

ACTIVE RC SYNTHESIS USING RIGHT  
HAND PLANE PHANTOM ZEROS

by

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## ABSTRACT

A Twin-T circuit using all equal capacitors is described and the required design information is given for a wide range of right half plane zeros. Using this equal capacitor Twin-T as the feedback network around a negative gain amplifier, a bandpass filter is designed. This active RC filter is analyzed in greater detail with respect to passive and active element sensitivities. The effects of non-ideal amplifier and generator are also analyzed and explained.

The circuit's advantages are its moderate gain amplifier with reasonable Q sensitivities. This circuit should find applications at IF frequencies where other active RC filters are not practical.



## CHAPTER 1

### INTRODUCTION

In recent years much work has gone into the development of an inductorless bandpass filter. Many designs have been examined, the most common using the work of Sallen and Key (1955). The desire for high Q, low gain and low sensitivity has been the goal.

One circuit that appears quite reasonable is a modified Twin-T used as the feedback network in combination with a negative gain amplifier, as described by Kerwin and Shaffer (1968). This circuit achieves high Q with reasonably low gains and the amplifier sensitivities are reasonable. The conventional Twin-T requires high amplifier gains and shows unreasonably high sensitivities to the passive elements. Another drawback to the Twin-T is the three unequal capacitors required. For a practical design, it is much better from the standpoint of construction to have all the capacitors equal.

The solution to the latter problem would be achieved if an equal capacitor Twin-T could be designed, but no one had ever shown that an equal capacitor Twin-T was possible except for  $b = k = \alpha = 1$  (Fig. 1), if indeed it was. I investigated the problem and found that the equal capacitor Twin-T was possible. This work is detailed in Chapter 2.

Using the equal capacitor Twin-T, the bandpass amplifier was then analyzed. The Q sensitivity to each element was found for various Q's. The details of this investigation are given in Chapter 2.

To complete the study of the bandpass amplifier, the effect of nonideal amplifier input and output impedances was analyzed, as well as the effect of generator source resistances. The results of these studies are given in Chapters 4 and 5.

To verify the theory developed in Chapters 2 through 5, an equal capacitor bandpass amplifier was built. The details of this and the results of the data taken are detailed in Chapter 6.

## CHAPTER 2

### THE EQUAL CAPACITOR TWIN-T

To develop an equal capacitor Twin-T, a familiarity with the normal Twin-T is required. The usual form of the Twin-T is shown at the top of Fig. 1. The transfer function is

$$T(s) = \frac{s^2 + \alpha s + 1}{s^2 + \beta s + 1} \quad (1)$$

where  $\alpha = b + \frac{b}{k} - 1$  (2)

$$\beta = b + \frac{b}{k} + \frac{1}{k} + \frac{1}{b} \quad (3)$$

The definition of  $b$  and  $k$  are shown in the figure. The equal capacitor Twin-T must have a transfer function of the same form as Eq. (1). Since the circuit produces a third order transfer function, one pole and one zero must cancel to give Eq. (1). With this in mind, we can rewrite the transfer function as

$$T(s) = \frac{s^2 + \alpha s + 1}{s^2 + \beta s + 1} \cdot \frac{s + X}{s + X} \quad (4)$$

or

$$T(s) = \frac{s^3 + s^2(X + \alpha) + s(X\alpha + 1) + X}{s^3 + s^2(X + \beta) + s(X\beta + 1) + X} \quad (5)$$

where  $X$  is the cancellation frequency. Using straight forward nodal analysis on the completely general Twin-T circuit, shown at the bottom of Fig. 1, the transfer function is found to be

$$\begin{aligned}
 T(s) = & \frac{s^3 + s^2 \left( \frac{1}{R_1 C_3} + \frac{1}{R_2 C_3} \right) + s \left( \frac{1}{R_1 R_2 C_1 C_3} + \frac{1}{R_1 R_2 C_2 C_3} \right) + \frac{1}{R_1 R_2 R_3 C_1 C_2 C_3}}{s^3 + s^2 \left( \frac{1}{R_1 C_3} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_3} + \frac{1}{R_3 C_1} \right)} \\
 & + s \left( \frac{1}{R_1 R_2 C_1 C_3} + \frac{1}{R_1 R_2 C_2 C_3} + \frac{1}{R_1 R_3 C_1 C_3} + \frac{1}{R_2 R_3 C_1 C_2} + \frac{1}{R_2 R_3 C_1 C_3} \right) \\
 & + \frac{1}{R_1 R_2 R_3 C_1 C_2 C_3} \quad (6)
 \end{aligned}$$

Now the impedance level of the circuit must be set, as well as setting all the capacitors equal. This is done by setting  $C_1 = C_2 = C_3 = 1$  farad. Now we can equate the coefficients of Eqs. (5) and (6), rearranged slightly, and develop the following set of equations.

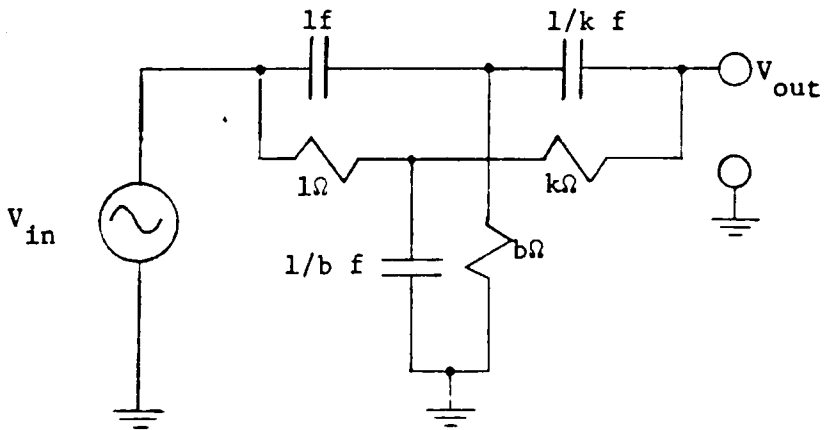
$$R_1 R_2 (X + \alpha) - R_1 - R_2 = 0 \quad (7)$$

$$R_1 R_2 (X\alpha + 1) - 2 = 0 \quad (8)$$

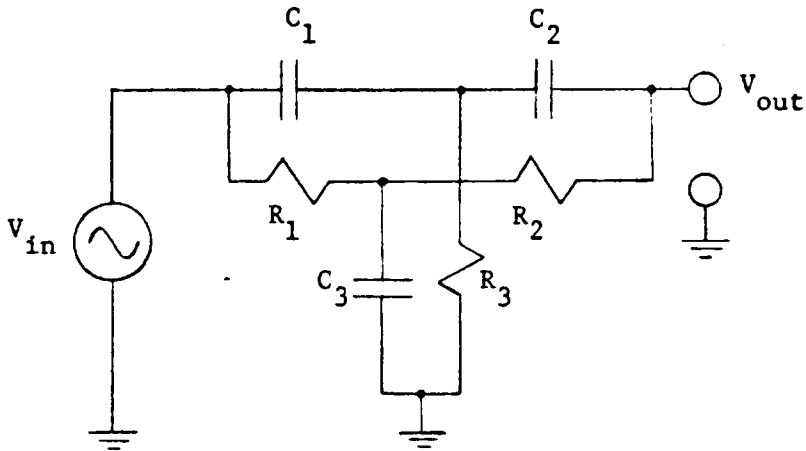
$$R_1 R_2 R_3 X - 1 = 0 \quad (9)$$

$$R_1 R_2 R_2 (X + \beta) - R_1 R_2 - R_2 R_3 - 3R_1 R_3 = 0 \quad (10)$$

$$R_1 R_2 R_3 (X\beta + 1) - 2R_1 - R_2 - 2R_3 = 0 \quad (11)$$



(a) Classic Twin-T



(b) General Twin-T

Figure 1 The Twin-T Circuit

This set of five nonlinear equations in six unknowns must be solved in order to find the resistor values for the Twin-T. The position of the zeros is the most important parameter, so for a given  $\alpha$ , there will be a set of values for  $R_1$   $R_2$   $R_3$   $\beta$  and  $X$ .

To solve Eq. (7) through (11) for various  $\alpha$ 's, the digital computer was used. A software optimization package by Huelsman (1968), named GOSPEL, was used. The results of these calculations are presented in Table 1 and Fig. 2.

Comparing the results with the unequal capacitor Twin-T shows that the value of  $\beta$  is no longer an independent parameter. As  $\alpha$  becomes more negative,  $\beta$  becomes more positive. It is interesting that the cancellation frequency,  $X$ , is constant, as is  $R_2$ , for  $\alpha$  more negative than  $-.025$ .

When the Twin-T is used as a notch filter, that is with  $\alpha = 0$  ( $j\omega$  axis zeros), the most common configuration used is Fig. 1, with  $b = .5$  and  $k = 1$ . This gives  $\beta = 4.0$ , compared to the equal capacitor Twin-T just found with  $\beta = 4.344$ . The frequency response of both circuits is plotted in Fig. 3 for comparison.

One important consideration in building a filter is what happens when the values of the elements change a little. This is usually expressed as a sensitivity. For the notch filter, however, it is very hard to find an expression of sensitivity that has real meaning. For that reason, another way of showing the affects of small element changes was chosen. Figs. 4 and 5 show the frequency response of the equal and unequal capacitor Twin-T circuits with and without a 1% increase in any one element. From these graphs it appears that

Table 1

Design Information for the Equal Capacitor Twin-T

$\alpha$	$R_1$	$R_2$	$R_3$	$\beta$	X
0.000	1.509	1.325	0.3528	4.344	1.417
-.010	1.480	1.370	0.3484	4.320	1.415
-.025	1.466	1.414	0.3410	4.321	1.414
-.050	1.522	1.414	0.3286	4.408	1.414
-.075	1.582	1.414	0.3160	4.503	1.414
-.100	1.647	1.414	0.3036	4.608	1.414
-.125	1.718	1.414	0.2910	4.725	1.414
-.150	1.795	1.414	0.2786	4.854	1.414
-.175	1.879	1.414	0.2660	4.998	1.414
-.190	1.934	1.414	0.2586	5.092	1.414
-.200	1.972	1.414	0.2536	5.158	1.414
-.210	2.012	1.414	0.2486	5.227	1.414
-.220	2.053	1.414	0.2436	5.300	1.414
-.225	2.074	1.414	0.2410	5.338	1.414
-.230	2.096	1.414	0.2386	5.376	1.414
-.240	2.141	1.414	0.2336	5.456	1.414
-.250	2.188	1.414	0.2286	5.539	1.414
-.260	2.237	1.414	0.2236	5.627	1.414
-.270	2.288	1.414	0.2186	5.720	1.414
-.275	2.314	1.414	0.2160	5.768	1.414

Table 1, Continued

$\alpha$	$R_1$	$R_2$	$R_3$	$\beta$	$X$
-.280	2.341	1.414	0.2136	5.817	1.414
-.290	2.397	1.414	0.2086	5.919	1.414
-.300	2.456	1.414	0.2036	6.027	1.414
-.310	2.518	1.414	0.1986	6.141	1.414
-.320	2.583	1.414	0.1936	6.261	1.414
-.325	2.617	1.414	0.1910	6.323	1.414
-.330	2.652	1.414	0.1886	6.388	1.414
-.340	2.724	1.414	0.1836	6.522	1.414
-.350	2.800	1.414	0.1786	6.665	1.414
-.360	2.881	1.414	0.1736	6.816	1.414
-.370	2.966	1.414	0.1686	6.977	1.414
-.375	3.011	1.414	0.1660	7.061	1.414
-.380	3.057	1.414	0.1636	7.148	1.414
-.390	3.154	1.414	0.1586	7.331	1.414
-.400	3.256	1.414	0.1536	7.527	1.414
-.410	3.366	1.414	0.1486	7.736	1.414
-.420	3.483	1.414	0.1436	7.960	1.414
-.425	3.545	1.414	0.1410	8.079	1.414
-.430	3.609	1.414	0.1386	8.202	1.414
-.450	3.889	1.414	0.1286	8.743	1.414
-.475	4.308	1.414	0.1160	9.556	1.414
-.500	4.828	1.414	0.1036	10.57	1.414



Table 1, Continued

$\alpha$	$R_1$	$R_2$	$R_3$	$\beta$	$X$
-0.525	5.491	1.414	0.09105	11.87	1.414
-0.550	6.365	1.414	0.07855	13.59	1.414
-0.575	7.570	1.414	0.06605	15.98	1.414
-0.600	9.336	1.414	0.05355	19.49	1.414

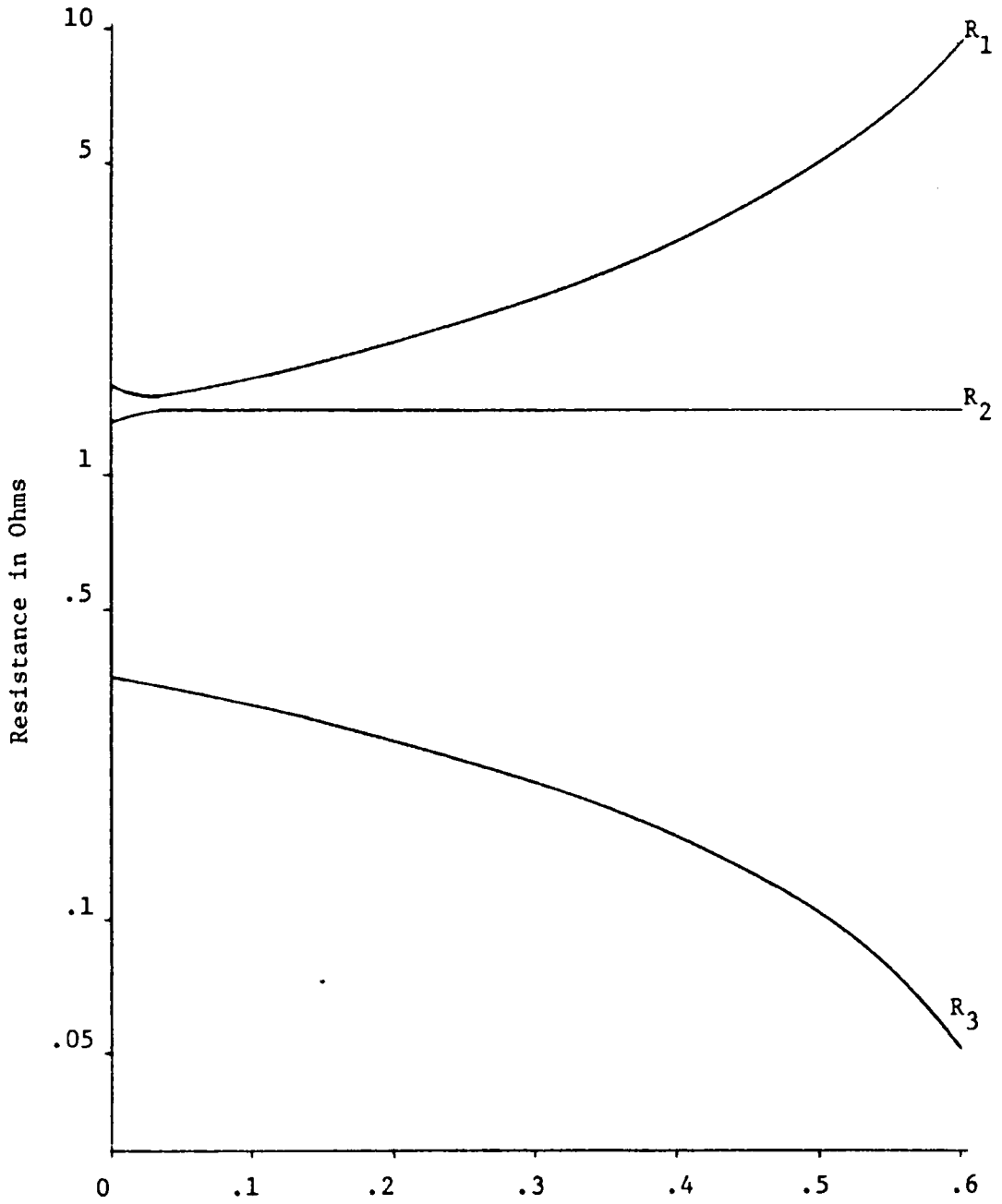


Figure 2 Equal Capacitor Twin-T Resistor Values vs.  $\alpha$

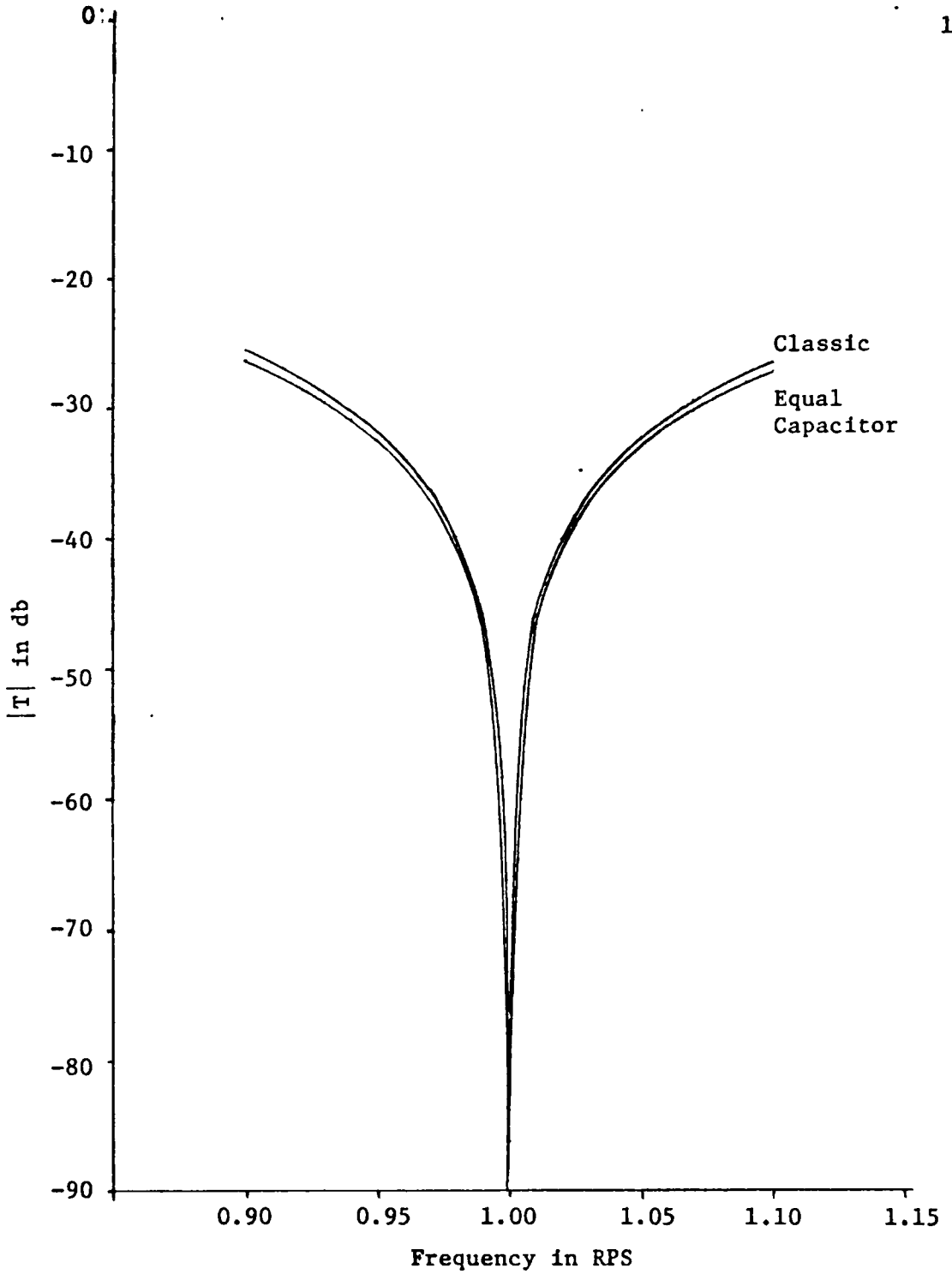


Figure 3 Frequency Response Comparison of Classical Twin-T and Equal Capacitor Twin-T

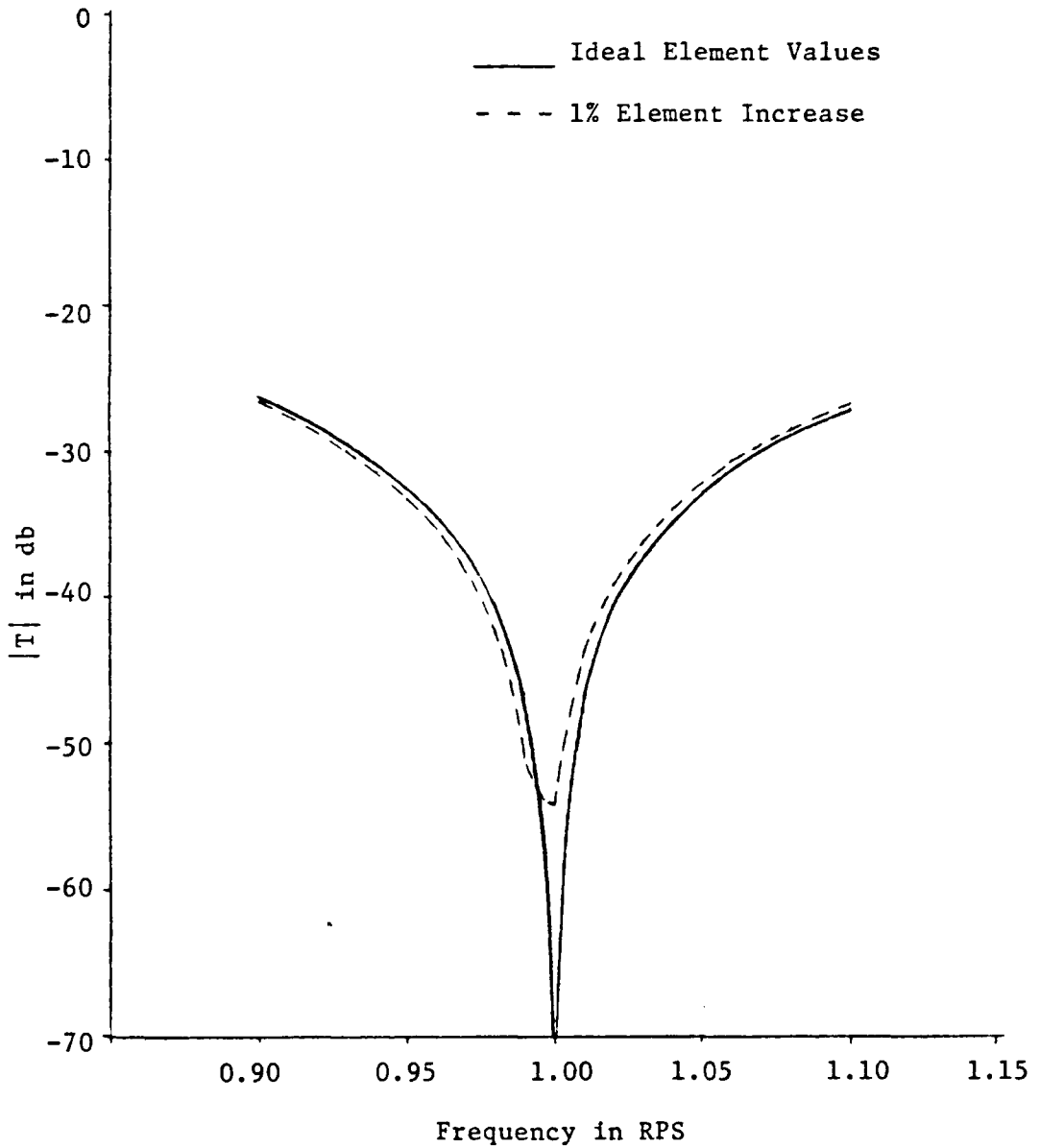


Figure 4 Effect of a 1% Increase in Any one Passive Element of the Equal Capacitor Twin-T (all elements affect the response equally)

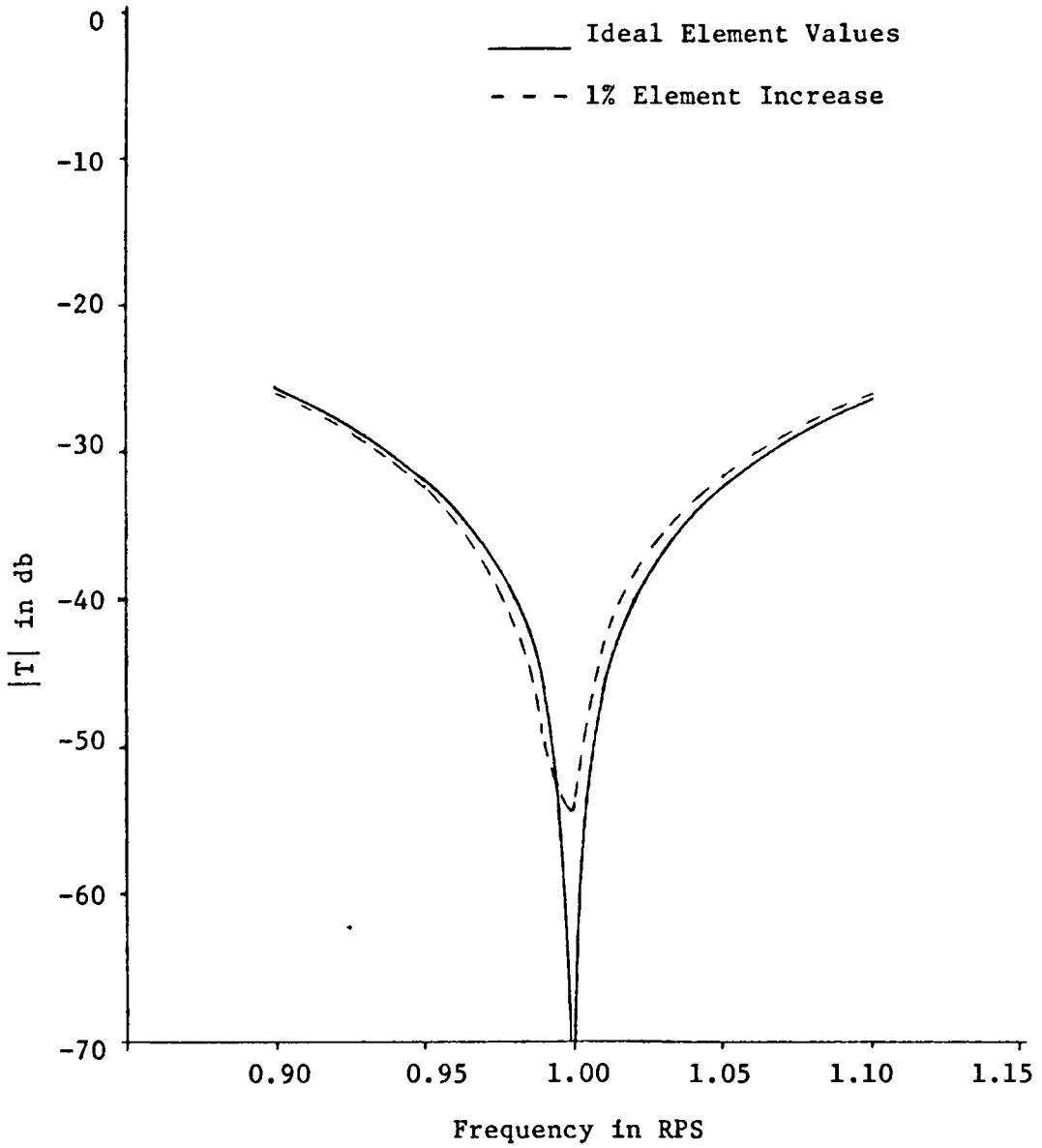


Figure 5 Effect of a 1% Increase in Any One Passive Element of the Classic Twin-T,  $b = 1/2$  and  $k = 1$  (all elements affect the response equally)

a small change in any one element will do the same thing to either circuit.

In summary, the equal capacitor Twin-T can replace the conventional Twin-T except when  $\beta$  must be kept very low.

## CHAPTER 3

### THE BANDPASS FILTER

A negative gain voltage controlled voltage source (VCVS) with a Twin-T as the feedback network, as shown at the top of Fig. 6, will give a pair of complex poles, a zero at the origin and at infinity (Kerwin and Shaffer, 1968). This circuit is therefore a true bandpass filter. The root locus of the circuit, shown in Fig. 6, is circular with a radius of 1 and the poles terminate on the zeros of transmission (phantom zeros). So, if the zeros of the Twin-T are on the  $j\omega$  axis, ( $\alpha = 0$ ), the circuit is stable at any gain. However, the gain required for high Q is also very high. It has been shown by Kerwin and Shaffer (1968) that putting the phantom zeros into the right half plane reduces the gain required. This can be seen by looking at the transfer function of the circuit

$$T(s) = \left(\frac{-K}{1+K}\right) \frac{(\beta - \alpha)s}{s^2 + \left(\frac{\beta + \alpha K}{1+K}\right)s + 1} \quad (11)$$

$\alpha$  and  $\beta$  are the same as in the Twin-T and K is the magnitude of the amplifier gain. The Q of the poles in Eq. (11) is

$$Q = \frac{1+K}{\beta + \alpha K} \quad (12)$$

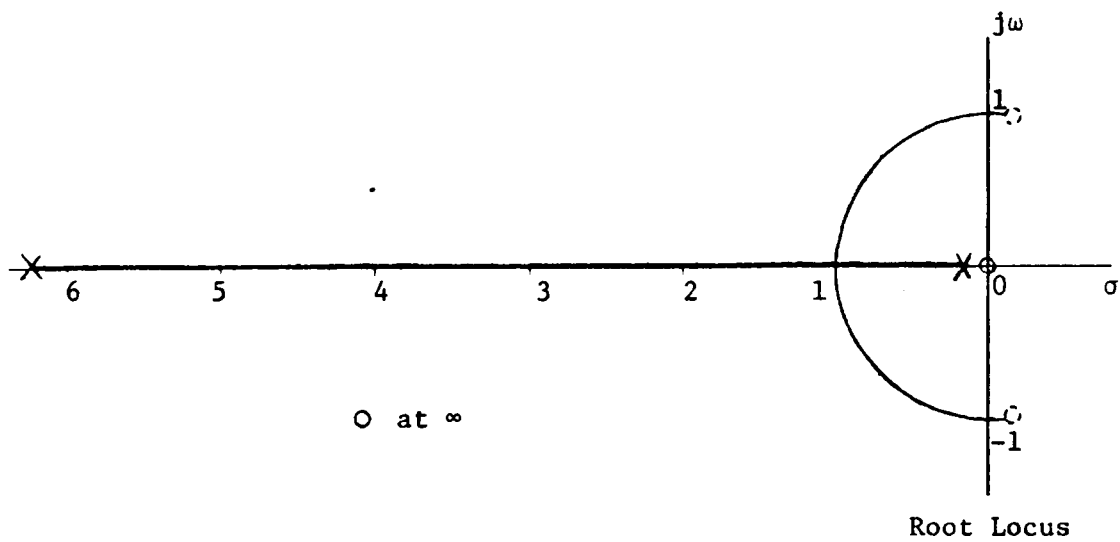
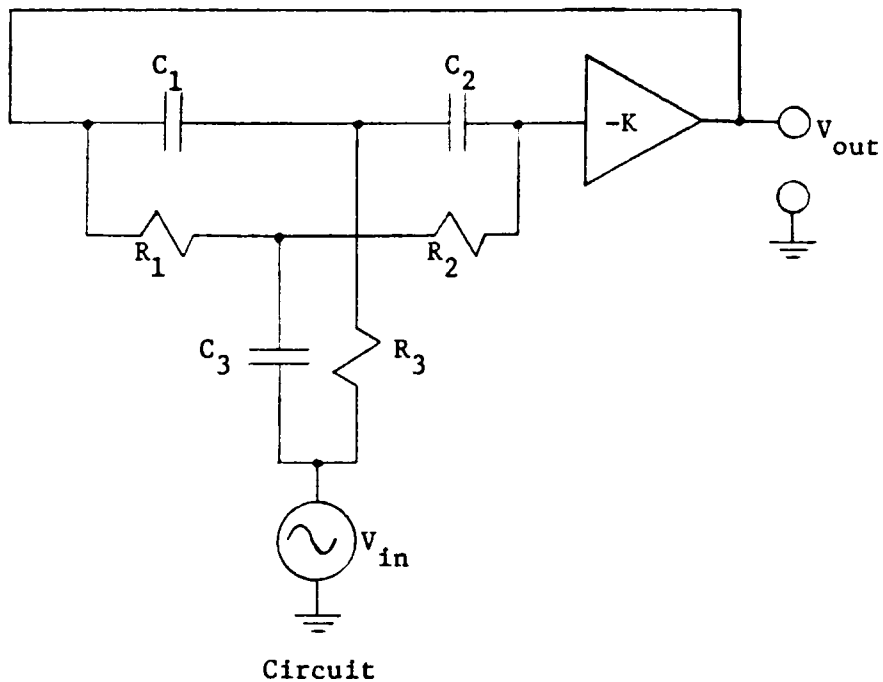


Figure 6 The Bandpass Filter



and therefore the gain magnitude required for a given  $Q$  is

$$K = \frac{\beta Q - 1}{1 - \alpha Q} \quad (13)$$

From this we see that negative  $\alpha$  and small  $\beta$  are the way to reduce  $K$ . The gain at center frequency,  $\omega_0 = 1$  rps, is

$$T(s) \Big|_{s=j1} = \frac{-K(\beta - \alpha)}{\beta + \alpha K} \quad (14)$$

So, from this we see that negative  $\alpha$  also gives a high center frequency gain.

Using Eq. (12) we can find the  $Q$  sensitivity to amplifier gain

$$S_K^Q \triangleq \frac{\partial Q/Q}{\partial K/K} = \frac{K}{Q} \frac{\partial Q}{\partial K} = \frac{K}{1 + K} \left( \frac{\beta - \alpha}{\beta + \alpha K} \right)$$

The  $Q$  sensitivity to gain therefore increases with more negative  $\alpha$  and with increasing  $Q$ . Because of the circular root locus, the center frequency sensitivity to  $K$  is equal to zero for high  $Q$ . Finding the sensitivities to the passive elements is much more difficult because there is no expression for  $Q$  in terms of them. The smallest change in the passive elements can cause  $\alpha$ ,  $\beta$  or  $X$  to change. So, to find  $Q$  sensitivity to passive elements, numerical methods and the digital computer must be used (Feugate cited in Kerwin, 1972).

The first step in this analysis is to find the transfer function in terms of all the passive elements and the amplifier gain. This was

done by simple node analysis and the resulting transfer function of Fig. 6 is

$$T(s) = \frac{-Ks(R_1C_2 + R_1C_3 + R_2C_2 + s(R_1R_2C_2C_3 + R_1R_3C_1C_3 + R_1R_3C_2C_3))}{s^3(1+K)R_1R_2R_3C_1C_2C_3}$$

$$+s^2[(1+K)(R_1R_3C_1C_2 + R_2R_3C_1C_2) + R_1R_2C_2C_3 + R_1R_3C_1C_3 + R_1R_3C_2C_3]$$

$$+s[(1+K)(R_3C_1 + R_3C_2) + R_1C_2 + R_1C_3 + R_2C_2] + (1+K) \quad (15)$$

This is of the general form

$$T(s) = \frac{-K(As^2 + Bs)}{Cs^3 + Ds^2 + Es + F} \quad (16)$$

For steady state analysis we have

$$|T(j\omega)|^2 = \frac{K(A^2\omega^4 + B^2\omega^2)}{\omega^6C^2 + \omega^4(D^2 - 2CE) + \omega^2(E^2 - 2DF) + F^2} \quad (17)$$

It is most convenient to work with Eq. (17), the Magnitude-Squared Function. To find the center frequency, we differentiate

the magnitude-squared function with respect to  $\omega$  and solve for the  $\omega$  that makes it equal to zero.

Or,

$$F(\omega) = \frac{d|T(j\omega)|^2}{d\omega} = \frac{K^2(-2A^2C^2\omega^8 - 4B^2C^2\omega^6) + \omega^4(2A^2E^2 - 4A^2DF - 2B^2D^2 + 4B^2CE) + 4A^2F^2\omega^2 + 2B^2F^2}{\text{DENOMINATOR}} = 0 \quad (18)$$

Solving Eq. (19) for  $\omega_0$ , is easily done using the Newton-Raphson iteration technique. That is,

$$\omega_0 = \omega - \frac{F(\omega)}{F'(\omega)} \quad (19)$$

where  $F(\omega)$  is the function in Eq. (18).

This technique converges rapidly when a good starting point is known, in this case  $\omega_0 \approx 1.00$  rps.

Now that the center frequency,  $\omega_0$ , is known, we can find the squared magnitude at this center frequency. The -3db frequencies occur where

$$|T(j\omega_{-3db})|^2 = \frac{|T(j\omega_0)|^2}{2} \quad (20)$$

Using Eq. (20) and an appropriate search technique, the upper and lower -3db frequencies can be found. Now the effective Q can be defined as

$$Q \triangleq \frac{\omega_0}{\omega_H - \omega_L} \quad (21)$$

where  $\omega_H$  is the upper -3db frequency  
and  $\omega_L$  is the lower -3db frequency.

Finally, the Q sensitivity to any parameter, P, can be found by using

$$S_P^Q \triangleq \frac{\partial Q/Q}{\partial P/P} \approx \frac{\frac{Q - Q_0}{Q_0}}{\frac{P - P_0}{P_0}} \quad (22)$$

where  $P_0$  and  $Q_0$  are the original values. Using this technique the Q sensitivity to  $R_1$ ,  $R_2$ ,  $R_3$ ,  $C_1$ ,  $C_2$ ,  $C_3$  and K for Q equal to 5, 10, 30, 50 and 70 were found. These results are plotted in Figs. 7 through 11. The plots are not as smooth as expected and this is due to the numerical analysis method of finding the Q sensitivities.

It should be noticed that more negative  $\alpha$  causes  $S_K^Q$  to increase but that passive element sensitivity is not as simply described. It is interesting that  $S_{R_1}^Q$ ,  $S_{R_3}^Q$ , and  $S_{C_1}^Q$  all go through zero. Depending on a particular application, any one of these three points might be the most advantageous operating point. Since capacitor sensitivity is often the most critical, Fig. 12 shows the  $\alpha$  needed to have  $S_{C_1}^Q = 0$  for a given Q. It is fortunate that for all Q greater than 20,  $\alpha \approx -.35$  gives  $S_{C_1}^Q = 0$ . For this reason most circuits can be designed with  $\alpha = -.35$ . It is also convenient that  $S_{R_1}^Q$  and  $S_{R_3}^Q$  go through zero at  $\alpha \approx -.25$  and  $\alpha \approx -.45$ , respectively, for all Q greater than 20.

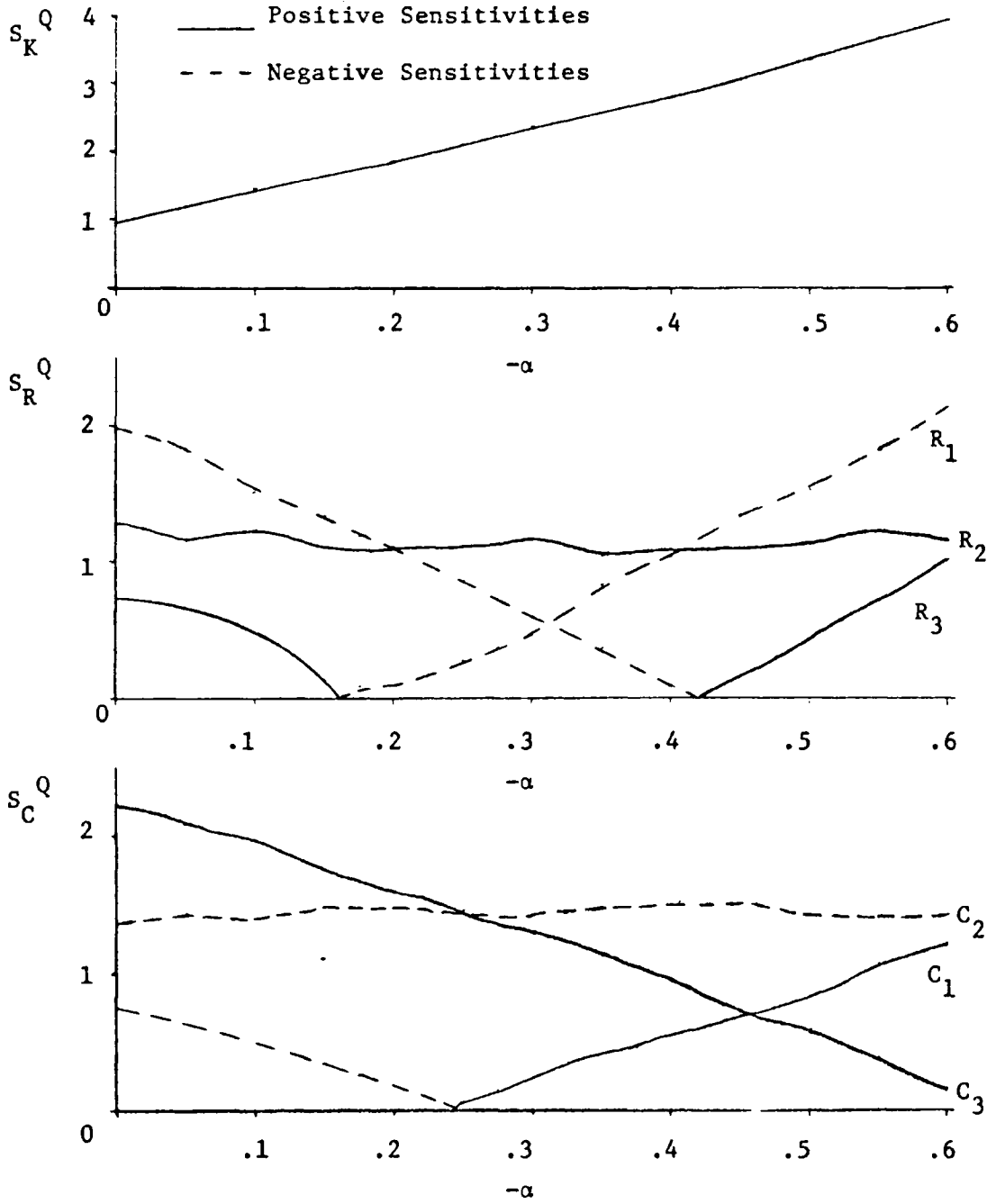


Figure 7 Q Sensitivity vs.  $\alpha$  for  $Q = 5.0$

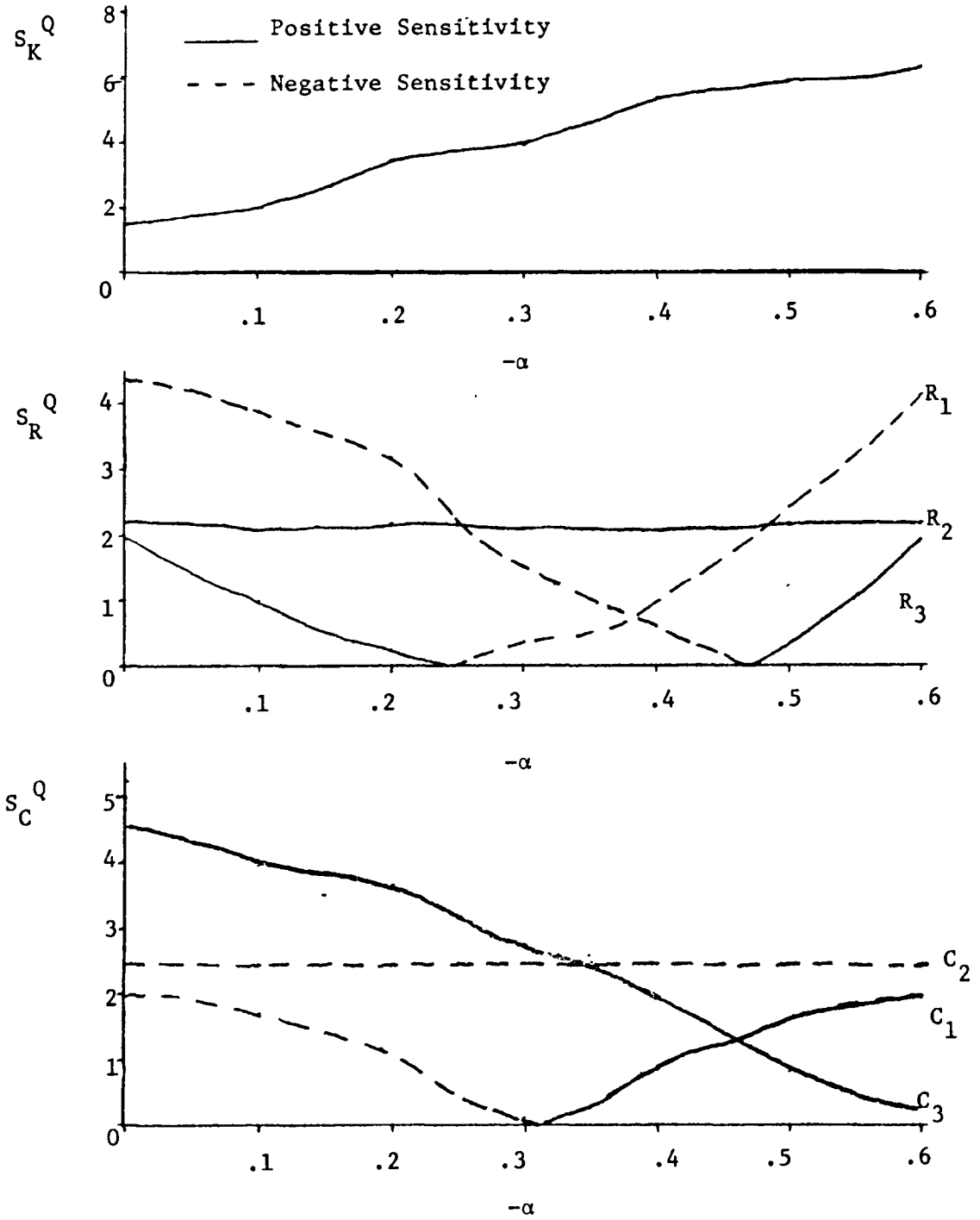


Figure 8 Q Sensitivity vs.  $\alpha$ ,  $Q = 10$

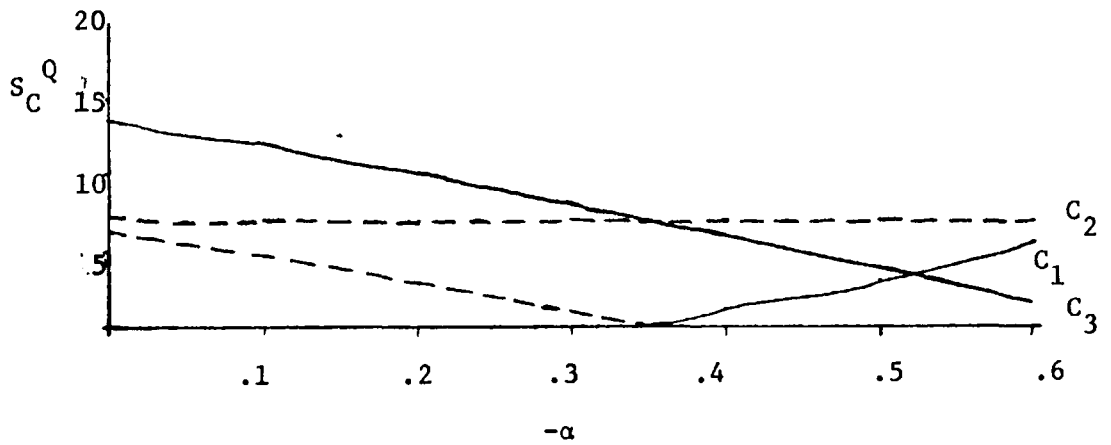
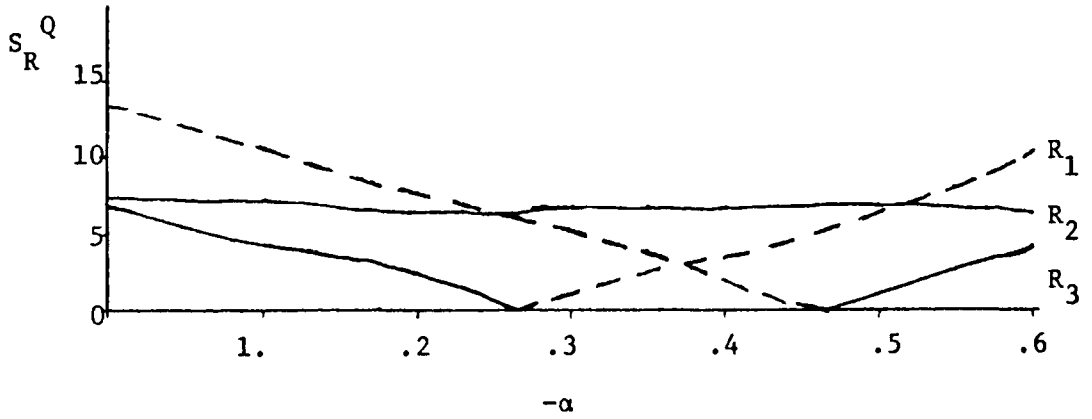
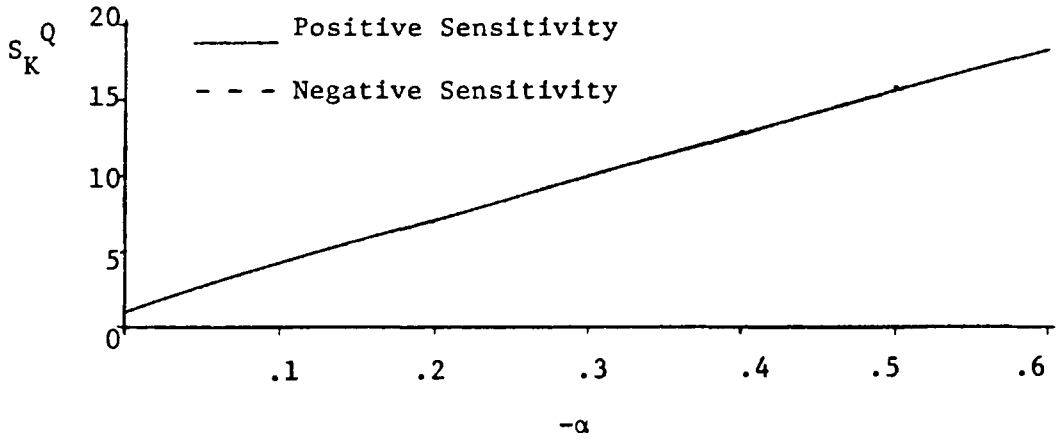


Figure 9 Q Sensitivity vs.  $\alpha$ ,  $Q = 30$

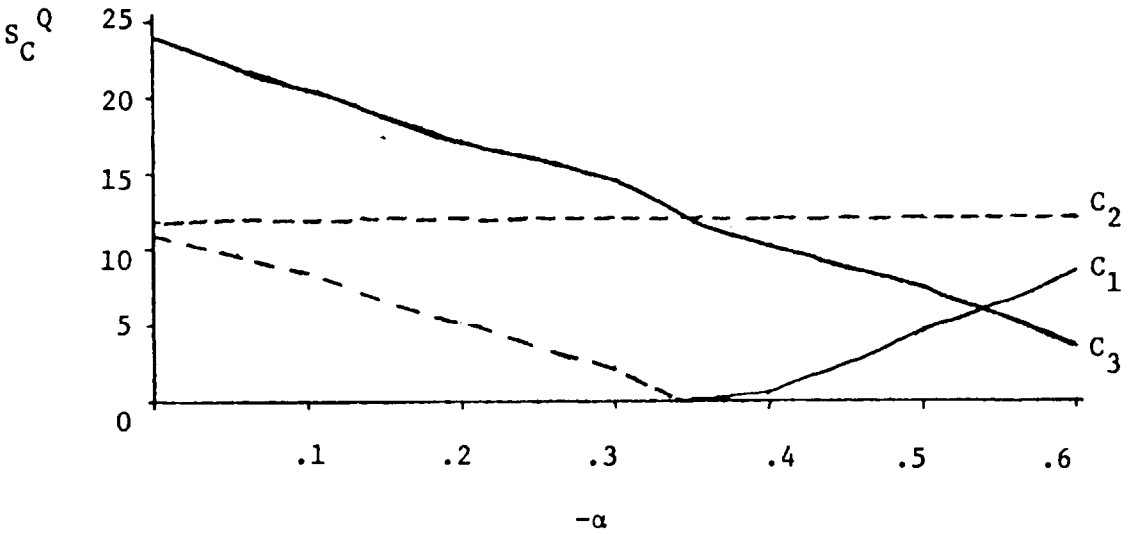
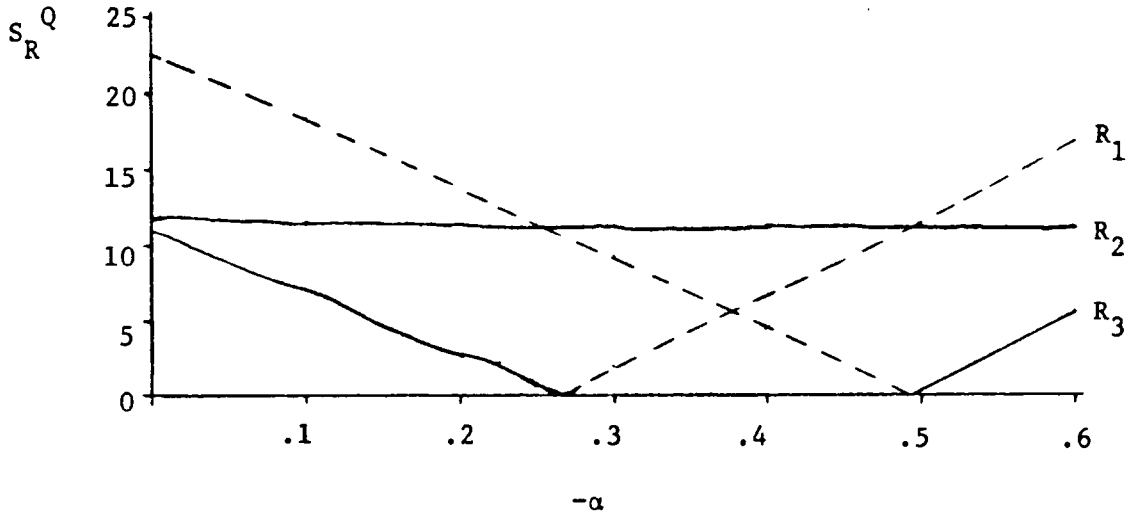
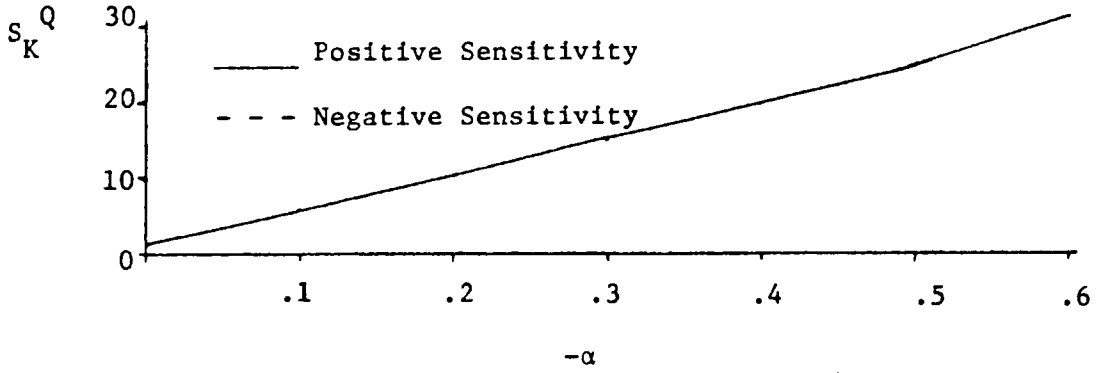


Figure 10 Q Sensitivity vs.  $\alpha$ ,  $Q = 50$



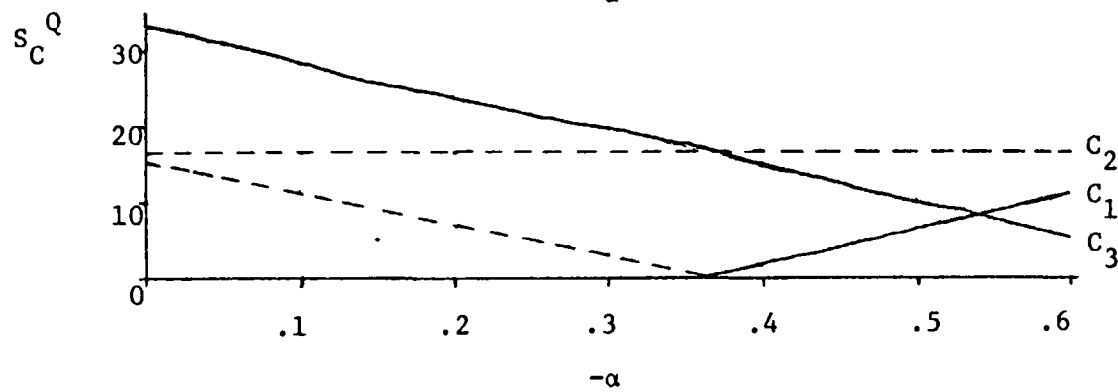
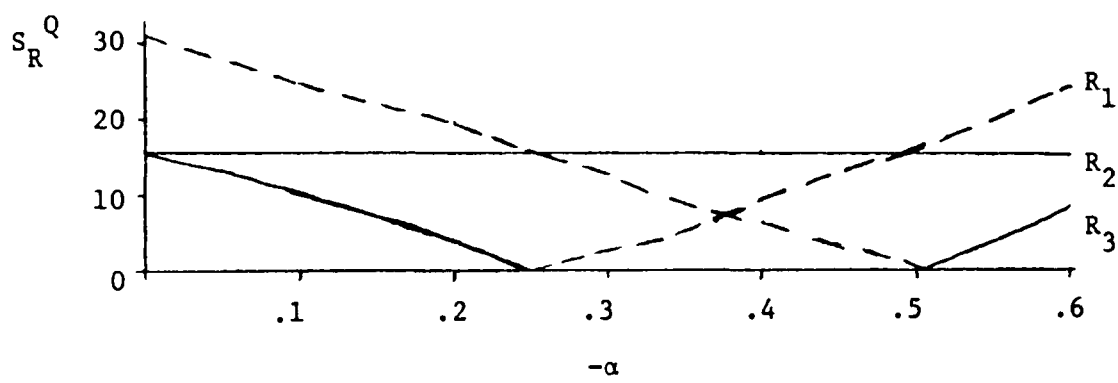
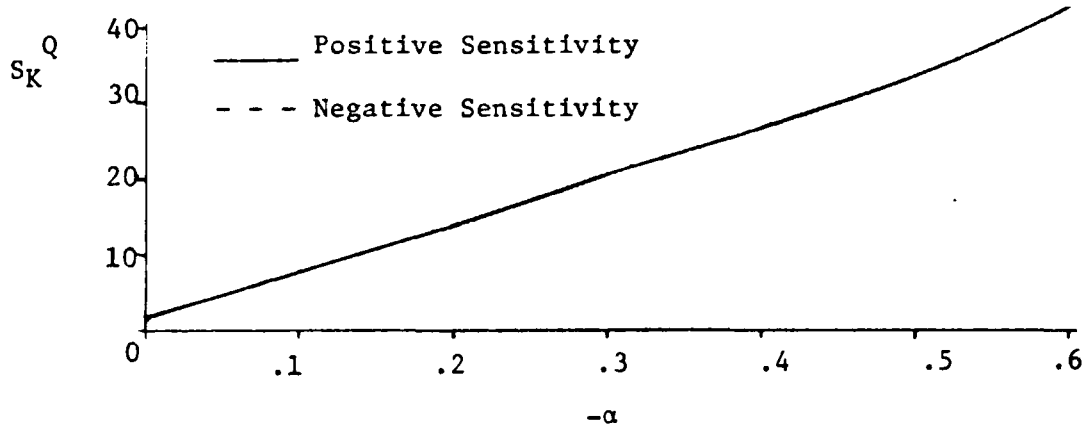


Figure 11 Q Sensitivity vs.  $\alpha$ ,  $Q = 70$

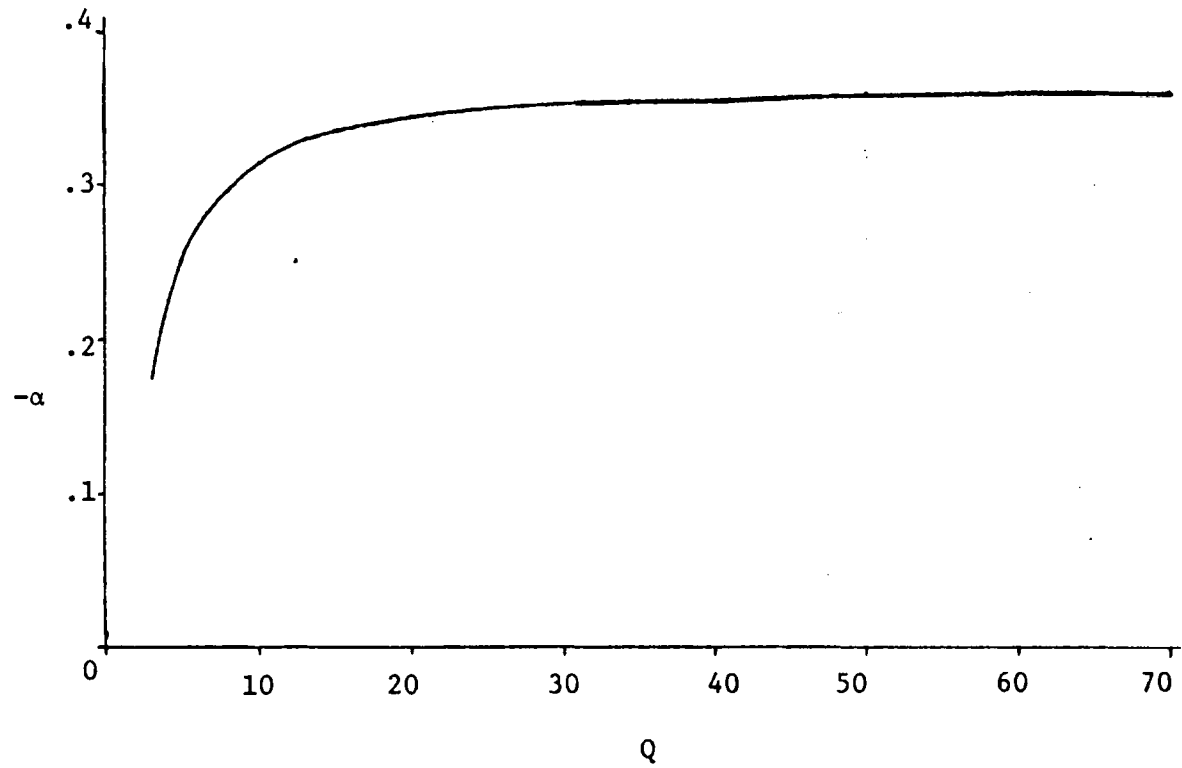


Figure 12  $-\alpha$  vs.  $Q$  for  $S_{C_1}^Q = 0$

## CHAPTER 4

### NON-IDEAL AMPLIFIER EFFECTS

The effects of amplifier input and output impedances must be investigated in order to see if either one critically influences circuit performance. These effects were analyzed by using the circuits shown in Fig. 13.

Fig. 13 (a) shows an amplifier output resistance  $R_0$ . Using node analysis it can be shown that the transfer function is

$$\begin{aligned}
 T(s) = & \frac{s^3 R_0 (R_1 R_2 + R_1 R_3 + R_2 R_3) + s^2 [2R_0 (R_1 + R_2 + R_3) - K(R_1 R_2 + 2R_1 R_3)]}{s^3 [(1+K)R_1 R_2 R_3 + R_0 (R_1 R_2 + R_1 R_3 + R_2 R_3)]} \\
 & + s^2 [(1+K)(R_1 R_3 + R_2 R_3) + R_1 R_2 + 2R_1 R_3 + 2R_0 (R_1 + R_2 + R_3)] \\
 & + s[2R_3(1+K) + 2R_1 + R_2 + 3R_0] + (1+K) \quad (23)
 \end{aligned}$$

This shows that the zero at infinity has moved toward the origin along the negative real axis. Also, the other real axis zero has moved as have all the poles. Using the same numerical techniques previously described, the Q for various gains and values of  $R_0$  were calculated. This data is presented for  $\alpha = -.35$ , in Fig. 14. This

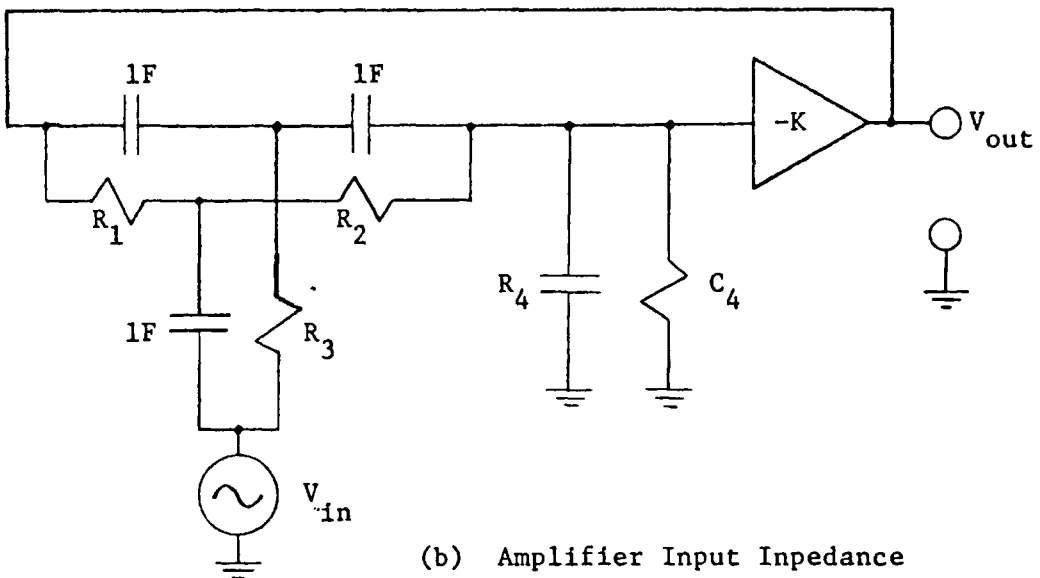
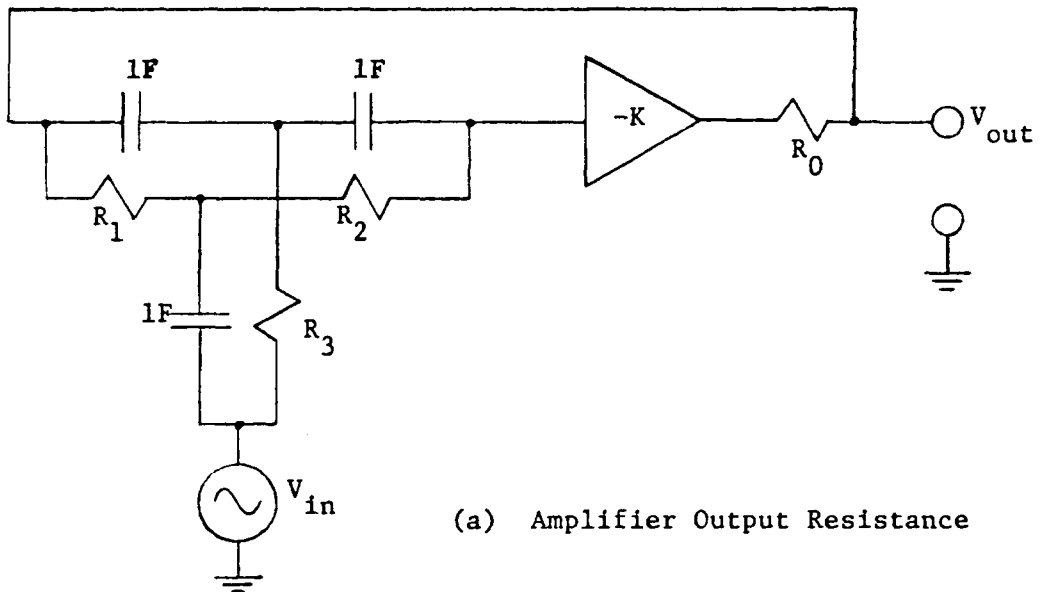


Figure 13 Non-Ideal Amplifier Effects

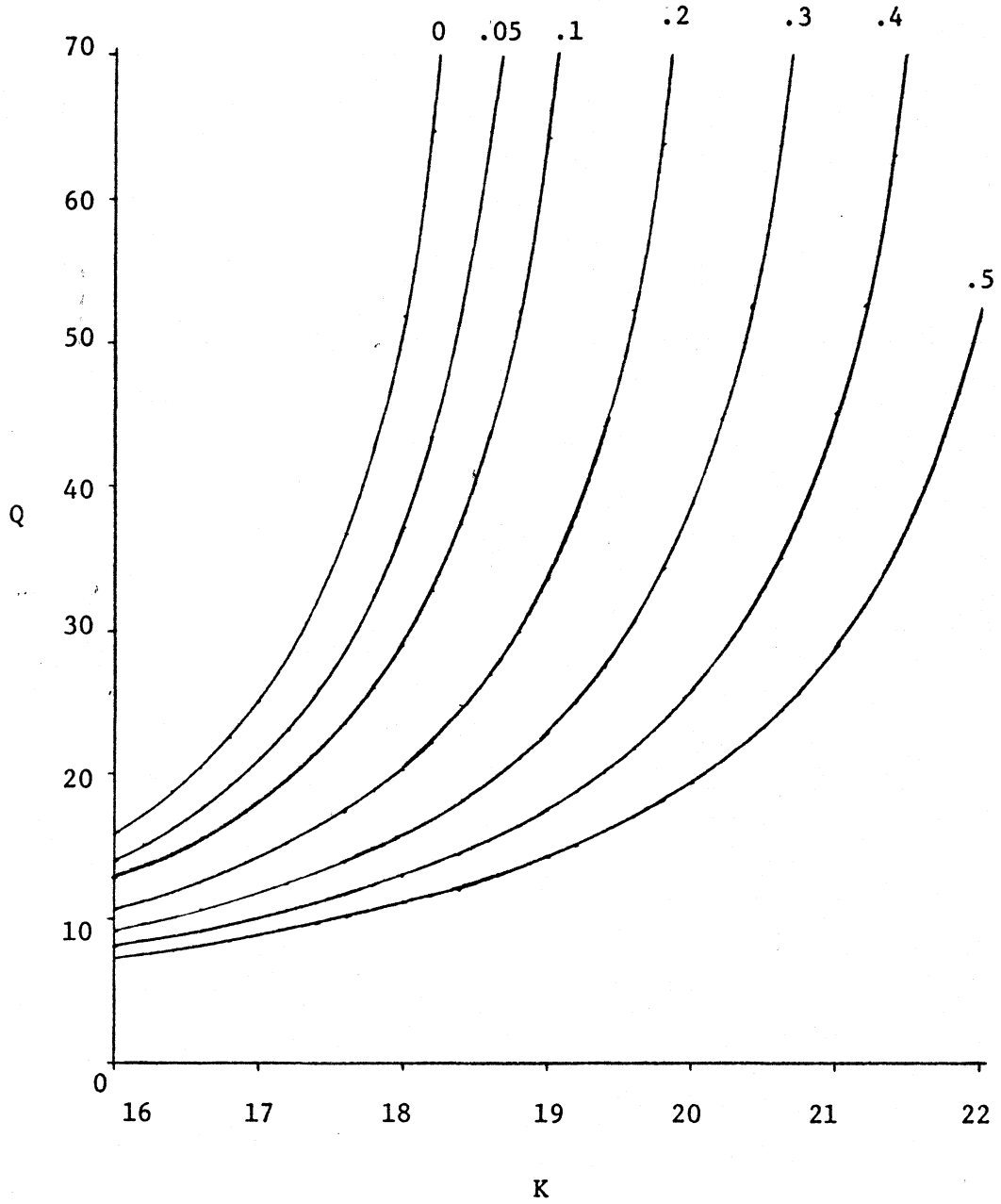


Figure 14 Calculated Q vs. K for various  $R_0$   
 $\alpha = -.35$

data shows that the larger  $R_0$ , the more amplifier gain is required for a given  $Q$ . As long as amplifier output resistance is less than a normalized value of .5, a very high output resistance, the desired  $Q$  can be obtained by increasing  $K$ .

The effects of amplifier input resistance and capacitance are very important at higher frequencies. The lower circuit of Fig. 13, with  $R_4$  and  $C_4$  representing the input impedance of the amplifier, has the following transfer function:

$$\begin{aligned}
 T(s) = & \frac{-K[s^2(R_1R_2+2R_1R_3) + s(2R_1+R_2)]}{s^3[R_1R_2R_3(1+K) + 2C_4R_1R_2R_3]} \\
 & + s^2[(1+K)(R_1R_3+R_2R_3) + R_1R_2 + 2R_1R_3] \\
 & + C_4(R_1R_2 + 2R_1R_3 + 2R_2R_3) + \frac{1}{R_4}(2R_1R_2R_3)] \\
 & + s[(1+K)(2R_3) + 2R_1 + R_2 + C_4(R_1 + R_2) \\
 & + \frac{1}{R_4}(R_1R_2 + 2R_1R_3 + 2R_2R_3)] + (1+K) + \frac{1}{R_4}(R_1 + R_2)
 \end{aligned} \tag{24}$$

This shows that  $R_4$  and  $C_4$  do not affect the zeros, only the pole positions. The over-all effect is to lower the  $Q$  for a given  $K$ . The numerical analysis of this was done as before and the data for  $\alpha = -.35$  is presented in Figs. 15 and 16.

It was found that the effect of both  $R_4$  and  $C_4$  together could be found from the graphs of their individual effects. The amount of

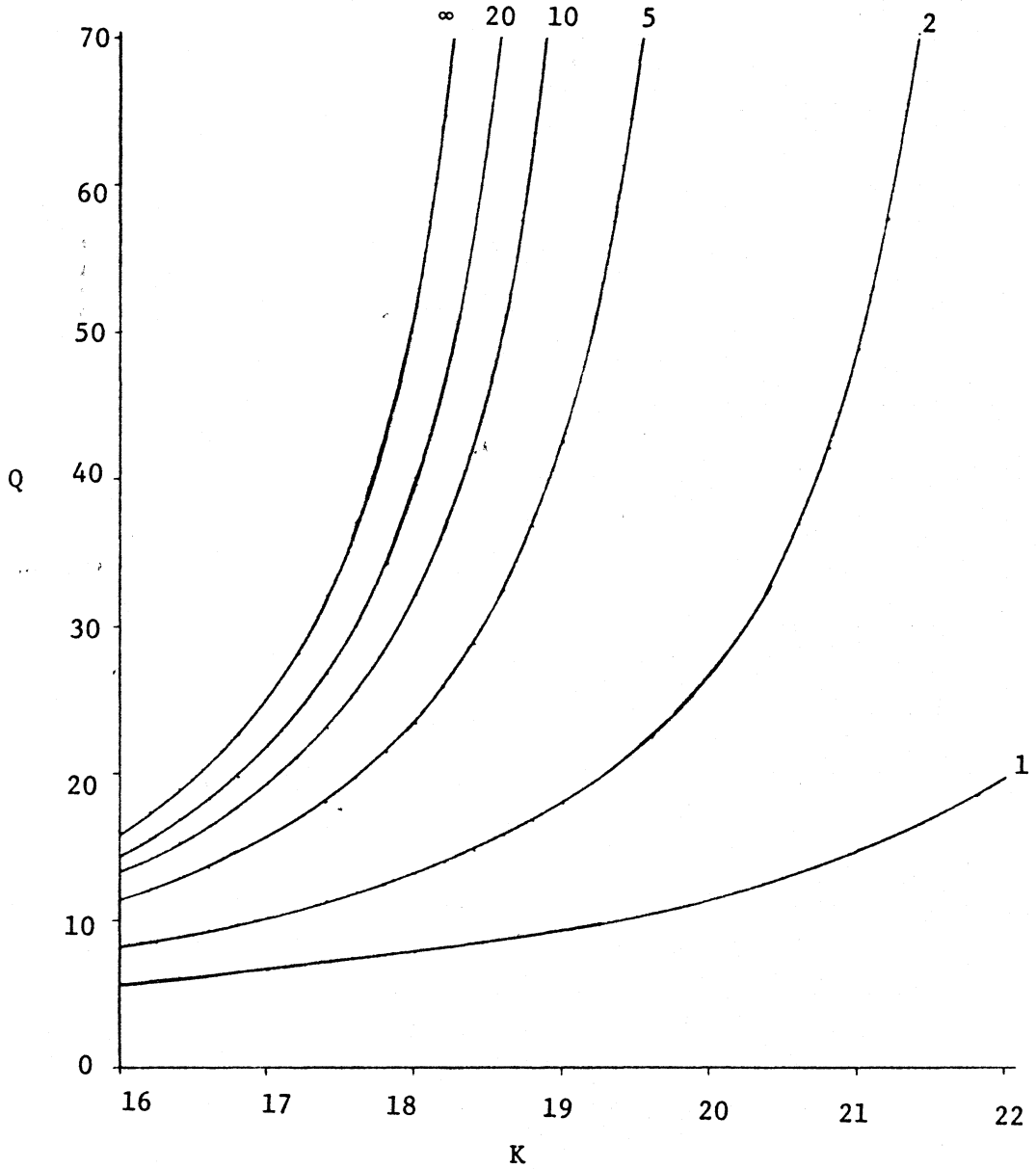


Figure 15 Calculated Q vs. K for Various  $R_4$   
 $C_4 = 0$   $\alpha = -.35$

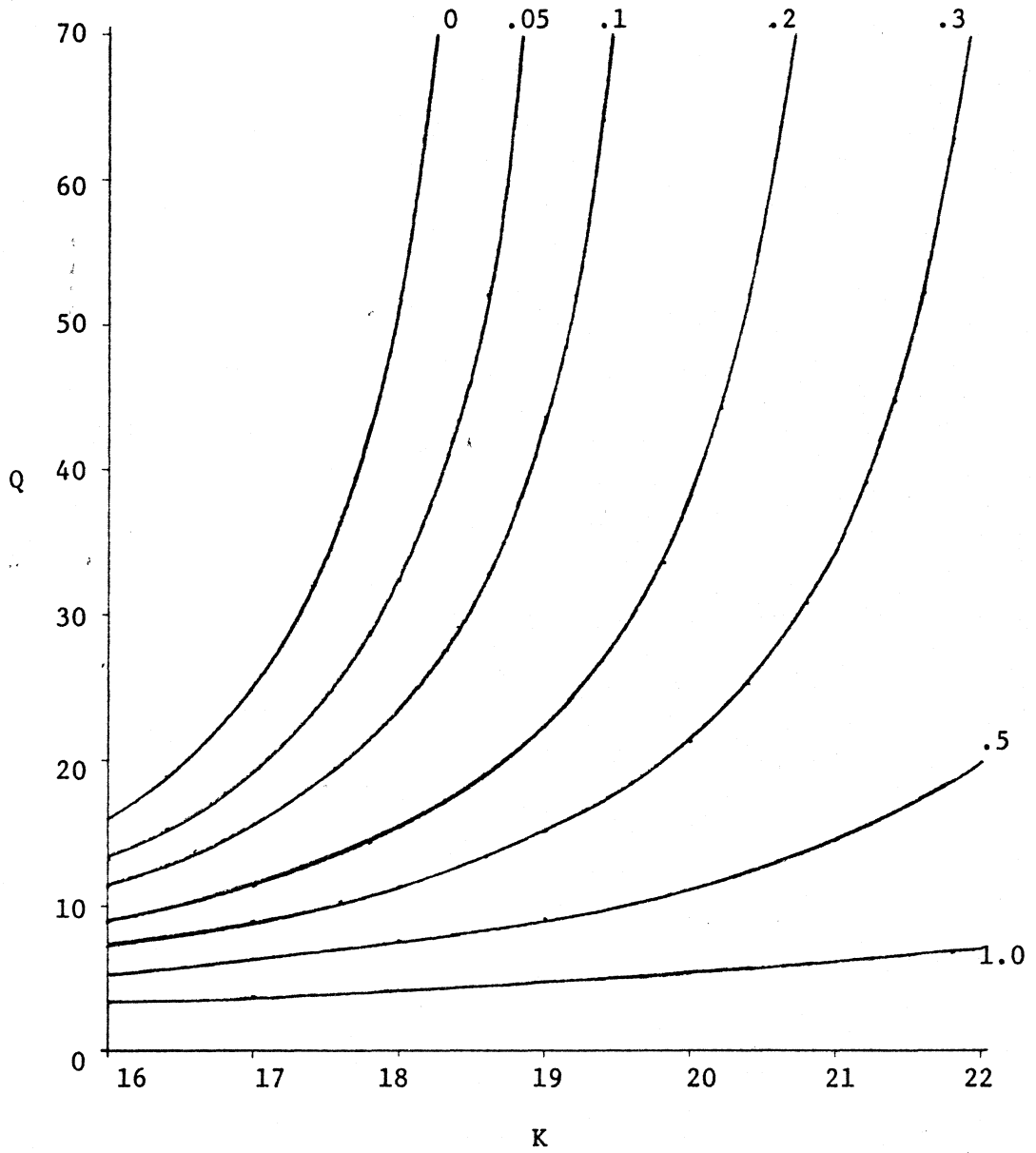


Figure 16 Calculated  $Q$  vs.  $K$  for Various  $C_4$

$$R_4 = \infty \quad \alpha = -.35$$



increase in  $K$  due to  $R_4$ , plus the amount of increase in  $K$  due to  $C_4$  is the total amount of increase in  $K$  required when both are present. For example, if a  $Q$  of 30 is desired, while the normalized value of  $R_4$  and  $C_4$  are 10 and 1.0 respectively, then the  $\Delta K$  due to  $R_4$  is .6 and the  $\Delta K$  due to  $C_4$  is 1.2. So the  $K$  required is

$$K = 17.3 + 0.6 + 1.2 = 19.1$$

for  $Q = 30$ ,  $R_4 = 10$  and  $C_4 = 0.1$ . Thus, if the amplifier input impedance is reasonably high, the  $Q$  desired can be obtained by increasing the ideal amplifier gain calculated from Eq. (13).

## CHAPTER 5

### GENERATOR SOURCE RESISTANCE

The effect of a real generator can be analyzed by using Fig. 17 where  $R_S$  represents the normalized source resistance of the generator. The transfer function of this circuit is

$$\begin{aligned}
 T(s) = & \frac{-K[s^2(R_1R_2 + 2R_1R_3) + s(2R_1 + R_2)]}{s^3[R_1R_2R_3(1+K) + R_S(1+K)(R_1R_2 + R_1R_3 + R_2R_3)]} \\
 & + s^2[(R_1R_3 + R_2R_3)(1+K) + R_1R_2 + 2R_1R_3] \\
 & + 2R_S(1+K)(R_1 + R_2 + R_3)] \\
 & + s[2R_3(1+K) + 2R_1 + R_2 + 3R_S(1+K)] + (1+K)
 \end{aligned} \tag{25}$$

The effect of  $R_S$  is to move the poles deeper into the left half plane. By using numerical analysis the  $K$  required for a given  $Q$ , when there is a generator source resistance  $R_S$ , was found. The results are shown in Fig. 18, for  $\alpha = -.35$ . These curves show that an increase in  $K$  will compensate for a normalized source resistance of less than  $.02\Omega$ .

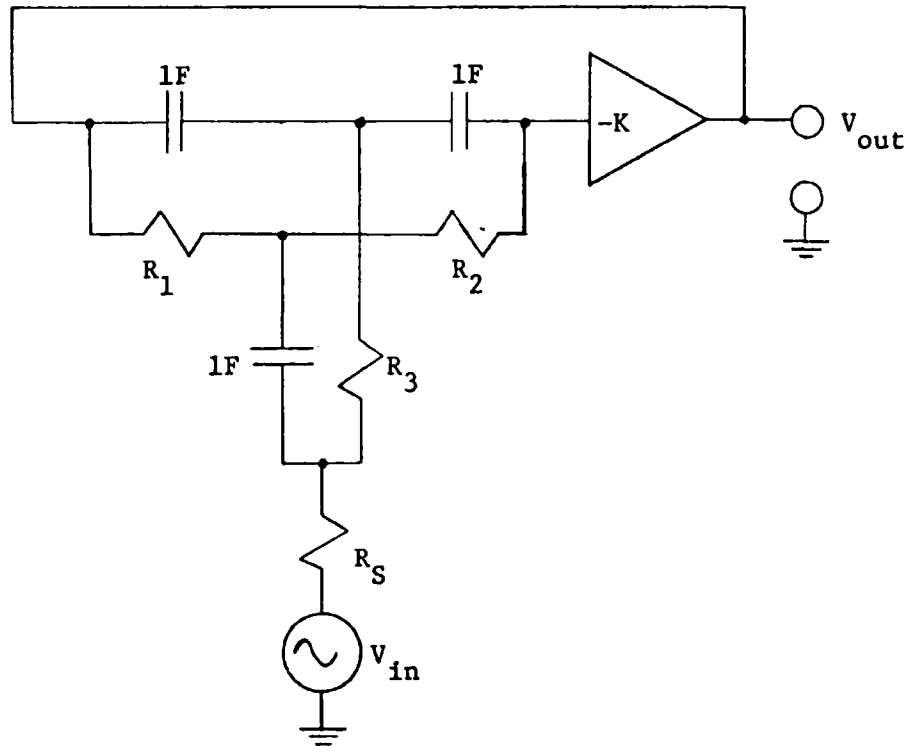


Figure 17 Effects of Generator Source Resistance

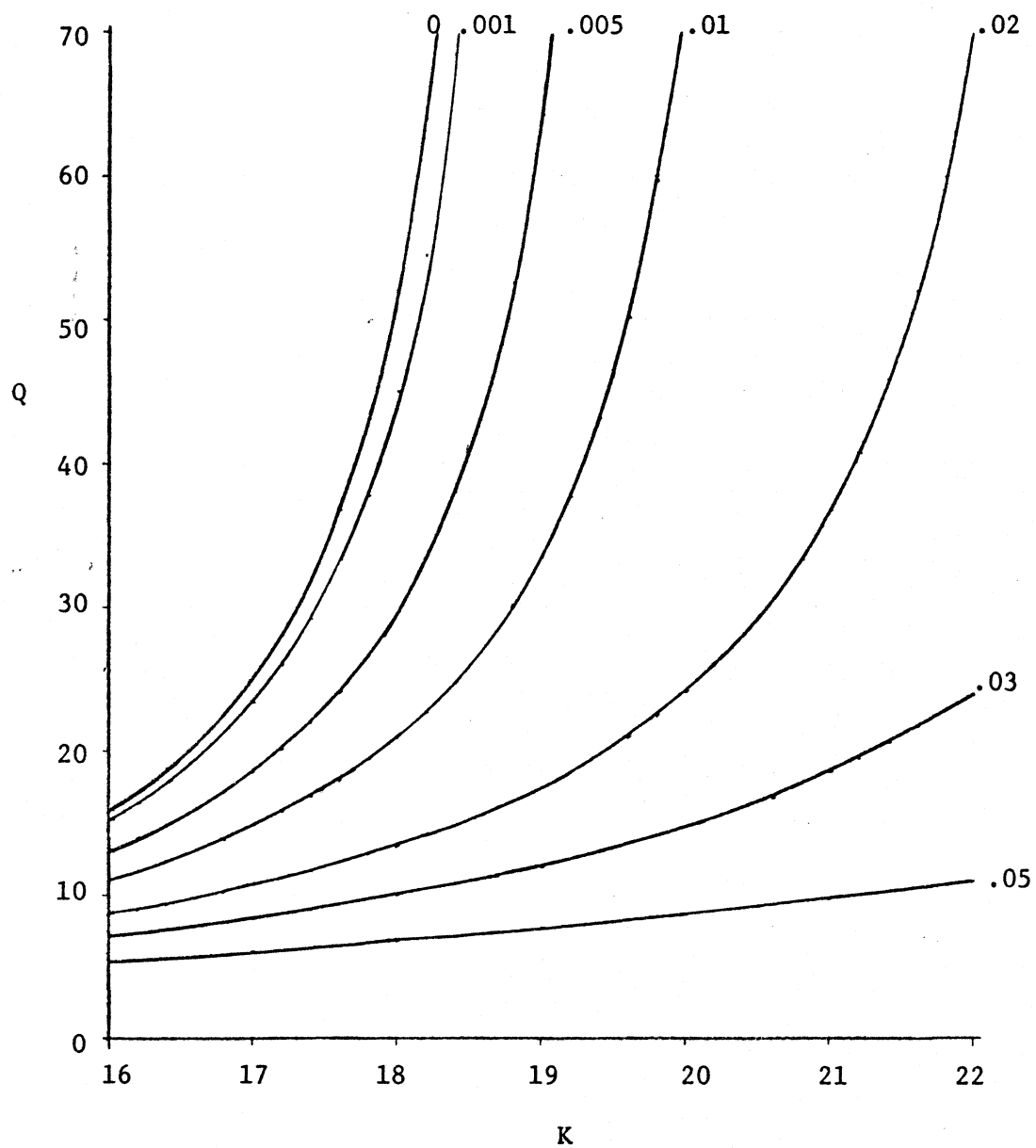


Figure 18 Calculated Q vs. K for Various  $R_S$   
 $\alpha = -.35$

The smallest resistor in the feedback circuit, ( $R_3$ ), is .1786  $\Omega$  for  $\alpha = -.35$ . For reasonable increases in K (less than 25%) the generator source must be less than about one tenth  $R_1$ . This is a very important design criterion.

## CHAPTER 6

### MEASURED RESULTS

To confirm the theory that has been presented, a working model was designed and built. Using an  $\alpha$  of  $-.35$  and impedance scaling by  $5.6K$  while frequency scaling up to  $610$  Hz ( $3,825$  rps) we get the following set of values:

$$\begin{array}{ll} R_1 = 15.7 \text{ K}\Omega & C_1 = 0.0470 \text{ }\mu\text{f} \\ R_2 = 7.92 \text{ K}\Omega & C_2 = 0.0470 \text{ }\mu\text{f} \\ R_3 = 1.00 \text{ K}\Omega & C_3 = 0.0470 \text{ }\mu\text{f} \end{array}$$

The actual values available and the values that were used are:

$$\begin{array}{ll} R_1 = 15.8 \text{ K}\Omega & C_1 = 0.0471 \text{ }\mu\text{f} \\ R_2 = 7.86 \text{ K}\Omega & C_2 = 0.0468 \text{ }\mu\text{f} \\ R_3 = 1.00 \text{ K}\Omega & C_3 = 0.0468 \text{ }\mu\text{f} \end{array}$$

The amplifier used was made up of two, three transistor operational amplifiers, and an emitter follower. The details are shown in Fig. 19. The first operational amplifier has an input resistance in excess of  $100 \text{ M}\Omega$  as a result of the low collector current in the input transistor and the feedback around the operational amplifier. The second operational amplifier supplies the negative gain,  $K \approx \frac{R_f}{R_i}$ , and the emitter follower, in the feedback loop, gives

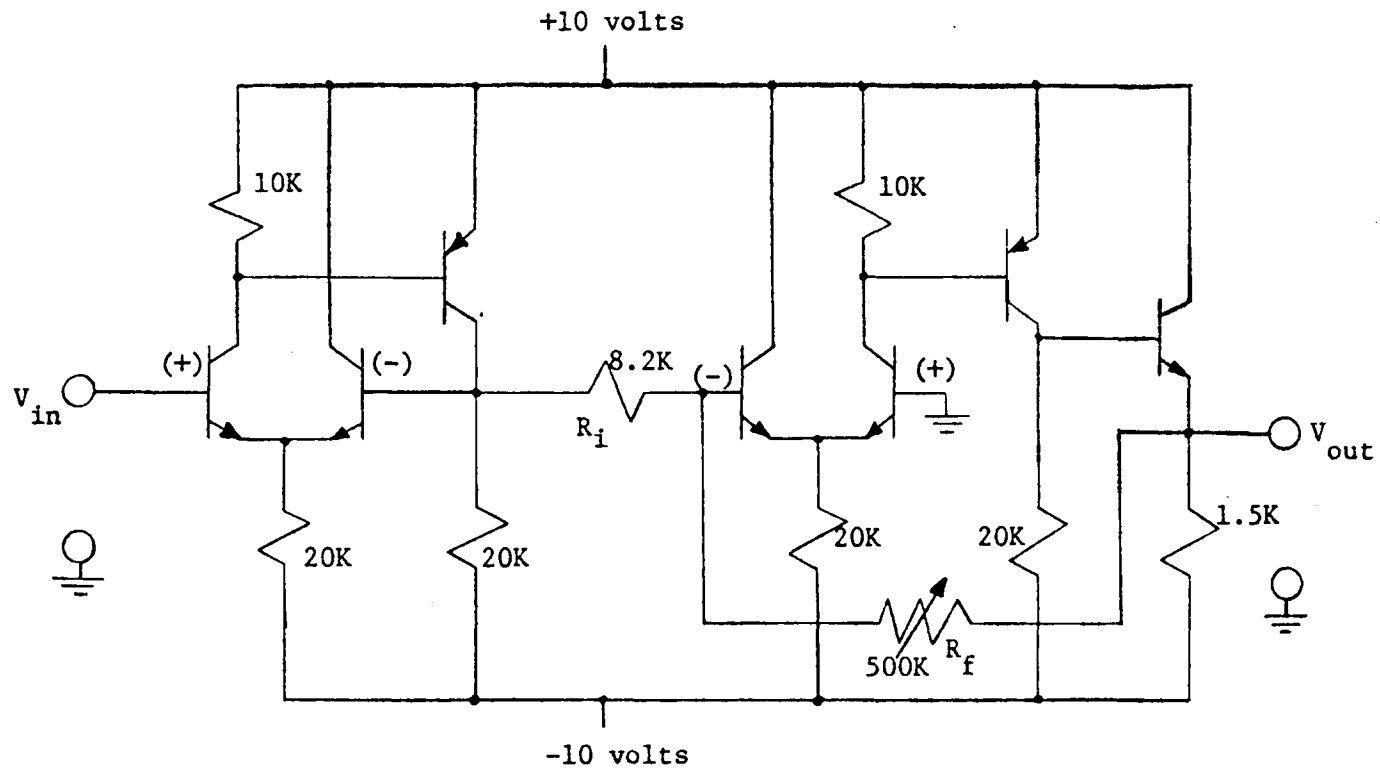


Figure 19 Amplifier Used in Test of Theory

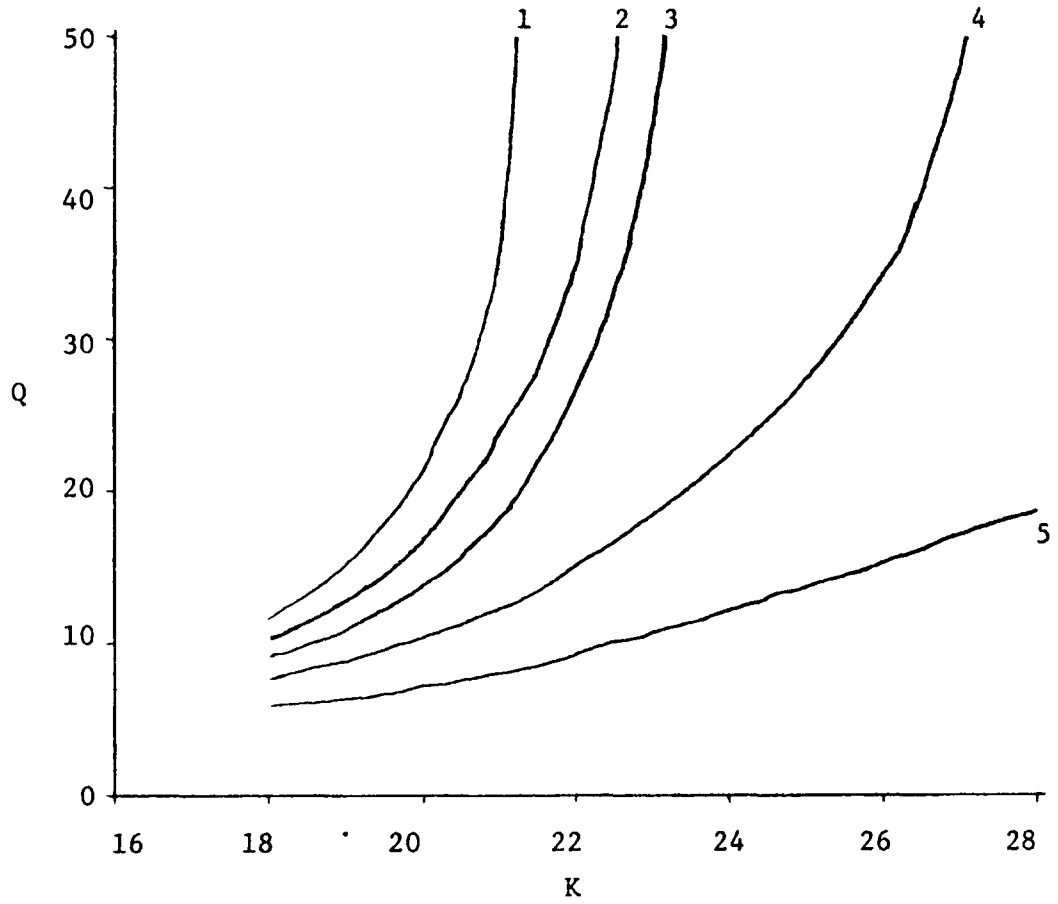
an output resistance of about  $1\Omega$ . The amplifier will give a K as large as 29, with a break frequency of 160 K Hz. When K is set equal to unity, the bandwidth is about 3.5 MHz. Since the operating frequency of the circuit is about 600 Hz there should be no adverse effects due to the amplifier pole.

The circuit operation is summarized in Fig. 20. It should be noted that the normalized values of  $R_0$  and  $R_5$  affected the circuit as predicted, that is to say that an increase in K was required for a given Q. However, the results do differ from the theory in one respect. To achieve a given Q, K had to be increased by about 3.3 over the value calculated using Eq. (13) and the curves for  $R_4$ ,  $R_0$  and  $R_5$ .

The reason for this discrepancy is not clear. The values for  $R_4$ ,  $R_0$  and  $R_5$  were accounted for and  $C_4$ , although not measured, could not have been on the order of 5,000 pf as the data would indicate.

In summary, this circuit offers an alternative to the standard RC active filters. It is a compromise between the high gain, low sensitivity circuits and the low gain, high sensitivity circuits. Because the VCVS used in this filter requires a gain of less than 30, it should find applications at IF frequencies where other active RC filters are not realizable.





1.  $R_S = 0.00087$      $R_0 = 0.0002$
2.  $R_S = 0.0087$      $R_0 = 0.0002$
3.  $R_S = 0.00087$      $R_0 = 0.2$
4.  $R_S = 0.02$      $R_0 = 0.0002$
5.  $R_S = 0.04$      $R_0 = 0.0002$

Figure 20 Measured Data

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