THE EFFECTS OF FORCED CONVECTION
ON MACROSEGREGATION

by

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STATEMENT BY AUTHOR

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This thesis has been approved on the date shown below:

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Professor of Metallurgical Engineering

[Date]
October 1, 1979
To my Parents
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ABSTRACT

Laboratory-scale apparatus was used to study effects of forced convection on the macrosegregation and structure of remelted ingots. Analyses also include the effect of the forced convection on the flow of interdendritic liquid in the mushy zone during solidification. The apparatus was used to make a series of Sn-Pb ingots (9-12% Pb) with solidification rates varied from $1.7 \times 10^{-3}$ to $1.0 \times 10^{-2}$ cm/sec.

Forced convection in the liquid pool was introduced by rotating a set of heating elements which produced primary tangential flow around the axis of the ingot. However at rotational speeds greater than 40 rpm, the tangential nature of flow was sufficiently disturbed by thermocouple tubes to cause severe localized macrosegregation. Apart from the interaction with thermocouple tubes, the rotation of the heating elements had the effect of producing a convex shaped liquidus isotherm.

Macrosegregation is calculated numerically by a computer model based on the equations that describe the flow of interdendritic liquid in the mushy zone. The effect of convection in the liquid pool is simulated by changing the prescribed pressure at the liquidus isotherm which is used as a boundary condition.
CHAPTER 1

INTRODUCTION

Electrometallurgical remelting techniques have been developed during the past 25 years for numerous alloys among which are alloy steels and superalloys of high quality required by the aircraft, nuclear and space industries [1]. These electrometallurgical processes are plasma arc melting and remelting, electroslag remelting (ESR), and vacuum arc remelting (VAR). In these methods, except ESR, electric arc is used as a heat source, melting takes place under reduced pressures, and impurities are separated by vaporization. In ESR a molten slag is employed for resistance heating and the slag also serves to remove inclusions and dissolved gases and to improve ingot surface quality.

Although there are a number of objectives which determine the acceptability of ingots produced by the above methods, generally the most important are controlled ingot structure and homogeneity, since these interdependent variables affect the forgeability of ingots and the properties of the final product. Both structure and homogeneity are affected by the solidification rate and the convecting patterns prevailing during solidification in the mushy zone and liquid pool. Variations in structure and homogeneity are also sensitive to alloy composition and ingot size and geometry. Much experimental work [2-5] has been done on the effects of solidification rate and control on convecting patterns in static and remelted ingots, but in all cases the objective was to control interdendritic liquid flow or to minimize convection in the liquid pool.
The effects of forced convection in the liquid pool and solidification rate on macrosegregation and ingot structure in remelted ingots are assessed in this investigation. Specifically, a number of ingots were cast under different solidification rates and with various degrees of forced convection within the liquid pool. Experimental results are compared to calculated results using a computer code which models macrosegregation.
CHAPTER 2

LITERATURE SURVEY

Convection in the liquid ahead of the mushy zone affects the structure and homogeneity of ingots [2-6]. When solidification takes place in a gravitational field, convection is due to the presence of temperature gradients (thermal convection) and concentration gradients (solute convection). Johnston and Griner [7], using NH$_4$Cl-H$_2$O as a metal model, have demonstrated that columnar growth results in the absence of gravitational field whereas an equiaxial zone, due to dendrite fragmentation caused by fluid flow, results in the presence of gravitational field.

Many investigators [8-10], in their attempts to determine the nature of columnar to equiaxial transition and crystal multiplication mechanisms, have studied the effects of convection and how ingot structure is controlled through externally applied forces. Homogeneous magnetic fields constrain the fluid from moving in a plane normal to the direction of field and flow due to Lorentz forces. Rotation has the same effect as a homogeneous magnetic field, but the additional Coriolis force deflects fluid particles in a direction normal to the axis of rotation and normal to the direction of particle motion. In both cases heat flow by convection is diminished, and the Nusselt number approaches that for pure conduction at sufficiently high magnetic fields or rotation. Rotating magnetic fields and oscillation, on the other hand, enhance convection.
Uhlman et al. [11] have shown that by using homogeneous magnetic fields to suppress convection, columnar growth is enhanced. Cole and Bolling [12] demonstrated that rotation of the ingot decreases the natural flow and convective heat transfer by increasing the net temperature gradient; oscillation of the ingot had the opposite effect. The temperature gradient in the liquid has been shown to be the most important variable affecting the columnar to equiaxial transition. The same authors [13], in a later investigation, found that the columnar grain diameter decreases with an increase of solute content independently of the degree of ingot oscillation.

Remelted ingots differ from static ingots not only in the chemical changes that take place during melting but also in the heat transfer and liquid pool convection conditions under which solidification takes place [14,15]. Growth takes place in the presence of a positive temperature gradient ahead of the mushy zone as heat is introduced in the liquid pool constantly. Besides natural convection in the liquid pool there is mixing due to the liquid entering the pool. Nakamura et al. [16] demonstrated that complete mixing takes place in the liquid pool during electroslag remelting. Using a transparent model, Campbell [17] made a qualitative study of the stirring patterns in the slag and metal pool during remelting by visual observation. He concluded that at slow melting rates the droplet size depends on the electrode radius and complete mixing takes place in the slag and liquid pools due to droplet velocity. Campbell attributed the difference in macrostructures observed between ESR and VAR ingots to the fact that in ESR divergence of electric currents occurs primarily in the slag layer,
with the Lorentz effect causing rapid stirring in the slag and very little stirring in the metal pool, whereas in VAR vigorous stirring occurs in the metal pool, which aids dendrite fragmentation.

The effects of externally applied forces to control heat flow and ingot structure in continuous casting and remelted ingots have been documented by many authors [18-20]. Temperature distributions in stirred ESR ingots, measured by Edwards and Spittle [18], indicate an increase in pool depth and a decrease in the temperature gradient ahead of the solid-liquid interface upon stirring. From observed macrostructures it was concluded that a convex-shaped solid-liquid interface had been established. This is contradictory to the increase of pool depth, since with lower temperature gradients, other things being equal, the solidification rate must increase.

Takahashi et al. [21,22] studied the effects of forced convection on macrosegregation. In their experiments the melt was contained between two concentric cylinders with the inner cylinder cooled and rotating. They showed that there was an increase of the deflection angle of dendrites with an increase in the rate of rotation of the inner cylinder and a decrease in the solidification rate. In their analysis of segregation, the "washing effect" of solute due to the presence of convection was considered; however, the effect of the gravity field and the centrifugal force on the convection of interdendritic liquid within the mushy zone was neglected.

Kou et al. [23] demonstrated, experimentally and by a computer model based on equations that describe interdendritic liquid flow, that macrosegregation is caused by solidification shrinkage and gravity, and
is affected by solidification parameters such as solidification rate and depth of mushy zone. Kou et al. [24] also demonstrated how the macrosegregation profile is influenced by an applied centrifugal force when the ingot is subjected to rotation. When macrosegregation is severe, due to low solidification rates, rotation reverses the concentration profile from positive at the centerline to negative, and there is an optimum rotational speed which minimizes the segregation across the ingot. Calculations performed by Keane [25], assuming unidirectional solidification in the horizontal direction, indicate that when convection exists ahead of the mushy zone the severity of macrosegregation increases. Other quantitative work on the effect of natural convection in the liquid on the convection within the mushy zone has been carried out by Szekeley and Jassal [26]. By combining thermal energy equations written for the solid, solid-liquid and liquid regions with the appropriate equations of motion, they were able to predict the temperature profile and the velocity fields in the mushy zone and the liquid region in a system of 30% ammonium chloride in water solidified at a nonsteady rate. However, their analysis is not directly applicable to metallic systems since the local volume fraction within the mushy zone was assumed to depend only on temperature. Their analysis also did not account for density variations in the interdendritic liquid within the mushy zone, nor did it include the solidification contraction (or expansion) which accompanies eutectic solidification at the solidus isotherm. These effects undoubtedly alter the convection in the mushy zone, but their analysis and experimental measurements did indicate that natural convection in the mushy zone induces convection within the mushy zone.
No calculations of macrosegregation resulting from their flow fields were performed.
CHAPTER 3

MACROSEGREGATION ANALYSIS

3.1 General Background

In the past twelve years, it has been shown that flow of interdendritic liquid, and in some cases flow of solid, causes almost all types of segregation in castings and ingots [23-26]. Interdendritic liquid convects as a result of solidification shrinkage, the force of gravity, and penetration of bulk liquid in front of the liquidus isotherm, due to fluid motion in this region, into the mushy zone.

The density of the interdendritic liquid within the mushy zone of a solidifying ingot varies and because of gravity there is natural convection. Convection of interdendritic liquid also occurs due to solidification shrinkage since liquid "feed" metal must flow towards regions where the solidifying solid has a density greater than the local interdendritic liquid. This contribution is called the "solidification induced" or "shrinkage induced" convection.

In previous work on macrosegregation [23-36], the effect of bulk convection in the liquid metal pool, above the liquidus isotherm, on the interdendritic flow velocity was not considered, even though flow in the bulk liquid can be appreciable especially in the presence of an electromagnetic force field as in the ESR process [37]. Any effect of convection in the bulk liquid on the flow of interdendritic liquid and macrosegregation was thought to be small since the penetrating bulk liquid encounters a dense dendritic network, which greatly impedes the
bulk flow, at a short distance behind the advancing dendrite tips. A major part of this thesis is to more critically examine this assumption.

Historically, "solidification induced" convection was probably first treated by Flemings and Nereo [28], who presented the "local solute redistribution equation" (LSRE) which is a key relationship in macrosegregation theory. Because of the importance of the LSRE, its development is reviewed here.

3.2 Theory

The severity of macrosegregation in a given alloy for specified thermal and fluid flow conditions existing at the time of solidification is described quantitatively by the local solute redistribution equation. Figure 1 shows the mushy zone in a unidirectionally solidified ingot and the temperature, liquid composition and fraction solid distribution in the direction of solidification. A small volume element is considered within the mushy zone; it is large enough that the volume fraction of solid within it is equal to local average, but small enough to be treated as a differential element. Solute enters or leaves the element only by liquid flow. Mass flow in and out of the element by diffusion is neglected; therefore dendrite geometry need not be specified. Liquid composition and temperature are uniform within the volume element, and they are related to each other according to the appropriate phase diagram. It is also assumed that during solidification there is negligible solid diffusion and no undercooling of the interdendritic liquid. With these assumptions, solute redistribution within the volume element [28] is given by
Figure 1. Schematic illustration of solidification model

(a) shape of mushy zone in unidirectional solidification;
(b) temperature distribution in the mushy zone;
(c) liquid composition distribution in the mushy zone;
(d) fraction solid distribution in the mushy zone;
(e) insert is a magnified section of the mushy zone as it would appear at approximately 23X in ingots cast in this study.
where

\[
\frac{\partial g_L}{\partial C_L} = - \left( \frac{1}{1 - K} \right) \left[ 1 + \frac{\mathbf{v} \cdot \nabla T}{\varepsilon} \right] \frac{g_L}{C_L}
\]

(1)

where

\[ g_L = \text{volume fraction liquid}, \]
\[ C_L = \text{composition of liquid}, \]
\[ \beta = \frac{(\rho_S - \rho_L)}{\rho_S} \text{ (i.e., solidification shrinkage)}, \]
\[ \rho_S = \text{density of solid}, \]
\[ \rho_L = \text{density of liquid}, \]
\[ K = \text{partition ratio}, \]
\[ \mathbf{v} = \text{velocity vector of interdendritic liquid}, \]
\[ \nabla T = \text{local temperature gradient}, \text{ and} \]
\[ \varepsilon = \text{local cooling rate (rate of temperature change)}. \]

In addition to assumptions given above, other assumptions are constant solid density and no pore formation, i.e.,

\[
g_S + g_L = 1
\]

(2)

where \( g_S \) = volume fraction liquid.

In order to calculate the extent of macrosegregation it is necessary to determine the velocity field of interdendritic liquid, which is a result of the combined effects of "solidification induced" and "gravity induced" convections [34]. The calculation of the flow of interdendritic liquid due to solidification shrinkage combined with "gravity induced" flow was first performed by Mehrabian et al. [32], who used D'Arcy's Law to calculate the flow of interdendritic liquid within
the mushy zone by considering the gravity effect as a body force on the interdendritic liquid. D'Arcy's Law is

\[ \mathbf{v} = -\frac{K}{\mu g_L} (\nabla P + \rho_L \mathbf{g}) \] (3)

where

- \( K \) = specific permeability,
- \( \mu \) = viscosity of the interdendritic liquid,
- \( P \) = pressure, and
- \( \mathbf{g} \) = acceleration due to gravity.

For flow through the volume element, assuming no movement of solid, the continuity equation is

\[ \frac{\partial}{\partial t} (\rho_S g_S + \rho_L g_L) = -\nabla \cdot \rho_L g_L \mathbf{v} \] (4)

where \( t \) is time.

In the LSRE, equilibrium is assumed at the solid-liquid interface; therefore with the chain rule it is shown that

\[ \frac{\partial C_L}{\partial t} = \frac{dC_L}{dT} \frac{\partial T}{\partial t} = \frac{\varepsilon}{m} \] (5)

and

\[ \frac{\partial \rho_L}{\partial t} = \frac{d\rho_L}{dC_L} \frac{\partial C_L}{\partial t} = \frac{d\rho_L}{dC_L} \frac{\varepsilon}{m} \] (6)

where \( m \) is the slope of the liquidus of the phase diagram.
Equations (1-6) were first given and applied by Mehrabian et al. [32]. They calculated pressure, interdendritic liquid flow and macro-segregation in an ingot solidified horizontally with unidirectional heat flow. More recently, the same equations have been applied to cylindrical remelted ingots [23, 24] in which Equations (1-6) were combined and then expanded into cylindrical coordinates (r,z) resulting in the following equation for the pressure distribution within the mushy zone:

$$\frac{\partial^2 P}{\partial r^2} + \frac{\partial^2 P}{\partial z^2} + A \frac{\partial P}{\partial r} + B \frac{\partial P}{\partial z} + C = 0$$

(7)

where A, B, and C are defined as follows.

$$A = \frac{1}{r} + \frac{2}{g_L} \frac{\partial g_L}{\partial r} + \frac{1}{\rho_L} \frac{\partial \rho_L}{\partial r} + \alpha \frac{\partial C_L}{\partial r}$$

(8)

$$B = \frac{2}{g_L} \frac{\partial g_L}{\partial z} + \frac{1}{\rho_L} \frac{\partial \rho_L}{\partial z} + \alpha \frac{\partial C_L}{\partial z}$$

(9)

$$C = g_L \rho_L \left[ \frac{2}{g_L} \frac{\partial g_L}{\partial z} + \frac{2}{\rho_L} \frac{\partial \rho_L}{\partial z} + \alpha \frac{\partial C_L}{\partial z} \right] - \frac{\epsilon \mu}{\gamma g_L} \left[ \frac{1}{\rho_L} \frac{dp_L}{dC_L} + \alpha \right]$$

(10)

where

$$\alpha = \frac{\beta}{(1 - K) C_L}$$

The pressure distribution equation can be solved with the appropriate boundary conditions if the temperature field within the mushy zone is known or if this equation is coupled with the appropriate
thermal energy equation. This was done by Kou et al. [23,24], and similar equations have been used by Ridder et al. [27].

The boundary conditions are as follows:

Centerline: \( v_r = 0 \) because of symmetry;

Mold wall: \( v_r = 0 \) because the mold wall is impermeable;

Solidus isotherm: \[
\frac{v_r - \rho_{SE} \rho_{LE}}{\rho_{LE}} u_{Er} = v_z - \frac{\rho_{SE} \rho_{LE}}{\rho_{LE}} u_{Ez} \]

Liquidus isotherm: \( P = P_0 + \rho_{LE} g \)h if no liquid velocity is assumed in the bulk liquid above the mushy zone; otherwise the stagnation pressure is specified;

\( v_r \) and \( v_z \) are the velocity components of interdendritic liquid, \( u_{Er} \) and \( u_{Ez} \) are the velocity components of solidus isotherm, and \( \rho_{SE} \) and \( \rho_{LE} \) are the densities of eutectic solid and liquid, respectively. The velocity components at the solidus isotherm satisfy continuity when the eutectic liquid solidifies.

When the pressure is known throughout the mushy zone, Equation (3) is applied to determine local velocity, \( \dot{v} \), of the interdendritic liquid. With the obtained velocity, Equation (1) is integrated to
establish the distribution of $g_L$ within the mushy zone. Finally when the fraction liquid distribution in the mushy zone is known, the local average composition is calculated [28] by

$$
\overline{C}_S = \frac{\frac{1-g_E}{\rho S K} \int_0^{g_L} C_L dz_S + \rho_{SE} g_E C_E}{\rho_S (1-g_E) + \rho_{SE} g_E}
$$

where

$g_E = \text{volume fraction of eutectic liquid, and}$

$C_E = \text{eutectic composition.}$

For a cylindrical ingot, in which isotherms move upward at a steady velocity, the integration can be carried out by integrating from the liquidus ($g_S = 0$) down to the solidus ($g_S = 1-g_E$) at a given radius. By doing this for different radial positions within the ingot, $\overline{C}_S$ versus radius is determined which can be plotted to give the pattern of macro-segregation.

To sum up, the analytical model of Kou et al. [23,24] (given a temperature distribution) can be used to predict:

1. the pressure distribution within the mushy zone, Equation (7);
2. the velocity of interdendritic liquid flow within the mushy zone, Equation (3);
3. the distribution of volume fraction liquid within the mushy zone, Equation (1); and
(4) the local average composition after solidification is complete, Equation (11).

3.3 Numerical Calculation of Macrosegregation

From experimentally obtained thermal data and physical properties for the alloy under consideration, all variables involved in the coefficients of the pressure distribution equation are known except $g_L$. With a computer code based on finite difference approximations, computations are initiated by first approximating $g_L$ using the Scheil Equation (i.e., Equation (1) with $\beta$ and $\vec{v}$ equal to zero). Ultimate values of $g_L$ are calculated through an iteration process which proceeds as follows. The pressure distribution in the mushy zone is calculated from Equation (7) with the appropriate boundary conditions. Equation (3) is used to calculate the velocity field, and then a new value of $g_L$ is calculated with Equation (1). This process is repeated, each time upgrading $g_L$, until the calculated local average composition (Equation (11)) converges to initial average composition $C_0$.

The computer code used for these calculations was based upon a code written and used by Kou [39]. However, his code was modified by approximately 65 percent in order to (1) treat the unusual shapes of liquidus isotherms encountered in this work (i.e., the convex upward shapes); (2) take account of the effect of convection, in the bulk liquid, on the boundary condition at the liquidus isotherm; and (3) minimize computer execution time.

The finite difference equations for different types of grid points are given in Appendix A. A simplified flow chart and the
computer notations are given in Appendices B and C, respectively. The computer program itself is listed in Appendix D.
CHAPTER 4

APPARATUS AND EXPERIMENTAL PROCEDURE

4.1 Apparatus

A sketch of the apparatus used to make experimental ingots is shown in Figure 2. It is a modified version of the apparatus used by Kou et al. [23,24] to simulate solidification behavior of ingots by the ESR process. After modification for this work it consisted of four major components: (a) a mold for ingot solidification, (b) a heated container to maintain a superheated melt, (c) a driving system to control the vertical solidification rate, and (d) a rotating set of resistance heaters for heating and convecting the liquid pool. The apparatus was used to make ingots of Sn-Pb alloys containing about 8 to 12 percent Pb.

Three stainless steel tubes, 3 mm in diameter, were located inside the mold at different radii; each contained one chronel-alumel thermocouple. The thermocouples were free to be manually moved up and down within the tubes. The mold was a stainless steel tube 8.3 cm in diameter and 33 cm long.

The melt was maintained in a stainless steel container which was heated by two wide band electric heaters connected in parallel. The temperature of the melt was controlled by a thermocouple connected to a temperature controller. A stirrer was used to insure uniformity of composition and temperature. To prevent oxidation of tin from the Sn-Pb alloys, a layer of carbon powder was placed on top of the liquid metal.
Figure 2. Apparatus used to make experimental ingots
The flow rate of liquid metal was controlled by the adjustable long-stem valve.

The driving system for vertical motion, to which a cooling jacket and the set of cylindrical resistance heaters were attached, had a variable speed range of $1 \times 10^{-3}$ to $2.3 \times 10^{-1}$ cm/sec. The vertical distance between the cooling jacket and the heaters was adjustable. Either water or air could be used for cooling.

The resistance heaters were mounted on stainless steel fixtures, 3.8 cm in diameter and 7.6 cm long. Two different types of fixtures were used; they are shown in Figure 3. They were immersed in the liquid metal above the mushy zone and the power to them was controlled with a variable transformer. Convection in the liquid pool was induced by rotating the heaters by means of the pulley-belt system which was also mounted on the driving system. The input power to the motor that drove the heaters was adjustable such that the heaters could rotate from 7 to 150 rpm. In this work, the maximum rotational speed was 60 rpm.

4.2 Ingot Casting Procedure

Tin and lead of commercial purity were melted in an electric furnace. The cooling jacket was positioned in the lower end of the mold, and the liquid alloy was poured into the melt container. The stirrer, internal resistance heaters and cooling water were turned on. The valve for supply of liquid metal was opened until the liquid level in the mold rose to 1 cm below the top of the internal heaters. The three thermocouples were moved up and down, and the shape of the mushy zone was determined. By varying the position of the cooling jacket and
Figure 3. Types of fixtures used to hold the electric resistors

(a) open type
(b) closed type
the power input to the internal heaters, the desired shape and size of
the mushy zone were obtained.

When a stable mushy zone was obtained (which took as long as
one hour), the driving system was turned on, and the flow rate of
liquid metal was adjusted so that the liquid level inside the mold rose
at the same rate as the cooling jacket and the internal resistance
heaters. The positions and shapes of the solidus and liquidus isotherms
were monitored with the three thermocouples as solidification proceeded.
Rotation of the heaters started at a predetermined angular velocity
either at the beginning of ingot casting or when approximately one-half
of the ingot was cast.

4.3 Metallography

After casting, each ingot (with the exception of Ingots 1 and 2)
was sectioned longitudinally along the axis of symmetry and in a plane
containing the three thermocouple tubes. First one section was polished
and etched with nitric and acetic acids in glycerol (1:1:8) to reveal
its dendritic structure. Then the same section was etched in concen­
trated hydrochloric acid to reveal its grain structure.

Horizontal slices, 0.5 cm thick, were then removed from each
ingot section at selective heights, and were polished for chemical
analysis by x-ray fluorescence. Subsequently, these samples were etched
in the nitric and acetic acids in glycerol so that the secondary
dendrite arm spacings across the ingot could be measured. This pro­
cedure was followed for most of the ingots, but in some cases additional
slices were removed in order to examine planes at various angles from the planes containing thermocouples.

4.4 Chemical Analysis

Chemical analyses of the macrosegregation profiles across the ingots were determined by x-ray fluorescence. A General Electric XRD-5 x-ray spectrometer was used with a platinum tube. The primary white radiation from the tube fluoresced the samples on an area of 2.8 mm in diameter. The size of this area insured that the measured compositions were local average compositions since the secondary dendrite arm spacings encountered were in the range of 35 to 120 microns.

The secondary radiation from the sample was diffracted from a lithium fluoride single crystal, and the intensity of the lead L characteristic line was compared to a standard intensity curve to determine the composition. Standard curves were determined before and after the analysis of each sample to insure that variations in voltage and amperage did not occur during the analysis of each sample. Standards of known composition were those prepared by Kou et al. [23]. All analyses were done by collecting counts for 100 seconds using a scintillation counter. Typical counts were 22,300/100 sec for a composition of 5% Pb and 87,500/100 sec for a composition of 30% Pb.
EXPERIMENTAL RESULTS

The initial mushy zone of Ingot 1 was established, and then the heaters were rotated at 60 rpm. The vertical movement of the isotherms proceeded at a rate of $1.7 \times 10^{-3}$ cm/sec. The positions of solidus and liquidus isotherms at three different radii, as function of time, are given in Figure 4a. From this figure the shape of the mushy zone at 65 minutes from the beginning of solidification is constructed as shown in Figure 4b.

Unexpectedly, it was found that the shape of the liquidus isotherm has a convex shape, for which there are two possible explanations. First, dendrite arms from near the mold wall are fragmented due to forced convection and brought to the center of the ingot where they survive complete remelting and collect together, providing the liquid in the ingot center was slightly supercooled. Second, the flow pattern in the liquid pool is such that liquid metal not only moves in the direction of rotation, but also moves downward at the mold wall and upward at the center. Hence the heat generated by the electric resistors is transferred by convection to the mold wall, such that positive radial temperature gradients are established. As to which of the two mechanisms predominated will be discussed later.

A horizontal slice, approximately 1.5 cm thick, was removed from this ingot at a distance of 11 cm from the bottom. This slice was cut through the plane of the center of the thermocouples and then etched to reveal its grain structure, which is shown in Figure 5. In the
Figure 4. Thermal data for Ingot 1

(a) positions of solidus and liquidus isotherms

(b) shape of mushy zone at 65 minutes
Figure 5. Grain structure of Ingot 1 (Mag 0.69X)

(a) transverse section, top view
(b) longitudinal section
longitudinal section of Figure 5b, the structure consists of some equiaxed grains at the center and columnar grains forming out from the center of the ingot towards the surface. Since grain growth direction is determined by heat and fluid flow conditions [38], the grains of this ingot grew approximately perpendicular to the liquidus isotherm. However, as seen on the left side of Figure 5a, the two thermocouple tubes, not centrally located, have definitely modified the structure. Surface examination of this ingot revealed that grains grew in a spiral fashion, which is an indication that grains tend to grow in the upstream direction of fluid flow [40].

The measured macrosegregation profile across the ingot, in the plane containing the thermocouple tubes, and at a distance of 11 cm from the bottom, is given in Figure 6. The macrosegregation profile is not symmetrical, and this result was not expected at the onset of this work. The thermocouple tubes have a profound effect on the convection in the liquid and, in turn, on the resulting macrosegregation. This is discussed in more detail in a later chapter. Macrosegregation is strongly positive at the center of the ingot (15% Pb) and even more so at a radius of 2.5 cm (20% Pb), where the second thermocouple tube is located. The relative solute content from center to surface is evident in the photomicrographs of Figure 7.

Figure 8 shows the macrosegregation profile at different angles with respect to the plane containing the thermocouple tubes. The segregation peak at \( r = 2.5 \) cm appears at all angles and is due to the wake formed behind the thermocouple tube. In the wake there is a slight pressure reduction which dissipates with increasing \( \theta \). Just behind the
Figure 6. Measured segregation profile of Ingot 1 at a distance of 11.0 cm from the bottom (12.37% Pb)
Figure 7. Microstructures of Ingot 1 (Mag 23X)

(a) center
(b) near second thermocouple tube
(c) near surface
Figure 8. Measured segregation profiles of Ingot 1 at different angles

(a) plane from which angle $\theta$ is measured
(b) $\theta = 2$ degrees
(c) $\theta = 20$ degrees
(d) $\theta = 40$ degrees
(e) $\theta = 60$ degrees
(f) $\theta = 358$ degrees
thermocouple tube (Figure 8b), the composition is 32% Pb. This drops to approximately 13.5% Pb at $\theta = 60^\circ$ (Figure 8e), to 12% Pb at $\theta = 180^\circ$ (left side of Figure 6), and then rises slowly to 15% Pb at $\theta = 358^\circ$ (Figure 8f), which is almost back to the original plane of the thermocouple.

Ingot 2 was cast with the heaters rotating at 43 rpm. At steady state the isotherms moved at a rate of $3.5 \times 10^{-3}$ cm/sec (Figure 9a). The shape of the mushy zone at 40 minutes is given in Figure 9b. Again the liquidus isotherm has a convex shape. The measured macrosegregation profile across the ingot, in the plane containing the thermocouple tubes and at a distance of 15 cm from the bottom, is given in Figure 10b. The extent of positive segregation at the center and at the radial position of the intermediate thermocouple tube is substantially less compared to Ingot 1; otherwise the thermocouple tubes had the same effect as in Ingot 1. Figure 11 shows the macrosegregation profile at different angles with respect to the plane containing the thermocouple tubes. The macrostructure of this ingot is given in Figure 12, which shows the same characteristics as the macrostructure of Ingot 1.

Ingot 3 was cast with the heaters rotating at 10 rpm and at a solidification rate of $4.5 \times 10^{-3}$ cm/sec. The mushy zone at 25 minutes from the beginning of solidification is given in Figure 13b. The shape of the liquidus isotherm is concave, and there is no solute accumulation associated with the intermediate thermocouple tube as observed for Ingots 1 and 2. However, the ingot center exhibited positive centerline
Figure 9. Thermal data for Ingot 2 (11.14% Pb)
(a) positions of solidus and liquidus isotherms
(b) shape of mushy zone at 40 minutes
Figure 10. Results obtained for Ingot 2

(a) calculated flow pattern
(b) segregation profile at 15 cm from the bottom
   (11.14% Pb)
Figure 11. Measured segregation profiles of Ingot 2 at different angles

(a) plane from which angle $\theta$ is measured
(b) $\theta = 2$ degrees
(c) $\theta = 320$ degrees
(d) $\theta = 340$ degrees
(e) $\theta = 358$ degrees
Figure 12. Grain structure of Ingot 2 (Mag 0.69X)

(a) transverse section, top view
(b) longitudinal section
Figure 13. Thermal data for Ingot 3

(a) positions of solidus and liquidus isotherms
(b) shape of mushy zone at 25 minutes
segregation of 12.5% Pb (Figure 14) due to the depth of the solidus isotherm.

After 30 minutes of solidification from the bottom, rotation was stopped; at this point remelting took place and both the solidus and liquidus isotherms moved downward. This ingot and all subsequent ingots were sectioned to reveal the complete vertical plane containing the thermocouples. Grain and dendritic structures of this ingot are given in Figure 15. The bottom of the ingot consists of equiaxed grains which were formed when the first liquid entered the mold and solidified rapidly. In the bottom center the equiaxed grains are coarser, indicating that in this region the original solid remelted and then resolidified as the initial mushy zone was being established. During the period of solidification at a constant rate, columnar grains grew and were aligned approximately parallel to the ingot axis. Equiaxed grains in the upper portion formed in the region where the remelting took place when rotation ceased. An increase in grain width with a decrease in convection in the liquid pool is evident by comparing Figure 15 to Figures 5 and 12. The horizontal bands which are present in the macrostructure of Ingot 3 are due to the sudden change in heat and flow conditions which took place upon remelting.

Ingot 4 was cast with a solidification rate of $9.2 \times 10^{-3}$ cm/sec and without rotation of the heaters. The positions of solidus and liquidus isotherms for this ingot are given in Figure 16a. The shape of the mushy zone at 15 minutes is given in Figure 16b, and the measured macrosegregation profile at 14 cm from the bottom is given in Figure 17. This ingot did not exhibit substantial macrosegregation since it
Figure 14. Results obtained for Ingot 3

(a) calculated flow pattern
(b) segregation profile at 11.0 cm from the bottom (8.96% Pb)
Figure 15. Macrostructures of Ingot 3 (Mag 0.5X)

(a) grain structure
(b) dendritic structure
Figure 16. Thermal data for Ingot 4
(a) positions of liquidus and solidus isotherms
(b) shape of mushy zone at 15 minutes
Figure 17. Results obtained for Ingot 4

(a) calculated flow pattern
(b) segregation profile at 14 cm from bottom (10.33% Pb)
solidified with a relatively high solidification rate. The bottom of the macrostructure consists of equiaxed grains and the rest of the ingot consists of columnar grains (Figure 18), which grew until the heaters were removed and the experiment ceased.

The lower half of Ingot 5 was cast with a solidification rate of \(5.6 \times 10^{-3}\) cm/sec and no rotation of the heaters. When the liquidus isotherm reached a distance of 19 cm from the bottom, rotation of the heaters was started and maintained at 18 rpm. At this time the vertical speed of the isotherms increased rapidly, and after approximately 1.5 minutes a steady rate of \(5.9 \times 10^{-3}\) cm/sec was obtained (Figure 19a). Associated with the change in casting speed there was a change in the shape of the mushy zone, as illustrated by Figure 19b. Very little macrosegregation resulted in the lower half of the ingot (Figure 20b). With rotation in the upper half of the ingot, an increase of approximately 1% in lead was present at the centerline (Figure 20d). A banded region with positive segregation was present across the ingot that corresponds to the change in solidification rate, and it is visible on the macrostructures of this ingot (Figure 21). Lead content along this band was not measured systematically as a function of radius since it does not lie exactly on a horizontal plane and it was not visible without etching, thus preventing exact positioning for x-ray fluorescence. However, from a few measurements made at approximately midradius it was possible to verify that the band had a solute content of approximately 1% greater than above and below the band.

Figures 22 and 23 show typical dendrite arms for Ingots 3 and 4 respectively. There is a slight decrease in dendrite arm spacing from
Figure 18. Macrostructures of Ingot 4 (Mag 0.5X)

(a) grain structure
(b) dendritic structure
Figure 19. Thermal data of Ingot 5
(a) positions of solidus and liquidus isotherms
(b) shape of mushy zone at 15 minutes
(c) shape of mushy zone at 30 minutes
Figure 20. Results obtained for Ingot 5

(a) calculated flow pattern in the lower half
(b) segregation profile in the lower half at 14.0 cm from bottom (10.23% Pb)
(c) calculated flow pattern in the upper half
(d) segregation profile of the upper half at 21.0 cm from bottom (10.12% Pb)
Figure 21. Macrostructures of Ingot 5 (Mag 0.5X)

(a) grain structure  
(b) dendritic structure
Figure 22. Microstructures of Ingot 3 (Mag 23X)

(a) center  
(b) mid-radius  
(c) near surface
Figure 23. Microstructures of Ingot 4 (Mag 23X)

(a) center
(b) mid-radius
(c) near surface
center to surface for both ingots with an obvious difference in dendrite arm spacing due to the difference in vertical solidification rates \(4.5 \times 10^{-3} \text{ cm/sec for Ingot 3 versus } 9.2 \times 10^{-3} \text{ cm/sec for Ingot 4}\).

Figure 22a also shows a patch of eutectic along the centerline of Ingot 3 which corresponds to a "freckle" at the segregation peak shown in Figure 14b.

Ingot 6 was cast with the heaters mounted on the stainless steel fixture shown in Figure 3b. This fixture was solid with a hole 0.7 cm in diameter, to accommodate the central thermocouple tube, and with holes drilled at the periphery to hold the electric resistors. The purpose of this was to determine if heat and fluid flow conditions are affected by the design of the heater. The lower half of this ingot was cast with the heaters rotating at 10 rpm and a solidification rate of \(3.7 \times 10^{-3} \text{ cm/sec}\); when the liquidus isotherm reached a distance of 14 cm from the bottom, rotation was increased to 40 rpm. At this time the vertical rate of isotherms increased rapidly, and after a period of approximately 1.5 minutes they moved with a steady rate of \(5.4 \times 10^{-3} \text{ cm/sec}\) (Figure 24a). The shape of the mushy zone corresponding to the lower half of the ingot at 10 minutes is constructed in Figure 24b and that of the upper half at 23 minutes in Figure 24c. The lower half of the ingot exhibited positive centerline segregation, measured at 11 cm from the bottom (Figure 25b). Macrosegregation in the upper half, measured at 16 cm from the bottom, exhibited the same characteristics as Ingots 1 and 2, but without any centerline segregation (Figure 25d). Grain and dendritic structures of this ingot are given in Figures 26a and 26b, respectively. In the region where the rotational speed was changed,
Figure 24. Thermal data for Ingot 6

(a) positions of solidus and liquidus isotherms
(b) shape of mushy zone at 10 minutes
(c) shape of mushy zone at 23 minutes
Figure 25. Results obtained for Ingot 6

(a) calculated flow pattern in the lower half
(b) segregation profile of the lower half at 11.0 cm from bottom (9.74% Pb)
(c) calculated flow pattern in the upper half
(d) segregation profile of the upper half at 16.0 cm from bottom (10.55% Pb)
Figure 26. Macrostructures of Ingot 6 (Mag 0.5X)

(a) grain structure
(b) dendritic structure
there is very slight evidence of banding in the macrostructure although a strong band is apparent across the lower half of the ingot. The latter formed when the initial mushy zone was being established before solidification proceeded upward at a constant rate.

Ingot 7 was cast similarly to Ingot 5 but with the second type of fixture holding the heaters. The lower half was cast with a solidification rate of $7.3 \times 10^{-3}$ cm/sec and no rotation; rotation of 23 rpm was initiated when the liquidus isotherm reached a distance of 16 cm from the bottom, and after a transient period of approximately 1.5 minutes solidification proceeded at a steady rate of $9.0 \times 10^{-3}$ cm/sec (Figure 27).

Ingot 8 was cast the same way as Ingot 7, the lower half with a solidification rate of $9.2 \times 10^{-3}$ cm/sec and no rotation and the upper half with a solidification rate of $1.0 \times 10^{-2}$ cm/sec and rotation of 25 rpm. Thermal data, measured macrosegregation profiles and photomacrostructures for Ingots 7 and 8 are given in Figures 28 to 32. In the transition region between no rotation and rotation, banding is not evident in either ingot. Only slight surface to center segregation exists in these ingots; there is no effect of rotation on macrosegregation in either ingot because of the relatively high solidification rate (approximately $10^{-2}$ cm/sec) employed.

In order to better understand the formation of the convex liquidus isotherms encountered in the macrosegregation experiments, a number of thermal measurements were taken from ingots with stationary mushy zones, i.e., no vertical movement of the isotherms, and with the heaters rotating at different rates and increased amount of superheat.
Figure 27. Thermal data for Ingot 7  
(a) positions of solidus and liquidus isotherms  
(b) shape of mushy zone at 5 minutes  
(c) shape of mushy zone at 15 minutes
Figure 28. Results obtained for Ingot 7

(a) calculated flow pattern in the lower half

(b) segregation profile of the lower half at 12.5 cm from bottom (11.49% Pb)

(c) calculated flow pattern in the upper half

(d) segregation profile of the upper half at 17.5 cm from bottom (11.42% Pb)
Figure 29. Macrostructures of Ingot 7 (Mag 0.5X)

(a) grain structure
(b) dendritic structure
Figure 30. Thermal data for Ingot 8

(a) positions of solidus and liquidus isotherms

(b) shape of mushy zone at 7 minutes

(c) shape of mushy zone at 17 minutes
Figure 31. Results obtained for Ingot 8

(a) calculated flow pattern in the lower half
(b) segregation profile of the lower half at 13.0 cm from bottom (11.05% Pb)
(c) calculated flow pattern in the upper half
(d) segregation profile of the upper half at 19.0 cm from bottom (11.78% Pb)
Figure 32. Macrostructures of Ingot 8 (Mag 0.5X)

(a) grain structure
(b) dendritic structure
Shapes of mushy zones corresponding to different angular velocities are given in Figure 33. Once the first mushy zone had been established with no forced convection in the liquid pool, rotation of the heaters was initiated and after a period of approximately 15 minutes a new stationary mushy zone was established. This procedure was repeated for each mushy zone by increasing the degree of rotation each time.

From Figure 33 it is seen that the shape of liquidus isotherm in the central region changes from slightly concave to convex as the speed of rotation is increased. In order to determine this mechanism the temperature distributions in the solid, mushy zone and bulk liquid were measured from ingots with stationary mushy zones for both cases of no rotation and rotation. Figures 34a and 34b give the isotherms with the heaters stationary and with rotation at 73 rpm, respectively. In the case of no rotation the radial temperature gradients are negative despite the natural convection due to superheat. When the heaters are rotated, the radial temperature gradients remain negative in the completely solidified region (less than 183° C); but at increased rates of rotation they become progressively positive in the mushy zone and the liquid pool, and towards the top of the resistors they become negative again.

From these measurements it is evident that the heat flow conditions in the bulk liquid and therefore in the mushy zone are altered by the forced convection. As to how heat flow conditions change with rotation of bulk liquid will be discussed later in conjunction with the formation of the convex liquidus isotherm.
Figure 33. Shape of stationary mushy zones at different angular velocities

(a) $\omega = 0$ rpm
(b) $\omega = 22$ rpm
(c) $\omega = 45$ rpm
(d) $\omega = 85$ rpm
Figure 34. Shape of isotherms in stationary mushy zones

(a) $\omega = 0$ rpm
(b) $\omega = 73$ rpm
The secondary dendrite arm spacings measured from all ingots are plotted as function of local solidification time in Figure 35. Local solidification time is defined as

\[ t_f = \frac{Z_L - Z_S}{R} \]

where

- \( Z_L \) = height of liquidus isotherm,
- \( Z_S \) = height of solidus isotherm, and
- \( R \) = casting speed.

The equation describing the curve of Figure 35 is

\[ d = bt^n \]

where \( b = 7.15 \) and \( n = 0.35 \). Values reported for \( b \) and \( n \) by Ridder et al. [27] for Sn-15% Pb are 7.47 and 0.346, respectively. These data were examined to see if effects of rotation (i.e., convection) on dendrite arm spacings were apparent. No such effect was detected and so all data are shown in Figure 35 as a single set.
Figure 35. Secondary dendrite arm spacing as function of local solidification time
CHAPTER 6

CALCULATION OF PRESSURE DUE TO CONVECTION

When forced convection is present above the mushy zone, the pressure along the liquidus isotherm is calculated by assuming that the rotating electric resistors and the holding fixture approximate a cylinder rotating inside the mold. In this case the Navier-Stokes equation for flow in the \( \theta \)-direction reduces to

\[
\frac{d}{dr} \left[ \frac{1}{r} \frac{d}{dr} (rv_\theta) \right] = 0
\]

where \( v_\theta \) = peripheral velocity.

This assumption was applied to both types of heaters shown in Figure 3; no significant difference in heat and flow conditions was detected between the two types of fixtures. Further, if the bottom of the heater is assumed to behave as a disc rotating in an infinite fluid [41], the maximum axial velocity component of fluid near the disc is less than 0.3 cm/sec when the disc is rotating at 60 rpm, which is negligible compared to the peripheral velocity produced from the rotation of the inner cylinder.

Integration of Equation (12) with the boundary conditions:

\[
v_\theta = KRw \text{ at } r = KR, \text{ and } v_\theta = 0 \text{ at } r = R
\]

where

\[
K = \frac{r_i}{R}, \quad r_i = \text{radius of inner cylinder},
\]
\( R = \) radius of outer cylinder (mold), and
\( \omega = \) angular velocity of inner cylinder

gives the following equation for the velocity in the \( \theta \)-direction:

\[
\nu_\theta = \frac{\omega K R^2}{1-K^2} \left( \frac{1}{r} - \frac{r}{R^2} \right). 
\] (13)

The Navier-Stokes equations for the \( r \) and \( z \) directions are

\[
\frac{\partial P}{\partial r} = \frac{\nu_\theta^2}{r}, \quad \frac{\partial P}{\partial z} = \rho g_z. 
\] (14) (15)

The pressure distribution within the liquid metal must satisfy Equations (14) and (15) and the total differential

\[
dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz. 
\] (16)

After substitutions of Equations (13-15) into Equation (16) and integrating, the integration constant is evaluated at

\[
r = R, \ z = 0 \quad P = P_0
\]

where the coordinate point \( r = R, \ z = 0 \) corresponds to the free surface of the liquid metal at the mold wall and \( P_0 = \) ambient pressure.

The equation for the pressure distribution in the region

\[
KR < r < R
\]

becomes

\[
P = \frac{\rho}{2} \left( \frac{\omega K R^2}{1-K^2} \right)^2 \left( \xi^2 - \frac{1}{\xi^2} - 4 \ln \xi \right) + \rho g_z + P_0
\] (17)

where \( \xi = r/R \) (radial dimensionless coordinate).
In the region \( 0 \leq r \leq KR \) (i.e., below the bottom of the electric resistors) it is assumed that

\[
v_{\theta} = \omega r.
\]  

(19)

After substitution of Equations (13), (14) and (19) into Equation (16) and integrating, the constant of integration is evaluated at

\[
r = KR, \quad z = z_{KR} \quad \text{where} \quad P = P_{KR}.
\]

\( z_{KR} \) is the axial distance between the free surface of the liquid metal and a point below the bottom of the electric resistors, and \( P_{KR} \) is calculated from Equation (17) by substituting \( r = KR \) and \( z = z_{KR} \).

Finally, the equation for the pressure distribution in the region \( 0 \leq r \leq KR \) becomes

\[
P = \frac{\rho \omega^2 R^2}{2} \left( (\xi^2 - K^2) + \left( \frac{K^2}{1-K^2} \right)^2 \left( K^2 - \frac{1}{K^2} - 4 \ln K \right) \right) + \rho g_z + P_0.
\]  

(20)

Either pressure or pressure gradients (i.e., derivatives of Equations (17) and (20) with respect to \( r \) and \( z \)) are used as boundary conditions. Appendix A gives the finite difference equations when pressure gradients are specified. However, as it will be seen below, it is more difficult to account for the effects of the thermocouple tubes when pressure gradients are specified. Figures 36 and 37 show the variation of peripheral velocity and dynamic pressure (i.e., Equations (17,20) without the last two terms) with respect to \( \xi \).
Figure 36. Tangential velocity as function of radial dimensionless coordinate
Figure 37. Dynamic pressure as function of radial dimensionless coordinate
The mathematical model takes into account (in a semiquantitative manner) the case where the flow in the liquid pool was strong enough to cause solute accumulation behind the thermocouple tubes. In this case the pressure, at the liquidus isotherm and at the position of the thermocouple tube, is reduced to the point where the value of the dimensionless group $\frac{\nu \cdot vT/e}{\varepsilon}$ of Equation (1) approaches -1. When $\frac{\nu \cdot vT/e}{\varepsilon} < -1$, quantitative calculation of macrosegregation is no longer valid.

The pressure reduction due to thermocouple tube effect relative to dynamic head of the free stream velocity is indicated by the dimensionless group

$$\frac{P - P_0}{1/2 \rho v_\infty^2}$$

where

- $P$ = stagnation pressure, calculated from Equation (17),
- $P_0$ = static pressure, equal to ambient pressure plus the metallostatic head, and
- $v_\infty$ = free stream velocity, calculated from Equation (13).

It is only an approximate indication of pressure reduction since flow is assumed to be unidirectional.

The values of this dimensionless group used in calculations for the ingots where the thermocouple tubes had an effect on the macrosegregation profile are given in Tables I and II. These values lie between -1.58 to -2.66; they are within the limits predicted for ideal flow.
TABLE I

Solidification Parameters of the Lower Half of Ingots

<table>
<thead>
<tr>
<th>Ingot</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Solidification rate (cm/sec)</td>
<td>1.7x10^{-3}</td>
<td>3.5x10^{-3}</td>
<td>4.5x10^{-3}</td>
<td>9.2x10^{-3}</td>
<td>5.56x10^{-3}</td>
<td>3.7x10^{-3}</td>
<td>7.3x10^{-3}</td>
</tr>
<tr>
<td></td>
<td>Rotation (rpm)</td>
<td>60</td>
<td>43</td>
<td>10</td>
<td>--</td>
<td>--</td>
<td>10</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>C_o (% Pb)</td>
<td>12.37</td>
<td>11.14</td>
<td>8.96</td>
<td>10.33</td>
<td>10.23</td>
<td>9.74</td>
<td>11.49</td>
</tr>
<tr>
<td></td>
<td>Range of 2πY DAS (μ)</td>
<td>118-100</td>
<td>84-60</td>
<td>75-70</td>
<td>40-35</td>
<td>43-37</td>
<td>53-43</td>
<td>39-36</td>
</tr>
<tr>
<td></td>
<td>γ_o (cm²)</td>
<td>1x10^{-7}</td>
<td>2x10^{-7}</td>
<td>2x10^{-7}</td>
<td>5x10^{-7}</td>
<td>1x10^{-7}</td>
<td>7x10^{-7}</td>
<td>2.5x10^{-7}</td>
</tr>
<tr>
<td></td>
<td>P-P_o</td>
<td>-1.58</td>
<td>-2.66</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>1/2ρV∞²</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Ingot</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
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<tr>
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<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Solidification rate (cm/sec)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>5.86x10^{-3}</td>
<td>5.4x10^{-3}</td>
<td>9.0x10^{-3}</td>
<td>1.0x10^{-2}</td>
</tr>
<tr>
<td>Rotation (rpm)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>18</td>
<td>40</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>C_o (%Pb)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>10.12</td>
<td>10.55</td>
<td>11.42</td>
<td>11.78</td>
</tr>
<tr>
<td>Range of 24V DAS (μ)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>44-37</td>
<td>50-40</td>
<td>40-36</td>
<td>42-37</td>
</tr>
<tr>
<td>γ_o (cm^2)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>2.5x10^{-7}</td>
<td>2.5x10^{-7}</td>
<td>1x10^{-7}</td>
<td>1x10^{-7}</td>
</tr>
<tr>
<td>P-P_o</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>-2.5</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
around circular cylinders and they are close to experimentally measured values as seen in Figure 38.
Figure 38. Pressure distribution around a circular cylinder, measured during the process of acceleration from rest
CHAPTER 7

COMPARISON BETWEEN EXPERIMENTAL
AND CALCULATED RESULTS

Calculation of macrosegregation profiles were done using experimental data and the phase diagram and density data for the Sn-Pb system as shown in Figures 39a and 39b, respectively. The value of viscosity used was 2.2 centipoises [47].

The width of the mushy zones and therefore the dendrite arm spacing varies from center to surface. The mathematical model takes into account this variation by making permeability a function of the secondary dendrite arm spacing, d, and fraction liquid. First, permeability, $K$, is assumed to vary with fraction liquid according to

$$K = \gamma g_L^2.$$  

There are some experimental data obtained by Piwonka and Flemings [48] and by Apelian et al. [49] which show that this applies for $0 \leq g_L < 0.35$, and the relationship has been used in calculation of macrosegregation by previous workers [23-25,27,32]. Second, Streat and Weinberg [50] measured permeability in Pb-Sn alloys and found that permeability varies with dendrite arm spacing; thus a value of $\gamma$ is selected at the centerline ($\gamma_0$) and $\gamma$ varies according to

$$\gamma = \gamma_0 \left(\frac{d}{d_0}\right)^n$$

where
Figure 39. Tin-lead system

(a) phase diagram
(b) densities of solid and liquid during solidification
\[ n = 2 \text{ for } 25 < d < 45 \text{ microns,} \]
\[ n = 5 \text{ for } 45 < d \text{ microns, and} \]

\[ d_0 \text{ is the secondary arm spacing at the center of the ingot.} \]

The value of \( \gamma_0 \) at the centerline used for calculations in each ingot was that which gave best agreement with results for macrosegregation. Only in the case where the flow in the bulk liquid and the thermocouple tubes have influenced the segregation profile, a value of \( \gamma_0 \) was selected such that any further increase in \( \gamma_0 \) would have produced "freckles" (i.e., \( \nabla \cdot \mathbf{V}/\varepsilon < -1 \)). When \( \nabla \cdot \mathbf{V}/\varepsilon < -1 \), the model for calculating macrosegregation is unstable in that the computer code does not converge to a fixed set of values of \( g_L \) throughout the mushy zone. Thus, a calculation of the flow field of the interdendritic liquid gives meaningful results only up to the point of predicting the formation of "freckles". Such a calculation gives the directions of flow lines, but underestimates the magnitude of velocity particularly within regions where \( \nabla \cdot \mathbf{V}/\varepsilon < -1 \).

In order to determine the effect of the convex shaped liquidus isotherm of Ingot 1, the flow pattern and segregation were calculated by ignoring the presence of convection in the liquid pool and the thermocouple, Figures 40a and 40b. Only metallostatic head was used as boundary condition at the liquidus isotherm. The measured macrosegregation profile is not axisymmetric (Figure 6), due to the presence of thermocouple which causes three-dimensional flow, but the calculated segregation profile is axisymmetric because the model only accounts for two-dimensional flow. The effect of the convex liquidus isotherm is to
Figure 40. Calculated results for Ingot 1

(a) flow pattern with no convection in the liquid pool; (b) segregation profile with no convection in the liquid pool; (c) flow pattern with convection in the liquid pool rotated at 60 rpm; (d) segregation profile with convection in the liquid pool; (e) flow pattern resulting from convection in the liquid pool and the presence of thermocouple tubes; (f) segregation profile resulting from convection in the liquid pool and the presence of thermocouple tubes.
cause downward flow from the center which fans outward, i.e., flow is from hotter to cooler regions, except in the upper region near the wall. The spacing of the flow lines in Figure 40a is approximately proportional to the inverse of the velocity magnitude. Interdendritic flow velocity is larger near the center of the ingot, resulting in negative segregation in this region. If the liquidus isotherm had the more usual concave shape, the segregation would be positive at the centerline. The flow reverses direction near the abrupt change in slope of the liquidus isotherm, and there is slight positive segregation.

In the flow pattern of Figure 40c the effect of convection in the bulk liquid is taken into account, by specifying pressure along the liquidus isotherm according to Equations (17) and (20). Due to the rotation, the pressure near the mold wall is now great enough to cause upward flow of interdendritic liquid in the central region in the upper half of the ingot, resulting in positive segregation at the center, Figure 40d. Slight positive segregation occurs at the midradius where the flow is converging towards the center and the axial velocity component diminishes.

When both effects of convection and thermocouple tube are taken into account, the calculated flow pattern is approximately the same as if the presence of thermocouple tubes is neglected, Figure 40e. The only difference is that there is localized flow towards the region of the thermocouple tube, and an additional peak of positive segregation results, Figure 40f. The velocity magnitude in the above calculated flow patterns lies in the range of $10^{-4}$ cm/sec in most of the mushy zone.
and $10^{-3}$ cm/sec in the upper central region just below the liquidus isotherm.

From the calculated flow patterns it is seen that the effect of convection in the liquid pool is to alter the flow pattern of interdendritic liquid, causing a complete flow reversal in the upper half of the mushy zone which results in the centerline segregation peak measured in this ingot (Figure 6). A pressure reduction of approximately 1.58 times the dynamic head causes the formation of positive segregation of 14% Pb (Figure 40f). But as the pressure reduction increases with $\theta$ along the wake, the segregation peak decreases, as shown by the measured segregation profiles (Figures 6 and 8).

The calculated flow pattern and macrosegregation profile of Ingot 2 are shown in Figures 10a and 10b, respectively. Again the measured segregation profile is not axisymmetric, but centerline segregation is less compared to Ingot 1. The shapes of the mushy zones of Ingots 1 and 2 are approximately the same, but their segregation profiles differ by the centerline segregation peak which is present only in Ingot 1, and the positive segregation due to the thermocouple tube is stronger in Ingot 1 than in Ingot 2. Segregation in Ingot 2 is less than in Ingot 1 because rotation of the bulk liquid is reduced (from 60 rpm for Ingot 1 to 43 rpm for Ingot 2) and vertical solidification rate is greater ($3.5 \times 10^{-3}$ cm/sec versus $1.3 \times 10^{-3}$ cm/sec).

To show the effect of a reduced rotation, the flow pattern of Ingot 1 was calculated assuming that the heaters were rotating at the same rate as in Ingot 2, i.e., 43 rpm. The calculated flow pattern and
segregation profile are shown in Figures 41a and 41b, respectively. The flow pattern is identical to Figure 40a, but the velocity of interdendritic liquid which moves towards the liquidus isotherm is reduced. Segregation at the centerline is reduced to 0.6% Pb. Figure 41c shows the combined effect of reduced rotation and increased solidification rate for Ingot 1. The flow pattern and segregation profile were calculated assuming that the electric heaters were rotating at 43 rpm and that the casting speed was the same as in Ingot 2, i.e., \(3.5 \times 10^{-3}\) cm/sec. The rate of flow towards the solidus isotherm is increased, since more liquid is solidified per unit time, and the effect of convection in the liquid pool is minimized resulting in only 0.3% Pb centerline positive segregation (Figure 41d).

In Ingot 3 (Figure 14a), interdendritic liquid moves down from the surface and towards the center; at approximately half the mushy zone height it is reversed upward resulting in positive centerline segregation (Figure 14b). Gravity has a strong effect in determining the flow pattern due to the large height of the mushy zone, and the relatively slow solidification rate of \(4.5 \times 10^{-3}\) cm/sec. Although calculations include the effects of rotation, those effects are small since the rotational speed is only 10 rpm. Agreement between measured and calculated segregation profiles is good.

The movement of interdendritic liquid in the mushy zone of Ingot 4 is downward and perpendicular to solidus isotherm. The gravity effect is minimum since the vertical solidification rate is high (\(9.2 \times 10^{-3}\) cm/sec). No flow reversal occurs (Figure 17a), and agreement between calculated and experimental results is excellent (Figure 17b).
Figure 41. Calculated results for Ingot 1 with reduced convection in the liquid pool

(a) flow pattern with the heaters rotating at 43 rpm;
(b) segregation profile with reduced convection;
(c) flow pattern with reduced convection (43 rpm) and increased solidification rate (3.5 x 10^{-3} cm/sec);
(d) segregation profile with reduced convection and increased solidification rate.
Ingot 5 was solidified at an intermediate rate (approximately \(6 \times 10^{-3}\) cm/sec) with no rotation. Flow lines in the mushy zone of the lower half point downward and perpendicular to the solidus isotherm, resulting in slight positive segregation (Figures 20a and 20b). In the mushy zone of the upper half of the same ingot they point towards the center and out towards the liquidus isotherm, resulting in positive segregation of 1% Pb (Figures 20c and 20d). Although the modest rotational speed is enough to produce a convex mushy zone, the effects of gravity and convection result in some positive segregation at the ingot center.

The calculated flow patterns and segregation profiles for both halves of Ingot 6 are shown in Figure 25. As in Ingot 3 (Figure 14), a rotational speed of 10 rpm has little effect on the flow of interdendritic liquid and macrosegregation (Figures 25a and 25b). In the upper half of this ingot, which was rotated at 40 rpm, the calculated flow in the mushy zone is upward in the central region and in the location of the second thermocouple tube with the segregation profile exhibiting peaks at these regions.

Calculated flow patterns and segregation profiles for Ingots 7 and 8 are given in Figures 28 and 31, respectively. The effects of gravity and convection in the liquid pool are minimized since both of these ingots solidified with a high casting speed (approximately \(10^{-2}\) cm/sec). It is apparent from these results that the effects of rotation at 23-25 rpm, when the casting speed is this high, is minimal, although the shape of the mushy zone in both ingots is changed somewhat.
8.1 Effect of Convection on Liquidus Isotherm Shape

Thermal measurements with stationary mushy zones (Figures 33, 34) preclude the mechanism of "dendrite break off" as being responsible for the formation of a convex shaped liquidus isotherm, since positive radial temperature gradients exist above the mushy zone while no new dendrites are formed. Fluid flow is responsible for the formation of the convex shaped liquidus isotherm, since when fluid flows between two concentric cylinders with the inner cylinder rotating at a certain Taylor's number, the purely tangential flow becomes unstable and there appear vortices whose axes are located along the circumference and which rotate in alternately opposite directions [41]. In this case two such vortices were established, one approximately 1 cm above the bottom of the electric resistors (Figure 34b), and one below such that in the first vortex the fluid moved upward in the mold wall and downward next to the heaters; in the second vortex the fluid moved in the opposite direction, thereby giving up heat generated by the electric resistors at the mold wall.

The amount of heat transferred by convection at the mold wall increases with increased rotation and becomes more important in determining the shape of the liquidus isotherm. As rotation increases, heat transferred by convection increases with the radius of the mold such that at approximately 30-40 rpm positive radial temperature gradients are established in the liquidus isotherm region.
8.2 Effect of Pressure Gradients on Macrosegregation

Macrosegregation depends strictly on the convecting conditions prevailing in the mushy zone during solidification. The velocity direction of the interdendritic liquid and its magnitude determine the type and extent of the resulting macrosegregation respectively. Interdendritic liquid flow is controlled by the pressure gradients resulting from density variation of liquid in the mushy zone and the presence of convection ahead of the mushy zone. Although in the mathematical model pressure is specified along the liquidus isotherm, it is the pressure gradients that determine the velocity direction and magnitude of interdendritic liquid (D'Arcy's Law). The degree of effectiveness of these pressure gradients depends on the resistance offered to the flow by the interdendritic porous network and the time available for flow. When there is convection in the liquid pool of the same pattern as in the present study, the imposed pressure gradient at the dendrite tips has the same effect as the liquid density gradient in the mushy zone. However, when the liquidus isotherm has a convex shape it has the opposite effect, and when convection in the liquid pool is strong enough it reverses the direction of interdendritic flow. This is seen in the calculated flow patterns for Ingot 1 (Figures 40a and 40c). In the first case when there is no convection in the liquid pool, flow lines fan outward and segregation is negative; but in the presence of convection flow lines fan inward and upward resulting in positive segregation. However, when the pressure gradient at the dendrite tips is reduced (Figure 41a), its effect is to cancel the driving force of the density gradient, resulting in minimum segregation. As to which of the two
gradients will predominate in a particular case depends upon the strength of the convection in the liquid pool and the slope of the liquidus isotherm.

Because the magnitude of the interdendritic velocity depends on the resistance offered to flow, small pressure gradients at the dendrite tips will affect the flow, since in this region the value of permeability is high \((g_L = 1)\). This is also true for the center region where variation of dendrite arm spacing is small, and becomes important for ingots having large dendrite arms with a large change in local solidification time from center to surface. In this case permeability varies with the fifth power of secondary dendrite arm spacing.

Small pressure reductions (i.e., increased pressure gradients) at the dendrite tips cause the development of localized flow patterns in the presence of wakes, as observed in this study. Liquid metal enters the mushy zone and becomes enriched in solute as it reverses direction and moves towards the low pressure region of the wake, causing positive segregation. As the pressure reduction is decreased with increased \(\theta\), the effect of thermocouple tubes is decreased. The localized flow patterns become weaker and the segregation peak decreases (Figure 8).

Since the effectiveness of the pressure gradients is time dependent, their effect is decreased with solidification rates above \(5 \times 10^{-3} \text{ cm/sec}\) for the ingots encountered in this study in which the electric resistors were rotating at less than 25 rpm, despite the fact that the shape of the liquidus isotherms was altered. Interdendritic liquid must "feed" solidification shrinkage before pressure gradients have any effect on its flow. As solidification rate is increased, more
liquid moves towards the solidus isotherm per unit time, thereby decreasing the effect of the pressure gradients.
CHAPTER 9

CONCLUSIONS

1. Increased rotational convection in the liquid pool increases the dynamic pressure along the liquidus isotherm, and it is alterations in the dynamic pressure which significantly affect the flow of interdendritic liquid in the mushy zone. In this study, the variations in the dynamic pressure were produced by rotating the immersed heaters. However, forced convection produced by other means would, of course, alter the dynamic pressure along the liquidus isotherm and so the findings are not restricted to forced convection in liquid pool produced by rotation. Thus when the dynamic head at the mold wall in the liquid pool is increased (due to increasing rotation), it has the effects of

(a) increasing the solidification rate and the depth of the mushy zone at any solidification rate in the range of \(10^{-3}\) to \(10^{-2}\) cm/sec when the rate of rotation is greater than 10 rpm;
(b) changing the shape of the liquidus isotherm from concave to convex when the rate of rotation is greater than approximately 18 rpm;
(c) changing the growth direction of columnar grains and decreasing their width above 40-60 rpm.

2. The presence of thermocouple tubes influences the segregation profiles such that in the case of high rates of rotation, localized segregation with compositions as rich as 32% Pb behind the thermocouple
tubes were found. The effect of the thermocouple tubes in causing positive segregation is decreased with decreased convection in the liquid pool (less than 25 rpm), and increased solidification rates (above $5 \times 10^{-3}$ cm/sec). The localized segregation associated with a thermocouple tube dissipates as distance, at a given radius, from the tube increases. The effect is really due to the local decrease of the dynamic pressure at the position of the thermocouple tube at the liquidus isotherm.

3. The effect of the pressure gradients at the dendrite tips due to rotational convection in the liquid pool complements the effects of the density gradients in the mushy zone, if the liquidus isotherm has a concave shape. When the shape of the liquidus isotherm is convex, the effect of these pressure gradients is to oppose the density gradients and, if convection in the liquid pool is strong enough, to invert the segregation profile from negative centerline segregation to positive centerline segregation.

4. Numerical calculations, which predict the extent of macrosegregation in remelted ingots, compare well with experimental results. The calculated flow patterns of the interdendritic liquid in the mushy zone demonstrate the effect of the forced convection in the liquid pool on the pressure and pressure gradients at the dendrite tips, and the resulting flow field of the interdendritic liquid in the mushy zone.
APPENDIX A

FINITE DIFFERENCE FORMS OF
PRESSURE DISTRIBUTION EQUATION

The solution of the pressure distribution equation in finite difference for node \((I,J)\) is

\[
P(I,J) = \frac{(RIG + TOP + LEF + BOT + KON)}{CON} \quad (A1)
\]

The expressions for the variables on the right side depend on the position of the grid points relative to the boundaries. The following notations are used in the subsequent derivation of finite difference equations.

- \(HC\) = specified radial spacing increment
- \(HA\) = radial distance between points \((I,J)\) and \((I+1,J)\)
- \(HB\) = radial distance between points \((I,J)\) and \((I-1,J)\)
- \(KC\) = specified axial spacing increment
- \(KA\) = axial distance between points \((I,J)\) and \((I,J+1)\)
- \(KB\) = axial distance between points \((I,J)\) and \((I,J-1)\)

1. Regular Interior Grid Points

Grid points are designated as regular interior if \(HC = HA = HB\) and \(KC = KA = KB\). They are classified as type 1.

\[
\frac{\partial P}{\partial r} \bigg|_{I,J} = \frac{(P(I+1,J) - P(I-1,J))}{(2HC)} \quad (A2)
\]
\[ \frac{\partial^2 p}{\partial r^2}_{I,J} = \frac{(p(I+1,J) + p(I-1,J) - 2p(I,J))}{HC^2} \]  
(A3)

\[ \frac{\partial p}{\partial z}_{I,J} = \frac{(p(I,J+1) - p(I,J-1))}{(2KC)} \]  
(A4)

\[ \frac{\partial^2 p}{\partial z^2}_{I,J} = \frac{(p(I,J+1) + p(I,J-1) - 2p(I,J))}{KC^2} \]  
(A5)

Substituting these equations in the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).

\[
\begin{align*}
RIG &= \frac{1}{HC} + \frac{A}{(2HC)} \ p(I+1,J) \\
TOP &= \frac{1}{KC} + \frac{B}{(2KC)} \ p(I,J+1) \\
LEE &= \frac{1}{HC^2} - \frac{A}{(2HC)} \ p(I-1,J) \\
EOT &= \frac{1}{KC} - \frac{B}{(2KC)} \ p(I,J-1) \\
KON &= C \\
CON &= 2/HC^2 + 2/KC^2
\end{align*}
\]  
(A6-A11)

2. Centerline Grid Points

Centerline grid points are those points that lie on the centerline and HA = HC, HB = 0, KA = KB = KC. They are classified as type 2.

All derivatives of the pressure distribution equation are the same as in the case of regular interior grid points (Equations (A2-A5)). But because of symmetry, \( P(I-1,J) = P(I+1,J) \) and \( \frac{\partial p}{\partial r} = \frac{\partial^2 p}{\partial r^2} \).

Substituting these into the pressure distribution equation (Equation
(7)), the following expressions are obtained for the variables of the left side of Equation (A1).

\[
\begin{align*}
\text{RIG} & = (4/HC^2) \, P(I+1,J) \quad \text{(A12)} \\
\text{TOP} & = (1/KC^2 - B/(2KC)) \, P(I,J+1) \quad \text{(A13)} \\
\text{LEF} & = 0 \quad \text{(A14)} \\
\text{BOT} & = (1/KC^2 + B/(2KC)) \, P(I,J-1) \quad \text{(A15)} \\
\text{KON} & = C \quad \text{(A16)} \\
\text{CON} & = 4/HC^2 + 2/KC^2 \quad \text{(A17)}
\end{align*}
\]

3. Wall Grid Points

Grid points are designated as wall grid points if they lie on the wall boundary and \( HB = HC, HA = 0, KC = KA = KB \). They are classified as type 3. All derivatives of the pressure distribution equation are the same as in the case of regular interior points (Equations (A2-A5)). But because of the wall boundary, \( P(I-1) = P(I+1) \). Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).

\[
\begin{align*}
\text{RIG} & = 0 \quad \text{(A18)} \\
\text{TOP} & = 1/KC^2 + B/(2KC)) \, P(I,J+1) \quad \text{(A19)} \\
\text{LEF} & = (2/HC^2) \, P(I-1,J) \quad \text{(A20)} \\
\text{BOT} & = (1/KC^2 - B/(2KC)) \, P(I,J-1) \quad \text{(A21)} \\
\text{KON} & = C \quad \text{(A22)} \\
\text{CON} & = 2/HC^2 + 2/KC^2 \quad \text{(A23)}
\end{align*}
\]
Grid points are designated as solidus interior if they lie just next to the solidus isotherm and either $KA \neq KB$ or $HA \neq HB$. They are classified into three types as follows.

Type 4 $HA < HB$ but $HB = HC$ and $KB < KA$ but $KA = KC$

Type 5 $HA = HB = HC$ and $KB < KA$ but $KA = KC$

Type 6 $HA < HB$ but $HB = HC$ and $KA = KB = KC$

The finite difference equations for each type are derived as follows.

Type 4

The pressure gradients at the solidus isotherm are needed for the calculation of pressures at the above grid points. These pressure gradients are obtained from D'Arcy's Law, and they are evaluated at the grid points which lie on the solidus isotherm.

\[
\frac{\partial P}{\partial r} \bigg|_{\text{SOLIDUS}} = \frac{\mu}{\gamma_{LE}} \left( \frac{\rho_{SE}}{\rho_{LE}} - 1 \right) u_{rE}
\]

\[
\frac{\partial P}{\partial z} \bigg|_{\text{SOLIDUS}} = \frac{\mu}{\gamma_{LE}} \left( \frac{\rho_{SE}}{\rho_{LE}} - 1 \right) u_{zE} - \rho g_z
\]

The pressure gradients at grid point $(I,J)$ are calculated by the "Level Rule" as follows.

\[
\left. \frac{\partial P}{\partial r} \right|_{I,J} = \frac{(HB)\text{SOLHZ} + (HA/2HB)(P(I,J) - P(I-2,J))}{(HA+HB)}
\]

\[
\left. \frac{\partial^2 P}{\partial r^2} \right|_{I,J} = \frac{(\text{SOLHZ} - (P(I,J) - P(I-2,J))/2HB)}{(HA+HB)}
\]
\[
\frac{\partial P}{\partial z}|_{I,J} = \frac{((KA)\text{SOLVE} + (KB/2KA)(P(I,J+2) - P(I,J)))/(KA+KB)}{(A28)}
\]

\[
\frac{\partial^2 P}{\partial z^2}|_{I,J} = \frac{((P(I,J+2) - P(I,J))/2KA - \text{SOLVE})/(KA+KB)}{(A29)}
\]

Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (Al).

\[
\text{RIG} = 0 \quad \text{(A30)}
\]

\[
\text{TOP} = \frac{1 + (KB)/2KC(KA+KB)}{2KC} P(I,J+2) \quad \text{(A31)}
\]

\[
\text{LEF} = \frac{1 - (HA)/2HC(HA+HB)}{2HC} P(I-2,J) \quad \text{(A32)}
\]

\[
\text{BOT} = 0 \quad \text{(A33)}
\]

\[
\text{KON} = \frac{1 + (HC)/HA+HB}{2HC} \text{SOLHZ} + \frac{(KC)-1}{2KC(KA+KB)} \text{SOLVE} + C \quad \text{(A34)}
\]

\[
\text{CON} = \frac{1 - (HA)/2HC(HA+HB)}{2HC} + \frac{1 + (KB)/2KC(KA+KB)}{2KC(KA+KB)} \quad \text{(A35)}
\]

Type 5

\[
\frac{\partial P}{\partial r}|_{I,J} \quad \frac{\partial^2 P}{\partial r^2}|_{I,J}
\]

are the same as in the case of regular interior grid points (Equations (A2,A3)); and \( \frac{\partial P}{\partial z} \), \( \frac{\partial^2 P}{\partial z^2} \) and SOLVE are the same as type 4 (Equations (A25,A28,A29)). Substituting these into the pressure distribution equation (Equation (7)), the following expressions for the variables of the left side of Equation (Al) are obtained.

\[
\text{RIG} = \frac{1/HC^2 + A/2HC}{P(I+1,J)} \quad \text{(A36)}
\]

\[
\text{TOP} = \frac{1 + (KB)/2KC(KA+KB)}{P(I,J+2)} \quad \text{(A37)}
\]

\[
\text{LEF} = \frac{1/HC^2 - A/2HC}{P(I-1,J)} \quad \text{(A38)}
\]
\[
\begin{align*}
\text{BOT} &= 0 \\
\text{KON} &= \frac{(KC)B - 1}{KA + KB} \\
\text{CON} &= \frac{2}{HC^2} + \frac{1 + KB}{2KA(KA + KB)}
\end{align*}
\]

Type 6

\[
\begin{align*}
\frac{\partial P}{\partial z}_{I,J}, \quad \frac{\partial^2 P}{\partial z^2}_{I,J} \quad \text{are the same as in the case of regular interior grid points (Equations (A4,A5)); and } \frac{\partial P}{\partial r}_{I,J}, \quad \frac{\partial^2 P}{\partial r^2}_{I,J} \quad \text{and SOLHZ are the same as type 4 (Equations (A24,A26,A27)). Substituting these into the pressure distribution equation (Equation (7)), the following expressions for the variables of the left side of Equation (A1) are obtained.}
\end{align*}
\]

\[
\begin{align*}
\text{RIG} &= 0 \\
\text{TOP} &= (1/KC^2 + B/2KC) \ P(I,J+1) \\
\text{LEF} &= (1 - (HA)A)/(2HC(HA+HB)) \ P(I-2,J) \\
\text{BOT} &= (1/KC^2 - B/2KC) \ P(I,J-1) \\
\text{KON} &= (1 + (HC)A)/(HA+HB) \ \text{SOLHZ + C} \\
\text{CON} &= 2/KC^2 + (1 - (HA)A)/(2HC(HA+HB))
\end{align*}
\]

5. Liquidus Interior Grid Points

Grid points are designated as liquidus interior if they lie just next to the solidus isotherm and either \( KA \neq KB \) or \( HA \neq HB \). They are classified into five types as follows.

Type 7 \( HA > HB \) but \( HA = HC \) and \( KA < KB \) but \( KB = KC \) (i.e., positive liquidus slope)

Type 8 \( HA < HB \) but \( HB = HC \) and \( KA < KB \) but \( KB = KC \) (i.e., negative liquidus slope)
Type 9  \[ \begin{align*} &HA = HB = HC \text{ and } KA < KB \text{ but } KB = KC \end{align*} \]
\[ (i.e., \text{positive or negative liquidus slope}) \]
Type 10  \[ \begin{align*} &HA > HB \text{ but } HA = HC \text{ and } KA = KB = KC \end{align*} \]
\[ (i.e., \text{positive liquidus slope}) \]
Type 11  \[ \begin{align*} &HA < HB \text{ but } HB = HC \text{ and } KA = KB = KC \end{align*} \]
\[ (i.e., \text{negative liquidus slope}) \]

The finite difference equations derived below for these types of grid points are utilized by the computer code when pressure gradients are specified for the points that lie on the liquidus isotherm.

\[ \frac{\partial^2 P}{\partial r^2} \bigg|_{\text{LIQUIDUS}} = \frac{P(I,J+2) - 2P(I,J) + P(I,J-2)}{2HA} \quad (A49) \]

Type 7

The pressure gradients at point \((I,J)\) are calculated by the "Lever Rule" as follows.

\[ \frac{\partial P}{\partial z} \bigg|_{\text{LIQUIDUS}} = \frac{(HA)LIQHZ + (HB/2HA)(P(I+2,J) - P(I,J))}{HA+HB} \quad (A50) \]

\[ \frac{\partial^2 P}{\partial z^2} \bigg|_{\text{LIQUIDUS}} = \frac{(P(I,J+2) - P(I,J))/2HA - LIQHZ}{HA+HB} \quad (A51) \]

\[ \frac{\partial P}{\partial z} \bigg|_{\text{LIQUIDUS}} = \frac{(KB)LIQVE + (KA/2KB)(P(I,J) - P(I,J-2))}{KA+KB} \quad (A52) \]
\[ \frac{\partial^2 p}{\partial z^2}_{I,J} = \frac{(LIQVE - (P(I,J) - P(I,J-2))/2KB)/(KA+KB)}{I,J} \] (A53)

Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).

\[ \text{RIG} = \frac{(1 + (HB)A)/(2HC(HA+HB))}{P(I+2,J)} \] (A54)

\[ \text{TOP} = 0 \] (A55)

\[ \text{LEF} = 0 \] (A56)

\[ \text{BOT} = \frac{(1 - (KA)B)/(2KC(KA+KB))}{P(I,J-2)} \] (A57)

\[ \text{KON} = \frac{((HA)A - 1)/(HA+HB)LIQHZ + ((KB)B + 1)/(KA+KB)LIQVE + C}{HA+HB} \] (A58)

\[ \text{CON} = \frac{(1 + (HA)A)/(2HC(HA+HB)) + (1 - (KA)B)/(2KC(KA+KB))}{HA+HB} \] (A59)

**Type 8**

\[ \frac{\partial p}{\partial z}_{I,J} \quad \frac{\partial^2 p}{\partial z^2}_{I,J} \] are the same as in type 7 (Equations (A52, A53)), and

\[ \frac{\partial p}{\partial r}_{I,J} \quad \frac{\partial^2 p}{\partial r^2}_{I,J} \] are derived using the "Lever Rule".

\[ \frac{\partial p}{\partial r}_{I,J} = \frac{((HB)LIQHZ + (HA/2HB)(P(I,J) - P(I-2,J)))/(HA+HB)}{HA+HB} \] (A60)

\[ \frac{\partial^2 p}{\partial r^2}_{I,J} = \frac{(LIQHZ - (P(I,J) - (P(I-2,J))/(2HB)))/(2HB)/(HA+HB)}{HA+HB} \] (A61)

Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).
RIG = 0  \hspace{1cm} (A62) \\
TOP = 0  \hspace{1cm} (A63) \\
LEF = \frac{1 - (HA)A}{2HC(HA+HB)} P(I-2,J)  \hspace{1cm} (A64) \\
BOT = \frac{1 - (KA)B}{2KC(KA+KB)} P(I,J-2)  \hspace{1cm} (A65) \\
KON = \frac{1 + (HC)A}{(HA+HB)}LIQHZ + \frac{1 + (KC)A}{(KA+KB)}LIQVE + C  \hspace{1cm} (A66) \\
CON = \frac{1 - (HA)A}{2HC(HA+HB)} + \frac{1 - (KA)B}{2KC(KA+KB)}  \hspace{1cm} (A67) \\

Type 9

$$\frac{\partial P}{\partial r} \bigg|_{I,J}, \frac{\partial^2 P}{\partial r^2} \bigg|_{I,J}$$ are the same as for regular interior grid points (Equations (A2,A3)), and $$\frac{\partial P}{\partial z} \bigg|_{I,J}, \frac{\partial^2 P}{\partial z^2} \bigg|_{I,J}$$ are the same as for type 7 (Equations (A52,A53)). Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).

RIG = \frac{1}{HC^2} + A/2HC \ P(I+1,J)  \hspace{1cm} (A68) \\
TOP = 0  \hspace{1cm} (A69) \\
LEF = \frac{1}{HC^2} - A/2HC \ P(I-1,J)  \hspace{1cm} (A70) \\
BOT = \left(1 - (KA)B\right)/(2KC(KA+KB)) \ P(I,J-2)  \hspace{1cm} (A71) \\
KON = \left(1 + (KC)B\right)/(KA+KB) + C  \hspace{1cm} (A72) \\
CON = 2/HC^2 + \left(1 - (KA)B\right)/(2KC(KA+KB))  \hspace{1cm} (A73)
Type 10

\[ \frac{\partial^2 P}{\partial z^2}_{L,J}, \frac{\partial^2 P}{\partial r^2}_{L,J} \] are the same as in the case of regular grid points (Equations (A4,A5)), and \( \frac{\partial P}{\partial r}_{I,J}, \frac{\partial^2 P}{\partial r^2}_{I,J} \) are the same as in type 7. Substituting these into the pressure distribution equation (Equation (7)), the following expressions are obtained for the variables of the left side of Equation (A1).

\[ \begin{align*}
RIG & = \frac{(1 + (HB)A)}{2HC(HA+HB)} P(I+2,J) \\
TOP & = \frac{1}{KC^2} + \frac{B}{2KC} P(I,J+1) \\
LEF & = 0 \\
BOT & = \frac{1}{KC^2} - \frac{B}{2KC} P(I,J-1) \\
KON & = \frac{(HC)A - 1}{(HA+HB)}LIQHZ + C \\
CON & = \frac{2}{KC^2} + \frac{(HC)A + 1}{2HC(HA+HB)}
\end{align*} \]

(A74) \quad (A75) \quad (A76) \quad (A77) \quad (A78) \quad (A79)

Type 11

By substituting LIQHZ for SOLHZ in Equations (A42-A47) of type 6, the representative expressions for type 11 are obtained.

The grid points referred to in this appendix are shown diagrammatically on the following page.
Different Types of Grid Points
APPENDIX B

FLOW CHART

START

INPUT

CALCULATE SHAPE OF MUSHY ZONE

CONSTRUCT GRID MESH

DO 251 LOOPS

CALCULATE AXIAL AND RADIAL SPACINGS BETWEEN GRID POINTS

DO 301 LOOPS

CLASSIFY GRID POINTS ACCORDING TO THEIR POSITION RELATIVE TO BOUNDARIES

DO 74, 70 LOOPS

ASSIGN VALUES OF T, CL AND ρ TO EACH GRID POINT

INITIALIZE PRESSURE AND FRACTION LIQUID FOR EACH GRID POINT
DO 300, 820, 825, 830 LOOPS

CALCULATE $T$, $c_l$ AND $\rho$ GRADIENTS
ALSO COOLING RATES FOR EACH
GRID POINT

DO 500 LOOPS

CALL SUBROUTINE
COEFF
(CALCULATE $A$, $B$, $C$)

DIRECTORY
(SELECT APPROPRIATE EQUATION
TO CALCULATE GRID POINT PRESSURES)

CALCULATE PRESSURE
FOR EACH GRID POINT

DO 730 LOOPS

DIRECTORY
(SELECT APPROPRIATE EQUATION
TO CALCULATE PRESSURE GRADIENTS
FOR EACH GRID POINT)

CALCULATE INTERDENDRITIC
LIQUID VELOCITY FOR EACH
GRID POINT

DO 1020, 810 LOOPS
CALCULATE LOCAL FRACTION LIQUID FOR EACH GRID POINT

CALCULATE AVERAGE INGOT COMPOSITION

IS $C_o = \bar{C}_{\text{INGOT}}$ NO

GO TO LOOP 500

YES

STOP

END
APPENDIX C

LIST OF COMPUTER NOTATIONS

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<tr>
<td>RIG</td>
<td></td>
<td>Coefficient of finite difference equation</td>
</tr>
<tr>
<td>RLE</td>
<td></td>
<td>Density of liquid eutectic</td>
</tr>
<tr>
<td>RLN(J)</td>
<td></td>
<td>Radius of liquidus isotherm having negative slope at row J</td>
</tr>
<tr>
<td>RLP(J)</td>
<td></td>
<td>Radius of liquidus isotherm having positive slope at row J</td>
</tr>
<tr>
<td>RO</td>
<td>$\rho_{Lo}$</td>
<td>Density of bulk liquid</td>
</tr>
<tr>
<td>Computer Notation</td>
<td>Algebraic Notation</td>
<td>Explanation of Symbols</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------------</td>
<td>-----------------------------------------------------------</td>
</tr>
<tr>
<td>ROR(I)</td>
<td></td>
<td>Radius at column I</td>
</tr>
<tr>
<td>RR</td>
<td></td>
<td>Ratio of inner cylinder radius to outer cylinder radius</td>
</tr>
<tr>
<td>RS(J)</td>
<td></td>
<td>Radius of solidus isotherm at row J</td>
</tr>
<tr>
<td>RSE</td>
<td>$\rho_{SE}$</td>
<td>Density of eutectic solid</td>
</tr>
<tr>
<td>SOLHZ</td>
<td></td>
<td>$\partial P/\partial r$ at solidus isotherm</td>
</tr>
<tr>
<td>SOLVE</td>
<td></td>
<td>$\partial P/\partial z$ at solidus isotherm</td>
</tr>
<tr>
<td>T(I,J)</td>
<td>T</td>
<td>Temperature at (I,J)</td>
</tr>
<tr>
<td>TE</td>
<td>$T_E$</td>
<td>Eutectic temperature</td>
</tr>
<tr>
<td>TL</td>
<td>$T_L$</td>
<td>Liquidus temperature</td>
</tr>
<tr>
<td>TM</td>
<td></td>
<td>Melting point of pure solvent</td>
</tr>
<tr>
<td>TOP</td>
<td></td>
<td>Coefficient of finite difference equation</td>
</tr>
<tr>
<td>TTHETA</td>
<td>$\Theta$</td>
<td>Angle</td>
</tr>
<tr>
<td>TYP(I,J)</td>
<td></td>
<td>Type of grid point</td>
</tr>
<tr>
<td>UZCL</td>
<td></td>
<td>Casting speed</td>
</tr>
<tr>
<td>VR</td>
<td>$v_r$</td>
<td>r-component of $\vec{v}$</td>
</tr>
<tr>
<td>VTOT</td>
<td>$</td>
<td>\vec{v}</td>
</tr>
<tr>
<td>VZ</td>
<td>$v_z$</td>
<td>z-component of $\vec{v}$</td>
</tr>
<tr>
<td>W</td>
<td>$\omega$</td>
<td>Angular velocity</td>
</tr>
<tr>
<td>ZAT(I)</td>
<td>$d/d_o$</td>
<td></td>
</tr>
<tr>
<td>ZILIQ</td>
<td></td>
<td>Height of liquidus at centerline</td>
</tr>
<tr>
<td>ZISOL</td>
<td></td>
<td>Height of solidus at centerline</td>
</tr>
<tr>
<td>ZL(I)</td>
<td></td>
<td>Height of liquidus isotherm at column I</td>
</tr>
<tr>
<td>ZLMAX</td>
<td></td>
<td>Maximum height of liquidus isotherm</td>
</tr>
<tr>
<td>ZLMIN</td>
<td></td>
<td>Minimum height of liquidus isotherm</td>
</tr>
<tr>
<td>Computer Notation</td>
<td>Algebraic Notation</td>
<td>Explanation of Symbols</td>
</tr>
<tr>
<td>------------------</td>
<td>--------------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>ZOZ(I)</td>
<td></td>
<td>Height of column I</td>
</tr>
<tr>
<td>ZS(I)</td>
<td></td>
<td>Height of solidus isotherm at column I</td>
</tr>
</tbody>
</table>
APPENDIX D

LIST OF COMPUTER PROGRAM
**GENERAL DATA INPUTS******************BEGIN**

BOUNDARY CONDITIONS AT LIQUIDUS ISOTHERM

**1. METALLOSTATIC PRESSURE, NO ROTATION**
FLOW.EQ.0.  AV.EQ.0.  W.EQ.0.

**2. METALLOSTATIC PRESSURE, LIQUID POOL ROTATION**
FLOW.EQ.0.  AV.GT.0.  W.EQ.0.

**3. PRESSURE GRADIENT DUE TO LIQUID POOL ROTATION**
FLOW.GT.0.  AV.GT.0.  W.EQ.0.

**4. PRESSURE DUE TO INGOT ROTATION**
FLOW.EQ.0.  AV.EQ.0.  W.EQ.0.

**GENERAL DATA INPUTS******************BEGIN**

**GENERAL DATA INPUTS******************BEGIN**

BOUNDARY CONDITIONS AT LIQUIDUS ISOTHERM

**1. METALLOSTATIC PRESSURE, NO ROTATION**
FLOW.EQ.0.  AV.EQ.0.  W.EQ.0.

**2. METALLOSTATIC PRESSURE, LIQUID POOL ROTATION**
FLOW.EQ.0.  AV.GT.0.  W.EQ.0.

**3. PRESSURE GRADIENT DUE TO LIQUID POOL ROTATION**
FLOW.GT.0.  AV.GT.0.  W.EQ.0.

**4. PRESSURE DUE TO INGOT ROTATION**
FLOW.EQ.0.  AV.EQ.0.  W.EQ.0.

GA05 = 1.00-7
AV = 0.29
UZCL = 1.70-3
PEXP = 5.
FLOW = 1.
RR = 0.45
CE = 38.1
CSE = 2.6
TM = 232.0
RE = 7.25
RO = 7.28
CO = 12.37
ATMP = 1.006
w = 0.0
MU = 0.0212
TE = 183.0
PI = 3.14156
RSE = 0.6
RLE = 8.1
GRAV = 980.
ZISOL = 0.0
RADIUS = 4.0
HC = 0.25
KC = 0.15
ZILIQ = 4.7
NTDP = 67
CF = 1.3
ACCR = 0.02
ADCS = 5.0 * CO
LITER = 50
TEST = 0.0
WRITE(6, 9) GA0S
IF (FLOW.EQ.0.) WRITE(6, 25)
IF (FLOW.NE.0.) WRITE(6, 30)
9 FORMAT(1H1"GAMMA AT CENTERLINE"', 25)
25 FORMAT(5X, "B.C. AT LIQUIDUS...PRESSURE")
30 FORMAT(5X, "B.C. AT LIQUIDUS...PRESSURE GRADIENT")

INPUT OF (SECONDARY O.A.S./SECONDARY O.A.S. AT CENTER LINE)

ZAT(1) = 1.0
ZAT(2) = 0.98
ZAT(3) = 0.98
ZAT(4) = 0.98
ZAT(5) = 0.96
ZAT(6) = 0.94
ZAT(7) = 0.93
ZAT(8) = 0.90
ZAT(9) = 0.89
ZAT(10) = 0.39
ZAT(11) = 0.39
ZAT(12) = 0.39
ZAT(13) = 0.38
ZAT(14) = 0.37
ZAT(15) = 0.37
ZAT(16) = 0.36
ZAT(17) = 0.35

GENERAL DATA INPUTS

IMAX = RADIUS / HC * 0.001
IMAX = IMAX + 1
IMAXP = IMAX + 1
IMAXN = IMAX - 1
DRODCL = (RO - RLE) / (CO - CE)
KAY = CE/CE
KON1 = (1.000/CD)**(1./(KAY - 1.))
KON2 = 1.000/(KAY - 1.000)
KON3 = KON2 - 1.000
KON4 = -KON2
SE = (CE/CD)**(1./(KAY - 1.))
EM = (TE - TM)/CE
TL = EM*(CD - CE) + TE
CONTR = RSE/RL - 1.
HCRAT = R0 - RL
TRAT = TL - TE
CRAT = CO - CE
KIS = 1./KC**2
HIS = 1./HC**2
KIST = 2.*KIS
HIST = 2.*HIS
KTI = 1./(2.*KC)
HTI = 1./(2.*HC)
DO 6 1, IMAX
G LEUT(I) = GE
CONTINUE
C
CALCULATION OF HEIGHTS OF LIQUIDUS POINTS, AND HEIGHTS & RADIUS OF
C SOLIDUS POINT***BEGIN
C
Z S(I) = ZISOL
Z L(I) = ZILIQ
WRITE (6, 6925)
6925 FORMAT (12X, "*********", 10X, "*********", 8X, "***")
WRITE (6, 9000)
9000 FORMAT (12X, "Z-SOLIDUS", 10X, "Z-LIQUIDUS", 9X, "I")
WRITE (6, 6926)
6926 FORMAT (12X, "*********", 10X, "*********", 8X, "***")
AL(I) = 0.
AIL(I) = 0.
AIL(IMAXP) = 0.
ROR(I) = 0.
DO 10 I = 2, IMAX
C
INPUT OF SLOPES OF SOLIDUS AND LIQUIDUS ISOTHERMS***BEGIN
SLOPES AT CENTERLINE ARE EQUAL TO ZERO
C
IF (I.GT.1).AND.(I.LE.11) MSOL = 0.
IF (I.GT.11) MSOL = 0.27
IF (I.GT.1).AND.(I.LE.3) MLIQ = 0.
IF (I.GT.11) MLIQ = -0.65
C
INPUT OF SLOPES OF SOLIDUS AND LIQUIDUS ISOTHERMS***END
C
AIL(I) = MSOL
AIL(I) = MLIQ
ROR(I) = ROR(I-1) + HC
ZL(I) = ZL(I-1) + AL(I) + HC
ZS(I) = ZS(I-1) + AIL(I) + HC
10 CONTINUE
DO 120 I = 1, IMAX
WRITE (6, 6000) ZS(I), ZL(I), I
CONTINUE
DO 11 J=1,NTOP
IF (J.EQ.1) GO TO 4
GO TO 5
11 CONTINUE
Z0Z(J)=ZCZ(J-1)+KC
GO TO 6
5 CONTINUE
Z0Z(J)=Z0Z(J-1)+KC
GO TO 6

INPUT OF PRESSURE GRADIENTS AT THE LIQUIDUS ISOTHERM

DO 2 I=1,IMAX
PGZ(I)=RO*GRAV
PC1=RO*(AV*RAADIUS)**2
PC2=RR**2/(1.-RR**2)**2
PRI=RO*(AV*RAADIUS)**2
IF (PRI.LE.RR) PGR(I)=PC1*PRI
IF (PRI.GT.RR) PGR(I)=PC1*PC2*(1.-PRI**2)/PRI**3
WRITE(6,179)PGZ(I),PGR(I),I
2 CONTINUE
AIS(IMAXP)=AIS(IMAX)
PGR(IMAXP)=PGR(IMAX)
PGZ(IMAXP)=PGZ(IMAX)

CALCULATION OF HEIGHTS OF LIQUIDUS POINTS, AND HEIGHTS & RADIUS OF SOLIDUS POINT***END

CONSTRUCTION OF THE GRID MESH***BEGIN

THIS PARTICULAR SECTION DEALS WITH THE SETTING UP OF THE GRID MESH FOR THE MUSHY ZONE

DO 312 I=1,IMAX
ZLT(I)=ZL(I)
312 CONTINUE
DO 310 I=1,IMAXN
N=I+1
DO 310 J=N,IMAX
IF(ZLT(I).GE.ZLT(J)) GO TO 310
TEPM=ZLT(I)
ZLT(I)=ZLT(J)
310 CONTINUE
ZLMAX=ZLT(1)
ZLMIN=ZLT(IMAX)
WRITE(6,110)IMAX,ZLMAX,ZLMIN
110 FORMAT(3X,"IMAX=",I3,3X,"ZLMAX=",F7.2,3X,"ZLMIN=",F7.2)
DO 40 I=1,IMAX
HEIT(I)=ZLMAX-ZL(I)
HTT(I)=ZL(I)-ZS(I)
40 CONTINUE
CALCULATION OF RADIAL AND AXIAL DISTANCES BETWEEN GRID POINTS

```
C K*LMAX/KC+3
DO 291 I=1,IMAX
DO 291 J=1,K
TYP(I,J)=1
IF(I.EQ.1) TYP(I,J)=2
IF(I.EQ.IMAX) TYP(I,J)=3
GPR(I,J)=HC
GPZ(I,J)=KC
DECLR(I,J)=2.0*HC
DECLZ(I,J)=2.0*KC
291 CONTINUE
DO 255 I = 2,IMAX
SUM=KC
DO 255 J = 2, K
ZS=SUM-ZS(I)
IF ((DZS.LE.KC).AND.(DZS.GT.0.0)) GO TO 257
GO TO 260
257 GPZ(I,J-1)=DZS
DECLZ(I,J-1)=DZS
JMIN(I)=J
260 DZL=ZL(I)-SUM
IF ((DZL.LE.KC).AND.(DZL.GT.0.0)) GO TO 262
GO TO 264
262 GPZ(I,J+1)=DZL
DECLZ(I,J+1)=DZL
NT(I)=J
IF((DZL.LE.0.05*KC)) NT(I)=J-1
264 IF((ZS(I).GE.SUM).AND.(ZS(I-1).LT.SUM)) GO TO 268
GO TO 270
268 RS(I)=SUM-ZS(I-1)/A1S(I)+(I-2)*HC
GPR(I,J)=RS(J)-(I-2)*HC
DECLR(I,J)=GPR(I,J)
270 IF((ZL(I).GE.SUM).AND.(ZL(I-1).LT.SUM)) GO TO 274
IF((ZL(I).GE.SUM).AND.(ZL(I-1).GT.SUM)) GO TO 276
GO TO 278
274 RL(P(J)=(SUM-ZL(I-1))/A1L(I)+(I-2)*HC
GPR(I-1,J)=GPR(I,J)
DECLR(I-1,J)=GPR(I,J)
278 IF(I.EQ.IMAX) TYP(I,J)=32
IF(I.EQ.1) TYP(1,J)=43
GO TO 278
276 RLN(J) = SUM-ZL(I-1)/A1L(I)+(I-2)*HC
GPR(I,J)=RLN(J)-(I-2)*HC
DECLR(I,J)=GPR(I,J)
278 SUM=SUM+KC
255 CONTINUE
```

CLASSIFICATION OF THE GRID POINTS ACCORDING TO THEIR POSITION RELATIVE TO THE Boundaries

```
JMIN(1)=2
TYP(1,2)=21
FRL=ZL(1)/KC+2
LFR=FRL
GPZ(I,LFR)=FRL-LFR*KC
NT(I)=LFR-1
TYP(I,LFR-1)=22
IF(NT(2).LT.NT(1)) TYP(1,LFR-1)=24
IF(JMIN(2).GT.JMIN(1)) TYP(1,2)=23
```
J = NT(IMAX)
TYP(IMAX, J) = 3
IF (NT(IMAX) .GT. NT(IMAX-1)) TYP(IMAX, J) = 34
J = JMIN(IMAX)
TYP(IMAX, J) = 31
DO 301 I = 2, IMAXN
MNT = JMIN(I)
NNT = NT(I)
DO 301 J = JMIN(I), NNT
IF (DECLR(I, J) .LE. HC) TYP(I, J) = 6
IF (DECLR(I+1, J) .LE. HC) TYP(I, J) = 5
IF (DECLR(I+1, J) .LE. HC) TYP(I, J) = 4
IF (DECLR(I-1, J) .LE. HC) TYP(I, J) = 10
IF (DECLR(I+1, J) .LE. HC) TYP(I, J) = 7
IF (DECLR(I-1, J) .LE. HC) TYP(I, J) = 11
IF (DECLR(I+1, J) .LE. HC) TYP(I, J) = 8
IF (I .EQ. 2) GO TO 331
IF (TYP(I, J) .LT. 4) TYP(I, J) = 41
IF (TYP(I, J) .EQ. 3) TYP(I, J) = 43
301 CONTINUE
DO 79 I = 1, IMAXP
NNT = NT(I)+1
DO 79 J = 1, NNT
DECLR(I, J) = 0.
DECLZ(I, J) = 0.
79 CONTINUE
WRITE (6, 309)
309 FORMAT (14X, "RS(J)", 10X, "RLP(J)", 10X, "RLN(J)", 13X, "J")
WRITE (6, 307)
307 FORMAT (11X, "**********", 7X, "**********", 7X, "**********", 12X, "****")
DO 303 J = 1, K
WRITE (6, 305) RS(J), RLP(J), RLN(J), J
305 FORMAT (5X, 3E15.4, /)
303 CONTINUE
WRITE (6, 6929)
6929 FORMAT (10X, "**********", 4X, "**********", 7X, "****")
WRITE (6, 9010)
9010 FORMAT (10X, "VALUE OF JMIN", 4X, "VALUE OF NT", 8X, "I")
WRITE (6, 6930)
6930 FORMAT (10X, "**********", 4X, "**********", 7X, "****")
DO 88 I = 1, IMAX
WRITE (6, 9010) JMIN(I), NT(I), I
8010 FORMAT (1X, 2I15, 1114/)
88 CONTINUE
C C CONSTRUCTION OF THE GRID MESH***END
C
C **********************************************************************
C TEMP AND TEMP RELATED ITEMS FOR EACH NODE***BEGIN
C **********************************************************************
C SCHEIL 'GL' AND HYDROSTATIC 'P' ARE ASSIGNED
C
DO 74 I = 1, IMAX
GAMMA(I) = GADS*(ZAT(I))*PEXP
PA = ATMPS*GRAV*RO*((NTOP-1)*KC-ZL(I))
P = 0.
IF (FLOW.NE.0.) GO TO 72
IF (AV.NE.0.) GO TO 73
GO TO 72

CALCULATION OF PRESSURE DUE TO CONVECTION

73 PR1=ROR(I)/RADIUS
PC1=(R0/2.)*((AV*RADIUS+PR)*2)/(1.-RR**2))**2
PC2=(R0/2.)*(AV*RADIUS)**2
IF(PR1.GT.RR) PF=PC1*(PR1)**2-1./PR1**2-4.*ALOG(PR1))
IF(PR1.LE.RR) PF=PC2*(PR1**2-RR**2)*PC1*(RR**2-1./RR**2-14.*ALOG(RR))
72 CONTINUE
PL(I)=PF
PT(I)=PF+PA
MNT=MIN(I)
NNT=NT(I)
DO 70 J=MNT,NNT
DEPMZ=ZL(I)-Z0(J)
HTMZ=HTT(I)-DEPMZ
HTFRAC=HTMZ/HTT(I)
T(I,J)=TAT*HTFRAC+TE
DIFTEM=T(I,J)-TE
RHO(I,J)=(MGRAT/TRAT)*DIFTEM+RLE
CL(I,J)=(CRAT/TRAT)*DIFTEM+CE
GL(I,J)=KDN1*(CL(I,J)**KON2)
P(I,J)=SO0*(RHO(I,J)*R)*GRAV*DEPMZ
P(I,J)=P(I,J)+0.5*RQ*(W**2)*(ROR(I)**2)
70 CONTINUE
NNT=NT(I)
NPP=ZLMAX/KC+2
DO 60 J=NNT,NPP
P(I,J)=ATMPR+GRAV*R0*(NTOP-J)**KC+PL(I)
GL(I,J)=1.
T(I,J)=TL
RHO(I,J)=R0
CL(I,J)=CO
TYP(I,J)=0
60 CONTINUE
WRITE(6,85)I,PL(I),PA
85 FORMAT(5X,"I",13,5X,"PRESSURE DUE TO CONVECTION="
1"METALLOSTATIC HEAD="",E12.5,5X,
1"METALLOSTATIC HEAD="",E12.5/)
74 CONTINUE
PF=0.
DO 65 I=1,IMAX
J=M1(MIN(I))-1
DO 65 J=1,IM1
T(I,J)=TE
RHO(I,J)=RLE
CL(I,J)=CE
TYP(I,J)=0
65 CONTINUE

VARIABLES AT LIQUIDUS

DO 300 I=1,IMAX
NNT=NT(I)
J=NNT+1
Z(I,J)=(TL-TE)/(ZL(I)-ZS(I))
VARIABLES AT INTERIOR GRID POINTS

\[ GZ(I,J) = \frac{GZ(I,J) \times A1L(I)}{EPPS(I,J) - GZ(I,J) \times UZCL} \]

\[ 300 \text{ CONTINUE} \]

VARIABLES AT CENTERLINE

\[ I = 1 \]
\[ NNT = NT(1) \]
\[ MNT = JMIN(1) \]

\[ 820 \text{ CONTINUE} \]

VARIABLES AT WALL

\[ I = IMAX \]
\[ NNT = NT(I) \]
\[ MNT = JMIN(I) \]

\[ 830 \text{ CONTINUE} \]

TEMP AND TEMP RELATED ITEMS FOR EACH NODE

\[ DO 100 I = 1, IMAX \]
\[ NNT = NT(I) + 1 \]
\[ MNT = JMIN(I) - 1 \]
\[ DO 100 J = MNT, NNT \]

\[ WRITE(6,110) I, J, GPR(I,J), GPZ(I,J), TYP(I,J), GL(I,J), T(I,J) \]
\[ P(I,J), RL(I,J), CL(I,J) \]

\[ 110 \text{ CONTINUE} \]
CONTINUE
C
C * ITERATION SEQUENCE FOR 'P', VELOCITY, 'GL' AND CCPM ** BEGIN
C
ITER=0
1000 ITER=ITER+1
IFREC=0
XITER=ITER
PRNT=XITER/25.
DO 15 KP=1,5
&PNTR*KP
IF(PRNT.EQ.APNTR) GO TO 12
15 CONTINUE
IPR=0
GO TO 13
12 CONTINUE
IPR=ITER
13 CONTINUE
DO 105 I=1,IMAX
MNT=JMIN(I)-1
DO 105 J=MNT,NNT
GL(I,J)=GPEUT(I)
105 CONTINUE
C
C * CALCULATION OF PRESSURE DISTRIBUTION ** BEGIN
C
DO 500 I=1,IMAX
M=I
IF(I.EQ.1) M=I+1
MNT=JMIN(I)
NNT=NT(I)
VD=MNT-MNT
DO 500 J=MNT,NNT
KA=GPZ(I,J-1)
KB=GPZ(I,J-1)
HA=GPR(I+1,J)
HB=GPR(M-1,J)
KK=KA+KB
HH=HA+HB
IF(I.EQ.IMAX) HH=HB
CALL CGEFF(I,J,A,B,C)
IF(TYP(I,J).EQ.1) GO TO 390
IF(TYP(I,J).EQ.2) GO TO 355
IF(TYP(I,J).EQ.3) GO TO 315
QH=MTI/HH
QK=KTI/KK
JHA=HA*QH
JHB=HB*QH
QKA=KA*QK
QKB=KB*QK
<1=RHO(I,J)*W**2
$K_2 * K_1 * R_Q(R(I))$

$V_F = (J - M_T) / V_O$

IF ($V_F \geq 0.5$) GO TO 347

IF ($I \geq I_{MAX}$) GO TO 320

$R_{FR} = H_A / H_C$

$G_L = R_{FR} * (G_EU(T(I+1)) - G_EU(T(I))) + G_EU(T(I))$

$S_{OLHZ} = -\mu * C_{ONTR} * U_Z(C) / (G_{AMMA} * G_L * (1. + A_{IS}(I+1) * \mu)^2)$

320 $SOLVE = \mu * C_{ONTR} * U_Z(C) / (G_{AMMA} * G_L * (1. + A_{IS}(I+1) ** 2)) = R_{LE} * GRAV$

IF ($TYP(I,J) = 5$) GO TO 360

IF ($TYP(I,J) = 4$) GO TO 365

IF ($TYP(I,J) = 6$) GO TO 370

IF ($TYP(I,J) = 21$) GO TO 395

IF ($TYP(I,J) = 23$) GO TO 445

IF ($TYP(I,J) = 31$) GO TO 420

IF ($TYP(I,J) = 41$) GO TO 415

347 CONTINUE

IF ($FLOW = 0.$) GO TO 505

$L_IQVE = P(GZ(I))$

$N = M + 1$

IF ($AIL(I) \leq C.$) $N = M + 1$

$R_{FR} = G_{PR}(N) / H_C$

$L_{IQHZ} = (P(GR(N)) - P(GR(I))) \cdot R_{FR} * P(GR(I))$

IF ($TYP(I,J) = 9$) GO TO 375

IF ($TYP(I,J) = 10$) GO TO 380

IF ($TYP(I,J) = 11$) GO TO 385

IF ($TYP(I,J) = 12$) GO TO 390

IF ($TYP(I,J) = 13$) GO TO 395

IF ($TYP(I,J) = 14$) GO TO 400

IF ($TYP(I,J) = 15$) GO TO 405

302 IF ($TYP(I,J) = 32$) GO TO 400

IF ($TYP(I,J) = 33$) GO TO 405

IF ($TYP(I,J) = 34$) GO TO 410

WRITE(6,75) $I, J$

75 FORMAT(3X, 'THIS TYPE OF GRID POINT HAS NOT BEEN IDENTIFIED',/10X, $\$I = ', 13X, 'J = ', 13X)

GO TO 1001

C

350 CON = KIST * HIST

$R_I G = (H_I S + A_{HTI}) \cdot P(I+1,J)$

$T_O P = (K_I S + B_{KTI}) \cdot P(I,J+1)$

$L_E F = (H_I S - A_{HTI}) \cdot P(I,J+1)$

$B_O T = (K_I S - B_{KTI}) \cdot P(I,J-1)$

$K_O N = C$

GO TO 720

C

355 CON = 2 * HIST * KIST

$R_I G = 2. * (H_I S - P(I+1,J))$

$T_O P = (K_I S + B_{KTI}) * P(I,J+1)$

$B_O T = (K_I S - B_{KTI}) * P(I,J-1)$

$L_E F = 0.$

$K_O N = C$

GO TO 720

C

360 CON = KIST * HIST

$B_O T = (K_I S - B_{KTI}) * P(I,J-1)$

$T_O P = (K_I S + B_{KTI}) * P(I,J+1)$

$L_E F = H_I S \cdot P(I,J)$

$R_I G = 0.$

C
CON = RH0(I,J) * RCR(I) * W**2 * (A + 2. / HC) + C
GO TO 720
C

TYPE 4
365 CON = QH + QK - A * QHA + 3 * QK3
TOP = (QK + 3 * QKB) * P(I, J + 2)
LEF = (QH - A * QHA) * P(I - 2, J)
RIG = 0.
BOT = 0.
< CON = (1. + HC * A) * SOLHZ / HH + (KC * B - 1.) * SOLVE / KK + C
GO TO 720
C

TYPE 5
366 CON = QK + QHA + 3 * QK3
RIG = (HIS + A * HT1) * P(I + 1, J)
TOP = (QK + 3 * QKB) * P(I + 1, J + 2)
LEF = (HIS - A * HT1) * P(I - 1, J)
BOT = 0.
< CON = (B * KC - 1.) * SOLVE / KK + C
GO TO 720
C

TYPE 6 OR 11
370 CON = QH + HIST + B * QKB
RIG = (HIS - A * HT1) * P(I + 1, J)
TOP = (QK + 3 * QKB) * P(I, J + 2)
LEF = (HIS - A * HT1) * P(I - 1, J)
BOT = 0.
< CON = (A * HC + 1.) * SOLVE / HH + C
RIG = 0.
GO TO 720
C

TYPE 7
390 CON = QH + QK + A * QHA + 3 * QKA
RIG = (QH + A * QKB) * P(I + 2, J)
BOT = (QK - 3 * QKA) * P(I, J - 2)
LEF = 0.
TOP = 0.
< CON = (A * HC - 1.) * SOLVE / HH + (B * KC - 1.) * LIQVE / KK + C
GO TO 720
C

TYPE 8
380 CON = QH + QK - A * QHA + 3 * QKA
LEF = (QH - A * QKB) * P(I - 2, J)
TOP = (QK - 3 * QKA) * P(I, J - 2)
RIG = 0.
BOT = 0.
< CON = (A * HC + 1.) * SOLVE / HH + (B * KC + 1.) * LIQVE / KK + C
GO TO 720
C

TYPE 9
375 CON = HIST + QK - A * QHA
RIG = (HIS - A * HT1) * P(I + 1, J)
TOP = (QH + A * QKB) * P(I + 1, J)
LEF = (HIS - A * HT1) * P(I - 1, J)
BOT = 0.
< CON = (A * HC + 1.) * LIQVE / KK + C
GO TO 720
C

TYPE 10
385 CON = QH + HIST - A * QKB
TOP = (KIS + B * KTI) * P(I, J + 1)
BOT = (KIS - B * KTI) * P(I, J - 1)
RIG = (QH + A * QKB) * P(I + 2, J)
LEF = 0.
< CON = (A * HC - 1.) * LIQVE / HH + C
GO TO 720
C TYPE 21
395 CON*Z.*HIST+QK+3*QKB
TOP* (QK+B*QKB)*P(I,J+2)
RIG*Z.*HIST*P(I+1,J)
LEF*O.
BOT*O.
KON* (B*KC-1.)*SOLVE/<KK+C
GO TO 720
C TYPE 22
400 CON*Z.*HIST+QK-B*QKB
RIG*Z.*HIST*P(I+1,J)
BOT* (QK-B*QKA)*P(I,J-2)
TOP*O.
LEF*O.
KON* (B*KC+1.)*LIQVE/<KK+C
GO TO 720
C TYPE 31
420 CON*HIST+QK+B*QKB
LEF*HIST*P(I-1,J)
TOP* (QK+B*QKB)*P(I,J+2)
RIG*O.
BOT*O.
KON* (B*KC-1.)*SOLVE/<KK+B*RH0(I,J)*W**2+C
GO TO 720
C TYPE 24
435 CON*QK+B*QKA
BOT* (QK-B*QKA)*P(I,J-2)
RIG*O.
TOP*O.
LEF*O.
KON*2.*LIQVE/HA+(1.+3*KB)*LIQVE/<KK+C
GO TO 720
C TYPE 23
445 CON*QK+B*QKB
TOP* (QK+B*QKB)*P(I,J+2)
BOT*O.
RIG*O.
LEF*O.
KON*2.*LIQVE/<KK+A*LIQHZ+RH0(I,J)*W**2*(1.+A*ROR(I)+B)+C
GO TO 720
C TYPE 34
440 CON*QK
BOT*QK*P(I,J-2)
TOP*O.
RIG*O.
LEF*O.
KON*LIQVE/<KK+A*LIQHZ+RH0(I,J)*W**2*(1.+A*ROR(I)+B)+C
GO TO 720
C TYPE 32
425 CON*HIST
TOP* (KIS+B*KTI)*P(I,J+1)
BOT* (KIS-B*KTI)*P(I,J-1)
RIG*O.
LEF*O.
KON*A*LIQHZ+RH0(I,J)*W**2*(A*ROR(I)+1)+C
GO TO 720
C TYPE 41
415 CON*QK+B*QKB
TOP* (QK+B*QKB)*P(I,J+2)
BOT*O.
RIG = 0.
LEF = 0.
CON = (A*HC+1.)*SOLV/H/H+(B*KC-1.)*SOLVE/KK+C
GO TO 720
C
TYPE 43
410 CON = QK-B*QKA
BOT = (QK-B*QKA)*P(I,J-2)
TOP = 0.
LEF = 0.
RIG = 0.
CON = (A*HC+1.)*SOLV/H/H+(B*KC+1.)*SOLVE/KK+C
GO TO 720
C
TYPE 33
405 CON = 2.*HIS+QK-B*QKA
LEF = 2.*HIS*P(I-1,J)
BOT = (QK-B*QKA)*P(I,J-2)
RIG = 0.
TOP = 0.
CON = (B*KB+1.)*SOLVE/KK+A*RHO(I,J)*ROR(I)*W**2+C
GO TO 720
C
LIQUIDUS BOUNDARY CONDITIONS....PRESSURE
505 PF = P(I,J+1)
IF(J.EQ.NNT) PF = DT(I)
TOP = (2.*B*K3)/(KA*KK)*PF
BOT = (2.*B*KA)/(KB*KK)*P(I,J-1)
PC1 = P(I,J)
IF(HA-LT,H<0.005) PC1 = P(I+1,J)+(1.-HA/HC)*(PL(I)-PL(I+1))
IF(I.EQ.1) GO TO 310
PC2 = P(I-1,J)
IF(HB-LT,H<0.005) PC2 = P(I-1,J)+(1.-HB/HC)*(PL(I)-PL(I-1))
IF(I.EQ.MAX) GO TO 515
C
CON = (2.+A*(HB-HA))/(HA*KA)*(2.*B*(KB-KA))/(KA*KB)
RIG = (2.*A*HB)/(HA*HB)*P(I,J)
LEF = (2.*A*KA)/(HA*KB)*PC2
KON = C
GO TO 720
510 CON = 4./(HA**2+(2.*3*(KB-KA))/(KA*KB)
RIG = (4./(HA**2)*PC1
LEF = 0.
KON = C
GO TO 720
515 CON = 2./(HB**2+(2.*3*(KB-KA))/(KA*KB)
RIG = 0.
LEF = (2./(HB**2)*PC2
KON = RHO(I,J)*ROR(I)**2*(A*2./HB)+C
720 PR = (RIG+TOP+LEF+BOT+KON)/CON
P(I,J) = P(I,J)+CP*(PR-P(I,J))
500 CONTINUE
C
C CALCULATION OF Pressures****End
C
C CALCULATION OF INTERDENDRITIC FLUID VELOCITY****BEGIN
C
IF(ADCS.LE.ACCR) GO TO 6920
IF(ITER.EQ.IPR) GO TO 6920
GO TO 6921
6920 WRITE(6,6931)
IF(I.EQ.IMAX) GO TO 635
RFR = HA/HC
GLE = RFR*(GLEUT(I)-GLEUT(J)+GLEUT(I))
SOLVE = MU*CONTR*UZCL/(GAMMA(I)*GLEUT(I)*{1.+A1S(I)**2})-RLE*GRAV
IF(TYP(I,J),EQ.4) GO TO 665
IF(TYP(I,J),EQ.5) GO TO 560
IF(TYP(I,J),EQ.6) GO TO 570
IF(TYP(I,J),EQ.21),OR.(TYP(I,J),EQ.31)) GO TO 695
IF(TYP(I,J),EQ.23) GO TO 745
IF(TYP(I,J),EQ.41) GO TO 715

CONTINUE
IF(FLOW.EQ.0.) GO TO 755

LIOVE = PGZ(N)
N = M-1
IF(A1S(N).LT.0.) N=M+1
RFR = GPR(N)/HC
LIOVE = (PGR(N)-PGR(I))*RFR+PGR(I)
IF(J,EQ.NNT) GO TO 735
IF(TYP(I,J),EQ.9) GO TO 675
IF(TYP(I,J),EQ.3) GO TO 650
IF(TYP(I,J),EQ.10) GO TO 685

CONTINUE
DO 730 I=1,IMAX
M = I
IF(I.EQ.1) M = I+1
MNT = MIN(I)
NNT = NT(I)+1
VD = NNT-MNT
DO 730 J = MNT, NNT
IF(TYP(I,J),EQ.1) GO TO 650
IF(TYP(I,J),EQ.2) GO TO 653
IF(TYP(I,J),EQ.3) GO TO 654
KA = GPZ(I,J+1)
KB = GPZ(I,J-1)
HA = GPR(I+1,J)
HB = GPR(M-1,J)
KK = KA*KB
HH = KA+KA
HH = HB+HB
KK = KB*KB
KH = HB/KK
KH = HB/HH
K = KTI/KK
HA = HA*HH
HB = HB*HH
KA = KA*KK
KB = KB*KK
QH = HTI/KK
QK = KTI/KK
Q = KA+KA
Q = KB+KB
VF = (J-MNT)/VD
IF(VF.GT.0.5) GO TO 647
IF(I.EQ.IMAX) GO TO 635
RFR = HA/HC
SOLHE = MU*CONTR*UZCL/(GAMMA(I)*GLEUT(I)*{1.+A1S(I)**2})-RLE*GRAV
IF(TYP(I,J),EQ.4) GO TO 665
IF(TYP(I,J),EQ.5) GO TO 560
IF(TYP(I,J),EQ.6) GO TO 570
IF(TYP(I,J),EQ.21),OR.(TYP(I,J),EQ.31)) GO TO 695
IF(TYP(I,J),EQ.23) GO TO 745
IF(TYP(I,J),EQ.41) GO TO 715

CONTINUE
IF(FLOW.EQ.0.) GO TO 755

LIOVE = PGZ(I)
N = M-1
IF(A1S(N).LT.0.) N=M+1
RFR = GPR(N)/HC
LIOVE = (PGR(N)-PGR(I))*RFR+PGR(I)
IF(J.EQ.NNT) GO TO 735
IF(TYP(I,J),EQ.9) GO TO 675
IF(TYP(I,J),EQ.3) GO TO 650
IF(TYP(I,J),EQ.10) GO TO 685
IF I TYPE 1, EQ. 11) GO TO 670
IF I TYPE 1, EQ. 7) GO TO 690
IF I TYPE 1, EQ. 22) GO TO 700
IF I TYPE 1, EQ. 24) GO TO 735
IF I TYPE 1, EQ. 32) GO TO 750
IF I TYPE 1, EQ. 34) GO TO 740
IF I TYPE 1, EQ. 33) GO TO 740
IF I TYPE 1, EQ. 43) GO TO 710

C TYPE 1
650 PDR = (P(I+1, J) - P(I-1, J)) * QT
PDZ = (P(I+1, J) - P(I-1, J)) * KT
GO TO 780

C TYPE 2
653 PDR = 0.
PDZ = (P(I+1, J) - P(I-1, J)) * KT
GO TO 780

C TYPE 3
654 PDR = 0.
PDZ = (P(I+1, J) - P(I-1, J)) * KT
GO TO 780

C TYPE 4
665 PDR = HHA + QHA * (P(I, J) - P(I-2, J))
PDZ = KKA + QKA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 5
660 PDR = (P(I+1, J) - P(I-1, J)) * QT
PDZ = KKA + QKA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 6 OR 11
670 PDZ = (P(I+1, J) - P(I-1, J)) * KT
SHLOS = SHL0S
IF I TYPE 1, EQ. 11) SHLOS = LIQHZ
PDZ = HHA + SHLOS + QHA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 7
690 PDR = HHA + LIQHZ + QHA * (P(I+2, J) - P(I, J))
PDZ = KKA + LIQVE + QKA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 8
680 PDR = HHA + LIQHZ + QHA * (P(I+2, J) - P(I, J))
PDZ = KKA + LIQVE + QKA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 9
675 PDR = (P(I+1, J) - P(I-1, J)) * QT
PDZ = KKA + LIQVE + QKA * (P(I, J) - P(I-2, J))
GO TO 780

C TYPE 10
685 PDR = HHA + LIQHZ + QHA * (P(I+2, J) - P(I, J))
PDZ = KTI * (P(I, J+1) - P(I, J-1))
GO TO 780

C TYPE 21 OR 31
695 PDR = 0.
PDZ = KKA + SOLVE + QKA * (P(I, J+2) - P(I, J))
GO TO 780

C TYPE 22 OR 32
700 PDR = 0.
PDZ = KKA + SOLVE + QKA * (P(I, J-2) - P(I, J-2))
GO TO 780

C TYPE 24
735 PDR = 0.
C 743 PDR=0.
    PDZ=KKB*LIQVE+QKA*(P(I,J)-P(I,J-2))
    GO TO 780
C 740 PDR=0.
    PDZ=KKB*LIQVE+QKB*(P(I,J)-P(I,J-2))
    GO TO 780
C 750 PDR=0.
    PDZ=KT1*(P(I,J+1)-P(I,J-1))
    GO TO 780
C 719 PDR=MMB*SOLHZ
    PDZ=KKB*SOLVE+QKB*(P(I,J+2)-P(I,J))
    GO TO 780
C 705 PDR=LIQHZ
    IF((I.EQ.1).OR.(I.EQ.IMAX)) PDR=0.
    LIQVE=PGZ(I)
    GO TO 790
C 704 PDR=H8/(HA*HH)*(PC1-P(I,J))
    PDZ=LIQVE
    GO TO 790
C 755 IF(I.EQ.1).OR.(I.EQ.IMAX) GO TO 760
IF(J.EQ.NNT) GO TO 770
PC1=P(I+1,J)
PC2=P(I-1,J)
IF(HA.LT.HC-0.005) PC1=P(I+1,J)+(1.-HA/HC)*(PL(I)-PL(I+1))
IF(HB.LT.HC-0.005) PC2=P(I-1,J)+(1.-HB/HC)*(PL(I)-PL(I-1))
PDR=H8/(HA*HH)*(PC1-P(I,J))+HA/(HB*HH)*(P(I,J)-PC2)
GO TO 765
760 PDR=0.
    IF(I.EQ.NNT) GO TO 774
765 IF(J.EQ.NT(I)) GO TO 775
PDZ=(P(I,J+1)-P(I,J-1))/(KA*K8)
GO TO 790
770 NPM=1
    L=I+1
    IF(ZL(I).GT.ZL(L)) GO TO 772
    GO TO 773
772 L=I-1
    NPM=1
773 NH=ZL(L)/K8+2
    PK=PT(L)
    IF(I.EQ.NH) PK=PT(L)
    PDZ=NPM*(PT(I)-PK+(PK-P(L,J-1))*(1.-GPZ(I,J)/GPZ(L,J)))/HC
774 PDZ*(PT(I)-P(I,J-2))/(GPZ(I,J)+GPZ(I,J-1))
GO TO 790
775 PDZ=K8/(KA*K8)*(PT(I)-P(I,J))+K8/(KB*KK)*(P(I,J)-P(I,J-1))
780 VR(I,J)=-GAMMA(I)*GL(I,J)/MU*(PDR*RHO(I,J))*ROR(I,J)*W**2
    VZ(I,J)=-GAMMA(I)*GL(I,J)/MU*(PDZ+GRAV*RHO(I,J))
    IF(VR(I,J).EQ.0.) VR(I,J)=1.00-6
    IF(A0CS.LE.ACCR) GO TO 6911
    IF(ITER.EQ.IPR) GO TO 6911
GO TO 730
**Calculation of Intergeneric Fluid Velocity**

```fortran
VTOT = VR(I,J)**2 + VZ(I,J)**2)**0.5
RARA = VR(I,J)*GR(I,J) + VZ(I,J)*GZ(I,J)/EPS(I,J)
ANG = VZ(I,J)/VR(I,J)
TTHETA = (180.0/P) * ATAN(ANG)
IF (RARA.LT.-1.0) IFR = 1
WRITE(6,7500) 5L(I,J), P(I,J), VZ(I,J), VP(I,J), VTOT, TTHETA, RARA, $I,J, ITER
7500 FORMAT(1X,1E13.4,1E15.6,5E12.3,2I5,Z7)
```

**Calculation of Macrosegregation**

```fortran
IF(ADCS.LE.ACCR) GO TO 6919
IF(ITER.EQ.IPR) GO TO 6918
GO TO 6919
6913 CONTINUE
WRITE(6,6933)
6933 FORMAT(5X,"********************",4X,"********************",4X,$
"********************",5X,"********************",5X,"**********")
WRITE(6,9900)
9900 FORMAT(5X,"LOCAL SOLUTE COMP","4X,"FRACT LIQUID EUTECTIC","4X,$
"RADIUS OF INGOT",5X,"COLUMN NO",5X,"ITERAT")
WRITE(6,6934)
6934 FORMAT(5X,"********************",4X,"********************",4X,$
"********************",5X,"********************",5X,"**********")
6919 CONTINUE
SUM1 = 0.0
SUM2 = 0.0
DO 1020 I = 1, IMAX
NMT = NT(I)
SUM = 0.
NMT = JMIN(I)
DO 810 IT = MNT, NMT
J = IT + MNT
IFJ = EQ.NMT) NLGL(J+1) = 0.0
COEFT1 = KON4*RH0(I,J)/CL(I,J)*RE)
COEFT2 = KON4*RH0(I,J+1)/CL(I,J+1)*RE)
RATER1 = VR(I,J)*GR(I,J)*VZ(I,J)*GZ(I,J)
RATER2 = VR(I,J+1)*GR(I,J+1)*VZ(I,J+1)*GZ(I,J+1)
VZE(I) = VZCPL(UZCL(I-JALSI)**2)
VRE(I) = ALS(I)*VZE(I)
RATER = VRE(I)*(TL-TE)/(ZL(I)-ZS(I)) - VRE(I)*(TL-TE)*ALS(I)/$ZL(I)-ZS(I))
FNJ = COEFT1*(1.0*RATER1/EPS(I,J))
FNJ+1 = COEFT2*(1.0*RATER2/EPS(I,J+1))
NLGL(J) = 5.0*(CL(I,J)-CL(I,J+1))*(FNJ+FN(J+1)) + SUM
IF(NLGL(J) .GE. 0.0) NLGL(J) = 0.
SUM = NLGL(J)
GL(J) = EXP(NLGL(J))
ST(J) = 1.0 - GL(I,J)
```

**Integration of the Local Solute Distribution Equation**

*Followed through column-wise starting from the liquidus.*

```fortran
6911 CONTINUE
VTOT = VR(I,J)**2 + VZ(I,J)**2)**0.5
RARA = VR(I,J)*GR(I,J) + VZ(I,J)*GZ(I,J)/EPS(I,J)
ANG = VZ(I,J)/VR(I,J)
TTHETA = (180.0/P) * ATAN(ANG)
IF (RARA.LT.-1.0) IFR = 1
WRITE(6,7500) 5L(I,J), P(I,J), VZ(I,J), VP(I,J), VTOT, TTHETA, RARA, $I,J, ITER
7500 FORMAT(1X,1E13.4,1E15.6,5E12.3,2I5,Z7)
```
CS(J)*KAY*CL(I,J)
GO TO 910

810 CONTINUE
J=MNT

EPPSE=-(TL*TE)*UZCL/(ZL(I)-ZS(I))
NLGE=NLGL(J)+0.5*(CE-CL(I,J))*(1.-KONA*RLE/(CE*RSE))*(1.+RATEPE/$EPPSE)*FN(J))
GEE=EXP(NLGE)

LOCAL SOLUTE COMPOSITION FOR DESIGNATED RADII WITHIN THE INGOT IS INITIATED.

J=JMIN(I)
SUMMS=0.5*(CSE+CS(J))*((1.-GEE)-GS(J))
NNT=NNT(I)+1
J=NNT
GS(J)=0.0
CS(J)=KAY*C
PPP=JMIN(I)+1
DO 850 J=PPP,NNT
SUMMS=0.5*(CS(J)+CS(J-1))*(GS(J-1)-GS(J))+SUMMS
850 CONTINUE
GLEUT(I)=GEE
TNT*RE*SUMMS*RSE*CE*GEE
DDD*RE*(1.-GEE)*RSE*GEE
LOCCOM(I)=TNT/DDD

AVERAGE INGOT COMPOSITION ACROSS THE INGOT IS CALCULATED
IF(I.EQ.1) GO TO 880
IF(I.EQ.IMAX) GO TO 985
DELRAD=HC
GO TO 890

880 DELRAD=HC/2.
ROR(I)=HC/4.
GO TO 890

885 DELRAD=HC/2.

890 CONTINUE
SUUM1=ROR(I)*TNT*DELRAD+SUUM1
SUUM2=ROR(I)*DDD*DELRAD+SUUM2
CSINGT=SUUM1/SUUM2
ROR(I)=0.0
IF(AOCS.LE.ACCR) GO TO 6913
IF(ITER.LE.IPR) GO TO 6913
GO TO 1020

6913 CONTINUE
WRITE(6,7900) LOCCOM(I),GEE,ROR(I),I,ITER
7900 FORMAT(1X,E15.4,E123.4,E20.3,E113,E113/)
IF(I.EQ.IMAX) GO TO 6914
GO TO 1020

6914 WRITE(6,6935)
6935 FORMAT(5X,**************)
WRITE(6,6915)
6915 FORMAT(5X,AVERAGE INGOT COMPOSITION)
WRITE(6,6936)
6936 FORMAT(5X,**************)
WRITE(6,6916) CSINGT
6916 FORMAT(1X,E20.4//)
DTEST-CSINGT-TEST
ATEST=ABS(DTEST)
TEST=CSINGT
IF(IFREC.EQ.1) WRITE(6,1003)
IF((ADCS.GT.0.5).AND.(IFREC.EQ.1)) GO TO 1011
GO TO 1012
1011 WRITE(6,1003)
1003 FORMAT(15X,"**************FRECKLES************"//)
WRITE(6,6937)
GO TO 1001
1012 CONTINUE
IF(ATEST.LE.0.05) GO TO 1013
GO TO 1015
1013 WRITE(6,1014)
1014 FORMAT(15X,"************SLOW CONVERGENCE************"//)
WRITE(6,6937)
GO TO 1001
1015 CONTINUE
WRITE(6,6937)
6937 FORMAT(2X,"**************************************************
IF(ADCS.LE.ACCR) GO TO 1001
IF(ITER.EQ.LITER) GO TO 1001
C C CALCULATION OF MACROSEGREGATION****END
C C 1020 CONTINUE
C C CALCULATION OF MACROSEGREGATION****END
C DCS=CSINGT-CO
ADCS=ABS(DCS)
GO TO 1000
1001 CONTINUE
C C ITERATION SEQUENCE FOR 'P', 'VELOCITY', 'GL' AND COPM****END
C STOP
END
SUBROUTINE COEFF(I,J,A,B,C)
   
   THIS SUBROUTINE CALCULATES THE COEFFICIENTS A, B, AND C OF THE PRESSURE DISTRIBUTION EQUATION

COMMON/GRAD/GZ(22,70),GR(22,70),EPPS(22,70),DERHOR(22,70)
COMMON/GRADZ/DERZ(22,70),DECLZ(22,70),DECLR(22,70)
COMMON/CONST/RHO(22,70),CL(22,70),GAMMA(22),R0P(22)
COMMON/RCAL/GL(22,70),KK,HH
REAL KK
COMMON/INIT/GRAV,EM,DRDC,CON2,IZCL,IMAX
REAL MU,KON2
ALFA=(RHO(I,J)/RE-1.)*KON2/CL(I,J)
DEGLZ*(GL(I,J+1)-GL(I,J-1))/KK
IF(I.EQ.1) GO TO 5
DEGLR*(GL(I+1,J)-GL(I-1,J))/HH
IF(I.EQ.IMAX) DEGLR=(GL(I,J)-GL(I-1,J))/HH
A1=1.000/ROR(I)
A2=2.*DEGLR/GL(I,J)*DERHOR(I,J)/RHO(I,J)
A3=ALFA*DECLR(I,J)
A=A1+A2+A3
GO TO 8

A=0.
CONTINUE
B1=2.*DEGLZ/GL(I,J)*DERHOZ(I,J)/RHO(I,J)
B2=ALFA*DECLZ(I,J)
C1=ALFA*(DECLZ(I,J)+2.*DERHOZ(I,J)/RHO(I,J)
C2=ALFA*(DECLZ(I,J)-*(W**2)*DELR(I,J)*ROR(I)/GRAV)
C3=GRAV*RHO(I,J)*(C1+C2)
C4=DRDC/RHO(I,J)+ALFA
C5=EPPS(I,J)*MU*(EM*GAMMA(I)*GL(I,J))
C6=(W**2)*(2.*RHO(I,J)+2.*RHO(I,J)*ROR(I)*DEGLR/GL(I,J)+2.*ROR(I)*
*DERHOR(I,J))
B=B1+B2
C=C3-C4*C5-C6
RETURN
END
REFERENCES


