# ADAPTATION OF THE MULTIGROUP REACTOR EQUATIONS FOR THE UNIVERSITY OF ARIZONA'S TRIGA REACTOR TO THE IBM-650 COMPUTER

by

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## APPROVAL BY THESIS DIRECTOR

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## ABSTRACT

The multigroup equation employed in criticality calculations has been adapted to the TRIGA reactor at the University of Arizona in a computer program for the IBM-650. The program includes the use of the Selengut-Goertzel approximation for the slowing down of neutrons by hydrogen and is based on the condition of zero flux at the extrapolated outer boundary of the reactor. The program also generates the source terms in all fuel regions of the reactor.

A test program was conducted, using the reactor parameters of the TRIGA reactor at the University of Arizona, to determine if the computer program would closely approximate the radial flux values and the actual critical fuel mass of the reactor. Results of this test program revealed that the critical mass of fuel could be determined very well and that the radial flux values, based on a spherical equivalent reactor, were a reasonable approximation of the expected values

## TABLE OF CONTENTS

## CHAPTER 1

## INTRODUCTION

### 1.1 Computer Programs in Criticality Calculations

Over the past fifteen years a great deal of effort has been devoted to associating the capabilities of the analog and digital computers with a number of areas of scientific interest. One of these areas in which such association has proved to be helpful has been the use of digital computer machine programs to assist in the solution of the diffusion equation in criticality calculations for nuclear reactors. The following sections of this chapter will discuss this matter in more detail, to include a description of the thesis objectives of the writer in adapting a machine program of criticality calculations to the TRIGA reactor at the University of Arizona.

### 1.2 The Lethargy Group

The current nuclear engineering student is acquainted with the diffusion equation with which criticality computations are conducted. The integro-differential equation with which we work is quite obviously impossible to solve by conventional methods of calculus over the ranges of energy and the regions of different composition found in the typical reactor. For this reason, it has been found necessary to divide the entire energy range up into smaller increments over which the physical reaction characteristics of the materials in the various regions can be considered essentially constant. Since most reactor computations are performed in terms of lethargy instead of energy, these increments are referred to as "lethargy groups" during the rest of this work. Lethargy is a dimensionless quantity defined as

$$u = \ln \frac{E_o}{E}, \tag{1.2.1}$$

where E is the energy of interest, and $E_o$ is a constant reference energy which, for a given problem, is generally equal to or slightly greater than the source energy. Much of the literature uses a value for $E_o$ of 10 Mev, which is accepted as approximately the maximum energy of fission neutrons.

## 1.3 Advantage of Machine Programs in Reactor Calculations

As mentioned in Section 1.2, the lethargy groups are so selected that the nuclear characteristics of the reactor materials are essentially constant in any given lethargy group of neutrons. By this approximation, the integro-differential equation can be converted to a set of differential equations, summed over the separate, sequential lethargy groups. The accuracy of the result can be kept relatively high by using a large number of lethargy groups, each of which covers a small increment of energy. It is at this point that the utility of machine programs in calculations has become of great interest in the past seven or eight years. If the engineer desires to improve his results by having sixty to eighty lethargy groups, the hand solution of that same number of differential equations would take days, if not weeks. It can thus be seen that if one can compute the parameters for the reactor materials in the various regions, place them into a computer and let it do the mechanical work, a most rewarding saving in time can be achieved. This advantage in itself justifies the existence of the machine program.

1.4 Limitations in the Use of Machine Programs

The preceding section has pointed to the greatest advantage of machine programs in reactor criticality calculations. There are certain limitations in the use of such programs as well. One factor which may be a limiting influence is the storage capacity of the computer used. Once the actual coded program is stored in the computer, the remaining storage space may be so curtailed that the number of lethargy groups, reactor regions or mesh points must be reduced to an amount less than the optimum. Such reductions may markedly lessen the accuracy of the program. Limited storage space may also require the scientist to convert the program for a given geometrical reactor shape from its true configuration with several spatial variables to an equivalent sphere with only one spatial variable, again possibly adversely affecting the accuracy of the results. For example, the cylindrically shaped TRIGA reactor - with two spatial variables - was converted to an equivalent sphere for the work in this thesis, in order to ensure sufficient storage space in the 2,060 storage areas of the IBM 650 at the University of Arizona.

The second limitation of machine programs for reactor criticality calculations is associated again with the number of lethargy groups selected. Coupled with the reduction in accuracy when only a few lethargy groups may be used due to insufficient storage space, the condition also exists that there is an upper limit to the number of lethargy groups which can be used in the program, even if there is more than enough storage capacity. The reason for this factor is that there are certain limitations in the multigroup theory itself.[1] If the lethargy groups are made too narrow, neutrons slowing down into the ith lethargy group may not only be coming from the (i-1)th group-as required by the multigroup equations-but they also may be slowing down from groups of lower lethargy than the adjacent group. This causes the multigroup equations to change into the continuous slowing-down model of the age equation, which becomes quite inaccurate with hydrogeneous moderated reactors such as TRIGA.

The third limitation is that a digital computer can only give a fair approximation of the first and second derivatives encountered in the diffusion equation, again reducing accuracy. Typical approximations employed in the machine program are[2]

$$\frac{dW_n}{dr} = \frac{W_{n+1} - W_n}{\Delta r} \tag{1.4.1}$$

$$\frac{d^2W_n}{dr^2} = \frac{W_{n+1} - 2W_n + W_{n-1}}{(\Delta r)^2} \quad . \tag{1.4.2}$$

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[1] A. M. Weinberg and E. P. Wigner, The Physical Theory of Neutron Chain Reactors, 1st Ed., The University of Chicago Press, 1958, p. 510

[2] D. K. Holmes and R. V. Meghreblian, Notes on Reactor Analysis; Part II, Theory, (pamphlet), August, 1955, Number ORNL-CF-54-7-88 (Pt. II), Office of Technical Services, Department of Commerce, Washington 25, D. C., pp. 277-278

1.5 Utility of the Machine Programs

Comparing the one main advantage of computer machine programs for reactor calculations with the three main disadvantages or limitations to their use, it hardly seems possible that such programs would still have utility. However, despite the seemingly overpowering disadvantages, it has been found that the results provided by the machine programs agree quite favorably with experimental results, sometimes differing only in magnitude by some constant value. The program used by Knolls Atomic Power Laboratory is one which has been proven to be quite effective. Incidentally, it is also used by General Atomics, the maker of the University of Arizona's TRIGA reactor.

1.6 A Brief Description of "PROD II"

The Knolls Atomic Power Laboratory multigroup program[3,4], PROD II, was used as the primary reference in applying the general multigroup equations to a new machine program for the TRIGA reactor. PROD II is a program which will perform the machine calculations for problems involving spherical, cylindrical or slab geometry. The user may select the option of including or excluding the Selengut-Goertzel approximation for the slowing down of neutrons by hydrogen (see Chapter

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[3] G. J. Habetler, One-Space-Dimensional Multigroup for the IBM-650, Part I, Equations, (pamphlet), December 1, 1955, Number KAPL-1415, Office of Technical Services, Department of Commerce, Washington 25, D. C., 41 pp.

[4] V. A. Walbran, One-Space-Dimensional Multigroup for the IBM-650, Part II, Machine Program, (pamphlet), April 10, 1956, Number KAPL-1531, Office of Technical Services, Department of Commerce, Washington 25, D. C., 51 pp.

2); he may also choose between two different boundary conditions for the neutron flux at the extrapolated outer boundary of the reactor. Furthermore, the program will generate the source terms on its initial run for the subsequent flux calculations, if desired, thereby eliminating the need to make source estimates before the program is run.

### 1.7 Thesis Objectives

The first objective of this thesis has been to re-program the multigroup equations into a new machine program employing spherical geometry only, using PROD II as a guide. Since the new program was to be adapted to TRIGA, which is hydrogenously moderated to a great extent, the Selengut-Goertzel approximation was made an integral part of the program. The boundary condition of zero flux at the extrapolated outer boundary of the reactor was selected for this program, as was the source generation option.

A secondary objective was to run a test multigroup calculation with the new program, using a set of data calculated from the composition of the University of Arizona TRIGA Reactor, in order to evaluate the efficacy of the program. The factors to be evaluated are as follows:

a. The computed radial flux distribution should conform with that known to exist in TRIGA.

b. The assumed critical fuel mass which will cause the test multigroup program to converge at the desired point should not differ from the actual critical fuel mass of TRIGA by an amount greater than a factor of two.

### 1.8 Explanation of Commonly Used Terms

The discussion throughout the following chapters will frequently employ alphabetic or numerical "shorthand" designations for a variety of terms associated with the diffusion equation and with the physical conditions chosen by the user to apply the diffusion equation. Although generally defined at the first point used, it is deemed appropriate to consolidate the explanations of these terms for the reader's convenience.

The first of these factors is the number of lethargy groups into which the computational work will be subdivided. For convenience in referring to these groups, the lower case letter, i, will be used when referring to a randomly chosen group. Furthermore, the use of "i" as a subscript for a reactor parameter infers that the parameter is evaluated in the ith lethargy group. The capital letter, I, indicates the total number of lethargy groups in the problem. It also denotes the thermal lethargy group, since in a multigroup problem, the highest value of lethargy is in the thermal group. Arabic numerals are used to refer to a specific lethargy group.

The second factor is the number of major regions into which the reactor is separated. For general reference to one of these hypothetically homogenized regions, it will be designated as the kth region. Similarly, the use of the lower case letter, k, in the argument of a parameter or as the subscript to a term indicates evaluation in the kth region. The total number of reactor regions is designated by K. When a specific region is to be discussed, it will be designated by a Roman numeral.

The kth region of the reactor will generally be subdivided into spherical shells of radial thickness, $\Delta r_k$. Values of $\Delta r_k$ may be different in various regions, but they are equal within any given region. The radially outer face of each of the incrementally thick spherical shells is identified by an Arabic number, starting with zero at the center of the spherical reactor and numbered by successive integers outwards to the extrapolated outer boundary of the reactor. The lower case letter, n, is used in referring to any arbitrarily selected interface between two of these sub-regions. Such interfaces are also called mesh points. The total number of mesh points (which is also the value of n at the extrapolated outer boundary) is indicated by N. The radius from the center of the reactor to the nth mesh point will be called $r_n$. In addition, the thickness of the nth sub-region in some discussions will be designated as $\Delta r_n$. Naturally, this thickness will also coincide with the $\Delta r_k$ of that particular region in which the nth sub-region is situated.

## CHAPTER 2

## THE MULTIGROUP DIFFUSION EQUATION

### 2.1 Foreword

In this chapter, the purpose is to discuss in general terms the diffusion equation as used in multigroup calculations, with the approximations which will be employed in this work. It is not the intent of this discussion to derive the diffusion equation, since more thorough and more authoritative references, such as the works of Weinberg and Wigner[5], and Holmes and Meghreblian[6], go into much greater detail on this subject than is warranted by the scope of this work.

### 2.2 The Word Description of the Diffusion Equation

The general diffusion equation employed in multigroup work is a balance equation within a given lethargy group and a particular region of the reactor, which can be stated in words as follows:

> In a unit volume of a given region of the reactor per unit time and at steady state operation, the number of fission neutrons born in the ith lethargy group plus the number of neutrons which slow down into the ith group from the (i-1)th group must equal the number of neutrons absorbed from the ith group of neutrons plus the number of ith group neutrons which leak out of the region and which slow down out of the ith lethargy group.

### 2.3 The Slowing-Down Density Terms; Selengut-Goertzel Approximation

It was stated in Section 1.7 that the diffusion equation to be

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[5]A.M. Weinberg and E.P. Wigner, The Physical Theory of Neutron Chain Reactors, Chapter XI, 1st Ed., University of Chicago Press, 1958

[6]D.K. Holmes and R.V. Meghreblian, Notes on Reactor Analysis, Part II Theory, (pamphlet), August, 1955, Number ORNL-CF-54-7-88 (Pt II), Office of Technical Services, Department of Commerce, Washington 25, D.C., pp. 270-279

used in this program includes the Selengut-Goertzel approximation. As quite thoroughly discussed in Weinberg and Wigner[7], for a reactor which has hydrogenous moderation, the calculation of flux values is greatly increased in accuracy by taking the general slowing-down term from the (i-1)th lethargy group into the ith lethargy group and separating it into two different slowing-down terms. One of these terms is for the slowing down of neutrons by all the reactor materials except hydrogen. The second slowing-down density term involves only the slowing down of neutrons by the hydrogen nuclei in the various regions of the reactor. This technique of employing two separate slowing-down terms is the Selengut-Goertzel approximation. Justification for the use of the Selengut-Goertzel approximation is that the hydrogen nuclei are of essentially the same mass as the neutrons, and a single elastic collision of a neutron with a nucleus of hydrogen can effectively reduce the energy into the thermal lethargy group.

## 2.4 Minimum Width of Lethargy Groups

A brief statement was made in Section 1.7 to the effect that there is an upper limit to the number of lethargy groups which can be employed in the multigroup diffusion equation. Weinberg and Wigner[8] give an excellent coverage of the reasons underlying this limitation on the total number of lethargy groups for a hydrogenously moderated reactor. The essence of their coverage will be stated in this section.

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[7] A.M. Weinberg and E.P. Wigner, The Physical Theory of Neutron Chain Reactors, 1st Ed., University of Chicago Press, 1958, pp. 362-365

[8] A.M. Weinberg and E.P. Wigner, The Physical Theory of Neutron Chain Reactors, 1st Ed., University of Chicago Press, 1958, pp. 509-510

In the collision of a neutron with a nucleus of one of the reactor materials, the neutron may undergo a gain in lethargy, associated with the possible loss of some of its energy. The average lethargy jump per collision, $\xi$ , is fairly constant within a given region of a reactor.

If the lethargy groups for a computation are made narrower than $\xi$ , the source of neutrons entering a certain lethargy group may then be not only the next lower lethargy group, as required for the diffusion equation, but probably other lower lethargy groups beyond the adjacent group. This is an undesirable condition in the "bookkeeping" of the life of a neutron during its slowing-down process, since we will not know into which lethargy group it may have jumped. In a reactor which has a high hydrogen content, such as TRIGA, the value of $\xi$ becomes quite large. Thus, selection of too many lethargy groups would cause the diffusion equation to degenerate into the continuous slowing-down equation, which is not as accurate in this case. For this reason, $\xi$ is taken as the lower limit of the width of lethargy groups.

CHAPTER 3

THE MACHINE PROGRAM FOR SOLVING THE DIFFUSION EQUATION

## 3.1 The Diffusion Equation, Definition of Terms

The diffusion equation which is to be used in this computer program includes the Selengut-Goertzel approximation for the elastic scattering by hydrogen and ignores inelastic scattering. Since the space variable in spherical geometry is r, the equation can be expressed on a per unit volume basis as

$$-\nabla\cdot\frac{1}{3\Sigma_{tr}(r,u)}\nabla nv(r,u)+\Sigma_a(r,u)nv(r,u) \qquad (3.1.1)$$

$$=-\frac{\partial q(r,u)}{\partial u}-\frac{\partial\eta(r,u)}{\partial u}+\chi(u)P(r)$$

in which the terms are defined as follows:

| | | |
|---|---|---|
| $\Sigma_{tr}(r,u)$ | .. | the macroscopic transport cross section |
| $\Sigma_a(r,u)$ | .. | the macroscopic absorption cross section |
| $nv(r,u) = \phi(r,u)$ | .. | neutron flux per unit lethargy |
| $\eta(r,u)$ | .. | the slowing down density due to elastic scattering of neutrons by hydrogen |
| $q(r,u)$ | .. | the slowing down density due to elastic scattering of neutrons by reactor materials other than hydrogen |
| $\chi(u)$ | .. | the fission spectrum, so normalized that |

$$\int_0^{u_{th}}\nu\Sigma_f(u')du'+\nu\Sigma_f(u_{th})=1$$

P(r) .. the spatial distribution of fission neutrons over all lethargies, expressed as

$$P(r) = \int_0^{u_{th}} \nu\Sigma_f(r,u')\phi(r,u')du' + \nu\Sigma_f(r,u_{th})\phi(r,u_{th}). \quad (3.1.2)$$

Equation 3.1.1 is a modification of the more general equation of the same type which was used by Habetler[9] in his multigroup program work.

3.2 Subdivision Into Lethargy Groups

In order to handle Equation 3.1.1, it is necessary to divide the lethargy region up into several groups of not necessarily equal extent. In fact, they are best selected to provide the most accurate and fairly constant values for the reaction cross sections of the assumed homogenous mixture of reactor materials within each of the lethargy groups. Designating the upper limit of the fission energy spectrum as $E_o$, its lethargy counterpart would be $u_o$. Similarly $u_{th}$ is equivalent in lethargy to the energy of the thermal neutron, $E_{th}$.

We, thus, divide the lethargy range into "I" intervals, as follows:

$$0 = u_o < u_1 < u_2 \ldots\ldots\ldots\ldots < u_I = u_{th} \quad . \quad (3.2.1)$$

Neutrons whose lethargy is between $u_i$ and $u_{i-1}$ are referred to as being in the "ith group", and as seen above, i varies from 0 to I through consecutive integral values.

[9] G.J. Habetler, One-Space-Dimensional Multigroup for the IBM-650, Part I, Equations, (pamphlet) December 1, 1955, Number KAPL-1415, Office of Technical Services, Department of Commerce, Washington 25, D.C., pp. 14-30

In order to eliminate the partial derivatives with respect to lethargy in Equation 3.1.1, we now integrate both sides over some arbitrary lethargy group, $\Delta u_i$ (where $\Delta u_i = u_i - u_{i-1}$) :

$$-\nabla\cdot\int_{\Delta u_i} \frac{1}{3\Sigma_{tr}(r,u)}\nabla\phi(r,u)du + \int_{\Delta u_i}\Sigma_a(r,u)\phi(r,u)du =$$

$$-q(r,u_i) + q(r,u_{i-1}) - \eta(r,u_i) + \eta(r,u_{i-1}) + P(r)\int_{\Delta u_i}\chi(u)du. \tag{3.2.2}$$

3.3 The Slowing-Down Terms

To get the "eta" terms of Equation 3.2.2 in one variable, we define

$$\frac{\partial\eta}{\partial u}(r,u) = \Sigma_{sh}(r,u)\phi(r,u) - \eta(r,u) \quad , \tag{3.3.1}$$

where $\Sigma_{sh}$ is the macroscopic scattering cross section for hydrogen. We then solve the above equation for $\eta(r,u)$ by means of an integrating factor, and evaluating the integral over $\Delta u_i$, we have

$$\eta(r,u_i) = \eta(r,u_{i-1})e^{-\Delta u_i} + e^{-u_i}\int_{\Delta u_i} e^{u'}\Sigma_{sh}(r,u')\phi(r,u')du' \; . \tag{3.3.2}$$

We assume that the asymptotic slowing down relation holds approximately and further define

$$q(r,u) = \xi\Sigma_s(r,u)\phi(r,u) \text{ , or} \tag{3.3.3}$$

$$\phi(r,u) = \frac{q(r,u)}{\xi\Sigma_s(r,u)} \; . \tag{3.3.4}$$

## 3.4 The Diffusion Equation in Terms of Slowing-Down Densities

With the relations 3.3.2 and 3.3.4 substituted into Equation 3.2.2, the result is

$$-\nabla \cdot \int_{\Delta u_i} \frac{1}{3\Sigma_{tr}(r,u)\xi\Sigma_s(r,u)} \nabla q(r,u)du + \int_{\Delta u_i} \frac{\Sigma_a(r,u)q(r,u)}{\xi\Sigma_s(r,u)} du$$
$$+ e^{-u_i}\int_{\Delta u_i} \frac{e^u\Sigma_{sh}(r,u)q(r,u)}{\xi\Sigma_s(r,u)} du = -q(r,u_i) + q(r,u_{i-1})$$
$$+ (1 - e^{-\Delta u_i})\, \eta(r,u_{i-1}) + P(r)\int_{\Delta u_i} \chi(u)du \quad . \qquad (3.4.1)$$

## 3.5 The Lethargy-Averaged Diffusion Equation

In the previous selection of lethargy groups, they were so chosen that the reaction cross sections of the materials were essentially constant over any given lethargy group. Since the slowing-down densities are a function of the macroscopic scattering cross sections, they will not change radically over such a lethargy range and can be considered constant within any group. Thus, in an arbitrary lethargy group, $\Delta u_i$, Equation 3.4.1 can be fairly well approximated by

$$-\overline{\frac{1}{3\Sigma_{tr}\xi\Sigma_s}}\nabla^2\overline{q_i}(r) + \overline{\frac{\Sigma_a}{\xi\Sigma_s}}\,\overline{q_i}(r) + \overline{\frac{\Sigma_{sh}}{\xi\Sigma_s}}\,\overline{q_i}(r)\,\frac{(1 - e^{-\Delta u_i})}{\Delta u_i}$$
$$= \frac{1}{\Delta u_i}\left[- q(r,u_i) + q(r,u_{i-1})\right] + \frac{1-e^{-\Delta u_i}}{\Delta u_i}\,\eta(r,u_{i-1})$$
$$+ P(r)\overline{\chi_i} \quad . \qquad (3.5.1)$$

The bar over the various terms indicates quantities averaged over the ith lethargy group.

### 3.6 Consolidation of Terms in the Diffusion Equation

We further define the following terms:

a. $$\overline{q_i}(r) = \overline{\xi\Sigma_s \; \phi_i(r)} = \overline{\phi_i(r)} \Big/ \overline{\frac{1}{\xi\Sigma_s}} \qquad (3.6.1)$$

b. $$D_i = \overline{\frac{1}{(3\Sigma_{tr}\xi\Sigma_s)_i}} \Big/ \overline{\frac{1}{(\xi\Sigma_s)_i}} \qquad (3.6.2)$$

c. $$f_i(r) = \frac{\overline{q_i}(r)}{q(r,u_i)} \qquad (3.6.3)$$

d. $$A_i = \left[\overline{\frac{\Sigma_a}{\xi\Sigma_s}} + \frac{(1-e^{-\Delta u_i})}{\Delta u_i}\,\overline{\frac{\Sigma_{sh}}{\xi\Sigma_s}} + \frac{1}{\Delta u_i f_i(r)}\right] \Big/ \overline{\frac{1}{\xi\Sigma_s}} \qquad (3.6.4)$$

With the above definitions substituted into Equation 3.5.1, it becomes simply

$$-D_i\nabla^2\overline{\phi}_i(r) + A_i\overline{\phi}_i(r) = \frac{1}{\Delta u_i f_{i-1}(r)}\,\overline{q_{i-1}}(r) + \frac{(1-e^{-\Delta u_i})}{\Delta u_i}\,\eta(r,u_{i-1})$$
$$+ P(r)\overline{\chi_i} \qquad (3.6.5)$$

### 3.7 The Boundary Condition for Slowing-Down Densities

Note that both of the slowing-down density terms on the right side of Equation 3.6.5 involve functions of the (i-1)th lethargy group. This is an important key to the solution for the various fluxes. At $u_o$, or fission energy, we have the boundary condition

$$\eta(r,u_o) = q(r,u_o) = 0, \qquad (3.7.1)$$

since $u_o$ is taken to be the lowest lethargy of the fission neutrons, and thus, no neutrons exist to slow down past $u_o$.

With the above boundary condition as a starting point, the lethargy-coupled, space dependent sets of equations obtained from Equation 3.6.5 are solved sequentially starting with $u_0$ and working progressively through the lethargy groups to $u_{th}$. Thus the source of neutrons in any given lethargy groups is derived from its predecessor. It is for this reason that the lethargy groups have a minimum width, as was discussed in Chapter 2.

3.8 Modification of the Laplacian Operator to the Machine Program

The next step in the conversion of the diffusion equation to a form suitable for use in a digital computer involves modifying the Laplacian operator in Equation 3.6.5. The division of the energy range into lethargy groups has yielded a flux which only has spatial dependence within any given lethargy group. The Laplacian operator can be expressed in spherical geometry as

$$\nabla^2 \bar{\phi}(r) = \frac{d^2}{dr^2} \bar{\phi}(r) + \frac{2}{r} \frac{d\bar{\phi}(r)}{dr} , \qquad (3.8.1)$$

the theta and phi components not being expressed since there is no angular dependence involved.

Now the reactor in its sphericized form must be investigated. It is composed of concentric shells, each of which is considered to be homogeneous. The shells will be of varying thicknesses, depending on the actual volume of reactor materials included in that region. It is possible, however, to divide each shell (or region) into a set of thinner shells of equal thickness, $\Delta r_k$, the subscript k denoting those equal incremental radial shell thicknesses in region k. For example, let us say that Region III is 15 cm thick. We can divide it into 10 sub-regions of equal thickness, each of which would be 1.5 cm.

These sets of additional interfaces between sub-regions, as well as the interfaces between major regions, are known as mesh points. They are chosen sufficiently small that they do not introduce great error in the computations, usually from 1 to 2 cm in thickness. The end points of the subsets must coincide with the interfaces between the major regions of different homogeneous composition. The mesh points are numbered sequentially from the center of the reactor to the extrapolated outer boundary of the reactor (e.g. $r_o, r_1, r_2, \ldots, r_N$).

Returning now to Equation 3.8.1, it is necessary to put this in a different form for use with the digital computer. We multiply the equation by $r^2$, giving

$$r^2\nabla^2\bar{\phi} = r^2 \frac{d^2\bar{\phi}}{dr^2} + 2r \frac{d\bar{\phi}}{dr} \qquad (3.8.2)$$

The right side of Equation 3.8.2 is now integrated over arbitrary limits:

$$\int_a^b r^2 \frac{d^2\bar{\phi}}{dr^2}\, dr + \int_a^b 2r \frac{d\bar{\phi}}{dr}\, dr$$

$$= \int_a^b r^2 \frac{d^2\bar{\phi}}{dr^2}\, dr + r^2 \frac{d\bar{\phi}}{dr}\Bigg|_{r=a}^{r=b} - \int_a^b r^2 \frac{d^2\bar{\phi}}{dr^2}\, dr$$

$$= r^2 \frac{d\bar{\phi}}{dr}\Bigg|_{r=a}^{r=b} \qquad (3.8.3)$$

Note that in the integration by parts of the second term, an equal term of opposite sign to the second derivative term was developed.

Again returning to Equation 3.6.5, all other terms in the equation are lumped together, and a new variable is defined.

$$L_i(r) = -A_i(r)\overline{\phi}_i(r) + \frac{(1-e^{-\Delta u_i})}{\Delta u_i} \eta(r,u_{i-1})$$

$$+ \frac{1}{\Delta u_i f_{i-1}(r)} \overline{q}_{i-1}(r) + P(r)\overline{\chi}_i \quad . \qquad (3.8.4)$$

Equation 3.6.5 multiplied through by $r^2$ is now in the form

$$-D_i r^2 \nabla^2 \overline{\phi}(r) = r^2 L_i(r) \qquad (3.8.5)$$

If we now integrate the above equation at the nth mesh point from the center of the reactor, with the limits $r_n - \frac{\Delta r_n}{2}$ to $r_n + \frac{\Delta r_{n+1}}{2}$, and also use the result obtained in Equation 3.8.3, we have

$$-D_i r^2 \frac{d\overline{\phi}_i(r)}{dr} \Bigg|_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} = \int_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} r^2 L_i(r) dr \qquad (3.8.6)$$

Examining the left side of Equation 3.8.6, it can be shown by differentiation that

$$-D_i r^2 \frac{d\overline{\phi}_i(r)}{dr} = -D_i \left\{ r \frac{d}{dr} \left[ r\overline{\phi}_i(r) \right] - r\overline{\phi}_i(r) \right\} \qquad (3.8.7)$$

A new variable is now defined for the spatially dependent flux term at the nth mesh point within the ith lethargy group:

$$\phi_{n,i} = r_n \overline{\phi}_i(r_n) \quad . \qquad (3.8.8)$$

With the substitution of the above term in the right side of Equation 3.8.7, and with the further substitution of that changed term in Equation 3.8.6, the latter becomes:

$$-D_i \left[ r_n \frac{d}{dr} \phi_{n,i} - \phi_{n,i} \right] \Bigg|_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} = \int_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} r^2 L_i(r)\,dr \qquad (3.8.9)$$

The approximation for the first derivative of a function, as previously stated in Equation 1.4.1, is now put in a form appropriate to our variables,

$$\frac{d\phi_{n,i}}{dr_n} \simeq \frac{\phi_{n+1,i} - \phi_{n,i}}{\Delta r_n} \quad . \qquad (3.8.10)$$

Taking $\phi_{n,i}$ as the averaged value of the flux at the nth mesh point and in the ith lethargy group, and evaluating the derivative on the left side of Equation 3.8.9 at both sides of the mesh point, the left side of Equation 3.8.9 is developed into a machine-controllable solution, as follows:

$$-D_i(n^+) \left[ \frac{r_n\phi_{n+1,i} - (r_n + \Delta r_n)\phi_{n,i}}{\Delta r_n} \right] + D_i(n^-) \left[ \frac{(r_n - \Delta r_{n-1})\phi_{n,i} - r_n\phi_{n-1,i}}{\Delta r_{n-1}} \right]$$

$$= \int_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} r^2 L_i(r)\,dr \qquad (3.8.11)$$

The terms $D_i(n^+)$ and $D_i(n^-)$, not heretofore defined, represent the diffusion coefficients for the regions on the radially outer and inner sides of the nth mesh point, respectively. It is significant to note that when the nth mesh point is internally located in the kth homogeneous region, the $D_i$ terms on both sides of the mesh point are equal in magnitude, as are the geometrical components. In this case, the left side of Equation 3.8.11 reduces to

$$r_n \frac{D_i(k)}{\Delta r_n} \quad (2\phi_{n,i} - \phi_{n+1,i} - \phi_{n-1,i}) \quad , \qquad (3.8.12)$$

$D_i(k)$ being the general diffusion coefficient for the kth region; this situation is recognized in the machine program, so that several computational steps are thereby eliminated by a routine machine check of the mesh point location.

3.9 Modification of the Remaining Terms of the Diffusion Equation

Although the left side of Equation 3.8.6 has been simplified to a form amenable to the digital computer, no action has yet been accomplished with the integral on the right side. After substituting the $\phi_{n,i}$ term from Equation 3.8.8 in the $L_i(r_n)$ term, we define a new space variable,

$$M_i(r_n) = r_n L_i(r_n) \qquad (3.9.1)$$

By methods similar to those of Section 3.8, the right side of Equation 3.8.6 is developed into the final form.

$$\int_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} r_n'^2 L_i(r_n') dr_n' = \int_{r_n - \frac{\Delta r_n}{2}}^{r_n + \frac{\Delta r_{n+1}}{2}} r_n' M_i(r_n') dr_n'$$

$$= M_i(n^-)r_n \frac{\Delta r_{n-1}}{2} (1 - \frac{\Delta r_{n-1}}{3r_n}$$

$$+ M_i(n^+)r_n \frac{\Delta r_n}{2} (1 + \frac{\Delta r_n}{3r_n}) \tag{3.9.2}$$

As in Section 3.8, the $M_i(n^+)$ and $M_i(n^-)$ terms have the same connotation as did those of the $D_i(n^{\pm})$. Similarly, within the kth homogeneous region, the right side of Equation 3.9.2 reduces to

$$M_i(k)r_n\Delta r_n \tag{3.9.3}$$

## 3.10 Consolidation of Modifications to the Diffusion Equation

By assembling the results of Sections 3.8 and 3.9, the final result for the integration at any mesh point, n, can be expressed as

$$-D_i(n^+) \frac{\phi_{n+1,i} - \sigma_{2n}\phi_{n,i}}{\Delta r_n} + D_i(n^-) \frac{\sigma_{1n}\phi_{n,i} - \phi_{n-1,i}}{\Delta r_{n-1}}$$

$$= \tau_{2n}M_i(n^+) + \tau_{1n}M_i(n^-) \quad , \tag{3.10.1}$$

where the new terms are the geometry-dependent coefficients defined as follows:

$$\sigma_{1n} = 1 - \frac{\Delta r_{n-1}}{r_n} \tag{3.10.2}$$

$$\sigma_{2n} = 1 + \frac{\Delta r_n}{r_n} \tag{3.10.3}$$

$$\tau_{1n} = \frac{\Delta r_{n-1}}{2} (1 - \frac{\Delta r_{n-1}}{3r_n}) \tag{3.10.4}$$

$$\tau_{2n} = \frac{\Delta r_n}{2} (1 + \frac{\Delta r_n}{3r_n}) \tag{3.10.5}$$

The two sigma terms develop from factoring out the $r_n$ term from the left side of Equation 3.8.11, thus permitting the cancellation of the $r_n$ term on both sides of Equation 3.10.1.

3.11 The Diffusion Equation as Used in the IBM 650

Equation 3.10.1 may also be expressed in another form, in which the several components of $M_i(r)$ are separately considered. This is the final form in which it is programmed for use with the IBM 650. The result is

$$-a_{n,i}\phi_{n+1,i} + b_{n,i}\phi_{n,i} - c_{n,i}\phi_{n-1,i} = Q_{n,i} , \qquad (3.11.1)$$

where

$$Q_{n,i} = \chi_i(\tau_{1n}S_{n^-} + \tau_{2n}S_{n^+}) + \frac{1-e^{-\Delta u_i}}{\Delta u_i}\left[\tau_{1n}\eta(n^-,u_{i-1}) + \tau_{2n}\eta(n^+,u_{i-1})\right] + e_{n,i}\phi_{n,i-1} \qquad (3.11.2)$$

New terms used in the above equations are defined as follows:

(a) $a_{n,i} = \dfrac{K_{i_1}^{1}(n^+)}{\Delta r_{n+}}$ . . the diffusion coefficient of the region radially outside the mesh point, n. (3.11.3)

(b) $b_{n,i} = \sigma_{1n}c_{n,i} + \sigma_{2n}a_{n,i} + \tau_{2n}\left[K_{i_1}^{4}(n^+) + K_{i_1}^{3}(n^+)\right] + \tau_{1n}\left[K_{i_1}^{4}(n^-) + K_{i_1}^{3}(n^-)\right]$ (3.11.4) . . the combined terms for resonance escape probability, Selengut-Goertzel scattering by hydrogen and leakage at the nth mesh point.

(c) $c_{n,i} = \dfrac{K_{i_1}^{1}(n^-)}{\Delta r_{n-1}}$ . . the diffusion coefficient (3.11.5)

of the region radially inside the mesh point, n.

(d) $e_{n,i} = \tau_{2n} K_{i_2}^{3}(n^+) + \tau_{1n} K_{i_2}^{3}(n^-)$ . . . the (3.11.6)

averaged proportionality factors for the slowing down of neutrons by materials other than hydrogen at the nth mesh point.

(e) $S_{n+}$ . . the source of fission neutrons at the mesh point interface from the region forming the outer side of the interface.

(f) $S_{n-}$ . . the analagous fission neutron source term at the inner interface of the nth mesh point.

$$\text{(g)}\quad K_{i_1}^{1}(n^{\pm}) = \left\{ \int \frac{du}{3\xi\Sigma_s\Sigma_{tr}} \div \int \frac{du}{\xi\Sigma_s} \right\}_{n^{\pm},i} \qquad (3.11.7)$$

$$\text{(h)}\quad K_{i_1}^{3}(n^{\pm}) = \left\{ \left[ \int \frac{\Sigma_a du}{\xi\Sigma_s} + \frac{1}{f_i} \right] \div \int \frac{du}{\xi\Sigma_s} \right\}_{n^{\pm},i} \qquad (3.11.8)$$

$$\text{(i)}\quad K_{i_1}^{4}(n^{\pm}) = \frac{1-e^{-\Delta u_i}}{\Delta u_i} \left\{ \int \frac{\Sigma_{sh} du}{\xi\Sigma_s} \div \int \frac{du}{\xi\Sigma_s} \right\}_{n^{\pm},i}$$

(3.11.9)

$$\text{(j)}\quad K_{i_2}^{3}(n^{\pm}) = \frac{\Delta u_{i-1}}{\Delta u_i} \left\{ \frac{1}{f_i} \div \int \frac{du}{\xi\Sigma_s} \right\}_{n^{\pm},i-1} \qquad (3.11.10)$$

With reference to the $S_{n\pm}$ terms defined in (e) and (f) above, if the region in which the $n^+$ or $n^-$ interface is located is non-fuel, the terms will be identically zero. Furthermore, only one source term exists at mesh points interior to any homogeneous region.

3.12 The Recursion Formulas

With the integrated diffusion equation in the form of Equation 3.11.1, two recursion formulas[10] are developed for the three-term linear system.

$$Z_{n,i} = \frac{a_{n,i}}{b_{n,i} - c_{n,i}Z_{n-1,i}} \quad , \quad n = 0,1,2, \ldots\ldots N \quad Z_{-1,i} = 0 \qquad (3.12.1)$$

$$P_{n,i} = \frac{c_{n,i}P_{n-1,i} + Q_{n,i}}{b_{n,i} - c_{n,i}Z_{n-1,i}} \quad , \quad n = 0,1,2, \ldots\ldots N \quad P_{-1,i} = 0 \qquad (3.12.2)$$

where N indicates the value of the mesh point at the extrapolated outer boundary of the reactor.

Utilizing the terms computed by Equations 3.12.1 and 3.12.2, the fluxes, $\phi_{n,i}$, are calculated by means of the formula

$$\phi_{n,i} = P_{n,i} + Z_{n,i}\phi_{n+1,i} \quad , \quad n = N-1,\ N-2,\ \ldots\ldots,\ 0 \qquad (3.12.3)$$

with the boundary condition of zero flux at the extrapolated outer boundary of the reactor, or

$$\phi_{N,i} = 0 \qquad (3.12.4)$$

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[10] G.J. Habetler, One-Space-Dimensional Multigroup for the IBM 650, Part I, Equations, (pamphlet), December 1, 1955, Number KAPL-1415, Office of Technical Services, Department of Commerce, Washington 25, D.C., p. 12

## 3.13 Conditions for Parameters at Thermal Lethargy

Certain conditions are imposed upon the program for the computation of the fluxes in the thermal lethargy group, where i = I, the maximum value. They are as follows:

$$D_I = \frac{1}{3\Sigma_{tr}}\Bigg]_{u = u_{th}} \qquad (3.13.1)$$

$$A_I = \Sigma_a \Bigg]_{u = u_{th}} \qquad (3.13.2)$$

$$\Delta u_I = 1 \qquad (3.13.3)$$

$$e^{-\Delta u_I} = \frac{1}{f_I} = 0 \qquad (3.13.4)$$

$$\frac{\Sigma_{sh}}{\xi \Sigma_s}\Bigg]_{u = u_{th}} = 0 \qquad (3.13.5)$$

## 3.14 Sequence of Solution for Fluxes

Starting with the lowest lethargy group, where i=1, the $Z_{n,i}$ and $P_{n,i}$ terms are computed and placed in temporary storage. These calculations -- using Equations 3.12.1 and 3.12.2, and related equations -- are conducted for each mesh point sequentially from r=0 out to the next to the last mesh point, with separate terms computed on both sides of each region interface.

Once the $Z_{n,i}$ and $P_{n,i}$ terms are all stored for the lowest lethargy group, the flux calculations using Equation 3.12.3 are started at the extrapolated outer boundary of the reactor (where the assumed zero flux value exists) and then proceed inwards to the center of the equivalent sphere.

After the fluxes for the lowest lethargy group have all been computed, the results are punched out on IBM cards, and the procedures outlined above are repeated for each of the remaining lethargy groups, in order. The steps from "BEGIN" to "CLEAR" of the schematic flow chart in Appendix A display these processes.

### 3.15 The Flux Values at Zero Radius

In Equation 3.8.8, a new flux variable was defined to assist in making the diffusion equation compatible with the capabilities of the computer. This variable is the product of the spatially dependent flux term at a general mesh point, averaged over the ith lethargy group, times the radial distance to that mesh point. At the center of the equivalent sphere, therefore, a zero value appears for the flux, as will be seen in the table of flux values in Chapter 6.

To determine the approximate value for the flux at the center of the sphere, the mesh points of the innermost region (Region I) of the reactor should be selected close together. The flux curve connecting these points on a graph of flux level versus radius is then extrapolated to the zero radius ordinate, providing a good approximation of the flux value at that point.

## CHAPTER 4

## THE MACHINE PROGRAM FOR CHECKING CONVERGENCE OF THE MULTIGROUP SOLUTIONS

### 4.1 Convergence of the Iteration Process

The discussion in Chapter 3 developed the machine program processes by which the fluxes for all lethargy groups are computed. After the values of these fluxes have been punched out on the IBM cards, the second portion of the program uses these values as input data to determine if the proper flux values have been achieved for criticality of the reactor.

The unknown values for which the second part of the program is to be solved are the eigenvalue, $\nu_c$; the criticality factor, k; and a new set of source terms, $S_n$. The term $\nu_c$ is defined in Holmes and Meghreblian[11] as "..... a fictitious number of neutrons which must be produced with each fission in order to maintain the given system at a steady state". With k being the value for the criticality factor, and $\nu$ having its usual connotation of the average number of neutrons produced per fission, the relation of $\nu_c$, k and $\nu$ to each other is

$$k = \frac{\nu}{\nu_c} . \tag{4.1.1}$$

[11]D.K. Holmes and R.V. Meghreblian, Notes on Reactor Analysis; Part II, Theory, (pamphlet), August, 1955, Number ORNL-CF-54-7-88 (Pt II), Office of Technical Services, Department of Commerce, Washington 25, D.C., p. 368

4.2 Determination of the Eigenvalue, $\nu_c$

In order to utilize the relationship expressed in Equation 4.1.1, the program must obtain the value of $\nu_c$. To accomplish this task, the sets of fluxes for each of the lethargy groups which were just computed by the multigroup process are fed back into the read hopper of the computer in the same order in which they were punched. That is to say, the lowest lethargy group fluxes will be the first to be processed within the computer, with thermal fluxes processed last.

After the first set of fluxes are stored on the drum, the machine selects the flux for the first mesh point of the innermost core region of the reactor. This flux is multiplied times the product of the macroscopic fission cross section of that core region (as a function of lethargy for the same lethargy group) and the width of the lethargy group, $\Delta u_i$, or

$$\pi_i(n_c) = \phi_{n_c,i} \Sigma_f(k_{c,i}) \Delta u_i \, , \qquad (4.2.1)$$

where the above symbols are defined as follows:

$\phi_{n_c,i}$ . . the neutron flux at the nth mesh point of the reactor in a core region for the ith lethargy group.

$\Sigma_f(k_{c,i})\Delta u_i$ . . the product of the lethargy-dependent macroscopic fission cross section within the kth region of the reactor (the subscript c indicating that it is a core region) times the width of the ith lethargy group.

$\pi_i(n_c)$ . . the product of the two terms defined above.

In the sequence of operations through all the lethargy groups, the product obtained is added to all similar products of the other lethargy groups for the same mesh point, and these sums are then stored in another region on the drum.

At the completion of the process for each lethargy group at all mesh points of all fuel regions, the resulting quantities placed in storage for each individual fuel region mesh point will be

$$S(n_c) = \sum_{i=1}^{I} \pi_i \quad . \qquad (4.2.2)$$

At the interface between two fuel regions, there will be a separate summation term stored for each side of the mesh point. For example, if the ninth mesh point (n=9) coincides with the interface between Regions I and II, both of which were core regions, an $S(9^-)$ term and an $S(9^+)$ term will be generated, to conform with the fission parameters of the inner and outer fuel regions, respectively. Similarly, a $\pi_i$ term exists on each side of such an interface.

The program next enters the integration sub-routine. This subroutine uses three approximation equations to sum up the number of fissions which have occurred in each fuel region for all lethargy groups.

The first of these equations, employed at each mesh point internal to a fuel region but not at an interface with another region, is

$$N_f(\Delta r_n) = r_n \Delta r_k S(n_c) \ , \qquad (4.2.3)$$

where $r_n$ is the distance to the mesh point from the center of the reactor, $\Delta r_k$ is the width of the sub-regions in the kth core region, and $N_f(\Delta r_n)$ is the number of fissions at all lethargies computed in the incremental sub-region adjacent to the mesh point being considered. The other two equations used in the integration sub-routine are for similar computations at the inner and outer sides of the interfaces between different fuel regions. They are

$$N_f^-(\Delta r_k) = \left[\frac{r_n}{2} - \frac{\Delta r_k}{6}\right] \Delta r_k S^-(n_c), \text{ and} \qquad (4.2.4)$$

$$N_f^+(\Delta r_{k+I}) = \left[\frac{r_n}{2} + \frac{\Delta r_{k+I}}{6}\right] \Delta r_{k+I} S^+(n_c) \ , \qquad (4.2.5)$$

where the minus and plus superscripts and the k and k+I subscripts indicate values respectively at the inner and outer sides of the interface between different fuel regions.

The results of the above equations at all fuel mesh points of the reactor are accumulated as a grand total, $N_F$, which is stored in a separate storage location in the computer.

In actuality, Equations 4.2.3 through 4.2.5 are multiplying the $S(n_c)$ terms at each fuel mesh point times the area of the spherical surface which coincides with that mesh point. The $4\pi$ term has not been included in these equations, because the next step in the solution cancels out that term, as will be seen shortly.

After the grand total of all fissions throughout the reactor, $N_F$, has been obtained, the value of $\nu_c$ is calculated by the relation

$$\nu_c = \frac{V}{N_F} \quad . \tag{4.2.6}$$

The new term, V, in the above equation is equal to the total volume of all fuel regions in the reactor, divided by $4\pi$. It is this mutual cancellation of the $4\pi$ terms which was mentioned in the preceding paragraph.

## 4.3 Computing the Criticality Constant

By the relation expressed in Equation 4.1.1, the criticality constant, k, is next computed in the machine program. The value used for $\nu$ in this program is 2.50. Upon completion of this step, the computer punches one IBM card containing the values of $\nu_c$ and k for the current iteration.

## 4.4 Generation of New Source Terms

Following the computation of $\nu_c$ and k, the program next directs the computer to generate a new set of neutron source terms at each interface of the reactor so that the computer can return to the calculations previously described in Chapter 3 and iterate a new set of fluxes.

Comparable to the $S(n_c)$ terms described in Section 4.2, there will be a source term on each side of the interface between major regions and a single source term at every other mesh point. Naturally enough, in non-fuel regions all source terms will be zero.

To obtain the source terms $S_n'$, the $S(n_c)$ terms are multiplied by $\nu_c$ at each mesh point:

$$S_n' = \nu_c S(n_c) \ . \qquad (4.4.1)$$

The resulting source terms are then punched out on IBM cards, so that if machine operation need be halted at this point, the iterative procedure may be resumed at a later time by simply reloading these source cards in with the data deck. If iteration is to be continued immediately, these source cards need not be reloaded, as the source values are in the proper storage position in the computer for the next iteration.

### 4.5 Determining Convergence of the Solutions

We now consider two successive series of calculations. If the value of $\nu_c$ changes only by a very small amount - in the hundredths or thousandths decimal position, for example - on the two successive runs, one may consider that the problem has converged to the desired degree.

It is emphasized to the reader that due to the nature of the mathematics of the program, the values of $\nu_c$ will converge, even if the state of criticality is nowhere near reality. Consequently, it is necessary to have some other check on the calculations. The criticality constant, k, serves this purpose. For a reactor to be operating in the steady state, the same number of neutrons must be gained in a given generation as are lost in the preceding generation. Consequently, the criticality constant, k, should equal unity. Since so many factors involved in the calculations are not known to a high degree of accuracy, values of k very close to unity will be acceptable in determining the criticality.

The iterative process is completed when the conditions discussed above have been met. The last set of fluxes are adjusted to the proper dimensions by dividing the radial flux value at each mesh point by the spherical radius to that mesh point, in conformity with Equation 3.8.8.

CHAPTER 5

DATA CALCULATIONS FOR THE TRIGA MULTIGROUP TEST RUN

## 5.1 Introduction

To facilitate understanding of the calculations to be made using TRIGA as the test vehicle, the next three sections are devoted to brief descriptions of the various portions of the TRIGA reactor which are of interest in such calculations. For more detailed descriptions, the reader is referred to General Atomic's Pamphlet Number GA-296,[12] which was the source for the following information.

## 5.2 Description of the TRIGA Core

The core lattice of TRIGA is in the shape of a right circular cylinder, the top of which is approximately sixteen feet below the surface of the water tank in which it is immersed. At the top and bottom of the core, aluminum grid plates are positioned to hold the cylindrically shaped fuel elements in a vertical position. These grid plates have a central hole drilled in them to admit a vertical aluminum tube, called the Glory Hole, which extends above the water surface. The Glory Hole, also called the A ring, is one of the locations provided for irradiating samples, since the maximum thermal flux within the reactor exists in this region.

In the grid plates there are also five concentric rings of six,

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[12]General Atomic Division of General Dynamics Corporation, TRIGA Reactor Description, Pamphlet Number GA-296, San Diego, California, March 10, 1958, 16 pp

twelve, eighteen, twenty-four and thirty holes, respectively. These rings, in each of which the holes are spaced equally, are lettered alphabetically from B to F, the latter being the outermost ring. Of the ninety pairs of holes drilled in the grid plates, eighty-six are available as fuel element positions. Three of the remaining four positions are in the twelve positions of the C ring. They are equally spaced peripherally, and they contain the control rod guide tubes inside which the boron carbide control rods are operated. The fourth position not specifically available for a fuel element is located in the F ring. It contains the lower portion of the rabbit tube, a pneumatic device for producing isotopes with short half-lives.

### 5.3 The Fuel Elements

The fuel elements used in TRIGA are aluminum clad cylinders which are 23 inches long and approximately 1.47 inches in diameter. Their overall length is slightly greater because aluminum positioning spikes protrude from each end. At the top and bottom of each fuel element there is a one half inch aluminum plug and a four inch graphite reflector region. The remaining fourteen inches of the fuel element is composed of a homogenized mixture of zirconium hydride and uranium, the uranium enriched to 20% in $U^{235}$. Thin wafers of samarium oxide, a burnable poison, are positioned at the top and bottom of the fuel region, between the fuel and the upper and lower graphite reflectors. Their main purpose is to extend the life of the fuel elements by reducing excess reactivity during the early life of the fuel charge.

In the University of Arizona TRIGA reactor, only sixty of the eighty-six available fuel element positions are actually occupied by fuel elements. The remaining twenty-six positions located in the F ring are filled with graphite dummy elements, which are identical to the fuel elements, except that the zirconium hydride-uranium alloy is replaced with graphite.

Since the core is immersed in water, all of the fuel elements are surrounded by water, and the net volume of water throughout the fuel region is approximately 35% by volume. This water and the other applicable reactor materials serve as the moderating medium for the fission neutrons.

## 5.4 The Reflector Region of TRIGA

Surrounding the core of TRIGA is an annular cylindrical graphite reflector which is approximately twenty-two inches high and twelve inches in radial thickness. This reflector region is completely enclosed in an aluminum container. Within the reflector itself there is an annular void approximately 10 inches deep and 6.4 inches in radial thickness, in which as many as 40 samples can be placed for irradiation in a rotary specimen rack. This device is known as the Lazy Susan.

## 5.5 Selection of Regions for the Multigroup Calculations

In subdividing the TRIGA reactor into separate regions, there were three main portions of the reactor which were considered independently. These three portions were as follows:

a. The core volume from the vertical axis of the reactor radially outward to the inner vertical surface of the reflector

b. The entire reflector volume

c. An effectively infinite water region outside the reflector region.

The core volume was broken up into five separate regions, the innermost one being the volume occupied by the Glory Hole and the remaining four consisting of the fuel elements and the water surrounding them. The height used in calculating the volume for each region was the 23 inch (or 58.42 cm) height of the fuel elements in the core. In general, the radial midpoint between the concentric rings of fuel elements was arbitrarily taken to be the boundary between core regions.

The reflector volume was separated into three different regions. The inner volume was chosen as the inner portion of the reflector radially outward to the vertical center line of the Lazy Susan. The second region in the reflector is the Lazy Susan itself, and the third region is the remaining outer portion of the reflector.

After the above eight regions were converted to concentric shells of an equivalent sphere of equal volume, an effectively infinite water region surrounding the entire reactor was added as the ninth region.

After figuring the radial thickness of each equivalent spherical shell, the nine major regions were subdivided into an appropriate number of subregions, yielding 57 mesh points.

Definition of the nine major regions employed in multigroup calculations and the results of the calculations described above are given in Table I.

### 5.6 Homogenizing the Reactor Materials

The volumes of the various reactor materials are computed for each reactor region, and their volume percentages of the total region volume are then calculated. With this information, the number of atoms

TABLE I

THE NINE REGIONS OF TRIGA FOR TEST MULTIGROUP CALCULATION

| Region Number | Reactor Components | Actual Volume ($cm^3 x 10^3$) | Outer Radius, Equivalent Spherical Shell, (cm) | Radial Thickness, (cm) | Number of Sub-Regions |
|---|---|---|---|---|---|
| I | Glory Hole | 0.462 | 4.798 | 4.798 | 4 |
| II | B Ring | 6.183 | 11.663 | 6.865 | 6 |
| III | C Ring | 11.573 | 16.323 | 4.660 | 4 |
| IV | D & E Rings | 40.609 | 24.126 | 7.803 | 8 |
| V | F Ring | 29.004 | 27.588 | 3.462 | 3 |
| VI | Inner Part[a] of Reflector | 96.601 | 25.310 | 7.722 | 7 |
| VII | Lazy Susan | 43.330 | 37.883 | 2.573 | 2 |
| VIII | Outer Part[b] of Reflector | 312.651 | 50.528 | 12.645 | 8 |
| IX | Water | 1,769.155 | 82.000 | 31.472 | 15 |

[a]From inner boundary of reflector to vertical center line of the Lazy Susan

[b]From vertical center line of Lazy Susan to outer boundary of reflector

(or molecules) of each reactor material which would exist in a cubic centimeter of the homogenized mixture is easily calculated by multiplying the volume fraction of each material times the normal atomic or molecular number density of that material.

5.7 Selection of the Lethargy Groups

As mentioned in Section 1.2, one criterion for the selection of lethargy groups in a multigroup calculation is that the groups should be chosen so that the physical reaction characteristics of the reactor materials can be considered essentially constant within any given lethargy group. For a reactor composed of seven or eight different materials in some regions, the selection of the boundaries between lethargy groups is based upon a survey to determine the material, or materials, which will have the most dominant macroscopic reaction cross sections. With the atomic or molecular number densities of the homogenized reactor materials computed by the method described in Section 5.6, reference to the plotted curves of microscopic reaction cross sections as a function of energy, such as are found in BNL-325,[13] furnishes the necessary information upon which to base the decision on lethargy group boundaries.

In this test calculation for TRIGA, it was decided that four lethargy groups would be used. As would be anticipated in a reactor with a fairly large amount of $U^{238}$ in the core regions, the high resonance absorption peaks of $U^{238}$ in the energy range of 4.56 - 1234 ev

[13] D. J. Hughes and R.B. Schwartz, Neutron Cross Sections, (catalog), July 1, 1958, Number BNL-325, Office of Technical Services, Department of Commerce, Washington 25, D.C.

influenced the selection of lethargy group boundaries. The four lethargy groups employed in the test calculation and the $\Delta u_i$ for each group are listed in Table II.

5.8 Calculation of the Macroscopic Reaction Cross Sections

For each region of the reactor, it is necessary to calculate the macroscopic reaction cross sections for all lethargy groups used in the program. The specific macroscopic reaction cross sections which are required to prepare the data cards for the machine program are the macroscopic absorption, non-hydrogeneous scattering, hydrogeneous scattering, transport and fission cross sections. Recalling that the macroscopic cross section of an element is obtained by multiplying its microscopic cross section times the atomic number density of the element, we see that the macroscopic cross section for a homogenized region of the reactor will be the sum of such cross sections for all elements in the region. The atomic number densities used in the calculations are the modified ones based on the volume percentage of the element in the region. The average microscopic cross section for each element is obtained by taking the arithmetic mean of the energy dependent microscopic cross sections over the energy range associated with the desired lethargy group.

In the calculations for the test multigroup program of TRIGA, the writer used the values of microscopic reaction cross sections found in Table 3-25 of ANL-5800[14]. This table gives the microscopic

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[14]Argonne National Laboratory, Reactor Physics Constants Center, Reactor Physics Constants, Number ANL-5800, July 1, 1958, Lemont, Illinois, pp. 157-159

TABLE II

THE FOUR LETHARGY GROUPS FOR THE

TEST MULTIGROUP CALCULATION

| Group Number | Lower Energy Limit, E, (ev) | $u = \ln \frac{E_o}{E}$ [a] | $\Delta u_i$ [b] |
|---|---|---|---|
| 1 | 1234.000 | 9.00 | 9.00 |
| 2 | 4.564 | 14.6 | 5.60 |
| 3 | .0796 | 18.698 | 4.098 |
| 4 | - - - | - - - | 1.00[c] |

[a] $E_o$ = 10 Mev

[b] $\Delta u_i = u_i - u_{i-1}$

[c] $\Delta u_4$ is set equal to 1.00, one of the conditions for solution as stated in Section 3.13.

absorption, transport, fission and total cross sections for 20 common reactor elements in a 30 group set covering the energy range from 10 Mev down to the thermal energy threshhold of 0.07595 ev. For TRIGA, it provides the necessary information for all elements except boron. The calculations for boron were conducted by the methods described in the preceding paragraph. It was necessary to convert the microscopic cross sections from the values averaged over 30 lethargy groups to a new set of values averaged over the three fast groups employed in the test program. This was accomplished by multiplying each microscopic cross section of the 30 group set by the width of its lethargy group, adding up these products within the lethargy ranges of each of the test program's three fast groups, and dividing each of the three sums by the associated $\Delta u_i$ from Table II.

For the microscopic cross sections in Group 4, the thermal lethargy group, all of them except the cross sections of $U^{235}$ and boron were obtained from BNL-325[15]. All thermal cross sections were then corrected for the Maxwell-Boltzmann energy distribution to give more realistic average reaction cross sections.

5.9 Selection of the Weighting Factor, $f_i(r)$

In Equation 3.6.3, we defined the term, $f_i(r)$, as being the ratio of the slowing-down density for materials other than hydrogen, averaged within a certain region and lethargy group, to the actual non-hydrogeneous slowing-down density at the upper lethargy boundary

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[15]D.J. Hughes and R.B. Schwartz, Neutron Cross Sections, (catalog), July 1, 1958, Number BNL-325, Office of Technical Services, Department of Commerce, Washington 25, D.C.

of the same lethargy group and in the same region. As stated by Ehrlich and Hurwitz[16], $f_i(r)$ is "a number chosen by trial and error for each region and group so that $[f_i(r)]$ times the spatial average of $[q(r,u_i)]$ gives a reasonable value for the spatial average of $[\bar{q}_i(r)]$."

According to the above cited reference, $f_i(r)$ is selected as somewhat greater than unity in non-fuel regions, and may be less than unity in fuel regions of the reactor.

In several experimental runs of the test multigroup machine program conducted by this writer, using $f_i(r)$ terms which varied by as much as a factor of two, it was found that the resulting fluxes were essentially insensitive to the variations imposed on this term.

The values of $f_i(r)$ selected for the test multigroup calculation for TRIGA are given in Table III.

5.10 Calculation of the Average Logarithmic Energy Decrement

In the two preceding sections, the techniques for computing the macroscopic reaction cross sections for the reactor regions in all lethargy groups and for selecting the $f_i(r)$ terms have been discussed. Additional data which must be calculated are the values of the average logarithmic energy decrement, $\bar{\xi}$, for each region.

For any element with an atomic mass greater than 10, a good approximation[17] for the value of its logarithmic energy decrement is

$$\xi = \frac{2}{A + 0.667} \qquad (5.10.1)$$

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[16] R. Ehrlich and H. Hurwitz, Jr., "Multigroup Methods for Neutron Diffusion Problems", Nucleonics, Vol. 12, No. 2, February, 1954, p. 24

[17] S. Glasstone, Principles of Nuclear Reactor Engineering, 1st Ed., D. Van Nostrand Company, Inc., New York, 1955, p. 152

TABLE III

TEST VALUES FOR THE NON-HYDROGENEOUS SLOWING-DOWN

DENSITY WEIGHTING FACTOR, $f_i(r)$

| Region | $f_1(r)$ | $f_2(r)$ | $f_3(r)$ | $f_4(r)$ |
|---|---|---|---|---|
| I | 1.5 | 1.8 | 2.1 | 2.4 |
| II | 0.96 | 1.15 | 1.8 | 2.5 |
| III | 0.96 | 1.15 | 1.8 | 2.5 |
| IV | 0.99 | 1.3 | 2.0 | 2.7 |
| V | 1.0 | 1.4 | 1.6 | 1.8 |
| VI | 1.2 | 1.4 | 1.6 | 1.8 |
| VII | 4.2 | 4.3 | 4.4 | 4.5 |
| VIII | 1.2 | 1.4 | 1.5 | 1.7 |
| IX | 1.7 | 2.0 | 2.3 | 2.6 |

where A has the usual connotation of the atomic mass number of the element.

To get the average logarithmic energy decrement for an entire region the method employed was to multiply the calculated values of $\xi$ for each element in the region times the atomic number density of that element in the homogenized region. These products for all the elements in a region were summed up and then divided by the total atomic number density of all elements in a homogenized cubic centimeter of the reactor to yield $\bar{\xi}$.

5.11 Calculation of the Fission Spectrum Term, $\overline{\chi_i}$

Another set of data which must be determined is $\overline{\chi}_i$, the values of the fission spectrum averaged over each of the lethargy groups. The lethargy dependent form of the fission spectrum, $\chi(u)$, is defined in Section 3.1. Since the definition states that the area under the fission spectrum curve must equal unity, it follows that the sum of the terms for all lethargy groups must also equal unity.

A reasonably accurate method to calculate the fission spectrum is stated by Weinberg and Wigner[18] as

$$f(E) = 2 \times 0.775\ (0.775E/\pi)^{0.5}\ e^{-0.775E} \qquad (5.11.1)$$

where f(E) is the energy-dependent fission spectrum analagous to $\chi(u)$. Values of E (in Mev) are chosen at the boundaries of the lethargy groups and at intermediate energies if deemed necessary. A set of values of f(E) are then calculated, which are then plotted against the

[18] A.M. Weinberg and E.P. Wigner, The Physical Theory of Neutron Chain Reactors, 1st Ed., The University of Chicago Press, 1958, p. 112

energy in Mev to give a curve of the probability of a neutron being born at a certain energy. With the area under the curve normalized to unity, the fraction of this area between the energies corresponding to the boundaries of a certain lethargy group represents the probability of fission neutrons being born in that particular lethargy group.

The values for $\overline{\chi}_i$ used in the test program were obtained from Table 3-25 of ANL-5800[19], the same source from which the microscopic reaction cross sections were taken.

5.12 Preparing the Data for the Machine Program

After computing the required macroscopic reaction cross sections and the average logarithmic energy decrement, and further selecting a set of values for $f_i(r)$, the calculation of the four terms defined in Equations 3.11.7 through 3.11.10 is the next major step in preparing the information for the data cards to be used in the multigroup machine program. Since each of the four terms is a function both of the lethargy group and of the region considered, the number of each of these terms calculated is the product of the number of lethargy groups used times the number of reactor regions chosen.

Since the methods described in Sections 5.8 and 5.10 obviate the need for the averaging techniques employing integrations in the numerators and denominators, as in the four equations cited in the above paragraph, those equations are closely approximated by a new set

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[19]Argonne National Laboratory, Reactor Physics Constants Center, Reactor Physics Constants, Number ANL-5800, July 1, 1958, Lemont, Illinois, p. 157

of equations which are easier to handle in preparing the data for the machine run. The relations used in preparing the data are as follows:

$$K_{i_1}^{1}(n^{*}) = \left.\overline{\frac{1}{3\Sigma_{tr}}}\right]_{n^{*},i} \qquad (5.12.1)$$

$$K_{i_1}^{3}(n^{*}) = \left[\overline{\Sigma_a} + \overline{\frac{\xi\Sigma_s}{f_i(r)}}\right]_{n^{*},i} \qquad (5.12.2)$$

$$K_{i_1}^{4}(n^{*}) = \left.\frac{1-e^{-\Delta u_i}}{\Delta u_i}\ \overline{\Sigma_{sh}}\right]_{n^{*},i} \qquad (5.12.3)$$

$$K_{i_2}^{3}(n^{*}) = \frac{\Delta u_{i-1}}{\Delta u_i}\left[\overline{\frac{\xi\Sigma_s}{f_i(r)}}\right]_{n^{*},i-1} \qquad (5.12.4)$$

The reader should note that all of the $K_{i_1}^{1}(n^{*})$ values in Equation 5.12.1 are equal to each other for a given lethargy group and <u>within</u> any homogenized region. They differ only when the value of n is that of a mesh point between two different regions of the reactor. This same relationship is applicable to the remaining three $K_i(n^{*})$ terms in Equations 5.12.2 through 5.12.4.

5.13 <u>Preparing the Data Cards and Conduct of the Program</u>

After all the calculations described in the preceding section have been completed, the data cards for the machine program are punched in the formats described in Appendix A. The necessary steps to follow in operating the IBM 650 for a machine run are also explained in the same appendix.

Tables showing the actual data used in the test multigroup calculations are listed in Appendix B.

The results of the test program employing the data tabulated in Appendix B are discussed in the next chapter.

## CHAPTER 6

## THE TEST MULTIGROUP PROGRAM FOR THE TRIGA REACTOR

### 6.1 Conduct of the Test Calculations

The multigroup program was made up by conventional programming procedures developed for the University of Arizona's SOAP X program. The material discussed in Chapters 3 and 4 of this thesis was used to prepare this test program. Reactor constants and other data used for test runs of the multigroup program were calculated by the general procedures discussed in the preceding chapter.

Several experimental runs of the program were conducted to determine the ability of the program to achieve the desired objectives. Various approximations were found to be necessary in the calculations of the data, before the program results were considered satisfactory. These approximations are discussed in the next section.

### 6.2 Approximations Used for the TRIGA Test Multigroup Program

As has been previously discussed, this multigroup program is designed to deal with a reactor in a spherical configuration. When one starts with a cylindrical reactor such as TRIGA and sphericizes the concentric cylindrical regions into spherical shells, it is to be anticipated that certain approximations must be made in order for the calculated results to agree substantially well with the empirical results.

If one further homogenizes the reactor materials within each region into a theoretical slurry of water, graphite, fuel, poisons and aluminum, the reader will recognize that quite a drastic change from reality has occurred. It is because of such factors as these that the actual and the calculated critical fuel masses might differ by as much as a factor of two without the program being considered invalid.

Recalling the descriptions of the core volume and the fuel elements in Sections 5.2 and 5.3, it can be seen that the principal approximations to be made in obtaining satisfactory agreement between actual and calculated critical fuel masses will involve the poisons introduced into the fuel regions of the reactor. These poisons are the boron carbide regulating rod, and the samarium oxide wafers situated at the top and bottom of the fuel region in each fuel element. A discussion of the approximations made for each of the two poisoning materials follows.

a. The Boron Carbide Control Rods

When the reactor is brought up to criticality and is operating in the steady state, the only control rod which is still inserted in the core is the regulating rod. Exact information on how much of the regulating rod is still inserted in its position in the C ring for a given power level could not be determined through any available sources of information. Consequently, an initial estimate was made that approximately 18 cm. of the regulating rod would still be in the core when the reactor was operating at some intermediate power level. Experimental runs of the machine program using data based on the above estimate revealed that a drastically large depression occurred in the thermal neutron flux distribution within Region III. The flux level was reduced to such an extent that it was considered to be unrealistic. Further experiments revealed that the equivalent boron poisoning of approximately 8 cm of the boron carbide control rod, homogenized with the other reactor materials of Region III, produced flux distributions which appeared to more nearly agree with the real conditions within the reactor. Although probably as much as 12 to 16 cm of the regulating rod may actually be inserted into the fuel region of TRIGA while operating at an intermediate power level of 10 kilowatts, the artificial homogenization of Region III changes the poisoning effect of the boron from its relatively localized position at one point in the C ring to an overall poisoning of the entire region. Furthermore, part of the actual poisoning effect at the upper end of the fuel elements by that portion of the regulating rod in proximity thereto has been modified to a poisoning effect in a region of much higher neutron flux, due to the averaging effect which accrues from homogenization of the fuel regions. As a result of this feature, it was necessary to reduce the estimated length of regulating rod inserted in the fuel region to approximately 45 per cent of its probable actual length.

b. <u>The Samarium Oxide Wafers</u>

Early experimental runs of the test multigroup program included the absorption effects of the samarium oxide within each of the homogenized regions. It was observed that this was also causing a particularly adverse depression on the flux distribution of thermal neutrons throughout the entire reactor. As with the boron regulating rod, the sphericizing and homogenizing of the reactor materials within the regions chosen contributed to this lowering of the flux values.

Considering the location of the samarium oxide wafers at the top and bottom of the fuel elements, it is seen that their primary absorption effect is on those vertical components of the neutron flux in their vicinity. The wafers will have relatively little effect on the radial flux from the fuel elements. In addition, the magnitude of the flux in the vicinity of the top and bottom of the fuel elements will be much lower than it is at the vertically-measured midpoint of the elements.

Based on the above considerations, the poisoning value for the samarium oxide wafers was deleted in the calculation of the relative radial thermal flux distribution shown in Figure 6.1. This factor brought the flux values up to a realistic level for the program after which minor variations in the fuel terms produced a satisfactory result.

### 6.3 Test Results

The distribution of the thermal neutron flux, as resulted from a series of trial runs of the program is portrayed in Figure 6.1. After the necessary number of machine runs were conducted, the thermal neutron flux term for each mesh point was divided by the radial distance to that mesh point to convert the term back to the proportionate neutron flux for the thermal lethargy group. It should be noted that these flux

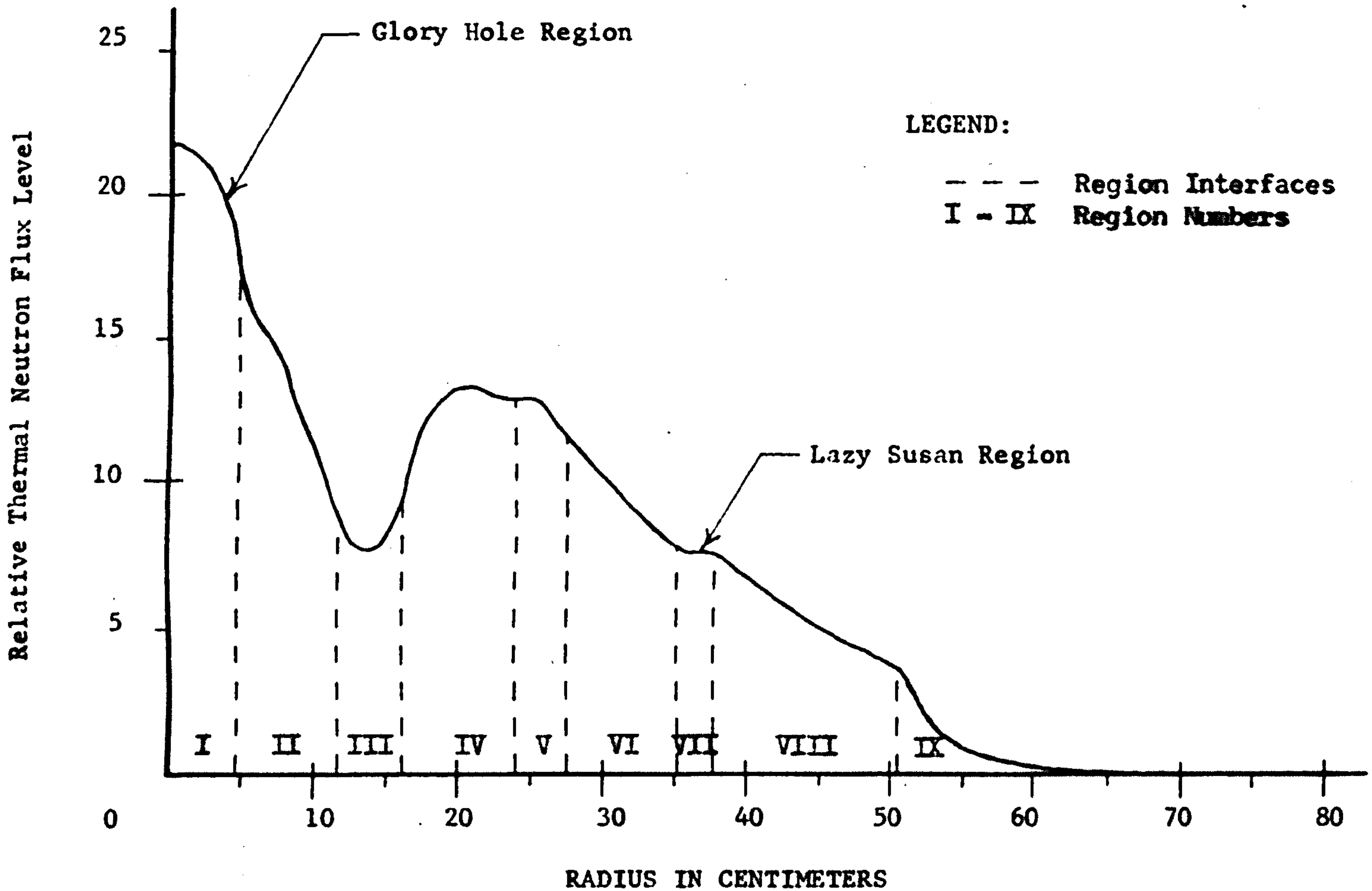


Fig. 6.1. Thermal Neutron Radial Flux Distribution, Test Multigroup Program for TRIGA.

TABLE IV

RELATIVE MESH POINT FLUX VALUES FOR TRIGA

FROM TEST MULTIGROUP PROGRAM

| Mesh Point | Radius (cm) | Relative Flux | Mesh Point | Radius (cm) | Relative Flux | Mesh Point | Radius (cm) | Relative Flux |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 20 | 22.2 | 13.1 | 40 | 47.4 | 4.4 |
| 1 | 1.2 | 21.7 | 21 | 23.2 | 12.9 | 41 | 48.9 | 4.0 |
| 2 | 2.4 | 21.1 | 22 | 24.1 | 13.0 | 42 | 50.5 | 3.6 |
| 3 | 3.6 | 19.9 | 23 | 25.3 | 12.9 | 43 | 52.6 | 1.7 |
| 4 | 4.8 | 17.7 | 24 | 26.4 | 12.3 | 44 | 54.7 | 0.8 |
| 5 | 5.9 | 15.9 | 25 | 27.6 | 11.5 | 45 | 56.8 | 0.4 |
| 6 | 7.1 | 14.9 | 26 | 28.7 | 11.0 | 46 | 58.9 | 0.2 |
| 7 | 8.2 | 13.6 | 27 | 29.8 | 10.4 | 47 | 61.0 | 0.08 |
| 8 | 9.4 | 12.2 | 28 | 30.9 | 9.8 | 48 | 63.1 | 0.04 |
| 9 | 10.5 | 10.7 | 29 | 32.0 | 9.2 | 49 | 65.2 | 0.02 |
| 10 | 11.7 | 8.9 | 30 | 33.1 | 8.6 | 50 | 67.3 | 0.01 |
| 11 | 12.8 | 7.7 | 31 | 34.2 | 8.1 | 51 | 69.4 | |
| 12 | 14.0 | 7.7 | 32 | 35.3 | 7.7 | 52 | 71.5 | |
| 13 | 15.2 | 8.5 | 33 | 36.6 | 7.5 | 53 | 73.6 | a |
| 14 | 16.3 | 10.1 | 34 | 37.9 | 7.5 | 54 | 75.7 | |
| 15 | 17.3 | 12.0 | 35 | 39.5 | 7.0 | 55 | 77.8 | |
| 16 | 18.3 | 12.6 | 36 | 41.0 | 6.4 | 56 | 79.9 | |
| 17 | 19.2 | 13.2 | 37 | 42.6 | 519 | 57 | 82.0 | 0.00 |
| 18 | 20.2 | 13.4 | 38 | 44.2 | 5.4 | | | |
| 19 | 21.2 | 13.4 | 39 | 45.8 | 4.9 | | | |

[a]Values approaching zero.

values are relative values only, and do not in themselves describe a specific flux level in the reactor. A list of these relative flux values obtained by the aforementioned process is given in Table IV. The zero value for the flux at $r = 0$ is discussed in Section 3.15.

After the approximations discussed in Section 6.2 were made, the terms involving the product of the lethargy dependent fission cross section for each core region times the lethargy width for that lethargy group were varied until sufficient convergence of the iterative process was achieved. With the last set of such critical fuel mass values employed, the value of k reached 0.9985 after the fifth iteration. At the same time, $\nu_c$ converged to the value of 2.504.

6.4 Analysis of Test Results

One of the thesis objectives was to run a test multigroup calculation with the new program to determine if the computed radial flux distribution would agree with that known to exist in TRIGA. The construction of TRIGA is such that radial flux measurements are only possible in the Glory Hole (Region I) and at the Lazy Susan (Region VII). Consequently, the ratio of the fluxes at these two points was selected as the criterion for determining the validity of the test calculations for radial flux distribution. In the "Technical Data Summary" of General Atomics' pamphlet [20] describing TRIGA, the ratio of the thermal neutron flux in the Glory Hole to that in the Lazy Susan is reported to be 2.50:1

[20] General Atomic Division, General Dynamics Corporation, TRIGA REACTOR DESCRIPTION, Pamphlet Number GA-296, San Diego, California, March 10, 1958, p. 14

at a power level of 10 kilowatts. Taking the extrapolated value of the relative thermal neutron flux level at the center of the Glory Hole and comparing it with the value found at the Lazy Susan, the ratio obtained was approximately 2.85:1. This figure was considered to be in reasonable agreement with the desired ratio.

An additional criterion for determining the validity of the multigroup program was that the critical fuel mass used in the test data should be within a factor of two of the actual fuel mass of the TRIGA reactor at the time of program convergence. Several adjustments of the $\Sigma_f(k_{c,i})\Delta u_i$ terms were required to achieve the desired values of k and $\nu_c$, as was mentioned in the preceding section. After convergence was achieved, similar terms were computed for each fuel region and for each lethargy group using the actual masses of fuel in each region of the reactor. A comparison of the two sets of values is provided in Table V. The reader will note that very good results were obtained, and that the ratio is well within the required factor of two.

## 6.5 Conclusions

Based on the analysis of test results presented in Section 6.4, the following conclusions are made:

a. The program as assembled will give a reasonable approximation of the radial thermal flux distribution within the TRIGA reactor.

b. Transformation of the cylindrical TRIGA into an equivalent sphere provides better multigroup program results in determining critical mass of the reactor than it does in calculating the flux distribution.

TABLE V

COMPARISON OF ACTUAL VALUES AND TEST RESULTS

FOR CRITICAL FUEL MASS TERMS, $\Sigma_f(kc,i)\Delta u_i$

| Lethargy Group | Fuel Region | Actual Value | Test Result |
|---|---|---|---|
| 1 | II | .0021 | .0019 |
| 1 | III | .0017 | .0016 |
| 1 | IV | .0022 | .0020 |
| 1 | V | .0002 | .0002 |
| 2 | II | .0057 | .0053 |
| 2 | III | .0048 | .0044 |
| 2 | IV | .0060 | .0056 |
| 2 | V | .0006 | .0006 |
| 3 | II | .0379 | .0356 |
| 3 | III | .0316 | .0298 |
| 3 | IV | .0401 | .0376 |
| 3 | V | .0042 | .0040 |
| 4 (Thermal) | II | .0409 | .0431 |
| 4 | III | .0341 | .0360 |
| 4 | IV | .0433 | .0455 |
| 4 | V | .0046 | .0048 |

c. An interesting topic for further study would be an effort to obtain verification of flux levels at intermediate radial points within the TRIGA reactor, if a suitable technique can be developed for such measurements.

# APPENDIX A

## DATA CARD FORMAT AND OPERATION

## OF THE MULTIGROUP MACHINE PROGRAM

Section 1. Format of the Machine Program Input Data Cards

The majority of Chapter 5 was devoted to the choice of regions and lethargy groups for a multigroup calculation and to the preparation of the data to be used as input information for the multigroup machine program. In this section, the seven different types of input data cards will be described individually, in sufficient detail to facilitate the punching of the input cards in their proper format.

The standard IBM eight word card is composed of 80 vertical columns in which digital information may be entered. The columns are numbered from 1 through 80 and from left to right across the card. In each column, any one of the 10 digits from 0 through 9 may be punched. Ten of the vertical columns of digital information constitute one "word" on the card. The words are numbered from 1 through 8 across the card from left to right. Thus, Word 1 on a card will consist of Columns 1-10, Word 3 contains Columns 21-30, and similarly for the other words on the card. When referring to specific digit positions within a 10 digit word, the digit positions are numbered from right to left. Thus, digit position 1 of Word 1 corresponds to Column 10 of the card, digit position 4 of Word 8 is located in Column 77 of the card, and so forth. In Columns 10, 20, 30, 40, 50, 60, 70 and 80, provision is made for punching a plus or minus sign. All data used in this program will be positive and must be so identified by the plus, or "high", punch in the 10th column of each data word. Word 1 of all cards is used only as a card identification word, and the high punch may be ignored in Column 10 for that reason. Since a high punch in Column 1 indicates a type of card other than a data card, a wise rule to follow is to have no high punches at all in Word 1.

The multigroup computer program has been designed to use automatic floating-decimal arithmetic, which eliminates the need for using scaling factors, as must be done in fixed decimal arithmetic. One of International Business Machines' publications[22] has a good description of the floating-decimal operations. Briefly, the floating-decimal notation positions the decimal point just to the left of the extreme left non-zero digit of a number, and then multiplies this decimal by the appropriate power of 10 so that the number retains its original magnitude. The power of 10 is in digit positions 1 and 2 of each word and is equal to the algebraic sum of the number, 50, plus the exponent of 10 with its sign. Thus, $10^{-2}$ is expressed as 48, and the floating-decimal equivalent of $10^3$ is 53. All of the eight remaining digit positions in a word may be used for the number in its decimal form. As an example of the system, the floating-decimal equivalent of "0.00531" is "5310000048".

It was mentioned above that Word 1 of all data cards will be used for identification purposes. Column 1 to Column 4 inclusive are used only to identify the problem being run. Any four digits may be

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[22]International Business Machines Corporation, <u>Immediate Access Storage, Indexing Registers, Automatic Floating-Decimal Arithmetic, and Magnetic Tape</u>, June, 1959, IBM-650 Data Processing Bulletin Number G24-5003-0, New York, pp. 20-26

used for this purpose. As an Army officer, the writer chose the last four digits of his serial number, since those numbers have only been used to identify such a prosaic item as a bag of laundry. In the following descriptions of card types, "XXXX" will be used to indicate this problem identification number.

On all input data cards, the numbers identifying the type of card will be found in Columns 9 and 10 of Word 1. The other four columns of Word 1 may serve various identification purposes, as will be discussed at the appropriate points in the individual descriptions of the various data cards.

A. <u>The Type-01 Card</u>

There will be one Type-01 card for each data deck. The identification digits, 01, are situated in digit positions 1 and 2 of Word 1. Digit positions 3 through 6 will be zeroes, and "XXXX" is located in positions 7 through 10. The remaining words of the card are punched in floating-decimal form as follows:

1. Word 2 . . The number of lethargy groups (I)
2. Word 3 . . The number of mesh points (N)
3. Word 4 . . The number of regions (K)
4. Words 5, 6, and 7 . . Zeroes
5. Word 8 . . The total volume of the core regions only, divided by $4\pi$ (V)

Format of this card is depicted in Table VI.

B. <u>The Type-02 Cards</u>

The Type-02 cards furnish the data on the fraction of the fission spectrum in the lethargy groups of the problem, and also give the value of $e^{-\Delta u_i}$ for each group. The format of these cards is shown in Table VI. The total number of Type-02 cards in a data deck will be the largest integer equal to or less than the relation, $\frac{I + 2}{3}$ .

C. The Type-03 Cards

The Type-03 cards furnish three types of information concerning the reactor regions, as follows:

1. The number of the mesh point located at the interface between the kth region and the (k-1)th region ($N_k$)

2. The radial thickness of the sub-regions within the kth region ($\Delta r_k$)

3. The fuel sentinel, which identifies each region of the reactor as fuel or non-fuel.

The values of $N_k$ will be punched in Words 2, 4 and 6 of the first card, and of succeeding Type-03 cards as needed.

The $\Delta r_k$ and the fuel sentinel for each region are both stored in Words 3, 5 and 7 of the Type-03 cards. The reader will recall that $\Delta r_k$ is punched in floating-decimal notation, and that eight digit positions are theoretically available for $\Delta r_k$. It is unlikely that more than the highest five or six digit positions would be used for this term. In any event, the fuel sentinels are located in digit position 3 of each $\Delta r_k$ term, still leaving seven significant digit positions for $\Delta r_k$, if so desired. The fuel sentinel for a fuel region is the digit 8. For a non-fuel region, the digit 9 is the fuel sentinel. After the computer program has investigated to see whether a given region is fuel or non-fuel, the 8 or 9 is subtracted out, and $\Delta r_k$ is stored back in its original storage position. As an example of the position of the sentinel,

TABLE VI

SAMPLE FORMATS OF TYPE-01 THROUGH TYPE-06

INPUT DATA CARDS

| Word 1 | Word 2 | Word 3 | Word 4 | Word 5 | Word 6 | Word 7 | Word 8 |
|---|---|---|---|---|---|---|---|
| XXXX000001 | I | N | K | Zeroes | Zeroes | Zeroes | V |
| XXXX00cc02[a] | $\chi_1$ | $e^{-\Delta u_1}$ | $\chi_2$ | $e^{-\Delta u_2}$ | $\chi_3$ | $e^{-\Delta u_3}$ | Zeroes |
| XXXX00cc03[a] | $N_I$ | $\Delta r_I(s)$ | $N_{II}$ | $\Delta r_{II}(s)$ | $N_{III}$ | $\Delta r_{III}(s)$ | Zeroes |
| XXXttcc04[a,b] | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | $S_6$ |
| XXXX00cc05[a] | $K_{1_1}^{4}(I)$ | $K_{1_1}^{4}(II)$ | $K_{1_1}^{4}(III)$ | $K_{2_1}^{4}(I)$ | $K_{2_1}^{4}(II)$ | $K_{2_1}^{4}(III)$ | $K_{3_1}^{4}(I)$ |
| XXXX00cc06[a] | $\Sigma_f(I,1)\Delta u_1$ | $\Sigma_f(II,1)\Delta u_1$ | $\Sigma_f(I,2)\Delta u_2$ | $\Sigma_f(II,2)\Delta u_2$ | $\Sigma_f(I,3)\Delta u_3$ | $\Sigma_f(II,3)\Delta u_3$ | $\Sigma_f(I,4)\Delta u_4$ |

[a]The letters, cc, denote the card loading sequence, 01, 02, 03, etc.

[b]The letters, tt, indicate how many sets of source terms have been produced.

consider a fuel region in which the mesh points are 1.25 cm apart. The number punched in the appropriate word position of the data card would be $1250000\underline{8}51$. Similarly, a non-fuel region with sub-region thicknesses of 0.967 cm would appear as $9670000\underline{9}50$. In these examples, the underscore identifies the fuel sentinels.

The format of the Type-03 cards is depicted in Table VI. The "s" in parentheses after each $\Delta r_k$ is simply a reminder of the presence of the fuel sentinels. A program stop will occur if they are omitted when punching the cards.

The number of Type-03 cards in the data deck is the largest integer equal to or less than the relation, $\frac{K + 2}{3}$.

D. <u>The Type-04 Cards</u>

If the program user decides to start the multigroup calculations with a set of neutron source guesses, the number of source terms to be punched on the Type-04 cards will be equal to the sum of the number of mesh points and the number of regions in the reactor. This total will include a source term of zero at $r = 0$. Since these terms are fission neutron sources, the values for the sources will be zero at all mesh points other than those in a fuel region. At interfaces between adjacent fuel regions, there will be two separate fission neutron source terms, as explained in Section 4.4. The number of Type-04 cards will be the largest integer equal to or less than $\frac{N + K + 6}{7}$. Format of the Type-04 cards is shown in Table VI.

If the program user lets the program generate the first set of source terms, the Type-04 cards are entirely ignored. One even discards those source cards which are punched out at the same time as the values of $\nu_c$ and k are punched, unless it becomes necessary to discontinue the machine run until a later time. Only in this case (or in the case of initial source guesses) should there be any Type-04 cards in the input data deck.

E. The Type-05 Cards

The Type-05 cards have the $K_{i_1}^{4}(k)$ terms for all lethargy groups and all regions. They are the same terms that will be punched in Word 4 of the Type-07 cards. This data is sequentially punched on the cards with the terms for all regions assembled within the lethargy groups, the lethargy groups being arranged in ascending numerical order. The example in Table VI shows the format for a three region reactor with at least three lethargy groups. The total number of these cards is that of the largest integer equal to or less than $\frac{(I \times K) + 6}{7}$.

F. The Type-06 Cards

The information punched in the Type-06 cards is the product of the macroscopic fission cross section within a given lethargy group and fuel region times the width of the lethargy group. The terms are arranged by lethargy groups, and within lethargy groups the terms for the fuel regions are arranged in ascending numerical order. Table VI shows the format for a Type-06 card for a reactor in which Regions I and II are core regions, and the program has at least four lethargy groups. The number of Type-06 cards prepared is the largest integer equal to or less than $\frac{(I \times K_c) + 6}{7}$, where $K_c$ is the total number of fuel regions.

G. The Type-07 Cards

The last type of input data cards to be described are the

Type-07 cards, which are the ones that require the predominant amount of the computations discussed in Chapter 5. For each lethargy group in the problem, there must be a Type-07 card representing each region of the reactor. Consequently, the total number of Type-07 cards is the product of the number of lethargy groups times the total number of regions, I x K.

These cards are arranged by lethargy groups, with the region cards arranged in numerical order within each lethargy group. A sample set of Type-07 cards for a three group calculation using two reactor regions is depicted in Table VII. Note that digit positions 3 and 4 of Work 1 identify the regions, and that digit positions 5 and 6 define the lethargy group.

In running a number of machine runs, the writer has found it most convenient to punch the Type-07 cards on cards of a different color than the remainder of the data deck. This is advisable because these cards are fed back into the read hopper on each flux calculation run of the machine program, as will be further discussed in a subsequent section.

H. <u>Partially Filled Input Data Cards</u>

Frequently, in the preparation of the input data cards, the last card of a certain type may not have data in all seven available words. This may apply to the Type-02 through Type-06 cards. When this condition occurs, it is necessary to punch zeroes in all unused words of that last card.

Section 2. <u>Format of the Machine Program's Output Cards</u>

The three types of output cards from the multigroup machine program are the flux cards, the source cards, and the card with the values

TABLE VII

SAMPLE FORMAT OF TYPE-07 CARDS FOR THREE GROUP,

TWO REGION REACTOR

| Word 1 | Word 2 | Word 3 | Word 4 | Word 5 | Word 6 | Word 7 | Word 8 |
|---|---|---|---|---|---|---|---|
| XXXX010107 | $K_{1_1}^{1}(I)$ | $K_{1_1}^{3}(I)$ | $K_{1_1}^{4}(I)$ | $K_{1_2}^{3}(I)$ | $\Delta u_1$ | Zeroes | Zeroes |
| XXXX010207 | $K_{1_1}^{1}(II)$ | $K_{1_1}^{3}(II)$ | $K_{1_1}^{4}(II)$ | $K_{1_2}^{3}(II)$ | $\Delta u_1$ | Zeroes | Zeroes |
| XXXX020107 | $K_{2_1}^{1}(I)$ | $K_{2_1}^{3}(I)$ | $K_{2_1}^{4}(I)$ | $K_{2_2}^{3}(I)$ | $\Delta u_2$ | Zeroes | Zeroes |
| XXXX020207 | $K_{2_1}^{1}(II)$ | $K_{2_1}^{3}(II)$ | $K_{2_1}^{4}(II)$ | $K_{2_2}^{3}(II)$ | $\Delta u_2$ | Zeroes | Zeroes |
| XXXX030107 | $K_{3_1}^{1}(I)$ | $K_{3_1}^{3}(I)$ | $K_{3_1}^{4}(I)$ | $K_{3_2}^{3}(I)$ | $\Delta u_3$ | Zeroes | Zeroes |
| XXXX030207 | $K_{3_1}^{1}(II)$ | $K_{3_1}^{3}(II)$ | $K_{3_1}^{4}(II)$ | $K_{3_2}^{3}(II)$ | $\Delta u_3$ | Zeroes | Zeroes |

of $\nu_c$ and k.

The flux cards for each lethargy group constitute the first type. Their identification number in Word 1 is of the form, "XXXXyyiilc". The "yy" portion of this identification word is the iteration index, which tells the program user how many iterations have been run. The first set of fluxes punched when starting with a source guess will have zeroes in these positions, and on subsequent machine runs, it will increase in integer steps. The "ii" part of the identification denotes the lethargy group to which the fluxes belong. The fluxes for the first lethargy group will have "01" in this position, and the number will increase by successive integers for each of the other lethargy groups. The "lc" designation is the sequence number of the flux cards which are punched for any lethargy group. The first flux card in each lethargy group will be numbered 11, the second will be numbered 12, and so forth. The total number of flux cards for each lethargy group will be the largest integer equal to or less than the relation, $\frac{N+6}{7}$.

The second type of output card is the Type-04 card, which has been described in sufficient detail in Section 1, above.

The third type of output card is the Type-00 card, on which the values of $\nu_c$ and k are punched upon completion of the second portion of the machine run. The identification number in Word 1 will be "XXXXyy0000", and the "yy" has the same connotation as it had for the flux card identification. However, the value of "yy" on the Type-00 cards will always be one greater than the value on the preceding flux cards. Thus, the first such card obtained will have the number, 01, in the "yy" position.

Section 3. Operating Instructions for the Computer Run

When all data cards have been prepared in the prescribed format, the multigroup calculations with the IBM-650 computer may be conducted. The following instructions outline the operating procedures in the sequence in which they must be performed.

A. Arrangement of Cards in the Read Feed Hopper

At the start of a machine run, the cards which are to be placed in the read feed hopper of the IBM-533 Input-Output Unit are arranged in the following order:

1. The 8103 multigroup coding deck (the first two cards of the deck will place zeroes in all storage locations of the drum and core; the third and fourth cards are a load routine to store the program in the computer).

2. One blank (unpunched) card.

3. The input data cards for the program to be run, sorted by type on Column 10 in the following order:

a. Type-01
b. Type-02
c. Type-03
d. Type-04 (if required)
e. Type-05
f. Type-06
g. Type-07

The Type-02 through Type-06 cards must also be sorted on Columns 7 and 8, in that order. The Type-07 cards are sorted first by lethargy groups using Columns 5 and 6, and then sorted within lethargy groups on Columns 7 and 8. As has been previously discussed in Section 1 of this appendix, the Type-04 cards may be completely omitted when the multigroup program is to generate the source terms.

B. Console Settings on the IBM-650

Console settings to start the loading of the program and the first flux run are as follows:

1. Set storage entry switches to "70 1951 1999+".
2. Set "Programmed" and "Error" switches to "STOP".
3. Set "OVERFLOW" switch to "SENSE".
4. Depress "COMPUTER RESET" and "PROGRAM START" keys.

C. Sequence of Actions in the Machine Run

When the coded program deck has been loaded into the computer, the instruction program automatically begins the calculations of the fluxes for the first lethargy group. When this step is completed, the flux cards indicating the fluxes at all mesh points are punched out by the IBM-533 Input-Output Unit. The program automatically continues this same procedure through the remaining lethargy groups. On the last Type-07 card of the thermal lethargy group, the computer will stop, waiting for the machine operator to depress the "END OF FILE" key on the IBM-533. After this action, the thermal flux cards will be punched.

Next, the operator removes the entire stack of flux cards from the punch output, discards the top and bottom cards (which should be blank ones), and places the remainder of the stack into the read feed hopper. During this time, the program has been preparing the necessary storage areas on the computer drum; the operator depresses the "START" key on the read feed of the IBM-533 to continue the program.

When the "START" key has been depressed, the computer continues the calculations to produce the values of $\nu_c$ and k on the Type-00 card, and a new set of source terms on the Type-04 cards. Since these source terms are also stored within the computer, the Type-04 cards may be discarded immediately, unless the continuation of the machine run must be postponed until a later time at which the program would have to be reloaded into the computer.

The operator checks the Type-00 card to see if the value of $\nu_c$ and k have reached the required values for the program. If further machine runs are required, he reloads only the Type-07 cards in the read feed hopper. Again the computer program has performed the necessary modifications of the storage areas within the computer, preparatory to the calculations for the next set of flux terms. Depressing the read feed "START" key will initiate this process.

The same steps of alternately loading the successive stacks of flux cards and the same deck of Type-07 cards are continued until the results are considered satisfactory. If satisfactory results have not been achieved after six or seven cycles of the program, it would be advisable to recheck the reactor data, particularly the values of the critical fuel mass on the Type-06 cards, before continuing the program. When the program is terminated, the last set of fluxes used to produce the desired value of $\nu_c$ and k should be printed out with the use of the IBM-407 Accounting Machine for convenience in reading.

D. <u>Programmed Stops</u>

The writer has included seven programmed stops in the multigroup program. These stops are provided to insure that the input data cards are loaded into the read feed hopper in the proper sequence. When a programmed stop occurs, a ten digit number will appear in the program register lights on the IBM-650 console. These numbers are associated with the incorrect sequence of the various types of input data cards, as follows:

1. 01 1022 1025: Type-02 cards
2. 01 1033 1025: Type-03 cards
3. 01 1044 0254: Type-04 cards
4. 01 1055 0829: Type-05 cards
5. 01 1066 1006: Type-06 cards
6. 01 1077 1276: Type-07 cards
7. 01 1011 0051: Flux cards

For programmed stops 1 and 2 above, the operator must run out the data deck, correct the sequence of the indicated type of cards, replace the entire data deck in the read feed hopper, and depress the "PROGRAM START" key.

With the other five programmed stops -- numbers 3 through 7 above -- the operator must run out the remainder of the data deck and correct the sequence of the cards indicated by the ten digit number in the program register. He then reloads only that portion of the data deck starting with the type of cards which were out of sequence and continuing to the end of deck. Reloading lower numbered types of cards will cause a program error which only can be remedied by starting the entire machine run at the very beginning. As with the case in the preceding paragraph, the operator can restart the program after the necessary corrections have been made by pushing the "PROGRAM START" key.

The corrective steps stated above for the Type-07 cards and the flux cards will only be effective if those cards which were out of order were in the first lethargy group. If they were in any other lethargy group, it is necessary to reload the entire program as described in the opening paragraphs of this section.

Due to the occasional "crankiness" of the computer, even the corrective steps described above may not resolve the problem of a machine stop. In this case, the writer has found that considerable time will be saved by reloading the entire program, rather than attempting to force the computer or the program to take up where it left off.

E. Capacity of the TRIGA Multigroup Program

The program as it was prepared is designed for use with a computer array of the IBM-650 which must include as a minimum the basic computer with the IBM-533 Input-Output Unit, magnetic core (immediate access) storage, and indexing registers. Although not required for the operation of the program, an IBM-407 Accounting Machine is desirable for preparing listings of the flux output cards, as discussed earlier.

One of the features achieved in the new program adapted from PROD II (see Section 1.6) is that the entire program can be loaded into the computer at one time, whereas the instruction cards for each part of PROD II had to be reloaded into the computer each time that portion of the program was to be run. Furthermore, additional storage space has been made available for the storage of temporary data. These features have naturally accrued from the reduction of the various options of PROD II, as well as from the fact that devices such as the indexing registers conserve storage space for the programmer when determining how many times a particular program loop is to be executed.

Drum storage space presently available for the storage of data permits the user a maximum number of regions, mesh points, and lethargy groups, as follows:

1. Regions: A maximum of fifteen (15)

2. Mesh points: One hundred and twenty one minus the number of regions selected for the program

3. Lethargy groups: A maximum of thirty (30).

In addition, there are 197 unused storage spaces in the computer-137 on the drum and 60 in the magnetic core -- which can be used to increase the capacity of the program, if desired. To increase the capacity, the program would have to be reassembled by use of the symbolic optimum assembly program, SOAP X, of the University of Arizona. New region cards would have to be prepared for the storage regions to be enlarged; the program deck with the new region cards included must then be reprocessed by the SOAP X program. A complete listing of the program prepared for this thesis is available from the Nuclear Engineering Department of the University of Arizona for the purpose of preparing the new program deck in one instruction per card format required for reassembling the program.

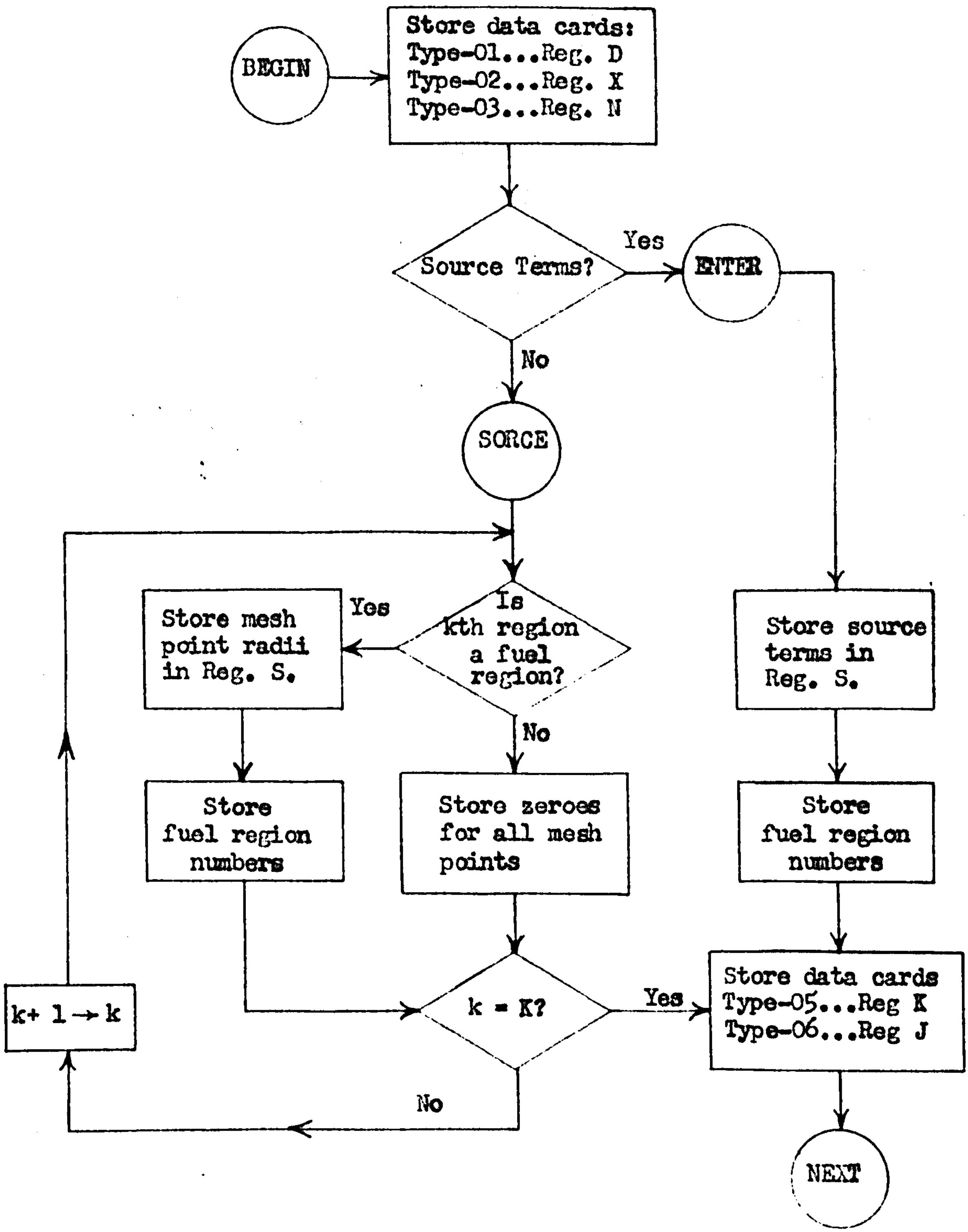
Section 4. Schematic Flow Chart of the Machine Program

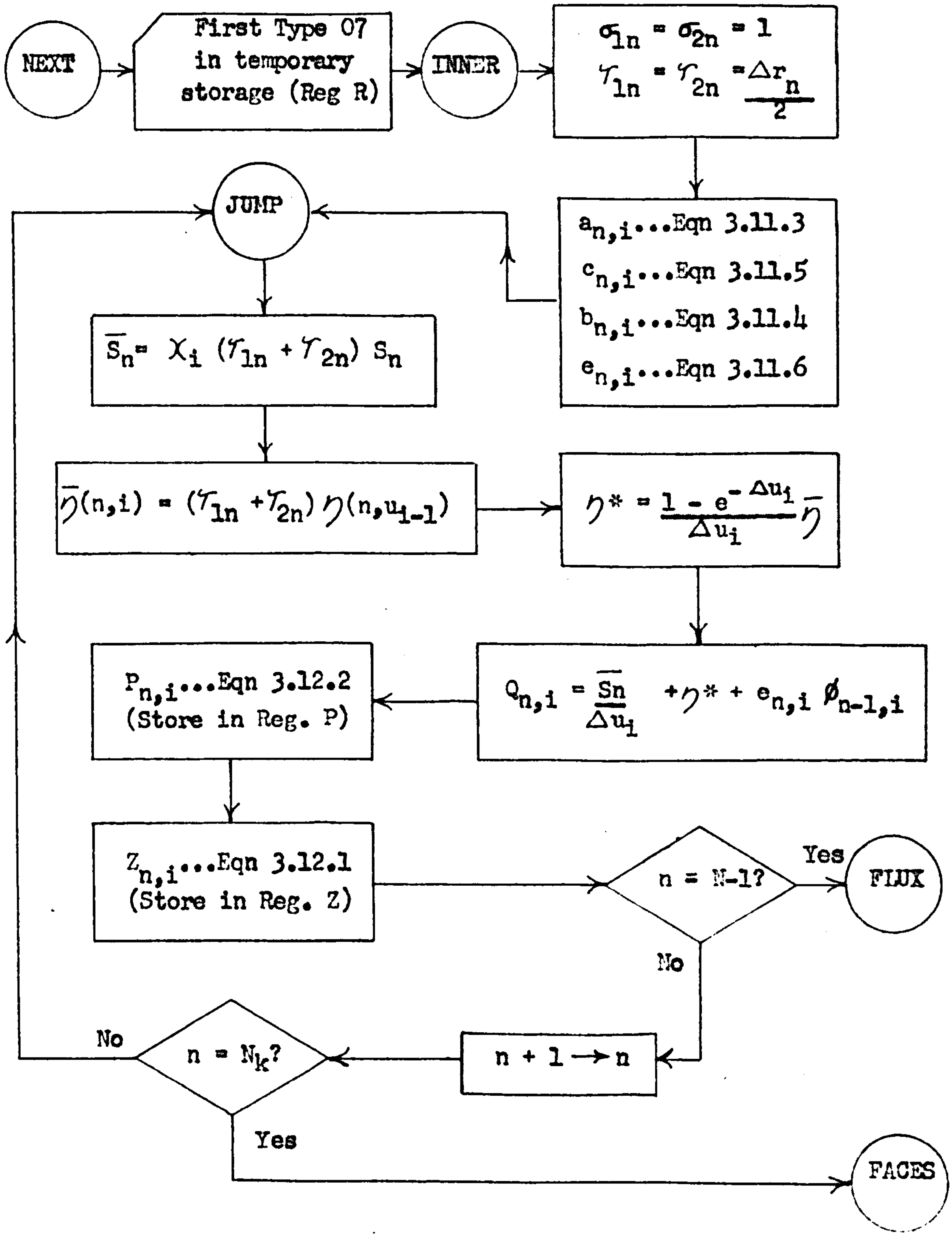
The charts on the following pages graphically depict the overall actions performed by the machine program in the criticality calculations. In the interest of brevity and clarity, no attempt was made to include every individual program step. Significant program points throughout the entire program are designated by capitalized words, such as "BEGIN", typed within the circles. All such points may be compared with the complete program listing for specific information on every program step.

Interrogation points throughout the flow chart are designated by diamonds. The answer to the question within the diamond determines if the program should enter or exit from a certain computational loop.

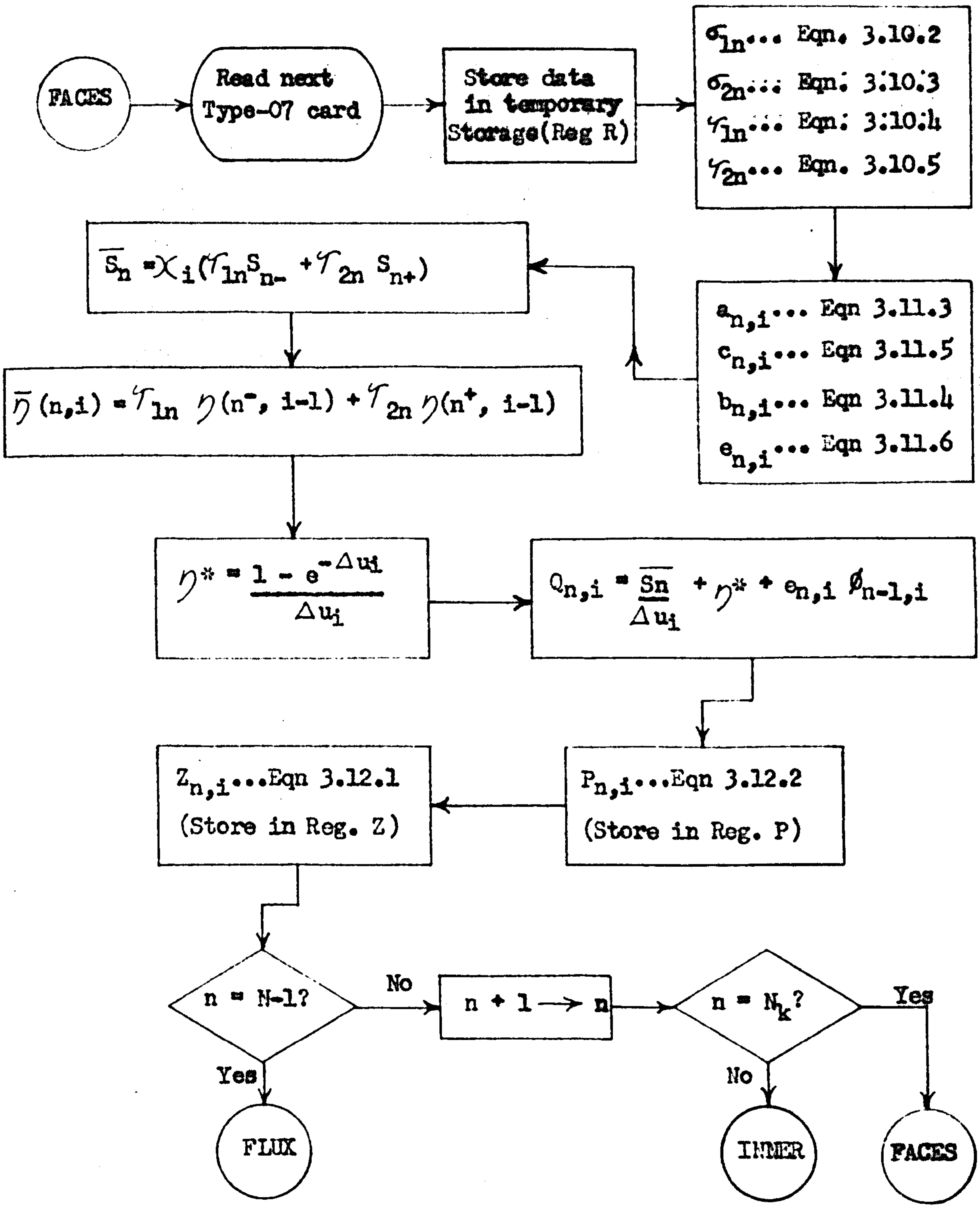
Symbols and terms used within the boxes and diamonds conform with those employed throughout the thesis. Reference to the text will provide further information on their meaning.

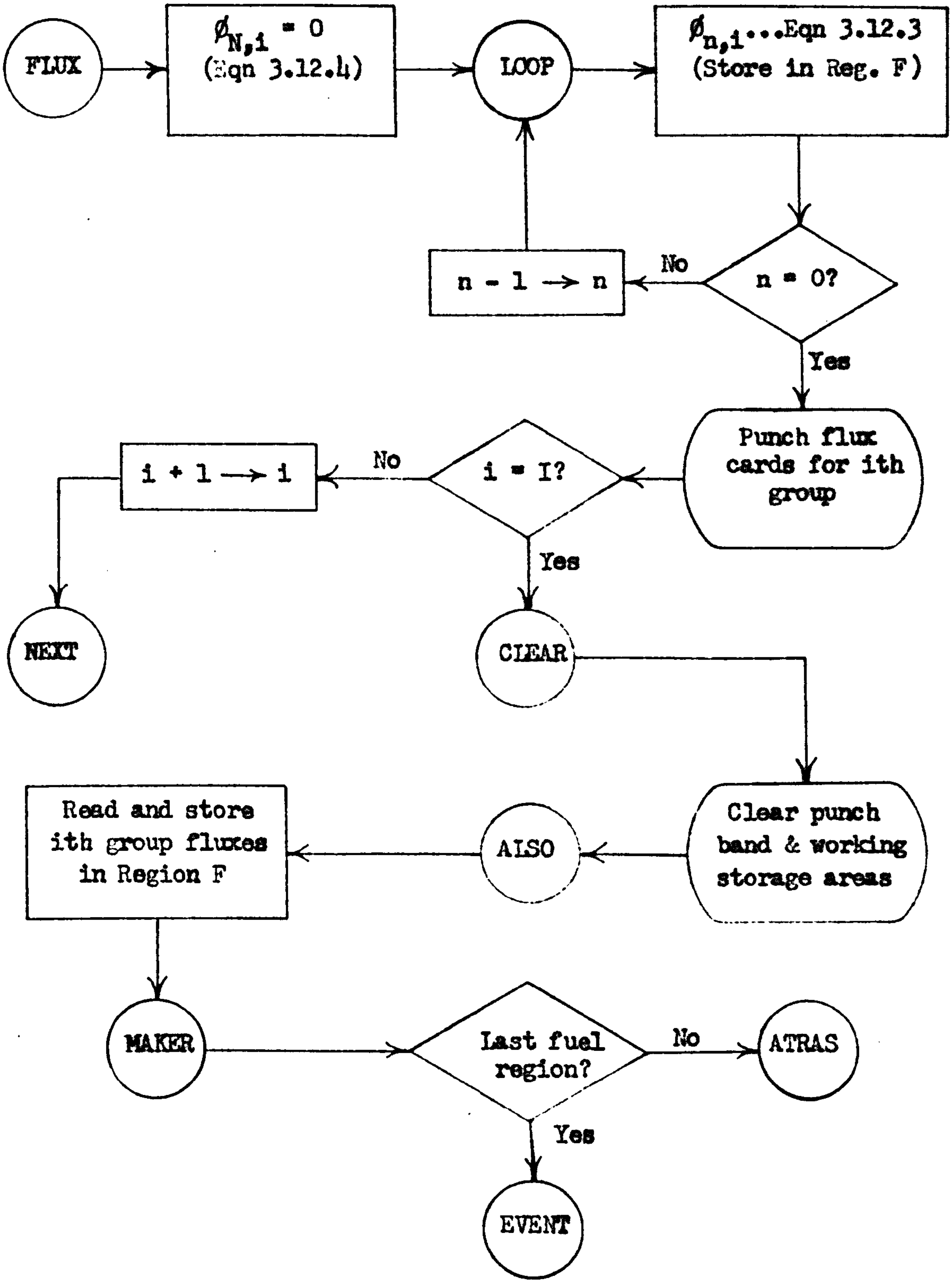
SCHEMATIC FLOW CHART OF THE MULTIGROUP PROGRAM

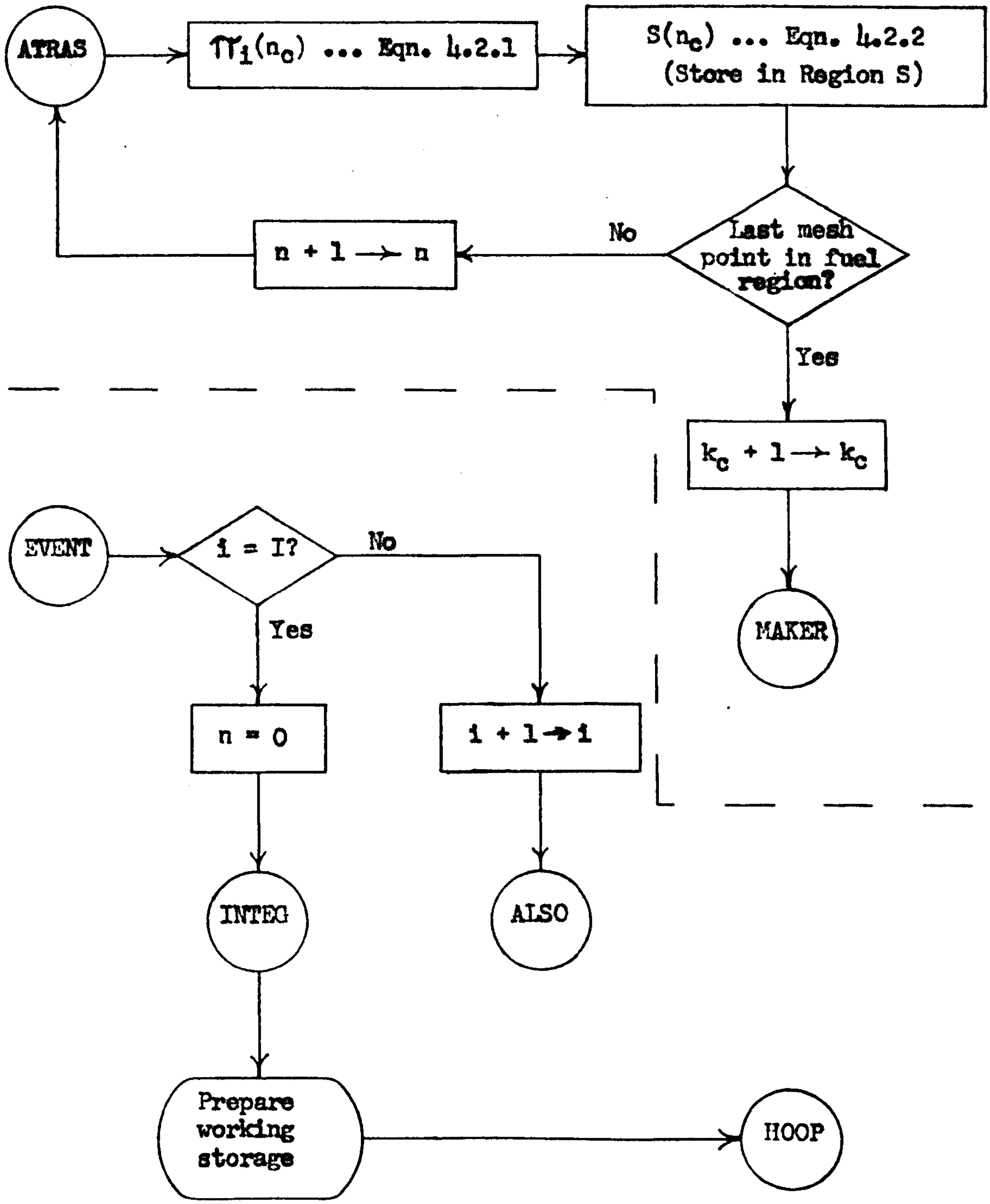




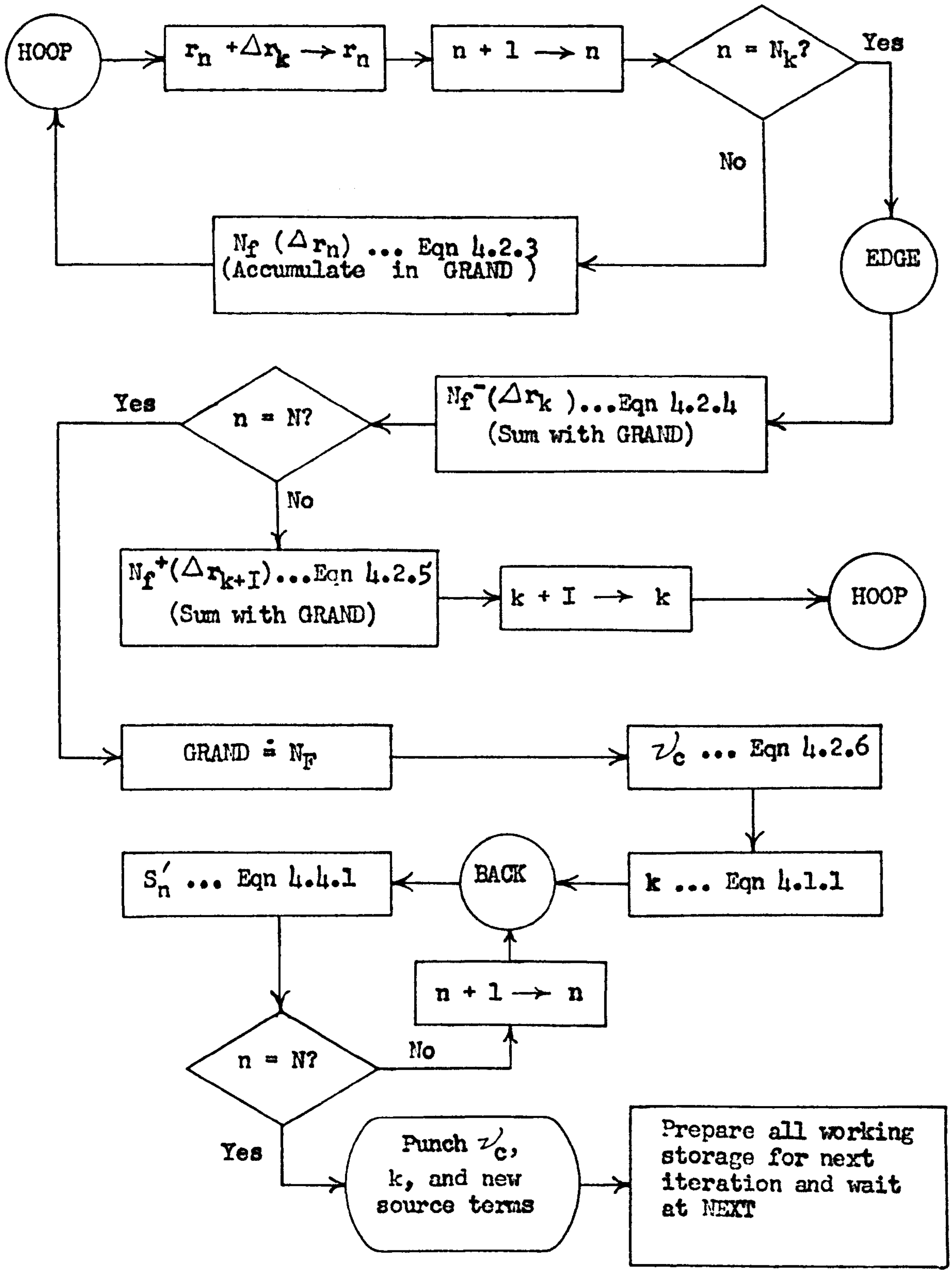
NEXT
First Type 07 in temporary storage (Reg R)
INNER
σ1n = σ2n = 1
τ1n = τ2n = Δrn/2
an,i...Eqn 3.11.3
cn,i...Eqn 3.11.5
bn,i...Eqn 3.11.4
en,i...Eqn 3.11.6
JUMP
S̄n = χi (τ1n + τ2n) Sn
η̄(n,i) = (τ1n + τ2n) η(n,ui-1)
η* = (1 - e^-Δui)/Δui η̄
Qn,i = S̄n/Δui + η* + en,i ϕn-1,i
Pn,i...Eqn 3.12.2
(Store in Reg. P)
Zn,i...Eqn 3.12.1
(Store in Reg. Z)
n = N-1?
Yes
FLUX
No
n + 1 → n
n = Nk?
No
Yes
FACES


FACES
Read next Type-07 card
Store data in temporary Storage(Reg R)
$\sigma_{1n}$... Eqn. 3.10.2
$\sigma_{2n}$... Eqn. 3.10.3
$\tau_{1n}$... Eqn. 3.10.4
$\tau_{2n}$... Eqn. 3.10.5
$a_{n,i}$... Eqn 3.11.3
$c_{n,i}$... Eqn 3.11.5
$b_{n,i}$... Eqn 3.11.4
$e_{n,i}$... Eqn 3.11.6
$\overline{S_n} = \chi_i(\tau_{1n} S_{n-} + \tau_{2n} S_{n+})$
$\bar{\eta}(n,i) = \tau_{1n}\,\eta(n^-, i-1) + \tau_{2n}\,\eta(n^+, i-1)$
$\eta^* = \frac{1 - e^{-\Delta u_i}}{\Delta u_i}$
$Q_{n,i} = \frac{\overline{Sn}}{\Delta u_i} + \eta^* + e_{n,i}\,\phi_{n-1,i}$
$P_{n,i}$...Eqn 3.12.2
(Store in Reg. P)
$Z_{n,i}$...Eqn 3.12.1
(Store in Reg. Z)
n = N-1?
No
Yes
$n + 1 \rightarrow n$
$n = N_k$?
Yes
No
FLUX
INNER
FACES


FLUX
$\phi_{N,i} = 0$
(Eqn 3.12.4)
LOOP
$\phi_{n,i}$...Eqn 3.12.3
(Store in Reg. F)
n = 0?
No
$n - 1 \rightarrow n$
Yes
Punch flux cards for ith group
i = I?
No
$i + 1 \rightarrow i$
NEXT
Yes
CLEAR
Clear punch band & working storage areas
ALSO
Read and store ith group fluxes in Region F
MAKER
Last fuel region?
No
ATRAS
Yes
EVENT



ATRAS
$\pi_i(n_c)$ ... Eqn. 4.2.1
$S(n_c)$ ... Eqn. 4.2.2 (Store in Region S)
Last mesh point in fuel region?
No
$n + 1 \rightarrow n$
Yes
$k_c + 1 \rightarrow k_c$
MAKER
EVENT
i = I?
No
Yes
n = 0
$i + 1 \rightarrow i$
INTEG
ALSO
Prepare working storage
HOOP



HOOP
$r_n + \Delta r_k \rightarrow r_n$
$n + 1 \rightarrow n$
$n = N_k$?
Yes
No
EDGE
$N_f (\Delta r_n)$ ... Eqn 4.2.3
(Accumulate in GRAND)
$N_f^- (\Delta r_k)$ ... Eqn 4.2.4
(Sum with GRAND)
$n = N$?
Yes
No
$N_f^+ (\Delta r_{k+I})$ ... Eqn 4.2.5
(Sum with GRAND)
$k + I \rightarrow k$
HOOP
GRAND $\doteq N_F$
$\nu_c$ ... Eqn 4.2.6
k ... Eqn 4.1.1
BACK
$S_n'$ ... Eqn 4.4.1
$n + 1 \rightarrow n$
$n = N$?
No
Yes
Punch $\nu_c$, k, and new source terms
Prepare all working storage for next iteration and wait at NEXT

# APPENDIX B

## DATA CARD LISTINGS

This appendix is composed of two tables, VIII and IX, of the actual data cards used in the solution of the test multigroup program for the University of Arizona TRIGA reactor. Both tables are typewritten copies of the exact format in which such data cards are listed, or printed out, by the IBM-407 Accounting Machine.

TABLE VIII

TYPE-01 THROUGH TYPE-06 INPUT DATA CARDS FOR TEST PROGRAM

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8103000001 | 4000000051+ | 5700000052+ | 9000000051+ | 0000000000+ | 0000000000+ | 0000000000+ | 6952200054+ |
| | | | | | | | |
| 8103000102 | 9999800050+ | 1234000047+ | 2000000046+ | 3698000048+ | 0000000000+ | 1660000049+ | 0000000000+ |
| 8103000202 | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | | |
| | | | | | | | |
| 8103000103 | 4000000051+ | 1199390951+ | 1000000052+ | 1144000851+ | 1400000052+ | 1165000851+ | 0000000000+ |
| 8103000203 | 2200000052+ | 9754000850+ | 2500000052+ | 1154000851+ | 3200000052+ | 1103000951+ | 0000000000+ |
| 8103000303 | 3400000052+ | 1285000951+ | 4200000052+ | 1581000951+ | 5700000052+ | 2098000951+ | 0000000000+ |
| | | | | | | | |
| 8103000105 | 1064000050+ | 7760000049+ | 7660000049+ | 7810000049+ | 5740000049+ | 0000000000+ | 0000000000+ |
| 8103000205 | 0000000000+ | 1412000050+ | 1704000050+ | 1243000050+ | 1227000050+ | 1251000050+ | 9190000049+ |
| 8103000305 | 0000000000+ | 0000000000+ | 0000000000+ | 2260000050+ | 2298000050+ | 1676000050+ | 1655000050+ |
| 8103000405 | 1687000050+ | 1240000050+ | 0000000000+ | 0000000000+ | 0000000000+ | 3050000050+ | 0000000000+ |
| 8103000505 | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ |
| 8103000605 | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ |
| | | | | | | | |
| 8103000106 | 1930000048+ | 1620000048+ | 2040000048+ | 2180000047+ | 5330000048+ | 4440000048+ | 5640000048+ |
| 8103000206 | 5970000047+ | 3560000049+ | 2980000049+ | 3760000049+ | 3970000048+ | 4310000049+ | 3600000049+ |
| 8103000306 | 4550000049+ | 4800000048+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ | 0000000000+ |

TABLE IX

TYPE-07 DATA CARDS FOR TEST PROGRAM

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8103010107 | 4970000050+ | 4770000049+ | 1064000050+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010207 | 5170000050+ | 1190000050+ | 7760000049+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010307 | 5570000050+ | 1060000050+ | 7660000049+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010407 | 5290000050+ | 1090000050+ | 7810000049+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010507 | 6500000050+ | 1080000050+ | 5740000049+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010607 | 1110000051+ | 4800000049+ | 0000000000+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010707 | 1320000052+ | 3700000048+ | 0000000000+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010807 | 1000000051+ | 4960000049+ | 0000000000+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103010907 | 3890000050+ | 7200000049+ | 1412000050+ | 0000000000+ | 9000000051+ | 0000000000+ | 0000000000+ |
| 8103020107 | 2850000050+ | 4000000049+ | 1700000050+ | 7540000049+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020207 | 3160000050+ | 1200000050+ | 1243000050+ | 1900000050+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020307 | 2120000050+ | 1070000050+ | 1227000050+ | 1650000050+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020407 | 3250000050+ | 1050000050+ | 1250000050+ | 1740000050+ | 5600000051+ | 0000000000+ | 0000000000+ |

(Continued on next page)

Table IX (Continued)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8103020507 | 4070000050+ | 7920000049+ | 9190000049+ | 1730000050+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020607 | 9130000050+ | 4200000049+ | 0000000000+ | 7770000049+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020707 | 1890000052+ | 3830000048+ | 0000000000+ | 5830000048+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020807 | 9000000050+ | 4260000049+ | 0000000000+ | 7980000048+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103020907 | 2310000050+ | 6120000049+ | 2260000050+ | 1140000050+ | 5600000051+ | 0000000000+ | 0000000000+ |
| 8103030107 | 3910000050+ | 3680000049+ | 2298000050+ | 5340000049+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030207 | 4030000050+ | 6600000049+ | 1676000050+ | 1350000050+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030307 | 4410000050+ | 9890000049+ | 1655000050+ | 1170000050+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030407 | 4130000050+ | 6800000049+ | 1687000050+ | 1130000050+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030507 | 5190000050+ | 6250000049+ | 1240000050+ | 1050000050+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030607 | 8780000050+ | 3700000049+ | 0000000000+ | 5670000049+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030707 | 1790000052+ | 4380000048+ | 0000000000+ | 4840000048+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030807 | 8500000050+ | 3740000049+ | 0000000000+ | 5820000049+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103030907 | 3400000050+ | 5540000049+ | 3050000050+ | 8300000049+ | 4098000051+ | 0000000000+ | 0000000000+ |
| 8103040107 | 4190000050+ | 1780000049+ | 0000000000+ | 1370000050+ | 1000000051+ | 0000000000+ | 0000000000+ |

(Concluded on next page)

Table IX (Concluded)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8103040207 | 4810000050+ | 7100000049+ | 0000000000+ | 2570000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040307 | 5330000050+ | 1510000050+ | 0000000000+ | 2240000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040407 | 5190000050+ | 7350000049+ | 0000000000+ | 2190000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040507 | 6670000050+ | 1630000049+ | 0000000000+ | 2460000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040607 | 9360000050+ | 8200000047+ | 0000000000+ | 1500000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040707 | 2100000052+ | 2500000048+ | 0000000000+ | 1420000049+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040807 | 9250000050+ | 5900000047+ | 0000000000+ | 1520000050+ | 1000000051+ | 0000000000+ | 0000000000+ |
| 8103040907 | 1640000050+ | 1960000049+ | 0000000000+ | 2150000050+ | 1000000051+ | 0000000000+ | 0000000000+ |

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