A Statistical Study of Cyclogram Analysis

With Application to Sun-spot Numbers,

the Variable Star SS Cygni, and Tree Growth

by

Edmund Schulman

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Table of Contents

Figures

PREFACE

Any work in Tree-Ring Analysis must bear acknowledgement of indebtedness at the very outset to the originator of most of the methods of tree-ring interpretation, Dr. A. E. Douglass. This is particularly true in that portion of the study which concerns the analysis of tree-ring data for the cycles they contain. It has been a privilege to work under his direction in Tree-Ring Cycle Analysis.

The present paper describes a formal study of the limitations of the Douglass Cyclograph, and makes use in large part of cycle analyses accumulated by Dr. Douglass in the course of an investigation now lasting over thirty years. The results of his investigations in the cycle analysis of tree-rings he has extensively published, particularly in Climatic Cycles and Tree Growth. In 1931 he obtained a magnificent collection, consisting of some twenty-five full sections, of the California Coast Redwoods. The complete reduction of these specimens, from the preparation, dating, and measuring, to the formation of the final periodograms, was carried through by the writer in the period 1932-34. All of the original data has been placed in the files of the Tree-Ring Laboratories at the Steward Observatory.

I. The Development of Tree-Ring Analysis[1]

When Dr. A. E. Douglass first turned his attention to the study of the annual growth-rings of trees in 1901, it was with the hope of finding a general variation in growth that might be traced back through the medium of climate to variations in solar radiation, and thus greatly extend the known history of the latter. Early work consisted largely in the acquisition of data; at the same time the best methods of collection, preparation, and reduction of specimens were established. It soon became evident that trees undergo not only a general variation in growth but that time leaves an indelible signature on the individual rings. Through the medium of crossdating part of an undated specimen with part of a dated one, an exact date could be assigned to any rings in a sequence. A precise and ever-lengthening chronology for Northern Arizona and similar ones for other regions began to emerge.

As it developed, the investigation began to make increasingly insistent calls on other sciences; botany, geology, meteorology, and astronomy are some of these. In particular, the precision in dating found a fertile domain in archaeology. The dating[2] of prehistoric

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1. Douglass, A. E. Climatic Cycles and Tree Growth. Carnegie Institution of Washington. Vol. 1, 1919; Vol. 2, 1928. These two volumes contain the major portion of the work in tree-ring analysis up to 1928; they will be referred to further simply as Vol. 1 and Vol. 2
2. Douglass, A. E. The Secret of the Southwest Solved by Talkative Tree Rings. National Geographic Magazine, December 1929, pp. 736-770.

Southwest Indian ruins through their building timbers offered inviting prospects, and proved enormously fruitful.

From the standpoint of climatology, the archaeological activity in tree-rings was of great value in furnishing a continuous, accurate, and highly sensitive record of variation in tree growth over a period of some nineteen hundred years in Arizona. Together with the records of growth of the California Sequoias extending back over three thousand years, and the tree-growth data from other parts of the world, it forms a most promising subject for an investigation of the climatological problem. This problem, which will be developed in the next section, is now again the uppermost consideration.

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The fundamental problem of tree-ring analysis may be formulated as follows, in three phases: Are there recurrence phenomena in past climate, what are the elements of such recurrence phenomena, and is there sufficient law and order in these elements to make possible the long-range prediction of future climate?

Now one evidence of at least an approach to law and order that we are acquainted with in nature is found in sun-spots, in the well-known sun-spot cycle of about eleven years. Since the phenomena of sun-spot variation are intimately connected with variations in solar radiation, it is evident that some effect of this cycle may well exist in climate. But meteorological records are on the whole much shorter than sun-spot records; probably much too short to give any real insight into what is happening in climate. On the other hand, the long records in trees offer a proper study of the problem.

Dr. Douglass discovered in the early years of the investigation that the faithfulness with which tree-rings in Northern Arizona recorded in the more favorable cases the annual fluctuations in rainfall was truly remarkable. The extension to other areas, in some cases with different dominating climatic factors than rainfall, was no simple task and is yet far from complete.

But regardless of the climatic interpretation of the cycles found in tree-growth, we are confronted with the problem of determining the nature of the cycles that may exist in climatic data or tree-growth variation. Consideration of a plot of the annual tree-growth in, let us say, Arizona or California, or even of the short meteorological records, shows that if cycles are operating they are probably quite complex. There is furthermore no particular *a priori* reason why the cycles should be regular in form or period, like the sine curve. The comparatively simple sun-spot curve is an example. Ordinary mathematical methods become extremely cumbersome, or fail completely, when the factor of unknown variability is introduced into the data on which they are to be employed.

It is for this reason that Dr. Douglass developed in 1914-15 an instrument which, after undergoing considerable evolution, is now called the cyclograph[3].

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3. Vol. 1, pp. 85-97.
Vol. 2, pp. 45-50.

The cyclograph has been in continuous use for twenty years, and has yielded information about the nature of recurrence phenomena in tree-ring and other data obtainable in no other comparable way. The confidence with which the derived cycle complex in tree-growth has been viewed by Dr. Douglass has rested on a number of general tests of the instrument. It has been known in a general way for some considerable time that all the variables affecting the cycle results obtained with the use of the cyclograph (such as smoothing, standardizing, position of the cutting line, errors of the instrument, and personal equation) did not seriously affect those results.

The last major proof of this, particularly with reference to the latter two factors mentioned, was carried through in January, 1928[4]. The cycles existing in three groups of Western United States tree-ring growth curves (Rocky Mountains, Arizona, and Pacific Coast) had been obtained and put in periodogram* form in 1926. The Western Groups comprise 305 trees, in 42 sub-groups, which fall into three general areas as already stated. The cycles found in each of the sub-groups were summed with their proper weights and formed into three periodograms. The higher the curve, the more often and the stronger the cycle appeared in the area.

Now here the general test came in. The original curves of the 42

4. Douglass, A. E. Cycles in Tree Growth. Reports of the Conferences on Cycles, Washington, 1929, p. 40.

* A periodogram is simply a plot with cycle lengths as abscissae and weight or importance of the cycles as ordinates. The characteristics determining importance are discussed below.

groups were replotted by an assistant on a scale unknown to Dr. Douglass. Cycleplots* were made, and a complete series of analyses carried through, the cycle lengths so obtained being, of course, out-of-scale by an unknown amount, since the calibration of the instrument for normal scale was used in taking the readings. The results were then corrected for scale change, and put into periodograms for each area as before. The agreement found between the two sets of periodograms was quite beyond question.

There remain, however, many questions regarding the limitations of the cyclograph which this general test and others like it have not answered, at any rate not completely. For instance, by exactly how much do the cycle lengths which an observer finds at one time differ from those which he finds at another time? What is the difference between observers in the matter of results? How sure is one of getting by the cyclographic method of analysis all the cycles that actually exist in the data, and how closely do the cycles thus found correspond to the true ones?

In the following pages will be found a detailed investigation of these and similar problems carried through under the direction of Dr. Douglass in the Tree-Ring Laboratories at Tucson.

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*A cycleplot is simply a plot on heavy opaque paper 48"x4" in size; the maxima are isolated and cut out.

II. The Douglass Cyclograph: Theory

A. Description of the Instrument

No attempt will be made here to follow through all the steps in the evolution of the cyclograph from its beginning to its present form. Three stages may, however, be recognized. First came the use of the multiple plot[5]. Second, the development of the periodograph was carried through. Third, the final concentration on the differential pattern determined the form of the instrument as a cyclograph[6]. We will briefly consider the first and last of these. For a detailed discussion, the reader is directed to the references cited.

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The multiple plot, the understanding of which means the understanding of the basic principle behind the cyclograph, is quite simple. Dr. Douglass worked out in 1913 an arrangement for rapid cycle analysis to which he gave this term. Consider any curve that it is desired to subject to cycle analysis, and let the maxima be isolated by darkening them or otherwise. The multiple plot is simply an arrangement of a number of duplicates of this isolated-maxima curve, placed one below the other, each one displaced by a fixed amount. If a regular period exists, then

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5. Douglass, A. E. A Photographic Periodogram of the Sunspot Numbers. Astrophysical Journal, Vol. 40, No. 3, pp. 326-331, October 1914.
6. See reference 3.

there will be alignments of maxima; if the displacement is exactly equal to the length of the period, it is evident that the alignments will be vertical. If the recurrence is not exactly periodic, the alighments will not be straight and the individual deviations will all be apparent.

If the length of the cycle is known, then the displacement necessary in order to get a vertical alignment is known. Suppose, however, that the length is unknown, but its magnitude is suspected. It is necessary then to test through a whole range of displacements in order to determine the displacement that gives a vertical alignment which determines the length of the cycle.

But it is a considerable task to make a series of multiple plots with a complete sequence of displacements. The next step was therefore an attempt to develop an optical method for doing the same thing. The final result, the Douglass Cyclograph, possesses a power far beyond that of the original multiple plot, as will become evident in the following pages.

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The cyclograph as used at present in the Tree-Ring Laboratories is substantially the same as described by Dr. Douglass in 1928, although some minor modifications were introduced in 1931-32. In skeleton outline, the instrument has three major parts.

1. The Light Source. This consists of a horizontal series of light bulbs, in back of which is a mirror, and in front of which is a diffusing screen. This screen forms a window, measuring 44 inches horizontally by 3 inches vertically, in which may be placed for analysis a

cycleplot of any curve. All the holes in the cycleplot therefore receive a uniform illumination.

2. Movable Mirror Carriage. The mirror carriage is suspended from a two-rail track, seven feet high, which runs for about forty feet horizontally, beginning some six and a half feet from the window. The carriage holds two mirrors, 5.5 inches by 25 inches, each inclined 45° to the vertical with the reflecting surfaces toward each other and facing the window. Since the top mirror is on a level with the light source, and the lower one is on a level with the object glass of the analyzing box, running the carriage out or in has the sole effect of changing the object distance without disturbing the optical axis. The carriage runway is calibrated in terms of standard scale (see below).

3. The Analyzing Box. Immediately in front of the window is found the analyzing box, which contains most of the working parts of the instrument.

The object glass is a Tessar II B lens of six inches focal length, directly in front of a negative cylindrical lens of six inches focal length, with its axis horizontal. Therefore in the focal plane all rays in the horizontal planes come to a focus, while there is no focussing action in the vertical planes. The image formed by this system consists then of an exact reproduction of the holes in the cycleplot with respect to horizontal spacing of the centers of gravity of the holes, but with the images in the form of parallel bars of light; the width of any bar depends upon the width of the corresponding maximum, and its light intensity upon the amplitude of the maximum. This image of parallel bars is called by Dr. Douglass a "sweep".

In the focal plane is placed an analyzing plate. This consists of two glass plates having alternate transparent and opaque rulings, each pair of lines being 0.02 inches in width. Superposing the two plates makes possible any size of transparent line up to 0.01 inches, the total spacing of a pair of successive transparent and opaque lines always remaining constant at 0.02 inches. As arranged at present, the width of a transparent line is 0.002 inches. The analyzing plate is inclined at an angle of 12° to the vertical; that is, to the bars of light, in the plane perpendicular to the optical axis. It is then easily seen that the analyzing plate breaks up every vertical bar of light into segments, some of which are permitted to pass through, and some of which are held back. The resulting pattern is a series of light images or dots in several duplicate horizontal bands or sets. With a total effective rectangular aperture of 2 x 1 inches, the longer side horizontal, an inclination of 12° yields with the analyzing plate used almost exactly five of these horizontal bands in the pattern.

Behind the analyzing plate is a condensing lens system, consisting of two cylindrical lenses of six inches focal length, with their axes vertical. Near this system is a small mirror set at an angle of 45°, which throws the light to one side to an observer who views the pattern through an eye lens. The observer then has the cyclograph window to his left and the movable mirrors to his right, while he is out of the direct line of the rays. For photography, the small secondary mirror is swung out of the way and the light allowed to proceed directly to a camera compartment, containing a single set of lenses, a focussing ground glass and plate holder.

When the mirror carriage is moved, the object distance changes, and, of course, the focal plane will suffer a corresponding slight change. To keep the position of the focal place constant, the object system of the analyzing box, consisting of Tessar lens and negative cylinder, is mounted on a movable frame which receives its motion from an arm resting against a spiral wheel. This wheel turns by means of a gear meshing with a toothed runway, which in turn moves at a rate that is a small ratio of the motion of the mirror carriage. The resulting motion of the objective is then such that the change of object distance is exactly counterbalanced and the focal plane remains fixed.

To summarize: the light from the holes in the cycleplot proceeds to mirror A of the movable carriage, then to mirror B, which reflects it back, to the analysing box below the window. Here the rays are gathered by the objective, brought to a focus in one co-ordinate after passing through the negative cylinder and being neutralized in the other co-ordinate, pass through an analyzing plate and a condensing lens system and are reflected off at right angles, to an observer at the side who views the image through an eye lens; or, no secondary mirror is used, and the pattern is photographed after the rays of light pass through the lens of a camera.

B. Scale Calibration

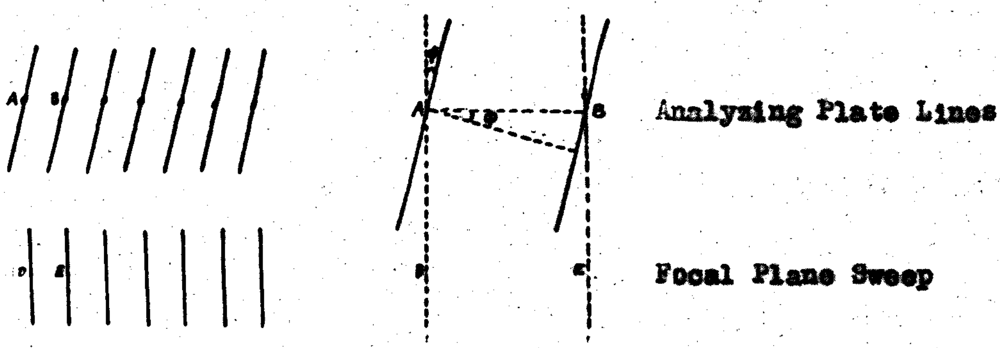
To understand how the cyclograph scale is calibrated, let us consider how the pattern or cyclogram is formed. If the maxima in the cycleplot are evenly spaced, and if the spacing is equal to the distance between two transparent lines of the analyzing plate as measured along a horizontal cross-hair, it is evident that there will be a horizontal alignment of dots. A moment's thought shows that this bears a strong resemblance to the vertical alignment in the case of the multiple plot. However, any different period in the cycleplot will give a horizontal alignment of dots in the pattern by the simple process of changing the distance of the movable mirrors. With a series of artificial curves with regularly spaced maxima plotted on the normal scale of a unit every two millimeters, it is possible to calibrate the carriage runway so that the cycle corresponding to any horizontal alignment may be read directly. The cyclograph is calibrated from 5.0 to 42.0, the smallest division being 0.1 unit.

C. Harmonics in a Cyclogram

A cyclogram pattern indicating harmonics arises from two fundamentally different causes. In one case the harmonic actually exists in the data under analysis; in the second case the harmonic is only apparent and arises from a fractionization of the pattern due to a fractional setting. We define here a unit setting as a setting of the mirrors which allows the direct reading of the cycle in the data from the calibrated mirror runway. We shall consider each case separately.

1. Harmonics Due to the Cyclograph. We here propose a criterion for judging the fractionization of a setting. If the movable mirrors are set at a point on the calibrated runway i/j of a cycle existing in a normal scale cycleplot, then there will be i times the normal number of horizontal rows in the pattern, every row consisting of a series of light dots located one to every j successive lines of the analyzing plate.

To show the validity of the above criterion, let us consider what happens when the mirrors are moved. Let the mirrors be set at the calibrated scale reading of 12.0, with a standard cycleplot in the window, containing 26 regularly spaced maxima 12.0 units apart. Then the cyclogram pattern with the present arrangement will consist of five horizontal rows of light images, each row containing 26 images appearing through 26 successive lines of the analyzing plate. In other words, the distance DE between the vertical bars of light forming the image cast by the cylindrical lens, measured at right angles to these bars, is exactly equal to the distance AC between centers of successive analyzing plate transparent lines divided by the cosine of the inclination of these lines to the vertical (DE=AB in figure).



Let the object distance now be decreased by moving the mirrors toward the window. The distance between the bars of the image will increase approximately in proportion to the fraction of change, that is, the amount of decrease of the object distance divided by the original distance, measured in terms of the cyclograph calibration. It may be measured in this manner since the scale is very closely linear; furthermore, by the method of calibration (q.v.) the image size at any setting compared to its size at any other setting is inversely proportional to the ratio of the settings.

The transparent lines as seen in the pattern viewed through the eye-lens run from upper right to lower left. Since the distance between lines is constant, it is evident that an increase in the distance between bars will result in a slant upward to the right of the rows of lights. When the mirror is set at one-half of the original setting, or 6.0, the distance between bars of light is 2 AB, and hence the dots will appear along every other analyzing plate line, the intermediate ones being blank.

With the mirrors back at 12.0, let them be moved out, increasing the object distance. The phenomena discussed above are reversed in many features. The image grows smaller and the rows of dots of light now slant downward to the right at a greater and greater angle. At 24, the double of 12, the distance between bars of light in the image will be $\frac{1}{2}$ AB. Hence, there will be in the pattern double the standard number of rows, every other row being formed of intermediate points halfway between the fundamental rows.

It is now easy to follow through the general case. Let the mirrors be set at i/j of the original cycle setting (unit setting) which gave the value of the cycle present directly. Starting from any dot of light in the pattern, the next image along a horizontal will be j analyzing lines away. To see this, consider the focal plane sweep. Since the image is now j/i of the original size, and each line is distant from the next j/i AB, and since j/i is expressed in its lowest terms, then after i lines of light, represented in the pattern by i dots in a slant line-up, the $(i\ \ 1)$th line provides the next dot of the horizontal line-up at $j/i\ \overline{AB} \times i = j\ \overline{AB}$*. That is, for every i dots on the fundamental line-up, there are j analyzing plate spaces or lines. But we started with any dot. Therefore there will be i rows for every one in the standard pattern, or 5i rows altogether.

The slant is then downward to the right if $i > j$, upward to the right of $i < j$, horizontal if $i = j$.

It may be remarked that it has been found in practice that with the exception of settings at 1/2, 1/3, x 2, and rarely 1/4 and x 3, the effect of harmonics is very slight. It is possible that the less simple harmonies determine to some extent, however, the apparent presence in real data of subordinate, weak cycles which may not be real.

2. Harmonics Actually Present. Consider again the previous arrangement, with a 12.0 unit standard cycleplot in the window, the mirrors set at 12.0, and the pattern consisting of 5 horizontal rows of light dots. Suppose we replace the 12 unit standard with another cycleplot, which also contains a 12 unit cycle and, in addition, a set of secondary maxima of smaller amplitude spaced halfway between the major maxima. We now find double the number of rows of light images in the cyclogram, every other row being relatively faint. Two secondary maxima will give two

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*A slightly different way of looking at it may be presented. We have AB the distance between analyzing lines along a horizontal. Two light dots will both be on a horizontal when the total distance between the bars of light of which they are a part is exactly nAB, where n is an integer, not zero. But the distance between each of the bars of light in the image cast by the objective is j/i AB, expressed in lowest terms. The distance between i 1 lines is therefore j AB, and hence in this case the dots corresponding to image lines (1), (i+1), (2i+1), (3i+1), ... will fall along a horizontal at analyzing plate lines 1, (j+1), (2j+1), (3j+1) ....

fainter companion rows of images. The departure from a straight line-up of the rows of dots, of course, measures the departure of the cycle from a perfect cycle, or periodicity.

It is important to note that the secondary maxima will not be of as great a mean amplitude as the fundamental; they may exactly divide the space between the fundamental crests into 2, 3, 4, ...n parts depending on the number 1, 2, 3, ..(n-1) secondary maxima, in which case there will be 5n equally spaced rows in the pattern, or they may directly follow or precede the fundamental, in which case there will be 5 similar sets of rows, the arrangement of rows within the set depending on the positions of the secondary maxima. Again, departure from straight line-up will depend upon departure of the maxima from exact spacing.

We may summarize the discussion of cyclograph harmonics by the following general criterion. An actually existing simple harmonic of a cycle will be evidenced in the cyclograph pattern at the cycle setting by a series of subordinate rows whose mean amplitude will in general be less than that of the principal rows, the subordinate rows being in general not spaced exactly between the principal ones. A simple harmonic due to the setting may be recognized by the approximately equal spacing between the lines and the approximately equal mean amplitude of the rows.

D. Procedure in Noting Cyclogram Phenomena

1. Description.

   (a) Put each analysis on a separate sheet.

   (b) Make two complete analyses over total range of movable mirrors, both out and in.

   (c) Note cycleplot designation, date of analysis, and operator.

   (d) Note time range covered by cycleplot; i.e., beginning date to ending date.

   (e) Note any special feature, such as other than normal vertical or horizontal scale, units if not in years, special cutting line.

2. Notation.

   (a) Values: Note value to nearest tenth, near left margin of paper.

   (b) Emphasis: The range of emphasis is from half normal value for the very weak cycles to four times normal value for the very strongest cycles. The determining factors are regularity of period, number of crests, regularity of amplitude, strength of amplitude, kind of fractionization, if any. If the parentheses are taken to represent cycle values, then the notation as regards emphasis may be summarized in the following form:

Grade of Cycle	Notation	Value in Periodogram Table
Poor	x( )	1
Normal	( )	2
Good	$(\underline{\quad})$	4
Excellent	$(\underline{\underline{\quad}})$	6
Perfect	$(\underline{\underline{\underline{\quad}}})$	8

   (c) Fractionization: If cycle setting is at twice real value of cycle denote by (   ) ($\frac{1}{2}$). Similarly, for settings at other multiples, as $(\frac{1}{1})$, $(\frac{1}{3})$, $(\frac{1}{4})$. If setting is at half cycle value, denote by (   ) (x2). Similarly for other fractions, as (x3), (x4). If fractionization is present only in part, denote by (oc $\frac{1}{2}$) or (much $\frac{1}{2}$) (first notation means occasionally $\frac{1}{2}$), or note range.

(d) Range: For ordinary work it quite suffices to obtain range without reference to cycleplot. This is done by mentally dividing the width of the pattern into ten segments, and noting which segments are concerned; for example, (0-7). If the cycleplot is put in window in normal position (time increasing from left to right) the time scale in the pattern will increase from right to left; zero is therefore at the right, ten at the left as seen in the eyepiece. The range may be read directly from the cycleplot, by running a narrow opaque strip in front of the window, viewing its effect in the pattern and stopping the strip at the limits indicated.

(e) Remarks: Note special features of pattern and, if unusual, draw skeleton picture to right of preceding notation.

Sample Notes.

8.3		(0-8) weak in center.
x12.4	(cc½)	(3-6) intermittent and irregular.

E. The Reduction of Cycle Data

The formation of tables, plots, or curves representing a summation process applied to cycle data may take a number of forms. By summation of cycle data many general features become apparent which are otherwise hidden; it is therefore proper to examine some of the methods of formation and types of cycle diagrams.

The most common cycle summation is that of Schuster[7], who called his result a periodogram and applied it to sunspots. By the method of successive integrations he obtained a value for the amplitude of mean trial cycles over a considerable range; plotting these amplitudes as ordinates against cycle lengths as abscissae, he produced a curve whose

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7. Schuster, A. The Investigation of Hidden Periodicities. Terrestrial Magnetism, 1898, Vol. 3, p. 24.

maxima indicated the presence of cycles of strength corresponding to their height in the curve or periodogram.

To produce a periodogram representing the analyses of a number of curves, some additional features are necessary. The procedure in the Tree-Ring Laboratories is to first make up a cycle diagram. Each cycle in every analysis of the series is entered in the diagram with its weight, the numerical value of which is that given in the preceding section. To expedite the formation of the diagram, the cycle weights are represented by symbols as follows:

Type	Weight	Symbol	
Poor	1	-	dash
Normal	2	.	dot
Good	4	x	cross
Excellent	6	⊗	circled cross
Perfect	8	⊗	double-circled cross

A plot is made on the standard 2 mm. graph paper used in the Laboratories; cycle lengths, one scale unit per cm., are plotted as abscissae, and all the cycle values in any one curve analysis plotted along one ordinate, the smallest interval being one-tenth unit. It is then easy to sum up the columns, mentally transforming symbols into numbers, and obtain a set of periodogram numbers. These are then plotted in the usual Schuster periodogram form.

It is evident from the optical nature of the cyclogram method of cycle analysis that there is a logarithmically decreasing sensitivity in assigning a cycle value with increasing value of the cycle. Hence, the abscissae of the periodogram are taken to represent a logarithmically increasing range from 5 to 42; in practice at the Laboratories a range of 5% is used. The limiting values for every interval, to the nearest tenth, and the mean value of the interval, for both standard and intermediate points, are given in Table I. The periodogram number for any range may be obtained directly from the cycle table.

The method of compiling table I is as follows: First find the difference in the logarithms corresponding to a 5% range in the numbers; e.g., $\log_{10} 10.25 - \log_{10} 9.75 = 0.02172$. By adding 0.02172 to any lower limit logarithm, an upper limit is obtained which bounds (with the lower limit) a range of 5% about the mid-point of the inverval. The upper limit is taken to the nearest tenth, since this is the lowest division on the cyclograph. The table of intermediate points was obtained by approximation from the first table.

It is often of value to plot the individual cycle lengths into other types in addition to the one described above and called a cycle diagram. For instance, a long sequence of data may be segmented into several cycleplots, each representing an equal time interval; if the sets of cycle analyses are plotted in suitable symbols with the ordinates corresponding to the successive segments equally spaced, then slant alignments may become evident which would indicate cycle flow; i.e., gradual change of cycle values. Again, by using time as abscissae and cycle length as ordinate, it is possible to represent graphically the exact range of occurrence of the cycles in any data. Examples of this may be found in the graphs connected with the application to sunspots and SS Cygni in this paper.

Table I. Periodogram Limits

Standard Abscissae

Absc.	Range		Interval
5.10	5.0	5.2	0.3
5.40	5.3	5.5	0.3
5.75	5.6	5.9	0.4
6.15	6.0	6.3	0.4
6.55	6.4	6.7	0.4
6.95	6.8	7.1	0.4
7.40	7.2	7.6	0.5
7.90	7.7	8.1	0.5
8.40	8.2	8.6	0.5
8.90	8.7	9.1	0.5
9.45	9.2	9.7	0.6
10.05	9.8	10.3	0.6
10.65	10.4	10.9	0.6
11.30	11.0	11.6	0.7
12.00	11.7	12.3	0.7
12.70	12.4	13.0	0.7
13.45	13.1	13.8	0.8
14.25	13.9	14.6	0.8
15.10	14.7	15.5	0.9
16.00	15.6	16.4	0.9
16.90	16.5	17.3	0.9
17.85	17.4	18.3	1.0
18.85	18.4	19.3	1.0
19.90	19.4	20.4	1.1
21.05	20.5	21.6	1.2
22.25	21.7	22.8	1.2
23.50	22.9	24.1	1.3
24.80	24.2	25.4	1.3
26.15	25.5	26.8	1.4
27.60	26.9	28.3	1.5
29.15	28.4	29.9	1.6
30.75	30.0	31.5	1.6
32.40	31.6	33.2	1.7
34.15	33.3	35.0	1.8
36.00	35.1	36.9	1.9
37.95	37.0	38.9	2.0
40.00	39.0	41.0	2.1
42.05	41.1	43.2	2.2

Intermediate Abscissae

Absc.	Range		Interval
5.20	5.1	5.3	0.3
5.55	5.4	5.7	0.4
5.95	5.8	6.1	0.4
6.35	6.2	6.5	0.4
6.75	6.6	6.9	0.4
7.20	7.0	7.4	0.5
7.70	7.5	7.9	0.5
8.20	8.0	8.4	0.5
8.70	8.5	8.9	0.5
9.20	9.0	9.4	0.5
9.75	9.5	10.0	0.6
10.35	10.1	10.6	0.6
10.95	10.7	11.2	0.6
11.60	11.3	11.9	0.7
12.30	12.0	12.6	0.7
13.05	12.7	13.4	0.8
13.85	13.5	14.2	0.8
14.65	14.3	15.0	0.8
15.50	15.1	15.9	0.9
16.40	16.0	16.8	0.9
17.35	16.9	17.8	1.0
18.35	17.9	18.8	1.0
19.40	18.9	19.8	1.0
20.45	19.9	20.9	1.1
21.55	21.0	22.1	1.2
22.75	22.2	23.3	1.2
24.00	23.4	24.6	1.3
25.35	24.7	26.0	1.4
26.75	26.1	27.4	1.4
28.20	27.5	28.9	1.5
29.75	29.0	30.5	1.6
31.40	30.6	32.2	1.7
33.15	32.3	34.0	1.8
35.00	34.1	35.9	1.9
36.95	36.0	37.9	2.0
39.00	38.0	40.0	2.1
41.15	40.1	42.2	2.2

F. The Nature of the Pattern

If the cycle present in the data is not exactly periodic, that is, with all the maxima equally spaced, then since there exists a mean value of the cycle there will be a mean alignment in the pattern. Departures from this mean horizontal position of the dots indicate a slight lag or advance in the appearance of the maximum concerned; hence, the term differential pattern has been applied* by Dr. Douglass. The closer the clustering of the light dots about the straight line, the closer the cycle approaches a perfect cycle or periodicity. It is analogous to the clustering of dots about a regression line in a scatter diagram as a measure of correlation, though the cyclogram pattern itself is far more closely kin to the familiar O-C diagram. It is evident that a change of period will be represented by a different trend in the corresponding range in the pattern, demanding a different position of the mirrors to bring that range into horizontality. Logarithmically varying cycles will form a curved line pattern.

From what has been said it becomes evident that the cyclograph admits a determination of the elements of cycles having a range which does not cover the entire data, with a length not necessarily constant, with changes of phase at any point, and with variable amplitude.

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*About 1914-15.

III. The Douglass Cyclograph: Limitations

A. Errors of the Instrument

The errors of the instrument fall into two classes, those involved in the building of the cycleplot and those due to manipulation of the optical elements of the cyclograph.

Since the presence of mistakes and other gross errors in the reduction of tree-ring data to a curve will affect the values of the cycles found, it is well to consider the magnitude of the effect. Given a suitable selection of tree-ring specimens according to the principles of tree-ring science, measurements are made of ring widths; in the Tree-Ring Laboratories the standard measuring engine* is now used almost exclusively. Mistakes rarely occur in measurement, due to the checking of decade sums by means of a subordinate scale on the measuring engine; these together with plotting errors may quite conservatively be postulated to be only second order errors, a gross error in any single ring being exceedingly rare. And since the individual fluctuations are greatly diminished in importance by smoothing anyway, it is evident that this sort of error has a vanishingly small effect on the cycles found.

Many growth curves show trends due to ageing of the tree, sudden favorable or unfavorable change in environmental factors and the like,

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*This is a modification of the cathetometer described in Vol. I, pp. 58-9, employing the same principle.

which are meaningless in terms of cycles, and hence are removed by standardizing. This process is accomplished by transforming the actual measured values to percentage departures from a mean or standardising line containing the trends it is felt should be removed. This line is run through the data by estimation. It has been found that standardizing lines drawn by Dr. Douglass when compared with lines through the same data drawn by the writer differ to only a small degree; however, it is evident that even a relatively large difference will have practically no effect on the lengths of the cycles found, and only a small effect on the amplitudes. This is particularly true in light of the practice of the Laboratories to keep the standardizing line as free from bends or "kinks" as the data permits.

Smoothing[8] of data is accomplished by means of the formula $\frac{A_{n-1} + 2A_n + A_{n+1}}{4}$. Geometrically this is equivalent to taking a smoothed point as the midpoint of the altitude of the triangle whose apex is one ordinate and whose base is the line joining the two adjacent ordinates. The errors involved in the very rapid smoothing of a curve by an experienced operator are vanishingly small, and have been found to have no effect on either the length or the amplitude of any cycles present.

By far the most serious of the possible sources of error lies in the cutting line. After a curve is smoothed, it is transferred by means of carbon tissue to a strip of brown opaque paper called cycleplot paper, some 48 inches long by 4 inches wide, and therefore slightly larger than the window of the cyclograph. A base line is then drawn on the curve;

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8. Vol. 2, pp. 43-44.

this is called a cutting line, and the maxima that it isolates are cut out with a razor blade. In the earlier forms this cutting line was highly irregular in form. It often had the effect, realized only some years after its first use by Dr. Douglass, of removing some of the longer cycles present in the data. However, the present practice of the Laboratories is to use a cutting line that approaches or is a straight base line, usually parallel to the horizontal axis and approximately one-third of the distance from mean minimum to mean maximum. It is put in by estimation. Those minor maxima which lie below the cutting line are taken care of by cutting off only the very top of the maximum; those minima which lie above the cutting line are extended downward by means of narrow tongues. Tests show that cutting lines of different observers yield no definite difference as far as cycle lengths are concerned; the amplitudes of those cycles if determined optically from the cycleplot would, of course, be seriously affected by cutting lines departing from the usual procedure. At present amplitudes are obtained by arithmetic integration of the data, and are thus independent of the cutting line.

We may therefore conclude from a study of the errors in the cycleplot building process that when the standard methods of the Laboratories are used the resultant extraneous effects introduced into the derived cycle elements are too small to be of any consequence.

. . . .

We next consider the errors involved in the actual manipulation of the instrument.

The most important of these is the occasional slight change in the effective object distance due to changes in the flatness error of the theoretically plane mirrors. Slight curvatures develop, neither the mirrors nor the mounting being free from defects. The resultant effect is to throw the calibration slightly off. The practice in cycle analysis is, however, to test the calibration both before and after every analysis or short series with the regular standard cycleplots, any scale error becoming immediately evident; the mirrors are then adjusted, or if the error is small it is applied as a correction to the results.

Since the scale is calibrated in tenths of units, cycle values are at best correct only to the nearest tenth unit. In rare cases only, of outstanding cycles or in special study, is interpolation to hundredths of a unit made.

B. The Statistics of Errors in Cyclogram Analysis.

The general test described in section I, and others of its kind, had given some indication of the small magnitude of the differences in cycle analyses of the same data at different times. A knowledge of the exact amount of the differences in this and other cases awaited, however, a detailed statistical study. Such a study is described in the following pages.

The material used consisted on the one hand of a large number of cycle analyses of the 42 Western Groups carried through by Dr. Douglass in 1926-28, and on the other hand of a considerable series on the same 42 Groups and also on the California Coast Redwoods, made during 1933-34 by the writer. An intensive study of this material over a period of about two months resulted in a set of statistical "constants" and is summarized in table II.

Table II. A. Variations in Cycle Settings

	Cases	% Ave. Diff.
1. Differences between Out and In Settings, same Obser.		
(a) 42 Groups, unkn. scale, using Max. AED. Jan. 1928	216	0.8
(b) 42 Groups, unkn. scale, using Min. AED. Jan. 1928	271	1.0
2. Differences between two analyses of same material, same Obser.		
(a) 42 Groups, using Max. 1st-direct; 2nd-unkn. scale AED. 1926-8	190	2.4
(b) 42 Groups, using Min. 1st-direct; 2nd-unkn. scale AED. 1926-8	178	2.5
(c) California Coast Redwoods, using Max. 1st-direct; 2nd-unkn. scale. ES. 1933	326	1.9
3. Differences between two analyses of same material, different Obser.		
(a) 42 Groups (part), AED vs. ES, principal cycles	46	2.5
Same all cycles	95	3.0
4. Effect of inverting curves for analysis.		
(a) 42 Groups, AED, 1st-Max; 2nd Min. Norm. Scale 1926	166	2.8
(b) 42 Groups, AED, 1st-Max; 2nd Min. Unkn. Scale 1928	241	2.0

Standard procedure in cyclograph use demands an analysis over the entire range not only when the mirrors move out, but also when they are brought in. As far as possible, the second analysis was made independently of the first. Some remembrance of the pattern may, however, influence the assignment of a horizontal setting the second time, in many cases.

It was to be expected that this difference would be quite small. The causes apparently arise from two conditions. First, psychological differences in judging horizontality; these are usually quite small, though if conditions are uniform, it is possible that a plus or minus trend may result. Second, confusion in the pattern (possibly due to real factors in the data, perhaps due to artificial factors, probably due to a combination of the two) which makes one setting hardly better than several others in the immediate vicinity. Often only one or two dots (crests) will determine a significant alternative setting. When the pattern contains a strong cycle, alternative settings could be largely eliminated by an exact statement of the range of the cycle. But more often the setting difference is due to general confusion in the pattern; one is led to suspect that the cycles involved are in such cases quite possibly accidental in origin. Ambiguity arises in part also from the placing of emphasis on different portions of the cycle at different times. In many cases, two distinctly alternative settings may be made, only a portion of the individual dots being common to each. The average difference of these alternative settings was found in 31 cases to be about 5 or 6 per cent.

The remarks on confusion in the pattern apply to all the differences discussed below.

The second kind of difference is that resulting when the same material is subjected to analysis at two different times by the same observer. To make the test as rigorous as possible, the original analysis was compared with a second set in which the same data had been replotted on an unknown scale. Extensive analyses of this type in the 42 Western Groups by Dr. Douglass and in the California Coast Redwoods by the writer were available. After applying the proper correction factors to the second set, the average difference when using the maxima was almost exactly the same as that when the plots were inverted and the minima were used. The slightly smaller average difference found in the Coast Redwoods may perhaps be ascribed to the somewhat different cycle complex in the trees. The total number of cycle pairs compared was 694. An average difference of 2.5%, the largest found, is not very great when we consider that we are dealing with highly variable phenomena containing fluctuations due to some minor extent at least to random causes.

The third kind of difference considered, that between two observers, is the most important. The only available data that could be used was in the 42 Groups; the analyses had been made some eight years apart and were, of course, quite independent. The average difference of 3% is gratifying small; in this value are included cycles, the identification of some of which as corresponding to the ones paired with were highly doubtful. When only the strong cycles were used, the average difference is no larger than the difference of the second kind.

In ordinary procedure the maxima of a curve are isolated for analysis. What happens when the minima are isolated instead? Comparison of analyses using maxima with those using minima shows that the average difference is of the same order as before. The total of 407 pairs of cycles is certainly large enough so that the result may be viewed with confidence. No apparent change was found in the number of emphasized cycles. There was also no evidence for any systematic increase or decrease in cycle values on inversion.

Frequently an observer finds cycles in data which another observer, or even the same observer at a different time, does not find. Such cycles constitute a serious matter for investigation. The amount of these cycles present are given in table III.

Table III. Anomalous Cycles

Type of Difference (see table II)	Total Cycle Values=y	Anomalous Cycles=x	%=x/y
1a	507	75	15
1b	625	83	13
2a	506	126	25
2b	450	94	21
2c	841	191	23
3	250	60	24
4a	443	111	25
4b	665	183	28

It is significant to note that the Out-In differences contain a much smaller percentage of anomalous cycles than the remaining kinds of differences; there is evidently a dependence of one set on the other, as suggested already. The similarity in the size of the remaining percentages indicates the absence of any systematic factor; but an average of one anomalous cycle in every four is disturbing. It was found, however, that these are almost all weak cycles of little weight, distributed at random, which largely cancel out when the data in which they are found is summed into a periodogram.

Another question which may be considered here is the difference in the lengths of cycles analyzed at different times, as a possible function of the length of the cycle. The available data is summarized in table IV. When plotted there is evident a straight line relationship for each observer which may be expressed in the formula

$$\overline{\Delta n} = an$$

where $\overline{\Delta n}$ = difference in cycle lengths.
a = constant for observer.
n = cycle length.

Table IV. Cycle Differences With Cycle Length

Range - Years	No. of Cycles	Av. Diff. - Years	Obs.
5.1-10.0	112	0.173	ES
	77	0.195	AED
10.1-15.0	97	0.259	ES
	105	0.323	AED
15.1-20.0	45	0.336	ES
	76	0.389	AED
20.1-25.0	44	0.445	ES
	58	0.502	AED
25.0-and over	30	0.480	ES
	37	0.638	AED

From the above table we find the difference in cycle values, in per cent, to be as follows:

Mean Cycle	%Av. Diff.	
	AED	ES
7.5	2.6	2.3
12.5	2.6	2.1
17.5	2.2	1.9
22.5	2.2	2.0
29.5	2.2	1.7

The value of a is found to be 0.0249 for AED and 0.0216 for ES.

C. Probability Tests

We now take up such matters as the tendency of an observer to favor certain cycle lengths resulting in spurious emphasis on special values in the periodogram, and the recognition of real and unreal cycles. The best manner of investigating such problems is by means of probability curves.

A general method, used in the Tree-Ring Laboratories, for obtaining a lot drawing or probability curve is the following. Each year represented in a sequence of measurements of ring widths of a specimen or group to be used is marked on a slip of paper. All slips are then thoroughly mixed in a container, and drawn out at random without replacements. The values corresponding to the years drawn are put down in the order of drawing, and a curve is made from the series. This represents a quite accidental or probability arrangement, whose frequency distribution, however, is, of course, identical with that of the original curve. Each curve is then put into cycleplot form.

There were available about fifty such lot drawings made in previous years. These were put into three groups, as follows:

Group 1a FL 86, FL 88. Two lot drawings of each; average length 200 yrs.
1b FL 33, FL 41. Two lot drawings of each; average length 500 yrs.
2 FV, EP, W, BC, CVP, FLU, GC, NE. Two lot drawings of each; average length 200 yrs.
3 General group; average length 175-200 yrs., as follows:
Lot 1. BC, CVP, EP, W, FV, SH, RL, ST, LNT, HNT, BDF, PPB, Y, SB, C.
Lot 11. W, FV, CVP, BC, EP, FLU, SH, ST, LNT, BDF, PPB, Y, SB, C, RL, HNT.
Original. RL, NE, AE, PRESC, LPJ, CPA.

Recognition Tests. A test was made with the four 500 year curves, to which were added two 500 year real curves, and the analysis carried through by the rack* method of simultaneous analysis, every curve appearing for a moment in front of the cyclograph window and then giving place to its neighbor. Both real curves were recognized.

A second and more extensive recognition test was made with group 3 (described below) after eliminating several lot drawing cycleplots of odd type. The group, comprising 27 lot drawings and 6 genuine curves, were put through simultaneous analysis, five and six at a time, two real curves, FLU and ALL-Arizona, being used for comparison. FLU was put in rack at the top, ALL-Arizona at the bottom, and the unknowns in between. The analysis was made in six batches. It is to be noted that the real

*A frame vertically movable in front of the window, in which as many as 12 cycleplots may be placed, one below the other.

curves among the unknowns were from the same geographic area (and hence contained approximately the same cycles) as the two comparison curves. Nevertheless, the results were quite gratifying:

Of 6 called genuine, 5 were genuine.
Of 2 called possibly genuine, 1 was genuine.
Of 25 called false, 25 were false.

We may conclude from this that when the curves are subjected to simultaneous analysis as above, the probability of picking out the real ones from the false is quite high. While Dr. Douglass made some individual analysis tests in 1932 which were quite successful, no extensive ones have been made by the writer; the individual method is a more severe one than that described above.

Cycle Preference Tests. To test whether there was any preference in the matter of assigning particular cycle lengths, group 3 was made up with considerable care. The group as finally constructed consisted of 37 cycleplots of which six were genuine.

The attempt to get together a homogeneous group of mixed originals and lot drawings was made in order to put the lot drawings through an analysis such as they would receive if considered real. The group was homogeneous in the sense that all detailed features making it possible to recognize any member as an original or a lot drawing were eliminated. In carrying through the individual analyses, the cycleplots were turned over so that all names and other distinguishing marks were hidden; each cycleplot was put into the window without looking at the cut-out portion, to avoid the possibility of recognizing special characteristics of lot or genuine curves; each cycleplot was given a number after analysis so that it could be identified when the entire group was completed.

The results of the analysis of groups 1 and 3 are summarized in the periodograms of fig. 1. Visual examination shows no particular tendency to prefer any cycle length. A more sensitive examination is nevertheless important; the results were therefore examined statistically.

The periodogram values of the first 15 of the lot drawing curves in group 3 were compared with the values for the remaining 16. A correlation coefficient of 0.016 ± 0.117 was found, indicating completely random distribution of the cycles in lot drawing curves. It follows from this that the periodogram would approach a smooth curve if a very large number of lot drawings were analyzed and summed. As a comparison, the periodogram numbers of the analyses of 22 dated Coast Redwood specimens were correlated with the numbers for 19 undated specimens. The correlation coefficient turns out to be 0.56 ± 0.08, strong evidence then that the cycles found are present in the trees concerned and not accidental in origin. As a check, the correlation coefficient was obtained between the periodogram numbers representing analysis of group 2 and those representing group 1. The value is 0.047 ± 0.111, again indicating random distribution of cycles in lot drawings.

D. Synthetic Curves

One of the most important questions to be answered in regard to the cyclograph is this: What portion of and how well will the cycles present in a curve be recognized by the use of the instrument? Since we do not know what cycles are supposed to exist in nature, in general,

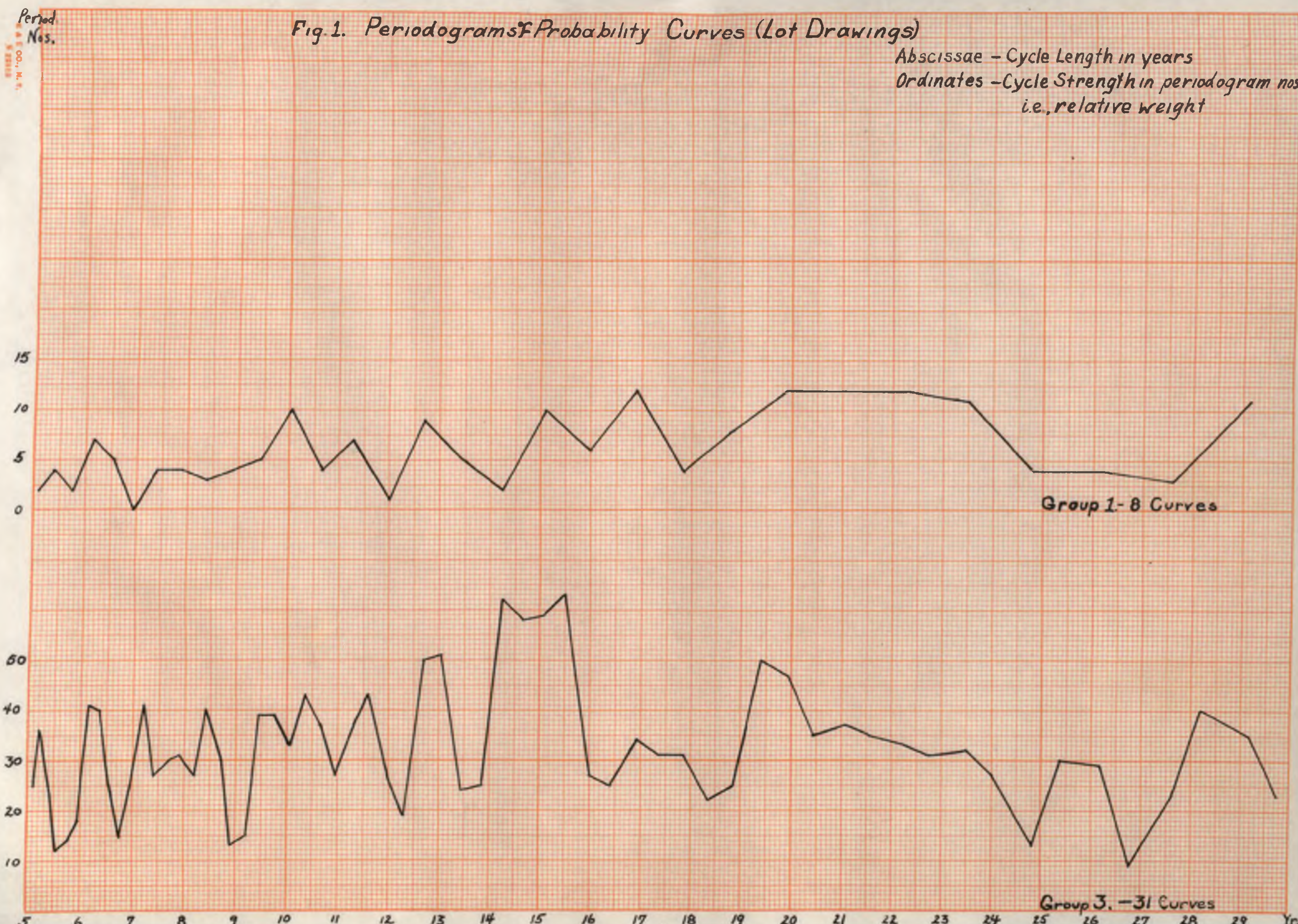


Fig. 1. Periodograms of Probability Curves (Lot Drawings)

recourse is had to synthetic curves. An extensive investigation has recently been made of this problem.

In 1932 Dr. W. S. Glock of the Tree-Ring Laboratories made up seven synthetic curves, each 300 years or units in length and containing cycles; these curves were put in cycleplot form at that time, the cycle-plots carrying no distinguishing marks other than a number. These cycleplots were turned over in November, 1934, for analysis, to the writer, who had no knowledge of what cycles were present, what type they were, over what range they existed in the curve, their amplitudes; in short, the investigation was begun with no knowledge of what the curves consisted of other than that they probably contained cycles. Each one was analyzed thoroughly, not only in a first rapid analysis with the instrument, which took about half an hour or less per curve, but also by the method of subtracting out the stronger cycles, and analyzing the residuals, a process which, of course, took much longer. The amplitudes of the cycles were then obtained by arithmetic integration.

After all work on these curves was completed, the original detailed data was obtained from Dr. Glock for comparison. The amplitudes for the cycles he put in were derived from the respective mean curve by arith-metic integration. The results are exhibited in tables V and VI. They are very satisfactory indeed. Out of a total of 35 cycles present in these 7 curves, 30 were recognized immediately, the average difference between the value of the cycle as analysed and the true value being 0.083 years, the mean value of the cycles being about 14 or 15 years. After complete analysis, including subtractions, 34 out of 35 cycles were recognized, the only one missed being a 20 year cycle which existed

Table V. Synthetic Curves. Recognition of Cycles Present

Curve	Real Value	Found on First Anal.	Found on Complete Anal.
#1	5.0	5.0	5.0
	11.0	10.9	11.0
	15.0	15.0	15.0
	16.0	16.0	16.0
	23.0	23.0	23.0
#2	7.5	7.5	7.5
	10.0	10.0	10.2
	13.0	12.2	13.0
	17.0	17.0	17.4
	25.0	25.2	25.0
#3	2.0	2.0	2.0
	6.0	6.0	6.0
	11.5	11.5	11.75
	12.0	11.7	12.1
	17.0	16.9	17.0
	20.0	----	----
	25.0	----	25.1
#4	3.33	3.33	3.33
	10.0	10.0	10.0
	18.0	18.0	18.0
	30.0	29.6	30.0
#5	3.0	----	3.0
	7.0	7.0	7.1
	9.0	9.0	9.3
	17.0	----	17.1
	27.0	27.2	27.0
#6	6.0	6.0	6.0
	7.5	7.5	7.5
	12.0	12.0	12.0
	15.0	15.2	15.0
	20.0	----	20.0
	24.0	24.0	24.0
#8	6.0	6.0	6.0
	12.0	12.0	12.0
	24.0	24.0	24.0

Table VI. Amplitude and Range Comparison

Curve	Cycle	Range WSG	Range ES	Amplitude WSG	Amplitude ES
#1	5.0	225-299	225-299	0.16	0.16
	11.0	42-125	43-108	0.28	0.38
	15.0	0-299	0-59	0.56	1.34
		-----	60-224	----	0.40
		-----	225-299	----	0.34
	16.0	0-64	1-64	1.60	0.58
	23.0	90-250	109-292	0.50	0.44
#2	7.5	0-300	112-299	0.15	0.17
	10.0	0-122	0-111	0.18	0.16
	13.0	156-300	0-85	0.18	----
		-----	150-260	----	----
	17.0	60-240	70-200	0.16	----
	25.0	0-300	0-299	0.24	----
#3	2.0	60-168	-----	----	----
	6.0	84-220	-----	0.08	----
	11.5	0-120	0-252	0.12	0.16
	20.0	60-168	-----	0.10	----
	25.0	0-299	-----	0.14	----
#4	10.0	0-299	144-297	0.08	0.17
	18.0	0-144	0-143	0.40	0.41
	30.0	0-299	0-143	0.16	0.10
		-----	144-293	----	0.14
#5	3.0	0-299	-----	----	----
	7.0	0-100	-----	0.06	----
	9.0	0-299	0-299	0.10	0.10
	17.0	204-299	-----	0.20	----
	27.0	0-299	0-296	0.10	0.13
#6	6.0	0-299	-----	0.04	----
	7.5	0-140	0-89	0.04	0.04
	12.0	0-299	-----	0.10	----
	15.0	90-220	90-209	0.05	0.09
	20.0	192-299	210-299	0.13	0.11
	24.0	0-299	0-299	0.16	0.18
#9	6.0	0-299	0-95	0.02	----
		-----	147-296	----	----
	12.0	0-299	0-95	0.05	----
		-----	96-146	----	----
		-----	147-296	----	----
	24.0	0-299	0-95	0.10	----
		-----	96-146	----	----
		-----	147-296	----	----

Amplitude is expressed as centimeters of normal scale, which is 2 cm. per mm. of ring width or 2 cm. per whole unit of standardized values; it is measured by the distance from minimum to maximum in these units.

for only one-third of the curve and was drowned out by a 25 year cycle and harmonics of 11.5 and 6 year cycles. The average difference was now 0.046, well within the limits of the smallest division of the cyclograph scale.

The degree of recognition of the elements of the cycles put in, while less good than the bare recognition of the cycles themselves, is still quite gratifying.

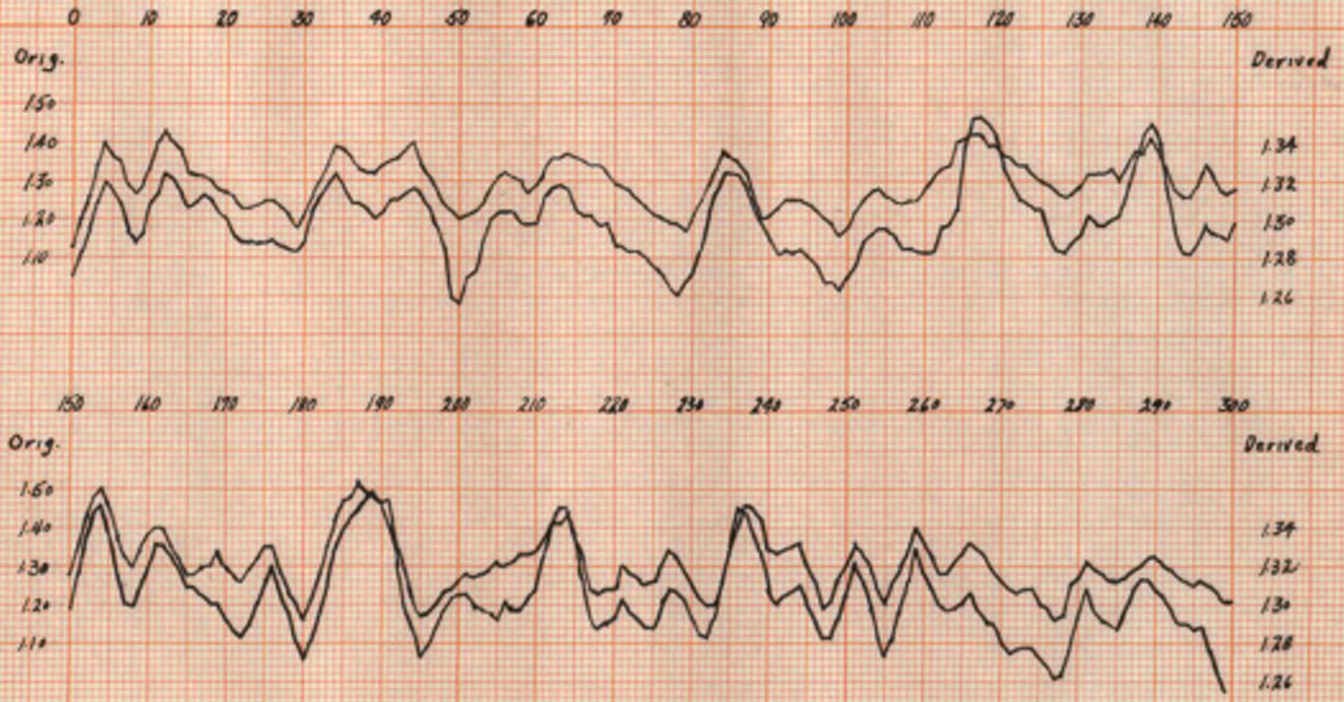
The cycle lengths found in mean curve #2, together with the amplitudes found over the ranges assigned, were synthesized into a resultant mean curve. The residuals remaining after subtraction of all cycles present were, of course, not included in the derived synthesis. The mean value of all the original values was used as the factor to add to the residuals (when the derived cycles were so expressed) in order to permit summation. This may be taken as standard procedure at present.

Comparison with the original mean values gave a correlation coefficient of 0.912 ± 0.007. The close similarity of the two curves is illustrated in fig. 2.

Synthetic Curve #1. An interesting feature is the very strong 14 year cycle which appears from 1 to 70 after all the major cycles have been taken out. This, of course, may be accounted for by the imperfect subtraction of the 15 year and the 16 year cycles, which were present in this interval, but the strength of the residual 14 year cycle remains puzzling, since the curve originally had none put in. The 17 year cycle found in the final residuals of interval 1-51, may be explained similarly and points to the danger in assigning any great validity to cycles in the residual curve neighboring those

Fig. 2. Synthetic Curve No. 2, Original and Derived Syntheses

Upper Curve - Original Synthesis



subtracted. The short intervals concerned as well as the small amplitudes involved should, however, be kept in mind as extenuating factors.

Notes on Methods of Synthesis. It became apparent after comparison of the derived with the original data that modifications in the methods of synthesis of synthetic curves were called for. It is evident that the component cycles should be expressed as residuals, the average value being zero. Furthermore, the maximum residual should be not much more than twice the mean residual, unless the cycle is superposed on another cycle. One may consider in this connection the following theorem, which hardly needs any proof: When a number of simple (or complex) curves, each containing a cycle and all measured from any origin, are added together and their arithmetic mean taken as the resulting curve, the original origin of each of the component curves is completely hidden, and it is impossible in general to determine by analysis of the mean curve what the cycles in it have as their origin or baseline. The amplitudes of these cycles may then be expressed only as departures from the mean value of the final summation curve.

Comparison of Synthetic and Real Curves. As a guide to any further synthetic curve making, several suggestions may be made.

Real curves have many more short variations; there is no regular progression as in curve #2. However, introduction of a two or three year cycle would probably eliminate this difference. Furthermore, a smoothed tree-ring curve is not much different from synthetic curve #2 also smoothed.

The writer has the feeling that cycles in real curves have more permanence than those in synthetic curve #2 (in terms of 300 year -

intervals), in the sense that some influence of each cycle is felt before and beyond the main range of its activity.

General Notes. Short cycles apparently are more important in concealing minor cycles present than long ones are, and should be subtracted out first. Furthermore, existing harmonics of long cycles when the long ones are taken out first are apt to be reduced in emphasis, possibly to the point of non-recognition in analysis of the residual curve.

The amplitudes of the derived cycles are in almost every case considerably less than those of the cycles put in, when the amplitudes of the latter are derived directly from the original cycle. However, it is evident that the method of synthesis used, involving arithmetic summation, had the effect of reducing each cycle amplitude by a factor equal approximately to the number of cycles summed in the interval; when the amplitude is derived from the original mean curve, the satisfactory results in table VI follow. Differences between the original and derived amplitudes arise from a number of causes: inexactness of range, shortness of data which has the effect on arithmetic integration of not allowing complete cancellation of other cycles than the one integrated for, carry-over of errors into the residuals which, when integrated, cannot possibly give true amplitudes even if the range is taken correctly.

As in the statistical comparisons of section III B, there were a number of additional cycles found which were not actually present in the data; 8 cycles all told on first analysis, and 9 cycles after complete analysis. An investigation of these anomalous cycles shows them in almost every case to be very weak, as found previously. It is possible that the less simple harmonics of strong cycles may exercise some influence in determining the patterns which yield these weak and spurious cycles.

IV. Applications

The cyclograph method of analysis has been applied with success not only to tree-ring growth curves, but also to sun-spot variation, variation in the solar constant, variable star light curves, geologic data such as varve sedimentations, and meteorologic data of all kinds.

The applications in the following pages not only illustrate the method of procedure, but emphasize the particular ability of the instrument to determine the elements of recurrences involving high variability.

A. Monthly Sun-Spot Numbers

The monthly sun-spot numbers[9], of which there is a continuous sequence extending back to 1749.0, offer a promising subject for study. When plotted, the dominating factor is, of course, the long cycle averaging about 11 years, which must be removed before the minor variations may be considered. A standardizing line run through the annual means serves to do this. However, since the variations in the minima will be enormously magnified by direct standardizing, a ten is added to both the observed Wolf Number and the standardizing line before making the division.

This had already been done at the Laboratories; six cycleplots, covering the period 1749 to 1932, derived according to the above method and plotted on 12 times normal horizontal scale had been analyzed by Dr.

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9. For earlier data, see World Weather Records, Washington, 1927, pp. 1181-1184. For recent data, see Astronomische Mitteilungen der Eidg. St., Zurich, Nos. CXVI to CXXI.

greater weights. This may undoubtedly be ascribed to the presence of the cycle in other members of the group, which are brought into the field of view of the cyclograph almost simultaneously.

The cycles found for the 16 groups analyzed individually are summed in the periodogram of fig. 6; the periodogram for the whole set of 42 Groups, derived by Dr. Douglass, is given for comparison. The resemblance in the major features, despite the errors involved in sampling, is strong.

Hellmann Cycle: Some evidence was found of this cycle in the Coast group, less in the Rockies region, and almost none in the Flagstaff area. These results are most uncertain, however, as the 33 cycle was not found very strong anywhere in the curves examined.

Remarks. The group method of analysis, while in its application to only a small number of groups can yield none too certain conclusions, admits at present of some tentative observations. The evidence in any one region for the presence of a particular cycle is in the field of view en masse. It is true a mean setting giving one periodicity is not infrequently little, if any, better than a mean setting at some slightly greater or smaller value. Nevertheless, the difficulty of deciding the validity of a weak cycle in an individual curve analysis is largely removed, granting the assumption that the weakness of a cycle throughout a region is an evidence of its accidental nature. It must be emphasized that this is an assumption, not an indisputable fact.

In general, one can say that with a large number of curves, individual analyses when plotted on a logarithmic or semi-logarithmic scale will yield a periodogram whose variations in amplitude are not so great as those in a curve representing group analysis. But it would

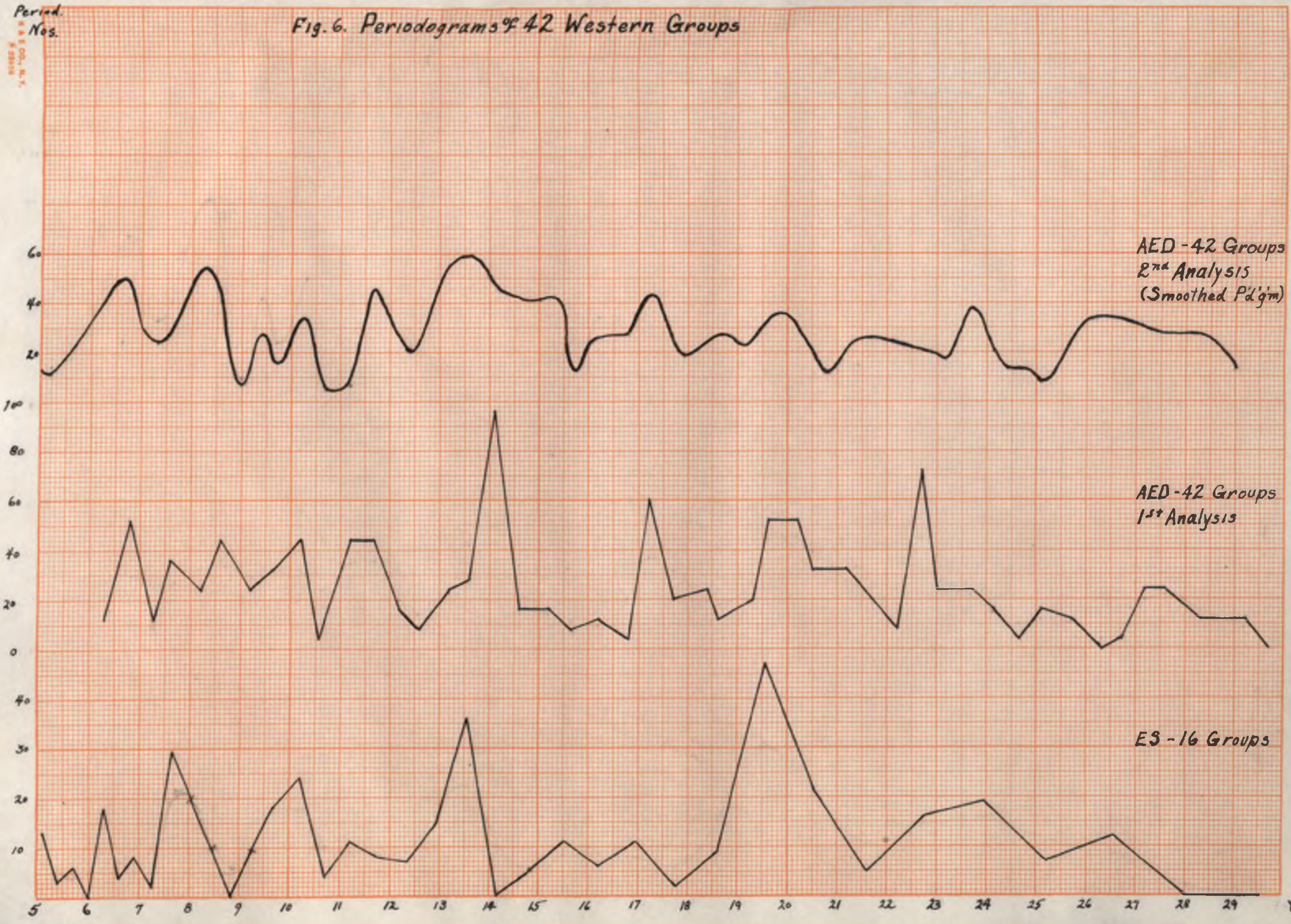
Period Nos.
Fig. 6. Periodograms of 42 Western Groups
AED - 42 Groups
2nd Analysis
(Smoothed Pd'g'm)
AED - 42 Groups
1st Analysis
ES - 16 Groups
Yrs.
52-A

Table VII. Cycles in Monthly Sun-Spot Numbers

Cycle of 11+ years removed by standardising

Time interval: Jan. 1905 to Sept. 1934

Cycle-mos.	Range	
<u>5.0</u>	(27.2-31.5)	
6.1	(08.0-14.5)	
7.2	(31.0-34.6)	
7.3	(05.2-08.2)	
<u>8.4</u>	(05.0-31.0)	general cycle, fails at 31.0; phase change at 15.5; (11.5-15.2) very weak or poss. absent; (24.5-31.0) weak.
8.7	(05.0-25.0)	alternative setting; mean, with no phase change considered.
10.45	(21.5-34.6)	from (25.5-34.6) the $\frac{1}{2}$ value is pretty nearly as strong as $\frac{1}{1}$; true cycle may thus be 5.22 here.
13.3	(05.0-34.6)	irregular mean; weak regions in amps. centering about sun-spot minima of 1914 and 1923.
16.1	(05.0-32.5)	possibly through rest, with phase change; from 13 to 24 the cycle varies, first becoming shorter by about a year, and then longer by about the same amount; much fractionization.
15.7	(05.0-34.6)	weak alternative; no regular change as in above; intermittent.
<u>17.2</u>	(05.0-34.6)	($\frac{1}{2}$ and $\frac{1}{3}$ present; $\frac{1}{1}$ most important) straight line-up.
25.3	(05.0-34.6)	weak; real?
34.1 ($\frac{1}{2}$)	(05.0-34.6)	

Table VII. Cycles in Monthly Sun-Spot Numbers

11 cycle removed by standardizing

Time interval: Jan. 1905 to Sept. 1934

Cycle	Range	
5.0	(27.2-31.5)	
6.1	(08.0-14.5)	
7.2	(31.0-34.6)	
7.3	(05.2-08.2)	
8.4	(05.0-31.0)	general cycle, fails at 31.0; phase change at 16.5; (11.5-15.2) very weak or poss. absent; (24.5-31.0) weak.
8.7	(05.0-25.0)	alternative setting; mean, with no phase change considered.
10.45	(21.5-34.6)	from (25.5-34.6) the $\frac{1}{2}$ value is pretty nearly as strong as $\frac{1}{1}$; true cycle may thus be 5.22 here.
13.3	(05.0-34.6)	irregular mean; weak regions in amps. centering about sun-spot minima of 1914 and 1923.
16.1	(05.0-32.5)	possibly through rest, with phase change; from 13 to 24 the cycle varies, first becoming shorter by about a year, and then longer by about the same amount; much fractionization.
15.7	(05.0-34.6)	weak alternative; no regular change as in above; intermittent.
17.2	(05.0-34.6)	($\frac{1}{2}$ and $\frac{1}{3}$ present; $\frac{1}{1}$ most important) straight line-up.
25.3	(05.0-34.6)	weak; real?
34.1 ($\frac{1}{2}$)	(05.0-34.6)	

Douglass. It was felt, however, that a new set of analyses with particular attention to range of occurrence would be of some value. In order to test the old plots for possible errors, and also to bring the material up-to-date, a replot and standardization was carried through for the period January 1880 to September 1934. The analysis covering the period January 1905 to September 1934 is given in detail in table VII. Comparison of the new cycleplots with the old show differences in minor features only, the new plots showing the short cycle variations slightly better. No definite differences could be discerned in the cyclograms.

Having carried out a detailed analysis, a periodogram was then computed, and is exhibited in fig. 3. During the analysis it became evident that no cycle stood out; the periodogram confirms this. The major points of emphasis are at 6.2, 7.9, 10.7, 13.5, 16.9, and 27.6 months.

A simultaneous analysis of the five cycleplots over the period 1749 to 1915 resulted in the following cycles, expressed in months:

6.0 quite good in first two, spotty in rest, fair at end.
6.7 best in #3 and #4, present in #1, spotty in others.
7.8 good in #1 and #5, poor or absent in others.
9.7 particularly evident in #1, #3, and #4.
10.4 pretty fair throughout.
11.3 fairly good in #2 and #3.
13.6 strong in #1 and #2, weaker but present in rest.
16.9 fair throughout.
24.6 not particularly good anywhere.

The range of each cycleplot is given in fig. 3.

The values obtained on simultaneous analysis for those cycles common to all or most of the segments agree tolerably well with the values obtained from the periodogram. There is, however, little justification for any assumption of constant period throughout. Rather, the principal feature of the shorter cycles in the monthly sun-spot numbers seems to

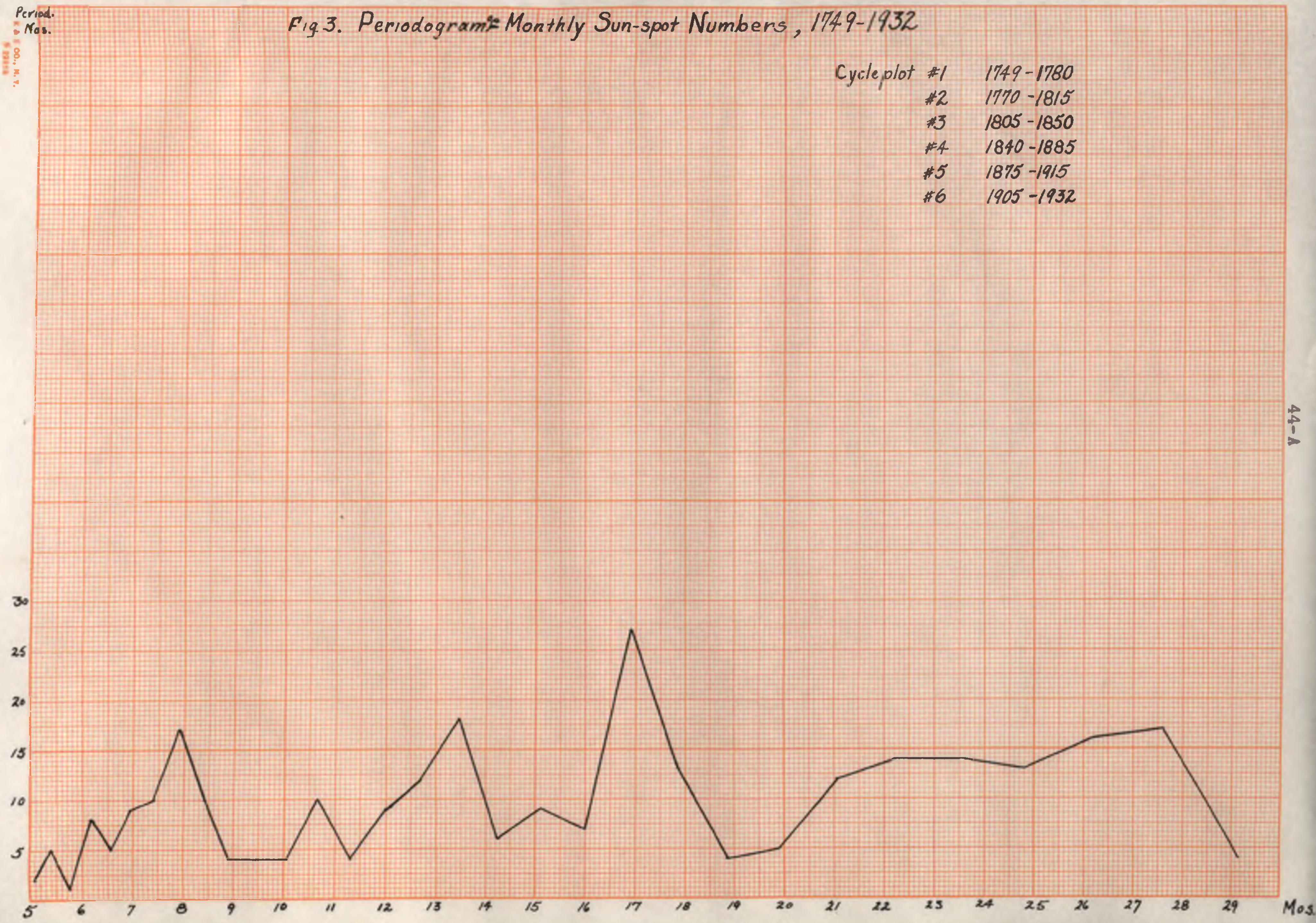
Fig. 3. Periodogram of Monthly Sun-spot Numbers, 1749-1932
Cycle plot #1 1749-1780
#2 1770-1815
#3 1805-1850
#4 1840-1885
#5 1875-1915
#6 1905-1932
Period. Mos.
30
25
20
15
10
5
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29
Mos.

44-A

be non-permanence. This is shown very clearly in fig. 4, representing the cycle pattern for the last 55 years obtained from a detailed analysis of the recently computed values mentioned above.

It is, of course, possible that some functional relation exists which will explain the changes taking place in these short cycles. The investigation is far from complete.

B. SS Cygni

The cycles in SS Cygni had been investigated by Dr. Douglass some ten years ago, cycleplots having been made with different horizontal scales. Lately, however, a new collection of data was published[10], representing photometric observations throughout the world on every maximum from 1896 to 1933. A further cycle investigation was therefore desirable.

By means of a pantograph, the small-scale plot of the data was transferred to cycleplots with a magnification factor of two. There were altogether 266 maxima, on a scale such that the calibrated scale reading of the cyclograph multiplied by eight and one-third gave the cycle value in days; four cycleplots were necessary, each one being made with some overlap of range. The cutting line used was the mean minimum of the light curve.

Modifications in the method of analysis for application to variable stars had been introduced by Dr. Douglass about 1919. They were necessary

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10. Campbell, Leon. The Light Curve of SS Cygni, 213843. Annals of Harvard College Observatory, 1934, Vol. 90, No. 3.

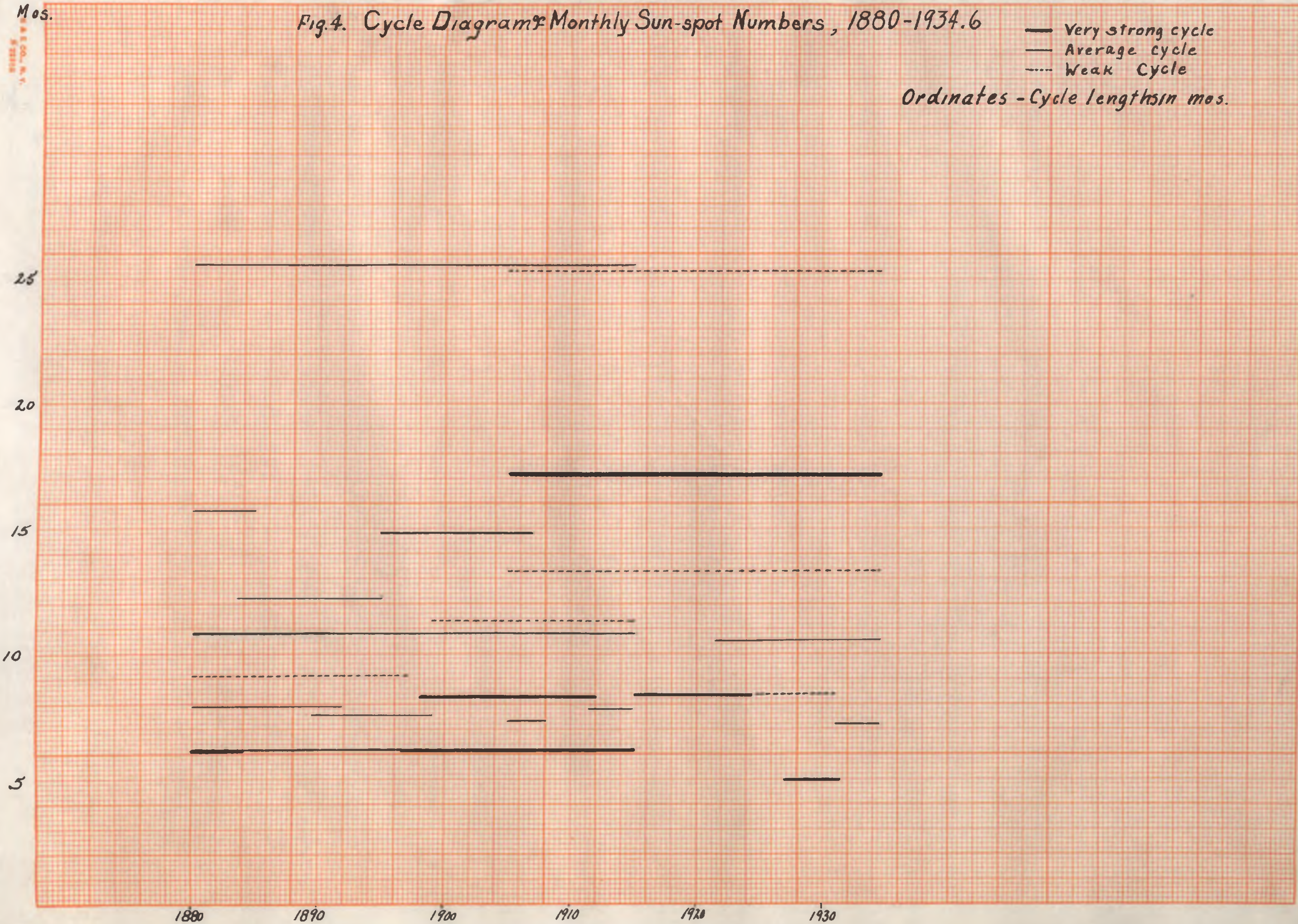

Fig. 4. Cycle Diagram of Monthly Sun-spot Numbers, 1880-1934.6

when the curve for analysis represented observations on the light changes of a long period variable star; observations which were not continuous, with the possibility of many missed maxima and minima. The cycleplot was cut so that all known maxima appeared above the cutting line, with heights equal to their amplitudes, and all known minima appeared below the cutting line, with the amplitudes taken care of similarly. The cutting line was centered in the cycleplot. When the latter was put into the cyclograph window, a transparent colored glass pane, half the vertical dimension of the window, was inserted to intercept the light through the minima. In the cyclogram, therefore, the maxima appeared as ordinary white light images, the minima as colored images. When a horizontal line-up contained images of only one color, a cycle was indicated.

By this method Dr. Douglass obtained cycle solutions for four variable stars which, although the time required was in only one case more than a few minutes, approximated the least squares solutions of Miss Cannon.*

However, since the observations on SS Cygni represented a continuous series, they were analyzed in the usual fashion for tree-ring curves. Table VIII gives the detailed results. The time required for the actual analysis was three hours. The cycle diagram of fig. 5 illustrates graphically the cycle complex of SS Cygni. It is evident both from inspection of the curve and from the cycle diagram that the range of variability in the cycle lengths is small, from 40 to 80 days. A possible cycle emphasis in the neighborhood of 120 days is apparently little more than the double of a strong cycle near 60 days.

While there is more order in the variability here than is found in

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*Read at Pasadena, June, 1919, A. A. A. S. Pacific Division meeting; unpublished.

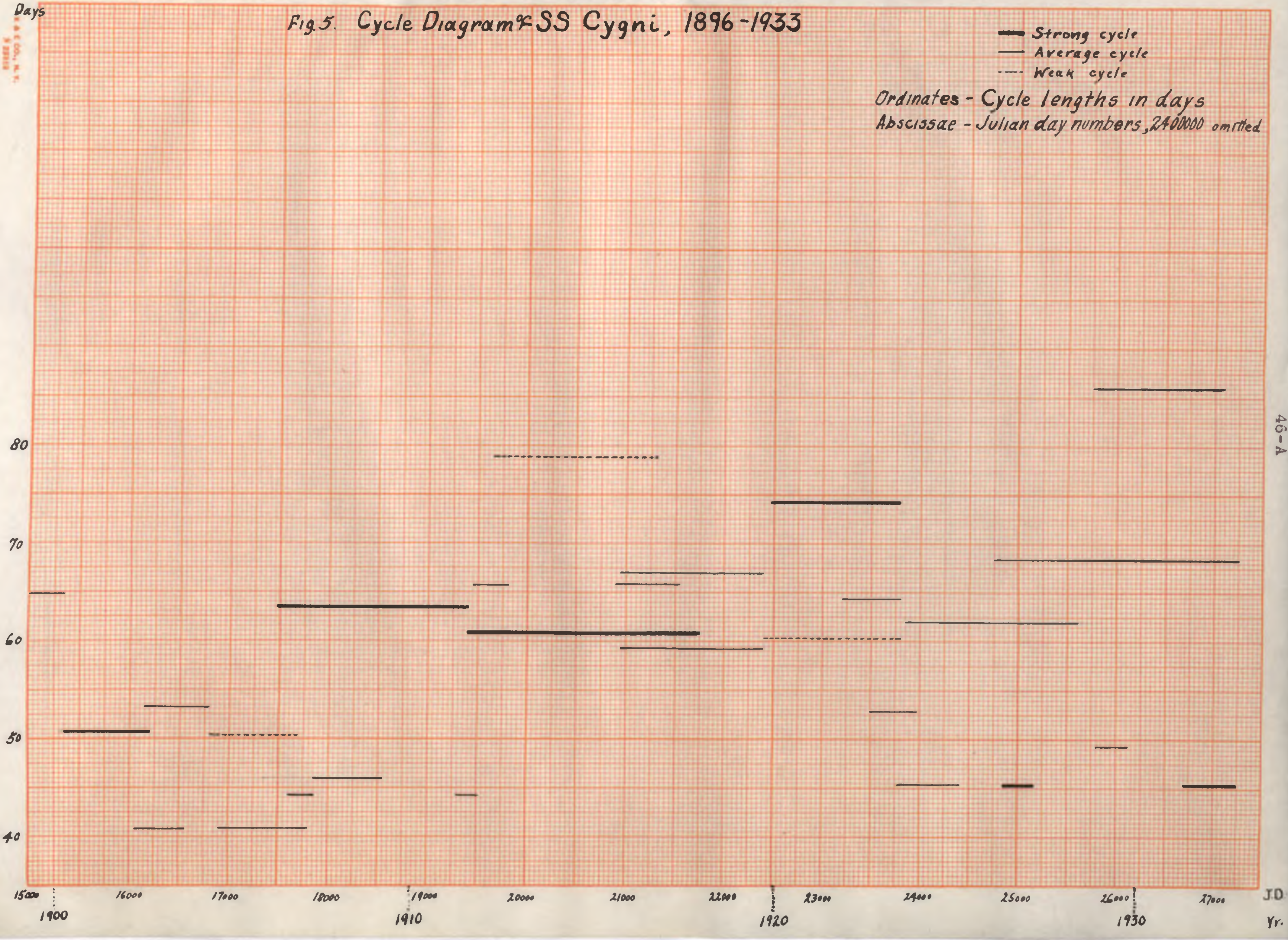

Fig. 5. Cycle Diagram of SS Cygni, 1896-1933

46-A

Table VIII. Cycles in SS Cygni

Cycleplot #1

Maxima Numbers 0-79, J. D. 2413829 to 2417800

Days	J. D. Range	Remarks
x 50.4	16800-17750	Very irregular.
50.8	15350-16200	Slight sinuosity.
53.3	16150-16800	Irregular.
58.2	14050-14900	
64.8	14850-15350	Phase change between Nos. 23 and 24, of 180°.
81.5 (½)	16050-16550 16900-17800	

Cycleplot #2

Maxima Numbers 73-160, J. D. 2417500 to 2421750

Days	J. D. Range	Remarks
44.3	17600-17850 19300-19500	
46.1 or	17850-18575	
46.8	17850-18100	
60.8	19400-21750	Varies about mean; irregular sinuosity, six waves.
63.5 (oc½)	17475-19400	Fractionization in early part; best late half.
65.8	20900-21550	
65.7	19450-19800	And irregularly during Nox. 108-116.
x 78.8	19650-21300	Irregular and very intermittent.
119.7 (½)	19400-21750	
125.8	17500-21750	Irregularly through all.

Table VIII (cont'd).

Cycleplot #3

Maxima Numbers 143-199, J. D. 2420950 to 2423800

Days	J. D. Range	Remarks
59.3	20950-22400	Irregular mean.
or		
66.0 (oc½)	20950-22400	Phase change between Nos. 156-157; maximum following No. 156 is 240° late.
or		
67.0	20950-22400	Mean of all points, disregarding phase change; two straight line-ups.
64.4	23200-23800	Irregular.
74.2	22475-23800	
120.7 (½)	20950-23800	Shows mean through all; hard to judge either range or length accurately.

Cycleplot #4

Maxima Numbers 193-266, J. D. 2423500 to 2427250

Days	J. D. Range	Remarks
45.3	23775-24400	Irregular.
	24850-25175	Good.
	26700-27250	Good.
49.3	25800-26100	
52.9	23500-23975	Mean.
62.0 (oc½)	23850-25600	
68.4	24750-27250	Very irregular and intermittent; best in last half.
86.1 (oc½)	25750-27100	
115.8	23500-27250	Mean; difficult.

the monthly sun-spot numbers, no law of variability of the cycles would be recognized. The cycle table shows some evidence of a sine wave type change in cycle length, with a period of about 45 years and an amplitude of 12 days about a mean of approximately 55 days. The strongest cycle throughout is a cycle of 60 days. While this value has not been particularly strong in the last 15 years, it should appear again, if the suggested cycle succession is correct, within the next five years. But this suggested succession should be viewed with great caution; the data are much too short to allow definite conclusions for a wave of this length.

C. 42 Western Groups

The principal divisions of this series are: Rocky Mountain or Pikes Peak area, Flagstaff or Northern Arizona area, Sierra Nevada or Coast area. The analysis was divided into two kinds, the first consisting of an individual examination for cycles of each cycleplot, and the second involving an examination all at once of four or six cycleplots, each set comprising the central elements of each area. All cycleplots represent original values, drawn directly from the growth curves.

Procedure: General Group Curve. The individual analyses of the first method cover the period from 1750 to 1900 . The cycleplots were taken from the complete set, no attempt being made at choice. Sixteen were examined. A later check-up showed that of these sixteen, 7 belonged to the Rocky Mountain area, of which 6 were central, 8 (4 of central region) belonged to the Flagstaff area, and only one (not central) to the Coast area.

Procedure: Central Regional Group Curve. In the second method, those cycleplots were chosen which had been found by Dr. Douglass to represent the central region of the three areas. Thus, the cycleplots used* were

Flagstaff: J, FL, NE, FV, FLU, SH.
Pikes Peak: C, PPB, BDF, HNT, LNT, ST.
Coast: EP, W, BC, CVP.

Each set was placed in the sliding rack in a group. Then, using the first one as the principal one, and checking with the others in the group, the successive apparent cycles were found by the usual method of alignment of pattern units of light. When the cycle appeared only in one-half or less of the group, it was carefully examined for strength, and if considered weak was counted as false and not included in the records. A visual mean of all the individual settings at any one period was made in all cases. Only the period 1750 to outside was used, as in the first method. During the check examination, when the movable mirrors of the cyclograph were taken from their farthest point out, back to the starting point, the last cycleplot in the rack was used as principal subject, check, however, being made with the others as before.

An index correction** of -0.2 years was applied to the scale readings in this second method.

Weights. If one-half or more, but not all of the cycleplots in the group showed the presence of a weak cycle, an average weight of one to each plot was assigned. If a cycle ran through all, each received more weight, the average weight now being two. The curves were, of course,

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*The description of these groups is published in vol. 2, appendix.
**The movable frame is some 2 inches nearer the mirror carriage than the window, thus shortening the object distance; hence the correction.

weighted individually within each region. If the cycle was considered quite good, it received a weight of 4., and a very strong one a weight of 6. There was one case of a cycle at 13.5 in SH which was so extraordinary it was given a weight of 8. Criteria for weighting were: low dispersion, or smallness of deviation of individual dots from horizontal alignment at cycle setting, number of dots and therefore of maxima concerned, uniformity of amplitude or, in other words, of light strength of individual dots.

<u>Results</u>: The following table gives the cycles that were found strong enough to be weighted in those curves which were analyzed by both methods.

	Group Method		Individual Analysis	
#	Cycle	Weight	Cycle	Weight
SH	13.5	8	13.5	6
SH	19.1	6	18.8	2
SH	27.3	4	26.8	4
FLU	6.7	4	7.0	1½
FLU	13.5	6	13.6	4
FV	6.7	4	6.9	1½
FV	19.1	2	20.4	3
NE	8.1	4	8.3	1½
C	7.8	4	7.8	4
C	9.8	4	9.8	2
PPB	9.8	4	10.1	2
BDF	7.8	4	7.9	2
BDF	9.8	4	10.0	4
BDF	19.9	2	19.9	4
LNT	9.8	4	10.1	2
LNT	19.9	4	20.1	2
ST	9.8	4	9.8	3
ST	19.9	4	19.5	2

It is quite evident, on comparing the weights assigned to the same cycles in the two methods, that the group method gives in almost every case more confidence in the "goodness" of the cycle, as shown by the seem that even in individual analysis accidental cycles average out, and maxima in the periodogram may be taken to a high order of probability as representing actually existing periods and their multiples.

V. Summary

The investigation of tree-ring data is primarily a study in climatology and astronomy, and is based on the connection between tree-growth, climate and solar radiation. The long history written in the yearly growth as expressed in the thickness of the annual rings provides a promising subject for investigation of the laws of change that may exist in rainfall, temperature and other climatic phenomena, in relation to sun-spots and other solar phenomena.

To meet the special problem of variability concerned, Dr. A. E. Douglass developed what is now called the Cyclograph, an instrument which makes possible the determination of hidden cycles that are variable in length and amplitude, unknown as to duration, and undergoing changes of phase. This instrument is described, and a study made of the light pattern from which the cycles are determined.

A statistical investigation of the limitations of the instrument indicates that the results obtained with its use have a high order of reliability. The personal equation of different experienced operators introduces an average difference into the cycle results of 2%. The power of the instrument in enabling the recognition of the hidden cycles in any data is shown to be of the order of 95% to 100%.

Applications are made to monthly sun-spot numbers, the variable star SS Cygni, and to the 42 Western Groups of trees. The short cycles in the sun-spot numbers, and the cycles in SS Cygni, are found to change apparently at random, although a law of cycle succession is suggested

for the latter. The cycles found in the 42 Western Groups confirm the results already published by Dr. Douglass.

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