HEAT LOSS CALCULATIONS FOR SMALL SUBTERRENE PENETRATORS

by

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ABSTRACT

Thermal analyses are discussed which illustrate that the heat losses to the rock surrounding small diameter, refractory metal, prototype Subterrene rock-melting penetrators can be predicted. A simplified right-cylinder model configuration is considered. The results of a parametric study of the heat loss as a function of penetrator diameter and length, penetration rate, rock-melting temperature, and rock thermal properties are presented. The demonstrated ability to predict the thermal performance of a penetrator establishes the possibility of optimizing the system. Increased thermal efficiency for higher penetration rates is predicted.
CHAPTER 1

INTRODUCTION

A Subterrene penetrator is a device which melts holes into rock. Unlike conventional drilling techniques, however, a Subterrene penetrator is a nonrotating device which is electrically heated. Penetration of rock is accomplished by melting a hole into the rock by the hot (approximately 1400 to 1700 C) penetrator tip. The melting temperature of rock is in the range of 1150 to 1250 C. Holes of virtually any geometry are possible, since the penetrator does not rotate. Subterrene penetrators which are presently being laboratory tested are about five centimeters in diameter. Penetration rates of 0.04 to 0.20 mm/sec have been attained.

The goals of the Subterrene Program (at Los Alamos Scientific Laboratory) call for the design and testing of melting penetrators which have higher penetration rates and which are slightly larger than those which are discussed in this thesis. From the standpoint of reducing the total penetrator power requirements, it may be advantageous to use annular (coring) penetrators for making large diameter holes. Also, it may be desired
to retrieve rock core samples for geological study during penetration operations. Thus a study of the heat losses for annular (coring) penetrators was also conducted. For this type of penetrator, only a relatively thin ring of rock need be melted. The solid rock core which remains will have the shape of a cylinder, and it can be extracted from the center of the penetrator. Large diameter holes can be formed in this fashion with greatly reduced power requirements, if the core material can be efficiently handled.

In the design and test analysis of future penetrators, it would be helpful if there were a simple method by which penetrator heat losses could be estimated. It is the purpose of this study to provide such a method.

The calculations presented here were obtained by the numerical solution of the simplified two-dimensional energy equation. The energy equation was solved by a digital computer which used the recently developed AYER code (Lawton, 1972). The computational method is described in Chapter 3.

The rock properties which were used in this study are summarized and discussed in Chapter 4.

Most of the results are presented graphically and in such a manner that the designer or analyst can quickly
calculate the heat losses of any small size penetrator (up to 15 cm in diameter and 10 cm in length) which has penetration rates that are currently attainable (up to 0.2 mm/sec). These results are given in Chapter 6.

Chapter 6 also gives factors which the designer may use to convert penetrator heat losses for obsidian to heat losses for basalt, granite, or tuff. These scaling factors are based on calculations which employ the respective rock properties.
CHAPTER 2

THE PROBLEM AND THE APPROACH TO ITS SOLUTION

The objective of this study is to determine the rate of heat transfer from a moving Subterrene penetrator to the surrounding rock. It is expected that the rate of heat transfer is a function of the penetrator dimensions, the rock type, the penetrator velocity, and the penetrator temperature.

Description of the Penetrator

The basic design of a solid front surface Subterrene penetrator, shown in Fig. 1, consists of an electrically heated penetrator tip of radius $R_h$. The penetrator tip is connected to a long stem (similar to that used in rotary drilling), through which electrical power and coolant pass. The entire assembly moves with a velocity $V$ in an infinite region of solid rock.

During operation, the heated tip melts the rock which is in the path of the penetrator. Most of the molten rock is removed, either through the stem or by viscous flow into the cracks and voids in the surrounding rock. The remainder of the molten rock flows around the
Fig. 1. Basic Solid Front Surface Subterrene Penetrator Configuration During Operation Showing Molten Rock Layer and Solid Glass Wall Liner
penetrator tip, is cooled by the stem and the ambient rock, and forms a glass wall lining of the hole. Here is one of the major advantages of this kind of penetration: the glass wall lining eliminates the need for a metal casing or liner which must be installed during conventional drilling procedures.

**Simplification of Penetrator Representation**

During operation, the penetrator is covered with a layer of molten rock. The temperature along the interface between the molten rock and the solid rock is the rock melting temperature. This interface and the temperature along it is a convenient boundary condition which is used to simplify the heat conduction analysis. Additionally, further analytical simplification is obtained by eliminating the effects of the stem and solid glass wall liner. Figure 2 shows the simplified representation of the solid front surface penetrator tip and the molten/solid rock interface. A similar representation of the annular penetrator tip is shown in Fig. 3.

The mathematical description of the heat transfer problem is given by the general energy equation:
Fig. 2. Simplified Solid Front Surface Penetrator Tip and Molten Rock Layer Used for Analysis
Fig. 3. Cross Sectional View of the Simplified Annular Penetrator and Molten Rock Layer
Rate of change of internal energy per unit volume = \begin{align*}
\text{Heat generation rate per unit volume} & - \text{Rate which heat leaves the unit volume by conduction} \\
- \text{Rate which heat leaves the unit volume due to the unit volume motion}
\end{align*}

or expressed in familiar differential form,

$$U'' - \nabla \cdot (k \nabla T) - \rho CV \cdot \nabla T = \rho C \frac{\partial T}{\partial t}$$

(1)

where:

- $k$ = thermal conductivity of solid rock (assumed to be isotropic but non-linearly dependent on temperature),
- $T$ = temperature,
- $U''$ = heat generation rate per unit volume,
- $\rho$ = solid rock density,
- $C$ = specific heat capacity of solid rock,
- $V$ = penetrator velocity, and
- $t$ = time.
Methods of Solution

An exact analytical solution of Eq. (1) would be difficult, if not impossible, to obtain. It is anticipated that so many simplifying assumptions would have to be made that an accurate description of the penetrator would not be preserved. Thus, the final results would not fulfill the intended goal of this study.

Consultation of the literature disclosed that this type of problem has not been analytically solved. Parametric studies, using dimensionless variables, have been conducted in simplified problems of this nature (Arpaci, 1966). However, due to the simplifications required, it was decided not to employ this method for the problem at hand.

One interesting study (Aamot, 1967) closely resembled this analysis of a Subterrene penetrator tip. The study involved the heat transfer from a self-propelled instrumentation probe to glacier pack ice. Propulsion was accomplished by melting the ice in the path of the probe as the probe moved downward by gravity. An electrical heater provided the necessary heat, and the electrical power was transmitted through a cable from the glacier surface to the probe (the cable was paid out from a spool within the probe
itself). The heat transfer analysis of the probe was carried out using parameter groupings as the variables in more familiar, zero velocity, solutions (Carslaw and Jaeger, 1959). In this representation, the velocity of the probe was not explicitly expressed as a variable as it is in Eq. (1). Rather, it was implicitly expressed in a "characteristic time (the ratio of the probe length to the probe velocity)." The characteristic time variable was used as the time variable in "conventional" stationary heat conduction equations. Again it was decided not to use such simplifications in the solution of the Subterrene penetrator problem.

The key to the accurate solution of the Subterrene penetrator problem was the development of a computer code which was able to solve the general energy equation for moving geometries. The code which was developed was the AYER code, and its method of solution is described briefly in Chapter 3.
CHAPTER 3

THE COMPUTATIONAL MODEL

In this chapter, the numerical solution of the penetrator heat conduction problem is described more fully.

The AYER Computer Code

The AYER computer code provides the methodology for solving the two dimensional energy equation (1) with the finite element method (Wilson, 1965 and Lawton, 1972). For the particular case of the Subterrene penetrator, the steady state heat loss was solved in two dimensions with rotational symmetry with the heat generation per unit volume assumed to be zero.

In order to preserve its generality, the AYER code was written in an incomplete format. As written, it provides only the necessary computer manipulations in order to solve Eq. (1) by the finite element method. In order to use the AYER code to solve a particular problem, six subprograms must be written and attached to the AYER code. These subprograms describe the particular problem completely.
For the problem at hand, the author wrote subprograms which described the geometry of the penetrator and the surrounding rock region, rock properties, boundary conditions, initial conditions, penetrator dimensions and velocity, and a series of constraints on the iteration procedure to be used by the main AYER program.

Using this method, the penetrator heat loss problem was uniquely solved.

The first step in the numerical solution of the problem is the division (by the computer) of the penetrator and a finite region of surrounding rock into small triangular elements, two of which form the rectangles shown in Fig. 4. The intersection of two lines in the "meshed" region is called a node point, and a spatially linear temperature relationship of the form

$$T = a_i + b_i R + c_i Z, \quad i = 1, n$$

(2)

is assumed to hold for the $n$ node points. Using an iterative procedure, the computer solves for the $a_i$'s, $b_i$'s, and $c_i$'s to obtain the temperature distribution for the entire meshed region. Then the computer calculates the heat flows through all boundaries that the author has specified in the subprograms. For this case, the computer solved for the heat flows through the front and side surfaces on the molten/solid rock interface.
Fig. 4. Typical Finite Element Region and Mesh Network
Figure 4 shows the finite region of surrounding rock with the molten/solid rock interface shown in the lower left-hand corner. The penetrator is stationary, and the rock in the region is assumed to move downward with a velocity $V$. The penetrator's outer surface is in an area of high mesh density, since it is in this area where the temperature gradients are largest (see Fig. 5).

The left-hand vertical axis of Fig. 4 is the extension of the centerline of the penetrator, and by symmetry, the heat flux across this boundary is zero.

It is assumed that the heat flux from the side surface of the molten/solid rock interface at $Z = 0$ is entirely radial, and thus the conductive heat flow through the bottom horizontal plane is zero. This assumption is not strictly true for finite penetrators, and it represents a source of error in the problem.

The remaining two region boundaries, the upper horizontal boundary and the right-hand vertical boundary, are assumed to be at an ambient temperature of 20°C, since this condition would exist at an infinite distance from the penetrator.

The variable mesh width technique was employed both to create a large mesh region (approximating infinite) and also to obtain an accurate solution in the vicinity of the penetrator. To create a finite mesh region large
Fig. 5. Typical Isotherm Plot for the Solid Front Surface Penetrator Provided by the Computer
enough to be considered "infinite," several test problems having different region sizes were solved. The region size was increased until the problem results no longer changed from one solution to the next. Notice that due to the poor heat conduction properties of solid rock, a region whose radius is greater than one meter and whose height is greater than 0.75 meters is sufficiently large to be considered "infinite" for the penetrators analyzed in this study.

The analytical model used for heat loss calculations for annular penetrators closely parallels the model used for solid front surface penetrators. The model used is an annular right circular cylinder, moving with a velocity $V$ in an infinite medium of solid rock. Figure 3 shows the annulus and the interface between the molten rock and the solid rock.

Figure 6 shows a typical annular geometry isotherm plot, which is a part of the computer output.

**Other Considerations**

To eliminate the uncertainties involved in including the molten rock layer, the AYER computer code was written so that the heat conduction from the front and side surfaces of the molten/solid rock interface would be calculated. These surfaces have dimensions $R_g$, the radius,
Fig. 6. Typical Annular Penetrator Isotherm Plot Provided by the Computer
and $H$, the heated length. Now the only material properties required in the calculations are those of solid rock. These properties are better known than those of molten rock.

Notice that with the above simplification, the penetrator velocity $V$ may be regarded as an independent variable. This freedom to choose the penetration rate does not exist during actual penetrator operation, however, because the power delivered to melt rock determines the maximum penetration rate. Hence, it is not an objective of this study to predict penetration rates, and so the designer must be careful in selecting penetration rates consistent with the melting power being considered.

When using the solid front surface penetrator in low density rock, such as tuff, no debris removal from the hole is required. Instead, the mass of material in the path of the penetrator is consolidated by the change of phase from low density rock to high density glass. In this case, $R_g$ and $R_h$ are related by (Rowley, 1972)

$$\frac{R_g}{R_h} = \sqrt{\frac{1}{1 - \frac{\rho_r}{\rho_g}}} \quad (3)$$

where $\rho_r$ is the density of the rock and $\rho_g$ is the density of the glass (usually considered to be obsidian).
For high density rocks where rock removal is necessary, the designer must estimate $R_g$ prior to using the data in this thesis.

Boundary Conditions of the Simplified Penetrator

The only boundaries of interest are the front and side surfaces (and the inner side surface of the annular penetrator) of the molten/solid rock interface. It is assumed that both the front surface temperature and the side surface temperature are uniform and equal to each other. This temperature corresponds to the melting temperature of the rock.

Effects of Variations in Melting Point

The temperature of the molten/solid rock interface was varied in order to study the effects of temperature on heat loss. Note that while the surface temperature of the penetrator may be 1500-1700 $^\circ$C, the temperature along the molten/solid rock interface is only 1150-1250 $^\circ$C. Thus, there is a considerable temperature gradient across the molten rock layer, a condition which indicates the importance of the molten rock layer in determining the overall performance of the actual penetrator.
CHAPTER 4

ROCK PROPERTIES

The literature provides a record of a wide variation in rock properties, principally because there are great variations in rock types throughout the world. To a lesser degree, differences in the methods of measurements of properties also account for variations in tabulated rock properties.

The rock properties shown in Table I and in Fig. 7 have been selected from the literature as being representative of the major rock types of interest. Therefore, the results of the calculations in this thesis should provide the Subterrene designer/analyst with representative penetrator heat losses for melting through nearly all major rock types.
<table>
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<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
<th>Specific Heat (cal/g·°C)</th>
<th>Melting Point (°C)</th>
<th>Latent Heat of Fusion (cal/g)</th>
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<tr>
<td>Basalt</td>
<td>2.8</td>
<td>0.24</td>
<td>1150</td>
<td>100</td>
</tr>
<tr>
<td>Granite</td>
<td>2.7</td>
<td>0.24</td>
<td>1250</td>
<td>80</td>
</tr>
<tr>
<td>Tuff</td>
<td>1.4</td>
<td>0.24</td>
<td>1150</td>
<td>100</td>
</tr>
<tr>
<td>Obsidian</td>
<td>2.8</td>
<td>0.24</td>
<td>1150</td>
<td>100</td>
</tr>
</tbody>
</table>

*Basalt and granite data from: Maurer, 1969.

Tuff and obsidian data from unpublished sources at Los Alamos Scientific Laboratory.
Fig. 7. Conductivity Data for Four Major Rock Types. (Obsidian and Basalt from Murase and Mc Birney, 1970; Tuff from Stephens, 1963; and Granite from Moiseyenko, Sokolova, and Istomin, 1970)
CHAPTER 5

PENETRATOR HEAT LOSS AND MELTING POWER

In this chapter, the relationships which will be used in calculating the penetrator heat loss and melting power are presented and discussed.

Heat Loss

The computer code was written so that the gross penetrator heat conduction would be calculated in several components. The first component is the heat conducted through the side surface of the molten/solid rock interface (see Fig. 2). This is also the heat conducted through the outer side surface of the annular penetrator in Fig. 3. All of this heat is considered to be lost, since it does nothing more than raise the temperature of the solid rock adjacent to the penetrator. The second component is the heat which is conducted through the front surface of the molten/solid rock interface. As we can infer from Fig. 5, the heat flux from the front surface is almost parallel to the direction of penetrator motion. Therefore, most of the heat conducted from the front surface raises the temperature of the rock in the
path of the penetrator to the melting point. This heat is called the sensible heat, and it is not considered to be a loss. The sensible heat is given by the relationship:

$$Q_{\text{sens}} = 4.19 \rho V (R_g)^2 C(T_m - T_o),$$

where:
- $Q_{\text{sens}}$ = sensible heat (watts),
- $\rho$ = solid rock density,
- $V$ = penetration rate (cm/sec),
- $R_g$ = molten/solid rock interface radius (cm),
- $C$ = specific heat of solid rock (cal/g·C),
- $T_m$ = melting point temperature (C),
- $T_o$ = ambient rock temperature (C), and
- 4.19 = 4.19 joules/cal.

From Fig. 5 we also note that some of the heat conducted from the front surface leaves the path of the advancing penetrator. This portion of the heat, called the front surface transverse conduction loss, is considered to be a loss, because it contributes nothing to the melting process.

A third component of the total heat conduction was calculated for the annular penetrator. This component is the heat conducted through the inner side surface (see Fig. 3). This component is also a loss.
Melting Power

The power required to effect a given penetration rate is a direct function of the volume of rock melted per unit time. For the solid front surface penetrator it is given by

$$Q_{melt} = 4.19\pi \rho V(R_g)^2 \left[ C(T_m - T_0) + H_f \right], \quad (5)$$

where: $Q_{melt}$ = melting power (watts), and

$$H_f = \text{latent heat of fusion of rock (cal/g)}.$$

The melting power, given by Eq. (5) is simply the sensible heat, given by Eq. (4), plus the heat required to cause the phase change from solid rock to molten rock.

Equation (5) will also be utilized in calculating the melting power of the annular penetrator.
CHAPTER 6

DISCUSSION OF RESULTS

Most of the heat loss computations were based upon the physical properties of obsidian. The results, shown in Figs. 8 through 15, are presented in such a manner that the designer/analyst can quickly calculate the heat losses of any small penetrator. Once the heat losses have been determined for obsidian, the heat losses for other rock types can be obtained by using suitable relationships which will be presented later in this chapter.

Calculation of Penetrator Heat Losses in Obsidian

In this section, the methods to be used in calculating the penetrator heat losses in obsidian are outlined. Example calculations illustrate the methods for both solid front surface penetrators and annular penetrators.

Example Calculation for Solid Front Surface Penetrators

Suppose we wish to calculate the heat losses of a solid front surface penetrator that creates a molten
Fig. 8. Front Surface Heat Conduction vs. Radius for Temperature = 1150 °C, Velocity = 0.04 mm/sec, and Obsidian
Fig. 9. Side Surface Heat Loss vs. Heated Length for Temperature = 1150°C, Velocity = 0.04 mm/sec, and Obsidian
Fig. 10. Variation of Front Surface Temperature Factor with Temperature for Velocity = 0.04 mm/sec and Obsidian
Fig. 11. Variation of Side Surface Temperature Factor with Temperature for Velocity = 0.04 mm/sec, and Obsidian
Fig. 12. Variation of Velocity Factor with Velocity for Temperature = 1150°C and Obsidian
Fig. 13. Annular Penetrator Inner Side Surface Heat Loss vs. Heated Length and Inner Radius
Fig. 14. Variation of Inner Side Surface Temperature Factor with Temperature for Velocity = 0.04 mm/sec and Obsidian
Fig. 15. Variation of Annular Penetrator Velocity Factor with Velocity for Temperature = 1150 °C and Obsidian
solid rock interface which has the following parameters: glass radius, \( R_g = 4.0 \text{ cm} \); heated length, \( H = 4.0 \text{ cm} \); penetration rate, \( V = 0.15 \text{ mm/sec} \); and melting temperature, \( T = 1180 \text{ C} \).

From Fig. 8, we find the heat conducted through the front surface, based on the 4 cm radius, is 860 watts. Figure 9 shows that the heat loss through the side surface, based on \( R_g = 4 \text{ cm} \) and \( H = 4 \text{ cm} \), is 1120 watts. However, since Figs. 8 and 9 were calculated on a basis of a penetration rate of 0.04 mm/sec and a melting temperature of 1150 C, we must apply some correction factors to account for our case of temperature = 1180 C and \( V = 0.15 \text{ mm/sec} \). These correction factors are given in Figs. 10 through 12, and they are:

Temperature Factors:

Front Surface (Fig. 10)

\[ = 1.03 \]

Side Surface (Fig. 11)

\[ = 1.04 \]

Velocity Factors:

Front Surface (Fig. 12)

\[ = 3.10 \]

Side Surface (Fig. 12)

\[ = 1.95 \]

Multiplying the heat conducted through the front and side surfaces by the corresponding factors, we find:
Heat conducted through front surface = 860(1.03)(3.1)
= 2745 watts, and

Heat loss through side surface = 1120(1.04)(1.95)
= 2271 watts.

In general, the total (neglecting stem heat losses and molten rock superheat) power required by a solid front surface penetrator is:

\[ Q_{\text{tot}} = \left[ (\text{Heat conducted through front surface}) - Q_{\text{sens}} \right] + Q_{\text{melt}} + (\text{Heat loss through side surface}). \] (6)

We have just calculated the heat conducted through the front and side surfaces. Let us now evaluate \( Q_{\text{sens}} \) and \( Q_{\text{melt}} \) for obsidian by using Eqs. (4) and (5) respectively:

\[ Q_{\text{sens}} = 4.19\pi(2.8)(0.015)(4.0)^2(0.24)(1180 - 20) \]
= 2461 watts, and

\[ Q_{\text{melt}} = 4.19\pi(2.8)(0.015)(4.0)^2[(0.24)(1180 - 20) + 100] \]
= 3345 watts.

Now we apply Eq. (6) to calculate the total required power:

\[ Q_{\text{tot}} = [2745 - 2461] + 3345 + 2271 \]
= 5900 watts.
Notice that the bracketed term in Eq. (6) is the front surface transverse conduction loss, defined earlier.

Example Calculation for Annular Penetrators

The method of calculating the power requirements of annular penetrators closely follows that used for the solid front surface penetrators. The heat conducted through the outer side surface of the annular penetrator is the same as that for the solid front surface penetrator. The heat conducted through the front surface of the annular penetrator is the same as that for the solid front surface penetrator, corrected by a ratio of areas. An additional heat loss is apparent in the annular penetrator, and that is the heat conducted through the inner side surface. Using the methods outlined in the previous section, this value of heat is evaluated by using Figs. 13, 14, and 15.

Let us now calculate the heat losses and the total power requirement for an annular penetrator which has the following parameters:

- Glass radius, $R_g = 4.0 \text{ cm}$
- Core radius, $R_i = 2.0 \text{ cm}$
- Heated length, $H = 4.0 \text{ cm}$
- Penetration rate, $V = 0.15 \text{ mm/sec}$, and
- Melting temperature, $T = 1180 \text{ C}$.
The rock type is obsidian. Notice that this penetrator will form a hole which is identical in all respects to the hole formed in the previous example.

The outer side surface heat loss is the same as that for the other example, 2271 watts.

The front surface heat conduction, $Q_{\text{sens}}$, and $Q_{\text{melt}}$ are the respective values for the previous example multiplied by the ratio of the annular front surface area to the solid front surface area:

\[
\text{Front surface heat loss} = 2745 \times (0.75) = 2059 \text{ watts};
\]
\[
Q_{\text{sens}} = 2461 \times (0.75) = 1846 \text{ watts}; \text{ and}
\]
\[
Q_{\text{melt}} = 3345 \times (0.75) = 2509 \text{ watts}.
\]

The inner side surface heat loss is calculated from the data given in Figs. 13, 14, and 15. From Fig. 13 we find the basic heat loss for $R_i = 2.0 \text{ cm}$ and $H = 4.0 \text{ cm}$ is 135 watts. From Figs. 14 and 15, the temperature and velocity factors are 1.03 and 3.47, respectively. Multiplying these values together we obtain:

\[
\text{Heat loss through inner side surface} = 135(1.03)(3.47) = 483 \text{ watts}.
\]
An estimate of the total power required by an annular penetrator (once again neglecting stem losses and molten rock superheat) is given by:

\[
Q_{\text{tot}} = [(\text{Heat conducted through front surface}) - Q_{\text{sens}}] + Q_{\text{melt}} + (\text{Heat loss through outer side surface}) + (\text{Heat loss through inner side surface}). \tag{7}
\]

Substituting the above values into Eq. (7), we find the total power requirement:

\[
Q_{\text{tot}} = [2059 - 1846] + 2509 + 2271 + 483 = 5476 \text{ watts,}
\]

which is less than the value of 5900 watts calculated in the previous solid front surface penetrator example.

Additional Considerations

The difference between the power requirements for solid and annular penetrators, although small for this case, rapidly increases as \( R_g \) increases (for a constant radial thickness of the annulus).

The powers required by actual penetrators are greater than the values predicted by Eqs. (6) and (7), since Eqs. (6) and (7) do not include either the stem losses or the molten rock superheat.
Calculation of Heat Losses for Other Rock Types

Equations (6) and (7) can also be used to estimate the heat losses of penetrators in rock other than obsidian. We must first estimate the quantities in the appropriate equation for the rock type of interest.

Calculations show that $Q_{\text{sens}}$ is about 80 to 90% of the heat conducted through the front surface for each of the four rock types considered in this study. Thus to estimate the heat conducted through the front surface for a particular rock type, first calculate $Q_{\text{sens}}$ for that rock type, and then divide the value obtained by 0.85.

Next, determine $Q_{\text{melt}}$ for the rock type of interest, using Eq. (5).

Finally, calculate the last term of Eq. (6) (last two terms of Eq. (7)) by first finding the heat loss through the side surface for obsidian as before. Then multiply this value by the appropriate factor given in Table II. Recall that the greatest heat transfer

<table>
<thead>
<tr>
<th>Material</th>
<th>Side Surface Heat Loss Compared to Obsidian (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>83</td>
</tr>
<tr>
<td>Granite</td>
<td>71</td>
</tr>
<tr>
<td>Tuff</td>
<td>48</td>
</tr>
</tbody>
</table>
Using the procedure outlined above, let us now evaluate the total heal losses of our example solid front surface penetrator in tuff. First, we know that the heat conducted through the front surface is $Q_{\text{sens}}$ for tuff, divided by 0.85. Thus,

$$Q_{\text{sens}} = 4.19 \pi (1.4)(0.015)(4.0)^2(0.24)(1180 - 20) = 1230 \text{ watts},$$

and the heat conducted through the front surface is 1448 watts. Using Eq. (5), $Q_{\text{melt}}$ is determined to be

$$Q_{\text{melt}} = 4.19\pi(1.4)(0.015)(4.0)^2[0.24(1180 - 20) + 100] = 1673 \text{ watts}.$$

From our previous calculations, we determined the side surface heat loss in obsidian to be 2271 watts. From Table II we find that the side surface heat loss in tuff is 48% of this value, or 1090 watts.

The total power requirement for melting into tuff is now found by using Eq. (6):

$$Q_{\text{tot}} = [1448 - 1230] + 1673 + 1090 = 2981 \text{ watts}.$$

Notice that the power requirements for tuff are roughly half of those for obsidian. This is because tuff has much smaller values of density and thermal conductivity.
Similar computations for annular penetrators may be performed by applying the values of Table II to the inner side surface heat loss as well as to the outer side surface heat loss.

**Accuracy of Hand Calculations**

The velocity and temperature correction factors presented in Figs. 10 through 12 and in Figs. 14 and 15 are based on data for molten/solid rock interface dimensions of $R_g = 4.5$ cm and $H = 3.5$ cm. Thus, it is necessary to estimate the accuracy of using these curves for other values of $R_g$ and $H$. To do this, a series of test problems were solved on the computer and compared with the results obtained from the hand calculations. The test problems spanned the ranges of size, temperature, and penetration rate which are indicated on Figs. 8 through 15. The comparison of results showed that the hand calculations were accurate to within $+2\%$ to $-5\%$ of the computer results.
CHAPTER 7

CONCLUSIONS

This thesis is an analysis of the heat losses of Subterrene penetrators which are now in experimental use. The following conclusions apply to the future study and use of more advanced penetrators.

**Penetrator Thermal Efficiency**

Table III presents representative power requirements and thermal efficiencies for several solid front surface penetrator configurations in obsidian. The thermal efficiency (neglecting stem colling losses, electrical losses, and molten rock superheat) is calculated from the ratio

\[
\text{Efficiency} = \frac{Q_{\text{melt}}}{Q_{\text{tot}}} \tag{8}
\]

Several interesting effects are apparent when Table III is studied.

First, penetrator thermal efficiency increases as the glass radius, \( R_g \), increases. This is due to the rapid increase in \( Q_{\text{melt}} \) compared to the increase in the heat loss.
### TABLE III

Representative Power Requirements and Thermal Efficiencies for Several Penetrator Configurations in Obsidian

<table>
<thead>
<tr>
<th>Glass Radius (cm)</th>
<th>Heated Length (cm)</th>
<th>V (mm/sec)</th>
<th>T (°C)</th>
<th>( Q_{\text{melt}} ) (watts)</th>
<th>( Q_{\text{tot}} ) (watts)</th>
<th>Thermal Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>6.0</td>
<td>0.04</td>
<td>1150</td>
<td>219</td>
<td>1529</td>
<td>14.3</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td>492</td>
<td>2592</td>
<td>19.0</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td>875</td>
<td>3834</td>
<td>22.8</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>1367</td>
<td>5307</td>
<td>25.8</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>0.04</td>
<td>1150</td>
<td>219</td>
<td>1209</td>
<td>18.1</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td>492</td>
<td>2152</td>
<td>22.9</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td>875</td>
<td>3294</td>
<td>26.6</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>1367</td>
<td>4727</td>
<td>28.9</td>
</tr>
<tr>
<td>2.0</td>
<td>6.0</td>
<td>0.04</td>
<td>1250</td>
<td>233</td>
<td>1679</td>
<td>13.7</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td>524</td>
<td>2864</td>
<td>18.3</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td>932</td>
<td>4223</td>
<td>22.1</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>1456</td>
<td>5826</td>
<td>25.0</td>
</tr>
<tr>
<td>2.0</td>
<td>6.0</td>
<td>0.15</td>
<td>1150</td>
<td>820</td>
<td>3973</td>
<td>20.6</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
<td>1846</td>
<td>7209</td>
<td>25.6</td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
<td>3282</td>
<td>11,193</td>
<td>29.3</td>
</tr>
<tr>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td>5128</td>
<td>16,056</td>
<td>31.9</td>
</tr>
</tbody>
</table>
Second, penetrator thermal efficiency decreases as the heated length, $H$, increases. This is to be expected, since a larger heated length creates a greater surface area for side surface heat losses.

Next, thermal efficiency decreases with increasing melting point. This effect is also obvious, because we expect greater heat transfer when there is a larger temperature gradient.

Finally, thermal efficiency increases as the penetration rate increases. Although the side surface heat loss increases with velocity (see Fig. 12), $Q_{\text{melt}}$ increases much more rapidly, resulting in higher thermal efficiencies with higher velocities.

**Reduction of Side Surface Heat Losses**

Table III shows that the thermal efficiency of the penetrator is improved if the heated length of the penetrator is reduced. Since the temperature distribution along the side surface affects the cooling rate of the glass liner, and hence the tempering of the glass, it is recommended that the heated length of the penetrator be reduced to a value consistent only with the side surface power input necessary to achieve a properly tempered glass wall liner.
Rock Properties

The mathematical model used herein could be made more realistic if there were more data on the properties of molten rock. In this case, the layer of molten rock could be included in the model, and the heat flow from the penetrator could be analyzed directly. This analysis would be very difficult, however. Since the penetration rate is a direct function of the rate of heat transfer from the penetrator through the molten rock layer, an upper limit on $V$ may well exist, depending upon the physical characteristics of the penetrator and the rock.

Geothermal Energy Exploration

The results presented in this study are based upon an ambient solid rock temperature of 20 C. Should the Subterrene penetrator be used for geothermal exploration, it is expected that the thermal efficiency will increase with depth (since the temperature of the ambient rock increases with depth).


