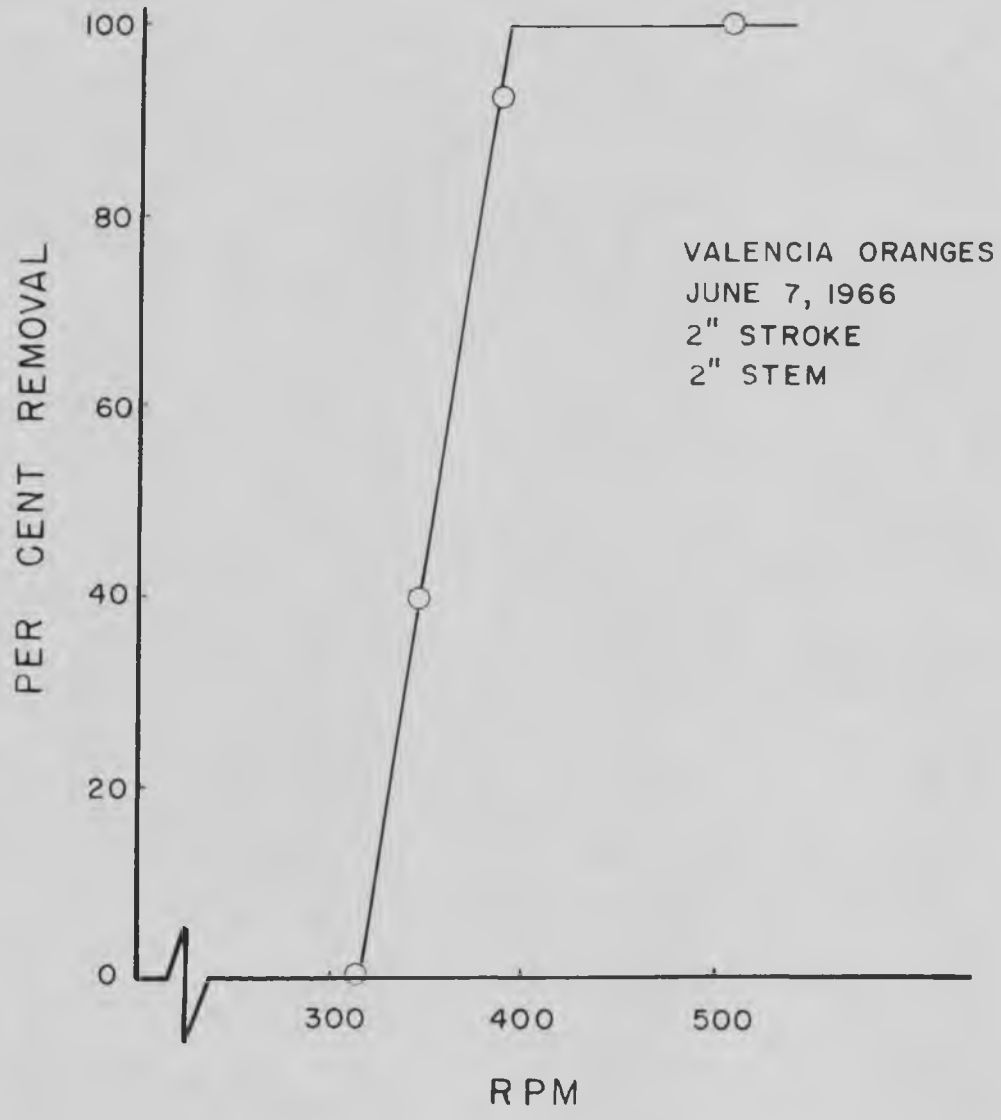

FIGURE 23: PER CENT REMOVAL VS. FREQUENCY OF VIBRATION

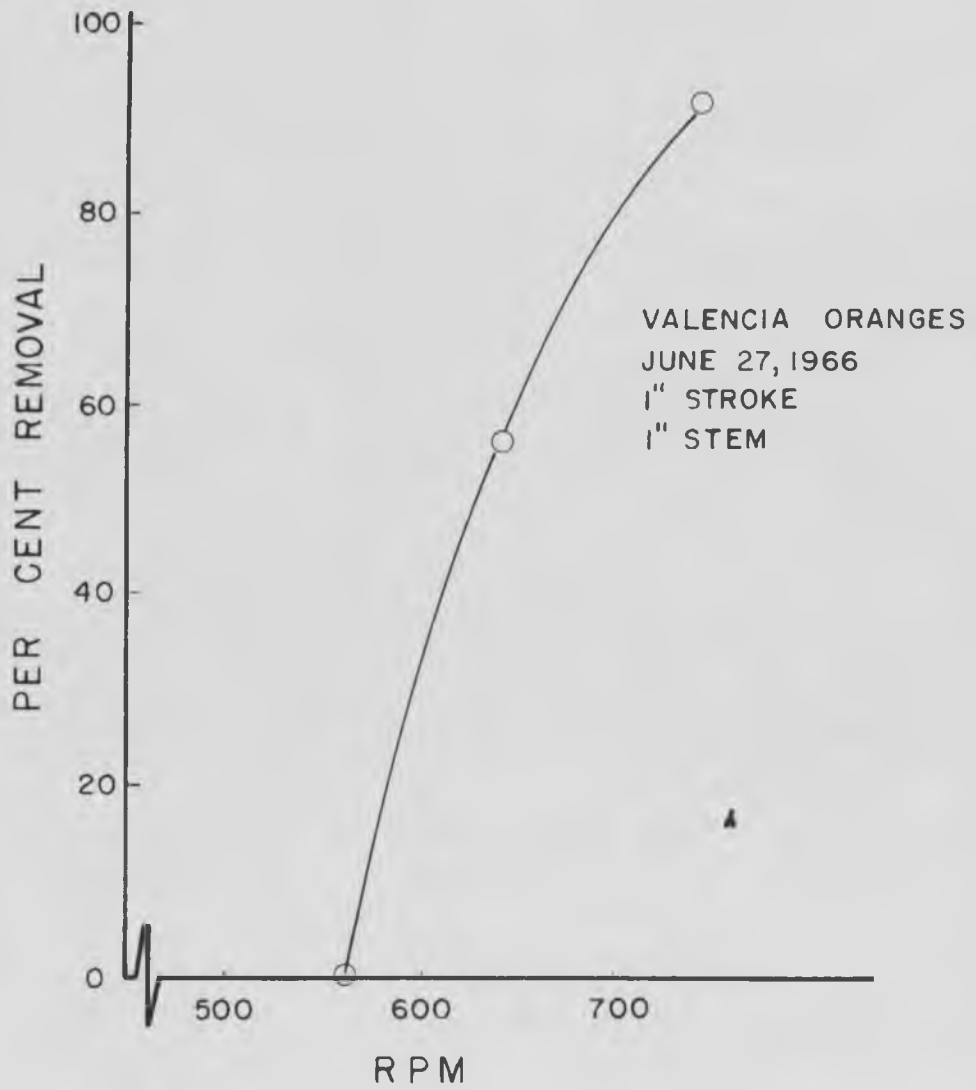

FIGURE 24: PER CENT REMOVAL VS. FREQUENCY OF VIBRATION

REGIMES OF VIBRATION OF THE FRUIT-STEM SYSTEM

Although considerable data has been taken to determine removal conditions for fruit under vibratory motion, the process of removal itself was not fully understood. What motion in the response of the fruit to vibration causes the fruit to be separated from the stem?

In order to visually study responses of the citrus to vibration, movies were made of the fruit undergoing a range of forcing frequencies. This was done for five typical fruit, three Valencia oranges and two Marsh grapefruit. Films were made of fruit with two and four inch stems. Results of observations of the films are recorded in Table 8. All fruit underwent the same characteristic vibrational responses to the exciting motion. At slow speeds, motion of the fruit-stem system was in phase with the exciting motion and the induced deflection (amplitude) of the fruit was small. The next regime of vibration was the first fundamental mode of vibration of the system. In this regime, amplitude of the system was extreme. By increasing the frequency by 15 to 30 rpm above the fundamental frequency of the system, another regime of vibration was encountered. At this particular frequency the fruit-stem system performed circular oscillations. The fruit-stem system performed movements that looked like a ball on the end of a string being whirled in a circular path. This

TABLE 8

REGIMES OF VIBRATION
(2 INCH STROKE)

Regime	Approximate Frequencies (RPM)	Removal Effectiveness
Very slow exciting frequencies	0-100	Will not remove fruit from stem
First fundamental frequency	100	Given sufficient time, the stem will fatigue at its fixed end due to excessive amplitude. The fruit will be removed with its stem intact.
Circular response	130	Removal will occur in same manner as above
Beat phenomena	145	This regime is poor in removal affectiveness. Relative motion between the fixed end of the stem and the stem is not sufficient to fatigue the stem at its fixed end and inertial forces induced at the button are not sufficient to remove fruit.
Unstable motion	300-400	Unstable motion of the fruit causes inertial and gravity forces to be applied at an angle to the stem causing fruit removal at the fruit-stem union.

unusual response was explained as follows. When a system undergoing forced vibrations enters the regime of its first fundamental mode of vibration the amplitude of the system increases linearly with time. As time progresses, the amplitude approaches infinity. When the fruit-stem system encounters forcing frequencies at its fundamental mode, the amplitude of the system can be seen to increase with time. At such time, the system goes into the circular mode response. As the system attempts to achieve infinite amplitude, imperfections such as unbalance of the hanging fruit and crooked stems become important. Such conditions cause the fruit to be displaced from its two dimensional plane of motion into the dimension perpendicular to the plane of motion. Once this lateral displacement has occurred, the ever increasing amplitude of the system tends to amplify the lateral movement. Hence the system enters a mode of circular oscillations.

An increase in frequency of from 15 to 30 rpm caused a transition into another regime of vibration. At this particular frequency, the fruit performed a combination of movements. Most noticeable was the transition of the system to vibration 180 degrees out of phase with the exciting motion. In addition, beats were clearly visible in the motion of the fruit. This beat, or stop and go phenomena, occurs when the exciting frequency is near the

natural frequency of an oscillating system. Combined with the beats and 180 degree phase shift was a twisting of the fruit about its major axis. This twisting can be explained by the mass center of the fruit being out of the plane of the exciting motion. The resultant inertial forces cause a twisting moment about the major axis. The latter regime of vibration occurred at approximately 150 rpm for the fruit filmed. The next definable regime of vibration of interest occurred at 400 rpm for a Valencia orange with a 4 inch stem and at approximately 350 rpm for a Valencia orange with a two inch stem. Similarly, the phenomena occurred at 250 rpm for a Marsh grapefruit with a four inch stem. This regime of vibration can best be described as a seemingly unstable vertical motion of the fruit while undergoing the horizontally applied exciting motion. The random motion of the fruit relative to the stem induces bending forces in the stem and causes inertial and gravity forces to be applied at an angle to the stem of the fruit. But data have shown that the separating force is lessened for angularly applied forces. Hence, when the fruit enters this regime of vibration, fruit separation is initiated. The regime of vibration described here is a characteristic response of the fruit-stem system. In other words, it is a definite, predictable response that does not occur randomly. It is determined by physical properties of the system and by the exciting function.

Data supports the theory that when the unstable regime of vibration is entered fruit separation will begin to occur. The unstable motion first occurred in the Marsh grapefruit film at approximately 250 rpm. Removal curves for grapefruit with the same 2 inch stem show that fruit removal is initiated near 200 rpm, with approximately 40 per cent of the fruit being removed at 250 rpm. Similarly, unstable motion was first observed at approximately 350 rpm for a Valencia orange with a 2 inch stem. Removal curves for Valencia oranges show that approximately 40 per cent removal would occur at this frequency. Hence, for both the one grapefruit and one Valencia orange, transition to the unstable regime fell near to the lowest frequencies which caused fruit separation.

PER CENT REMOVAL VS. STROKE

Figures 25 and 26 show the influence of stroke on fruit removal. Data from Figure 25 was for Marsh grapefruit with a two inch stem. A one and two inch stroke setting was used to obtain the curves. For a one inch stroke setting, 100 per cent fruit removal was obtained at 400 rpm. By comparison, the two inch stroke setting gave 100 per cent removal at 280 rpm. Results were even more dramatic with Valencia oranges in Figure 26. The stem length for these fruit were one inch. At a one inch stroke setting, the highest speed of the shaker, 720 rpm, removed 80 per cent

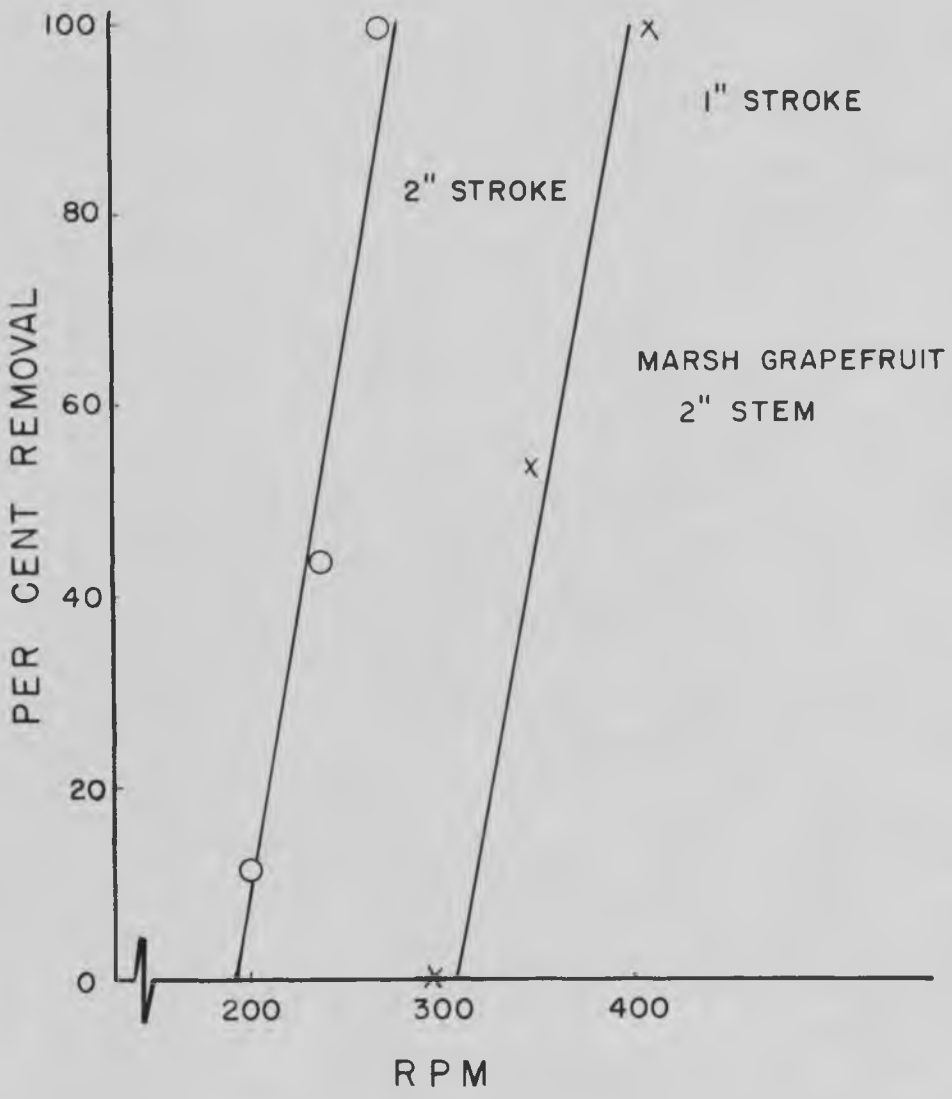

FIGURE 25: EFFECT OF STROKE OF VIBRATION ON PER CENT REMOVAL OF MARSH GRAPEFRUIT

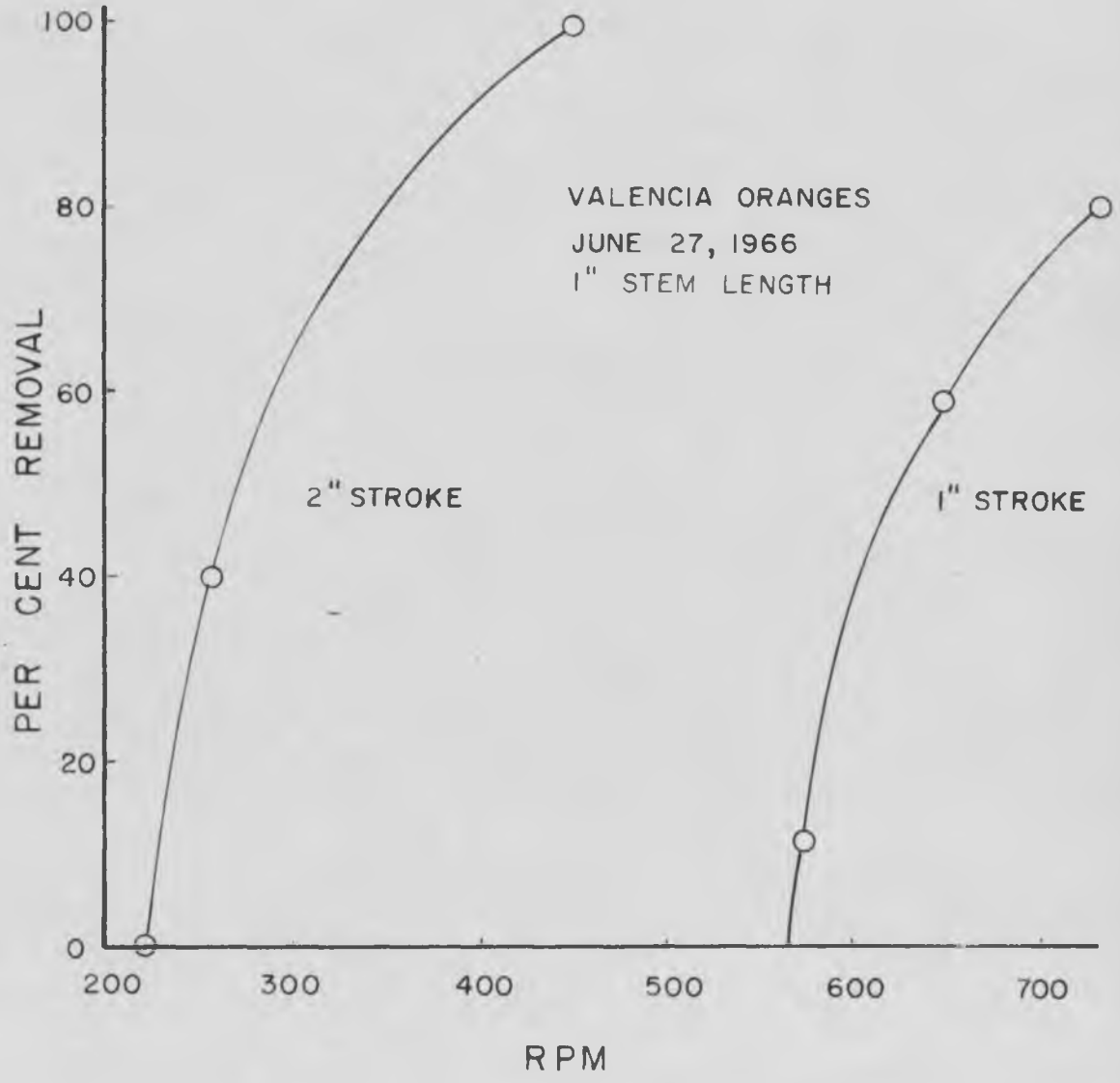

FIGURE 26: EFFECT OF STROKE OF VIBRATION ON PER CENT REMOVAL OF VALENCIA ORANGE

of the fruit. On the other hand, 80 per cent removal at the two inch stroke setting occurred at 350 rpm.

Stroke appears to have a marked affect upon citrus fruit removal. As mentioned in the literature review, several attempts have been made to correlate fruit removal with stroke and frequency. No attempt in this study was made at such a correlation. The marked effect of stroke on citrus fruit removal for the two strokes used indicates that further work in this direction should be pursued.

REMOVAL FREQUENCIES VS. TIME OF SEASON

Figures 27 and 28 show the effect of time of season on the frequency necessary to remove 100 and 50 per cent of the fruit. It is interesting to note the similarity between these two curves and the pull force vs. time curves in Figures 17 and 18. The bottom two curves of Figure 27 show 100 and 50 per cent removal data from Valencia oranges with a two inch stem and a two inch stroke set on the shaker. Note the relatively rapid change in the curves in late April and the leveling off through June. From the curve we see that in late April it took nearly 600 rpm to remove 100 per cent of the fruit. By late June it took only 400 rpm to remove 100 per cent. The upper curve shows a 50 per cent removal curve for oranges with a one inch stem and the shaking stroke set at one inch. This curve showed less change with time as compared with the lower two curves.

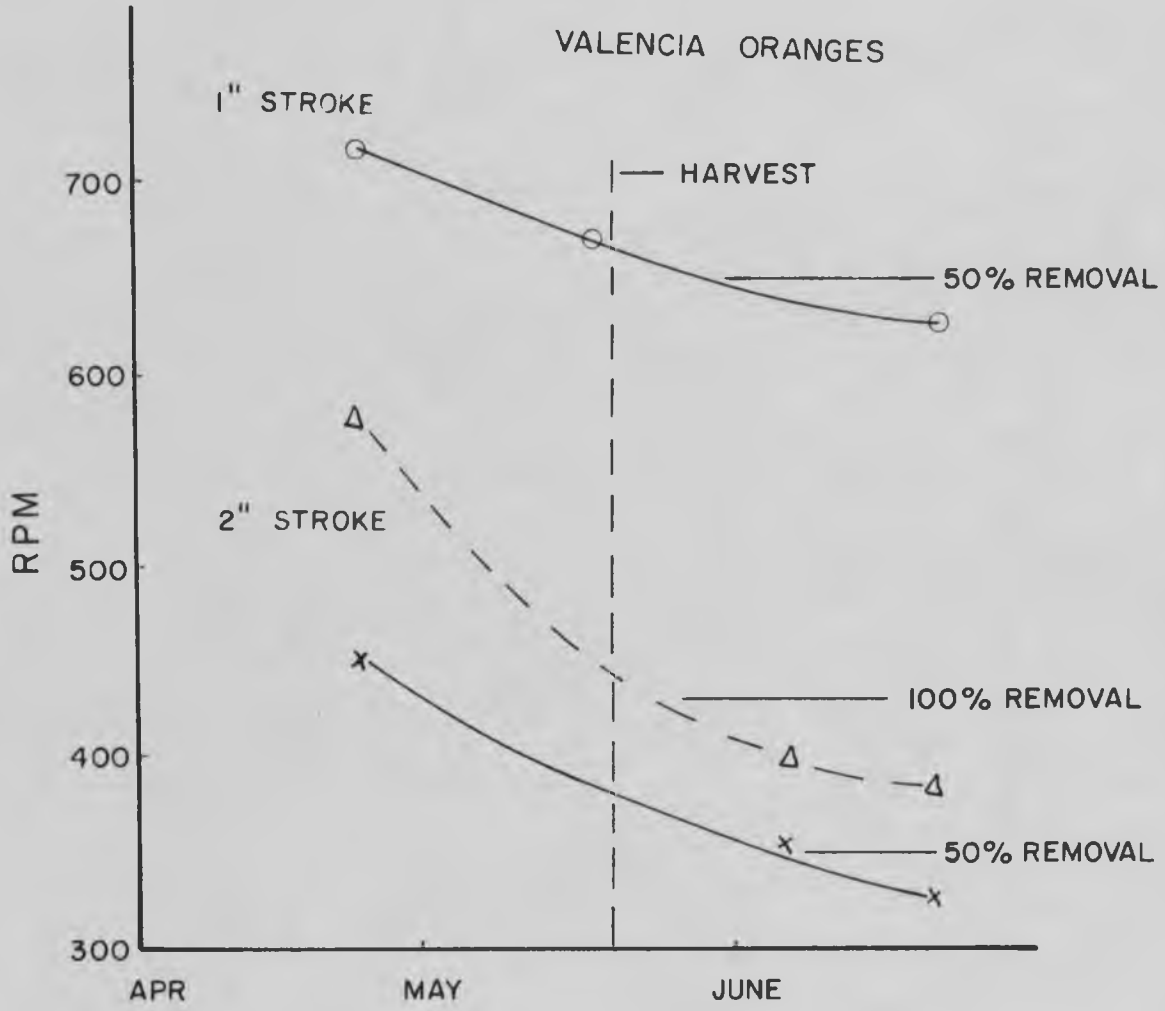

FIGURE 27: EFFECT OF TIME OF SEASON ON REMOVING FREQUENCIES FOR VALENCIA ORANGES

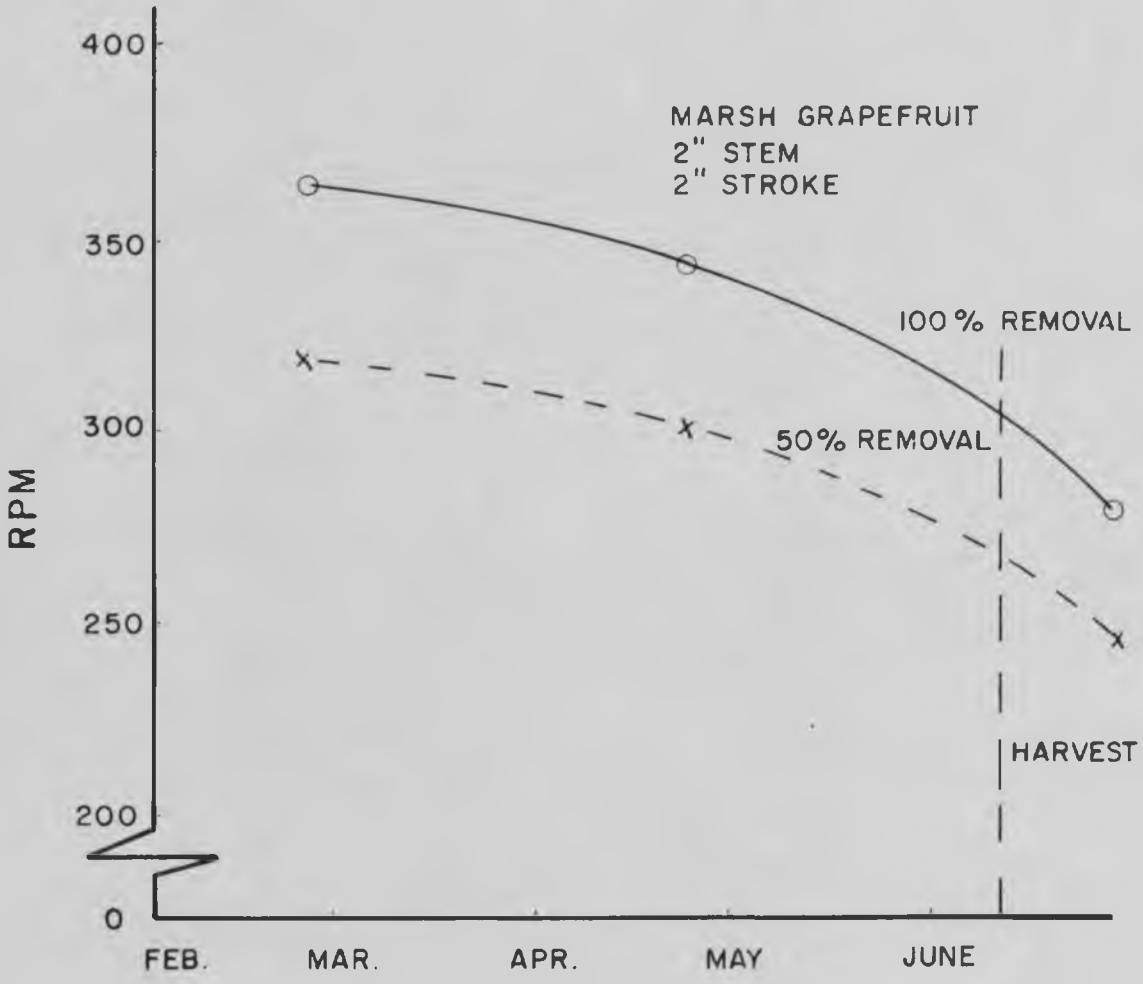

FIGURE 28: EFFECT OF TIME OF SEASON ON REMOVING FREQUENCIES FOR MARSH GRAPEFRUIT

Figure 28 shows the same plot for grapefruit. The curve has the same general shape as the pull force vs. time curve for grapefruit for this same time period. In February it took approximately 360 rpm to remove 100 per cent of the fruit with a two inch stem and utilizing a two inch stroke. By the end of June it took approximately 280 rpm to remove the same fruit with the same stroke.

MEAN ZERO DEGREE REMOVAL FORCE VS. 100 PER CENT REMOVAL FREQUENCY

A plot of mean zero degree pull force vs. 100 per cent removal frequency for Marsh grapefruit is shown in Figure 29. The points for the curve were determined by taking the mean pull force and the frequency determined to remove 100 per cent of the fruit from days in February, April, and June. The purpose of the curve was to determine if pull force was a sensitive indicator of removing frequencies. Statistical analysis has shown the ordinate value of the top point on the curve to be significantly different from the ordinate value of the bottom point. The curve shows zero degree pull force to be a relatively sensitive indicator of removal frequencies for Marsh grapefruit.

TORSIONAL REMOVAL TESTS

REMOVAL TORQUE VS. STEM TENSION

Torsional removal tests were conducted with 0, 8, and 10 pound stem tensions. Data from the tests is presented in Table 9.

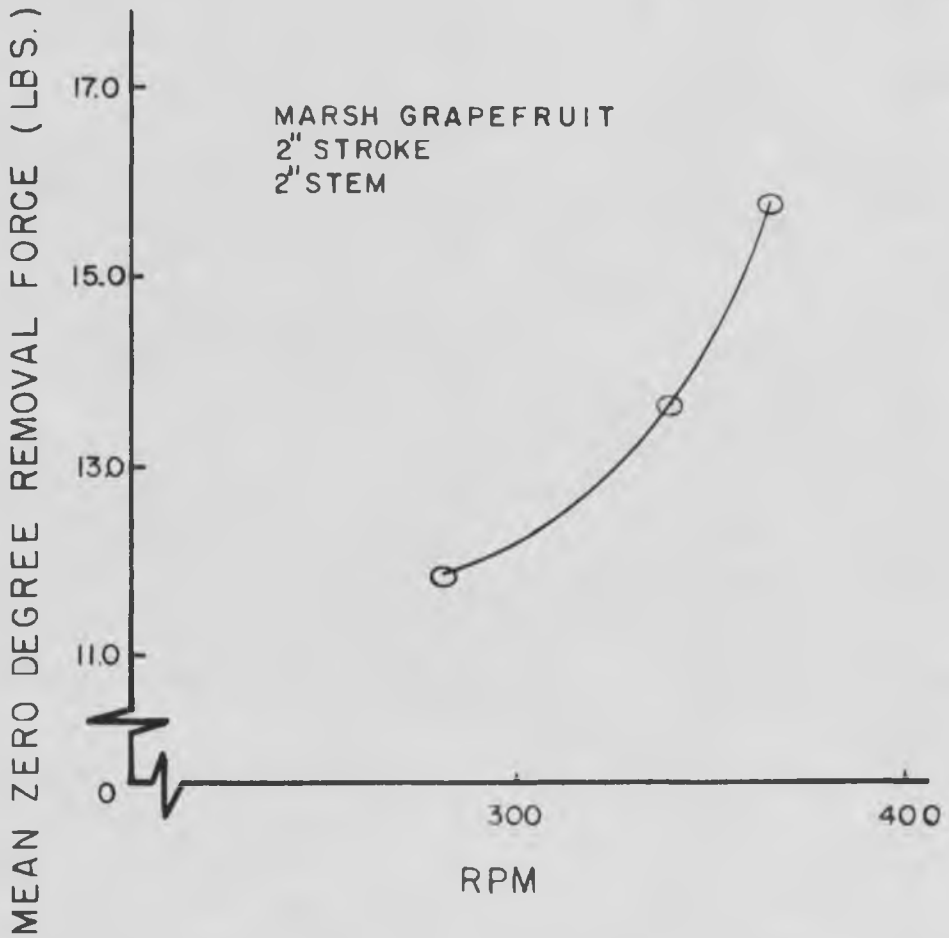

FIGURE 29: MEAN ZERO DEGREE REMOVAL FORCE VS. 100% REMOVING FREQUENCIES

TABLE 9
ANALYSIS OF VARIANCE OF NAVEL ORANGE, VALENCIA ORANGE, AND MARSH GRAPEFRUIT MEAN REMOVAL TORQUES VS. STEM TENSION

NAVEL ORANGE MEAN REMOVAL TORQUES (IN. LB.)

Date	0 Pound	8 Pound	F Test (1% level)
December 22	0.648	0.612	Not significant
December 29	0.703	0.847	Not significant
January 31	0.836	0.647	Not significant

MARSH GRAPEFRUIT MEAN REMOVAL TORQUES (IN. LB.)

Date	0 Pound	10 Pound	F Test (1% level)
February 25	0.590	0.457	Not significant
March 18	0.460	0.421	Not significant
April 24	0.443	0.432	Not significant
June 8	0.561	0.360	Not significant

VALENCIA ORANGE MEAN REMOVAL TORQUES (IN. LB.)

Date	0 Pound	10 Pound	F Test (1% level)
February 24	0.537	0.620	Not significant
March 18	0.604	0.553	Not significant
April 24	0.590	0.515	Not significant

Navel oranges were tested on three occasions. At each date of test, removal torques were determined for 0 and 8 point stem tensions. Of the three tests, none showed a significant difference in removal torque for the two stem tensions. Marsh grapefruit were tested at four different dates. Results show that for all four dates, no significant differences were found between the removal torques for the 0 and 10 pound stem tensions. The three Valencia tests gave no significant differences in removal torques for the two stem tensions.

REMOVAL TORQUE VS. TIME OF SEASON

The torsional removal data collected throughout the season for Marsh grapefruit is presented in Tables 10 through 12. In Table 10, Section 1, the means of the 0 and 10 pound tests on grapefruit are ranked in descending order. Computations and results of Duncan's New Multiple Range Test are shown in Section 2 of Table 10. Results show that mean removal torques determined in February, March, April, and June were not significantly different from one another. In other words, removal torque is independent of the maturity of the grapefruit.

Similar comparisons between means determined for Navel and Valencia oranges showed mean removal torque to be independent of time of season. The data and results of Duncan's New Multiple Range Test are presented in Table 11 and 12.

TABLE 10
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
GRAPEFRUIT MEAN REMOVAL TORQUE VS. TIME OF SEASON

1. RANKING OF MEANS

0 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
1. February 25	0.589
2. June 8	0.562
3. March 18	0.458
4. April 24	0.443

10 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
a. February 25	0.457
b. April 24	0.432
c. March 18	0.421
d. June 8	0.361

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
1-4	0.146	0.207	Not significant
1-3	0.131	0.201	Not significant
1-2	0.027	0.193	Not significant
2-4	0.119	0.201	Not significant
2-3	0.104	0.193	Not significant
3-4	0.015	0.193	Not significant
a-d	0.096	0.267	Not significant
a-c	0.036	0.259	Not significant
a-b	0.025	0.249	Not significant
b-d	0.071	0.259	Not significant
b-c	0.011	0.249	Not significant
c-d	0.060	0.249	Not significant

TABLE 11
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
NAVEL ORANGE MEAN REMOVAL TORQUE VS. TIME OF SEASON

1. RANKING OF MEANS

0 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
1. January 31	0.837
2. December 29	0.732

10 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
a. December 29	0.847
b. January 31	0.647

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
1-2	0.105	0.330	Not significant
a-b	0.200	0.447	Not significant

TABLE 12
DUNCAN'S NEW MULTIPLE RANGE TEST
FOR
VALENCIA ORANGE MEAN REMOVAL TORQUE VS. TIME OF SEASON

1. RANKING OF MEANS

0 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
1. March 18	0.604
2. April 24	0.590
3. February 24	0.538

10 POUND STEM TENSION

Date	Mean removal torque (in. lb.)
a. February 24	0.620
b. March 18	0.552
c. April 24	0.516

2. COMPARISON OF MEANS AT 1 PER CENT LEVEL

Means compared	Difference	SSR	Conclusion
1-3	0.066	0.258	Not significant
1-2	0.014	0.248	Not significant
2-3	0.052	0.258	Not significant
a-c	0.104	0.231	Not significant
a-b	0.068	0.221	Not significant
b-c	0.036	0.221	Not significant

PRACTICAL IMPLICATIONS OF DATA

A major goal of this thesis would go unfulfilled if the data taken could not be applied in a practical manner. Extensive data has been gathered on the physical properties of the citrus fruit-stem system under various removal actions. Now the question must be asked, "How can the results of this experiment be applied to mechanical harvest systems for use on citrus?"

PULL AND TORSIONAL TESTS

Devices which remove fruit by direct physical contact can use the pull and torsional test data outright as it is presented. Magnitude of the data (such as removal pull forces) could conceivably be used for design purposes.

Plugging of citrus fruit has been a problem in hand picked fruit in Florida and to some extent in California. Coppock (9) has commented that a direct, or zero degree pull on the fruit encourages plugging. However, of the 425 zero degree pull tests conducted, only eleven fruit plugged. No fruit plugged when the force was applied at an angle and none plugged under torsional or vibratory removal actions. Thus, data shows an inherent difference in one particular physical property of Arizona and Florida citrus. It appears that plugging would not be a problem to the desert grown citrus in Arizona.

VIBRATORY REMOVAL TESTS

Application of the vibratory removal data to practical utilization is not the same direct process. This situation arises directly from the assumptions made in setting up the experimental fruit-stem system. For laboratory tests, the fruit-stem system was idealized in two respects. First, the exciting motion was applied normal to the major axis of the fruit. Secondly, the initial end of the stem was clamped in a rigid support. These idealizations may or may not exist in the actual environment of the fruit-stem system.

The vibratory removal data are directly useful in a relative sense. Data have shown that the exciting stroke applied to the system has a great influence on fruit removal. In practical application this would indicate that exciting frequencies applied to trees could be decreased if the stroke of the exciting motion were increased.

Other data (Figures 27 and 28) have shown removal frequencies to be dependent upon time of season. Of special interest is the trend shown for grapefruit as shown in Figure 28. The curve shows the beginning of a rapid decrease in removal frequencies after the conventional harvest date. This indicates that delay of harvest would ease vibratory removal. It should be noted that the discussion of harvest delay is based upon vibratory removal data

only. Most certainly other factors such as fruit quality, maturity, and market conditions would have to be considered before delaying harvest.

It is of interest to compare removal frequencies for Valencia oranges and Marsh grapefruit. Comparing the two citrus species at their respective harvest dates (Figures 27 and 28) we see that it took approximately 480 rpm with a two inch stroke and stem length of two inches to remove the Valencia orange. Under similar shaking conditions it took approximately 300 rpm to remove the grapefruit at harvest. The magnitude of the difference is important here, as the numbers have no direct translation to field conditions. The relative ease of vibratory detachment of the grapefruit as compared to the Valencia orange is due in part to the lower strength of the fruit-stem union of the grapefruit at harvest. The greater mass of the grapefruit will also contribute to its easier removal because of the greater inertial forces which would be developed.

Marsh grapefruit and Valencia oranges are usually harvested during May and June, respectively, in Arizona. At this time of the season the fruit for next years crop is already present on the tree. This young fruit is small, being only one to two inches in diameter at harvest. The presence of the young fruit makes the magnitude of the

shaking actions extremely critical. It is conceivable that shaking conditions of sufficient magnitude to remove mature fruit would also remove the young crop of fruit. Observations by the author indicate that although the tensile strength of the fruit-stem union of young fruit is high, the strength of the young stem itself is low. In other words, young fruit would probably be removed with portions of the stem intact if shaking conditions were severe enough. Here the difference in removal frequencies between Marsh grapefruit and Valencia oranges becomes important. The comparative removal frequencies indicate that removal of young fruit in Valencia trees might be a problem. Conversely, the less severe shaking actions required to remove grapefruit indicates that removal of young grapefruit may not be a problem.

STEM AND BUTTON REMOVAL

Fresh-marketed grapefruit and oranges are hand-harvested with the button left on the fruit. The button is left on partly as protection against deterioration and partly because of its appeal to the consumer. It would be desirable that a mechanical harvest system remove the fruit with the button intact on the fruit.

The pull, torsional, and shaking tests showed that the fruit was removed from the stem in one of three ways. One mode of fruit removal left the fruit with no button.

On some tests, the complete stem was removed from the fruit with the button left intact on the fruit. Here separation of the stem occurred within the button. The third mode of fruit removal left part of the stem attached to the fruit. The third mode of removal is highly undesirable not only from the standpoint of appearance but from the fact that the portion of the stem left with the fruit is stiff. In transit, fruit with short stems would poke and lacerate other fruit thus adversely affecting the exterior quality of the fruit.

Tests have shown that 28.4 per cent of all Valencia fruits that were removed by vibratory action were removed with the button intact on the fruit. This figure was determined from 450 fruit that had been detached with removing frequencies ranging from 260 to 720 rpm. Similarly, 10.0 per cent of all Marsh grapefruit fruits that were removed by vibratory action were removed with the button intact on the fruit. This figure resulted from 300 fruit that had been detached with removing frequencies ranging from 190 to 510 rpm. One might expect that the number of buttons left intact on the fruit would be dependent upon shaking speed. Results have shown that 40.9 per cent of the Valencia fruit removed at frequencies of 390 rpm (50 per cent removing frequency at harvest) or greater were removed with the button intact on the fruit. Likewise,

16.0 per cent of Valencia fruit removed at less than 390 rpm were removed with the button intact. Results of grapefruit testing have shown that 12.0 per cent of the Marsh grapefruit removed at frequencies of 275 rpm (50 per cent removing frequency at harvest) or greater were removed with the button intact on the fruit. Only 8.0 per cent of the Marsh grapefruit removed at frequencies of less than 275 rpm were removed with the button intact. Therefore both the Valencia orange and Marsh grapefruit data show that the higher removing frequencies are more favorable to removing fruit with buttons intact on the fruit.

Removal of the fruit with small portions of the stem intact on the fruit was encountered in vibratory removal. At frequencies of 390 rpm and greater approximately 14 per cent of Valencia oranges were removed with stems left on the fruit. Conversely, at frequencies of greater than 275 rpm, essentially no Marsh grapefruit were removed with stems left on the fruit. However, for both fruits, removal with small stems on the fruit was highly encouraged when the fruit was shaken at or near their first fundamental frequencies.

Torsional removal tests showed substantial percentages of fruit removed with buttons intact on the fruit. Tests showed 63.7 per cent of Navels, 58.7 per cent of Marsh grapefruit, and 56.0 per cent of Valencia oranges

were removed with buttons intact on the fruit (stem failure occurring inside button). The remaining percentages of Valencia and Navel fruit removed by torsional action, i.e., those removed other than those removed with button on the fruit, were removed with portions of stem left on the fruit. As mentioned before, fruit harvested with stems left on are highly undesirable. The remaining percentage of Marsh grapefruit that were not removed with button on the fruit were removed with no button left on the fruit. Very few Marsh grapefruit were removed with portions of the stem left on the fruit. This indicates a difference in the response of Marsh grapefruit to torsional removal actions. The button of the Marsh grapefruit appears to be weaker in torsion than the Valencia and Navel fruit. The buttons of the Navel and Valencia fruit are stronger in torsion than their stem as shown by the fact the stem will fail before the button is removed under torsional action.

Pull tests, regardless of angle of pull, almost consistently resulted in the removal of the button from the fruit when the fruit was separated from its stem. Only twelve of the 425 zero degree pull tests resulted in buttons left intact on the fruit.

CONCLUSIONS

PULL TESTS

1. Significant differences were found in removal forces measured over the growth and harvest season of the fruit.

2. The force necessary to remove a fruit from its stem can be substantially reduced by pulling at an angle to the major axis of the fruit.

3. Navel removal forces decrease drastically after harvest, Valencia removal forces are leveling off during their normal harvest period.

4. Plugging was not a problem in pull tests.

VIBRATORY REMOVAL TESTS

1. A near linear relationship exists between per cent removal and shaking frequencies at and near harvest.

2. Increasing the stroke of the shaking motion applied to the fruit-stem system decreases the frequency necessary to remove the fruit from its stem.

3. Removal frequencies (at a given stroke) decrease as the fruit matures. The frequencies for Valencia oranges and Marsh grapefruit decrease with fruit maturity in the same manner as do their removal pull forces.

4. The fruit-stem system responds with definite mannerisms to a sinusoidal shaking action. Its responses (or regimes of vibrations) are useful in predicting how, why, and if fruit removal will occur.

5. Marsh grapefruit require a substantially less severe shaking action to cause removal than do Valencia oranges.

6. Approximately 30 per cent of Valencia oranges are removed with buttons intact on the fruit when shaken at minimum removing frequencies (at a given stroke). Only 10 per cent of Marsh grapefruit are removed with buttons intact under similar shaking conditions.

7. Removal of fruit with a portion of the stem left is encouraged when the fruit is vibrating at its first fundamental frequency. At frequencies which remove 50% or more fruit approximately 14 per cent of Valencia oranges were removed with stems left on the fruit. Conversely, at frequencies which remove 50% or more fruit no Marsh grapefruit were removed with stems left on the fruit.

TORSIONAL REMOVAL TESTS

1. In general, no significant differences at the one per cent level in removal torque were found when 0 and 10 pound stem tensions were used.

2. There were no significant differences between removal torques measured throughout the pre- and post-harvest season.

APPENDIX A

MARSH GRAPEFRUIT

ZERO DEGREE PULL TESTS (LBS.)

Feb. 2	Feb. 24	Mar. 17	Apr. 23	May 14	June 7	June 27
11.0	15.6	11.4	13.6	11.2	17.1	13.2
16.2	19.3	9.3	10.6	12.1	16.3	13.2
10.4	16.7	16.0	13.0	10.5	14.5	13.9
14.5	10.3	12.4	14.2	12.2	14.1	12.0
18.0	19.3	12.9	15.4	15.5	13.6	12.7
15.5	15.3	11.8	17.4	15.0	10.9	7.5
11.5	13.0	15.2	12.3	13.5	10.4	11.3
7.4	13.0	12.9	11.8	10.0	15.2	11.2
12.0	19.8	13.5	15.3	14.5	9.9	11.8
18.1	17.4	9.7	11.0	12.7	7.3	11.7
14.4	16.9	13.3	24.5	14.4	13.7	13.0
12.9	18.2	19.9	10.9	13.9	16.8	11.7
13.0	20.7	13.8	11.2	16.6	14.8	12.0
13.4	14.5	15.5	12.0	14.1	15.2	11.7
14.4	14.5	12.6	15.2	12.1	20.7	12.3
16.5	15.6	11.6	15.7	13.2	12.6	12.5
14.9	13.2	18.3	15.1	14.0	12.8	11.3
14.7	15.0	8.1	15.6	12.0	14.3	12.1
17.3	12.0	11.3	17.7	11.2	15.0	11.6
10.3	18.5	11.9	12.8	14.9	12.0	10.6
11.1	15.0	12.4	13.7	11.5	14.5	11.2
14.6	14.5	12.4	10.0	14.9	13.7	13.2
12.2	17.6	13.6	11.3	13.7	14.6	14.0
16.0	15.5	13.5	9.0	13.0	16.1	13.7
14.6	13.2	13.6	10.8	11.7	16.7	6.5
avg 13.8	15.8	13.1	13.6	13.1	14.1	11.8
s = 2.6	2.6	2.6	3.3	1.7	2.7	1.7

MARSH GRAPEFRUIT

FORTY-FIVE DEGREE PULL TESTS (LBS.)

Feb. 24	Mar. 17	Apr. 23
8.2	9.1	8.3
8.9	6.3	6.8
11.8	7.0	5.5
7.0	7.7	7.4
8.0	8.5	5.5
7.5	9.7	6.3
7.0	9.2	8.6
6.5	10.0	5.6
5.6	10.0	5.4
5.2	8.5	3.0
6.4	7.2	6.8
7.0	7.1	5.1
6.7	7.4	5.0
5.0	8.1	4.0
5.0	7.0	5.8
6.5	11.5	6.6
9.0	8.1	2.7
3.5	8.2	7.4
5.0	7.1	9.7
8.0	8.3	5.7
5.0	7.5	4.9
7.0	8.6	5.9
8.6	5.2	6.7
6.0	4.8	5.5
7.5	7.1	2.8
avg 6.9	avg 8.0	avg 5.9
s =1.70	s = 1.66	s = 1.03

MARSH GRAPEFRUIT

NINETY DEGREE PULL TESTS
(LBS.)

Feb. 24	Mar. 17	Apr. 23
9.4	9.2	8.3
7.2	6.7	6.8
10.5	4.8	5.5
10.0	7.5	7.4
7.2	6.5	5.5
8.2	5.7	6.3
9.8	9.8	8.6
3.1	12.0	5.6
9.8	9.7	5.4
4.3	10.2	3.0
6.4	8.9	6.8
9.5	11.6	5.1
9.7	9.4	5.0
5.1	11.6	4.0
4.0	8.3	5.8
7.2	7.6	6.6
5.0	10.9	2.7
5.4	9.3	7.4
8.1	9.0	9.7
11.6	6.8	5.7
10.0	9.0	4.9
9.4	7.0	5.9
4.3	2.8	6.7
4.5	9.8	5.5
8.7	8.2	2.8
avg 7.5	avg 8.5	avg 5.9
s = 2.42	s = 1.86	s = 1.33

NAVEL ORANGES

ZERO DEGREE PULL TESTS (LBS.)

Dec. 21	Dec. 29	Jan. 31	Feb. 24
22.1	24.3	20.1	12.0
18.4	21.7	19.9	15.6
17.3	19.3	22.2	12.3
20.8	25.3	20.3	12.1
18.0	18.4	22.6	10.1
25.0	19.7	18.2	17.5
18.0	17.0	18.2	23.6
22.7	18.3	20.3	11.6
22.4	21.6	14.3	7.0
14.6	23.5	22.0	13.0
24.5	20.0	22.6	12.3
22.2	7.4	22.0	12.3
20.0	23.0	12.2	15.0
25.9	21.6	14.0	7.9
21.0	21.5	14.6	14.9
17.0	17.9	18.0	10.5
19.8	21.1	14.5	15.0
19.0	24.6	16.0	11.8
24.2	22.0	18.3	7.9
27.4	19.5	12.0	12.5
19.0	23.0	17.0	16.0
21.8	24.5	27.2	7.0
19.5	19.3	21.2	14.0
22.3	19.0	17.0	12.7
24.4	15.7	23.0	13.1
avg 21.1	avg 20.4	avg 18.7	avg 12.7
s = 3.10	s = 3.70	s = 3.81	s = 3.57

NAVEL ORANGES

FORTY-FIVE DEGREE PULL TESTS
(LBS.)

Dec. 21	Dec. 29	Jan. 31
7.7	9.0	7.6
10.6	15.1	7.1
16.1	12.6	11.0
22.2	11.0	14.2
22.2	7.0	8.7
27.5	12.7	14.2
11.2	11.9	2.5
11.7	16.5	9.0
17.6	10.8	9.2
14.7	14.0	10.4
19.2	13.0	8.0
16.3	18.0	11.8
17.7	8.0	4.2
12.7	16.9	13.5
14.1	17.5	6.0
21.1	10.9	3.5
9.7	10.7	7.6
23.1	16.7	11.7
12.0	13.9	4.6
17.5	16.4	11.6
11.6	7.2	5.5
15.5	15.3	7.5
13.6	13.2	5.2
12.8	13.2	4.4
13.6	9.6	6.5
avg 15.7	avg 12.9	avg 8.2
s = 4.15	s = 3.18	s = 3.32

NAVEL ORANGES

NINETY DEGREE PULL TESTS (LBS.)

Dec. 21	Dec. 29	Jan. 31
13.0	8.2	5.9
8.4	8.5	6.7
26.2	7.6	6.6
14.1	6.5	6.8
12.0	9.2	4.9
13.5	9.2	14.5
7.5	10.8	9.1
7.5	10.5	7.2
9.6	11.4	10.0
14.5	11.6	7.5
14.5	8.5	8.9
14.0	10.5	14.4
10.9	15.0	11.8
7.5	10.5	7.6
20.3	19.0	5.6
8.6	16.4	6.7
10.6	15.6	13.0
21.6	9.2	7.8
10.8	18.4	7.6
7.9	9.5	9.6
13.8	10.4	5.5
15.8	11.8	9.6
10.6	17.8	10.2
10.2	14.5	10.0
17.6	5.6	8.6
avg 12.8	avg 11.5	avg 8.6
s = 4.63	s = 3.66	s = 2.56

VALENCIA ORANGES

ZERO DEGREE PULL TESTS
(LBS.)

Feb. 24	Mar. 17	Apr. 23	May 14	June 7	June 27
16.9	16.2	15.4	12.7	9.7	13.5
19.1	20.0	13.8	18.5	18.0	16.6
19.8	19.5	15.0	15.8	18.4	16.1
20.0	20.1	18.4	16.7	16.2	15.3
14.5	18.4	16.5	13.7	14.1	15.5
10.5	22.7	14.1	17.5	9.7	13.1
13.2	18.1	12.8	18.0	15.5	14.9
18.8	21.7	13.6	15.1	13.8	16.5
14.8	19.8	15.2	13.1	11.2	16.3
14.2	18.0	13.6	16.2	15.8	17.5
18.4	23.5	13.1	14.4	15.0	13.1
16.1	20.5	17.0	19.5	16.0	13.1
16.6	17.6	16.2	15.9	12.4	13.5
17.5	13.8	14.1	15.9	16.4	14.5
16.5	15.4	19.4	14.8	13.2	15.9
19.5	19.4	19.8	18.5	16.4	14.4
18.8	6.6	15.6	17.1	19.2	16.5
19.2	19.5	17.5	16.8	14.8	18.0
21.4	18.8	20.5	18.8	15.6	17.4
20.1	22.0	16.9	16.2	15.7	18.1
17.6	24.6	16.6	18.0	17.8	16.9
20.1	19.8	20.2	17.3	20.9	15.6
20.5	19.0	22.5	12.5	21.5	16.1
26.5	20.8	20.5	14.1	20.4	16.5
28.0	18.0	16.6	15.8	12.5	14.5
avg 18.3	avg 19.0	avg 16.6	avg 16.1	avg 15.6	avg 15.6
s = 3.7	s = 3.7	s = 2.7	s = 2.0	s = 3.2	s = 1.5

VALENCIA ORANGES

FORTY-FIVE DEGREE PULL TESTS
(LBS)

Feb. 24	Mar. 17	Apr. 23
13.3	7.3	9.4
9.0	9.5	11.6
12.0	9.1	8.6
8.2	9.3	8.0
9.8	12.2	11.7
10.4	7.4	11.5
10.3	9.0	8.1
7.0	14.5	8.5
8.1	11.5	10.6
11.0	9.5	11.1
11.0	12.9	6.1
12.0	9.5	9.5
7.0	10.7	9.2
11.2	10.7	9.2
13.6	12.6	9.0
11.0	10.6	7.8
12.3	9.9	9.6
12.0	11.2	7.4
11.0	9.0	8.6
11.2	13.1	8.8
9.6	4.5	4.5
11.2	10.1	9.1
10.5	9.2	11.5
8.3	5.9	8.2
10.5	8.6	6.6
avg 10.5	avg 9.9	avg 9.0
s = 1.72	s = 2.37	s = 1.74

VALENCIA ORANGES

NINETY DEGREE PULL TEST
(LBS.)

Feb. 24	Mar. 17	Apr. 23
13.5	13.2	13.0
12.7	11.5	9.7
12.0	10.9	9.8
13.5	13.1	16.5
11.3	12.4	4.4
9.0	6.5	10.1
15.5	19.0	13.1
11.0	18.3	13.3
15.2	16.1	10.5
19.1	13.8	10.4
15.2	11.5	14.0
12.6	15.0	13.3
7.0	13.5	16.2
14.6	13.0	16.8
10.4	8.4	14.7
7.8	12.0	8.2
10.9	9.6	11.6
13.1	9.5	14.7
11.5	13.4	9.8
8.5	11.8	9.7
11.5	11.6	16.3
7.5	14.0	13.1
8.6	19.0	12.2
15.1	12.1	12.3
11.8	17.0	12.6
avg 12.0	avg 13.0	avg 12.2
s = 2.89	s = 3.06	s = 2.87

VALENCIA ORANGES

TORSIONAL REMOVAL TESTS
(POUNDS ON 1/4 INCH RADIUS)*

Feb. 24		Mar. 18		Apr. 24	
0#	10#	0#	10#	0#	8#
3.5	2.0	2.0	2.3	1.9	2.1
2.7	2.6	4.6	1.9	1.9	1.4
2.6	2.5	1.9	1.6	2.4	2.0
1.9	2.8	2.1	2.4	1.7	1.9
1.4	3.2	2.3	2.0	3.0	1.3
1.6	1.6	2.8	3.0	2.9	1.9
2.3	3.6	3.7	2.9	2.6	3.4
1.9	2.3	4.1	1.3	2.8	3.2
1.5	2.5	3.2	1.4	1.9	1.7
2.0	3.1	1.8	2.0	1.7	1.9
1.5	3.1	2.7	2.4	3.0	2.0
2.6	2.5	1.6	2.4	1.6	1.6
1.4	3.1	3.5	2.8	2.3	2.2
3.1	0.0	2.5	2.5	2.3	2.3
2.6	2.6	2.3	1.6	2.1	2.0
1.7	1.6	2.4	2.8	2.6	2.1
2.6	3.2	1.2	2.6	2.1	1.5
2.7	2.5	1.3	2.1	2.5	2.2
2.4	1.9	1.5	1.6	3.2	2.0
2.8	2.4	3.1	2.6	2.0	2.5
2.0	2.4	1.8	2.0	2.0	2.4
1.3	2.9	1.9	2.7	2.6	1.4
1.8	1.7	1.8	2.0	2.3	2.0
1.9	3.7	1.7	2.5	2.8	2.1
2.1	2.2	2.6	1.9	2.8	2.5
avg 2.2	avg 2.5	avg 2.4	avg 2.2	avg 2.4	avg 2.1
s = 0.47		s = 0.50		s = 0.23	

*to convert data to torque, multiply by 1/4 inch

MARSH GRAPEFRUIT

TORSIONAL REMOVAL TESTS
(POUNDS ON 1/4-INCH RADIUS)*

Feb. 25		Mar. 18		Apr. 24		June 8	
0#	10#	0#	10#	0#	10#	0#	10#
2.2	2.5	2.0	0.0	2.2	1.9	3.2	1.7
2.1	2.7	1.8	1.6	1.9	1.4	2.5	3.6
2.1	2.0	2.5	0.0	3.0	2.1	1.7	2.1
2.5	1.5	1.4	1.6	1.7	1.2	2.4	0.0
1.6	1.9	2.4	2.0	1.7	1.5	1.4	1.5
3.0	2.2	2.2	2.5	2.4	1.4	2.1	1.4
2.3	1.5	1.6	2.5	1.9	1.7	1.8	1.0
2.8	1.2	2.0	1.2	2.1	1.9	1.7	0.0
2.6	1.4	2.4	1.3	1.6	1.3	2.0	2.5
2.7	1.8	1.6	1.5	2.5	2.2	2.3	1.0
3.1	1.3	1.5	2.3	1.6	1.5	3.4	1.4
1.8	1.1	1.7	1.5	1.6	1.8	1.8	1.4
2.5	1.5	1.7	2.0	1.5	2.6	2.4	0.0
2.7	2.0	1.5	1.7	1.6	1.6	1.4	1.6
1.8	2.6	1.3	1.7	1.7	2.5	2.2	1.8
3.4	2.6	1.6	2.2	1.4	1.6	1.9	1.7
2.2	1.7	2.0	1.7	2.1	1.4	1.6	1.3
2.5	1.6	1.4	2.4	1.3	1.5	1.7	0.0
2.0	2.0	1.5	1.1	1.4	1.5	1.5	1.4
1.4	1.9	2.1	2.2	1.4	2.2	2.9	1.1
2.0	1.5	1.7	2.6	2.0	1.7	1.9	2.1
1.8	1.7	2.0	1.5	1.3	1.6	2.3	3.4
2.5	1.9	2.5	4.9	1.1	1.0	4.2	0.0
2.8	1.6	1.8	2.8	1.4	2.0	2.6	1.4
2.6	2.0	1.6	1.3	1.9	2.1	3.3	2.7
avg 2.4	avg 1.8	avg1.8	avg 1.7	avg1.8	avg 1.7	avg2.3	avg1.4
s = 0.22		s = 0.32		s = 0.17		s = 0.73	

*to convert data to torque, multiply by 1/4-inch.

NAVEL ORANGES

TORSIONAL REMOVAL TESTS
(POUNDS ON 1/4 INCH RADIUS)*

Dec. 22			Dec. 29		Jan. 31	
0#	8#	0#	2#	8#	0#	8#
2.7	3.2	4.0	2.1	2.7	4.2	2.0
2.5	2.9	2.4	3.1	2.3	3.6	1.6
2.6	2.8	4.1	1.9	2.5	3.2	3.4
2.2	2.1	3.4	2.8	3.7	4.1	3.1
2.8	2.5	2.6	2.2	3.5	2.3	1.2
3.1	3.7	3.0	3.2	4.7	2.4	2.2
3.5	1.7	2.8	4.4	2.7	3.3	4.1
2.6	1.7	2.4	1.7	2.8	3.1	3.1
1.6	1.9	3.0	3.9	2.1	3.4	3.6
2.6	3.9	2.0	2.1	3.9	2.3	2.0
3.6	1.9	3.1	2.3	4.5	5.0	2.5
2.1	2.1	2.5	3.7	8.3	2.3	2.1
3.0	2.1		2.6	2.9	4.3	4.2
2.1	2.1		1.6	2.9	5.0	2.3
1.8	2.6		3.4	4.0	3.0	1.3
	1.8		3.3	2.7	2.4	0.0
	2.6		2.2	3.9	2.6	2.3
	1.7		2.9	3.9	3.6	1.6
	2.6		2.3	3.5	3.1	4.3
	3.2		2.3	1.4	5.0	3.4
	2.6		2.3	2.6	4.2	4.0
	2.1		4.4	3.3	2.1	2.4
	2.1		3.1	3.7	4.1	2.3
	2.2		2.9	2.9	2.0	3.5
	3.4		3.8	3.4	3.2	2.2
avg 2.6	avg 2.5	avg 2.9	avg 2.8	avg 3.4	avg 3.3	avg 2.6

s = 1.13 s = 1.01

*to convert data to torque, multiply by 1/4-inch.

VIBRATORY REMOVAL TESTS

MARSH GRAPEFRUIT

DATE	FREQUENCY	STEM LENGTH (W.)	PER CENT FRUIT REMOVED	% REMOVED WITH BUTTON ON FRUIT	% REMOVED WITH STEM ON FRUIT
4/23	270	2	16	4	4
4/23	330	2	92	32	0
4/23	510	2	100	-	-
5/15	450*	1	56	-	-
5/15	490*	1	72	4	0
5/15	720*	1	100	-	-
6/7	400*	1	0	0	0
6/7	600	1	96	4	4
6/7	220	3	28	4	0
6/7	260	3	92	8	0
6/7	310	3	100	0	0
6/7	460	3	100	32	0
6/27	200	2	12	-	-
6/27	240	2	44	16	0
6/27	270	2	96	-	-
6/27	300*	2	0	0	0
6/27	350*	2	48	8	0
6/27	400*	2	100	-	-
6/27	190	3	4	0	0
6/27	240	3	80	16	0
6/27	260	3	100	-	-

* one-inch stroke used on these tests. All other tests at two-inch stroke.

VIBRATORY REMOVAL DATA

VALENCIA ORANGES

DATE	FREQUENCY	STEM LENGTH	PER CENT FRUIT REMOVED	% REMOVED WITH BUTTON ON FRUIT	% REMOVED WITH STEM ON FRUIT
4/23	330	1	48	4	36
4/23	510	1	100	68	20
6/7	310	2	0	0	0
6/7	350	2	40	16	20
6/7	390	2	92	36	40
6/7	510	2	100	60	8
6/7	310	3	0	0	0
6/7	390	3	84	68	8
6/7	720	3	100	36	32
6/27	220	1	0	-	-
6/27	260	1	44	24	4
6/27	320	1	100	44	48
6/27	570*	1	16	0	0
6/27	680*	1	68	16	8
6/27	720*	1	92	28	12
6/27	260	2	0	-	-
6/27	330	2	44	24	8
6/27	390	2	100	-	-
6/27	310	3	12	12	0
6/27	350	3	72	20	0
6/27	390	3	84	56	4

*one-inch stroke used on these tests. All other tests at two-inch stroke.

APPENDIX B

NORMAL MODES OF VIBRATION

The differential equation for transverse oscillations of thin beams is,

$$\frac{\partial}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = -\frac{\rho a}{g}\frac{\partial^2 y}{\partial t^2}. \qquad \ldots (1)$$

This well known differential equation, (11) and (12), is derived from considering the forces on a differential element along the beam.

A system is said to be in a normal mode of vibration when every particle of the system is undergoing simple harmonic motion in the same period and phase. The amplitude of the particle, U, is a function of x, the distance along the beam. Thus, the motion of the beam can be represented by,

$$y = U\sin pt. \qquad \ldots (2)$$

Substitution of (2) into (1) allows the determination of U.

$$EI\frac{d}{dx^4}(U\sin pt) = -\frac{\rho a}{g}U\frac{d^2}{dt^2}(\sin pt)$$

$$EI\frac{d^4U}{dx^4}\sin pt = \frac{\rho a}{g}Up^2\sin pt$$

$$E a k^2 \frac{d^4U}{dx^4} = \frac{\rho a}{g} p_N^2 U \qquad \text{WHERE} \quad I = a k^2$$

$$\frac{d^4U}{dx^4} = \frac{\rho p_N^2}{E k^2 g} U \qquad \text{WHERE} \quad m^4 = \frac{\rho p_N^2}{E k^2 g} \quad \ldots (3)$$

To solve for U, we assume a solution of the form,

$$U = A e^{mx}.$$

In (3) this becomes,

$$A m^4 e^{mx} = m^4 A e^{mx}$$

and,

$$m = \pm m_1 \pm i m \qquad \text{WHERE } i = (-1)^{1/2}.$$

Therefore U becomes,

$$U = A_1 e^{mx} + A_2 e^{-mx} + A_3 e^{imx} + A_4 e^{-imx}.$$

Recognizing that,

$$A_1 e^{mx} = (\cosh mx + \sinh mx) A_1$$

$$A_2 e^{-mx} = (\cosh mx - \sinh mx) A_2$$

$$A_3 e^{imx} = (\cos mx + i \sin mx) A_3$$

$$A_4 e^{-imx} = (\cos mx - i \sin mx) A_4$$

we have,

$$U = (A_1 + A_2) \cosh mx + (A_1 - A_2) \sinh mx + (A_3 + A_4) \cos mx + i (A_3 - A_4) \sin mx .$$

Letting

$$(A_3+A_4)=A$$

$$i(A_3-A_4)=B$$

$$(A_1+A_2)=C$$

$$(A_1-A_2)=D$$

we have,

$$U=A\cos mx+B\sin mx+C\cosh mx+D\sinh mx. \qquad \ldots (4)$$

Equation (4) gives the amplitude of the normal mode vibration of a thin beam. The amplitude is a function of x, the distance along the beam, m, which is a function of the physical properties and normal mode frequency, and four constants A, B, C, and D. The four constants may be determined by boundary conditions for the particular beam problem. The boundary conditions are discussed in detail in the text and will be presented here as derived. They are,

$$U=0 \qquad \text{AT } X=0 \qquad \ldots (5)$$

$$\frac{du}{dx}=0 \qquad \text{AT } X=0 \qquad \ldots (6)$$

$$\frac{d^2U}{dx^2}=\frac{m^4RW}{\rho a}U \qquad \text{AT } X=l \qquad \ldots (7)$$

$$\frac{d^3U}{dx^3} = -\frac{m^4 W}{a\rho} U \qquad \text{AT } x = l. \qquad \ldots (8)$$

These four boundary conditions determine A, B, C, and D. Using boundary condition (5) we have,

$$0 = A + C.$$

Boundary condition (6) gives

$$0 = Bm + Dm.$$

Using the relation between A and C, and B and D, equation (4) becomes,

$$U = A(\cos mx - \cosh mx) + B(\sin mx - \sinh mx). \qquad \ldots (9)$$

Differentiating (9) twice with respect to x we have

$$\frac{d^2U}{dx^2} = Am^2(-\cos mx - \cosh mx) + Bm^2(-\sin mx - \sinh mx).$$

Using boundary condition (7),

$$\frac{d^2U}{dx^2} = -Am^2(\cos ml + \cosh ml) - Bm^2(\sin ml + \sinh ml) = \frac{m^4 l^2 W}{\rho a} U. \qquad \ldots (10)$$

From equation (9) we may evaluate U at $x = l$.

$$U = A(\cos ml - \cosh ml) + B(\sin ml - \sinh ml).$$

Substitution of this value of U into equation (10) gives

$$-Am^2(\cos ml + \cosh ml) - Bm^2(\sin ml + \sinh ml) = \frac{m^4 l^2 W}{\rho a}\left[A(\cos ml - \cosh ml) + B(\sin ml - \sinh ml)\right].$$

Combining under like co-efficients,

$$A\left[-(\cos ml+\cosh ml)+\frac{m^2RW}{a\rho}(\cosh ml-\cos ml)+\right.$$
$$\left.B\left[-(\sin ml+\sinh ml)+\frac{m^2RW}{a\rho}(\sinh ml-\sin ml)\right]=0.\right. \qquad \ldots (11)$$

Now, differentiating (9) three times, we have,

$$\frac{d^3U}{dx^3}=-Am^3(-\sin mx+\sinh mx)-Bm^3(\cos mx+\cosh mx).$$

From boundary condition (8) we obtain,

$$-Am^3(-\sin ml+\sinh ml)-Bm^3(\cos ml+\cosh ml)=\frac{-m^4W}{a\rho}U. \qquad \ldots (12)$$

Evaluation of U at x = 1 and substitution into (12) yields,

$$-Am^3(-\sin ml+\sinh ml)-Bm^3(\cos ml+\cosh ml)=\frac{-m^4W}{a\rho}\left[A(\cos ml-\cosh ml)+B(\sin ml-\sinh ml)\right].$$

Combining under like co-efficients we have,

$$A\left[(\sinh ml-\sin ml)+\frac{mW}{a\rho}(\cosh ml-\cos ml)\right]+$$
$$B\left[(\cosh ml+\cos ml)+\frac{mW}{a\rho}(\sinh ml-\sin ml)\right]. \qquad \ldots (13)$$

At this point consider equations (11) and (13). Solving for the ratio, A/B, in (11) and equating it to the ratio of A to B from (13), we eliminate A and B. From equation (11),

$$\frac{A}{B}=\frac{-\left[-(\cos ml+\cosh ml)+(m^2RW/\rho a)(\cosh ml-\cos ml)\right]}{\left[-(\sin ml+\sinh ml)+(m^2RW/\rho a)(\sinh ml-\sin ml)\right]}.$$

From equation (13),

$$\frac{A}{B} = \frac{-[(\sinh ml - \sin ml) + (mW/\rho a)(\cosh ml - \cos ml)]}{[(\cosh ml + \cos ml) + (mW/\rho a)(\sinh ml - \sin ml)]}.$$

Letting $z = ml$ and $C = \frac{W}{\rho a l}$ = weight of fruit/weight of stem, we have

$$\frac{-[-(\cos z + \cosh z) + (R/l)Cz^2(\cosh z - \cos z)]}{[-(\sin z + \sinh z) + (R/l)Cz^2(\sinh z - \sin z)]} = \frac{-[(\sinh z - \sin z) + Cz(\cosh z - \cos z)]}{[(\cosh z + \cos z) + Cz(\sinh z - \sin z)]} \quad (14)$$

We shall proceed to solve this equation for z. Let us first digress and reason why we wish to find z. We have defined,

$$z = ml$$

and,

$$m^4 = \frac{\rho R_\omega^2}{E k^2 g}.$$

Therefore,

$$R_\omega = \frac{z^2}{l^2}\left(\frac{gEk^2}{\rho}\right)^{1/2}. \quad \ldots (15)$$

The frequency of the normal mode vibration, , is proportional to z^2 and other physical properties of the fruit-stem system. Hence, if we can determine z, we may determine the normal mode or, as sometimes called, fundamental mode frequencies of the fruit-stem system.

Proceeding with equation (14),

$$-[(\sinh z - \sin z) + Cz(\cosh z - \cos z)][-(\sin z + \sinh z) + (R/l)Cz^2(\sinh z - \sin z)]$$
$$= -[-(\cos z + \cosh z) + (R/l)Cz^2(\cosh z - \cos z)][(\cosh z + \cos z) + Cz(\sinh z - \sin z)].$$

$$-(\sinh z-\sin z)(\sin z+\sinh z)+(\sinh z-\sin z)(R/l)cz^2(\sinh z-\sin z)-$$
$$cz(\cosh z-\cos z)(\sin z+\sinh z)+cz(\cosh z-\cos z)(R/l)cz^2(\sinh z-\sin z)=$$
$$-(\cos z+\cosh z)^2-(\cos z+\cosh z)cz(\sinh z-\sin z)+(R/l)cz^2(\cosh z-\cos z)(\cosh z+\cos z)+$$
$$(R/l)c^2z^3(\cosh z-\cos z)(\sinh z-\sin z):$$

$$[-(\sinh z-\sin z)(\sin z+\sinh z)+(\cos z+\cosh z)^2]+cz[-(\cosh z-\cos z)(\sin z+\sinh z)+$$
$$+(\cos z+\cosh z)(\sinh z-\sin z)]+(R/l)cz^2[(\sinh z-\sin z)^2+$$
$$-(\cosh z-\cos z)(\cosh z+\cos z)]+(R/l)c^2z^2[(\cosh z-\cos z)(\sinh z-\sin z)+$$
$$-(\cosh z-\cos z)(\sinh z-\sin z)]=0.$$

$$[-\sin z\sinh z-\sinh^2 z+\sin^2 z+\sin z\sinh z+\cos^2 z+2\cos z\cosh z+\cosh^2 z]+$$
$$+cz[-\cosh z\sin z-\cosh z\sinh z+\cos z\sin z+\cos z\sinh z-\cos z\sin z+$$
$$+\cosh z\sinh z-\cosh z\sin z]+(R/l)cz^2[\sinh^2 z-2\sin z\sinh z+\sin^2 z+$$
$$-\cosh^2 z-\cos z\cosh z+\cos z\cosh z+\cos^2 z]+(R/l)c^2z^3[0]=0.$$

Using,

$$\cos^2 z+\sin^2 z=1$$

$$\cosh^2 z-\sinh^2 z=1$$

further simplification yields,

$$[1+1+2\cos z\cosh z]+cz[-2\cosh z\sin z+2\sinh z\cos z]+$$
$$+(R/l)cz^2[-1-2\sin z\sinh z+1]=0.$$

This becomes,

$$[1+\cos z\cosh z]-cz[\cosh z\sin z-\sinh z\cos z]-(R/l)cz^2(\sin z\sinh z)=0.$$

Finally,

$$\frac{1+\cos z \cosh z}{\cosh z \sin z - \sinh z \cos z} = Cz + \left(\frac{R}{l}\right) Cz^2 \frac{\sin z \sinh z}{\cosh z \sin z - \sinh z \cos z} \quad \ldots (16)$$

As stated in the text, this equation was solved for z using an iterative procedure on the digital computer. Values of z determined can be placed in equation (15) to determine fundamental frequencies. Equation (16) is satisfied by an infinite number of "z"'s. The lowest value of z satisfying the equation determines the first fundamental frequency, the next value of z satisfying the equation determines the second fundamental frequency, and so on.

Values of z could have been determined from equation (14) just as easily as from equation (16). Prescott, (11) however, has derived a similar fundamental mode frequency equation for the same system which assumes the moment induced at the stem end, at x = l, is zero. This results in boundary condition equation (8) being,

$$\frac{d^3U}{dx^3} = 0 \qquad \text{AT } x = l$$

Prescott's frequency equation is,

$$\frac{1+\cos z \cosh z}{\cosh z \sin z - \sinh z \cos z} = Cz \quad \ldots (17)$$

Equation (16) predicts slightly lower values for first fundamental mode frequencies than does equation (17). This lowering effect is more pronounced for fruit-stem systems having short stem lengths.

STEADY-STATE FORCED VIBRATION ANALYSIS

Under steady-state forced vibration an additional inertial load per unit length is added to the system. This load is,

$$\left(\frac{\rho a}{g}\right) a_o p^2 \sin pt$$

The differential equation of motion for forced vibration now becomes,

$$EI\frac{\partial^4 y}{\partial x^4} + \frac{\rho a}{g}\frac{\partial^2 y}{\partial t^2} = \left(\frac{\rho a}{g}\right) a_o p^2 \sin pt \qquad \ldots (18)$$

At steady-state conditions, the solution of this equation is of the form,

$$y = U \sin pt \qquad \ldots (19)$$

Note that the assumed deflection, y, conforms with steady-state assumptions that all portions of the system are vibration at the same frequency and the amplitude is a function of a linear dimension in the system only. Substitution of (19) into (18) gives,

$$EI\frac{d^4 U}{dx^4} - \left(\frac{\rho a}{g}\right) p^2 U = \left(\frac{\rho a}{g}\right) a_o p^2 \qquad \ldots (20)$$

The homogeneous solution of equation (20) is derived in the same manner as the normal mode solution.

$$U_h = A\cosh mx + B\sinh mx + C\cos mx + D\sin mx$$

The particular solution is chosen as

$$U_p = -a_0.$$

The total steady-state forced vibration amplitude solution is,

$$U = A\cosh mx + B\sinh mx + C\cos mx + D\sin mx - a_0 \quad \ldots (21)$$

The boundary conditions are the same as those derived in the normal mode vibration problem. They are,

$$U = \frac{dU}{dx} = 0 \qquad \text{AT } x = 0 \quad \ldots (22)$$

$$\frac{d^2U}{dx^2} = \frac{m^4 RW}{\rho a} U \qquad \text{AT } x = l \quad \ldots (23)$$

$$\frac{d^3U}{dx^3} = -\frac{m^4 W}{\rho a} U \qquad \text{AT } x = l. \quad \ldots (24)$$

The boundary conditions of equations (22) and (23) substituted into (21) give,

$$A + C - a_0 = 0,$$

$$B + D = 0$$

Therefore, (21) becomes,

$$U = A(\cosh mx - \cos mx) + B(\sinh mx - \sin mx) + a_0(\cos mx - 1) \quad \ldots (25)$$

Using equation (23) and differentiating (25) twice with respect to x, we have,

$$\frac{d^2U}{dx^2} = Am^2(\cosh ml + \cos ml) + Bm^2(\sinh ml + \sin ml) - a_0 m^2 \cos ml \quad \ldots (26)$$
$$= \frac{m^4 RW}{\rho a} U.$$

Using equation (24) and differentiating (25) three times with respect to x, we have,

$$\frac{d^3U}{dx^3} = Am^3(\sinh ml - \sin ml) + Bm^3(\cosh ml + \cos ml) + a_0 m^3 \sin ml \quad \ldots (27)$$
$$= -m^4 WU/\rho a.$$

Using equation (25) for the right hand side of equations (26) and (27) respectively, we have,

$$A[(\cosh ml + \cos ml) - \frac{m^2 R W}{\rho a}(\cosh ml - \cos ml)] + B[(\sinh ml + \sin ml) - \frac{m^2 R W}{\rho a}(\sinh ml - \sin ml)]$$
$$+ a_0[-\cos ml - \frac{m^2 R W}{\rho a}(\cos ml - 1)], \quad \ldots (28)$$

and,

$$A[(\sinh ml - \sin ml) + \frac{m W}{\rho a}(\cosh ml - \cos ml)] + B[(\cosh ml + \cos ml) + \frac{m W}{\rho a}(\sinh ml - \sin ml)] +$$
$$+ a_0[\sin ml + \frac{m W}{\rho a}(\cos ml - 1)]. \quad \ldots (29)$$

At this point rewrite equations (28) and (29) and designate the co-efficients of A and B as,

$$A[W] + B[X] = K_1$$
$$A[Y] + B[Z] = K_2$$

Solving these two equations for A and B, we obtain,

$$A = \frac{(K_1[Z] - K_2[X])/[W]}{[Z] - \frac{[X][Y]}{[W]}} \quad \ldots (30)$$

$$B = \frac{(K_2[W] - K_1[Y])/[W]}{[Z] - \frac{[X][Y]}{[W]}} \quad \ldots (31)$$

We will now procede to solve for A and B so that equation (25) may be evaluated. Let,

$$K_1[Z] - K_2[X] = A_1$$

Therefore,

$$A_1 = \left[(\cosh ml + \cos ml) + \frac{mW}{\rho a}(\sinh ml - \sin ml)\right]\left[a_0 \cos ml + a_0 \frac{m^2 RW}{\rho a}(\cos ml - 1)\right] +$$

$$+\left[a_0 \sin ml + a_0 \frac{mW}{\rho a}(\cos ml - 1)\right]\left[(\sinh ml + \sin ml) - \frac{m^2 RW}{\rho a}(\sinh ml - \sin ml)\right].$$

Let $z = ml$ and $c = \frac{W}{\rho a l}$ = weight of fruit/weight of stem

Then,

$$A_1 = \left[a_0 \cos z(\cosh z + \cos z) + a_0 (z(\sinh z - \sin z)(\cos z) + \right.$$
$$+ a_0 (R/l)(z^2(\cos z - 1)(z(\sinh z - \sin z)$$
$$\left. + a_0 (R/l)(z^2(\cos z - 1)(\cosh z + \cos z)\right] +$$
$$+\left[a_0 \sin z(\sinh z + \sin z) - a_0 (R/l)(z^2 \sin z(\cosh z - \sin z) + \right.$$
$$\left. + a_0 (z(\cos z - 1)(\sinh z + \sin z) - a_0 (z(\cos z - 1)(R/l)(z^2(\sinh z - \sin z)\right].$$

$$A_1 = \left[a_0 \cos z \cosh z + a_0 \cos^2 z + a_0 \sin z \sinh z + a_0 \sin^2 z\right] +$$
$$+ a_0 (z\left[\sinh z \cos z - \sin z \cos z + \cos z \sinh z + \cos z \sin z - \sinh z - \sin z\right] +$$
$$+ a_0 (R/l)(z^2\left[\cos z \cosh z + \cos^2 z - \cosh z - \cos z - \sin z \sinh z + \sin^2 z\right] +$$
$$+ a_0 (R/l)(z^2(z\left[\cos z \sinh z - \cos z \sin z - \sinh z - \sin z - \cos z \sinh z + \cos z \sin z + \right.$$
$$\left. + \sin z + \sinh z\right].$$

$$A_1 = a_0\left[1 + \cos z \cosh z + \sin z \sinh z\right] + a_0 (z\left[1 \cos z \sinh z - (\sin z + \sinh z)\right] +$$
$$+ a_0 (R/l)(z^2\left[1 + \cos z \cosh z - \sin z \sinh z - (\cos z + \cosh z)\right] \quad \ldots (32)$$

In equation (30) let $A_2 = [W]$

$$A_2 = \left[\cosh z + \cos z - \left(\frac{R}{l}\right)(z^2(\cosh z - \cos z)\right] \quad \ldots (33)$$

In equation (30) let $A_3 = [Z] - \frac{[X][Y]}{[W]}$. Now, refer to equation (14) in the normal mode frequency analysis. Observation of the bracketed terms indicate that it may be rewritten as,

$$\frac{[W]}{[X]} = \frac{[Y]}{[Z]}.$$

Simplifying, we obtain,

$$[Z] - \frac{[X][Y]}{[W]} = 0. \qquad \ldots (34)$$

Equation (14) was solved for values of z so that the normal mode frequencies could be determined. In other words, equation (34) is true only at values of z which correspond to the normal mode frequencies. Therefore A_3 is zero at normal mode frequencies and is equal to the numerical value of $[Z] - \frac{[X][Y]}{[W]}$ at other frequencies. Equation (14) was simplified into,

$$[Z] - \frac{[X][Y]}{[W]} = \frac{1+\cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} - Cz - \left(\frac{K}{I}\right)\left(z^2 \frac{\sin z \sinh z}{\cosh z \sin z - \sinh z \cos z}\right) = 0 \qquad (35)$$

Therefore, at forcing frequencies not equal to a normal mode frequency of the system,

$$A_3 = \frac{1+\cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} - Cz - \left(\frac{K}{I}\right)\left(z^2 \frac{\sin z \sinh z}{\cosh z \sin z - \sinh z \cos z}\right). \qquad \ldots (36)$$

Referring to equation (30), and our definitions of A_1, A_2, and A_3, we may write,

$$A = \frac{A_1}{A_2 A_3} \qquad \ldots (37)$$

Now consider equation (31). Define B_1 as,

$$B_1 = [k_2[W] - k_1[Y]]$$

$$B_1 = a_0[-\sin z - cz(\cos z - 1)][(\cosh z + \cos z) - (R/l)cz^2(\cosh z - \cos z)] +$$
$$-a_0[\cos z + (R/l)cz^2(\cos z - 1)][(\sinh z - \sin z) + cz(\cosh z - \cos z)].$$

$$B_1 = [-a_0 \sin z(\cosh z + \cos z) + a_0(R/l)cz^2 \sin z(\cosh z - \cos z) +$$
$$-a_0 cz(\cos z - 1)(\cosh z + \cos z) + a_0 cz(R/l)cz^2(\cos z - 1)(\cosh z - \cos z)] +$$
$$-a_0[\cos z(\sinh z - \sin z) + cz\cos z(\cosh z - \cos z) +$$
$$+(R/l)cz^2(\cos z - 1)(\sinh z - \sin z) + (R/l)cz^2(cz)(\cos z - 1)(\cosh z - \cos z)].$$

$$B_1 = a_0[-\sin z \cosh z - \sin z \cos z - \cos z \sinh z + \cos z \sin z] +$$
$$+a_0 cz[-\cos z \cosh z - \cos^2 z + \cosh z + \cos z - \cos z \cosh z + \cos^2 z] +$$
$$+a_0(R/l)cz^2[\sin z \cosh z - \sin z \cos z - \cos z \sinh z + \cos z \sin z + \sinh z - \sin z] +$$
$$+a_0(R/l)c^2z^3[\cos z \cosh z - \cos^2 z - \cosh z + \cos z - \cos z \cosh z + \cos^2 z + \cosh z - \cos z].$$

$$B_1 = -a_0[\sin z \cosh z + \cos z \sinh z] +$$
$$+a_0 cz[-2\cos z \cosh z + \cosh z + \cos z] +$$
$$+a_0(R/l)cz^2[\sin z \cosh z - \cos z \sinh z + \sinh z - \sin z]. \quad \ldots (38)$$

Equation (31) then becomes, using definitions of A_2 and A_3,

$$B = \frac{B_1}{A_2 A_3} . \qquad \ldots (39)$$

The total solution for the amplitude of the system in steady-state vibration is, from equations (25), (37), and (39),

$$U = \frac{A_1}{A_2 A_3}(\cosh mx - \cos mx) + \frac{B_1}{A_2 A_3}(\sinh mx - \sin mx) + A_0(\cos mx - 1) \ldots (40)$$

Hence the deflection of the fruit-stem system in steady-state vibration as a function of distance along the beam and time is,

$$y = \left[\frac{A_1}{A_2 A_3}(\cosh mx - \cos mx) + \frac{B_1}{A_2 A_3}(\sinh mx - \sin mx) + A_0(\cos mx - 1)\right] \sin pt . \ldots (41)$$

BENDING STRESS ACROSS BUTTON

The moment induced across the button by inertial forces is,

$$M = \frac{W}{g} R \frac{d^2 y}{dt^2} = -\frac{W}{g} R U p^2 \sin pt$$

Therefore the maximum moment is,

$$M_{MAX} = -\frac{W}{g} R p^2 |U|$$

Assuming the fruit weights 300 grams and the fruit radius is 1.5 inches, this reduces to,

$$M_{MAX} = (2.58 \times 10^{-3}) p^2 |U|$$

From strength of materials equations, the maximum bending stress across the button is,

$$\sigma_{MAX} = \frac{M_{MAX}(J)}{I} = \frac{M_{MAX}(1/8)}{1.20 \times 10^{-5}} \qquad \frac{M_{MAX}}{9.6 \times 10^{-5}}$$

$$\sigma_{MAX} = \frac{(2.58 \times 10^{-3})}{(9.6 \times 10^{-5})} p^2 |U|$$

$$\sigma_{MAX} = (26.9|U|) p^2$$

Since U is a function of P/P_1, the equation was plotted as σ_{MAX}/p^2 vs. P/P_1.

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