A SYSTEMS ANALYSIS OF THE OPEN PIT MINE DESIGN PROBLEM

by

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STATEMENT BY AUTHOR

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ABSTRACT

In any attempt to optimize a design it is necessary that the parameters influencing the design in question be investigated to determine which parameters are variables and which are fixed. It is also necessary to investigate the methods of evaluation of comparative designs.

An open pit mine system is the subsystem of a corporate system and many of the design parameters are determined at the corporate level. The fundamental variable which the designer can manipulate is, in this case, the sequence in which the ore is mined.

An economic evaluation algorithm which discounts the future earnings and accrues the future investment requirements is the logical standard to use to evaluate different designs. This algorithm shows that the optimum design will be the one that produces the maximum return as close to the start of operations as possible.

A computer program to evaluate any particular design in the manner currently used in the industry was developed to act as a check against which any theoretically optimum design can be evaluated.

Any analytical model of a mine will require knowledge of the ore grade distribution function and the spatial ore distribution function associated with the particular orebody. The ore grade distribution function was found to be expressible as a Pearson Type IV curve, but the spatial ore distribution function could not be identified.

The tools necessary for a complete analytic analysis of the problem are not yet developed, but a combination of computer simulation and analysis of subsystems should lead to a design considerably closer to the optimum design than is possible by using techniques currently applied to the problem.

CHAPTER 1

BACKGROUND FOR THIS THESIS STUDY

It is the purpose of this thesis to investigate the practicality of optimizing an open pit mining system. Before attempting any analysis, it is necessary to define some terms and develop a background which will act as a framework for future discussion.

Figure 1.1 is an organizational chart representing a typical corporation which has mining as one of its major fields. The chart illustrates the fact that a mine system is composed of several different subsystems: a non-controllable distribution of some economic mineral in the ground; an earthmoving subsystem of shovels, trucks, bulldozers, etc.; a mineral concentration subsystem; a smelter subsystem; a transportation subsystem which links the three previous subsystems; the human subsystem of supervisors, operators, and technicians; and finally a fiscal policy, determined at the corporate level, which sets criteria and optimization goals to which the total mine system must adhere. The mine system is then itself a subsystem of the corporate system and at several points conditions are such that decisions regarding operations and policy must be referred out of the mine system to the corporate system.

It is the primary purpose of a mine to recover from some specific volume of ground a mineral of economic value, such as iron, diamonds, copper, lead, gold, uranium, or any of dozens of other minerals, metallic and non-metallic. There are two basic approaches to

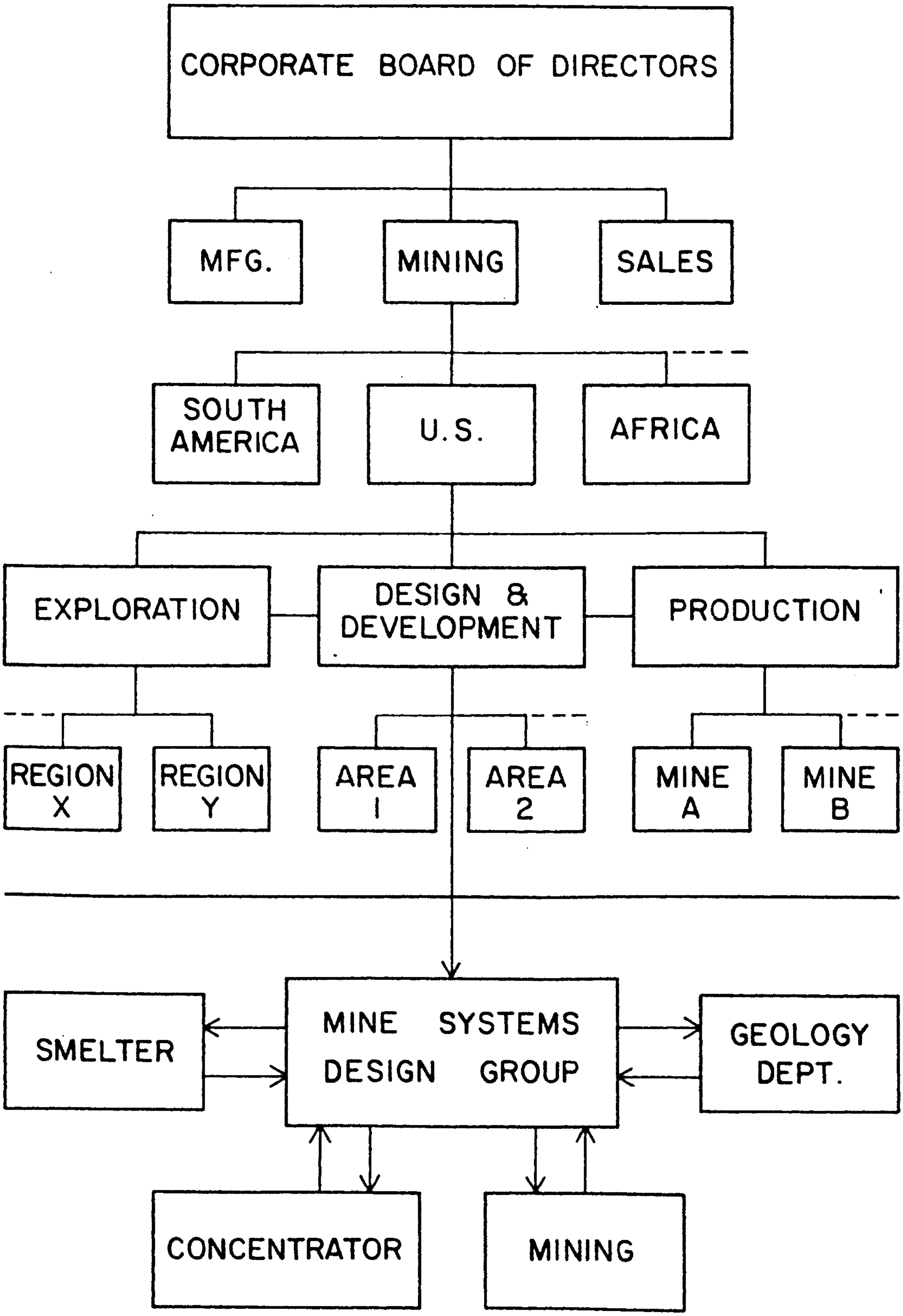

CORPORATE BOARD OF DIRECTORS
MFG.
MINING
SALES
SOUTH AMERICA
U.S.
AFRICA
EXPLORATION
DESIGN & DEVELOPMENT
PRODUCTION
REGION X
REGION Y
AREA 1
AREA 2
MINE A
MINE B
SMELTER
MINE SYSTEMS DESIGN GROUP
GEOLOGY DEPT.
CONCENTRATOR
MINING

Fig.1.1

recovering these minerals; one is the underground mine into which a vertical shaft is sunk to the level where the mineral occurs. The mineral bearing rock is then drilled and blasted and the ore is hoisted to the surface. This type of operation is practical only where the deposit is located far below the surface and is concentrated, i.e., one that has a high percentage of valuable mineral. The second method of mining is that of the open pit mine. In this method, any overlying rock, which is barren of mineral, is removed to a waste dump and the mineral bearing rock is exposed. The ore is then drilled and blasted, loaded into trucks, railroad cars, or onto a conveyor, and sent to a concentrating plant. Figures 1.2 and 1.3 show sketches of similar orebodies being mined by the two different methods. An orebody is defined as three dimensional region in the ground containing a mineral of economic importance in sufficient percentage and suitably located to make mining the orebody economically feasible.

Open pit mining is obviously applicable only when the orebody is basically a horizontal deposit and "fairly close" to the surface. "Fairly close" depends on the value and concentration of the mineral being mined.

In this thesis only the design of open pit type mine systems is to be considered, although plans will often be made to develop an orebody in both ways and a comparison made for cost and profits before the decision is made to mine in one manner or the other.

The open pit mine must go through a sequence of development stages before the mine becomes operational. Supposing that an orebody has been discovered, delineated, and analysis has proven it to

UNDERGROUND MINE

DIAGRAMMATIC

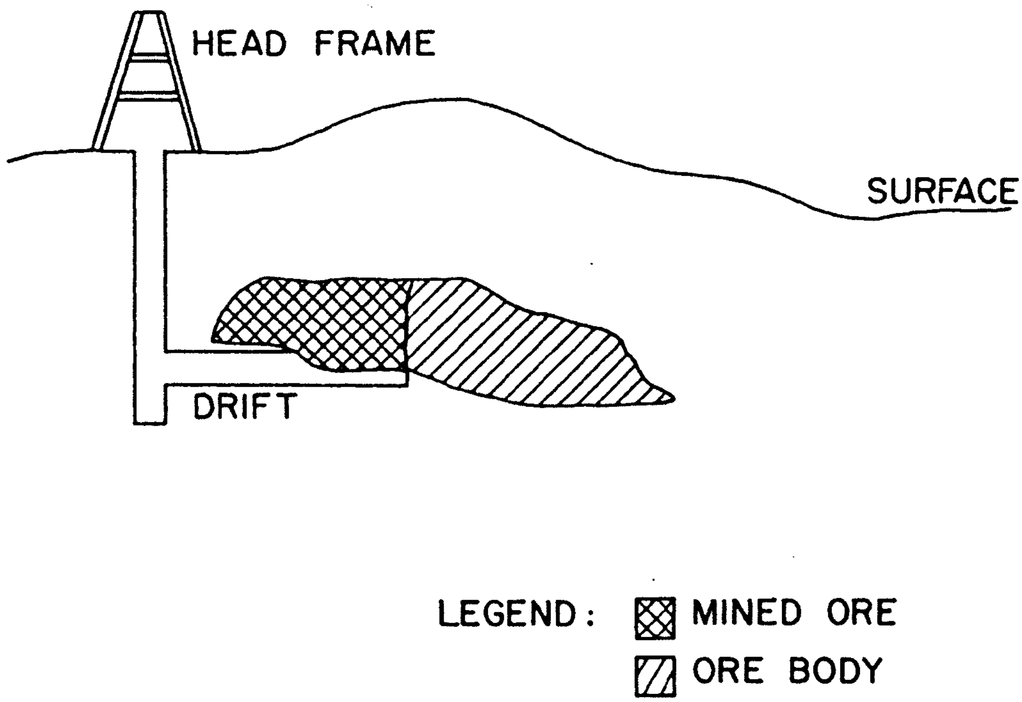


Fig. 1.2

OPEN PIT MINE

DIAGRAMMATIC

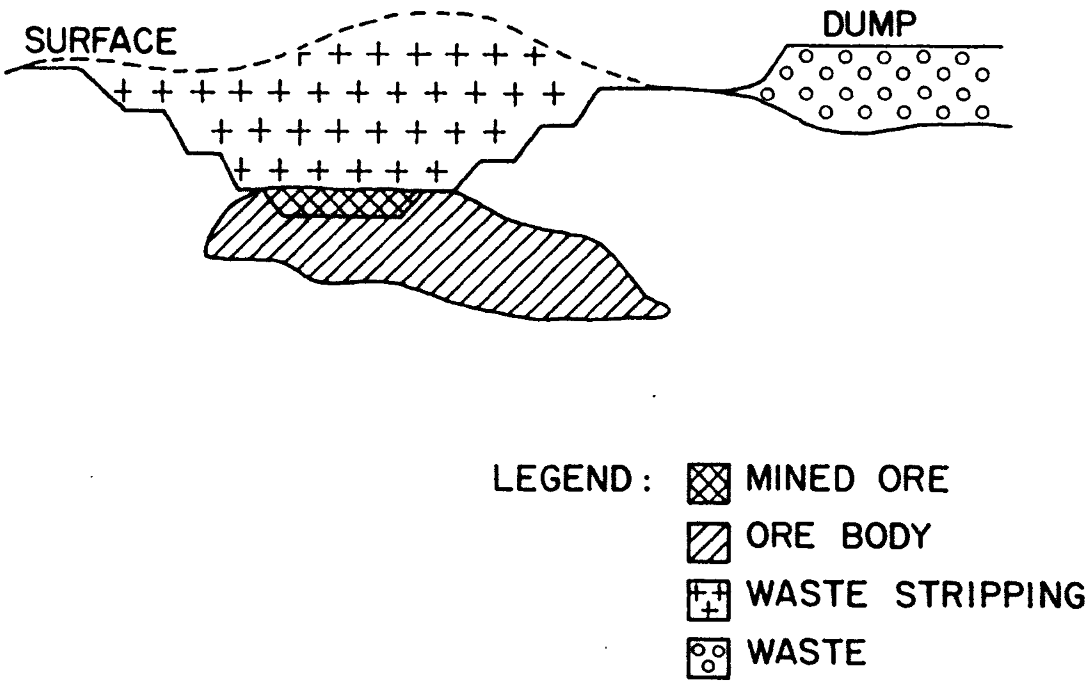


Fig. 1.3

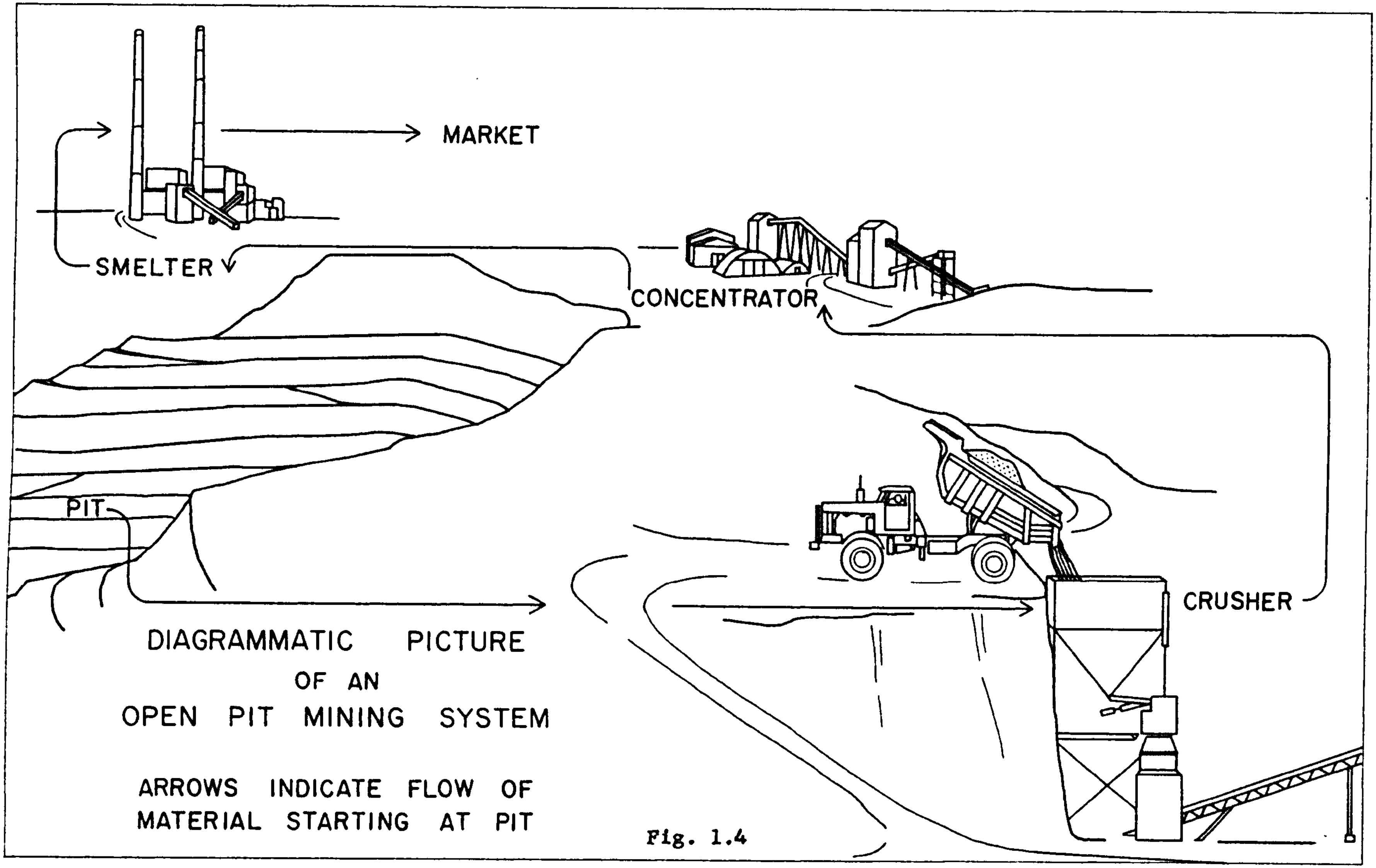
be of sufficient value to mine as an open pit, the first step can start. The waste material which usually overlies the orebody must be removed, at least enough of it to expose sufficient ore to assure a constant supply of ore to the mill from a specific time onward. The waste material removed before ore extraction begins is called "development stripping" and is treated as a capital expenditure. The amount of development stripping and the ore which it uncovers first determine, to a large extent, the sequence in which the ore will be mined. The orebody has a three-dimensional spatial distribution, in grade and tonnage, and the decision as to the sequence in which to mine the orebody is a function of the ore and tonnage distributions and a corporate financial policy which defines ore "optimum" financial return.

While development stripping progresses, the mill is being built, roads constructed, mine equipment purchased and assembled, and an operating crew is being assembled and trained.

Once material from the orebody starts being removed, the mine is considered operational and the development stripping becomes waste stripping, an operational cost. Most orebodies are not homogeneous and contain large amounts of both waste rock and ore of such low mineral content as to make concentrating unprofitable. The waste material is separated in the mining operation and sent to waste dumps while the low grade ore may be sent to a leaching process where acidified water is allowed to flow through the broken ore, dissolving the metallic content. This solution is then reclaimed and processed (in copper by passing the solution over scrap iron where the iron replaces the copper in solution) filtered and returned to the leaching process. Leaching is only done when the orebody contains large tonnages of ore whose mineral content is below profitable milling percentage. This value is called a mill cutoff grade. The percentage of mineral which separates leachable material from waste is called the leach cutoff grade.

When the daily production of ore has reached the design quantities, the mine system has reached full operational status and the waste stripping, mining of ore, leach and mining waste will continue at a fairly constant rate until the orebody has been exhausted. The mill cutoff and leach cutoff grade will vary during the life of the mine due to fluctuations in the market price of the mineral, to technological changes which affect the cost of milling, mining and leaching, modifications in the tax structure, and changes in labor cost.

Figure 1.4 is a schematic representation of an operating open pit mine system. The process starts at the pit where the waste necessary to uncover the ore is removed and sent to the waste dump. The ore is then mined, the leachable material being sent to the leach dump, the mining waste to the waste dump, and the millable ore to the concentrating facility. Depending on the distance of the pit from the concentrator the ore may be trucked or loaded into railroad cars. At the concentrator the ore is crushed and usually subjected to some type of flotation process which separates the mineral of value from the host rock. The host rock, being of no value, is sent to a "tailings pond" where the liquid is removed for recycling and the concentrated mineral is packaged for shipment to a processing center. In the case of copper the next step would be shipment to a smelter while uranium concentrate



DIAGRAMMATIC PICTURE
OF AN
OPEN PIT MINING SYSTEM

ARROWS INDICATE FLOW OF
MATERIAL STARTING AT PIT

Fig. 1.4

would be sent to Hanford, Washington or Oak Ridge, Tennessee, for further processing.

It is the system which contains all of the foregoing that we wish to optimize.

In the search for an optimum method of designing and operating a mining system, two basic approaches seem feasible. The first is to simulate the system under study by use of analogue and/or digital computers and then vary all the design parameters over their practical range and choose that combination of parameter values producing the highest (or lowest, as the case may be) value of the function being optimized. The approach is practical only in those situations having a small number of design parameters and a limited range of values for each parameter. If a system has M design parameters and each parameter has N possible values, then it will be necessary to operate the system under NM different conditions in order to arrive at the optimum. This approach has been used successfully by C. B. Manula in his "System Simulation - A Gaming Model for Mine Management" (1). Manula has simulated underground mining and haulage system in part of a coal mine.

Such techniques are very applicable to the problem of optimizing small segments of the entire system -- especially those segments composed of mechanical components. But the magnitude of applying such a technique to the entire mining system precludes its use as the basic solution tool.

The analytic approach to optimization of the mine system is the more desirable method, but this approach immediately raises the question of the development of equations that will adequately represent the various components of the system. Once the components of the system have been modeled, there still remains the problem of developing mathematical representations of the relations among the components. One approach that appears practical is to attempt the development of mathematical models for each of the components and their interrelationships and in the event that any component proves intractable to this type of analysis, then revert to the simulation approach.

One primary problem inherent in any attempt to simulate real world conditions with mathematical models or digital simulation is that of evaluating the performance of the model against the real world counterpart. This evaluation implies the presence of actual production and profit figures against which the models may be compared. The mining industry is notoriously wary of releasing any such data, especially if the possibility of publication exists.

In order to check any models, then, it will be necessary for the researcher to be able to create a synthetic orebody (by means of a Monte Carlo simulation) and then proceed to calculate tonnages, grades, to design a proposed pit and calculate the economic returns in a manner similar to that used by the engineering staff of a mining company.

In order for any optimization study to be proved practically, some means must be developed by which the synthetic orebody can be quickly evaluated by the same methods commonly used in the mining industry today. As part of this thesis, such a program was developed to operate on the IBM 7072 computer of the Numerical Analysis Laboratory of The University of Arizona. The program accepts, as input, the results of simulated assay results of diamond drill holes and outputs, first, the total tonnage of material above a specified grade contained in the orebody and, secondly, the maximum possible tonnage and grade which can be profitably extracted from the orebody by open pit methods, and, finally, the configuration of the final excavation. A detailed description of this program is given in Chapter 3. The output from this program can then be compared to the results obtained by optimizing the mathematical model. The model should produce approximately the same figures as the program when the model is restricted to the same output sequence as the program.

Chapter 2 describes the basic financial analysis of the mining system and the development of a model. The analysis of this model points up the vital role played by the spatial distribution of grade and tonnage of the economic minerals in any optimization attempt. Chapter 3 has been mentioned. Chapter 4 is the description of a series of programs to identify the basic type of distributors of the economic minerals. Successful identification of this distribution would allow the use of a probabilistic model to calculate the total tonnage and grade of material in the ore body. Also in this chapter is a description of the attempts to develop a unique predictor function which would allow prediction of grades of ore as a function of spatial coordinates.

CHAPTER 2

ECONOMIC ANALYSIS OF A MINE DESIGN

While it is true that standard economic evaluation techniques have been used for mine system evaluation in the past, the peculiar characteristics of a mining investment have caused many in the industry to discount these standard techniques and to develop criteria which, experience shows, provide better measures of the economic worth of a mining property.

The characteristics which make a mining venture unique are of three types. The first is the uncertainty regarding the exact amount and grade of ore in the deposit. As an example of this, consider a hypothetical orebody with the following characteristics: total material equal to 3 million tons, material above ore cutoff grade equal to 1 million tons, average grade of ore equal to 2.0%, standard deviation of the ore samples equal .1435%. All of the foregoing are estimates based on 70 drill holes which produced a total of 10,000 separate assays. There are two regions of uncertainty involved. First is the question of the accuracy of the estimate of ore tonnage and, secondly, the estimate of the average grade of the ore.

From the 10,000 original assays it was estimated that one-third of the total material contains sufficient mineral to be classified as ore. This situation can be represented as a binomial distribution where p is estimated to be $\frac{1}{3}$. With 10,000 samples the normal approximation to the binomial is appropriate and a 95% confidence limit about the estimate of p is

$$\hat{p} \pm Z_{.05}\sqrt{\frac{p(1-p)}{10,000}} = \frac{1}{3} \pm 1.96\sqrt{\frac{2/9}{10,000}} = \frac{1}{3} \pm .00925$$

Probability $(.32408 \leq p \leq .34258) = .95$ (Confidence Interval)

Translated into tons of ore this says that the true tonnage of ore in this orebody is between 97,224,000 tons and 102,879,000 tons, recognizing that there is one chance in twenty that the result will be in error due to sampling error.

The average grade of the ore is estimated from approximately 3,300 samples. From the Central Limit Therom it can be assumed that with a sample of this size the mean grade of the ore will be approximately normally distributed. Using the same confidence limit as used for tonnage, 95%, the true mean of the grade of ore can be expressed as follows:

$$\text{estimate of the mean} \pm Z_{.05}\left(\frac{\text{Sample Deviation}}{\sqrt{3,300}}\right)$$

$$= .02 \pm 1.96\ (.1435/\sqrt{3,300})$$

$$= .02 \pm .005$$

Probability $(.0195 \leq \text{mean grade} \leq .025) = 95\%$, recognizing that there is one chance in twenty that the result is in error due to sampling error.

If both the grade and tonnage are calculated at the low limits of the 95% confidence interval, the company can expect to recover 97,224,000 tons x .015% = 1,458,000 tons of mineral, compared to 2,560,000 tons of mineral recovered when the recovery is calculated at the high limits. This indicates that the amount of recovered mineral can vary by 1,102,000 tons which represents a dollar income of $682,000,000. If the upper limit calculations indicate a return less than the minimum acceptable, then the property is not economically feasible and no further work is required. On the other hand, if the return calculated at the lower limits exceeds the minimum return required, then the property should be put into production immediately. Unfortunately, the decision is seldom so clear-cut, and because of this there usually exists a high degree of uncertainty regarding the actual return from a mining venture. For this reason it is not unusual for a company to require a 25% rate of return on money invested in mining ventures.

A second unusual characteristic of an open pit mining venture is the inflexibility of the operation once production has commenced. The original overburden of waste has been removed to uncover one particular ore block, and this ore block must be first, the second block must be next to the first, and so on. No matter how other considerations change, the basic plan of mining is subject to very limited change once operations have started. If technological advances make the mill or transportation equipment obsolete, then the mine must either be shut down completely or further capital investment made, based on the unchanging return of the particular orebody that is being operated.

The third condition, also related to the long investment period, is the fact that the prediction of potential markets, in the face of almost certain technological advances, is almost impossible beyond a 10-year period. It is perfectly possible that the plastic industry may develop a material which will replace copper, in many of its uses, at a much lower price and suddenly make the mining of copper unprofitable except in those mines of exceptionally low operating cost and for high grade deposits.

With this background, it is intuitively obvious why most mining people have chosen rate of return and payout time as their basic evaluation criteria.

The evaluation of a potential mining system today is no longer the decision of a single man, but it is usually the considered opinion of a board of directors which includes many non-mining specialists, especially fiscal and marketing experts. For this reason the financial analysis of a proposed mine system should be evaluated using modern financial techniques, modified as necessary to conform to the situation existing in a mining system. It would also be desirable that this analysis be able to generate the rate of return and payout time for a specific design.

It is the object of a financial analysis to determine the "best" design for an open pit mining system. The modern trend in financial analysis has been to "present worth" concepts (2). The present worth concept differs from previously used standards, such as cash flow, total profit, payout time, rate of return, etc., in that it includes the concept of applying a discount rate to future earnings.

For the analysis of a mining system, it would also seem logical to discount future contingent liabilities. The investment of capital is not a single, one time thing. Money is invested in mining equipment, milling facilities and transportation systems as required to bring the entire system into operation at a given time, usually three to seven years in the future. It seems only logical that if the present worth of money which is to be earned in the future is discounted, then the present value of money to be spent in the future should also be accrued, though not, of course, at the same rate of interest.

The normal present worth calculations,

$$\text{P.S.} = \frac{\text{amount to be returned}}{(1+\text{interest rate})^{\text{time until return}}}$$

considers that the investor, requiring a specific rate of return on invested money, today is purchasing a future earnings function, say RET, which has a life expectancy of L times units. A mining system present worth calculation, allowing for both discount and accrual, should be the difference between the return function, RET, evaluated from today until time L, less the present worth of a schedule of capital investments, CAP, that the system will require. These investments must be discounted at an interest rate which is a function of the time in the future that the money will be required. Let us call this method a mine system Present Worth Evaluation of a particular design, MSPW for short. Then

$$\text{MSPW} = \sum_{t=0}^{L} \frac{\text{RET}\ (t)}{(1 + i)^t} - \sum_{t=0}^{L} \frac{\text{CAP}\ (t)}{(1 + \text{CRATE}\ (t))^t}$$

where RET is a dollar valued function whose value at time t is the profit to be made by the system during a unit time period t units in the future; CAP is a dollar valued function whose value, CAP (t), at time t is capital expediture to be incurred at time t; i is the rate of return that corporate fiscal policy requires on this investment; and CRATE is a function whose value, CRATE (t), at time t, is the discount rate to be applied to capital which will be required t time units in the future. CRATE will vary with time because money which will be required next year, for instance, can be invested at a very low rate of interest. Money which will not be required for 3 years can be invested at a higher rate of interest. Money which will not be needed for 10 or more years can be invested at a maximum rate of interest.

MSPW is a figure which represents an evaluation of a particular design of the proposed system at time zero. Suppose that the system has been operating for a period of time and changes in the operating parameters of the system (a sudden change in price, a new tax structure, etc.) make it desirable to re-evaluate the system. Having followed the original plan from time zero to time , the system is now in the financial position MSPW ( ). The original plan must now be compared to alternate future plans. The present worth evaluation function must then consider past capital expenditures and past income to create a present financial status of the system.

From the point of view of mathematical optimality, there is no reason to include the past performance of the system in any attempt to optimize the time from to L, as proven in Bellman's theorem (3) on optimality, but we are dealing with factors other than pure optimality.

An operating mine is, besides an assemblage of mechanical equipment and a piece of mining real estate, a highly integrated team of specialists. A very large part of the intangible assets of a mining company is the existence of this team of specialists. If a mine is to be closed, the company must either layoff the men employed at the mine or continue to pay them while waiting for an opportunity to utilize their skills. If discharged, these men will be reluctant to work for the company again and even if they are kept on the payroll, the men will soon look for other jobs rather than remain idle for long.

A very real and measurable cost is associated with the learning curve of a new operating crew. Two types of learning are involved: first, the learning associated with the physical accomplishment of the job, and, secondly, the learning to operate as a crew. Anytime that a mine is to be opened, or reopened, with a new operating crew, this cost must be included in the operating cost.

For this reason, if re-evaluation becomes necessary, it is essential that the decision maker have available the current financial position of the property as a necessary ingredient to making a decision consistent with the best interest of the company. If the property is in a position where the most optimal future policy will only result in a break-even situation, and there are no other properties ready for development, it is possible that the decision will be made to continue operation simply to prevent the loss to the company of a pool of highly skilled labor. On the other hand the property may already have shown some profit and the optimal policy for the future indicates a loss. By comparing the future loss against the past profit, it may be possible to return some of this profit to the mine and permit continued operation at a slight loss in order to keep the operating crew together. Being able to compare the results of the future optimal policy and the present financial position, management is in a position to evaluate properly the situation from an overall point of view. This decision is obviously one which must be made at the corporate level in a manner which will optimize the operation of the corporate system.

The capital invested in the past should have earned money at rate i, unless the income from the system was able to retire some, or all, of the capital invested. It would be possible to keep an accumulated capital invested, plus interest, and to subtract from this total the return function value for each period. The same thing is achieved if the capital invested is credited with i rate of interest from the time of expenditure to time $\mathcal{T}$. The income from each period is also credited with an i rate of interest from the time it was received until $\mathcal{T}$ and the difference between the two taken at time $\mathcal{T}$. Thus,

$$\text{Financial position of the system at time } \mathcal{T} = -\sum_{t=0}^{\mathcal{T}} \text{CAP}(t)(1+i)^t + \sum_{t=0}^{\mathcal{T}} \text{RET}(t)(1+i)^t$$

for the period of time between $\mathcal{T}$ and L the MSPW would be calculated exactly as before except that the summations would run from $\mathcal{T}$ to L and the exponent of (1+i) would be $(t-\mathcal{T})$, instead of t

$$\text{MSPW}(\mathcal{T}) = \sum_{t=\mathcal{T}+1}^{L} \frac{\text{RET}(t)}{(1+i)^{t-\mathcal{T}}} - \sum_{t=\mathcal{T}+1}^{L} \frac{\text{CAP}(t)}{(1+\text{CRATE}(t-\mathcal{T}))^{t-\mathcal{T}}}$$

In this manner the financial position of the system may be evaluated at any time τ and alternate return functions for the period from τ to L may be evaluated.

Loss is a term which can be defined in the following ways:

(a) Expected future earnings are less than operating cost.

(b) Expected future earnings are above operating cost but not sufficient to return capital.

(c) Expected future earnings will not produce the minimum return demanded by corporate fiscal policy.

(d) Expected future return is less than the return originally predicted for the property.

The decision as to which definition to use is a corporate policy decision.

As an example of the operation of the mine system evaluation function, three different return functions will be evaluated for a given CAP function and a given CAP function and a given CRATE function. L is set to be 15 years, and the financial position is calculated at a 5% interest level.

Figures 2.1 and 2.2 are graphical representations of the CRATE and CAP functions. Figure 2.3 is a series of three graphs of the three return functions used. Function 1 has a constant return of $5000 per time period from the 6th through the 15th time period. Function 2 is a monotonically increasing function from time period 6 to 15. Function 3 is monotonically decreasing from period 6 through period 15. All three functions return to the system the same amount of money over the

ACCRUAL FUNCTION

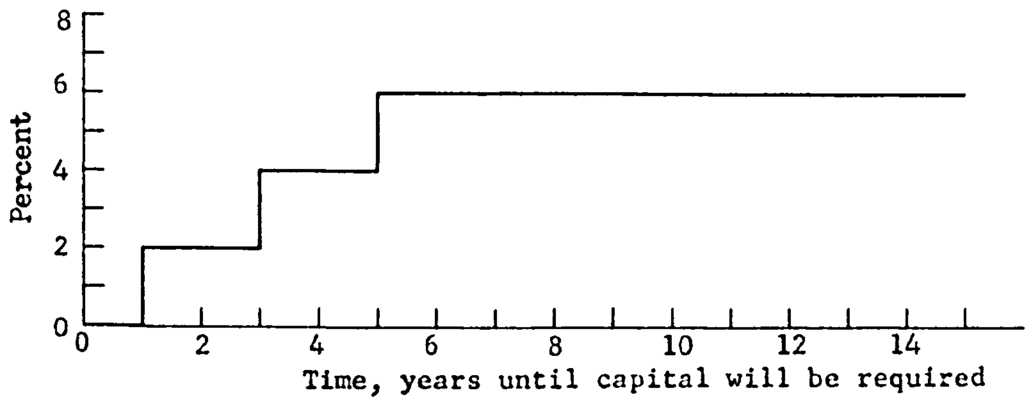


Fig. 2.1

10 time periods from 6 to 15, namely $50,000. Figure 2.4 shows the MSPW evaluation of these three return functions for varying rates of interest from 5% to 25%. The percent interest which has a MSPW of zero is the true rate of return for that function. Note that Function 2 has the lowest rate of return and also has the lowest MSPW evaluation at any given interest rate. Function 3 has the highest rate of return and also the highest MSPW evaluation at any given interest rate. Figure 2.5 is a plot of the financial position of each of the three return functions at each time period from 1 to 15. The time period during which the financial position of the system becomes positive for the first time is called the payout time. Note that as in the

CAPITAL INVESTMENT SCHEDULE

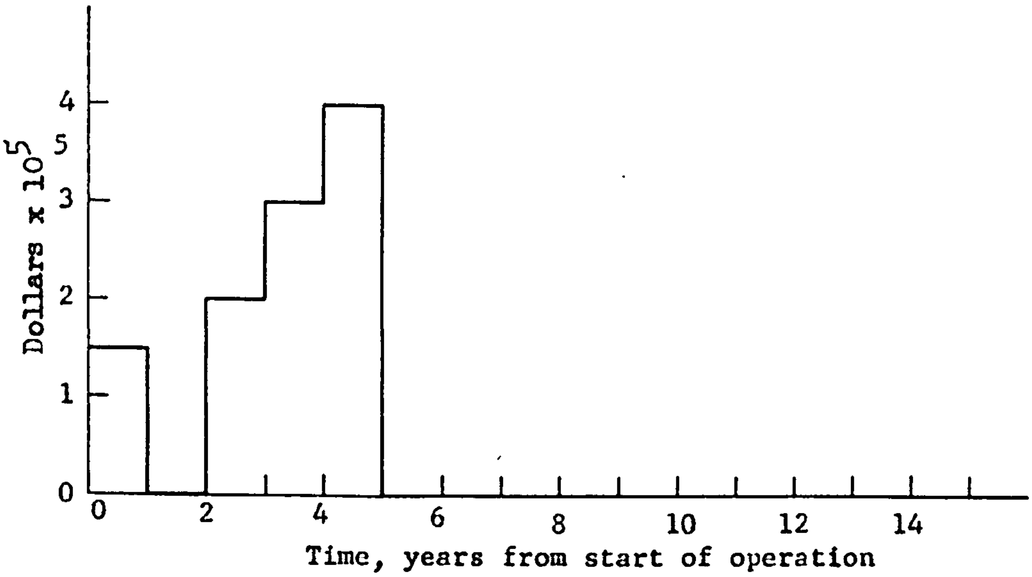


Fig. 2.2

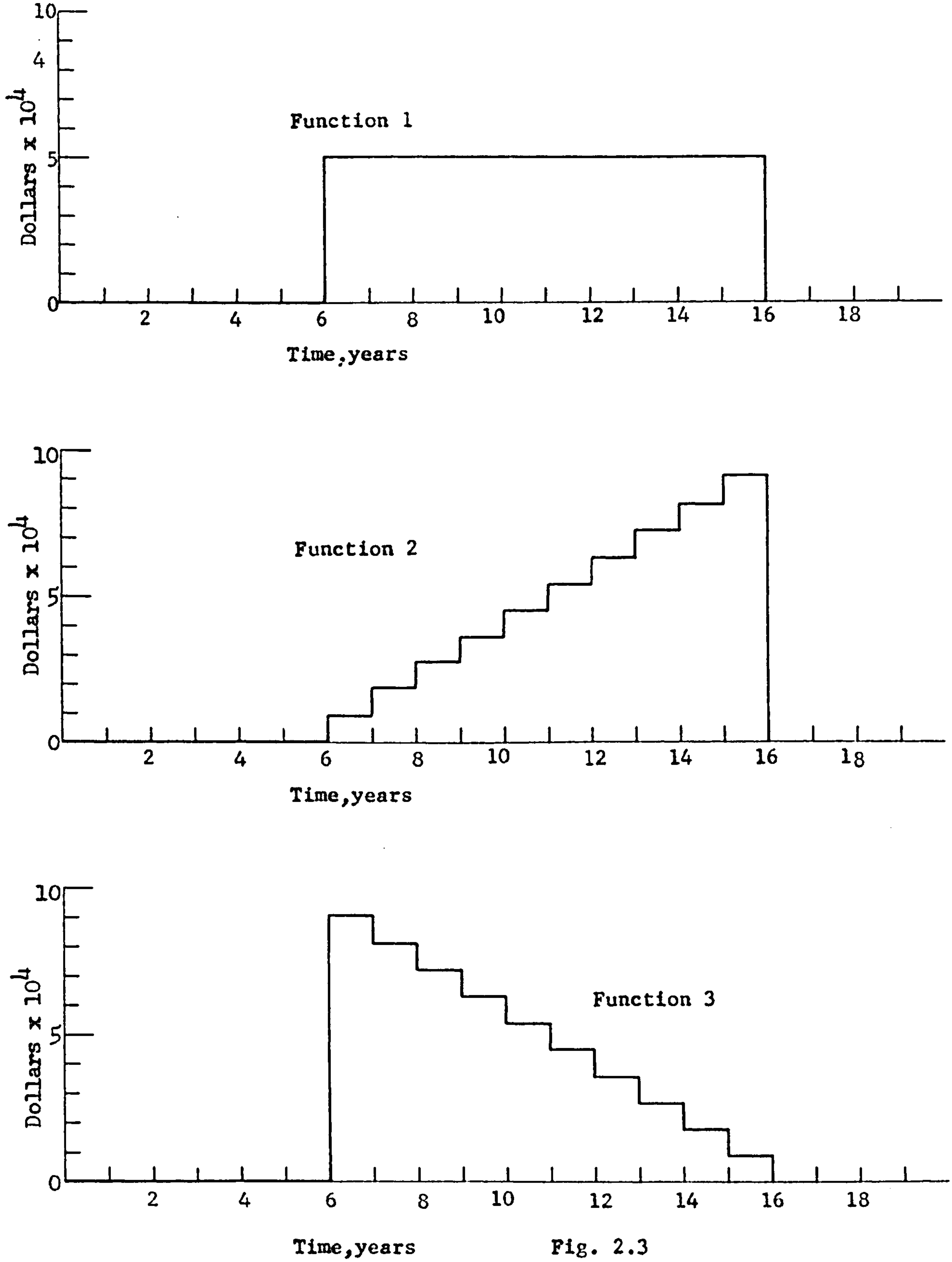

Function 1
Dollars x 10⁴
Time,years
Function 2
Dollars x 10⁴
Time,years
Function 3
Dollars x 10⁴
Time,years

Fig. 2.3

MINE SYSTEM PRESENT WORTH RESULTS

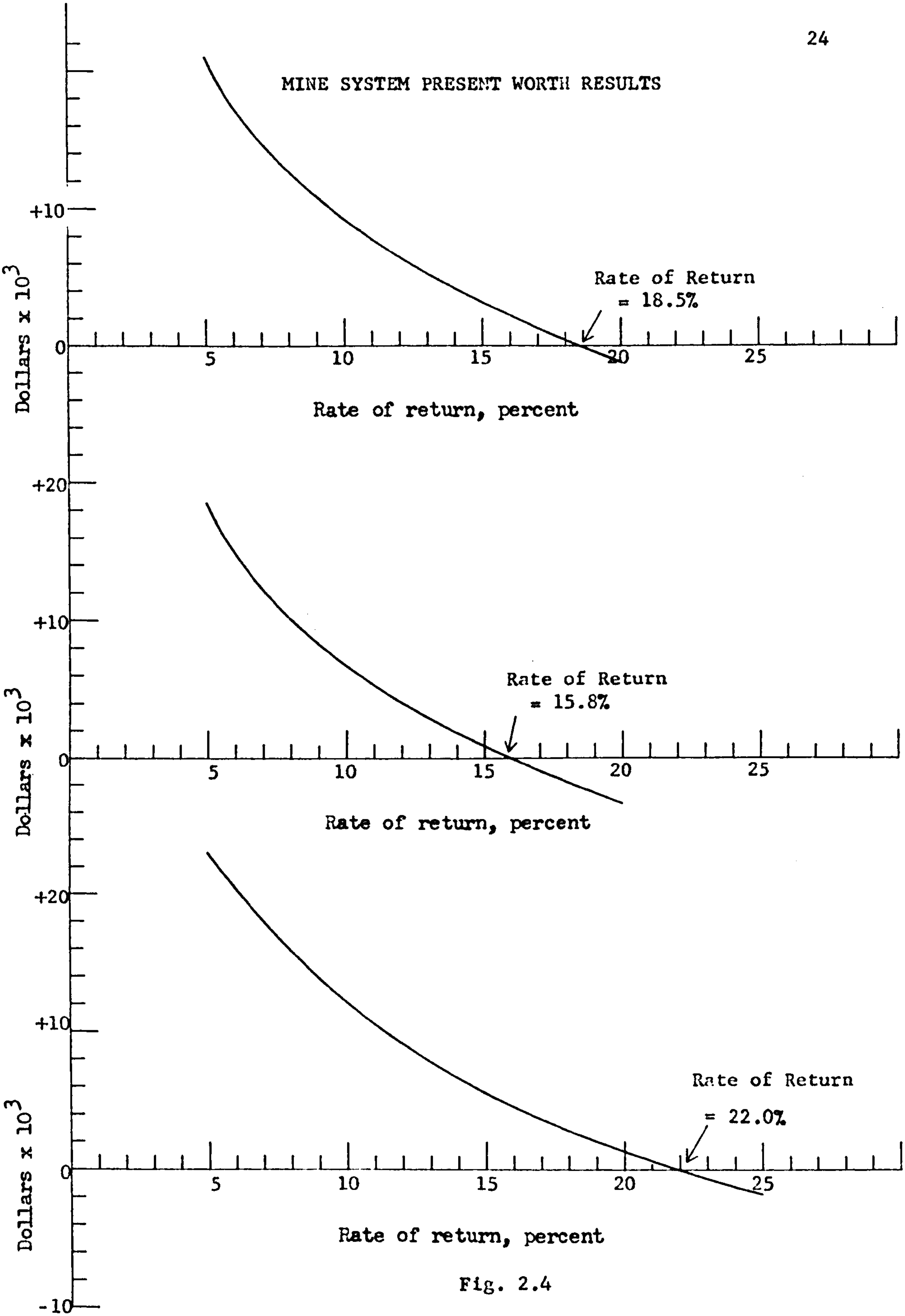


Fig. 2.4

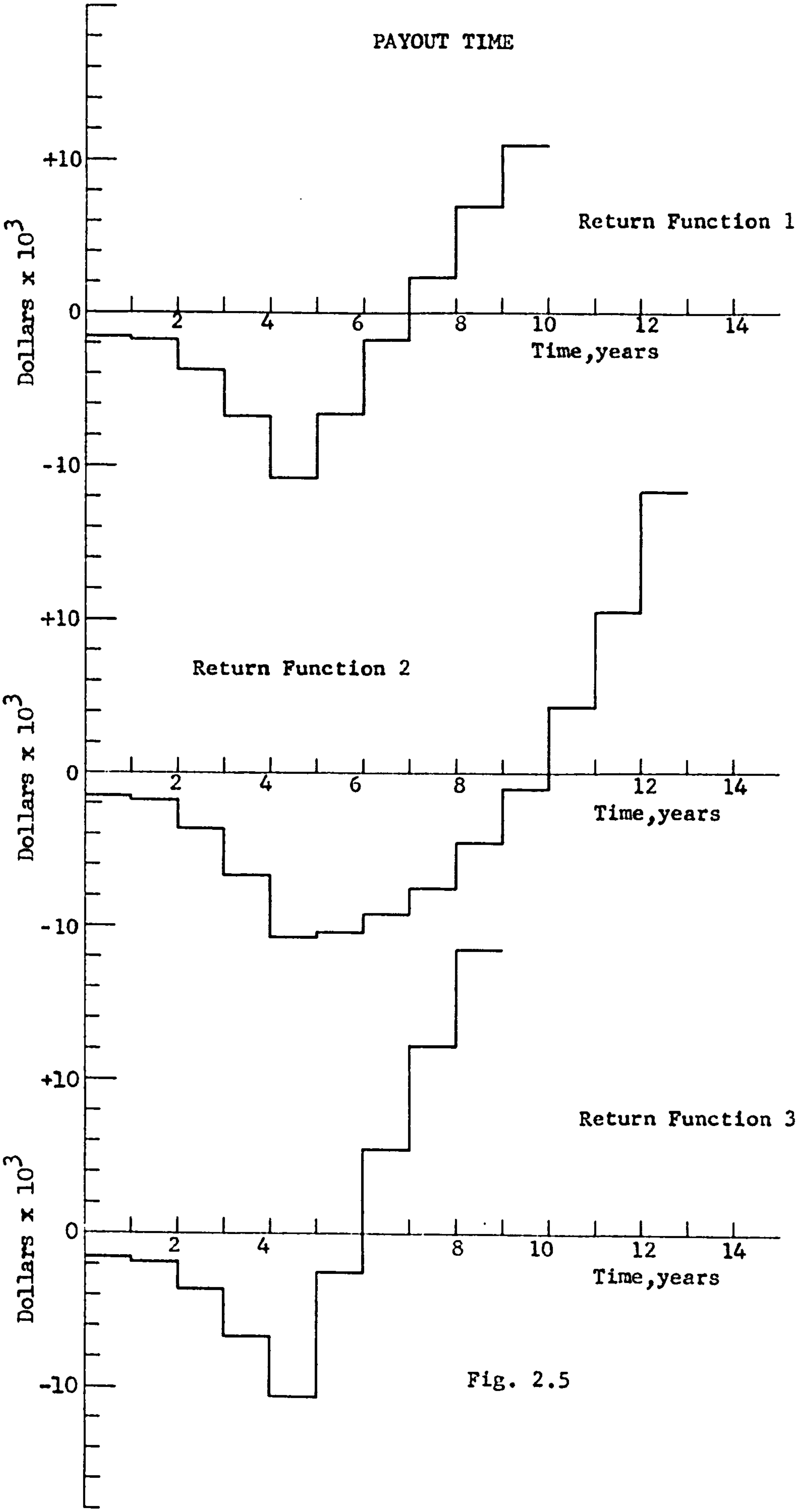

PAYOUT TIME
+10
0
-10
Dollars x 10^3
Return Function 1
2
4
6
8
10
12
14
Time,years
Return Function 2
Return Function 3

Fig. 2.5

previous graphs, Function 2 has the worst payout time, while Function 3 has the best (shortest).

The present worth evaluation function is simply a tool to be used to evaluate numerous possible "designs" and to choose the design which maximizes the present worth evaluation function (or minimizes the time until the financial position becomes positive, or maximizes the rate of return depending on the criteria considered of primary importance by the person evaluating the design).

Of the several segments of the MSPW only RET is truly flexible from the point of view of the system designer. CAP, the capital investment schedule has very definite limitations, imposed often by corporate financial policy and also by physical limitations. In a mining system the primary items in CAP will come under three classifications.

A. Equipment for mining and overburden removal

B. Mill construction

C. Cost of overburden removal (material which must be removed before mining operations can begin).

If the system design stage has been reached, then several factors have already been established.

A. The total amount of ore in the orebody

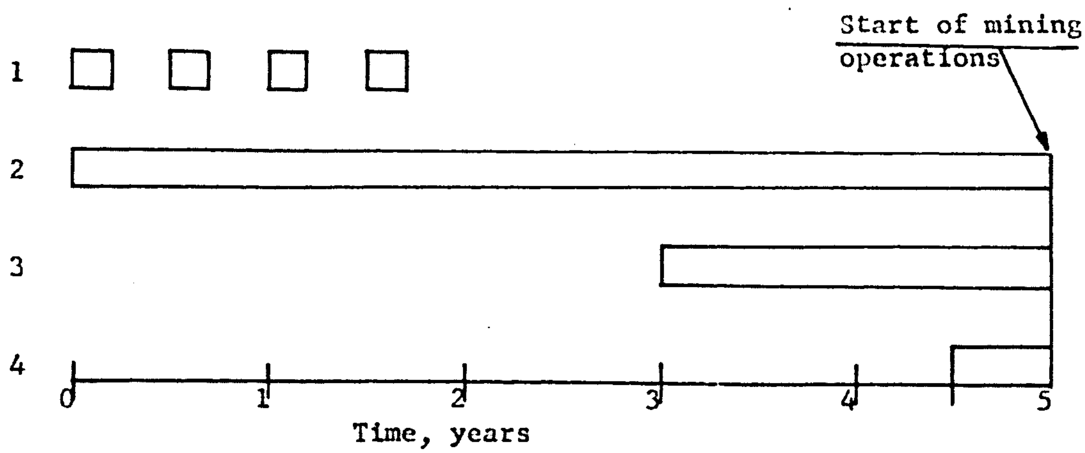
B. The amount of waste (or overburden) which must be removed to mine this ore

C. The length of time over which the ore is to be extracted (usually due to estimates of the markets capability to absorb the product)

D. A minimum amount of overburden which must be removed before mining can start

Item D implicitly defines a period of time which will necessarily pass before mining operation can begin. This automatically establishes a date at which the mill should be ready to process ore. Mill construction time is almost a linear function of size and this then implies a date at which mill construction must start. The purchase of ore mining equipment before ore mining is ready to start would needlessly tie up capital and should be delayed as long as possible. For these reasons, Capital Investment in a mining system will proceed in the following sequence.

1. Purchase overburden removal equipment
2. Start overburden removal
3. Start mill construction
4. Purchase mining equipment



Notice that the purchase of overburden removal equipment is spread over a period of time. This is necessary because of the time required to prepare places for the equipment to work, build roads, establish waste dumps, etc.

From the foregoing, it can be seen that the system's designer is extremely limited in his ability to alter CAP.

CRATE and i are both dictated by fiscal policy and the current state of the national economy and, hence, can be treated as constants for design optimization purposes.

The primary tool of the mine system designer must then be the return function RET. The elements which make up this function must be investigated. Some definitions are necessary.

1. Let a piece of mining real estate be called a mine and be defined as a three-dimensional rectangle over which is defined an assay function A, defining the percent of mineral of economic interest present at any given location and a density Function D, defining the number of cubic feet of material per ton for any location in the mine. The rectangle is divided into cubes of "suitable" dimensions, to each of which is assigned a unique integer between 1 and N.

2. Let a "mining plan" be defined as a piece of mining real estate and a set S of the following form: $S = \{(i_1 t_1), (i_2 t_2) \ldots\ldots (i_N t_N)\}$ where i_j is a cube designation and t_j represents the time period at which i_j is to be removed and $t_j < t_{j+1}$.

3. Let MILL be a function such that

$$\text{MILL}(t) = \left[\text{VOL}(i_j) \cdot A(i_j), D(i_j)\right] \text{ if } A(i_t) \geq \text{cutoff grade}, = 0 \text{ otherwise},$$

(cutoff grade is that assay value below which the economic mineral cannot be extracted at a profit).

4. Let \$ (t) be a function such that \$ (t) = MILL (1,t) $\cdot$ MILL (2,t) $\cdot$ Price/unit of mineral. Thus, \$ (t) is the amount of money earned by the system during the period t - 1 to t.

It is recognized that while each mine has N! possible mining plans, only a few of these plans are feasible; i.e., plans which could actually be implemented in a real mining situation. In theory, any of the N blocks could be taken first, while in practice only those blocks on the surface are eligible to be selected first. From this point on only those blocks next to or below a block already removed are possible next blocks. There is also the restriction that no block can be removed unless those directly above and on all sides have already been removed in order to maintain stable slopes on the sides of the pit. From this point on, when reference is made to a mining plan, it will be understood that the plan being referred to is a feasible mining plan.

Now define a function DIRT (t) such that DIRT (t) = $\sum_{t_j=0}^{t}$ VOL $(i_j) \cdot$ D (i_j) : $t_j \leq t$. DIRT is then a non-decreasing function which, when evaluated for any t, and a specific mining plan will develop the tonnage removed from the mine since the start of operations at $\tau = 0$ until time t.

It is assumed that

$$\frac{\text{DIRT } (t_j) - \text{DIRT } (t_k)}{t_j - t_k} \text{ is a constant for each } t_j - t_k$$

in our mining plan. This will be true once the mining operation has passed through the transient phase of fluctuating production early in the operation.

This assumption implies a constant operating cost and capital investment requirements during the operational life of the mine per ton of material removed.

This assumption also permits the mining plan to be redefined as an ordered set S of the form $S = \{i_1, i_2 \ldots . i_N\}$, which is some permutation of the integers 1, 2, ... N, if we take as the unit of time the period necessary to remove one cube of material.

Over the piece of mining real estate there is also defined an assay function A which defines the mineral content of each cube i_j as MILL (i_j). The recovery of this mineral content is the purpose of the mine. Every cube i_j then has an associated MILL (i_j) and an associated \$ (i_j) which is the dollar value of the cube's mineral content.

The mining plan, which has been defined as an ordered set $S = \{i_1, i_2 \ldots . i_N\}$ then implicitly defines an ordered set of triples $(i_j$, MILL (i_j), \$ (i_j) $\quad j = 1, N$. This sequence of triples can be designated MINE (t) for it defines all of the vital operational sequences in the mine system. The sequence of first elements of the triple defines the mining sequence of the cubes which go to make up the mine. The sequence of second elements defines the flow of material into the mill for concentrating and defines both tonnage and grade. The sequence of third elements defines the flow of money into the system.

It has been shown that a feasible mining plan defines a function MILL (t) and a function \$ (t). While each feasible mining plan defines one and only one MILL Function and one and only one \$ Function, several mining plans may produce the same MILL and \$ Functions.

The \$ (t) Function is a linear relation to the return Function RET (t). The difference between the two functions represents the operating cost per time period, plus the capital expenditure per time period, hence

$$\text{RET}(t) = \$(t) - k - \text{CAP}(t)$$

It is now clear that in order to optimize the mine system Present Worth Evaluation Function, it is necessary to investigate all feasible mining plans and to select that mining plan which produces the RET (t) Function which corresponds to an optimum MSPW value. The primary factor in the optimization of a mining system is the sequence with which the cubes are removed.

It has been the purpose of this chapter to show that the sequence in which the ore is removed from a mine is the critical aspect of mine system design and to develop a tool which will allow a particular mining plan to be compared to an alternate plan. Here the knowledge of the ore and density distribution functions has been assumed. Obviously, if these functions are not known, the MILL function and the return function RET cannot be developed from the mining plan. In Chapter 4 some possible methods of representing these functions will be investigated.

CHAPTER 3

A PIT LIMIT DESIGN COMPUTER PROGRAM

In Chapter 2 it was shown that to evaluate a potential mine it is necessary to know two functions, or at least to be able to approximate them; namely, the assay distribution function and the density distribution function. If these function are known, an arbitrary "cube system may be defined on the "piece of mining real estate," an excavation sequence selected, and a mine design would exist which could be evaluated. In actual practice the assay distribution function is an unknown; the density function is known. In an attempt to approximate these functions, a series of equally spaced vertical holes are drilled over the surface of the piece of mining real estate, and the material contained in these holes is analyzed to determine the percent of economic mineral present at various depths (usually not equally spaced). The material is examined mineralogically to identify the type of rock and determine the density.

The estimation of grade of ore and tonnage by the engineers of a mining company follows a procedure roughly analagous to the theoretical procedure described in Chapter 2. The surface of the property being investigated is divided into areas by using each drill hole as the center of a polygon whose sides are halfway between adjacent drill holes. This polygon is considered to extend vertically down into the earth. The vertical distance between assays (usually greater than 6 inches and less than 10 feet) is used as a horizontal dividing factor and generates a series of polygon-shaped wafers whose thickness varies with the interval between assays. These wafers are analagous to the cubes in Chapter 2. Each wafer has a density and an assay value; thus, if we assign each wafer an integer w_i, where w takes on the values between 1 and N, and N is the total number of wafers in the mine, each wafer is uniquely identified and a sequence (w_i), where $i = 1$ to N, determines a particular mining sequence.

The mining engineer assumes that the material contained in the wafer is of uniform assay and density equal to the assay grade and density of the drill hole sample at the center of the wafer. From these wafers an ore estimate is made by summing the tonnages and grades of all the wafers whose assay is greater than some predetermined cut-off grade. If the tonnage and assay grade of the ore estimate is sufficiently high to interest the company involved, the mining engineer proceeds to make a mining ore estimate; i.e., the tonnage and grade of the ore that would actually be recovered by a mining operation.

To accomplish this the mining engineer first divides the mine into equally spaced horizontal layers called benches. The vertical height of these benches is determined by the type and size of the equipment expected to be used. Intersecting these horizontal layers with the vertical extensions of the surface polygons, a new series of polygonal-sided wafers is defined, all of equal thickness. Assay values and density values for these new wafers are obtained by averaging the assays and densities of the original wafers contained in the new wafer. The usual vertical divisions for mining purposes are between

20 and 50 feet. The mining engineer now defines a sequence of wafer extraction, which conforms to a feasible mining plan and which intuition and experience tell him will produce the maximum amount of mineral for a minimum expenditure of money and work. This mining plan will, in turn, define a return function, RET, for this plan which can be financially analyzed by methods similar to the one described in Chapter 2.

The foregoing operation, although simple in theory, will consume thousands of man hours. To produce a single feasible mining plan for a 100 million ton orebody which has been drilled on 200 foot centers will take a 5 man crew engineering team about 18 months. The development of alternate mining plans will take from 6 to 12 months per plan, depending on how much the alternate plan differs from the original. From a practical viewpoint of option dates, lease agreements, and cost, it is obvious that only 2, or at the most 3, mining plans can be considered by a company evaluating a potential mine.

If a computer program could be developed that include a mining plan, or a series of mining plans in hours instead of months, it is obvious that many more plans could be considered and several very desirable effects would accrue. First, if the property were rejected as a potential mine, the company could be assured, with a higher probability than was possible previously, that no feasible mining plan existed that would make this property a profitable mine. If the decision were favorable there would be a much higher probability that the mining plan selected would be close to the absolute optimum mining plan for the property.

The existence of such a computer program would also benefit the theoretical research field. In any attempt to optimize a mining system it will be necessary to create a mathematical model of the real world mining system and to optimize this model. It is essential that any model be checked against reality before any confidence can be placed in the results obtained from the optimization of the model. It is obviously impractical for a researcher to undertake the calculations which would duplicate the work of an engineering staff. Yet these calculations must be made to establish a standard against which the model can be evaluated.

The remainder of this chapter is devoted to the description of a computer program which accomplishes the calculations just described and gives examples of the program's output.

Program Input Variables

The variables which must be entered into this program for each run are as follows:

A. Bench height

B. Cutoff grade of primary mineral at selected points:

1. Ore grade - material above this grade is shipped to a processing unit.

2. Intermediate grade - material which must be up-graded by some intermediate process before it can be sent to a processing unit; for example, leaching in Cu deposits or concentrating in iron ores.

3. Low grade - material which is not currently economic but should be kept separated for possible future use.

4. Waste

C. Slope - the maximum pit slope for this type of material (entire pit).

D. The price per pound of the principal mineral and the price per pound of up to 3 minerals of secondary importance for which assays have been obtained.

E. A reference elevation which is higher than the collar of any drill hole. (This variable, REFEL, is also used to position the benches as each bench will be a bench-height multiple below the reference elevation.)

F. Cost data for moving material of the 4 different ore grades.

G. The N and E coordinates of property corners (up to 6 corners).

H. Tape unit assignments for 7 tapes.

Basic input to the program is a magnetic tape listing of a sequence of punched cards, each card representing one assay interval from the original drill-hole core log. Each card must supply the following information for each assay interval:

1. The distance from the top of the assay interval to the collar of the hole (called ENTT in the program).

2. The distance from the bottom of the assay interval to the collar of the hole (called ENTB in the program).

3. An assay value for this interval.

4. Tonnage factor of the rock type of this assay interval.

Optional information which will be utilized if available:

1. Geologic rock type.

2. A second rock designation (metallurgical rating, grindability, etc.)

3. Assay values for up to 3 additional minerals of economic value.

CARD FORMAT

Format	Column Number	Contents
F3.0	1 - 3	Hole # (Hole, IHole)
I1	4	Second rock type designation (MATK) (metallurgical, grindability, etc.)
	5 - 9	Blank
I6	10	Geologic rock type (MATYP)
F10.2	11 - 20	Surface elevation of collar (SUREL)
F10.2	21 - 30	N coordinate (CARDN)
F10.2	31 - 40	E coordinate (CARDE)
F10.2	41 - 50	Distance to top of assay interval (from collar) (ENTT)
F10.2	51 - 60	Distance to bottom of assay interval (from collar) (ENTB)
F2.1	61 - 62	Assay of third auxillary mineral (ESSAY 4)
F2.1	63 - 64	Assay of second auxillary mineral (ESSAY 3)
F2.1	65 - 66	Assay of first uaxillary mineral (ESSAY 2)
F4.4	67 - 70	Assay of primary mineral (ESSAY 1)
F10.2	71 - 80	Tonnage factor of the material in this assay interval (SPGR)

Zero card follows last card of any hole. Card w/pos. value in ENTT and zero in ENTB follows last card of last hole.

Program - Part I

Part I of the program transforms the original drill-hole data, with its usual but not necessarily irregular assay intervals, into assay intervals of the desired bench height. In order to maximize recovery, the program is designed so that each bench can be selectively mined in such a manner that a minimum of one-third of the bench height and a maximum of two-thirds of the bench height can be separated from the remaining material for shipment to a different destination; for example, if a bench has a stringer of 8 feet of ore in a face of 50 feet, it is possible for the shovel operator to separate this ore if he is permitted to take an additional 9 feet of other material, thus diluting the ore. It is not recommended that any bench heights of less than 15 feet be used in this program. The ability of a shovel operator to separate material in a bench less than 15 feet in height is very questionable.

After calculating possible subdivisions of a bench, the grade of the divisions are checked to determine the effect of subdividing and each subdivision is classified by the grade of the material in it. At the same time each subdivision is assigned the the geologic material type equivalent to the majority of the material in the subdivision and also the metallurgical ore type of the majority of the material (if this information is available from the core logs; otherwise, material designations are set to zero). As each drill hole is completed, a new drill-hole record is produced as output (in both printed form and as a new magnetic tape which will be used as input in the second and third parts of the main program). Examples of this printed output are in Figure 3.1, which includes a description. A comparison between an original drill-hole log and the newly developed drill-hole record by bench height is shown in Figure 3.2.

As each hole is completed, this part of the program also outputs cumulative tonnages of the 3 ore types and their associated grades for the hole (Fig. 3.3). When the final hole has been calculated, the program outputs a total cumulative tonnage similar to those previously output for each hole but this time for the entire orebody (Fig. 3.4).

Because the bench height is often a variable at the early stages of pit design, an option has been included in this part of the program to allow a person operating the program to evaluate many different bench heights before selecting a final bench height to be used in later parts of the program. The use of this option involves three variables which are input to the program through the cardread (CDRD) subroutine. The first of these variables is BENHT, whose input value is the largest bench height that the person running the program wishes to consider. The second variable is BENMN and represents the minimum bench height that the person running the program wishes to consider. A third variable BENIN is a decrementing value which will be subtracted from BENHT after Part I of the program has been completed, unless BENHT is equal to BENMN, and the program will return to the start of Part I and recalculate the holes on the basis of the reduced BENHT. This cycle will continue until BENHT has been decremented to be equal to BENMN. The sequence operates as follows:

The first cycle will calculate blocks having a height equal to the maximum bench height indicated by the initial value read into the

FIGURE 3.1

Line 1: 82.(-10.)
The first number, outside the parenthesis, is the actual number assigned to the drill hole by the company; the number inside the parenthesis is the drill hole number assigned by the program. All program assigned numbers are negative to help distinguish them from block numbers within the hole.

Line 2, 3 and 4 are self-explanatory.

Line 5: 2 This represents the block number FROM the reference elev.

Line 7+; 8+; 9+:
25500. This is the tonnage in the first block interval in the first block of this hole.
10001001. This number represents that type of material in the first block interval of this hole. It is an 8 digit number which is actually a combination of three other numbers;

$$\underline{10}_{a} \quad \underline{001}_{b} \quad \underline{001}_{c}$$

a is the geologic rock type of the material in this block interval

b is the optional rock characteristic number of this same material

c is a representation of the grade of the ore contained in this block interval; 1 represents waste, 2 represents low grade, 3 represents intermediate grade, and 4 represents ore. All these grades are defined by the cutoff grades defined by the person running the program.

Line 9: 0.00 This is the dollar value of the minerals contained in all the block intervals in this block as calculated by the CALVAL Subroutine.

continuation of Line 7+; 8+; 9+:
The 4 numbers appearing to the right in the form X,XX are the assay values for up to 4 minerals, of the material in this block interval. The number appearing under the heading 1 is the assay of the mineral designated as the Primary Mineral.

The above described numbers are repeated for all the bench height blocks that this hole contains. Naturally, the first and the last blocks may not be full bench height blocks due to the varying surface elevation and depth of hole.

EXAMPLE OF CONVERTED DRILL HOLE DATA

```
Line Number

 1          DRILL HOLE NUMBER,        82.(-10)
 2          COORDINATES ARE     108825.92N       109723.75E
 3          SURFACE ELEVATION =   3210.64      BOTTOM ELEVATION =  2530.14
 4          AREA OF INFLUENCE =  58900.00                BENCHEIGHT = 50.0
 5                                             BLOCK NUMBER 2 TONNAGES
 6              TONS        MATERIAL           ASSAY    1         2         3         4
 7            25500.        10001001                  0.00      0.00      0.00      0.00
 8                0.               0                  0.00      0.00      0.00      0.00
 9                0.               0                  0.00      0.00      0.00      0.00
10                       VALUE OF THIS BLOCK IS $       0.00
11
12                                             BLOCK NUMBER 3 TONNAGES
13              TONS        MATERIAL           ASSAY    1         2         3         4
14           176453.        10001001                  0.00      0.00      0.00      0.00
```

Fig. 3.1

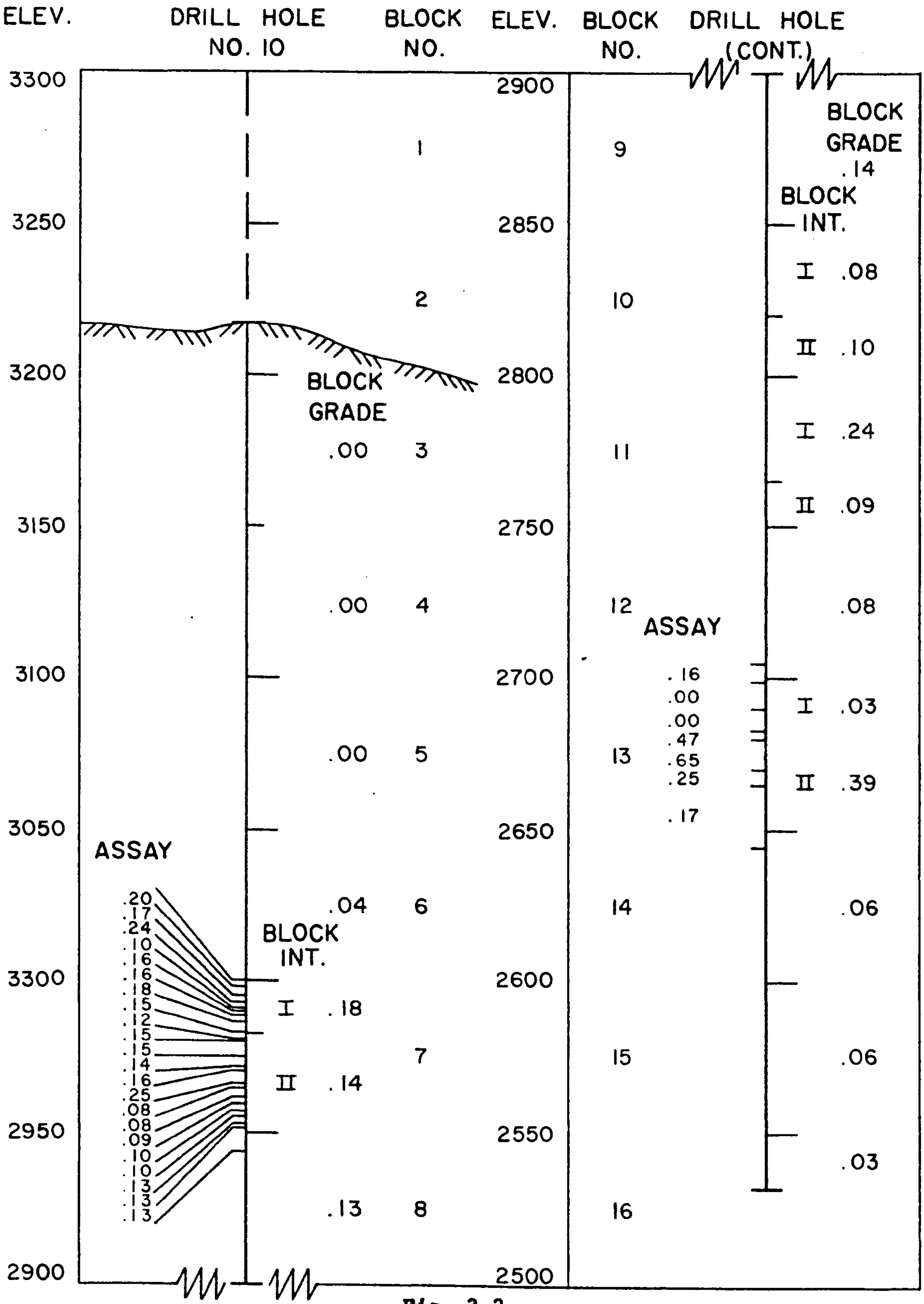

ELEV.
DRILL HOLE NO. 10
BLOCK NO.
ELEV.
BLOCK NO.
DRILL HOLE (CONT.)
3300
3250
3200
3150
3100
3050
3300
2950
2900
BLOCK GRADE
BLOCK INT.
ASSAY
.20 .17 .24 .10 .16 .16 .18 .15 .12 .15 .15 .14 .16 .25 .08 .08 .09 .10 .10 .13 .13 .13
I .18
II .14
.00 3
.00 4
.00 5
.04 6
7
.13 8
1
2
2900
2850
2800
2750
2700
2650
2600
2550
2500
9
10
11
12
13
14
15
16
ASSAY
.16 .00 .00 .47 .65 .25 .17
BLOCK GRADE
.14
BLOCK INT.
I .08
II .10
I .24
II .09
.08
I .03
II .39
.06
.06
.03

Fig. 3.2

FIGURE 3.3

Lines 1 through 4: same as in Figure 3.1

Line 6: Heading

Lines 7 through 11:

Line 7: Tons of material with assay of the Primary Mineral greater than or equal to the ore cutoff grade.

Line 8: The average assay value of tonnage in line 7 for the Primary Mineral.

Line 9: The average assay value of the mineral designated as the first secondary mineral (e.g., Moly in a Copper mine) for the tonnage in Line 7.

Line 10: Same as Line 9, but for a second secondary mineral.

Line 11: Same as Line 9, but for a third secondary mineral.

Line 12: Tons of material in this hole that fell between the assay cutoff grades of ore and low grade.

Lines 13 through 16: Same as Lines 8-11 but for the tonnage in Line 12.

Line 17: Tons of material in this hole whose assay of the primary mineral is above the waste cutoff and below the intermediate ore cutoff.

Lines 18 through 21. Same as Lines 8-11 for the tonnage in Line 17.

EXAMPLE OF CUMULATIVE DRILL OUTPUT

Line Number	
1	DRILL HOLE NUMBER, 82.(-10)
2	COORDINATES ARE 108825.92N 109723.75E
3	SURFACE ELEVATION = 3210.64 BOTTOM ELEVATION = 2530.14
4	AREA OF INFLUENCE = 58900.00 BENCHEIGHT = 50.0
5	
6	CUMULATIVE TONNAGES IN HOLE 82. FOR A BENCHEIGHT OF 50.
7	TONS OF ORE = 149495.
8	GRADE OF ORE = 0.39 PER CENT MINERAL 1
9	GRADE OF ORE = 0.00 PER CENT MINERAL 2
10	GRADE OF ORE = 0.00 PER CENT MINERAL 3
11	GRADE OF ORE = 0.00 PER CENT MINERAL 4
12	TONS OF LEACH = 187797.
13	GRADE OF LEACH = 0.24 PER CENT MINERAL 1
14	GRADE OF LEACH = 0.00 PER CENT MINERAL 2
15	GRADE OF LEACH = 0.00 PER CENT MINERAL 3
16	GRADE OF LEACH = 0.00 PER CENT MINERAL 4
17	TONS OF LOW GRADE = 820670.
18	GRADE OF LOW GRADE = 0.14 PER CENT MINERAL 1
19	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 2
20	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 3
21	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 4

Fig. 3.3

FIGURE 3.4

This output is identical to the output represented in FIGURE 3.3 except that the figures are cumulative for all the holes that have been evaluated by the program.

These figures correspond to what are usually designated as "Geologic Ore Reserves," that is, they represent the total amount of mineral available without reference to the ability of the mine operator to extract the mineral profitably.

EXAMPLE OF CUMULATIVE OREBODY OUTPUT

Line Number	
1	CUMULATIVE TONNAGES IN THIS OREBODY FOR A BENCHEIGHT OF 50.
2	TONS OF ORE = 10871344.
3	GRADE OF ORE = 0.56 PER CENT MINERAL 1
4	GRADE OF ORE = 0.00 PER CENT MINERAL 2
5	GRADE OF ORE = 0.00 PER CENT MINERAL 3
6	GRADE OF ORE = 0.00 PER CENT MINERAL 4
7	TONS OF LEACH = 3771890.
8	GRADE OF LEACH = 0.25 PER CENT MINERAL 1
9	GRADE OF LEACH = 0.00 PER CENT MINERAL 2
10	GRADE OF LEACH = 0.00 PER CENT MINERAL 3
11	GRADE OF LEACH = 0.00 PER CENT MINERAL 4
12	TONS OF LOW GRADE = 6967082.
13	GRADE OF LOW GRADE = 0.15 PER CENT MINERAL 1
14	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 2
15	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 3
16	GRADE OF LOW GRADE = 0.00 PER CENT MINERAL 4

Fig. 3.4

program as BENHT. When the complete orebody has been evaluated and the individual hole tonnages and grades printed out, along with the cumulative tonnages and grades, the program branches on whether or not the current value of BENHT is equal to BENMN. If BENHT is greater than BENMN, the value of BENHT is reduced by an amount equal to BENMN and the program is reinitialized and started again. This sequence continues until the value of BENHT is less than or equal to BENMN. When this occurs, the program proceeds into Section 2 of Part I.

It is anticipated that many operators will wish to run this part of the program and evaluate the effect of the various bench heights before executing the remainder of the program. This can be accomplished by running the program with the computer in "ADDRESS STOP" condition with stop address being the first command of Section 2 of Part I. The sequential operation previously described is as follows:

1. Calculate the block tonnages for a sequence of bench heights starting with a maximum called BENHT decreasing for each series of calculations by an increment BENMIN is reached.

2. Individual block tonnages output may be suppressed by turning switch 1 to "off" position, then only total cumulative tonnages will be output for each bench height.

In this way, the effect of different bench height and cutoff grades can be investigated and an optimal bench height selected. The optimal bench height and cutoff grades would then be input to the program and this part of the program would be rerun in the normal fashion as part of the longer program with the individual blocks being output.

Operational Logic of Part I

The logic behind Part I of the program which converts the raw assay data into bench height blocks is basically an attempt to duplicate the reasoning processes used by a competent mining engineer in his design of an open-pit mine. Modern open-pit mines like AS&R's Mission and Anaconda's Jackpile have demonstrated the practicality of selective pit mining within a single bench. It has been the experience of the author that a good shovel operator can, if some visual indicators such as color difference are present, select ore out of a mixed face, but it is not believed that this procedure can be done dependably on less than about one-third of a face. In order that this program calculate the available tonnage as accurately as possible, this selective ability must be considered. To do this, the program uses the following logic circuit.

A. Contained in any bench height block of any hole there may be several assay interval of the original drill-hole log. If any single assay interval, of the several which constitute the bench height block, is greater than two-thirds of the bench height, then the block is considered as having a single grade - that of the average of all the intervals contained in it.(see Fig. 3.5a).

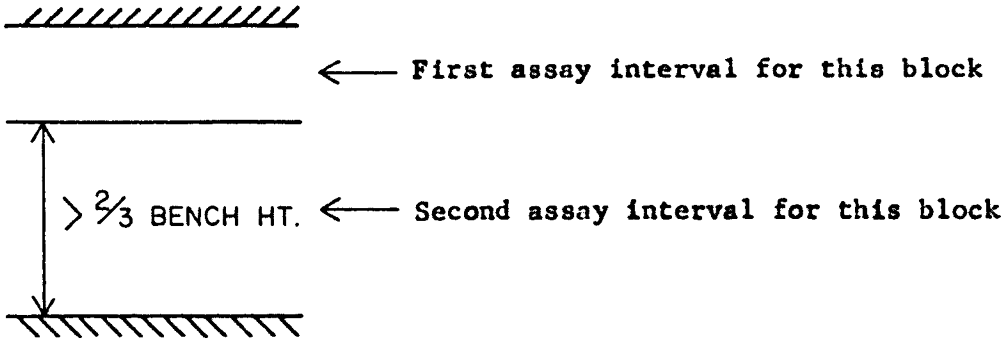
B. If any assay interval is less than one-third bench height, it cannot be separated and must be averaged with the previous assay interval unless it is the first interval, in which case it will be combined with the next assay interval (see Fig. 3.5b).

GRAPHIC REPRESENTATION OF BENCH HEIGHT LOGIC

Part 1

Text
Ref.

a.



b.

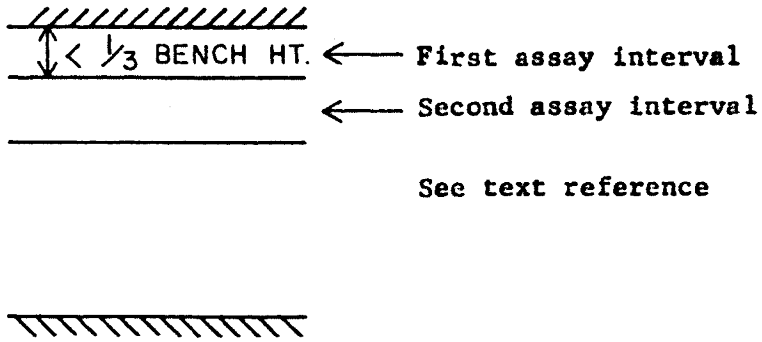


Fig. 3.5

C. If any assay interval (or several averaged assay intervals) is greater than one-third bench height but less than two-thirds bench height, the next assay interval is examined (see Fig. 3.6).

1. If this assay interval has the same ore classification as the previous interval (or average of intervals) the current interval is averaged with the previous ones, and all are called block interval 1.

2. If this assay interval has a different ore classification than the previous interval (or average of intervals) the current assay interval is calculated as a separate block interval (Number 2) and the next assay interval is investigated (see Fig. 3.6).

2.a. If the next assay interval has the same ore classification as block interval 2, the new interval and block interval 2 are averaged (see Fig. 3.6).

2.b. If block 2 and the next assay interval do not have the same ore classification:

2.b.I. When block interval 2 is greater than one-third bench height, then the current assay interval must be averaged with block interval 2 as it cannot be separated (see Fig. 3.6).

2.b.II. Where block interval 2 is less than one-third bench height:

GRAPHIC REPRESENTATION OF BENCH HEIGHT LOGIC

Part 2

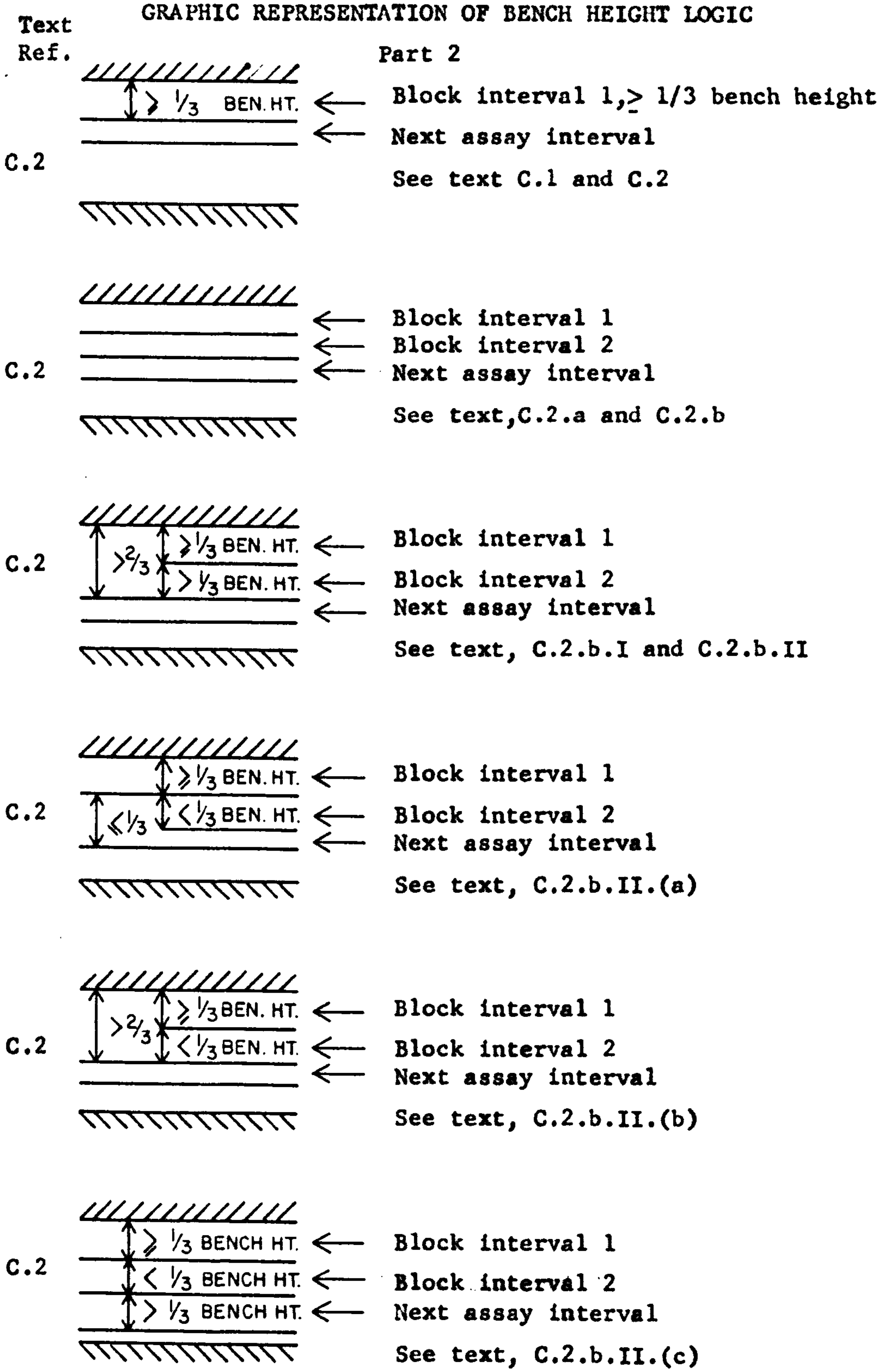


Fig. 3.6

2.b.II.(a) If block interval 2 plus the current assay interval is less than one-third bench height, average the current assay interval with block interval 2.

2.b.II.(b) If block interval 1 + block interval 2 is greater than two-thirds bench height, then the current assay interval must be averaged with block interval 2.

2.b.II.(c) If the current assay interval is greater than one-third bench height:

1. Calculate the assay difference between block interval 1 and block interval 2.
2. Calculate the assay difference between block interval 2 and the current assay interval.
3. If (1.) is greater than (2.), average the current assay interval with block interval 2. If (2.) is greater than (1.) average block interval 1 and block interval 2, call the average block interval 1 and call the current assay interval block interval 2.

This process is repeated for every bench height block calculated for every hole.

Program - Part II

Part II of the program is primarily a bookkeeping section which sets up a series of arrays needed by Part III.

Section 1 of Part II uses as input a tape developed by Part I which lists sequentially the hole number, east coordinate, north coordinate, surface elevation, number of the first block with non-zero tonnage (all blocks are numbered from the reference elevation, each bench height being one block), area of influence, and the block number of the last block in the hole.

Section 1 establishes and outputs a one-dimensional array for each hole whose sequential entries are the horizontal distances from the current hole to every other hole in the orebody. These arrays are output on tape as they are developed. These arrays will be used by Part III to determine whether or not a specific hole is within the perimeter of a given mine.

Section 2 of Part II requires as input the north and east coordinates of the property corners (up to 6 corners) and then establishes slopes and intercept values for each property line. When this has been done the program then calculates the perpendicular distance from each hole to each boundary line and establishes a one-dimensional array of these distances and then outputs these arrays on the same tape as the inter-hole distance arrays were written on. These arrays will be used by Part III to determine whether or not a particular mine extends beyond the mine property lines and, if so, how much.

Section 3 of Part II proceeds to define all possible conical mines which could exist in this orebody, supposing that each block which has a positive dollar value could be the apex of a conical mine. The method is as follows: using as input the block tonnage, grade, and value figures developed and output on tape in Part I.

The complete block information on the first hole is brought into memory, and, starting with the first non-zero tonnage block, each block is sequentially examined until the first block with a primary mineral grade greater than the one cutoff grade is found. When this block is found, the depth of the block from the reference elevation is calculated. This depth times the size of the slope angle determines a mine radius in the plane of the reference elevation.

From the tape output by section 1, the distance array for the first hole is brought into memory and the distance from the first hole to each of the other holes is compared to the mine radius. If the distance is greater than the mine radius, the distance to the next hole is investigated. If the distance is less than the mine radius, then the hole whose distance is being checked will be affected by the mine. The depth to which this hole will be mined is equal to the difference between the interhole distance and the mine radius times the cosine of the angle of slope. As the distance from the center hole to each of the other holes in the area is checked, two new arrays are developed which are associated with a unique mine number. These two arrays consist of a sequential list of those holes that will partially be removed by the excavation of this particular mine.

The second array matches the array of affected holes and contains the depth to which each of these holes will be mined. When every hole in the area has been checked for distance, a possible mine configuration has been delineated and the following information is output:

1. The mine number
2. The hole number containing the apex block
3. The block number of the apex block

4. The array containing the sequential list of those holes that would be affected by this conical mine.

5. The depths associated with each of the holes in the array of hole numbers.

6. Mine radius

Having completed one possible mine, the next lower block of the first hole is examined; if it is ore a new mine is designated and outlined, or if it is waste, the block is passed over and the next block is searched until an ore block is found or the end of the hole is reached.

When the end of the first hole is reached, the complete block information of the next sequential hole is brought into memory. Starting with the first non-zero tonnage block, each block in the hole is checked for ore. If it is an ore block, a conical mine is designated with this block as an apex and the two arrays for the mine are determined and output onto tape. This process continues until the last block in the last hole is examined for the possibility of being the apex of a conical mine. When this block has been examined and a mine determined or not, depending on the grade of this block, the process of defining all possible mines in this ore body, for this bench height, has been accomplished.

<u>Operational Logic of Part II</u>

<u>Section 1</u>

Distance from hole X to hole Y

$$= \sqrt{\left[(\mathrm{CORDN}(X)-\mathrm{CORDN}(Y)\right]^2+\left[(\mathrm{CORDE}(X)-\mathrm{CORDE}(Y)\right]^2}$$

Section 2

Let X(I) and Y(I) be the north and east coordinates, respectively, of the I^{th} property corner.

The equation for the line is $Y = Mx + b$. The slope, M, of the line between the I and the $[I + 1]^{st}$ corner

$$= [Y(I+1) - Y(I)] \;/\; [X(I+1) - X(I)]$$

The intercept, b, of the same line from the point slope form of the standard equation of a line

$$y - Y(I) = M(x - X(I))$$

for the intercept desired, $X = 0$, then $y = b$.

$$b = Y(I) - MX(I)$$

$$= Y(I) - \frac{Y(I+1) - Y(I)}{X(I+1) - X(I)} X(I)$$

$$= \frac{X(I+1)Y(I) - X(I)Y(I) - Y(I+1)X(I) + X(I)Y(I)}{X(I+1) - X(I)}$$

$$= X(I+1)Y(I) - Y(I+1)X(I) \;/\; X(I+1) - X(I) .$$

Calculation of the perpendicular distance from each hole to every property line where M (I) and b(I) are the slope and intercept of the i^{th} line.

For the j^{th} hole with the coordinates (X(J), Y(J), the perpendicular distance to the i^{th} property line is derived from the distance formula

$$\text{DIST.} = \left| \frac{AX + BY + C}{\sqrt{A^2 + B^2}} \right|$$

converting to $Y = MX + b$.

$$\text{DIST.} = \left| \frac{-MX + Y - b}{\sqrt{(-M)^2 + 1}} \right|$$

Perpendicular Dist. From the J^{th} hole to the I^{th} line $= \dfrac{Y(J) - \text{Slope}(I)X(J) - b(I)}{\sqrt{\text{Slope}(I)^2 + 1}}$

Section 3

It is conceptually possible (though not practically) that every bench height block in every drill hole might be used as the apex of a conical pit. To make this concept more nearly a reality, the blocks which can act as pit apices are limited to those blocks which have a dollar value in excess of some established minimum.

The dollar value of a block is calculated as follows:

Where

V(J) = dollar value of the j^{th} block

A = area of influence,

ENT(I) = the height of the i^{th} block interval
i = 1,2. ENT(1) + ENT(2) = bench height

ASSAY (i,k) = assay in percent by weight of the k^{th} mineral in the i^{th} block interval
k = 1,2,3,4

PR(k) = price per pound of the k^{th} mineral

SPG(I) = Cu ft ton of the material in the i^{th} block interval (tonnage factor)

MAT(i) = Geological material type of the i^{th} block interval

COORE = Cutoff grade of ore

COLEA = Cutoff grade of intermediate material

COLG = Cutoff grade of low-grade material

If ASSAY (i,1) $\geq$ COORE

$$V(J) = \sum_{i=1}^{2} \sum_{k=1}^{4} \frac{A \times INT(I)}{SPG(I)} \times 2000 \times ASSAY\ (i,k) \times PR(k)$$

If COLEA $\leq$ ASSAY (i,1) > COORE

$$V(J) = \sum_{i=1}^{2} \frac{A \times INT(I)}{SPG\ MAT(i)} \times 2000 \times ASSAY\ (i,1) \times \frac{PR(1)}{2}$$

Assuming that secondary processing will decrease the value of the mineral by one-half and will make recovery of other minerals impossible

if ASSAY(i,1) < COLEA

$$V(J) = 0 .$$

The minimum value for any block to be eligible to act as an apex is defined as a block whose metal value is in excess of the sum of the cost of removal of the block plus the mineral value of an identical block of exactly cutoff grade of the primary mineral.

Let

VALMN(j) = the minimum value of the j^{th} block

MAT(j,i) = type of material in the i^{th} interval of the j^{th} block

CST MAT(j,i) = cost per ton of removal of the type of material in the i^{th} interval of the j^{th} block

$$\text{VALMIN}(j) = \sum_{i=1}^{2} \frac{A \times \text{INT}(I)}{\text{SPG}(I)} \times \text{CST MAT}(j,i) + \sum_{i=1}^{2} \frac{A \times \text{INT}(I)}{\text{SPG}(I)}$$

$$\times\ 2000 \times \text{COORE} \times \text{PR}(1)^{*}$$

*Note that the minimum value is calculated only on the basis of the primary mineral and it is, therefore, possible for a block to be used as an apex if it has a high value due to the presence of other economic minerals.

Once a block is determined to have a value exceeding the minimum required, a theoretical mine is established using this block as an apex. This mine will be conical in shape and have a mine radius on the previously established reference elevation. The true apex of this mine will be below the apex block and this true apex distance from the surface is found as follows:

$$\text{Cone height} = \text{Apex block number} \times \text{bench height} + \sqrt{\frac{\text{Area of influence}}{\pi}}.$$

The mine radius is the radius of the circle defined by the intersection of the reference elevation plane and a right circular cone with base angle equal to the pit slope.

Let RADMN = the mine radius

J = block number of the apex block

A = area of influence of the hole containing the apex block

ANG = the pit slope

$$\text{RADMN} = \frac{\text{cone height}}{\text{TAN(ANG)}}$$

The next problem is to discover what other drill holes fall within this mine radius, because each hole that is within the mine radius will contribute to the total tonnage of material in the mine. From the distance arrays developed in section 2, it can be determined whether or not a hole will contribute to this mine. If it is within the mine radius, then the depth to which this hole will be excavated can be calculated as follows:

Depth (J) = depth to which the j^{th} hole will be excavated

Depth (J) = RADMN - Dist(J) x TAN(ANG)

In this way the depth to which any hole falling within the mine radius will be excavated can be calculated.

Program - Part III

While Part III of the program is lengthy and time consuming, the logic in this part is extremely simple and straightforward. Each of the conical "mines" of section 3 Part II contain a definite and accountable tonnage of known grade and of known material type. Each of these tonnages is, of course, an approximation because if a drill hole, which is approximately at the center of its area of influence is just outside the mine radius, then no material from the area of influence of that hole will be included; on the other hand, if the hole is just inside the mine radius, at least some part of the entire area of influence will be included which should not have been.

It is felt that this type of error will tend to be self-canceling.

If the area of influence is approximately 40,000 square feet for the average drill hole and the hole in question is 1,000 feet beyond the mine radius, then the tonnage of material not included which should have been can be approximated by calculating the volume included in the intersection of a right cone of base angle 45° and of a right cylinder of radius equal to $\sqrt{\frac{40000}{\pi}}$. The volume in this intersection is approximately 71,350 tons of alluvium. The total volume of the cone involved contains

$$\frac{\text{Area of Influence} \times 1/3 \times \text{cone ht.}}{12.85} = 104,700,000 \text{ tons.}$$

Thus, the error is less than .001 percent per hole on the low side.

The complementary error will contribute + .0001 + percent of the total volume, and it is believed that these errors are of less significance than the sampling error, assay interval measuring error, and the value and cost approximations used.

A valid question is that regarding the selection of the highest value mine as the first to be excavated and why not simply calculate the tonnages and values of each sequential "mine" and "excavate" it as the sequence appears. However, once any group of blocks has been removed, the value of the remaining block is, of course, altered by being so much more accessible. For this reason, it is necessary to reevaluate all the remaining mines whenever a series of blocks has been excavated by calculation. When, after a reevaluation cycle, no mine has a positive value, it means that then mineral values are all less than the cost of removing them.

Calculation Procedure of Part III

The purpose of Part III is to evaluate each of the possible mine configurations determined in section 3 of Part II.

The inputs used in Part III are:

1. The description of all possible mine configurations from section 3 of Part II.
2. The hole by bench height block output by Part I.

The description of mine 1 is brought into memory with its associated arrays of all holes affected by the mine and the depths of each of these holes. The block tape from Part I is now searched to locate the hole containing the apex block of this mine. When this hole is located, the tonnages of all material in all blocks, from the surface down to and including the apex block, are summed in the following classifications:

1. By ore grade classification: ore, intermediate ore, low-grade, or waste.

2. By geologic rock type by ore grade: ore in limestone, ore in sandstone, and so forth.

3. By mineral type by ore grade: oxide ore, oxide intermediate, oxide low-grade, sulphide ore, sulphide intermediate, and so forth.

4. Dollar value of minerals present.

This program is built to run for nine geologic types, three mineral types, and four ore grades.

Once the hole containing the apex block has been summed, the block tape is searched to find the hole number corresponding to the first hole listed in the array of holes affected by this mine. When this hole is found, the block information is brought into core and the depth to which this hole is to be removed is determined from the second array output by section 3, Part II. The tonnages of all blocks from the surface down to the depth indicated are summed and added to the cumulative tonnages for this mine. In most cases the depth will not correspond to an even block, and in this case tonnages equal to that fraction of the block above the excavation depth are added to the cumulative totals. When each hole in the affected hole array is complete, the block records of the next hole in this array are brought into memory, summed to the indicated depth and added to the cumulative totals. When the last hole in the array of affected holes has been summed, a set of cost figures are read into memory. These costs are in the form of dollars per ton for removing waste, dollars per ton for removing and stock-piling low-grade material, dollars per ton for mining of intermediate grade ore, and dollars per ton for mining ore. These cost figures are multiplied by the appropriate cumulative tonnages and then summed to get the total cost of mining. This cost is subtracted from the cumulative mineral value and a mine profit is represented by the remainder. This same process is repeated for all possible mines and that mine having the highest mine profit is selected as the first excavation of the orebody. The mine with the maximum mine profit is determined through the usual maximum-replacement routine where the current value is compared to the maximum of all previous mine profits. If the current value is less than the previous maximum, the program proceeds to the next mine. If the current mine profit is greater than the previous maximum, the current mine number and mine profit replace the previous maximum mine number and mine profit, and the program proceeds to calculate the profit for the next mine. This process continues until all possible mines have been evaluated and the maximum determined for all mines.

After the maximum profit mine has been determined, the configuration arrays (hole numbers affected by this mine) and associated depths of this mine are brought back into memory and the process of deleting the material included in this mine begins. The magnetic tape containing the block tonnages by hole number is rewound and the first hole number is compared to the first hole number in the array of affected hole numbers. If this hole number is not contained in the array of affected holes, the block tonnage, grade, value for the entire hole is copied onto a new tape and the next hole number is compared to the array. If the hole number of the blocks now in storage is found to be contained in the affected array, the tonnage of all blocks down to the affected depth of this hole is set to zero. The tonnages in the block containing the final affected depth are reduced by an amount proportional to that portion of the block above the affected depth. The block information for this (modified) is now copied onto the new tape and the hole number of the next series of blocks is compared to the affected array.

When all the holes of the orebody have been transferred to the new tape with the block tonnages modified or not as needed, the list of possible mines is again evaluated and a new mine profit figure is developed for each possible mine as contained in the list of all possible mines generated by section 3, Part II. In the event that the apex block of any mine is found to have zero tonnage (indicating that the excavation of some previous mine caused this block to be removed) this mine is deleted from the list of all possible mines. When all the currently possible mines have been reevaluated, that one having the maximum mine profit will again be excavated. All tonnages, grades, and profits of this new maximum profit mine will be added to the orebody cumulative totals, and then the listing of block tonnages by hole will again have those blocks contained in the new maximum profit mine deleted from it and be rewritten on a new tape (actually the tape that the original listing was on). This iterative process continues until the maximum profit mine has a zero or negative value. At this point there is no material remaining which can be mined at a profit and the process is complete.

The final output of the program is the cumulative tonnages, mineral content, and profit figures and the sequence of apex block numbers in the order in which they were selected.

CHAPTER 4

DEVELOPMENT OF AN ORE GRADE DISTRIBUTION FUNCTION

It was shown in Chapter 2 that the most important criterion in the financial analysis of a proposed mine design is the sequence in which blocks containing the economic mineral are to be mined and processed. In Chapter 3 a computer program was described which defines the approximate final pit configuration for an open-pit mine that will recover all of the mineral that can be mined at a profit, for a given set of cost parameters.

Before proceeding further in the search for an optimum mining sequence, it would be advantageous to be able to develop some limits on the financial return of a proposed system design. Especially useful would be an upper bound. If an upper bound could be determined and if this bound proved to be an unacceptable return, the system could be immediately rejected and the expense of developing a "best system" could be saved.

Analysis of the MSPW function, defined in Chapter 2, disclosed that the optimum system is the one which returns to the system the greatest amount of money as close to time 0 as possible, for any discount rate greater than zero. This requirement translates into mining terms in the following manner: Mine the highest grade ore as early as possible, leaving the lower grades to be mined during the latter years in the life of the mine. Whether or not this can actually be accomplished is a function of the distribution of the mineral in the particular piece of mining real estate, but if it is assumed that it can be calculated which will be an upper bound.

To evaluate this bound it will be necessary to develop a grade distribution function for the ore in this particular "mine". It would be convenient if this distribution was one of the better-known probability distributions. This would enable the investigator to use many of the published tables to find the percent of ore above or below a particular grade or within specific limits.

Drill hole assays from the Palo Verde mine of the Banner Mining Company were used to determine if such a function could be found. Figure 4.1 represents the distribution of ore grade by percent assay in this orebody. The distribution is so excessively skewed to the right that none of the standard distributions seemed applicable.

Statistics of the Ore Samples

N = 10,286

Mean = .3961% $\qquad \beta_1 = \mu_3^2 / \mu_2^3 = 72.3499$

Variance = .3749%

3rd Moment = 1.9525% $\qquad \beta_2 = \mu_4 / \mu_2^2 = 117.5901$

4th Moment = 16.5274%

This shows a curve positively skewed with a very long tail.

Krige (4), Hazen (5), Hewlett (6), and Miller and Kahn (7) all show examples of the distribution of various minerals normalized by the log transformation, but they never say that the transformation is a general one. Figure 4.2 is a plot of the log transformation applied

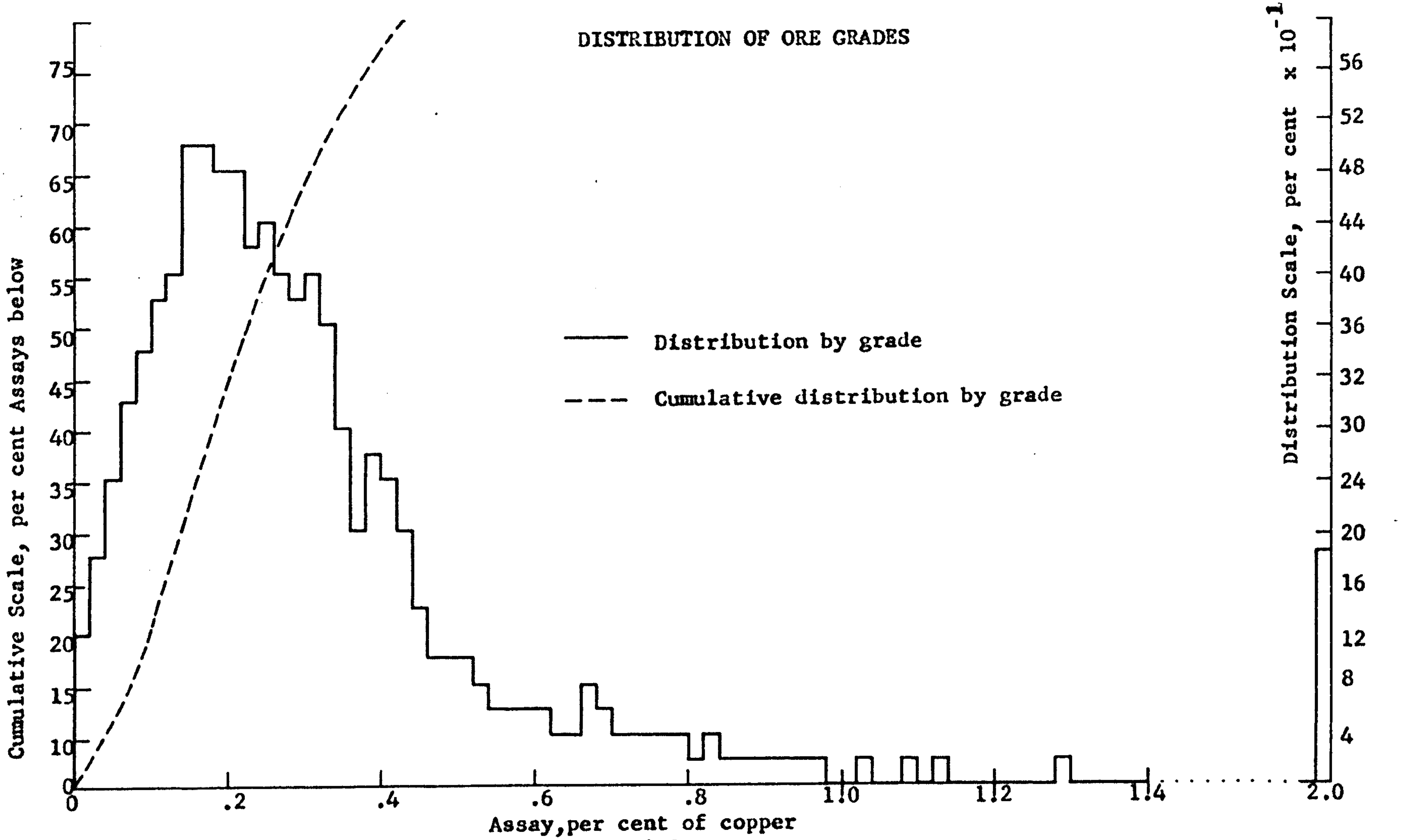

DISTRIBUTION OF ORE GRADES
Distribution by grade
Cumulative distribution by grade
Cumulative Scale, per cent Assays below
Distribution Scale, per cent x 10^{-1}
Assay,per cent of copper
0
.2
.4
.6
.8
1.0
1.2
1.4
2.0
0
10
15
20
25
30
35
40
45
50
55
60
65
70
75
4
8
12
16
20
24
30
32
36
40
44
48
52
56

Fig. 4.1

DISTRIBUTION OF LOG TRANSFORMATION
Log(Assay x 100 + 1)

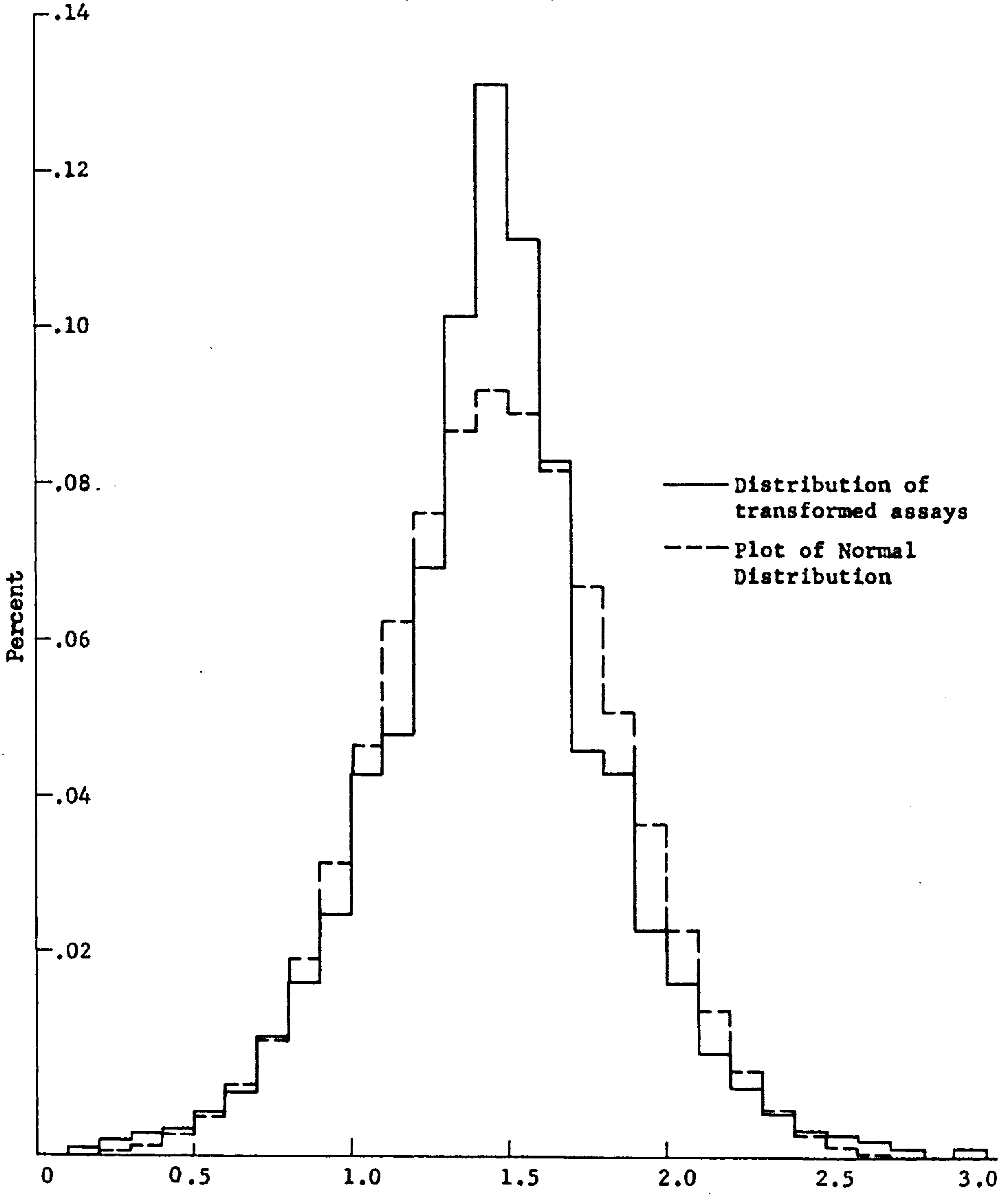


Fig. 4.2

to this drill-hole data plotted against a normal curve of the same mean and variance. Notice that no change of variance could cause the normal distribution to approximate closely the log-transformed assay distribution.

The statistical parameters of the log (assay x 1000 + 1) are as follows:

Mean	=	1.4214
Variance	=	.1500
μ_3	=	.0129
μ_4	=	.0852
β_1	=	.0495
β_2	=	3.7837

A normal curve has β_1 and β_2 values of 0 and 3, respectively, and it is apparent that the log transformation does produce a distribution which approaches the normal. The type of function indicated would seem to be of the same general type as the normal but with more parameters to increase its flexibility. The Pearson Type IV (8) curve is of the form $y = k_1 f_1 e^{-k_2 f_2}$, where $f_1(x) = (1 + \frac{x^2}{a^2})^m$, $f_2(x) = \tan^{-1}(\frac{x}{a})$ and a, m, k, and k_2 are all functions of μ_1, μ_2, μ_3, and μ_4. Thus, the Type IV Pearson curve is a four-parameter curve of the same general shape as the normal.

Figure 4.3 is a plot of the log-transformed assay data and the Type IV Pearson curve produced using the moments of the transformed assay distribution. A Chi Square value of 620 indicates a very bad fit, but closer examination of the plot indicates that most of this discrepancy is due to roughness of data rather than poor fit. Figure

COMPARISON OF LOG TRANSFORMATION OF ASSAY

AND PEARSON TYPE IV CURVE

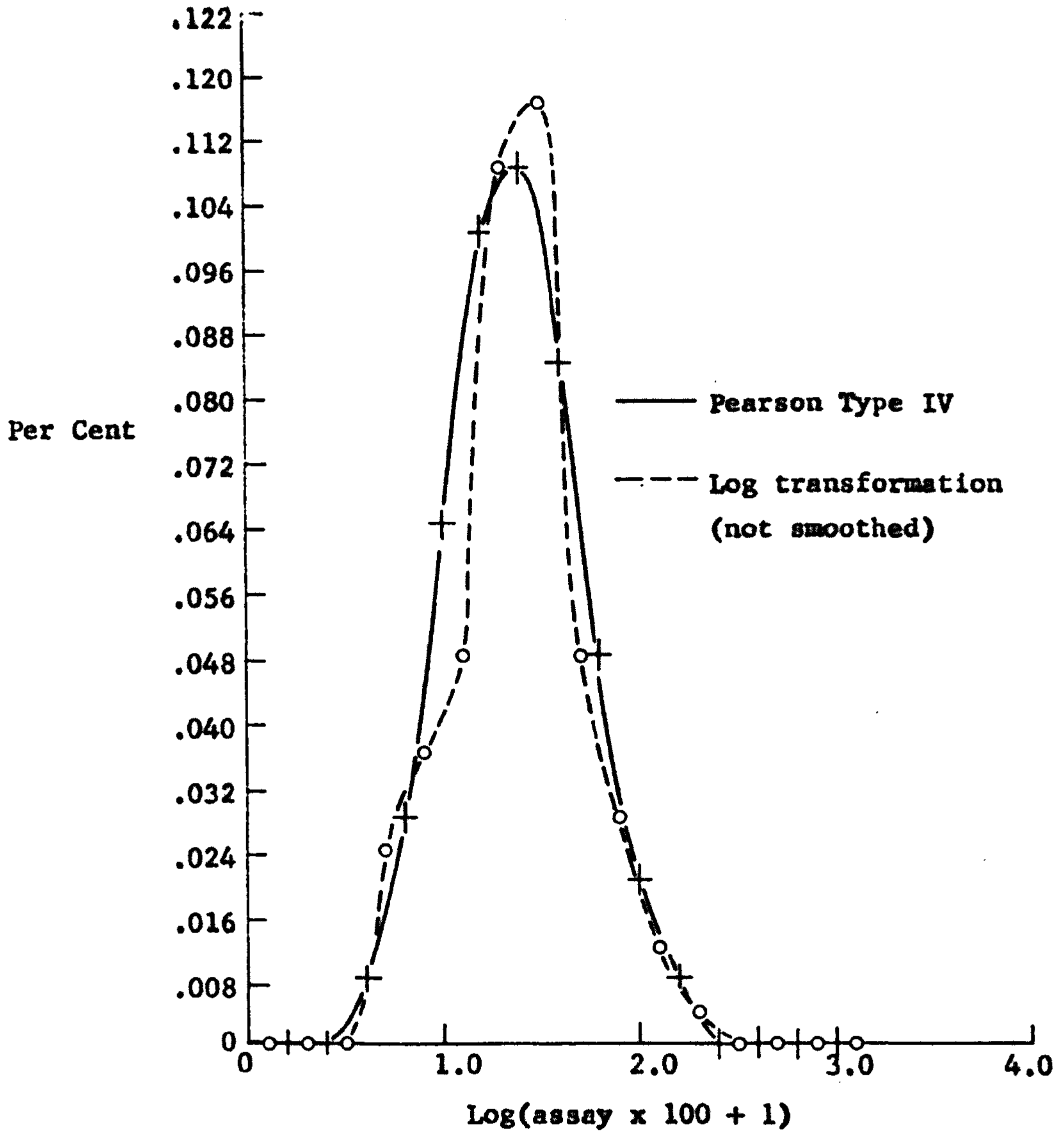


Fig. 4.3

4.4 is the same as Figure 4.3 except that the transformed assay data has been put through a smoothing function, $f(x_i) = f(x_{i-1}) + f(x_i) + f(x_{i+1}) / 3$. The Chi Square value for the smoothed distribution divided into 100 cells, and the appropriate Type IV Pearson curve is equal to 112. Using the equation $\chi^2 = \frac{1}{2}(Z_p + 2v-1)^2$ (9), where Z_p is the Cumulative Normal Distribution area to the left of p and v is the degrees of freedom, to calculate the tabular value of χ^2, the tabular value for 99 df at the 95% confidence limit = 124. Hence, a Chi Square value of 112 is not significant and the hypothesis that the data is represented by a Pearson Type IV curve cannot be rejected.

The problem of roughness in the original data, and transferred to any transformed data, is one that must be expected. Commercial assaying procedures are generally repeatable only within 10% in the regions above 1% assays, increasing to 100% as the assay decreases to the .05% range.

Further error is introduced by loss of care in drill holes. This loss will vary from 0% to 10% without any notations of loss being made. Losses in excess of 10% are usually noted and can be treated as missing data.

Accepting the Pearson Type IV curve as a functional representation of the log of the ore distribution, one can now generate an upper bound for the MSPW. Using 0.30% copper as an ore cutoff grade, Figure 4.5 (a tabulation of the ore distribution function in increments of .01 (on the log scale) shows that 57.65% of the material in the mine is below the cutoff grade; hence, there remains 42.35% to be mined over 20

COMPARISON OF LOG TRANSFORMATION OF SMOOTHED ASSAY DATA AND PEARSON TYPE IV CURVE

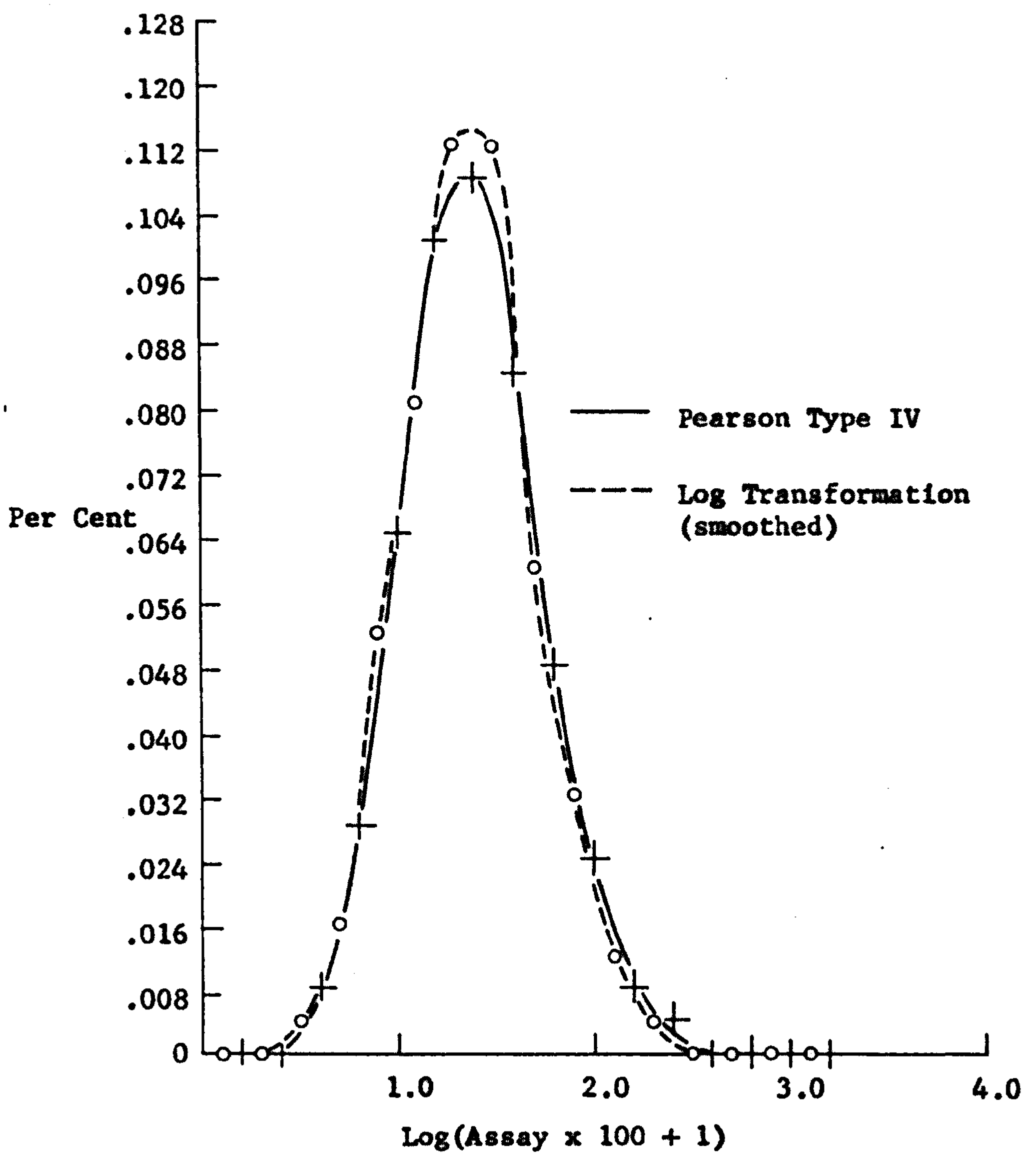


Fig. 4.4

FIGURE 4.5

TABULATION OF CUMULATIVE DISTRIBUTION PEARSON TYPE IV CURVE

PERCENTAGES AND APPROPRIATE ASSAY VALUES

Class Mark	Cl.Mk. Assay	CDF(X) (Percent)	Class Mark	Cl.Mk. Assay	CDF(X) (Percent)
0.005	0.00	0.0000	1.125	0.12	0.2169
0.205	0.01	0.0010	1.135	0.13	0.2252
0.425	0.02	0.0050	1.145	0.13	0.2337
0.535	0.02	0.0102	1.155	0.13	0.2423
0.595	0.03	0.0148	1.165	0.14	0.2512
0.645	0.03	0.0200	1.175	0.14	0.2602
0.685	0.04	0.0253	1.185	0.14	0.2694
0.715	0.04	0.0301	1.195	0.15	0.2787
0.745	0.05	0.0356	1.205	0.15	0.2882
0.765	0.05	0.0398	1.215	0.15	0.2979
0.785	0.05	0.0444	1.225	0.16	0.3077
0.805	0.05	0.0495	1.235	0.16	0.3176
0.825	0.06	0.0550	1.245	0.17	0.3277
0.845	0.06	0.0611	1.255	0.17	0.3379
0.855	0.06	0.0644	1.265	0.17	0.3482
0.875	0.06	0.0713	1.275	0.18	0.3586
0.885	0.07	0.0750	1.285	0.18	0.3692
0.895	0.07	0.0788	1.295	0.19	0.3798
0.905	0.07	0.0828	1.305	0.19	0.3905
0.915	0.07	0.0870	1.315	0.20	0.4013
0.925	0.07	0.0913	1.325	0.20	0.4121
0.935	0.08	0.0959	1.335	0.21	0.4230
0.945	0.08	0.1005	1.345	0.21	0.4340
0.955	0.08	0.1054	1.355	0.22	0.4450
0.965	0.08	0.1104	1.365	0.22	0.4560
0.975	0.08	0.1156	1.375	0.23	0.4671
0.985	0.09	0.1210	1.385	0.23	0.4781
0.995	0.09	0.1266	1.395	0.24	0.4892
1.005	0.09	0.1324	1.405	0.24	0.5002
1.015	0.09	0.1384	1.415	0.25	0.5112
1.025	0.10	0.1445	1.425	0.26	0.5222
1.035	0.10	0.1509	1.435	0.26	0.5332
1.045	0.10	0.1574	1.445	0.27	0.5441
1.055	0.10	0.1642	1.455	0.28	0.5550
1.065	0.11	0.1711	1.465	0.28	0.5658
1.075	0.11	0.1783	1.475	0.29	0.5765
1.085	0.11	0.1856	1.485	0.30	0.5872
1.095	0.11	0.1932	1.495	0.30	0.5978
1.105	0.12	0.2009	1.505	0.31	0.6082
1.115	0.12	0.2088	1.515	0.32	0.6186

Figure 4.5--Continued

Class Mark	Cl.Mk. Assay	CDF(X) (Percent)	Class Mark	Cl.Mk. Assay	CDF(X) (Percent)
1.525	0.32	0.6288	1.815	0.64	0.8589
1.535	0.33	0.6390	1.825	0.66	0.8642
1.545	0.34	0.6490	1.835	0.67	0.8694
1.555	0.35	0.6589	1.845	0.69	0.8745
1.565	0.36	0.6686	1.855	0.71	0.8793
1.575	0.37	0.6782	1.865	0.72	0.8841
1.585	0.37	0.6877	1.875	0.74	0.8886
1.595	0.38	0.6970	1.885	0.76	0.8930
1.605	0.39	0.7062	1.915	0.81	0.9054
1.615	0.40	0.7152	1.925	0.83	0.9093
1.625	0.41	0.7240	1.945	0.87	0.9166
1.635	0.42	0.7327	1.955	0.89	0.9200
1.645	0.43	0.7412	1.975	0.93	0.9266
1.655	0.44	0.7495	1.985	0.96	0.9297
1.665	0.45	0.7577	2.005	1.00	0.9355
1.675	0.46	0.7657	2.015	1.03	0.9383
1.685	0.47	0.7735	2.045	1.10	0.9459
1.695	0.49	0.7811	2.055	1.13	0.9483
1.705	0.50	0.7885	2.085	1.21	0.9548
1.715	0.51	0.7958	2.105	1.26	0.9587
1.725	0.52	0.8029	2.145	1.39	0.9656
1.735	0.53	0.8098	2.175	1.49	0.9701
1.745	0.55	0.8166	2.215	1.63	0.9752
1.755	0.56	0.8231	2.255	1.79	0.9795
1.765	0.57	0.8295	2.315	2.06	0.9847
1.775	0.59	0.8357	2.405	2.53	0.9902
1.785	0.60	0.8418	2.535	3.42	0.9950
1.795	0.61	0.8476	2.995	9.88	1.0000
1.805	0.63	0.8533			

years, or 2.12% per year. If this entire orebody represents 1.0×10^8 tons, this indicates an annual production of 2.12×10^6 tons per year.

If f is the PDF representing the distribution of ore grades within the mine, then

$$\int_{x_i}^{x_j} f(\tau)\, d\tau$$ will be the percentage of material in the mine

between grades of x_i and x_j. If the highest grade is to be mined first, then there exists an x_1, such that

$$\int_{x_1}^{100} f(\tau)\, d\tau = .0212,$$

i.e., all the material in the mine of a grade greater than, or equal to, x_1 will equal one year's ore production. Similarly, there exists an x_i, such that

$$\int_{x_i}^{x_{i+1}} f(\tau)\, d\tau = .0212, \text{ where } x_{i+1} > x_i,$$

x_i is the minimum grade, x_{i+1} is the maximum grade and all material such that $x_i \leq$ grade mined $< x_{i+1}$ will be mined during the i^{th} year. There also exists an $\bar{x}_i$ where $x_i \leq \bar{x}_i < x_{i+1}$ and

$$\int_{x_i}^{\bar{x}_i} f(\tau)\, d\tau = .0106 \quad .$$

Here, $\bar{x}_i$ will be the average grade mined during the i^{th} year. The assay corresponding to the 1.0000 - i x .0121 value in the cumulative value column of Figure 4.5 will give the average assay mined during the i^{th} year; i.e., 2.39% will be the average grade for the first year, 1.44% for the second year, etc.

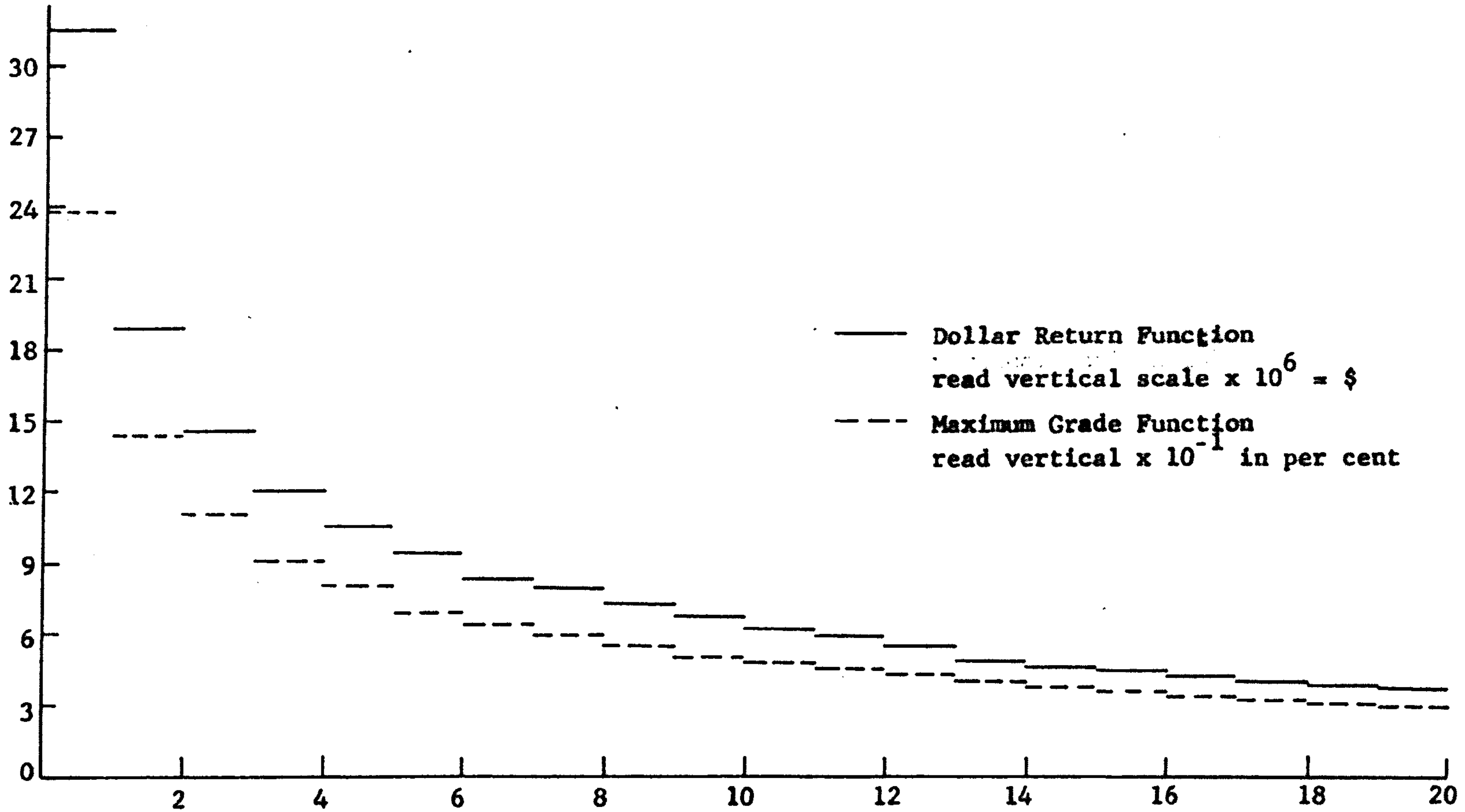
Figure 4.6 is a graph of this grade change by years and also of the dollar value of the ore mined each year.

Strictly as an example of its use, the dollar return function determined by the foregoing schedule was input to the MSPW program. This indicates a rate of return of 21.2% which constitutes an upper bound and the rate of return possible from this orebody, for a given investment schedule. How closely this bound can be approached if determined by the spatial distribution of the ore.

A mining plan was previously defined as $s = \{i_1, i_2, \ldots\ldots i_n\}$ $i = 1, 2, 3, \ldots\ldots n$ where i is the integer representation of a "suitably sized" block into which the piece of mining real estate was subdivided. Before any optimal mining sequence can be determined, the assay function A and the density function D must be developed.

Using 10,286 actual drill-hole assays from 68 holes, an attempt was made to develop a function capable of predicting assay grade for any given set of three-dimensional coordinates, within the drilling limits. Using a stepwise multiple regression technique described by Efroymson (10) and programmed originally by Richard H. Giering, a series of 37 computer runs failed to develop any function whose correlation to the drill hole assays was greater than .505%, a result that is very possibly a random occurrence.

MAXIMUM GRADE AND DOLLAR RETURN FUNCTIONS



Time,years

Fig. 4.6

From these results it is obvious that a blind searching for functions to represent ore distribution as a function of spatial location only, while possible, is not a practical method of approach. An approach which would seem to offer more hope would be the development of a successive series of functions; first, to describe the various rock types present, then their location and thickness. To each of these functions would be attached a probability distribution function describing the distribution of ore grade within that rock type. It might then prove practical to develop a spatial distribution of ore within each rock type and, in this manner, be able to evaluate the entire ore body.

In following such a program what would actually be done is that the complex process of paragenesis would be broken down into several, less complex sub-processes, which theoretically might be easier to model, and then the sub-models assembled into an overall model of the entire orebody. The investigation of this possibility would, judging from the length of time required in the development of the ore grade distribution function and the unsuccessful attempt at developing spatial distribution model, consume several years of research time.

The problem of the development of the function D, the spatial density distribution, is a subproblem of the spatial ore distribution problem. Instead of attempting to predict the percent of mineral present at a point in 3-dimensional space, it is desired to predict the density of the material present at that same point. The density of the material at any point is a function of the type of rock present at that point (assuming that each rock type has a constant density).

It can then be reasonably assumed that any technique which successfully predicts the spatial ore distribution of an orebody can also be used to develop a function that will predict the rock type and, hence, the density, at any point in the orebody.

CHAPTER 5

CONCLUSIONS

This investigation began as an attempt to determine the possibility of optimizing an open-pit mine system, starting with the data provided by the original drill hole assays. While the mine and the directly associated facilities were the system originally under consideration, it immediately became obvious that this point of view was too narrow. The mine system as conceived is actually a subsystem of a larger system which is the corporate entity of which the mine is only a part. As a result of this relationship, there are constraints immediately imposed on the design of the mine system. The financial policy of the superior system must govern the financial decisions of the mine system. The rate of return considered acceptable, discount rates on future earnings, and accrual rates on future capital expenditures are all policies which must be made at the corporate level. The MSPW function was developed in Chapter 2 to give the mine system designer a standard method of comparing several system designs within the constraints of corporate policy. In a detailed optimization program, the corporate labor policy would also be a constraint on the design, but this study does not delve this deeply into the details of such an optimization.

Analysis of the mine system shows that one essential piece of information required by the designer is the amount and grade of ore present in the orebody. The calculation of these figures was automated through the development of the computer program described in Chapter 3. An additional benefit derived from the use of this program is an approximate final-pit configuration for any given set of operating parameters; namely, the cutoff grade, cost of mining waste, cost of mining ore and dollar value of the economic mineral of the orebody.

When the designer has available the total tonnage and grade figures and also a function which will evaluate any design, a logical next step would be to the development of a theoretical maximum return possible from this orebody. To accomplish this it is necessary that the ore grade distribution be representable as a continuous function. In Chapter 3 it was shown that, in the particular case being studied, a Pearson Type IV curve approximated the ore grade distribution function sufficiently closely to permit the establishment of a maximum possible return. Only in the event that this maximum were greater than the minimum acceptable return would the design effort continue in a real-life situation.

Analysis of the characteristics of the MSPW function has shown that the factor of primary importance in the optimization of the mine system is the sequence in which the ore is removed from the ground. This fact points out the second major constraint that is present in this system - that of geology. In order to model the mine system, preparatory to optimization, it is necessary to model the spatial distribution of ore in the ground, a function called A.

Any attempt theoretically to model A must take cognizance of two vital facts. First is the fact that the theories of the paragenesis of the ore must remain exactly that - theories which are not subject to experimental verification. These theories cover a time lapse of millions of years and assume conditions which currently cannot be duplicated in a laboratory. Secondly, the presence, or absence, of the parameters of paragenitic theory can only be implied today by the presence, or absence, of 2nd, 3rd, or n^{th} generation effects of these parameters. Paragenitic theory (11) of the formation of deposits of geothermal origin* (such as copper, lead, zinc, etc.) require the presence of four basic parameters; a source of minerals, usually an igneous intrusive from the interior of the earth; an extremely high pressure which requires that the action take place tens of thousands of feet below the surface of the ground; a very high temperature which decreases in the direction of the ground surface and will allow a series of crystallization cycles as the intrusive solution moves upward and cools; lastly, the presence of a host rock which is favorable to the deposition of minerals, both from a structural and chemical point of view. If all these features are present, the first step in a long sequence takes place. The second step requires that this region be uplifted and the overlying rock eroded away, permitting the start of a third step. As erosion progresses and the mineral deposit gets closer to the surface, ground water, heated by the still-warm host rock, takes much of the ore mineral into solution and redeposits it, in a different mineral form and in a less concentrated manner. If all these sequences have taken place, and the deposit has been discovered, all the basic ingredients of a mine are present.

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*Geothermal Origin - a deposit whose origin was an igneous intrusive starting below the earth mantle and forcing its way upward into sedimentary rocks where cooling action caused the crystallization of metal bearing minerals, often in the form of sulphides and silicates.

Any mathematical model which will model this process must be capable of modeling the processes of physical chemistry, tectonic action, erosion, ground water action, all over a time scale of millions of years. This model must also be capable of micro analysis, while the theories on which it must be cased are micro theories. The modeling of this process is, today, beyond the state of the art.

The need here is to create methods of expressing mathematically the state of a micro portion of this system at a specific instant in time. All of the theoretical factors, taken individually, should be capable of being modeled in three-dimensional space in time. What needs representations, then, is the current state of the interactions of these functional parameters at a given time t. It seems reasonable to be able to expect this state to be expressible as a single function defined on Euclidean three space. The function would certainly be complex, but it also should exist and should be of similar form for orebodies of similar geologic origin. The U. S. Bureau of Mines, Kennecott Copper Mining Company and the Banner Mining Company have all conducted or sponsored research in this field. None has been successful on a large scale deposit, but R. Hewlett (6) has developed functions for two orebodies, one in Alaska and one in Canada, which have a statistical correlation of 95% between drill hole assays and predicted grade. These functions were developed from widely spaced (500' to 1000') exploration drill holes and how well these functions will predict the grades between the original drill holes can only be determined by further drilling, which is currently in progress.

While the mine design program may speed up a design selection, and an evaluation technique provides a measure of improvement, neither provides assurance that the actual optimum design has been achieved or even approached. In practical situations, it must be assumed that the optimum will at least be approached. The experience of the engineer selecting the sequences to be evaluated, developed by years of working with ore deposits and designing many pits, is reflected in the sequences he chooses. Experience has shown that these men are seldom more than a few percentage points from an optimal selection. It is these few percentage points that any analytic system must be able to detect.

Creation of an usable model is, however, not impossible. By introducing as parameters such factors as: rock type distribution, chemical composition of rock type, rock type age, and other physical characteristics of the different rock types, the use of alternate coordinate systems, and the use of smoothing functions to help eliminate variance due to assaying and core recovery errors, it should be possible to develop a very workable model.

Optimization of the mine design, or even of the mine system consisting of mine, concentrator, smelter, and transportation system, is not an end in itself. Usually such a mine system is part of an organization which also owns manufacturing facilities which utilize the output of the mine system. If it is possible to analyze the operation of the mine system itself, this analysis can then be introduced into the corporate structure and the operation of the mine system can be optimized as a function of a corporate policy where the type and grade of material produced is that which will most profitably allow the manufacturing facilities to utilize this output.

The analysis of a second level system, of which the mine system is only one of several first level systems, is the desirable next step. The implication of further levels, representing the corporate relation to the entire industry, and of specific industries to the national economic system is inherent.

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