PHYSICAL MODELLING TO ASSESS THE DYNAMIC BEHAVIOR OF ROCK SLOPES

by

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ABSTRACT

The design of rock slopes in seismically active areas should consider the risk of earthquake loads. Very few methods have been proposed to determine the response of slopes to dynamic loads, and until recently they had not been tested. This study involved testing simple models of rock slopes on a hydraulic shaking table and displaying the results on time versus displacement curves.

Model tests using wood and steel sliding blocks on an inclined ramp were tried and discarded before rock blocks on a rock base were found to give acceptable repeatability. Tests were performed at varying frequencies and amplitudes which produced maximum accelerations in the range of those measured during earthquakes.

The results were compared with predicted displacements computed using methods proposed by Newmark and Hendron, Cording, and Aiyer. At low frequencies and small amplitudes the predicted sliding rates compared very well with the rates observed during the testing. However, as the frequencies and amplitudes were increased, (and therefore, the accelerations increased) the equations overestimated the displacements by gradually increasing amounts. Generally, Hendron's method was slightly less conservative than Newmark's.

If the physical properties of the rock are known, the equations can be used to obtain a conservative estimate of the amount of displacement to be expected as a result of dynamic ground motions.

CHAPTER 1

INTRODUCTION

Slopes in seismically active areas are seldom designed to withstand dynamic loads. Very few methods have been proposed to handle such conditions and most engineers have not considered it to be feasible for designing such structures as open pit mines since methods have not been previously tested. However, the increasing use of open pit methods for mining and the growing awareness that earthquakes can pose a significant hazard makes it necessary to investigate the effects of various dynamic loading conditions on rock slopes.

Studies by Glass, Savely, and Call (1977), indicate that economic optimum slope design is sensitive to earthquake hazard. Since the risk of slope failure should be considered in assessing the economic feasibility of engineering projects, earthquake hazard is an important consideration in evaluating the stability of slopes. Therefore, an analysis to determine an economic optimum slope design should include the probability of occurrence of nearby earthquakes. In recent years, several techniques have been proposed to evaluate the dynamic response of rock slopes, but until now no attempts have been made to compare these by model testing or other methods. The pseudostatic technique which has been used in civil engineering for many years, treats the earthquake as a static force. Methods proposed by Newmark (1965) and

Hendron, Cording, and Aiyer (1971), involve simple equations to estimate displacements that would be caused by earthquake motions.

In an effort to determine the validity of some of these techniques for analyzing the dynamic stability of rock slopes, a simple model testing program was carried out. This program consisted of modeling idealized slopes on a hydraulic shaking table and monitoring the downslope displacement of blocks of rock and other materials. Control of the dynamic motions of the shaking table was maintained, and the physical properties of the model material were known.

Motions available from the shaking table are restricted to simple harmonic motions over a wide range of frequencies. This fact makes the testing program more idealized than desired, but does not cause major problems in the analyses.

CHAPTER 2

THEORETICAL BACKGROUND

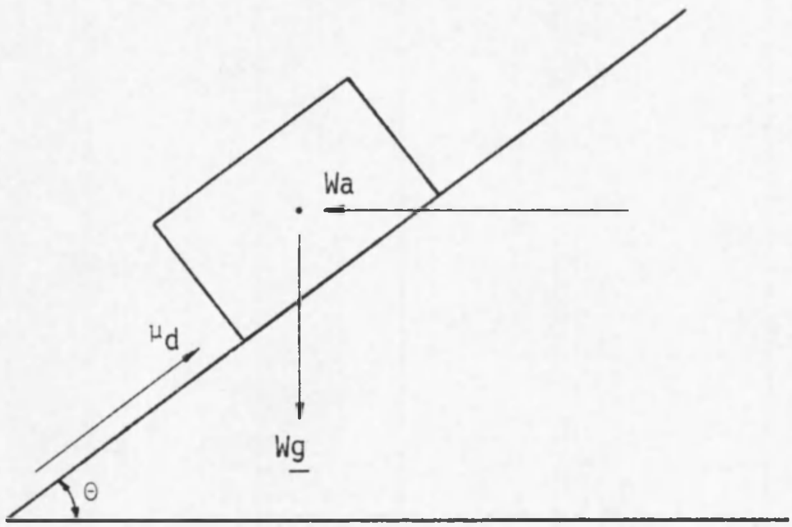
The recognition that engineering design in seismically active areas requires special attention has led to several advancements in the techniques used to deal with such problems. Large factors of safety have been required in many eningeering projects to account for earthquake loads, but it is impossible to quantitatively determine precisely how large the factor of safety should be. A slightly more reliable method, still frequently used today, involves including the maximum expected ground acceleration as an additional static force along with the standard static analysis.

More recent studies have attempted to estimate the amount of displacement an element of soil or rock would undergo in response to specific ground motions. Newmark (1965) developed simple equations to analyze the behavior of soil masses subjected to dynamic forces. Hendron et al. (1971), expanded on this theory and developed similar equations.

Pseudostatic Technique

The pseudostatic technique has been accepted for years on account of its simplicity and alleged conservatism. Using this technique, a design earthquake magnitude is decided upon, and its maximum ground acceleration at the slope is determined. The engineering stability analysis is carried out using the usual limit equilibrium methods (Jaeger and Cook, 1976), adding a horizontal force equal to the design acceleration multipled by the mass of the structure. The method was considered to be conservative because the ground acceleration is added as a constant static force, even though the maximum acceleration is only reached for a brief instant during an earthquake. Many structures which were designed using this method have withstood large earthquakes, thereby lending credence to the technique.

However, many earth materials are known to lose strength when subjected to dynamic movements. This fact has led to a further refinement of the pseudostatic techniques of designing for earthquake loads. Many soils may lose part or nearly all of their shearing resistance under dynamic conditions, either because of increased pore pressure or remolding of internal soil structure. The development of laboratory tests using cyclic loading has helped to make it possible to determine dynamic strength properties of earth materials. Using dynamic strengths in the analysis provides a reliable assessment of material behavior during earthquakes. For example, in determining the stability of a rock block on an inclined plane, the driving forces are the product of the mass times the component of acceleration due to gravity parallel to the plane, plus the mass times the component of maxium ground acceleration parallel to the plane. The resisting force is given by the dynamic coefficient of friction times the normal force. A simple example is presented in figure 1.



F.S. = factor of safety

W = weight of block

μ_d = dynamic coefficient of friction

g = acceleration of gravity

a = maximum ground acceleration developed during earthquake

Θ = slope angle

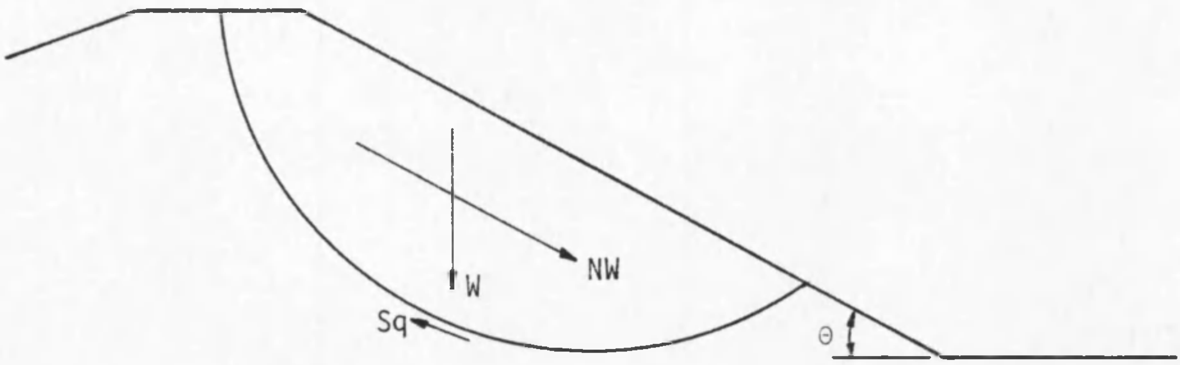
$$F.S. = \frac{(Wg \cos \Theta - Wa \sin \Theta)\ \mu_d}{Wg \text{ sine } \Theta + Wa \cos \Theta}$$

Figure 1. - Pseudostatic analysis of a rigid block on a plane sliding surface.

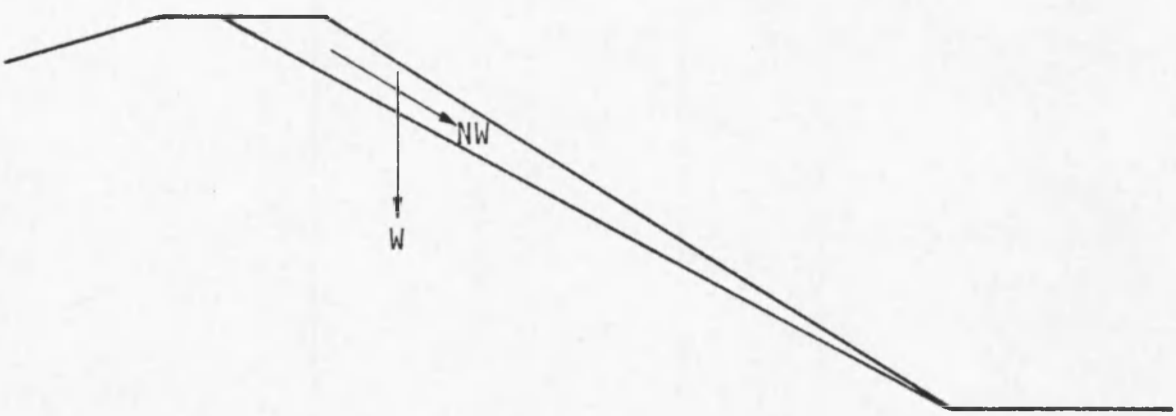
Newmark's Equations

Newmark (1965) developed equations to compute the displacement of discreet soil or rock elements as a function of the maximum ground velocity occuring during an earthquake. He considered cases of a rigid block of weight W, supported on a base which moves as a function of time. Although for many cases the resistance to sliding varies with the block displacement, this discussion centers on rigid-plastic resistance, where no displacement occurs until the yield point of the system is reached. Newmark considered three cases of sliding, a circular surface, a plane surface, and a block sliding horizontally (fig. 2).

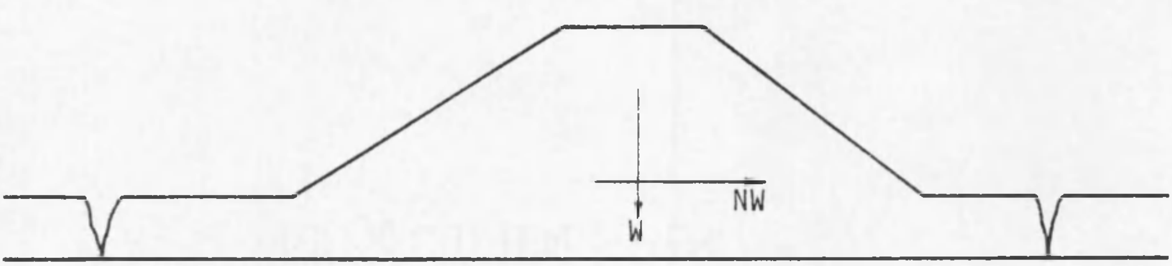
The resistance to earthquake ground motion of a block of soil or rock that slides on a surface is a function of the shearing resistance of the material under the conditions applicable during the earthquake. The magnitude of this resistance depends on the amount of displacement induced; however, very little displacement is necessary to mobilize the average yielding resistance normally considered in a stability analysis. In his analysis, Newmark used a frictional resistance value which is a steady force acting through the center of gravity of the sliding mass. This is the force which will just overcome the stabilizing forces and keep the mass barely moving after it has started to move, or after several pulsations of motion. In order to apply this resistance in the analysis, he introduced a coefficient, N, multiplied by the weight of the sliding mass, which can be a function of the amount of deformation, of time, or of any other parameter. Then the quantity Ng, where g is the acceleration of gravity, corresponds to that steady



(a) Circular Sliding Surface



(b) Plane Sliding Surface



(c) Block Sliding

Figure 2. - Three cases of sliding for an embankment.

acceleration, acting in the proper direction, which would just overcome the resistance to sliding of the element.

It is common practice to give man-made earth embankments a factor of safety against static failure, and this is equally important if the slope is to have dynamic resistance to sliding. The static factor of safety is computed by conventional means with no consideration for horizontal or inclined accelerations. Values of the dynamic factor of safety against sliding may be determined in a similar manner, but the dynamic physical properties of the materials must be applied in order to take into account any loss of shear strength which may occur as a result of rapid changes in pore pressures.

Before a value for N can be calculated it is first necessary to compute the dynamic factor of safety against sliding. Newmark presented equations for calculating safety factors for each of the three cases of sliding failure discussed. For a circular cylindrical sliding surface the dynamic factor of safety can be defined as the sum of the resisting moments divided by the sum of the driving moments:

$$F.S. = R\, \Sigma Sq\, ds / R\, \Sigma\tau\, ds = Sq\, ds/\Sigma\tau\, ds$$

where:

R = radius of slip circle

Sq = shearing strength of soil

τ = shearing force on soil

ds = width of slice of soil

For horizontal block sliding of the entire embankment between fissures or embankment surfaces, the factor of safety is found by summing forces rather than moments. The resisting force is the maximum shear strength which can be mobilized during an earthquake; that is, the undrained shear strength, Sq. The driving force is the effect of ground acceleration on the embankment.

The equation for the factor of safety of a block on a plane sliding surface such as was used in the model tests is easily applied. For cohesionless, free draining materials, and a block with a dynamic friction angle, $\emptyset$, between the sliding surface and the rigid base material, the factor of safety can be computed by:

$$F.S. = \tan \emptyset / \tan \theta$$

Equations to calculate the value of N for each slope were derived for the potential sliding surfaces by equating driving and resisting forces or moments using a method of slices. For the circular cylindrical sliding surface, N is found by:

$$N = (F.S. - 1)b/h$$

where: b = horizontal distance from center of slip circle to center of gravity

h = distance from center of circle to line parallel to slope surface passing through center of gravity

To find N for the case of horizontal block sliding the equation is:

$$N = Sq/p(1 - ru)$$

$$ru = \Sigma\, u_p\, ds / \Sigma\tau\ h\ ds$$

where: Sq = undrained shear strength

p = effective overburden pressure

u_p = pore pressure

τ = bulk density of the soil

h = height of element

ds = width of slice of soil

For a plane sliding surface, under conditions described above, it can be determined that the minimum value of N is:

$$N = (F.S. - 1) \sin \Theta$$

In deriving the equation to estimate the displacement incurred in a sliding block subjected to dynamic motions, Newmark based his calculations on the assumption that the whole mass moves as a single rigid body with resistance mobilized along the sliding surface. The acceleration considered is a single pulse of magnitude A_g_ acting on the mass M, lasting for a time interval t_o. The resisting acceleration is N_g_. Figure 3 shows the velocities as a function of time for both the accelerating force and the resisting force. The maximum velocity for the accelerating force has a magnitude V, given by the expression:

$$V = A\underline{g}\ t_o$$

Once time t_o is reached, the velocity due to the accelerating force remains constant. The velocity due to the resisting acceleration of the block has the magnitude N_g_ t. The maximum displacement of the mass relative to the ground is obtained by computing the shaded triangular area in figure 3. This is done by:

$$Um = 1/2\ V\ t_m - 1/2\ V\ t_o$$

$$Um = 1/2\ V^2/N\underline{g} - 1/2\ V^2/A\underline{g}$$

$$Um = V^2/2N\underline{g}\ (1 - N/A)$$

where: V = maximum ground velocity

A = maximum ground acceleration in percent _g_

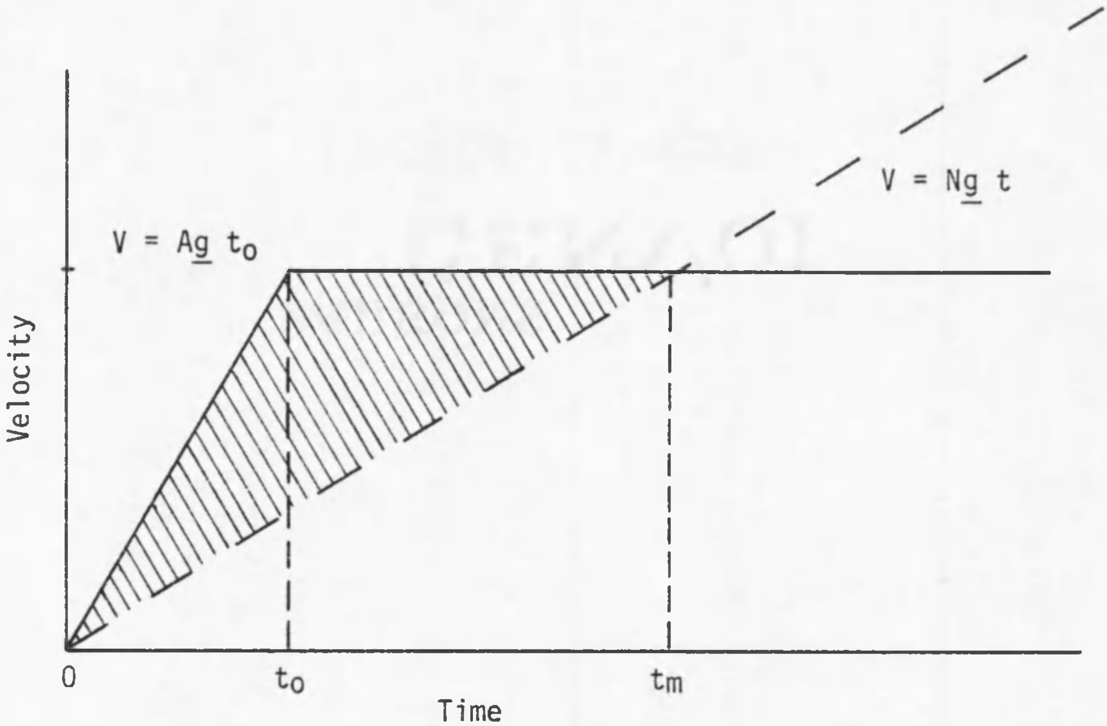


Figure 3. - Velocity response to rectangular block acceleration.

From Newmark, 1965, p. 153.

According to Newmark, this equation will generally overestimate the amount of displacement generated by earthquake ground motions because it does not take into account pulses in the opposite direction. The important fact indicated by the equation is that the displacement is proportional to the square of the maximum ground velocity. This supports Dr. Newmark's claim that ground velocity, rather than acceleration, is most important in determining behavior of materials subjected to earthquake motions.

It is very important in an earthquake analysis to consider the effective number of spikes of acceleration in order to compute the cumulative displacement. Newmark addressed this problem by studying four West Coast United States earthquakes for which strong motion accelerogram records are available. His results indicated that the effective number of pulses in the earthquakes considered is equal to A/N, the maximum acceleration divided by the resistance coefficient. However, his data indicated that when A/N is very large (greater than 10.0), the number of effective pulses in the earthquake is apparently no greater than six.

Newmark then applied these equations and procedures to model tests of a rockfill dam. The scale of the model was one three-hundredth of the prototype. The tests were performed by striking a shaking table with a heavy pendulum and allowing it to rebound and cause a second, lower acceleration. The measured displacements were in fair agreement with the results predicted by the equation.

Marek and Call, personal communications, (1977), have recently begun earthquake stability analyses of open pit mines in seismically active areas using Newmark's equations. Their method is simple, but appears to be a great improvement over the traditional method of applying a static load equal to the maximum acceleration of the design earthquake. The maximum acceleration which would be encountered in a mine from an earthquake of a given magnitude is first applied to the slope, and the factor of safety is calculated. If the safety factor is less than one, then displacement is said to occur along the fracture being examined and it is assumed that the strength along the fracture is reduced due to the breaking of intact rock bridges. The shear strength is then considered to be the residual strength value along the fracture, and it is conservatively assumed that the reduction in strength is immediate.

At this point, a new factor of safety is calculated using the residual strength values found from direct-shear test results, to determine the postquake stability of the slope. If the slope is still stable for these reduced strength conditions, the conclusion is that the earthquake causes displacement, but the slope stabilizes after the earthquake. The amount of displacement caused by the ground motions is approximated using the empirical relationships developed by Newmark. Since the true relationship of N/A is not known, the factor to account for the number of pulses is unknown. Therefore, the displacement is taken as the minimum of these functions:

$$Um = (V^2/2g\underline{N})\ (A/N)$$

$$\text{or } Um = 6\ (V^2/2g\underline{N})$$

where (A/N) and (6) are factors to account for the number of acceleration cycles.

The conclusions from this are that if the displacement is less than some arbitrary value, usually 1 meter, the stable postquake factor of safety is used. The block is assumed to have displaced without causing failure of the pit wall. If the displacement exceeds this value, the slope is considered to be at impending failure with a factor of safety equal to one.

Hendron Method

A study by Hendron et al. (1971), produced an equation very similar to Newmark's for approximating the relative movement of a rock block induced by dynamic ground accelerations. First the yield acceleration, Ky $\underline{g}$, must be determined either by graphical methods using stereographic projection, or by a static resolution of forces method. Ky $\underline{g}$ is the horizontal force applied to the center of gravity of the rock block which would just overcome the resisting forces. This factor is similar to Newmark's N.

The ground acceleration and its duration, or the ground acceleration and the particle velocity, must be known. When the acceleration reaches or exceeds Ky $\underline{g}$, relative displacement between the ground and the block begins to occur. This slip continues until the block acquires a velocity equal to the ground velocity, $V = A\underline{g}\, t_o$.

t_o = duration of ground acceleration ($A\underline{g}$)

At time t_m, the total displacement of the ground is:

$$Xg = (1/2)\, A\underline{g}\, t_o^2 + A\underline{g}\, t_o\, (t_m - t_o)$$

$$t_m = V/Ky\, \underline{g}$$

t_m = time at which block velocity equals ground velocity

The total displacement of the block at time t_m is:

$$Xb = (1/2)\ Ky\ \underline{g}\ t_m^2$$

Therefore, the total relative displacement between the block and the ground is:

$$\Delta X = Xg - Xb$$

$$\Delta X = (-1/2)\ A\underline{g}\ t_o^2 + A\underline{g}\ t_o\ t_m - (1/2)\ Ky\ \underline{g}\ t_m^2$$

and since $t_o = V/A\underline{g}$ and $t_m = V/(Ky\ \underline{g})$:

$$X = V^2/2\underline{g}\ (1/Ky - 1/A)$$

which is very much like Newmark's displacement equation. Hendron et al. (1971), stated that this equation is not strictly valid for an inclined plane since the direction of relative motion calculated is not necessarily the same as the downslope direction of slip. However, estimates of the severity of a particular impulse on a block with known yield acceleration, Ky $\underline{g}$, should be reasonable. As the model tests performed for this study show, it is generally a slight improvement over Newmark's equation.

CHAPTER 3

DYNAMIC TESTING PROGRAM

In order to assess the accuracy of the techniques used to predict the response of rock slopes subjected to dynamic forces, a simplified model testing program was designed, and the results were recorded and are presented in this thesis. The primary objective of the testing program was to observe the behavior of blocks of known mass sliding down an inclined slope of a predetermined angle under controlled dynamic motions. These motions were charted on an X-Y recorder and the block sliding results from each trial performed under similar conditions were plotted on a graph to observe variations. Wood and steel sliding blocks were tried with limited success before rock blocks were used successfully on a rock base. The final tests, using granite for modeling the slopes, gave fair repeatability of results for comparing with the displacements as predicted from analytical techniques.

Description of Equipment

The dynamic motions were achieved with the use of a Riehle-Los Universal Hydraulic Fatigue Testing Machine. The machine was designed to operate with loading in one direction (single action) or two directions (double action), for these tests it was operated in single action, providing a pulsating compressive load of varying frequency and displacement. Load is applied through two driving rams attached to a large wooden shaking table. Return action is provided by four steel springs connected to the opposite side of the table. The table is mounted on guiding rails ng rails to restrict sideward motions and rolls on ball bearings. The guiding rails, springs, and driving rams are securely bolted to the laboratory floor.

The Rhiele-Los Fatigue Testing Machine provides wide flexibility in the selection of cycling frequencies, but has a limited range for controlling the table displacement. Frequencies between 50 and 1,000 cycles per minute, and higher can be used but extremely high frequencies cause violent vibrations of the pressure lines and table causing damage. Usually operating between 200 and 400 cycles per minute provided good results in terms of earthquake accelerations. The stroke was never allowed to exceed 0.2 inch total displacement and was usually kept between 0.12 and 0.19 inch during the model testing program. The allowable limit of the system is approximately 0.25 inch.

Data Recording

The data recording system used in the study consisted of a Sanborn Two Channel recording system, Model 296, and two Moxon DC/DC Linear Displacement Transducers. Simultaneous recordings were made of the table motions and the downslope displacement of the sliding block. Acceptable results were obtained during the testing program, but due to the incompatibility of the recorder and the transducer, the curves were not linear and had to be calibrated by hand at the end of each day's testing.

The Sanborn Two Channel recording system is a self-contained direct-writing recording system, designed for operation with two plug-in preamplifiers. Inkless recordings are made on heat sensitive permapaper with heated wire writing arms on rectilinear coordinates. This recording system is mounted on a mobile cabinet for access to the shaking table operator (fig. 4).

Two Sanborn Carrier Preamplifiers, Model 250-1100B, were used as part of the two-channel data recording system. These preamplifers operate in conjunction with the external transducers to measure displacements for display on the recording system. A special circuit permits measuring either the instantaneous or the average value of the variable applied to the transducer. These tests required the instantaneous values for determining the displacements achieved by the shaking table and the sliding blocks. The preamplifier also supplies 5 volts input for the transducers, but this was not sufficient for operation of the transducers used in these tests.

The displacements were measured using Moxon DC/DC Transducers which require 6 volts input for each instrument. The voltage was supplied and regulated with a Moxon Model 3567 - 100 power supply. Each transducer was equipped with a spring and follower attachment to keep the measuring bar extended and allow for accurate tracking of all moving test materials. One transducer was tightly fitted into a woodblock which was secured to the floor to prevent sliding. This was placed against the shaking table, near the driving rams and measured the vibrating motions of the platform. The other transducer was strapped to a similar woodblock which was nailed to the adjustable ramp and measured

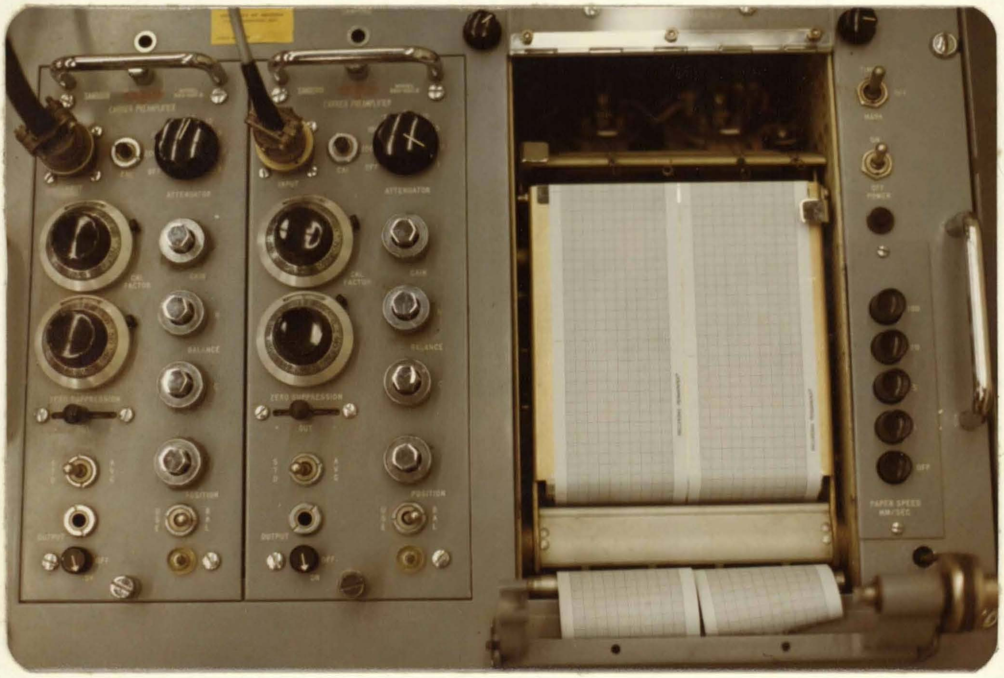


Figure 4. - Photograph of recording system.

the downslope displacement of the sliding blocks. The transducers were each wired to a separate preamplifier and gave independent tracks of displacement versus time on the recording system.

Slope Modeling

A wooden ramp was constructed out of 1-inch plywood, 2 by 4 planks, and door hinges (fig. 5). The extra thick plywood provided strength and stiffness for the rock tests, and also added mass to resist sliding as it sat on the table surface. Four steel ribs ran lengthwise across the surface of the shaking table, and the ramp was sized to fit snugly between these for extra prevention against independent movement. Three heavy duty door hinges were bolted to the two pieces of plywood for easy maneuverability. Lengths of 2 by 4 wood were nailed to the top and bottom of the sliding surface.

The most difficult problem was devising a method to hold the ramp steady at a controlled angle. This was solved by cutting two strips of aluminum angle rod to equal lengths and attaching them to each side of the ramp. The rods were screwed into threaded metal pieces which were bolted to the plywood so that they could be loosened easily to adjust the ramp's position. A slot was cut into each aluminum rod through which the top screw could fit and be tightened at any place along the cut. The bottom screws fit through a single hole and held the rods in place. The angle of the ramp could then be changed by loosening the two top screws and raising or lowering the sliding surface to the desired height. When all four screws were tight, the ramp was stiff and could only move as one piece with the shaking table.

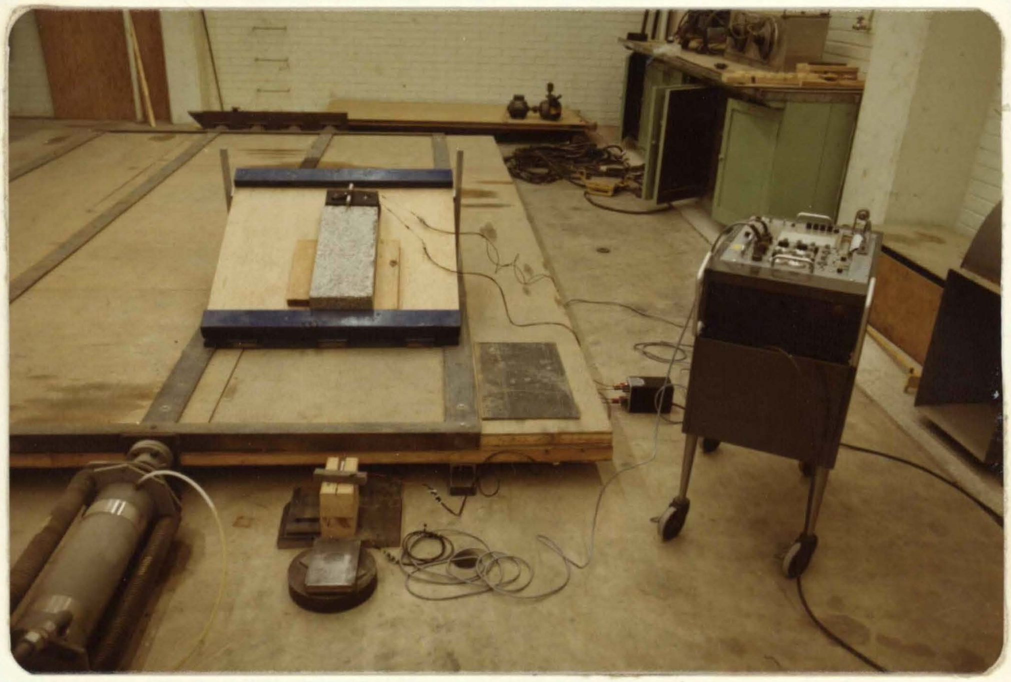


Figure 5. - Photograph of block sliding ramp.

For the rock-on-rock sliding block tests, a flat block of granite was placed on the ramp to act as a steady base. This block was secured tightly to prevent vibrations by nailing lengths of wood tightly against the sides of the block. The upslope block of wood supported the displacement transducer which measured the downslope movement of the sliding blocks.

Model Tests

The first series of tests were performed using a wood sliding block weighted with a piece of steel. The block was made with scrap wood and had a plywood bottom to give a good wood-to-wood contact. The bottom was sanded smooth as was the sliding surface of the ramp in order to eliminate any asperities which might cause the test results to vary. The aim of the model tests was to simulate the ideal condition of a perfectly flat surface with no roughness or waviness since these irregularities would be much more difficult to model mathematically. The steel weight was added to the block in order to provide enough mass so that the downslope push of the spring on the transducer arm could be neglected as a driving force. Although the push was small, it could make a difference as the slope approached a stable angle where the table motions alone could not cause displacement.

Approximately 75 trials were executed with the wood sliding block at three different conditions of table shaking. Three slope angles were used for two of the test series and five angles for the other. For purposes of definition, a series is a collection of trials run under varying slopes, but with the frequency and stroke of the shaking table held constant. A trial is one run of the block downslope, providing a single measured time versus block displacement curve.

Steel sliding blocks were also tried, but like the wood block, did not produce sufficiently consistent results.

The failure of the wood and steel blocks to provide useful results led to the decision to attempt rock-on-rock sliding block tests. In order to create a sturdy rock base on the ramp, a large, solid piece of gray granite was cut into a rectangular block 5 inches thick and secured on the ramp as previously described. Two smaller blocks, cut from the same granite, were used as sliding blocks in the tests. The granite was porphyritic, approximately 70 percent plagioclase, 20 percent quartz, and 10 percent minor minerals and was relatively free of fractures. It was cut with a water-cooled rock saw using a 20-inch-diameter blade. The surfaces were too large to be smoothed with the available lap wheels, so were left unfinished, leaving a fairly smooth sliding surface with some small ridges resulting from rearranging the position of the rock during cutting. These asperities gave the block an uneven surface which may have influenced the sliding rates of the blocks.

Two sliding blocks were used to provide a comparison between blocks with similar physical properties, but different size and shape (fig. 6). This also allowed for better testing of the methods of analysis. The first sliding block (block 1) was slightly smaller in volume but had a greater sliding area in contact with the base rock. It weighed 15.8 pounds and was somewhat triangular in shape, with a center

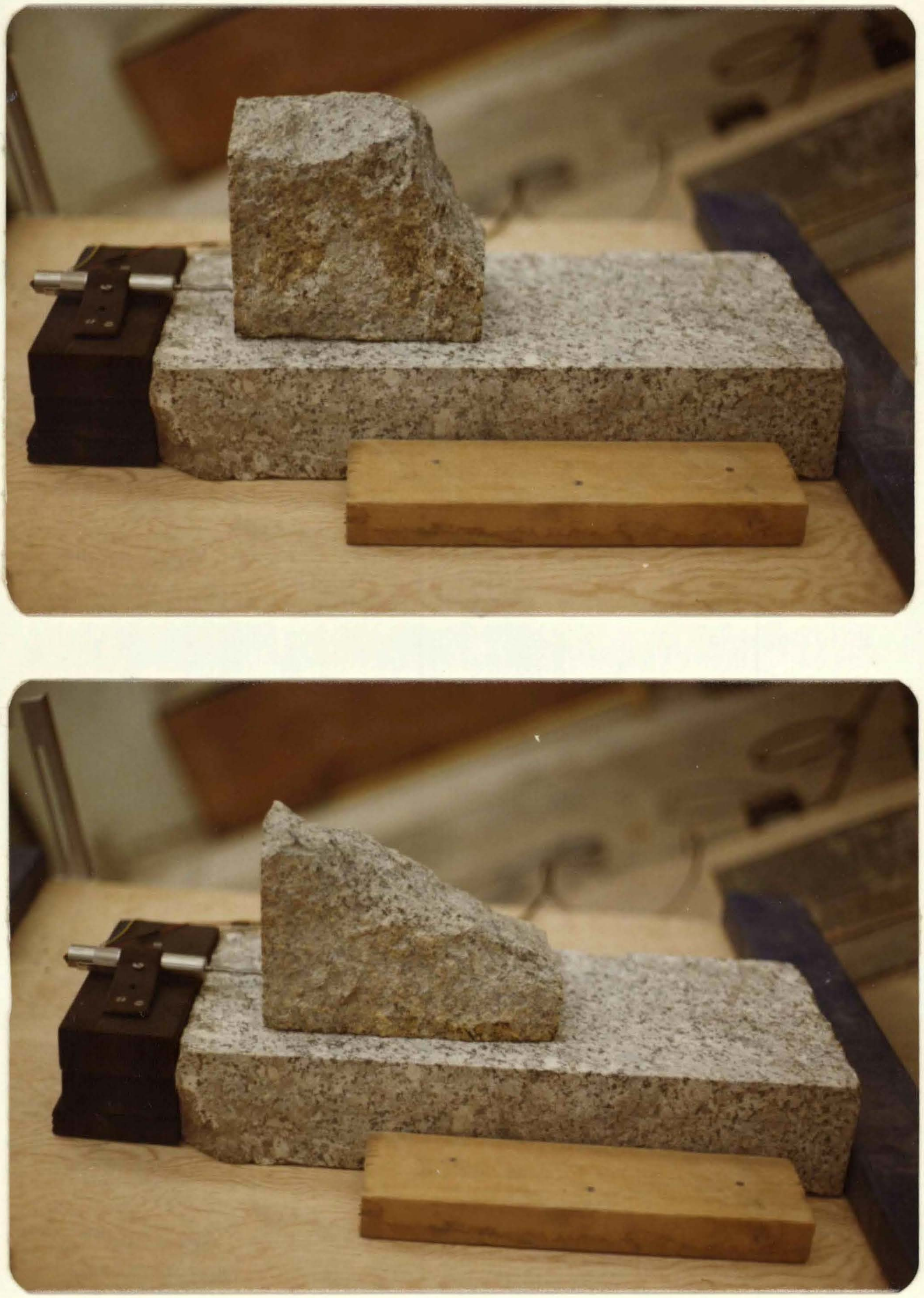


Figure 6. - Photographs of rock sliding blocks in place for testing.

of gravity near the upslope side of the block as used in the tests. Block 2 was slightly rectangular in shape and had a greater mass but less surface area contacting the base rock. This block weighed 18.1 pounds and had its center of gravity very near the midpoint of the sliding surface.

Determination of the angle of sliding friction accurately proved to be difficult. Simple sliding tests on the ramp and direct shear tests were performed, and a wide range of values were obtained from which to chose an angle for use in the analysis. The blocks used in the dynamic sliding tests were first used in simple sliding tests to find the lowest angle at which they started to slide. Direct shear tests were performed on 2-inch-diameter cores of the granite and these core pieces were then used in simple sliding tests.

The rock sliding blocks were placed on the ramp which was sitting at a flat angle. The ramp angle was then slowly raised using an automobile jack until the blocks started to slide. This angle was then measured with a Brunton compass and recorded, and the test was repeated 20 times. Friction angles were measured from 26.4 to 31.6° with a mean value of 28.1°. However, since such a wide range of angles was measured, direct shear tests were also performed.

Cores were cut from remaining pieces of the granite and were tested with normal loads of 203 pounds and 387 pounds. Peak and residual values were selected from the curves plotted during the direct shear tests and friction angles were calculated for both cases using a computer program written for this type of test. The angle of sliding friction was 17.7° from the peak values and 19.3° from the residuals. However, the peak angle should be higher, so not much emphasis was placed on these tests.

The core pieces used in the direct shear tests were then used in more simple sliding tests as described earlier. Friction angle values were measured from 24.5 to 27.8° with a mean of 26.2°.

Finally, the lower bound measured from the sliding tests on the large blocks, 26.4°, was decided upon. The higher values were attributed to asperities in the rock surfaces. The lower values measured from the direct shear tests and sliding tests on the core pieces help to support this. Any values lower than about 26.4° are inconsistent with average values for granite recorded in the literature (Jaeger and Cook, 1976).

CHAPTER 4

RESULTS OF LABORATORY TESTS

The results of the laboratory tests are displayed as time versus displacement curves for examination and comparison. They show that regardless of the materials used the rate of sliding on the ramp is rarely steady for a complete trial through the measurable range of the transducer. For many of the tests the blocks slow down after a comparatively quick start, causing the curves to flatten out on the graphs.

Wood Sliding Block

The wood sliding block moved with varying rates depending on where it was started on the sliding ramp. In the first test performed, at a slope of 14.5° and frequency of 5.67 hertz, two trials produced long, flat curves indicating slow movement, while two others produced steep curves indicating a much faster sliding rate (fig. 7). For the two trials which resulted in flat curves, the block was started about 4 inches to the right of where it was started for the other trials. This caused the block to slide directly over a very smooth, hard area on the ramp which lacked the graininess of most of the face of the ramp. The block slid at a much slower rate over this smooth area than over the rest of the ramp.

This wide range of sliding rates was also evident for tests run at 4.17 hertz. Figure 8 shows three groupings of curves for tests at

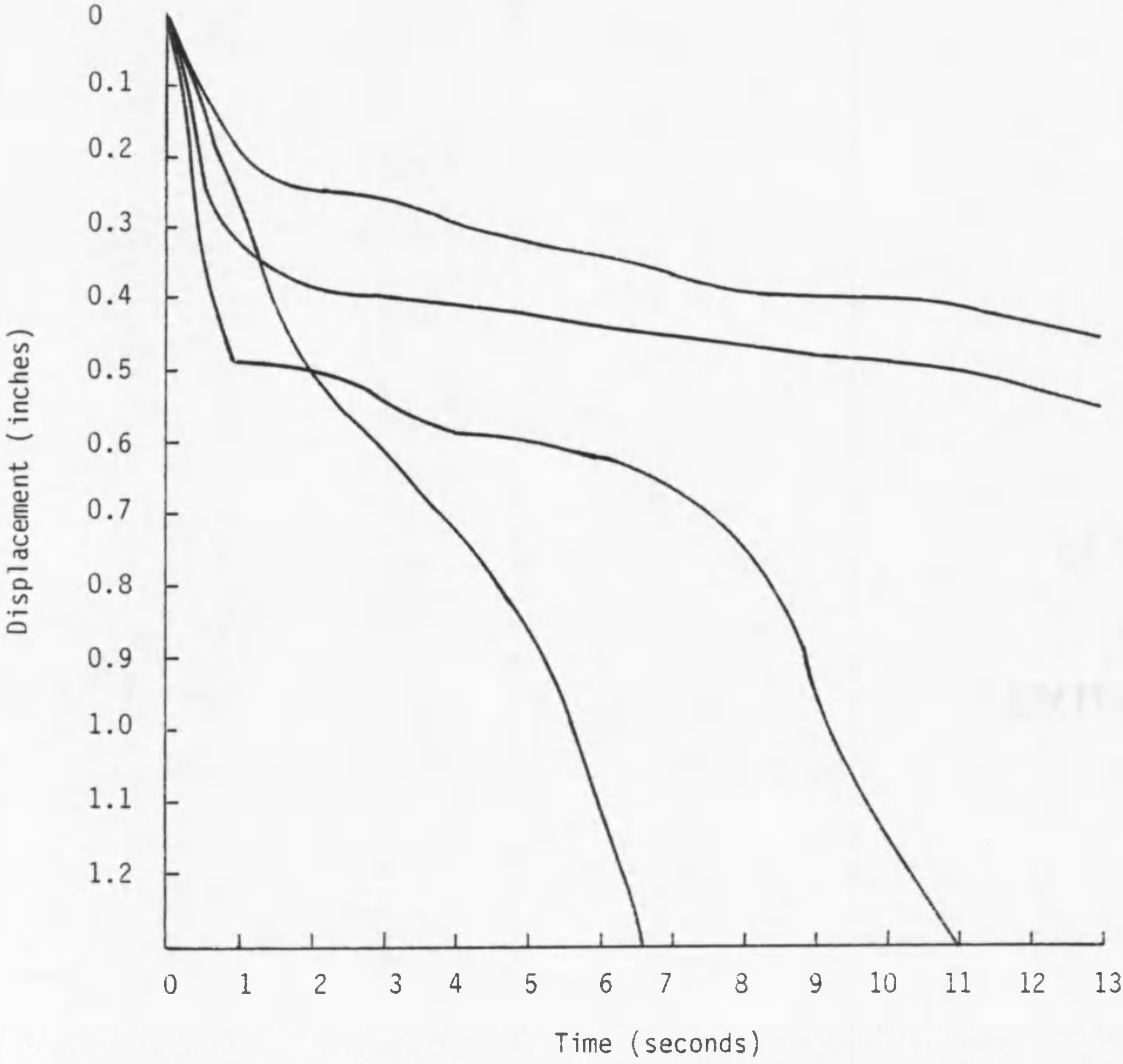


Figure 7. - Wood sliding block test curves, Test 1-A.

Slope = 14.5°
Frequency = 5.67 Hz

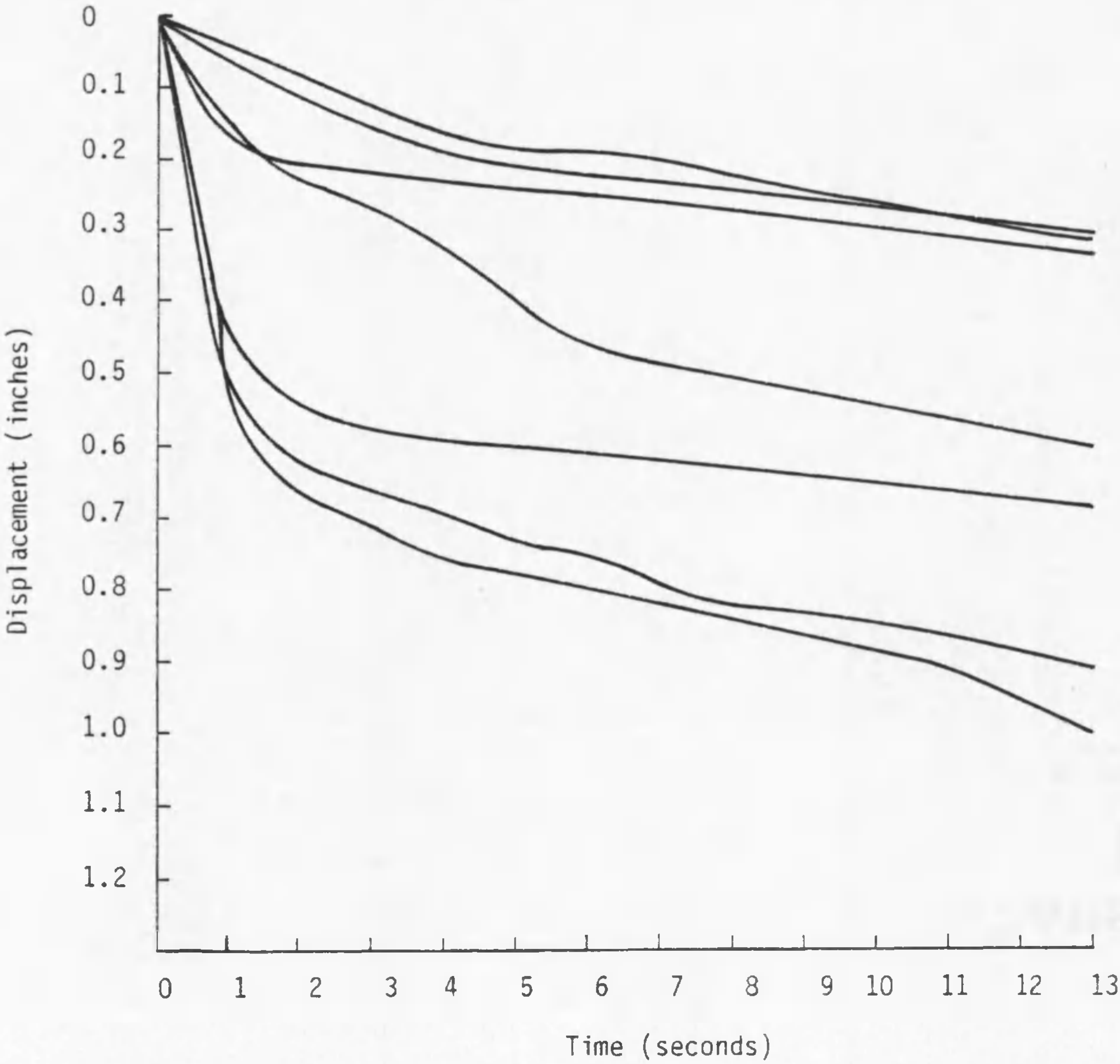


Figure 8. - Wood sliding block test curves, Test 2-B.

Slope = 20°
Frequency = 4.20 Hz

20° slope, all with fairly flat slopes for most of their lengths. However, they have steeper slopes at first and then flatten out after the first second or two of sliding. The three groups were also evident at 21° (fig. 9). In these trials, with one exception the blocks started out at about the same rate and then slowed to a smooth, steady pace. After about 12 seconds, for two cases, the block accelerated to a faster rate. The rate for the last few seconds was the same as for the last portion of the first run. This is shown by the similarity in slopes at the end of the curves.

Steel Sliding Blocks

The steel sliding blocks demonstrated more consistency and repeatability than the woodblock tests, but there were still irregularities and sudden changes in the rates of sliding. As in the woodblock test there appeared to be places on the surface of the ramp where the blocks would slide with greater resistance than others. In some cases the steel blocks would slow down, but in a few cases the blocks stopped completely after several seconds of sliding.

Figures 10 and 11 show that at steep angles the block slid at a steady, consistent rate. The curves fall into a tight group and only a small amount of flattening is visible at the end of some of them.

The abrupt halt of sliding is demonstrated in figures 12 and 13. In all cases the block stopped at the same location on the ramp, between 0.9 inch and 1.3 inches downslope from the transducer. This is over the same smooth area where the woodblock was being held up in the earlier tests.

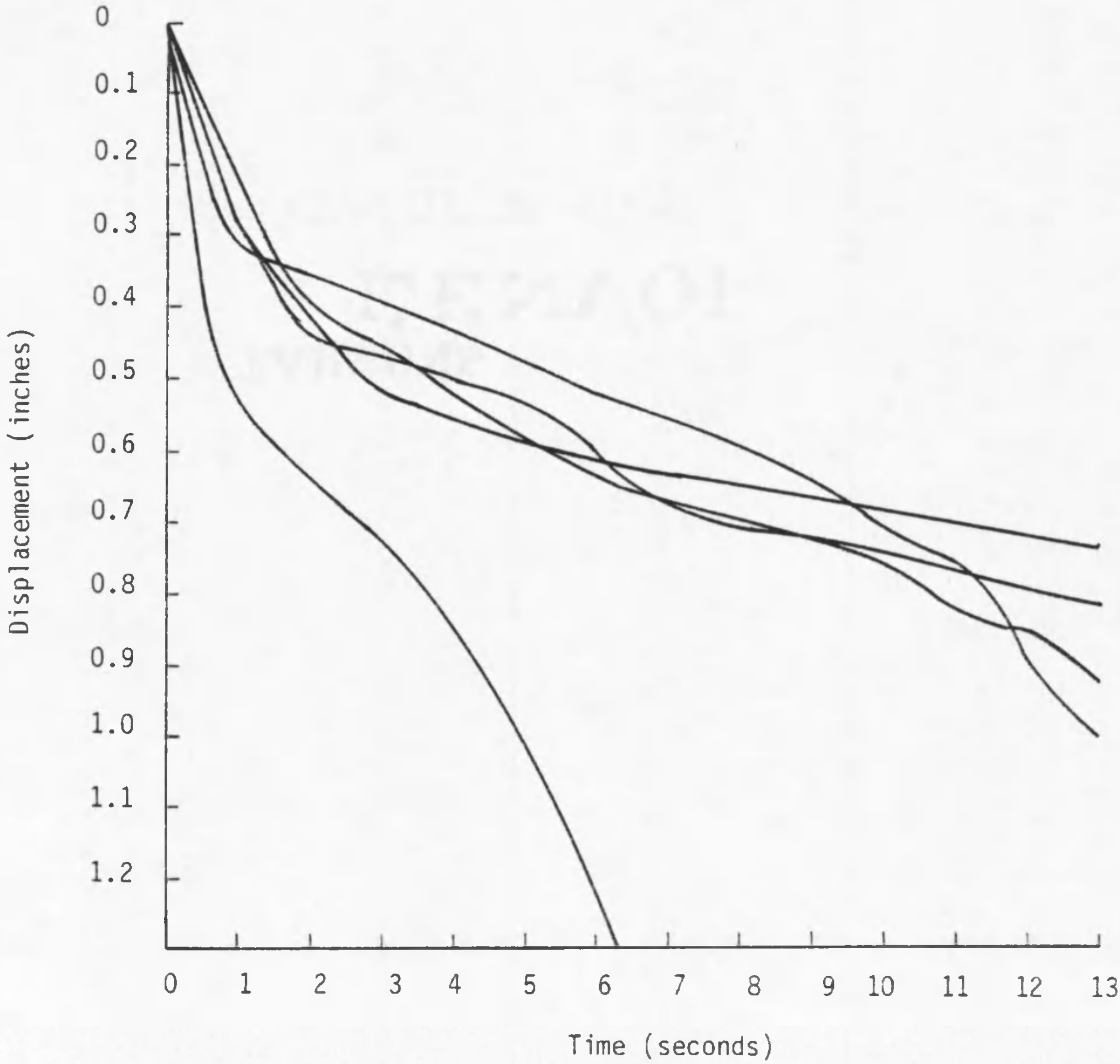


Figure 9. - Wood sliding block test curves, Test 3-B.

Slope = 21°
Frequency = 4.20 Hz

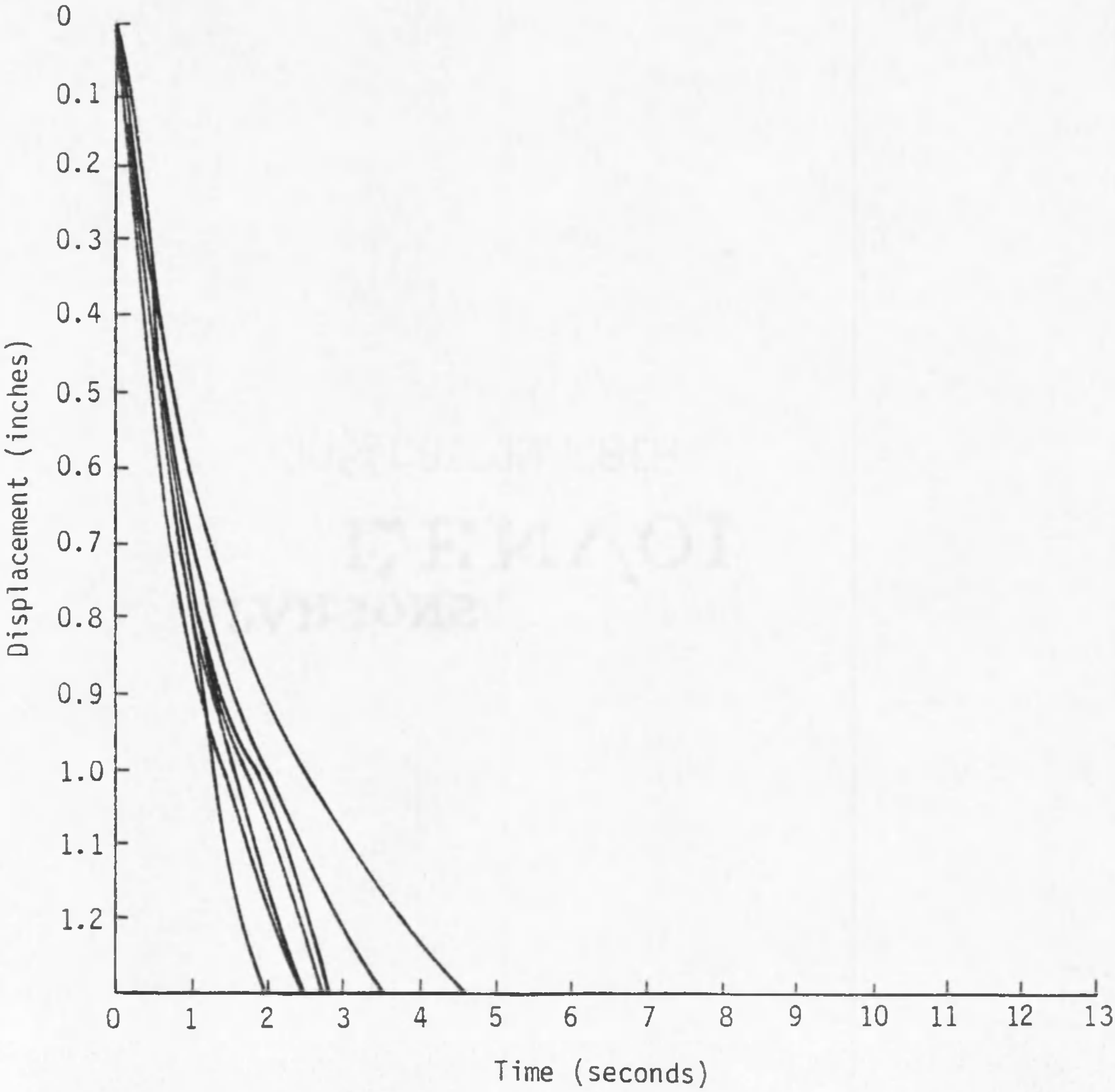


Figure 10. - Steel sliding block test curves, Test 2-H.

Slope = 23°
Frequency = 4.60 Hz

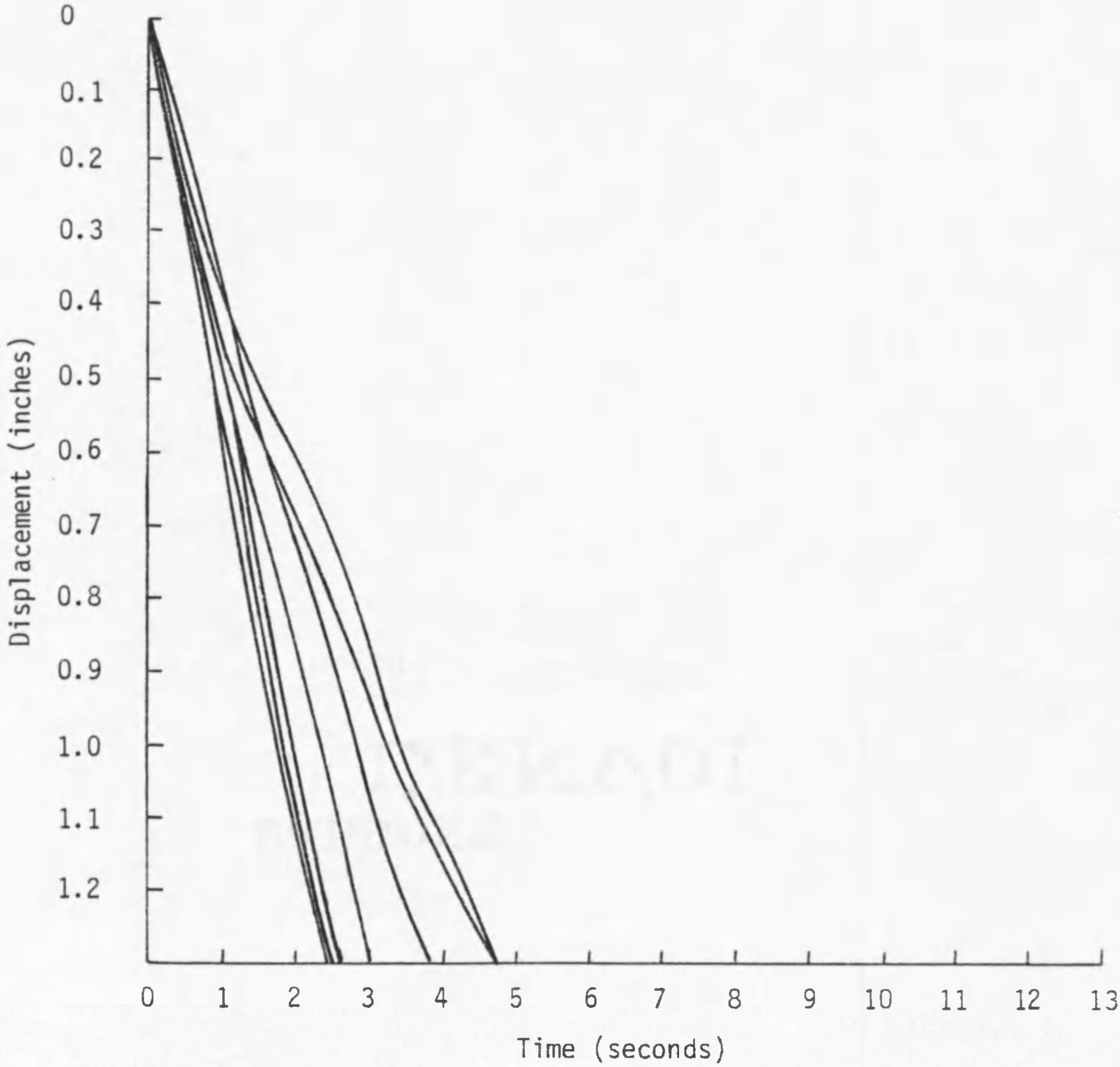


Figure 11. - Steel sliding block test curves, Test 4-J.

Slope = 22°
Frequency = 5.17 Hz

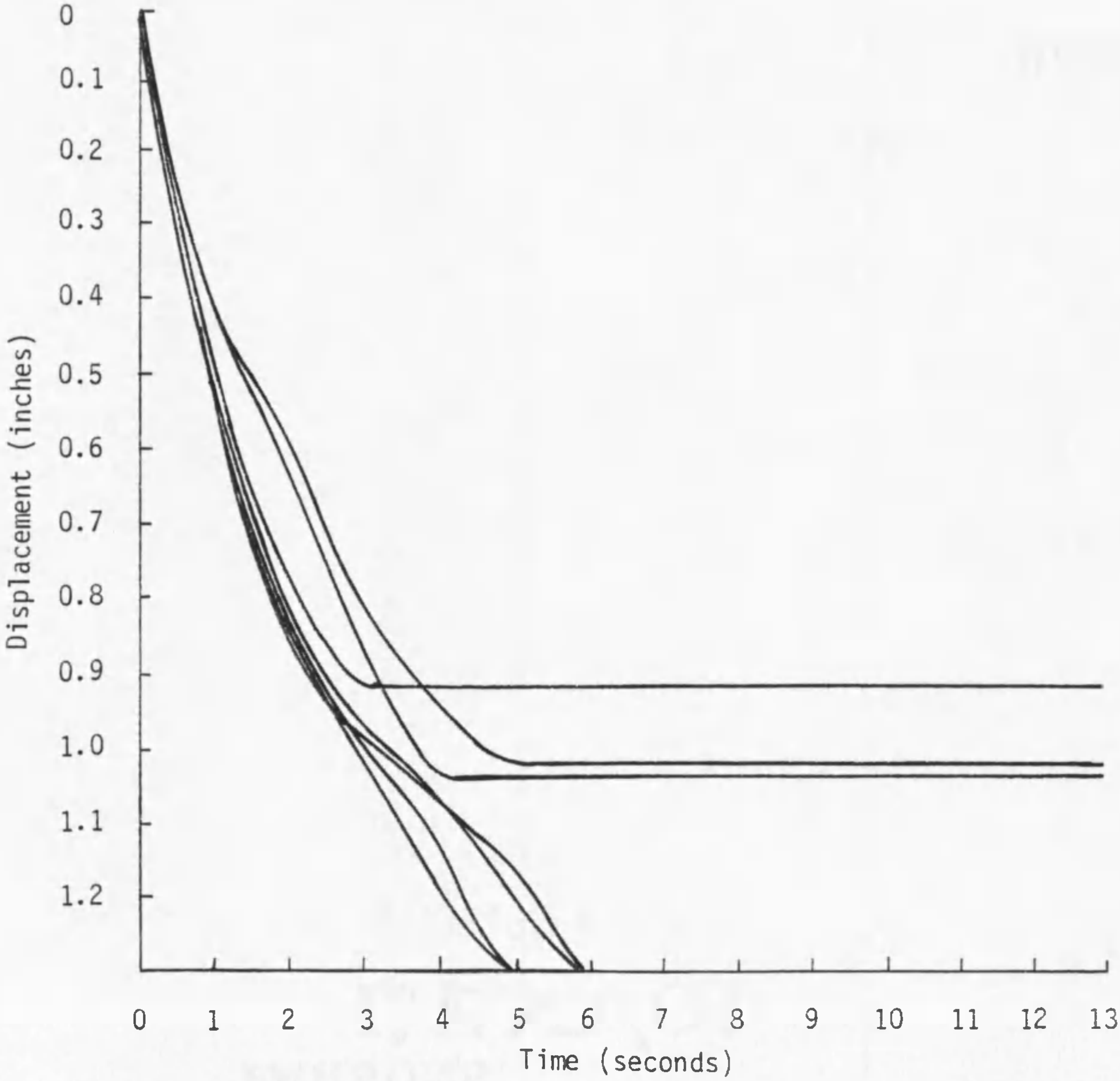


Figure 12. - Steel sliding block test curves, Test 3-H.

Slope = 22°
Frequency = 4.60 Hz

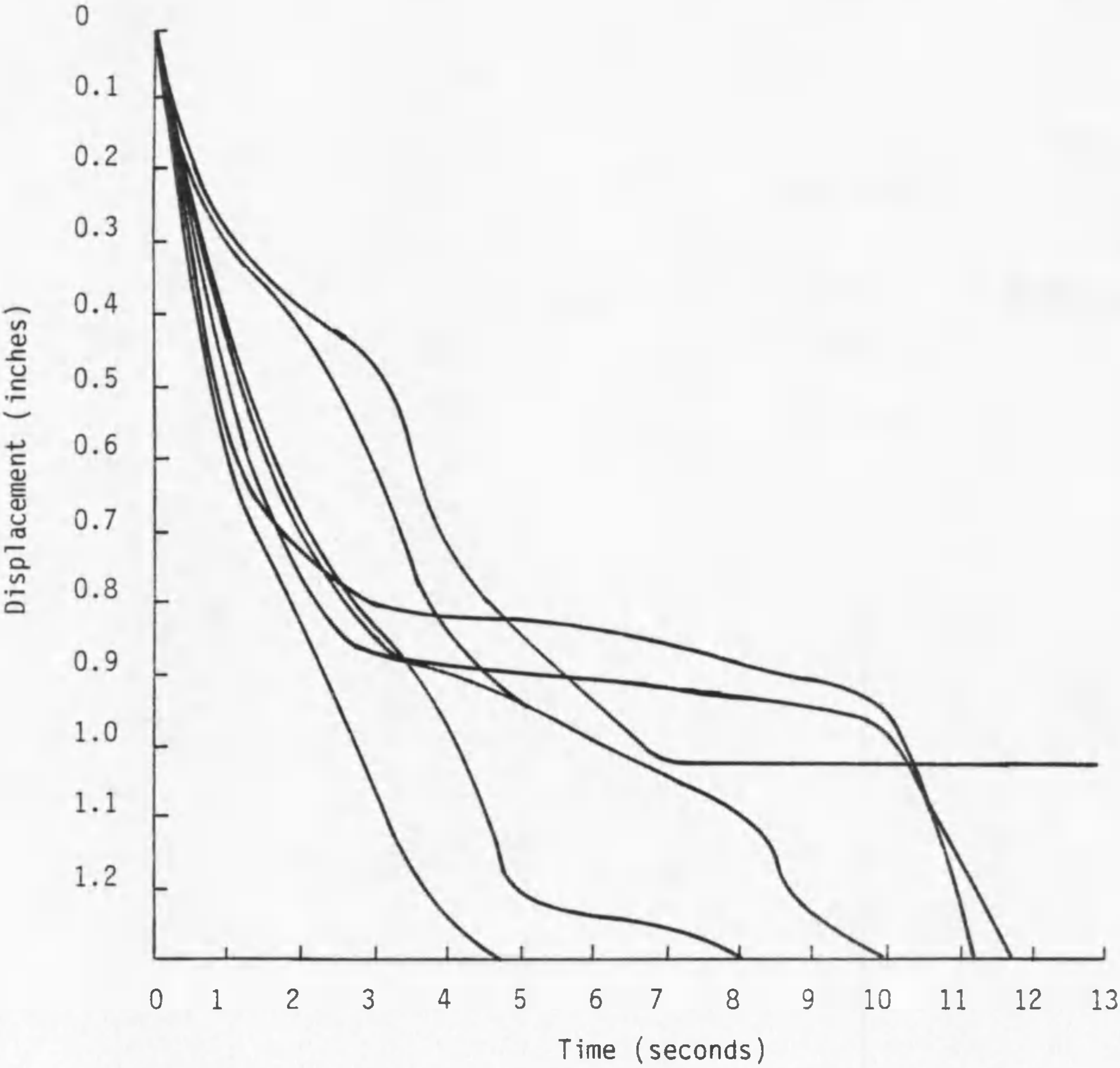


Figure 13. - Steel sliding block test curves, Test 4-H.

Slope = 21°
Frequency = 4.60 Hz

At flatter slopes the sliding rates became more irregular. Many of the curves are wavy due to changes in speed as the sliding block moved over areas of varying resistance on the surface of the ramp. Figure 14 shows the waviness which would not occur if the friction between the block and the ramp were constant over the entire length of each trial. In figure 15 the slopes are relatively constant throughout each trial, but the difference from trial to trial is too great to determine which is the correct (or most correct) description of sliding conditions caused by the motions of the shaking table.

Rock Sliding Blocks

The rock-on-rock tests were set up as described earlier with the large base block and two sliding blocks for observing differences which could be ascribed to variations in shape and/or surface area. These tests produced far more consistent results than the wood or steel sliding block tests and therefore were used to compare with the predicted displacements obtained from the analytical methods. However, many of the curves indicate by their waviness that the rate of sliding was not steady. Also, in the majority of trials the blocks decreased their rate of sliding after 1 or 2 seconds. For purposes of definition, block 1 weighs 15.8 pounds and has a larger surface area than block 2 which weighs 18.1 pounds.

The first series of tests were run with a table frequency of 5.23 Hz and displacement of 0.12 inch (maximum acceleration = 0.34 g). The tests for block 1 were started at a slope of 25° where the block slid smoothly and fairly rapidly (fig. 16). The sliding rate decreased

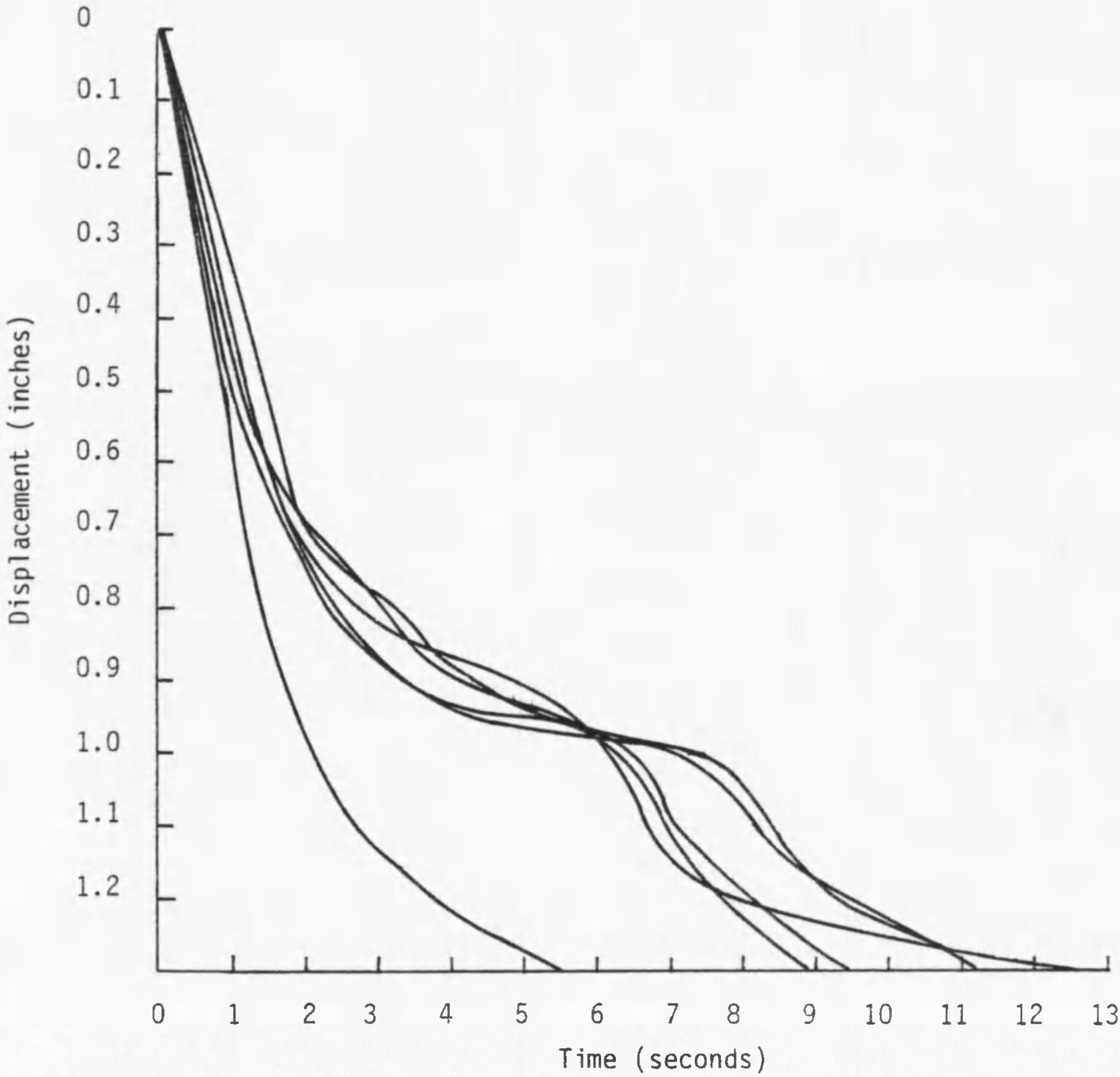


Figure 14. - Steel sliding block test curves, Test 5-H.

Slope = 20°
Frequency = 4.60 Hz

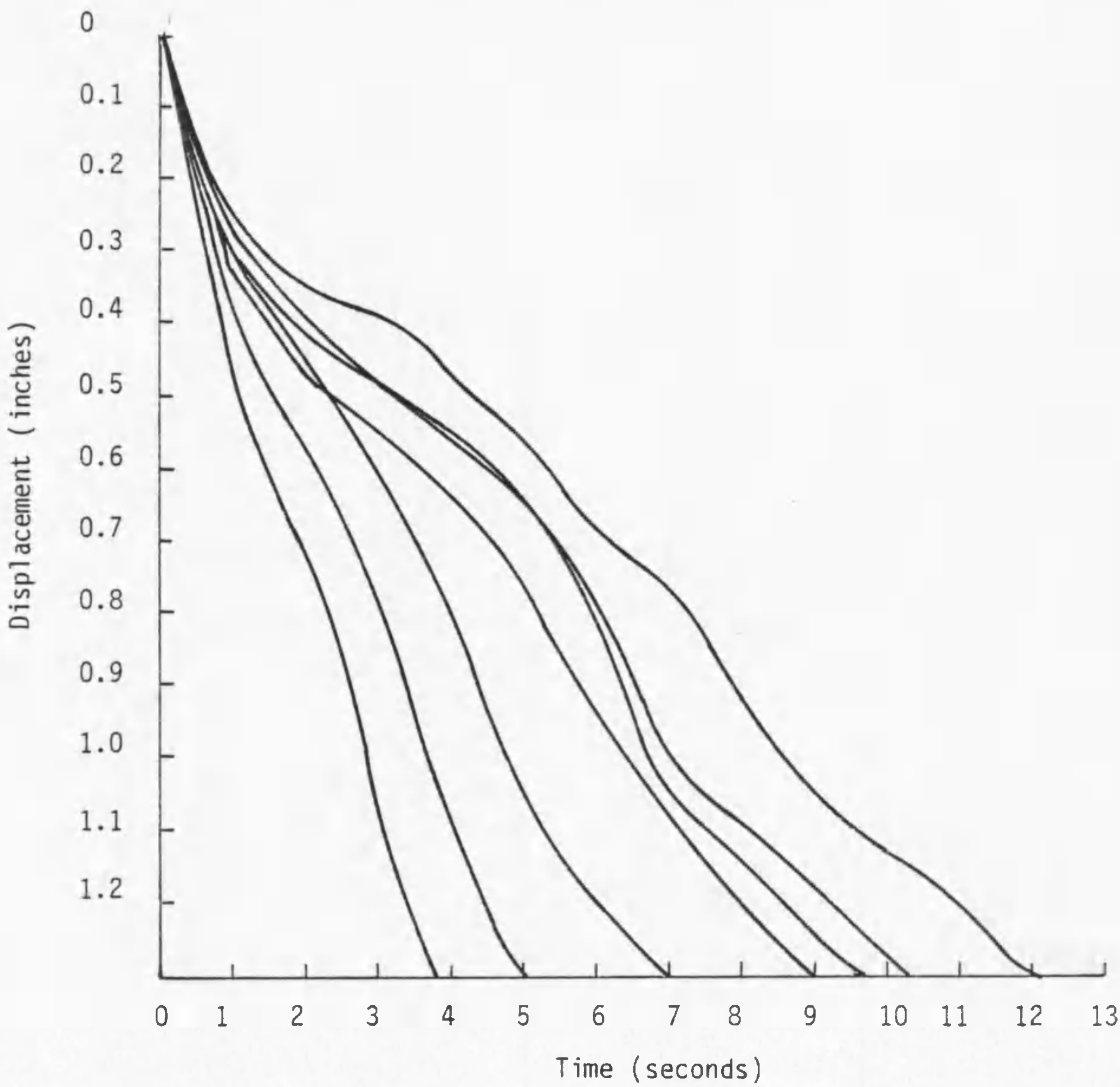


Figure 15. - Steel sliding block test curves, Test 3-J.

Slope = 21°
Frequency = 5.17 Hz

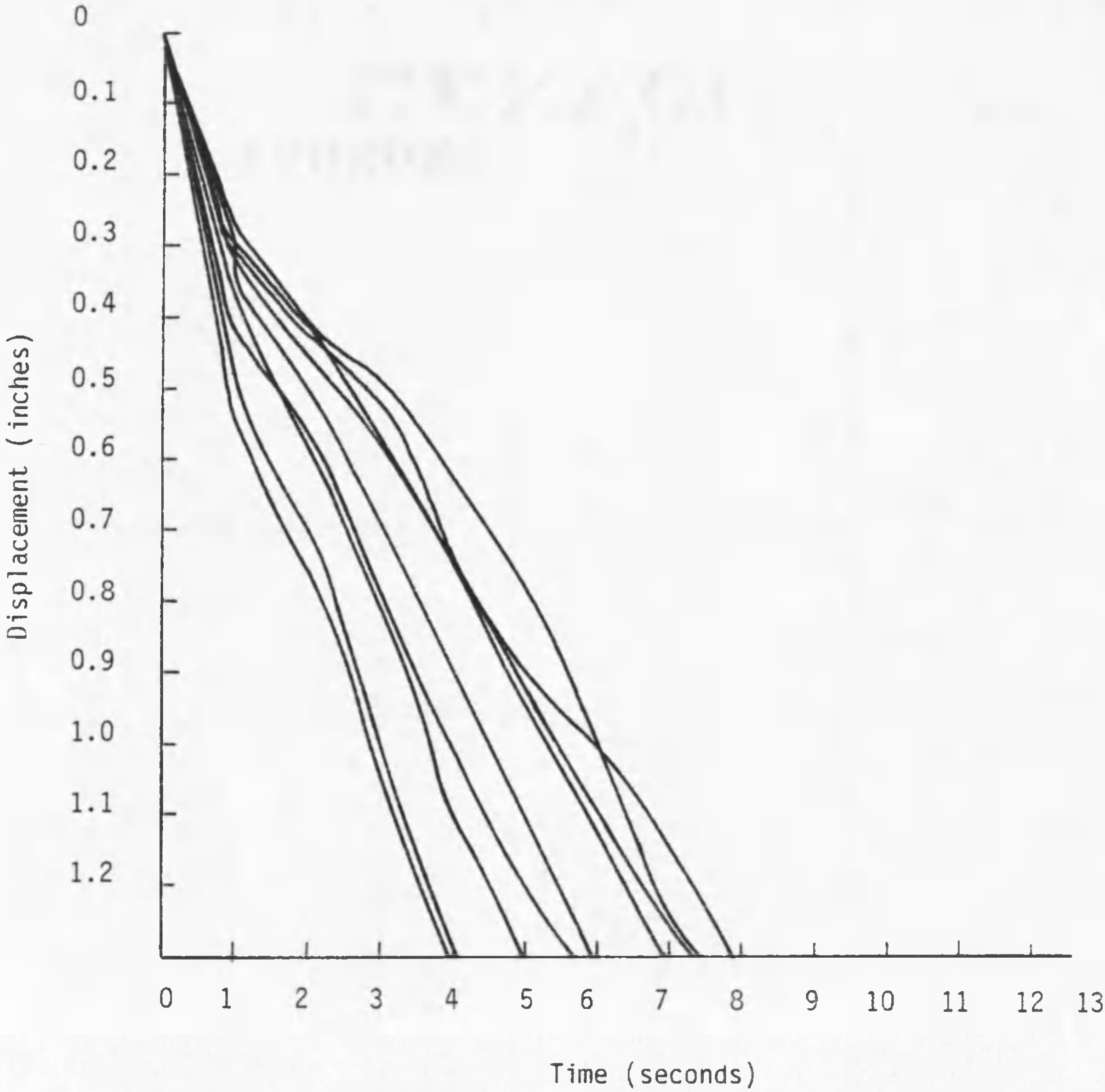


Figure 16. - Rock sliding block test curves, Test 1-L(1).

Slope = 25°
Frequency = 5.23 Hz

at angles of 24 and 23° (figs. 17 and 18). At 22° (fig. 19) the block moved very slowly, and at 21° it was stable and did not slide. Testing on block 2 was also started at 25° (fig. 20) and it moved faster than block 1 did at the same slope and for the same table motions. The rate of sliding for this block decreased for all slopes proceeding downward to 21° where it slid very slowly (figs. 21 through 24). Block 2 was stable at 20°. At steeper slopes the blocks slid at a steady consistent rate from trial to trial as is shown by the straight, tightly grouped curves. As the slope is flattened, the rate of sliding became irregular and the tests produced a more pronounced decrease in sliding rates after the first few seconds.

For the next series of tests, the table displacement was increased to 0.192 inch and the frequency was slightly decreased to 5.17 Hz. This produced a maximum acceleration of 0.52 *g* for each cycle of motion. The tests were started at a slope of 24° and the slope was lowered 1° at a time until the blocks would not slide when placed on the ramp (figs. 25 through 33). As before, block 1 slid noticeably slower than block 2 for each set of table motions. At this higher acceleration, the curves produced a tighter grouping than for the previous tests. The curves also indicate that the reduction in sliding rate with time, as observed for most tests was decreased.

Block 2 continued to slide faster for most cases, but the differences in rate were less for lower frequencies. Tests run at a frequency of 3.67 Hz with a stroke of 0.11 inch (maximum acceleration = 0.15 *g* showed only about a 5 percent difference in sliding rates.

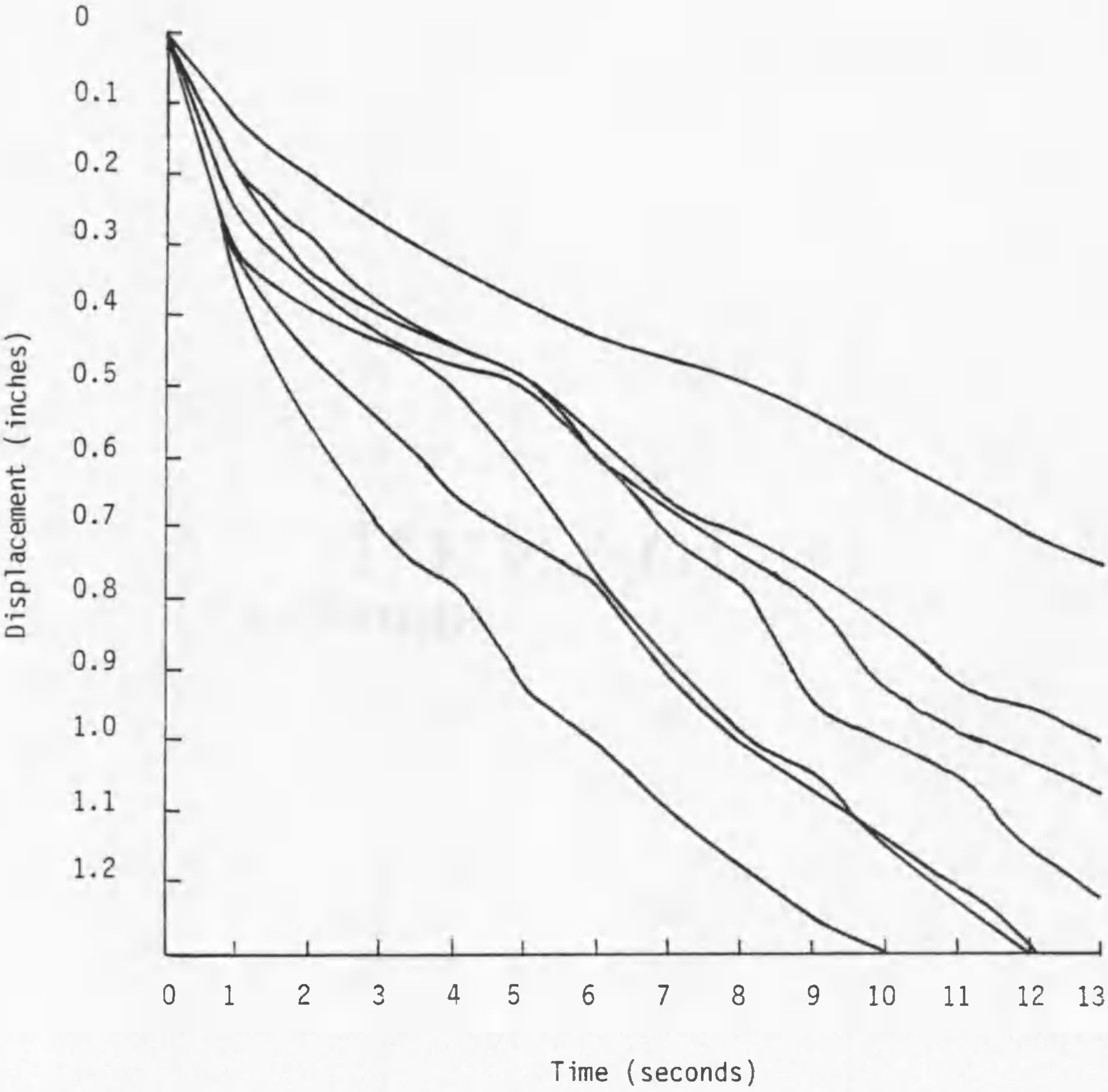


Figure 17. - Rock sliding block test curves, Test 2-L(1).

Slope = 24°
Frequency = 5.23 Hz

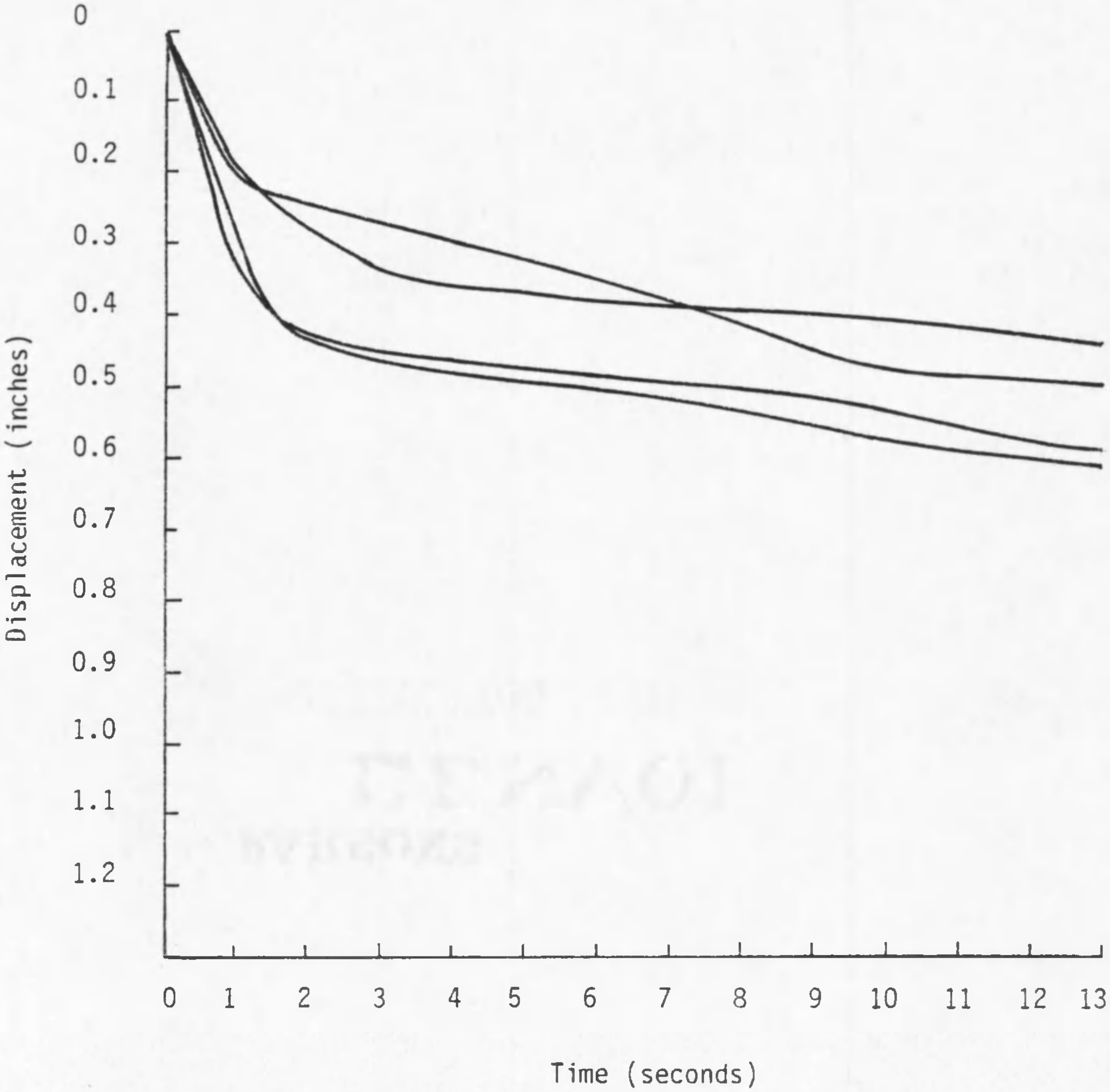


Figure 18. - Rock sliding block test curves, Test 3-L(1).

Slope = 23°
Frequency = 5.23 Hz

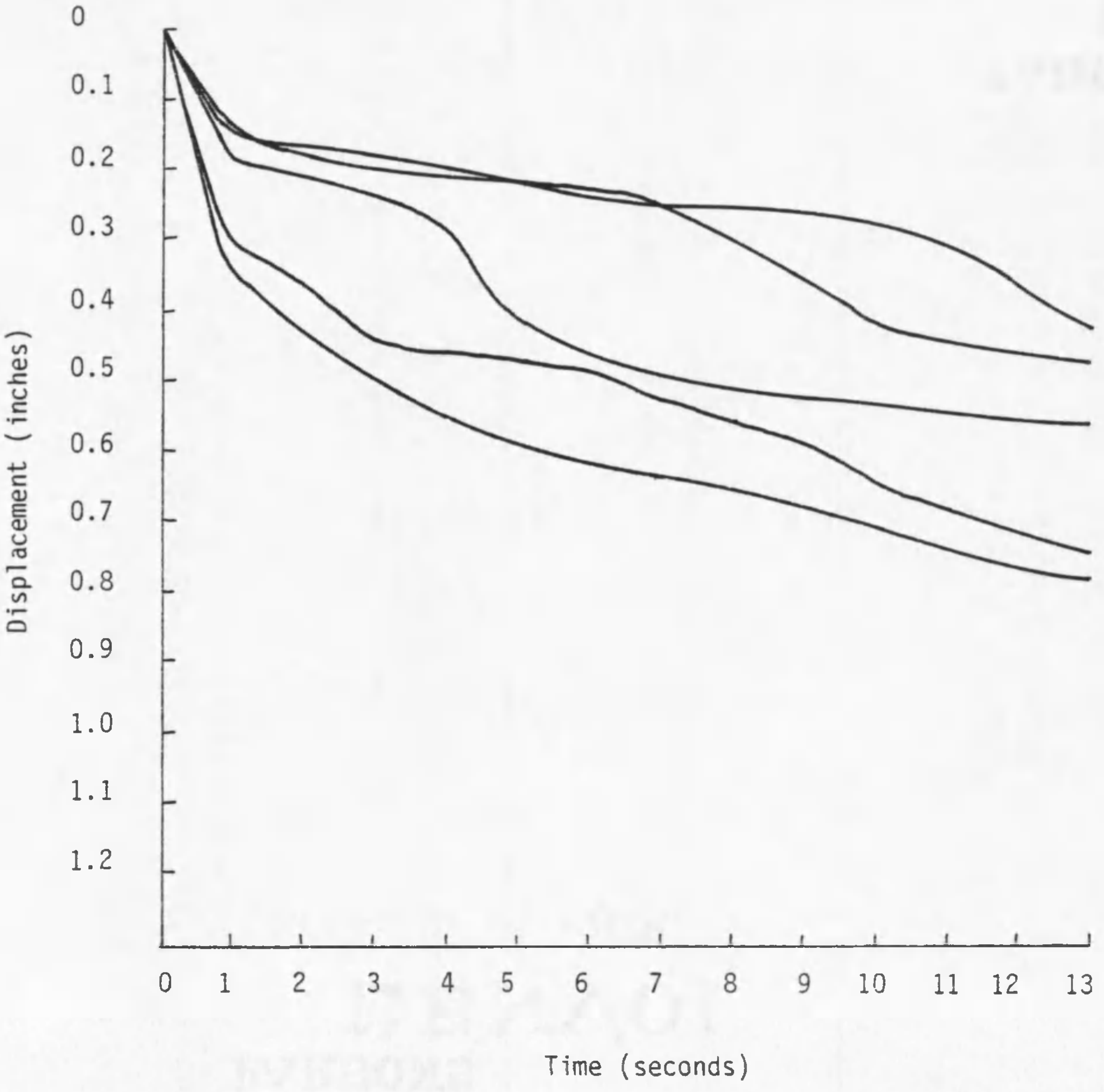


Figure 19. - Rock sliding block test curves, Test 4-L(1).

Slope = 22°
Frequency = 5.23 Hz

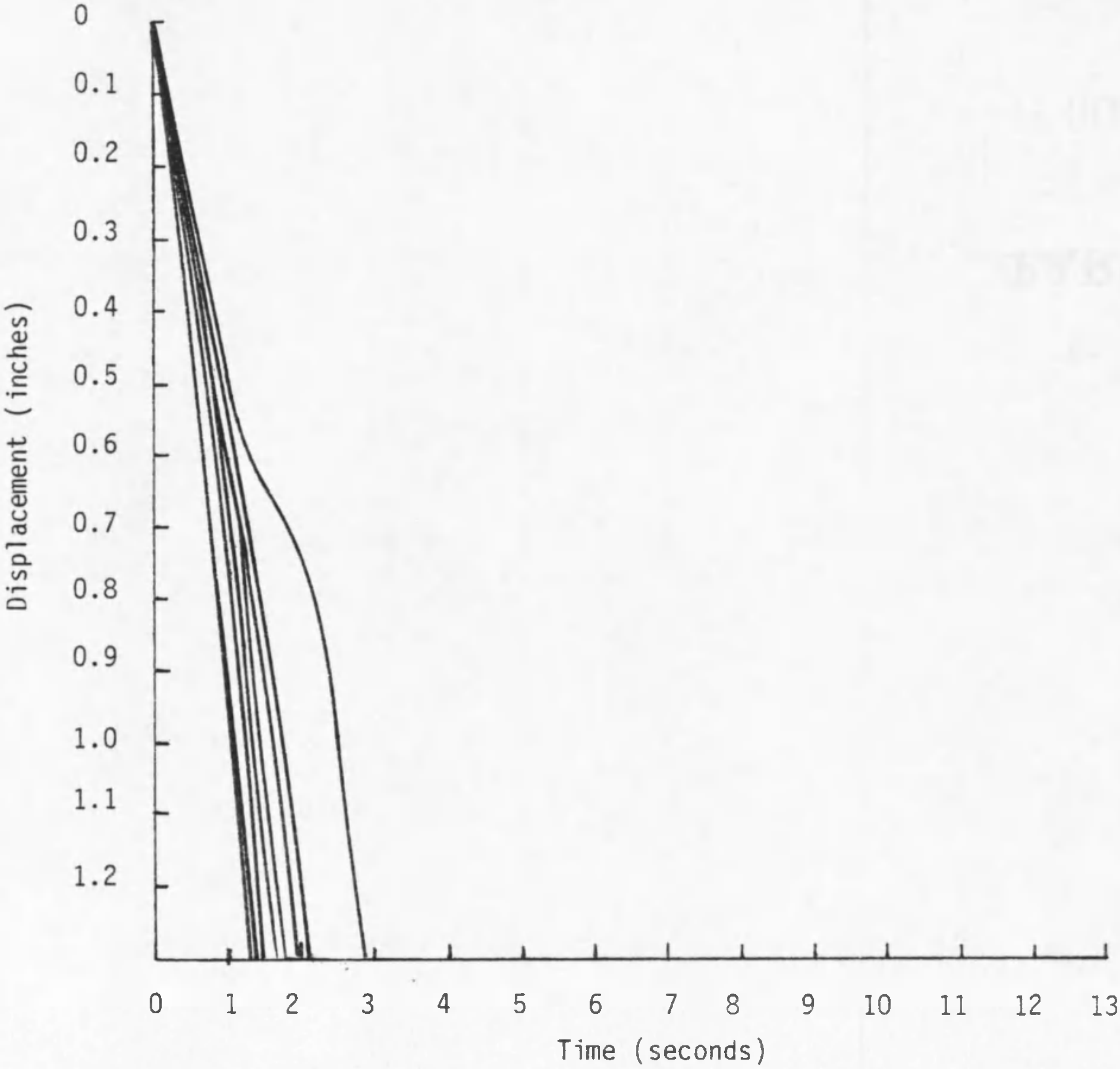


Figure 20. - Rock sliding block test curves, Test 1-L(2).

Slope = 25°
Frequency = 5.23 Hz

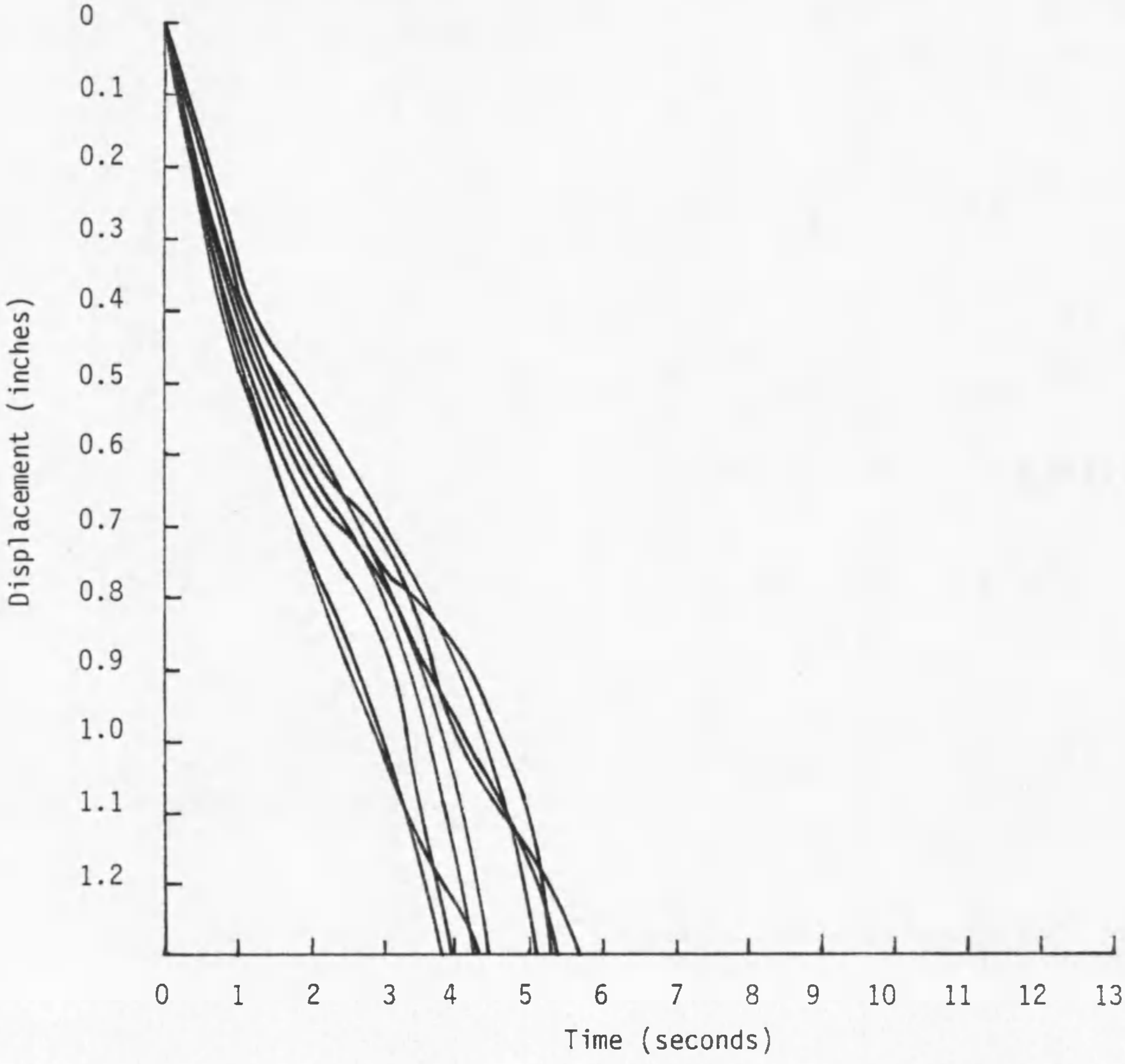


Figure 21. - Rock sliding block test curves, Test 2-L(2).

Slope = 24°
Frequency = 5.23 Hz

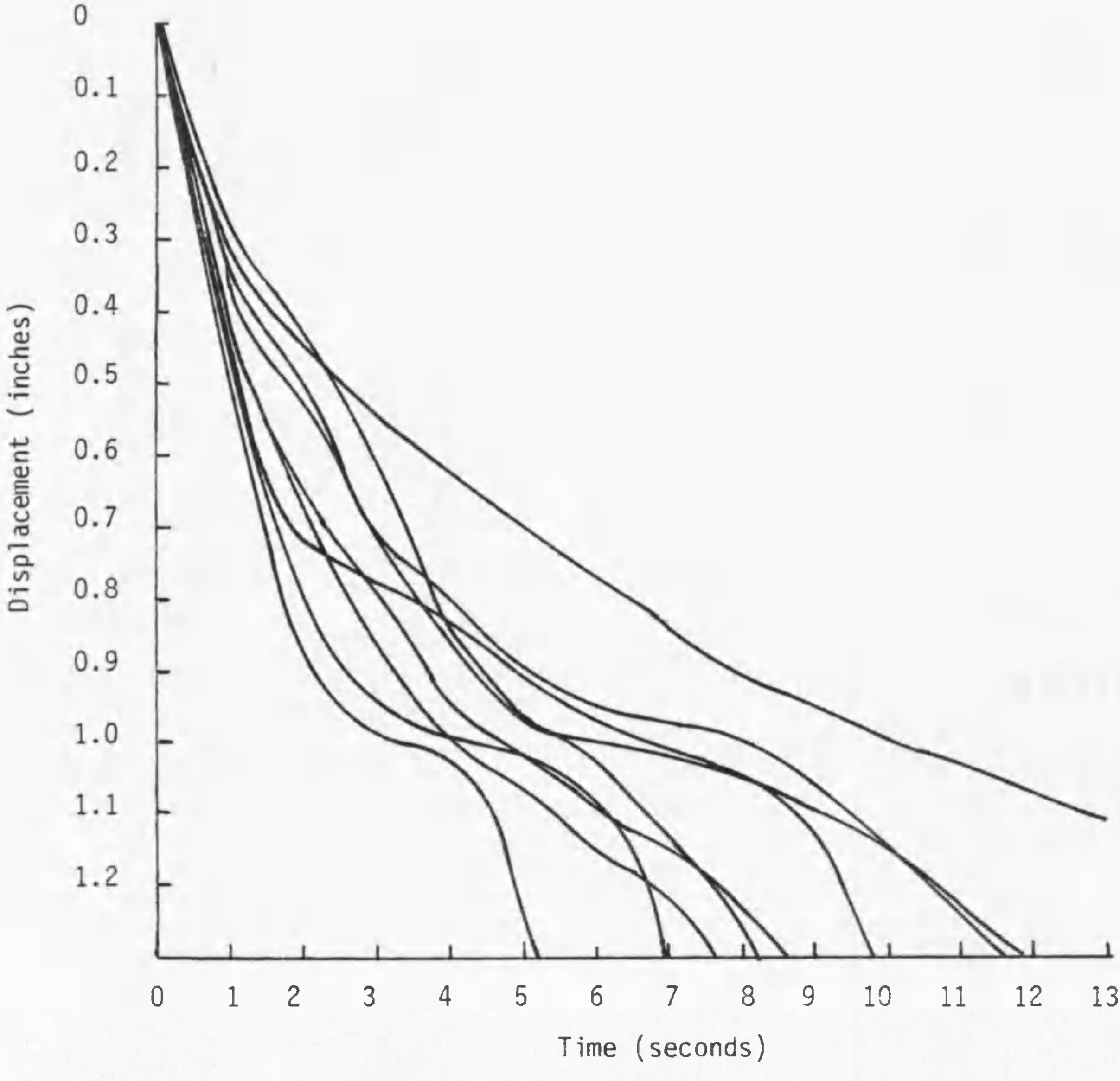


Figure 22. - Rock sliding block test curves, Test 3-L(2).

Slope = 23°
Frequency = 5.23 Hz

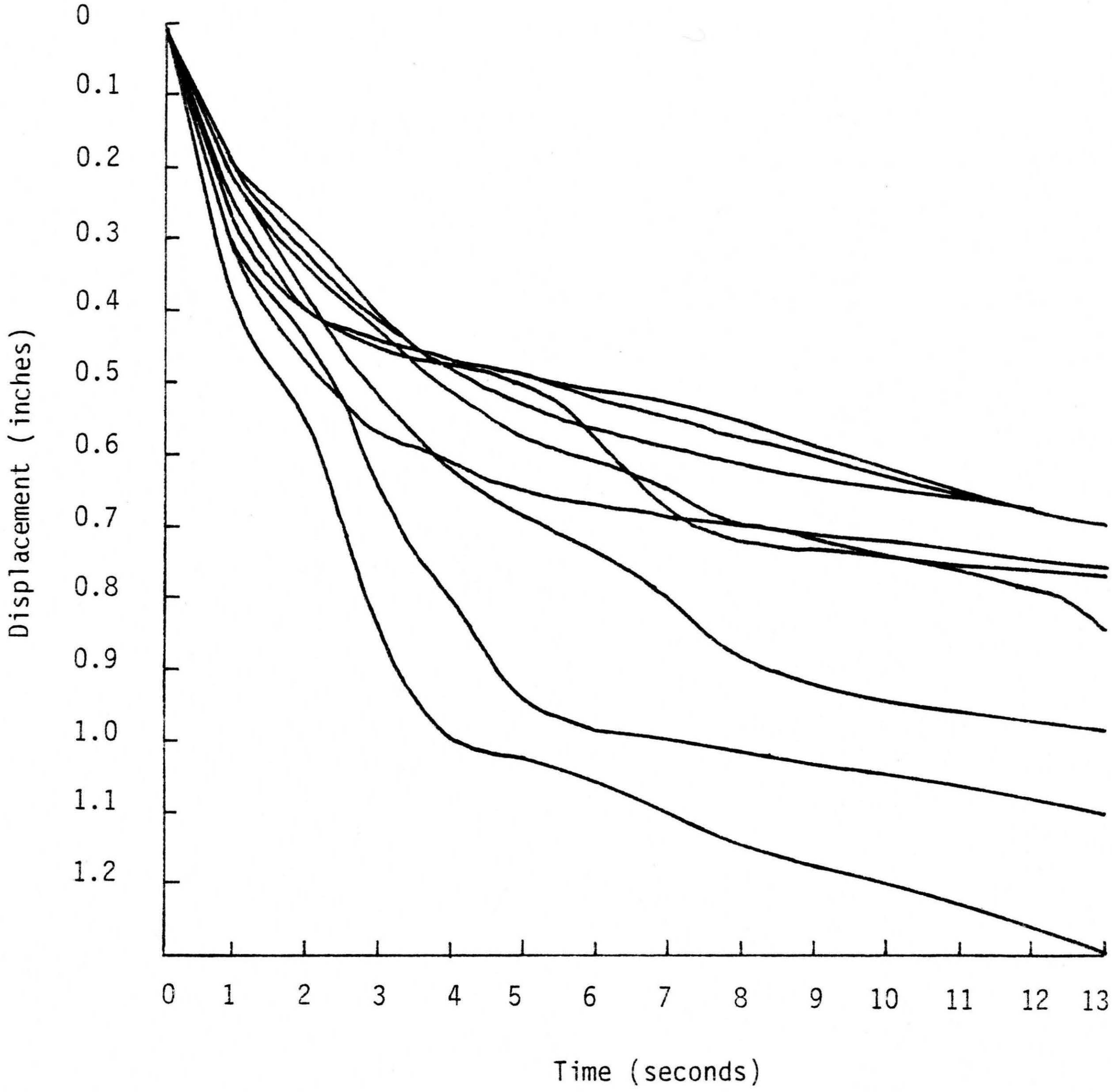


Figure 23. - Rock sliding block test curves, Test 4-L(2).

Slope = 22°
Frequency = 5.23 Hz

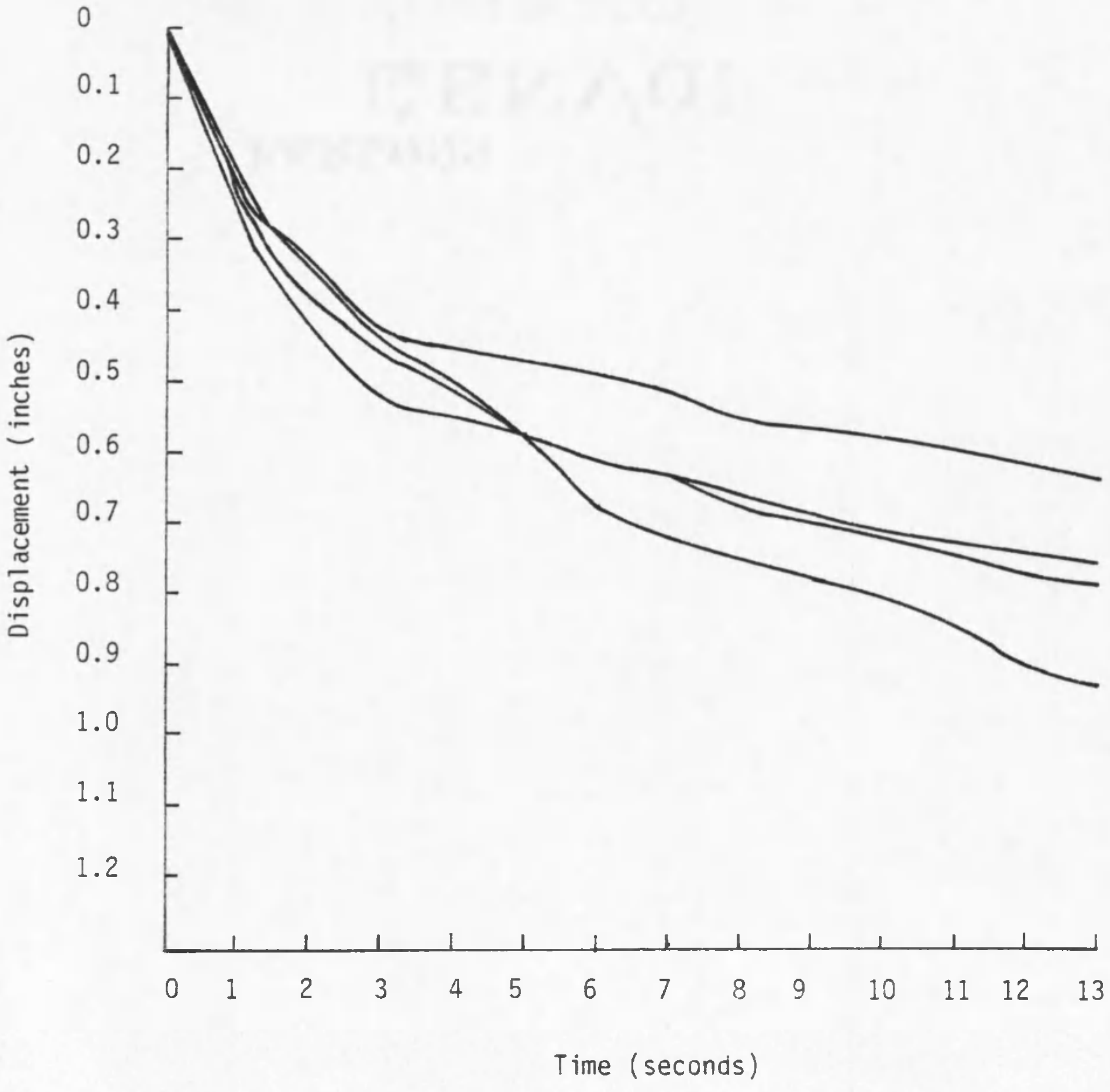


Figure 24. - Rock sliding block test curves, Test 5-L(2).

Slope = 21°
Frequency = 5.23 Hz

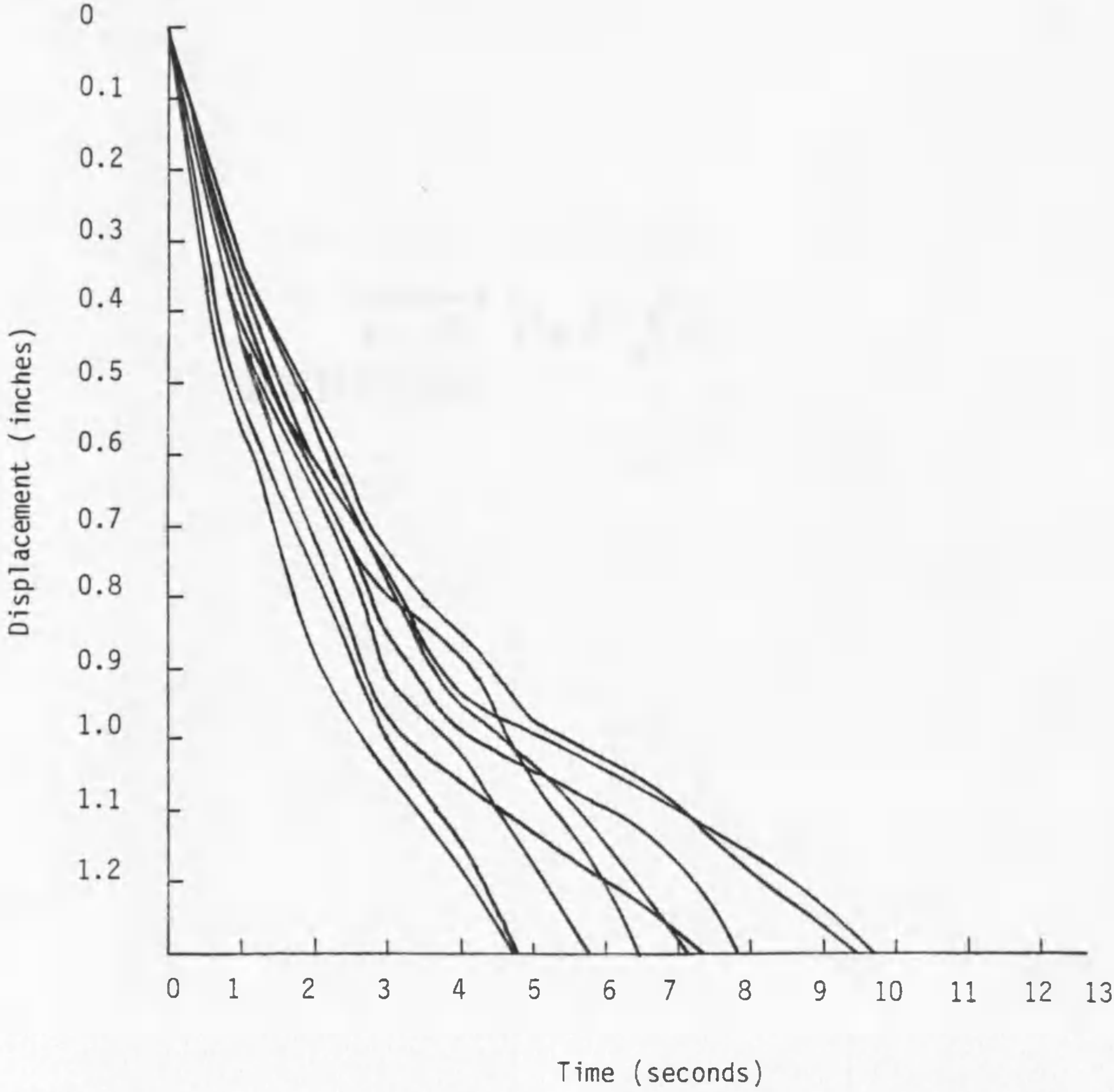


Figure 25. - Rock sliding block test curves, Test 1-M(1).

Slope = 24°
Frequency = 5.17 Hz

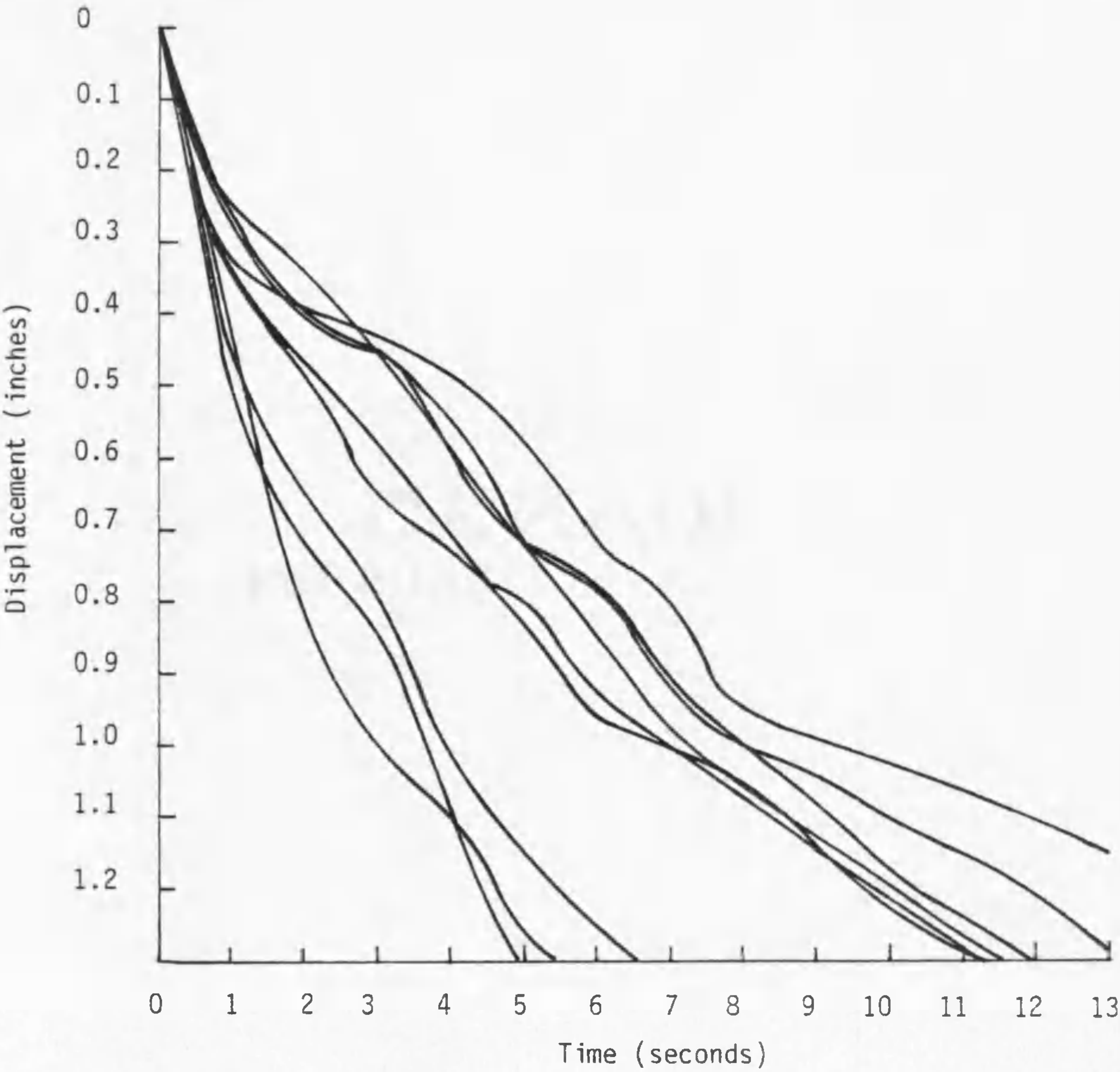


Figure 26. - Rock sliding block test curves, Test 2-M(1).

Slope = 23°
Frequency = 5.17 Hz

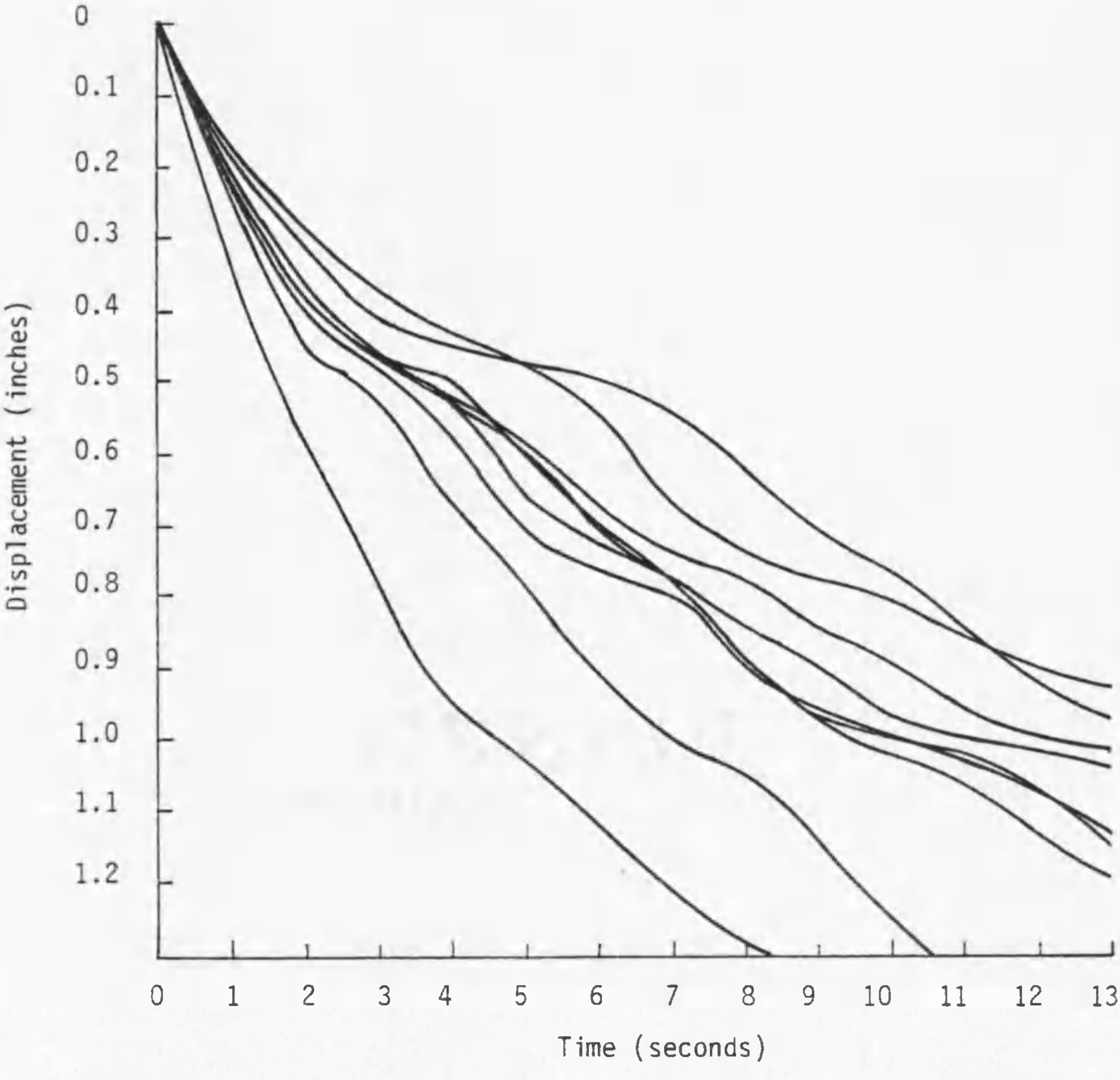


Figure 27. - Rock sliding block test curves, Test 3-M(1).

Slope = 22°
Frequency = 5.17 Hz

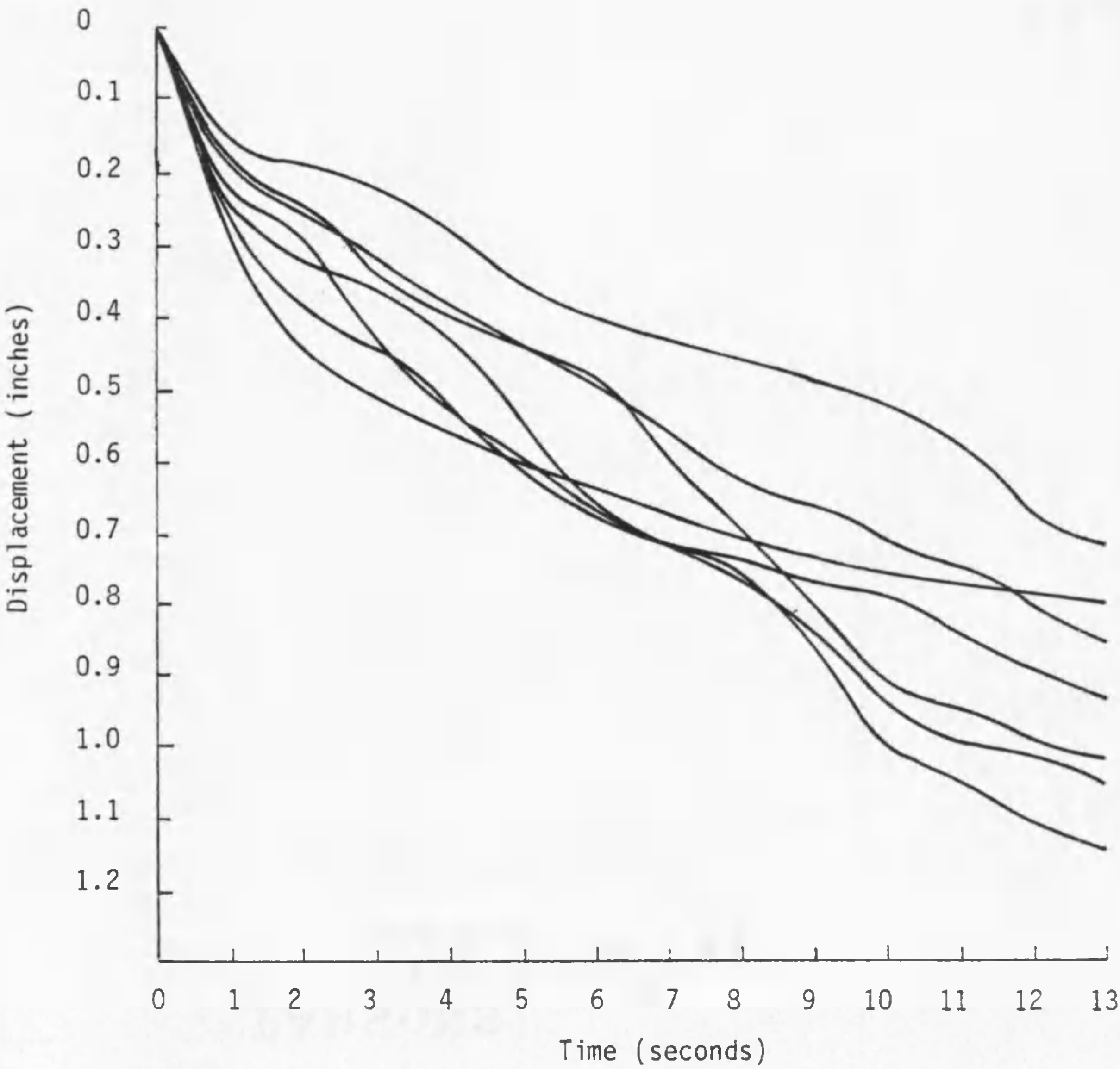


Figure 28. - Rock sliding block test curves, Test 4-M(1).

Slope = 21°
Frequency = 5.17 Hz

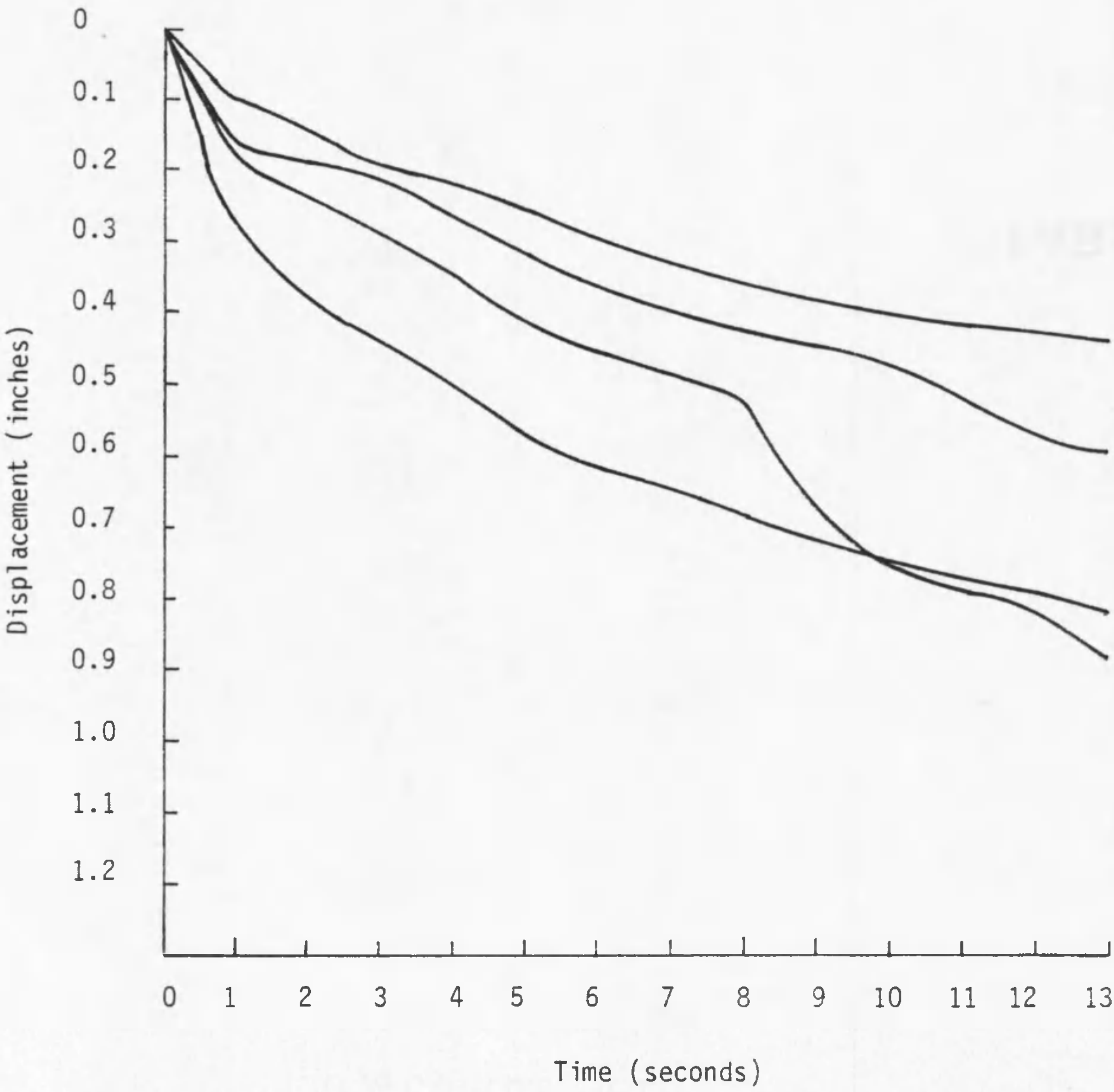


Figure 29. - Rock sliding block test curves, Test 5-M(1).

Slope = 20°
Frequency = 5.17 Hz

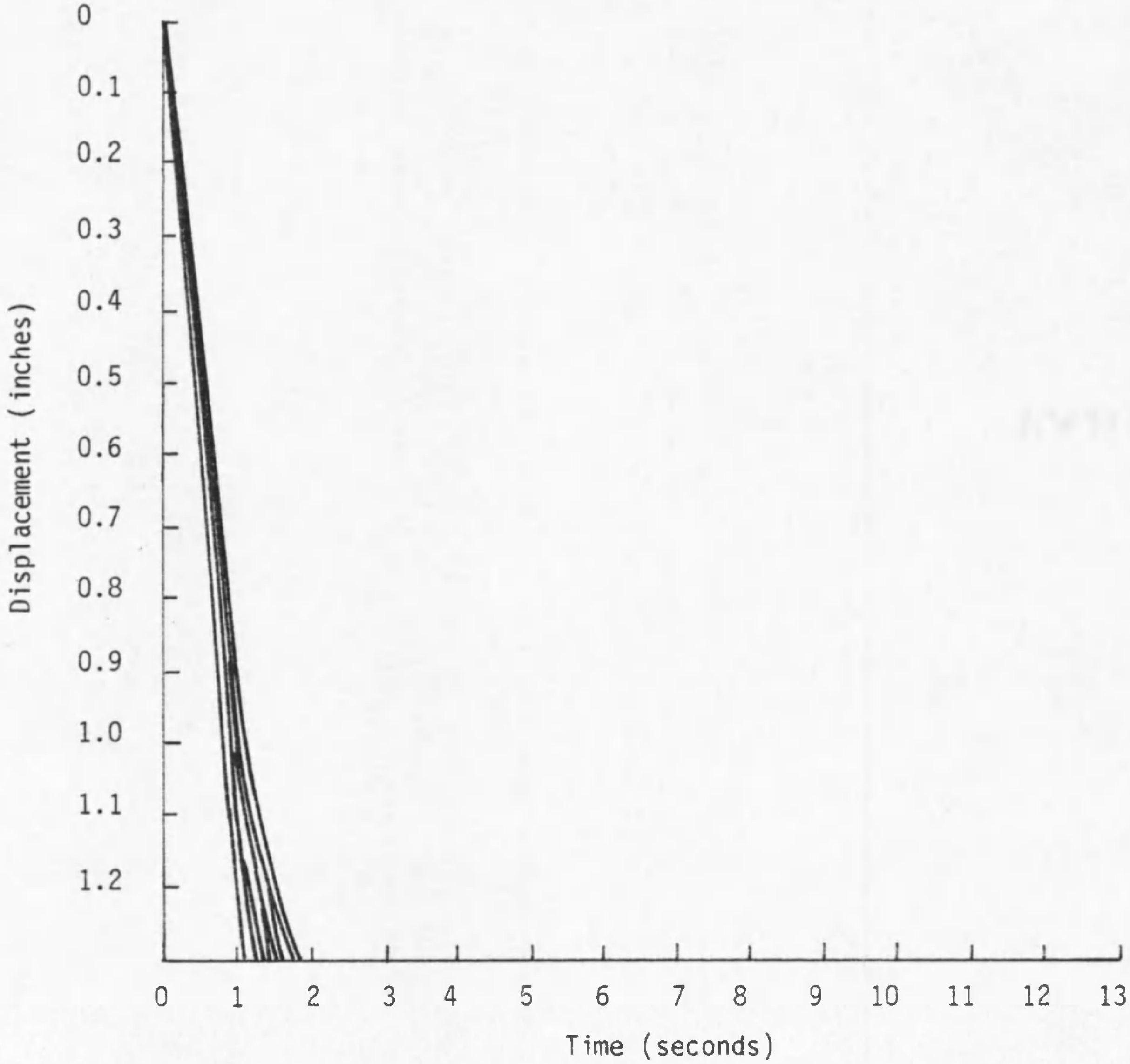


Figure 30. - Rock sliding block test curves, Test 2-M(2).

Slope = 23°
Frequency = 5.17 Hz

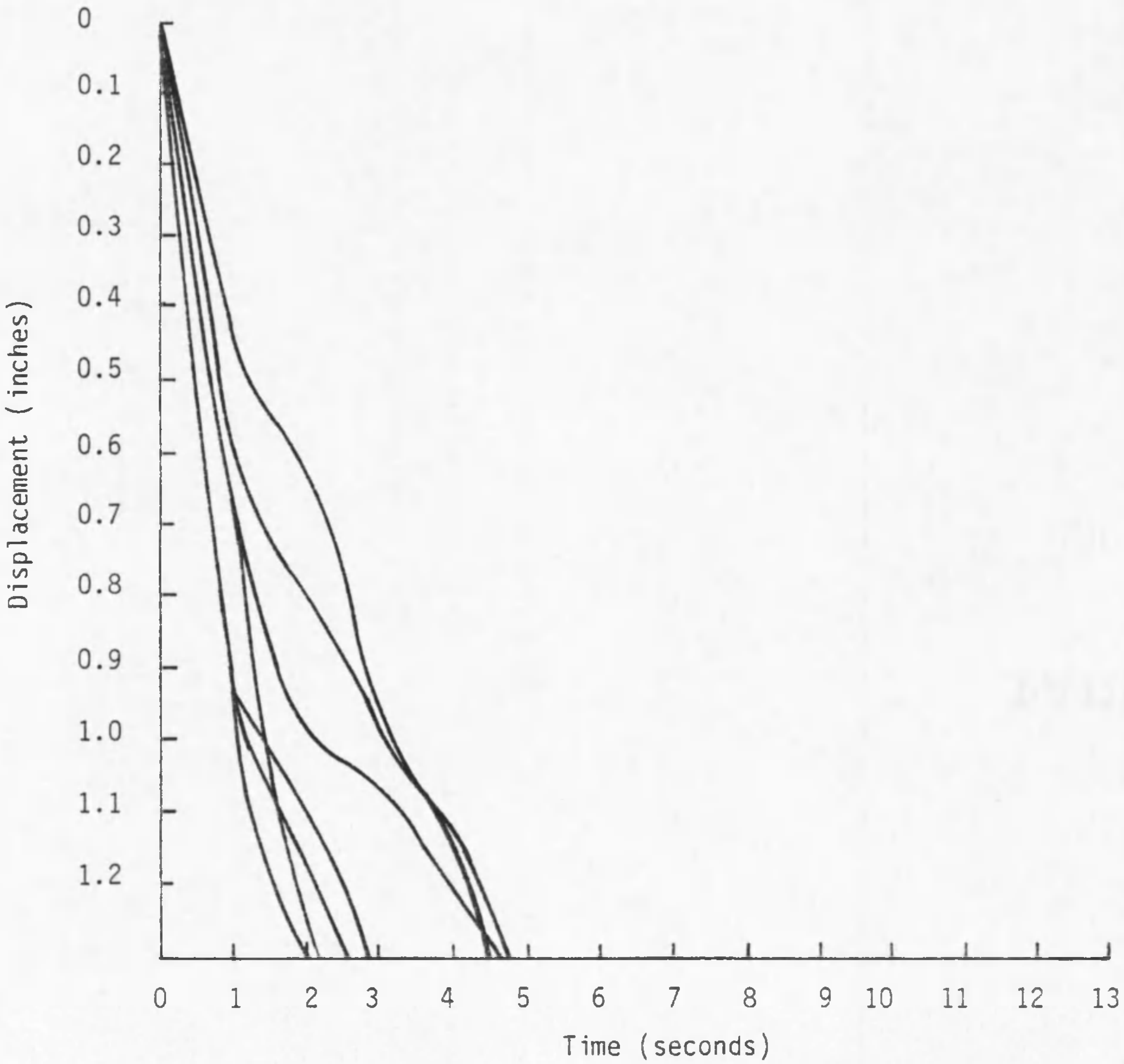


Figure 31. - Rock sliding block test curves, Test 3-M(2).

Slope = 22°
Frequency = 5.17 Hz

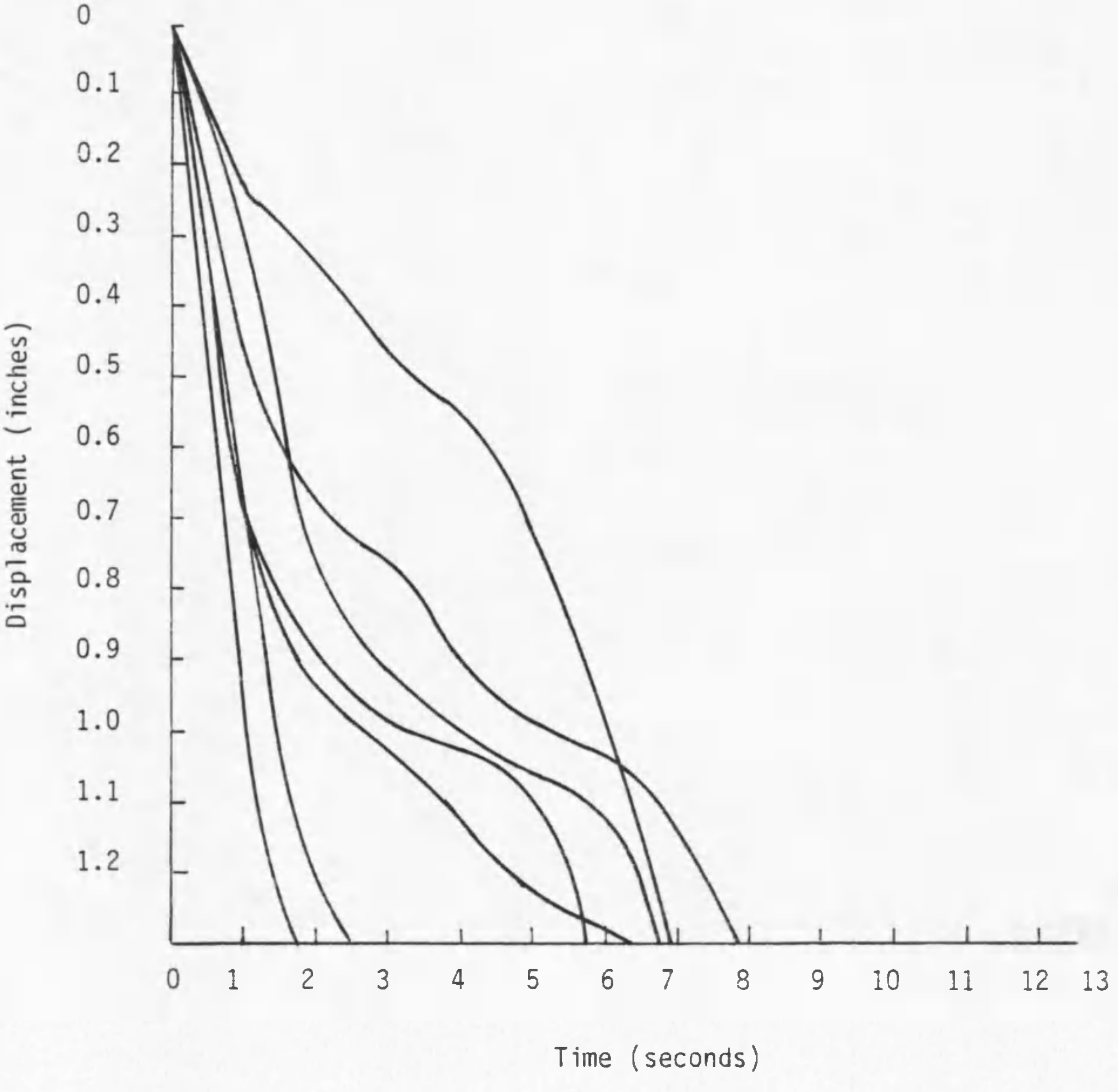


Figure 32. - Rock sliding block test curves, Test 4-M(2).

Slope = 21°
Frequency = 5.17 Hz

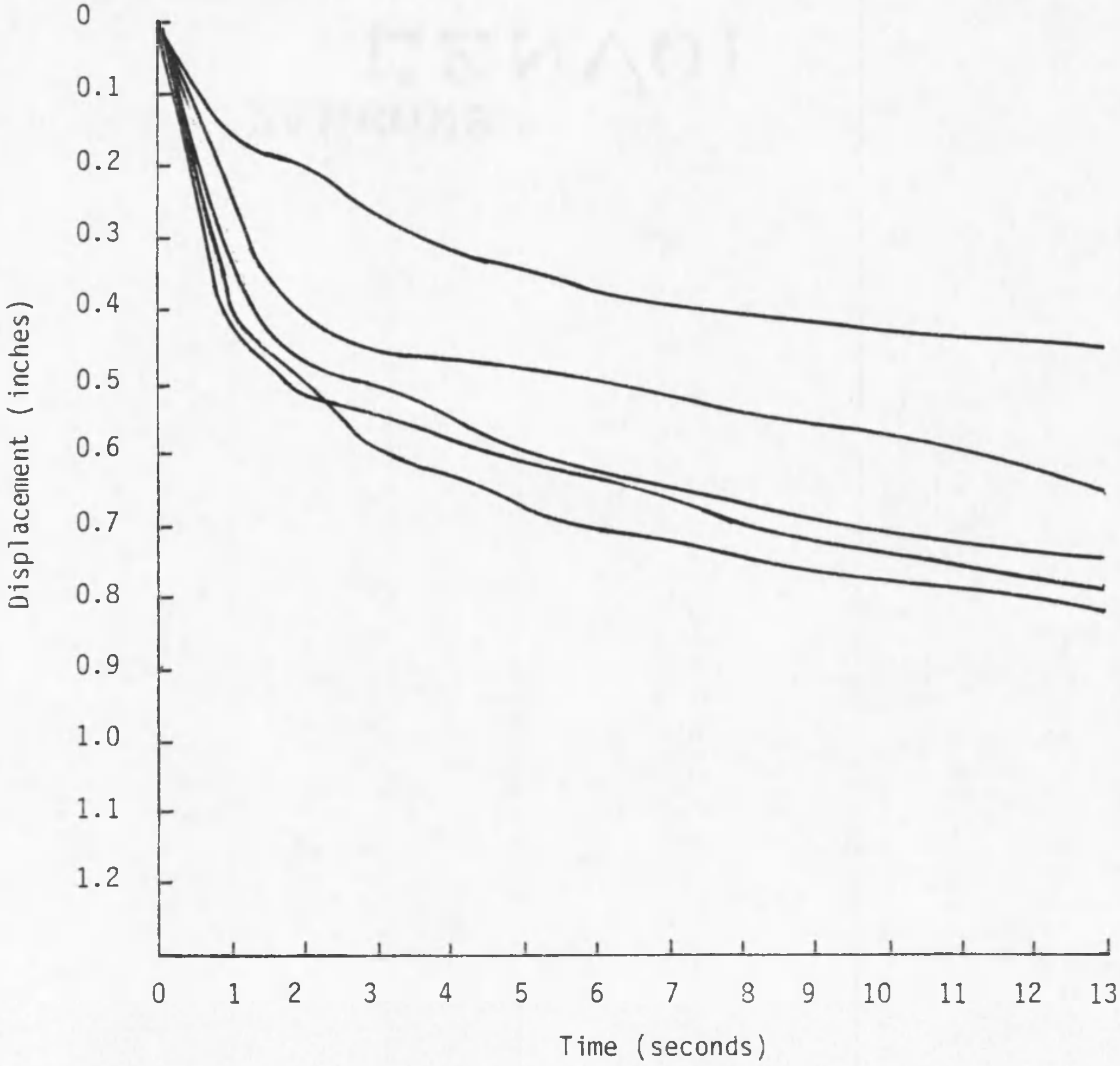


Figure 33. - Rock sliding block test curves, Test 5-M(2).

Slope = 20°
Frequency = 5.17 Hz

Figure 34 (block 1) and figure 35 (block 2) both have about the same average slope. The stroke was then increased to 0.18 inch and the frequency was held constant (maximum acceleration = 0.25 g). The two blocks continued to slide at approximately the same rate, however, in some cases the sliding rates increased after the blocks had already slid 0.5 to 0.7 inch. This is seen in figures 36 through 38. Several tests were run at a frequency of 4.7 Hz with a displacement of 0.172 inch, and maximum acceleration of 0.39 g. As before, the curves are tightly grouped at steep angles and become widely irregular at flatter angles.

As the table frequency was decreased, the ability to obtain acceptable repeatability of the test results also decreased. At high frequencies, the curves for the trials usually stayed within a small range, providing reliability in the accuracy of the results. However, the tests performed at low frequencies were not as consistent in their repeatability, thus resulting in a much wider range from which to select a "correct" block sliding motion. For all cases, the trials run at steep slopes fell into a tighter range of curves while those run at flatter slopes, and therefore moving more slowly, produced a wider range of curves.

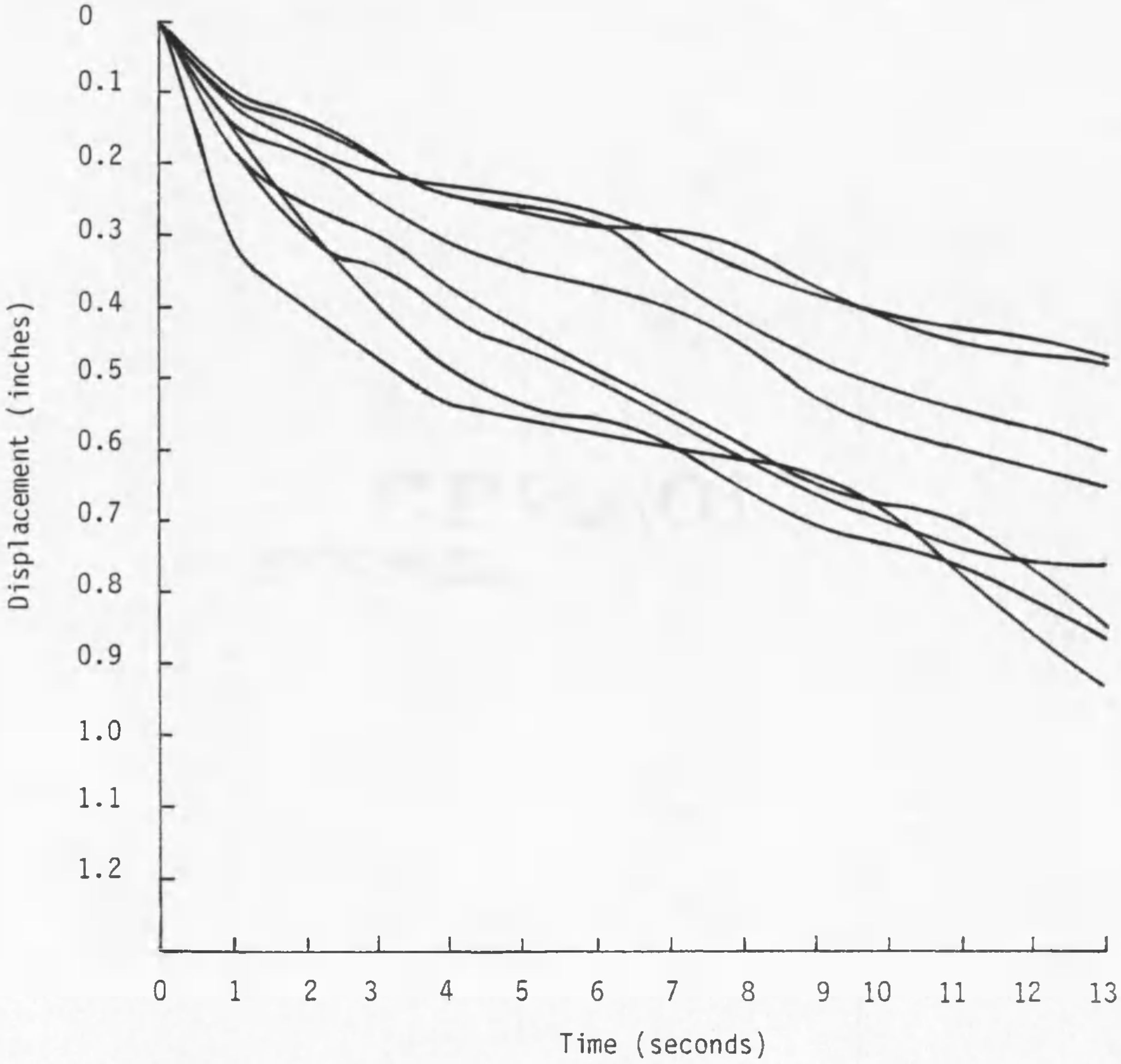


Figure 34. - Rock sliding block test curves, Test 1-N(1).

Slope = 27°
Frequency = 3.67 Hz

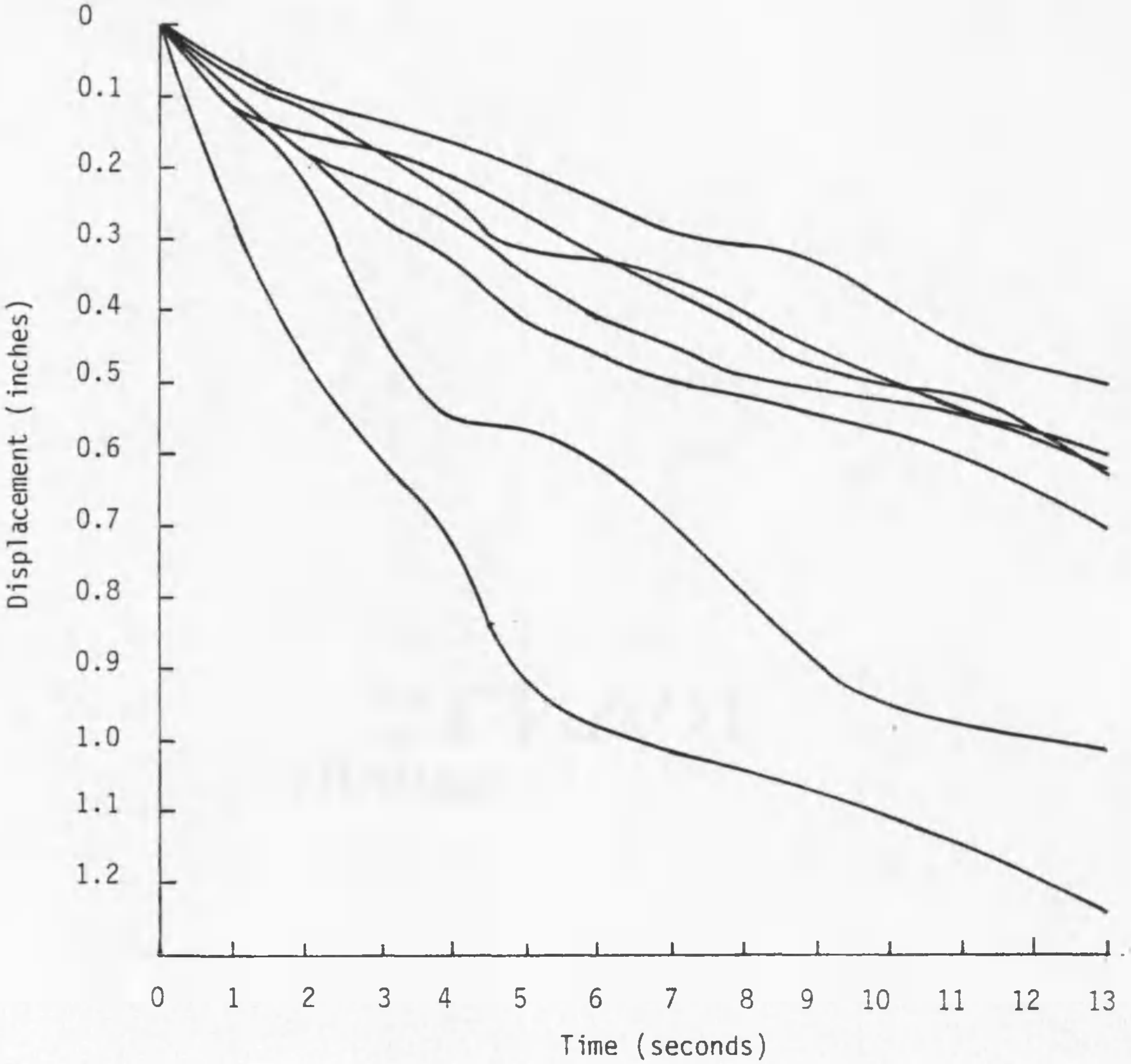


Figure 35. - Rock sliding block test curves, Test 1-N(2).

Slope = 27°
Frequency = 3.67 Hz

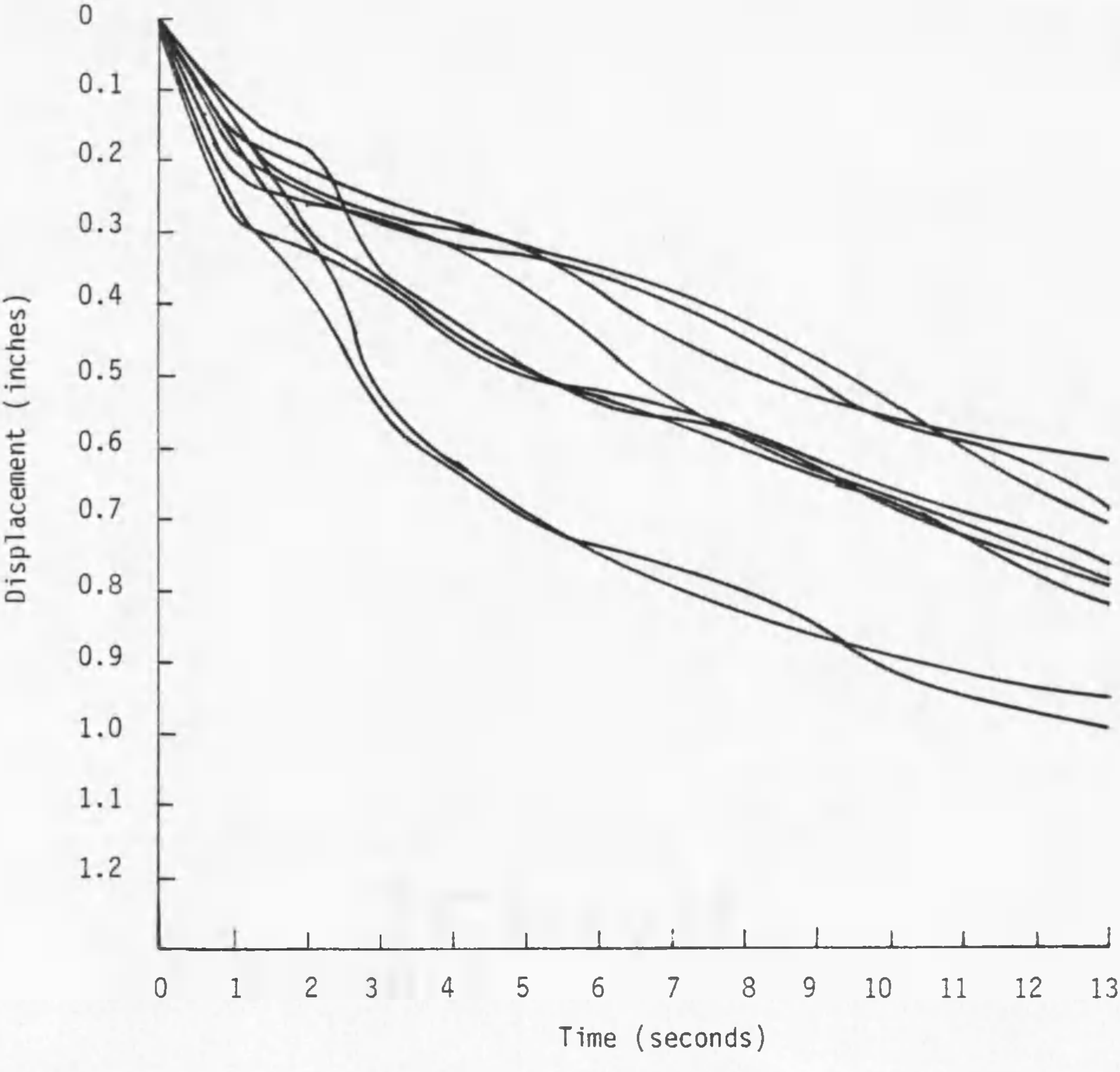


Figure 36. - Rock sliding block test curves, Test 3-P(2).

Slope = 26°
Frequency = 3.60 Hz

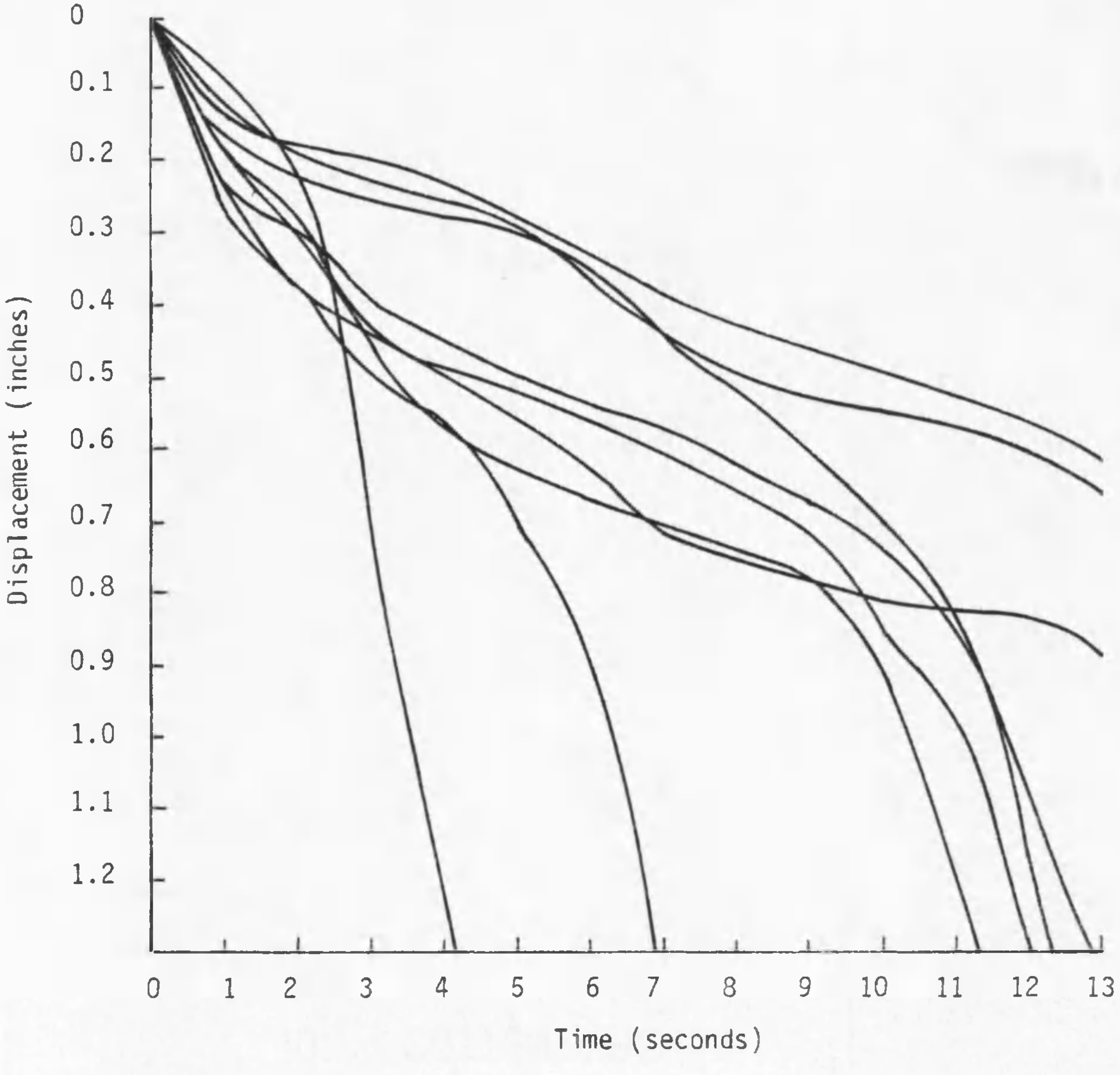


Figure 37. - Rock sliding block test curves, Test 2-P(2).

Slope = 27°
Frequency = 3.60 Hz

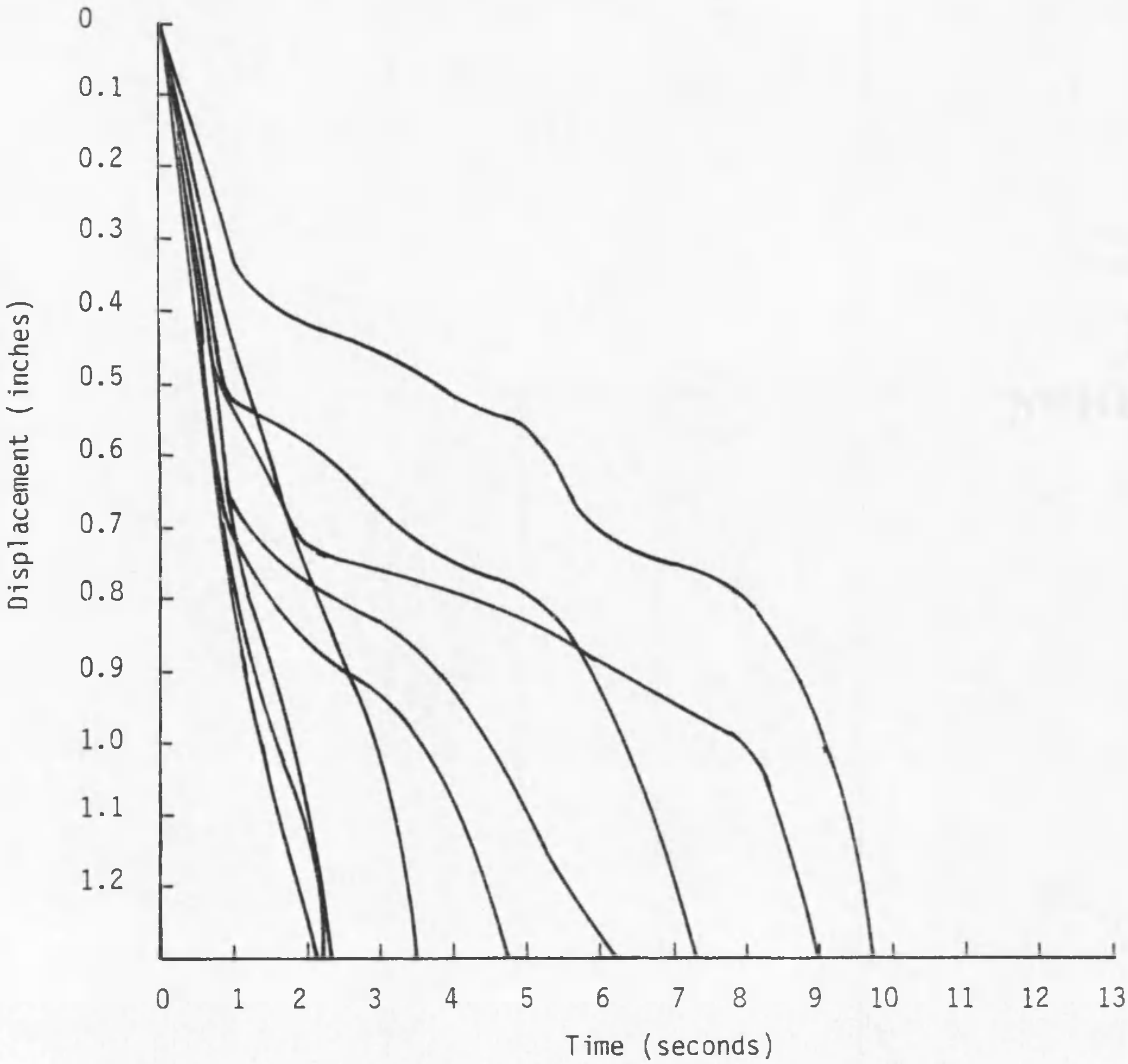


Figure 38. - Rock sliding block test curves, Test 1-P(2).

Slope = 28°
Frequency = 3.60 Hz

CHAPTER 5

COMPARISON OF PREDICTED DISPLACEMENTS WITH MEASURED DISPLACEMENTS

The results from the rock sliding block tests were compared with the results obtained from the analytical methods of predicting the downslope displacements. Newmark's method gave reasonable though conservative values for the displacements, and Hendron's method was less conservative than Newmark's. For the low frequency tests, all methods severely overestimated the displacment. Some results of the predictions, compared with the range of sliding motions obtained from the physical tests, are displayed in figures 39 through 51.

In the majority of the tests, the blocks started sliding fairly rapidly for the first second or two and then would slow down to a steady pace for the remainder of the test. This change in sliding rates could be due to two things. The spring on the follower transducer is definitely exerting a small force on the block which may be enough to show up on the curves. As the spring is opened outward, the force is decreased until it is insignificant. Therefore, the driving force is greater for the first few seconds at each trial. Also, the sliding surfaces of the blocks and the base are not polished, but are saw cuts. There are small asperities which the blocks must move over as it displaces downslope. At higher frequencies it would be easier for the

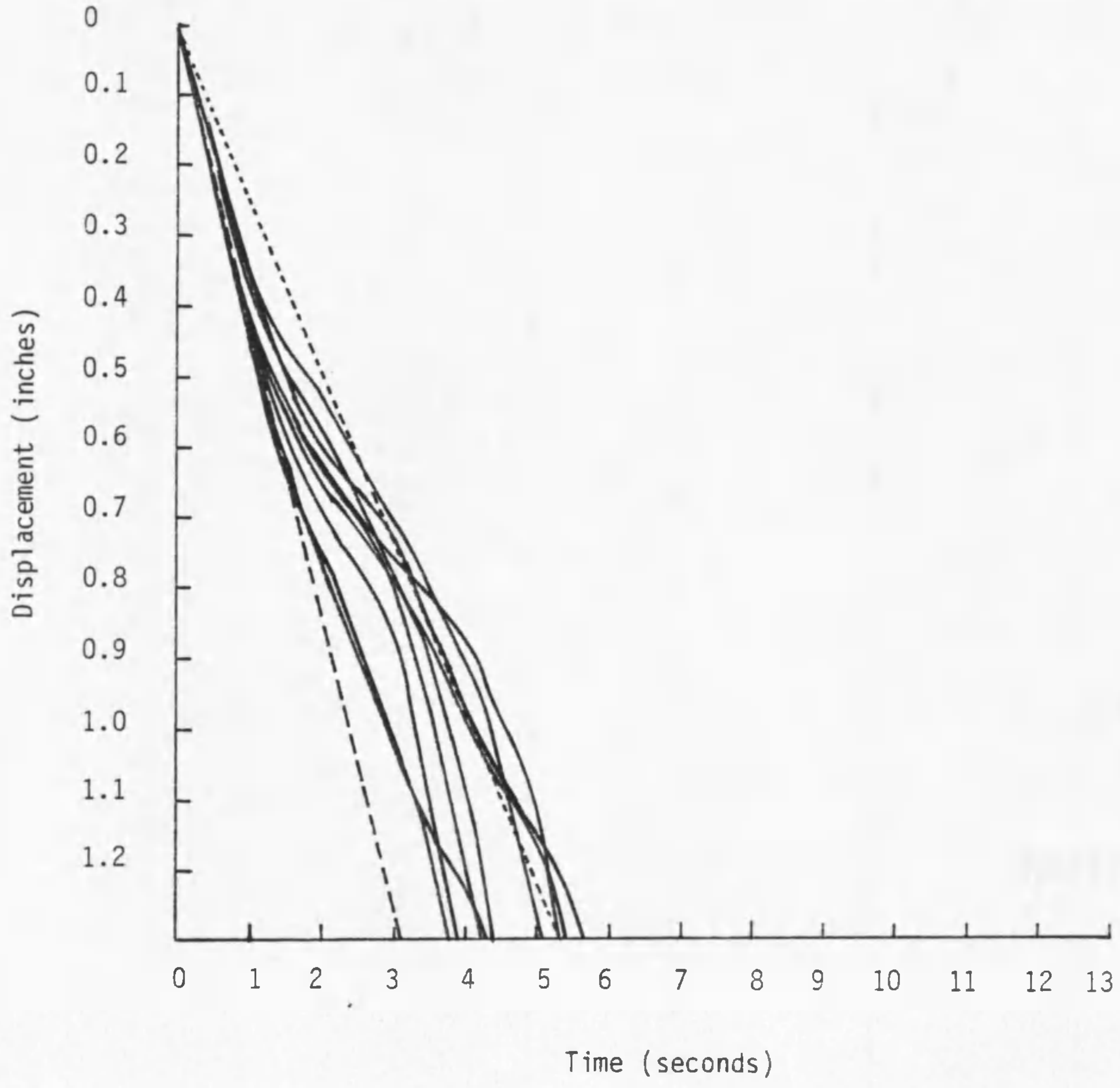


Figure 39. - Model test displacements versus predicted displacements, Test 3-P(2).

Slope = 24° — — Newmark
Frequency = 5.23 Hz - - - Hendron

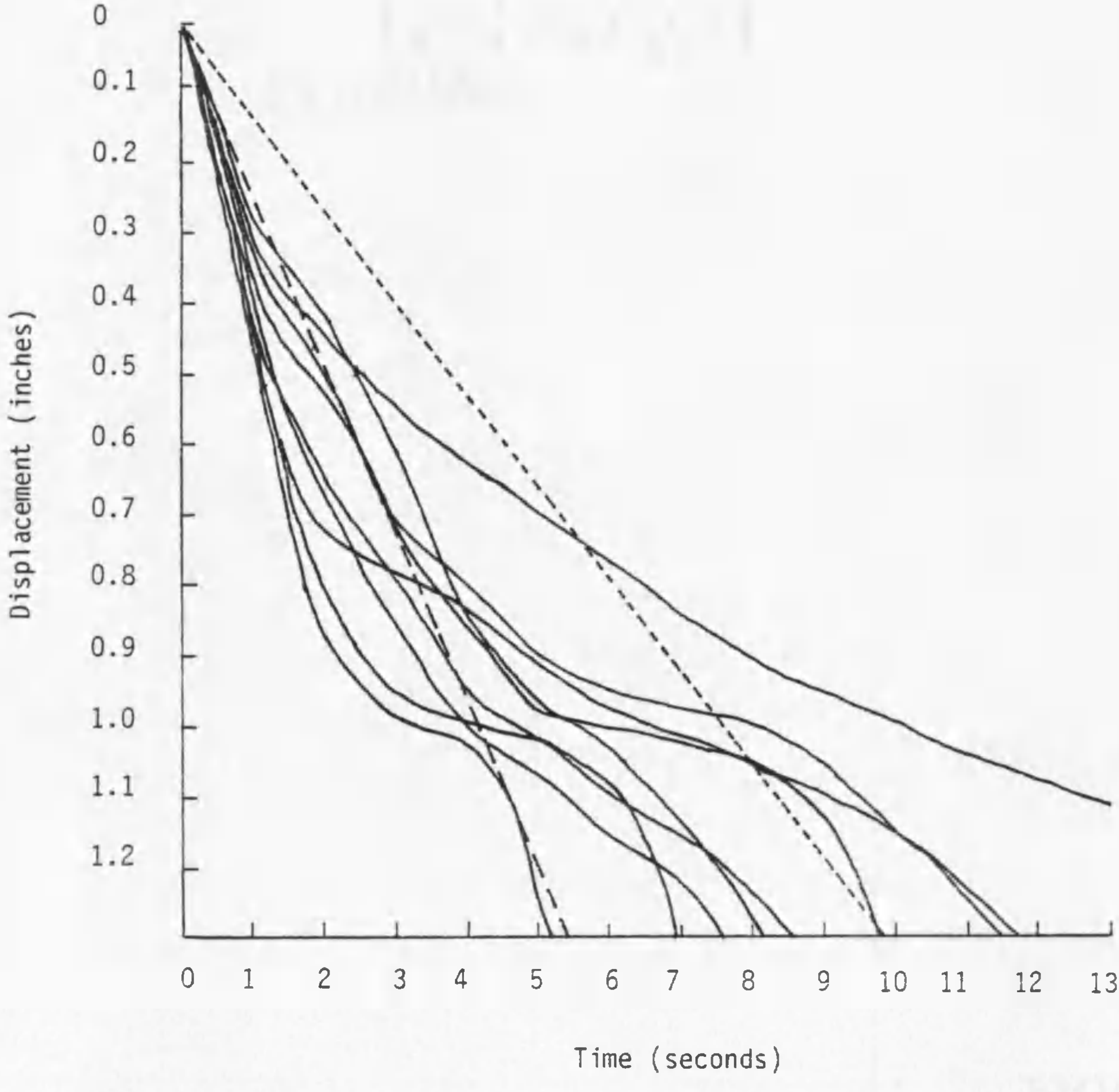


Figure 40. - Model test displacements versus predicted displacements, Test 3-L(2).

Slope = 23°
Frequency = 5.23 Hz

— — Newmark
- - - - Hendron

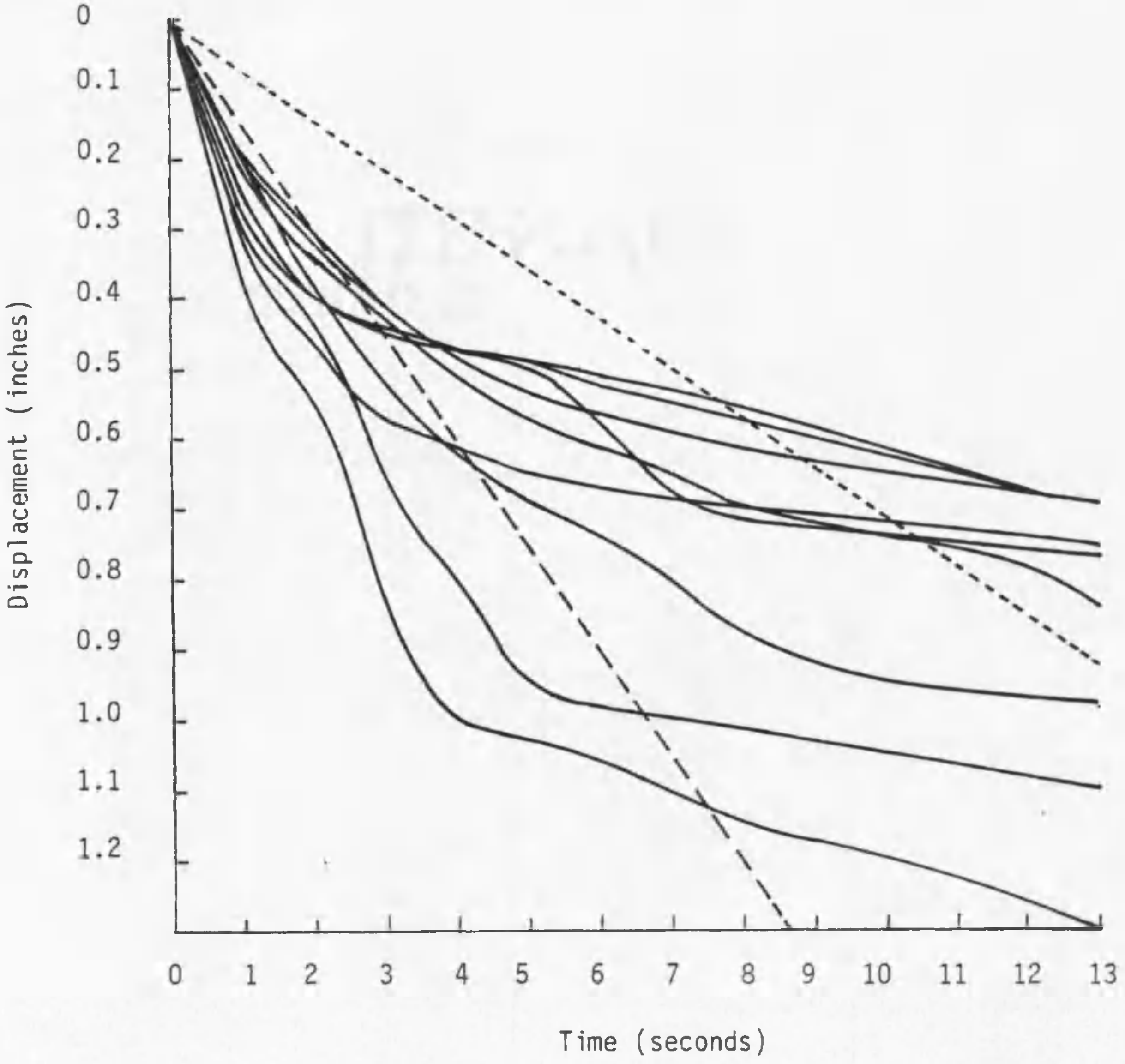


Figure 41. - Model test displacements versus predicted displacements, Test 4-L(2).

Slope = 22°
Frequency = 5.23 Hz

— — Newmark
- - - Hendron

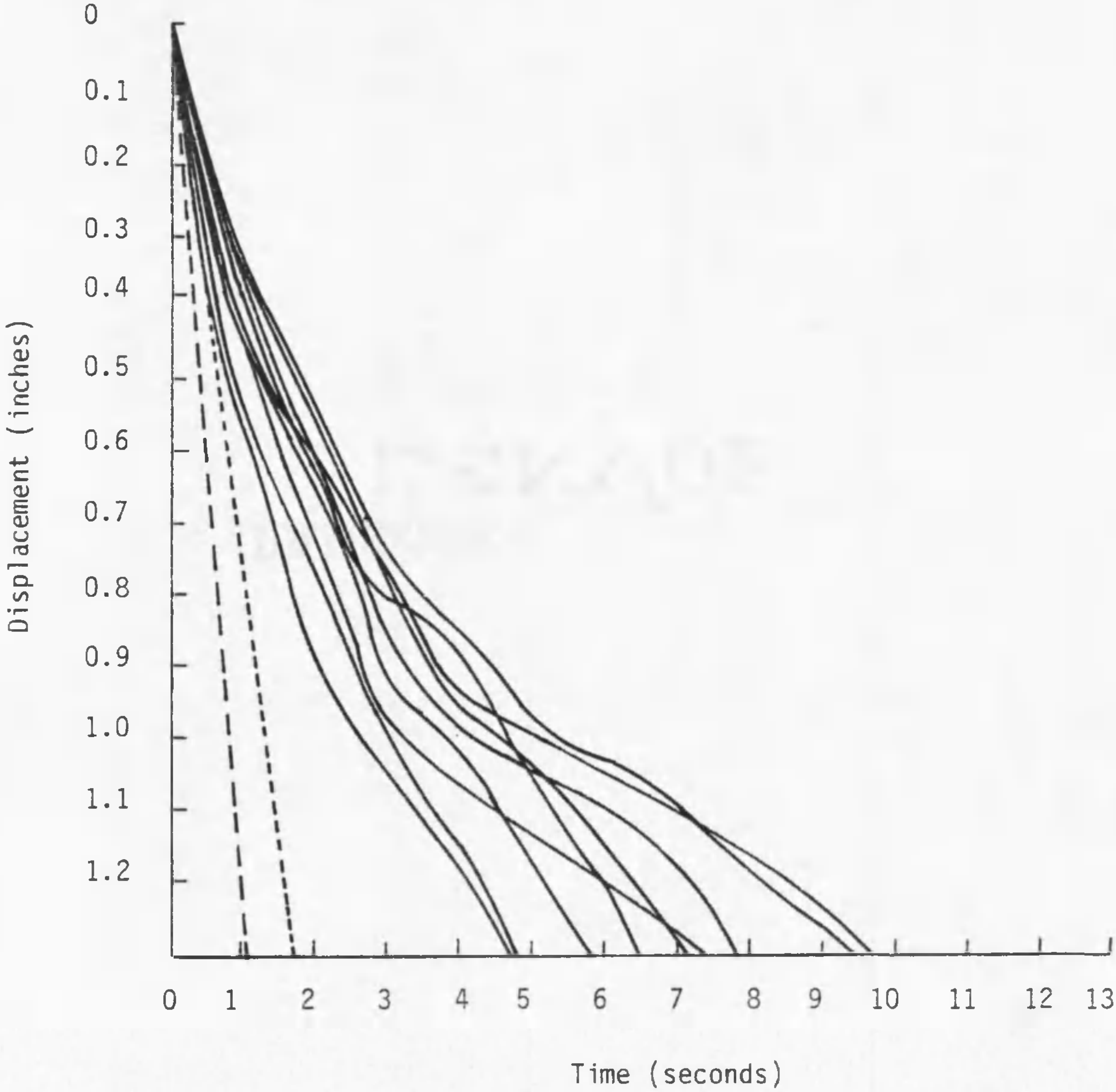


Figure 42. - Model test displacements versus predicted displacements, Test 1-M(1).

Slope = 24°
Frequency = 5.17 Hz

—— Newmark
---- Hendron

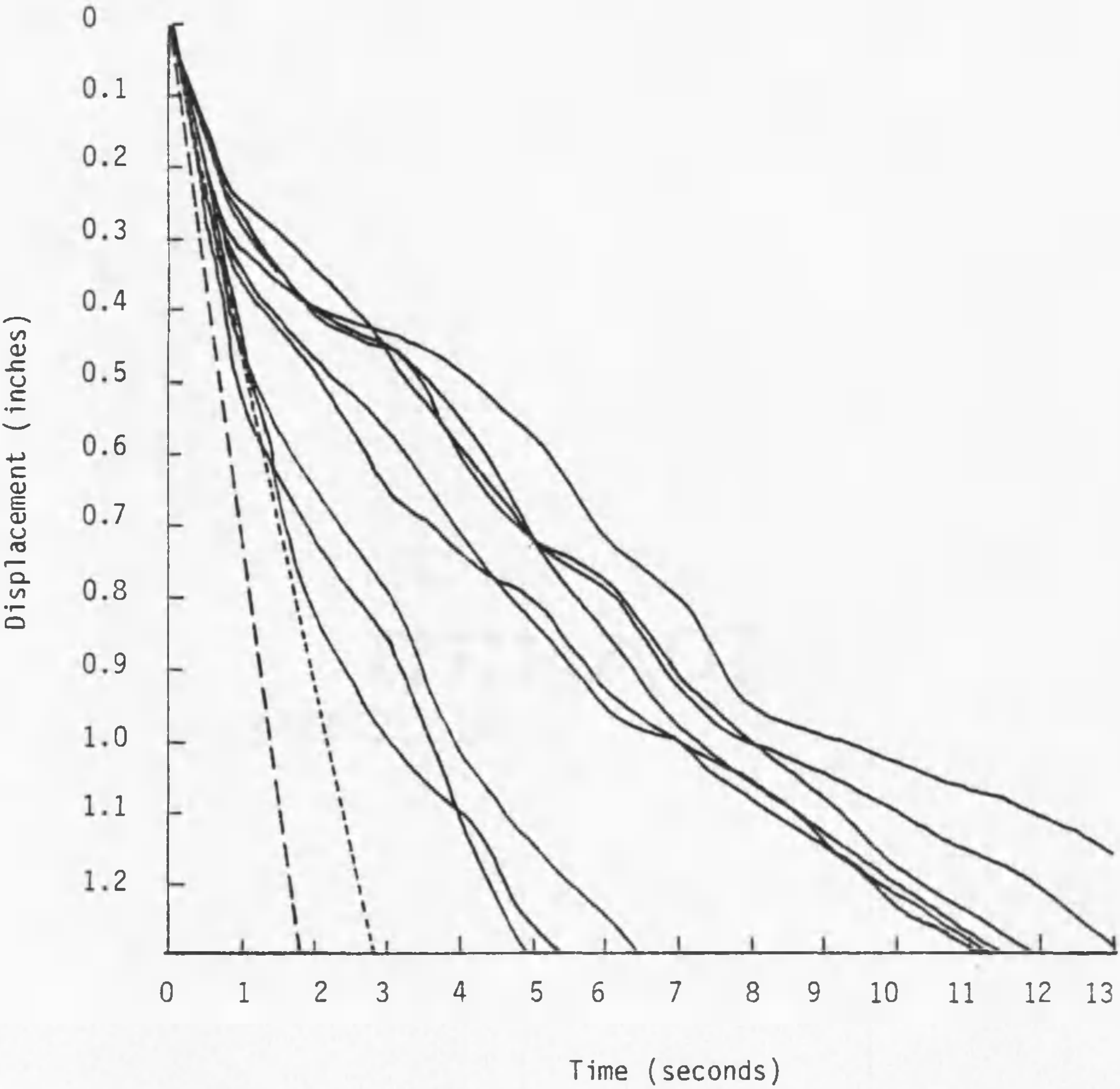


Figure 43. - Model test displacements versus predicted displacements, Test 2-M(1).

Slope = 23°
Frequency = 5.17 Hz

— — Newmark
--- Hendron

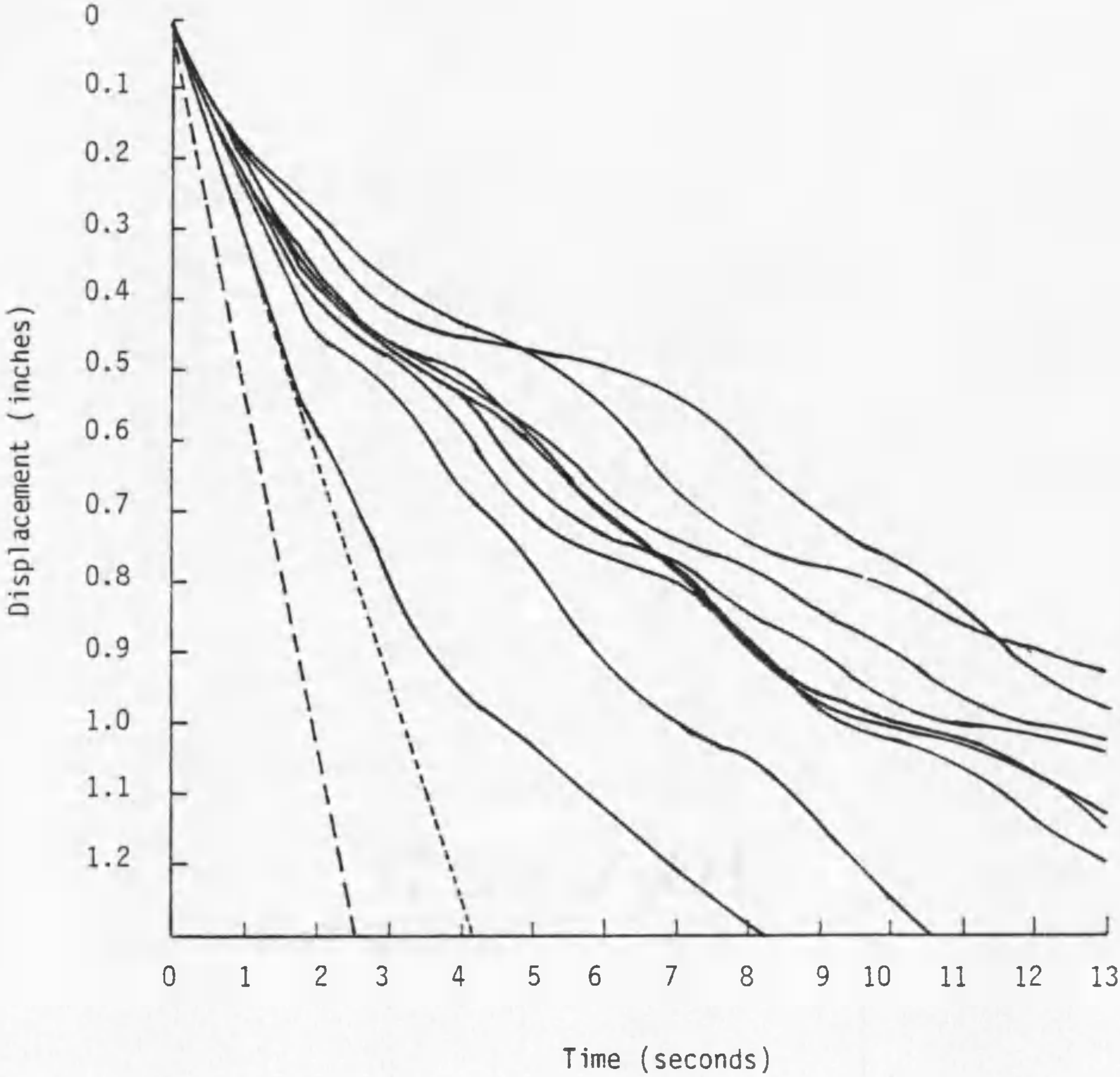


Figure 44. - Model test displacements versus predicted displacements, Test 3-M(1).

Slope = 22°
Frequency = 5.17 Hz

— — Newmark
--- Hendron

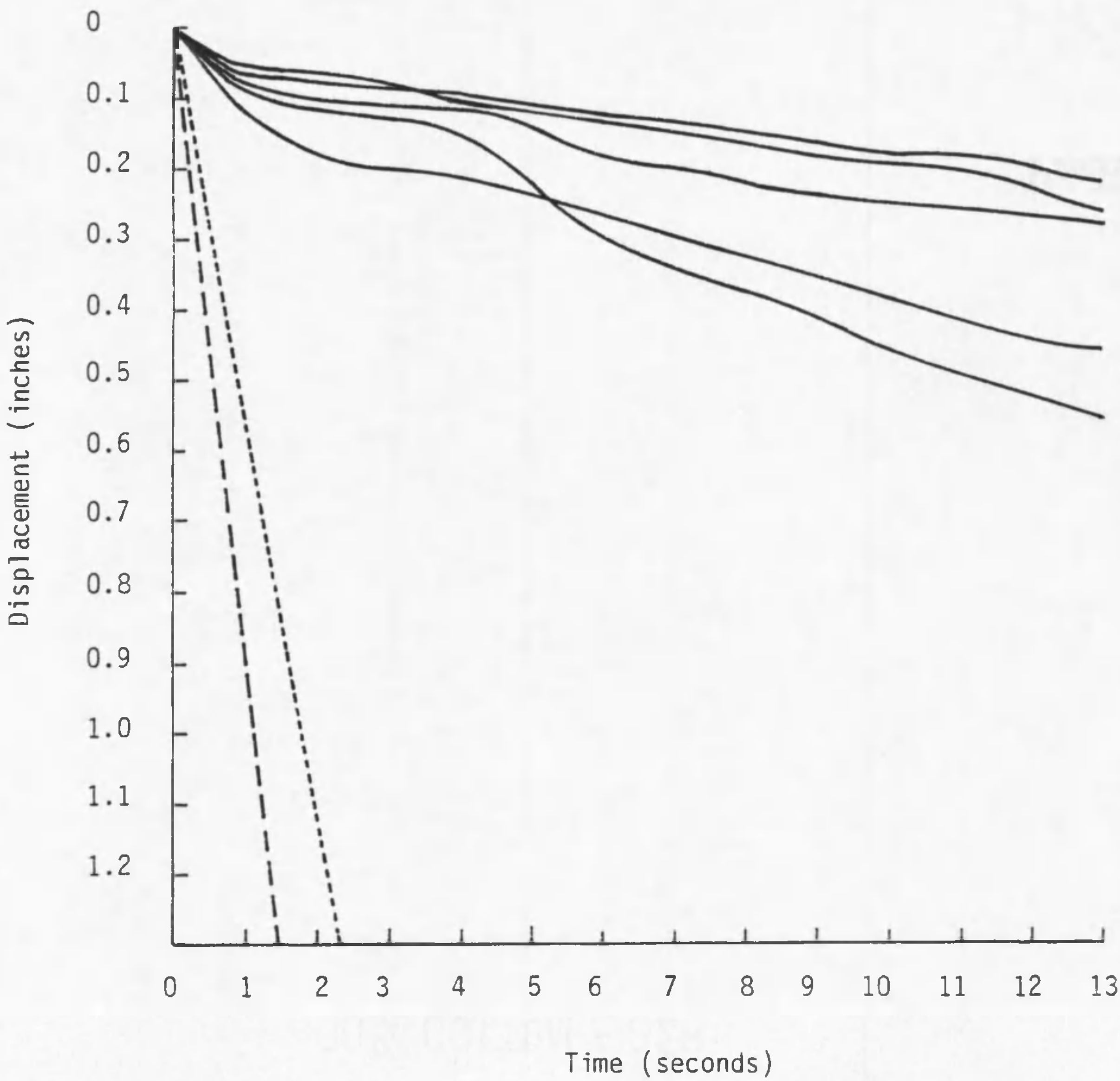


Figure 45. - Model test displacements versus predicted displacements, Test 2-N(1).

Slope = 26°
Frequency = 3.67 Hz

— — Newmark
- - - Hendron

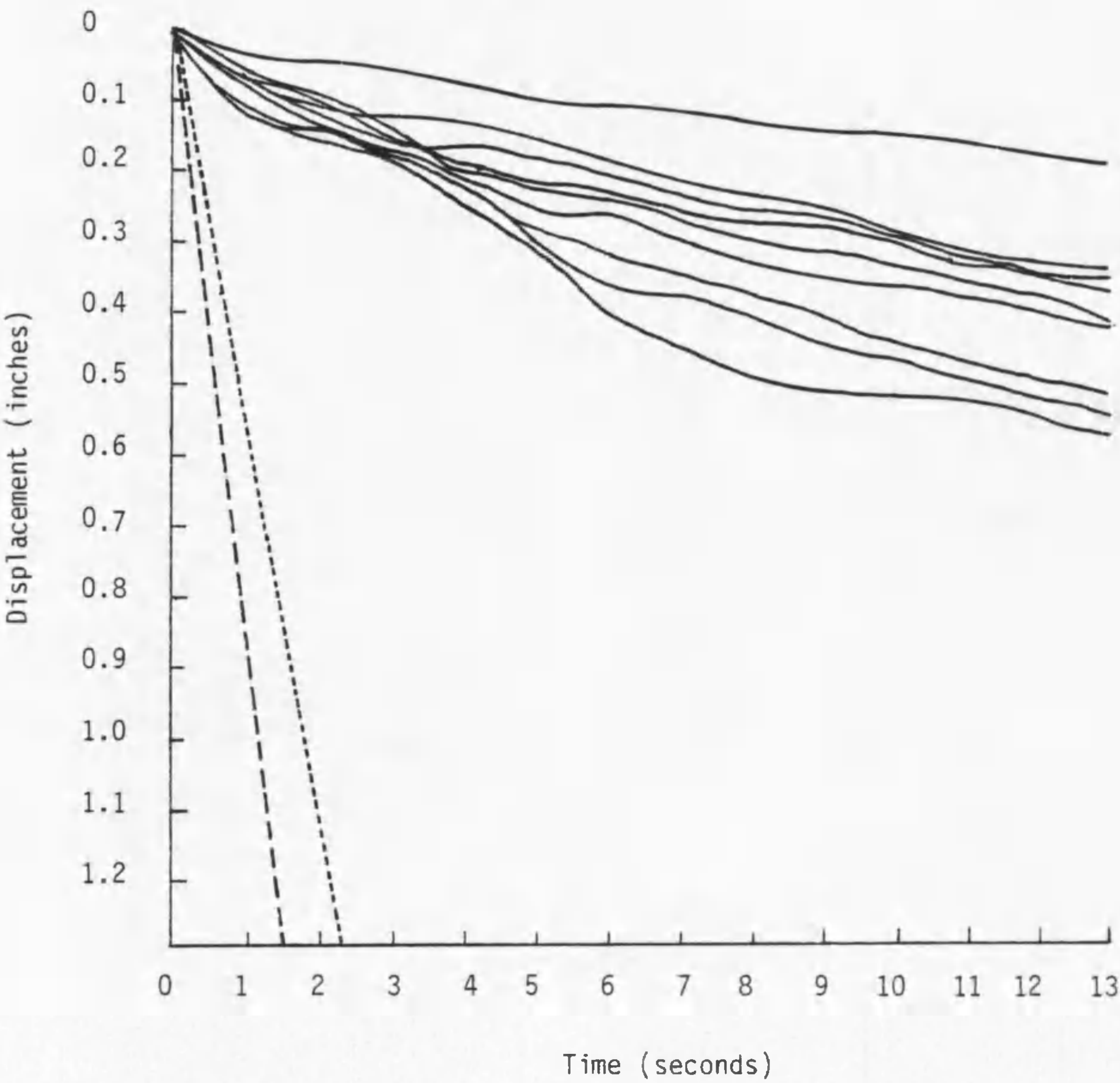


Figure 46. - Model test displacements versus predicted displacements, Test 2-N(2).

Slope = 26°
Frequency = 3.67 Hz

— — Newmark
--- Hendron

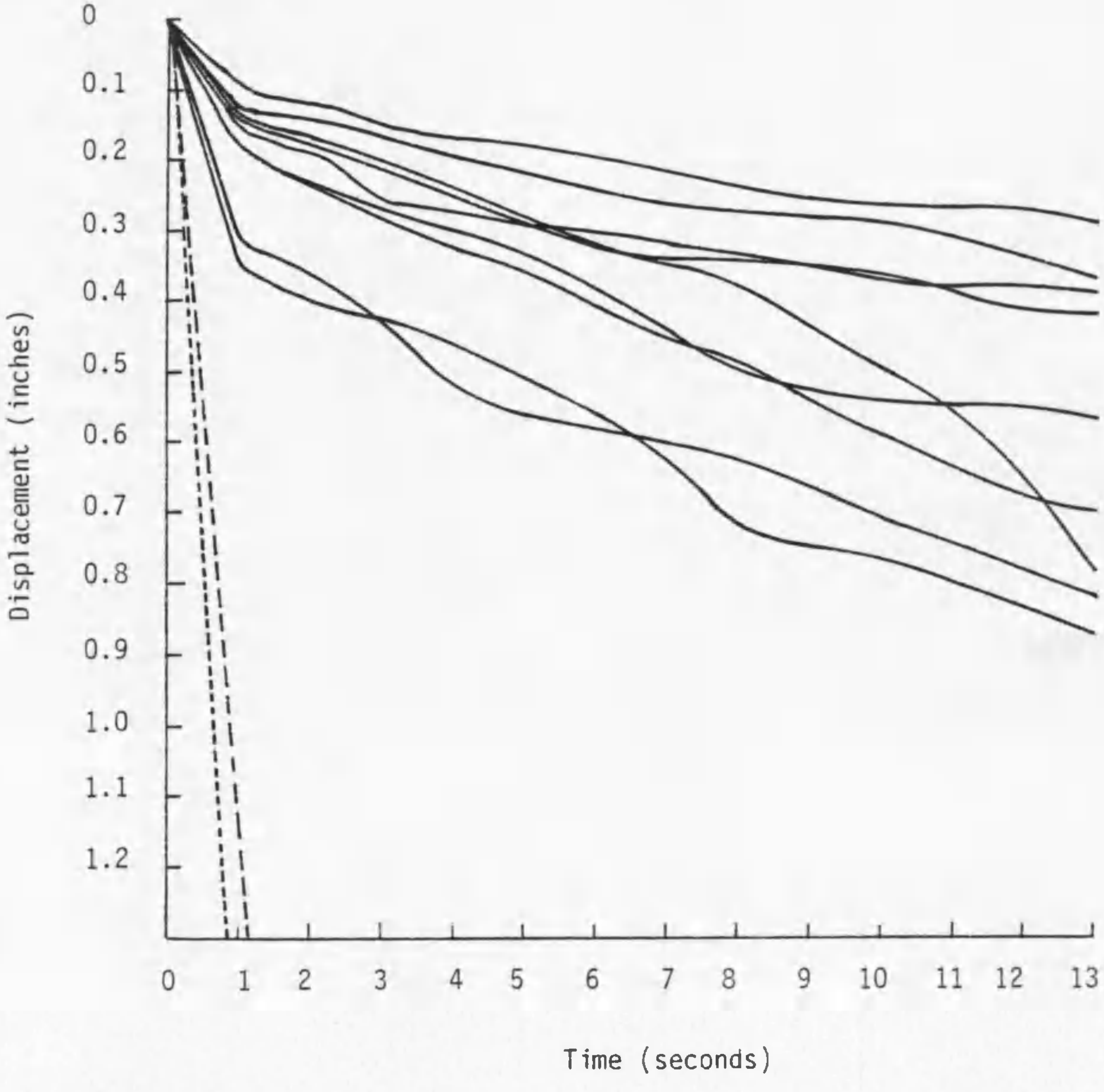


Figure 47. - Model test displacements versus predicted displacements, Test 3-P(1).

Slope = 26°
Frequency = 3.60 Hz

— —Newmark
----Hendron

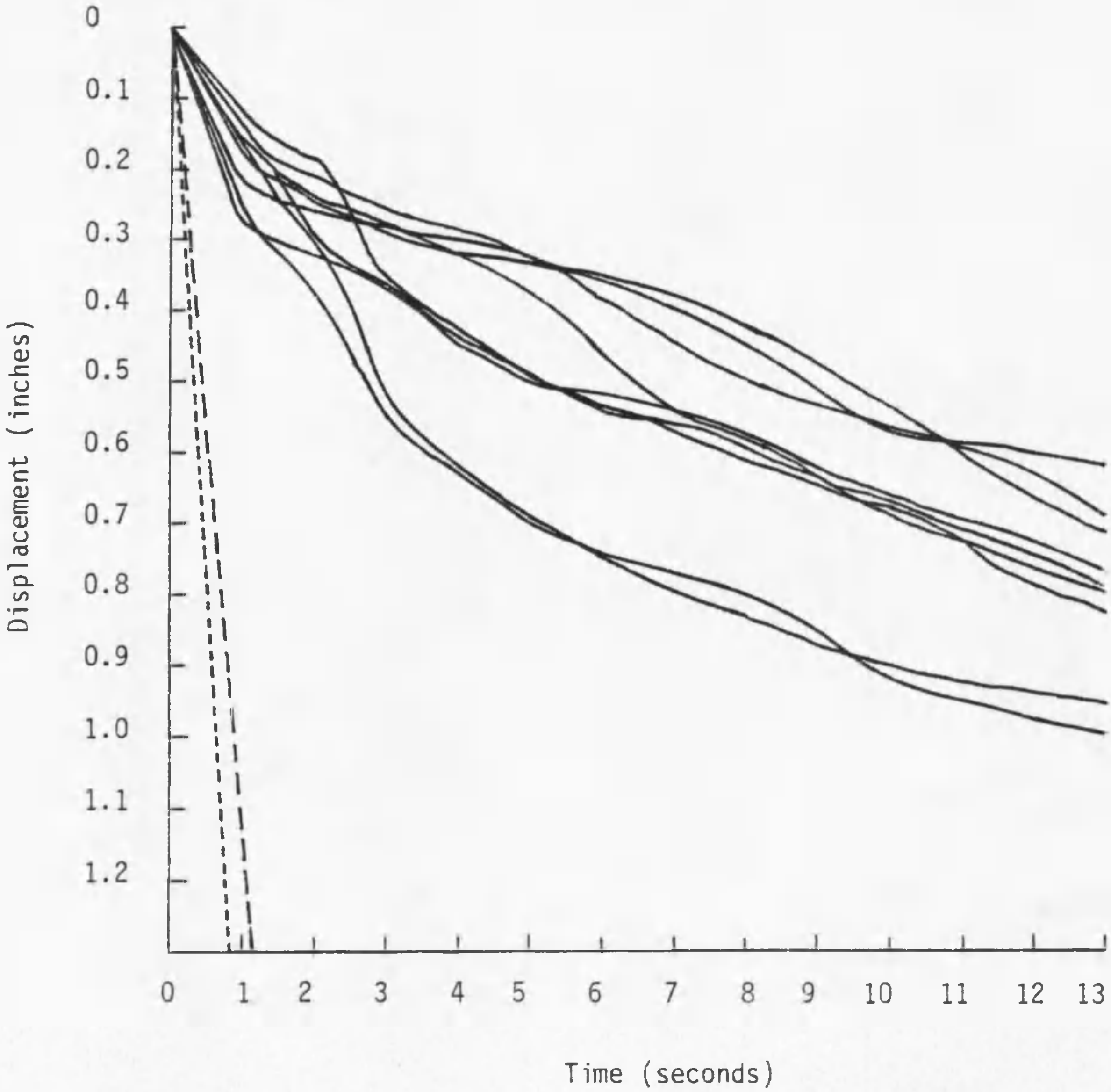


Figure 48. - Model test displacements versus predicted displacements, Test 3-P(2).

Slope = 26° ——Newmark
Frequency = 3.60 Hz ---Hendron

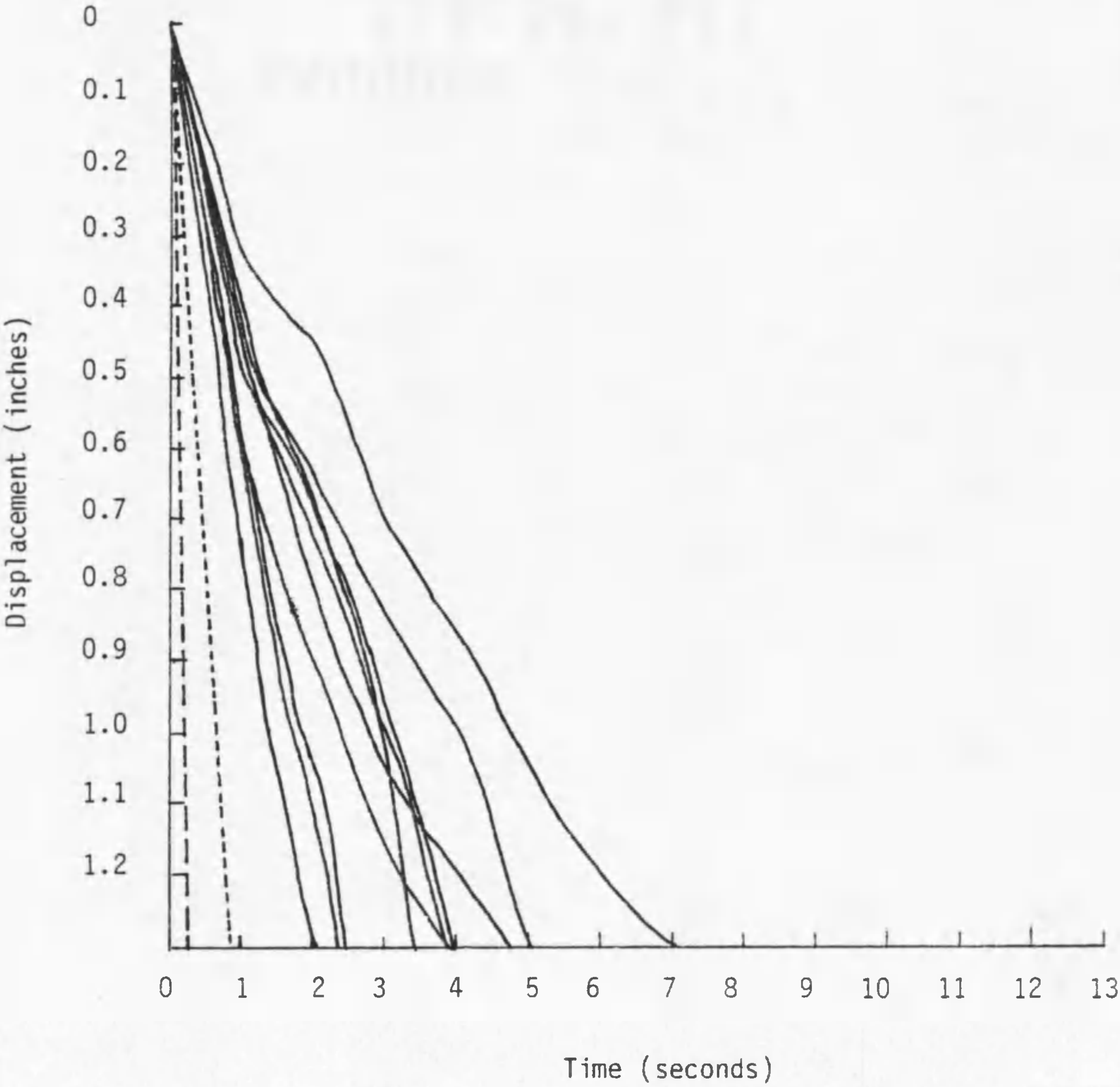


Figure 49. - Model test displacements versus predicted displacements, Test 2-V(1).

Slope = 26° —— Newmark
Frequency = 4.70 Hz - - - Hendron

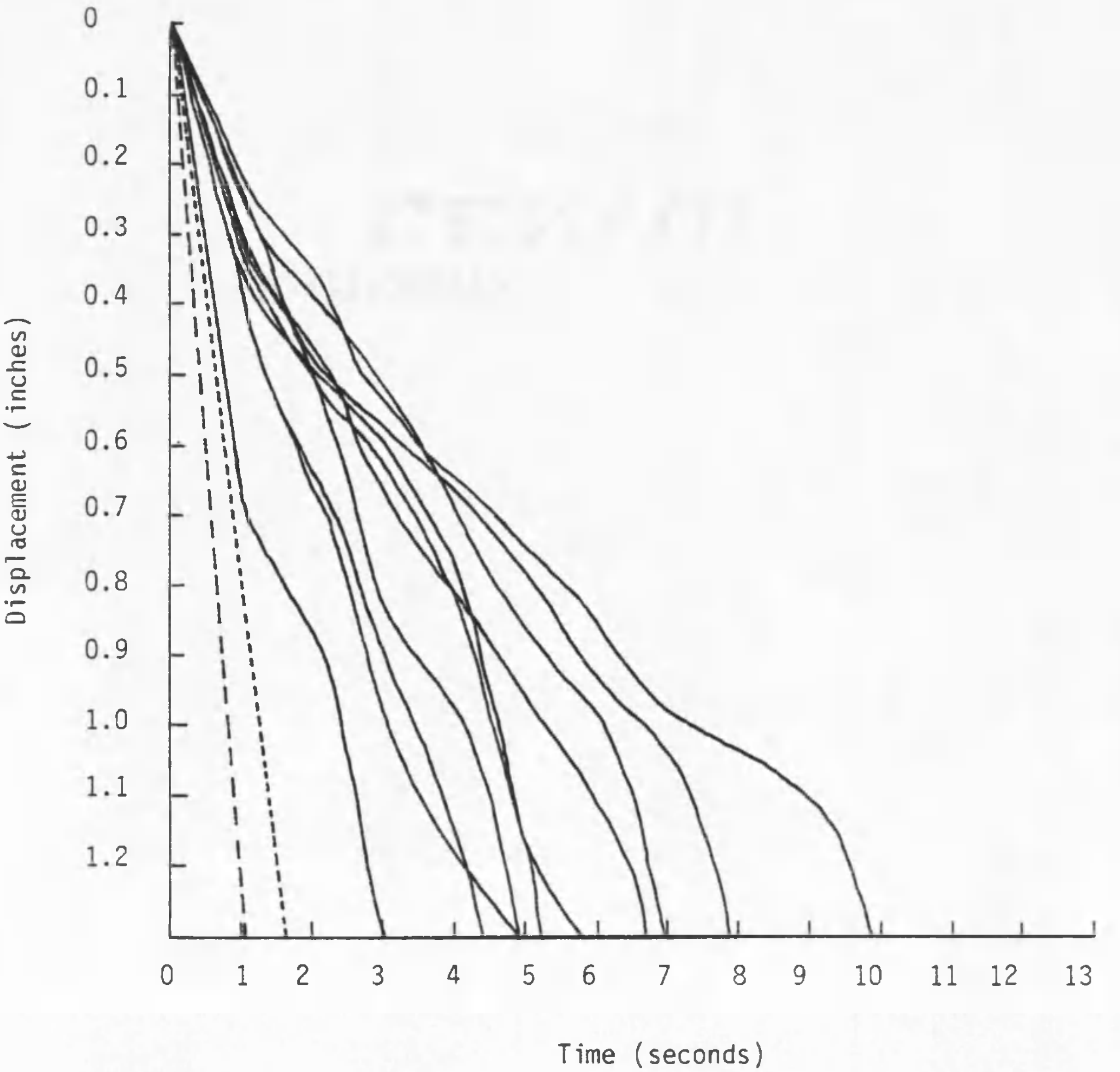


Figure 50. - Model test displacements versus predicted displacements, Test 3-V(1).

Slope = 25°
Frequency = 4.70 Hz

— — Newmark
- - - - Hendron

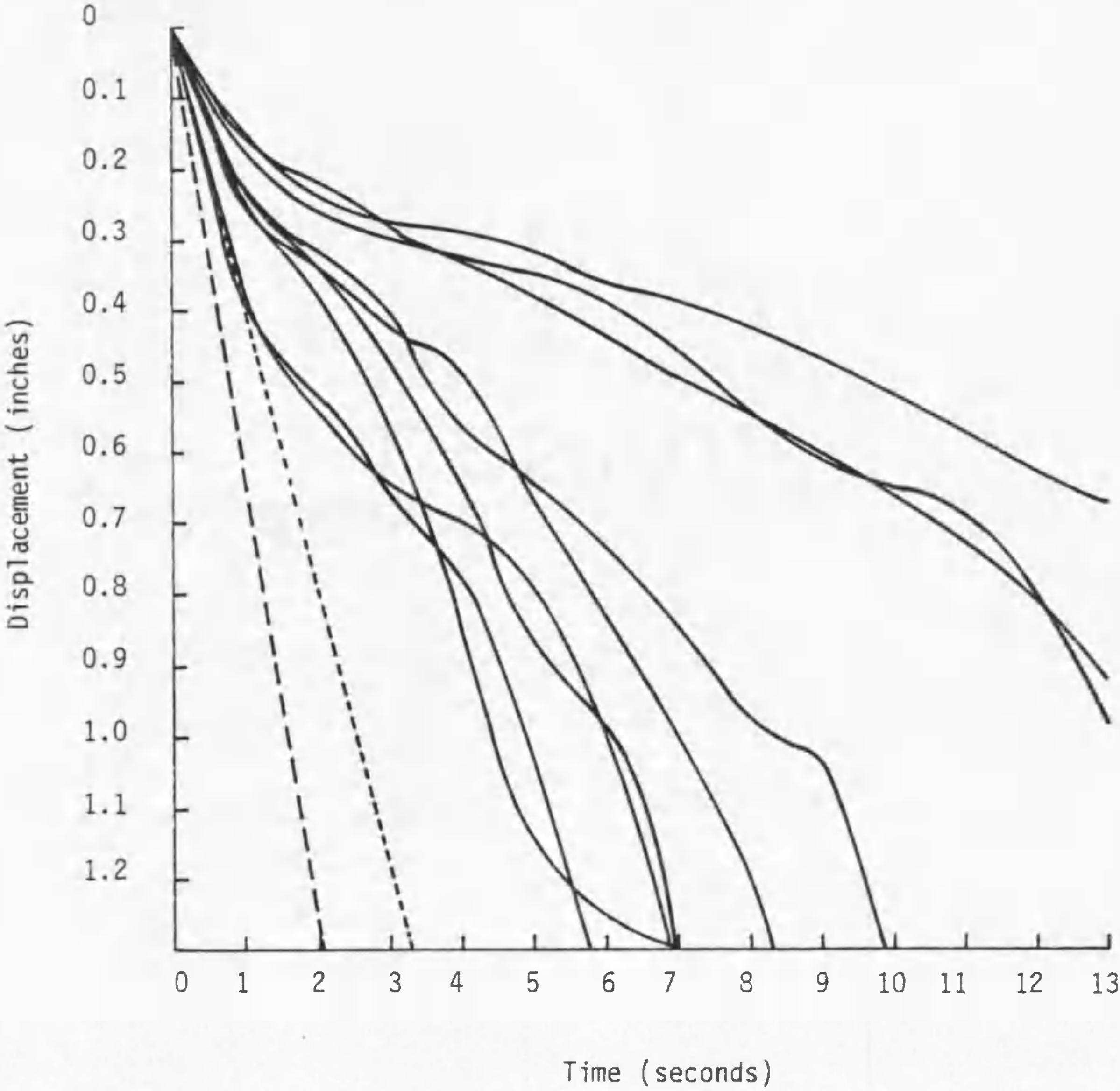


Figure 51. - Model test displacements versus predicted displacements, Test 4-V(1).

Slope = 24°
Frequency = 4.70 Hz

— — Newmark
– – – Hendron

sliding blocks to "ride" over these without as much grabbing effect. Also, as more tests have been run, there would be weathering of the rock surfaces and an increase in the amount and possibly size of the asperities.

Because of the additional force of the transducer spring, the first part of the curves reflects conditions which are not accounted for in the Newmark and Hendron equations. But the force of the spring may not be all. The block must exceed a static value to begin sliding, thus it would exceed by quite a bit the dynamic friction value and one would expect it to slow to some equilibrium value. The remainder of the curves, after the change in slope, is usually straight enough to be considered linear in overall shape. This portion is the best description of the downslope displacements which are the result of the table motions, gravity forces, and the friction between the sliding block and the base. The slope of this part of the curves is considered to be the true sliding rate of the blocks to be compared with the curves obtained from the Newmark and Hendron methods.

High Frequency Tests

Series L, Amax = 0.17 g, Freq. = 5.23 Hz

These tests, run at frequencies of 5.23 Hz and 5.17 Hz, compared better than any of the others with Newmark's and Hendron's methods. The maximum accelerations of 0.17 g and 0.26 g also compare well with maximum accelerations reached in moderate-sized earthquakes. Generally, for steeper slopes the curves compare well with the predicted results, but as the ramp slope is made more shallow, the predicted displacements become much greater than the measured displacements.

For the 5.23 Hz, 0.17 g tests on block 1, the Newmark and Hendron analytical curves compare well with the measured curves for slopes of 25 and 24°, but are conservative. At 23 and 22°, the measured curves show a pronounced flattening after the first second and depart widely from the straight lines produced by the analytical techniques. These calculated curves are much steeper than the smooth, steady portions of the measured curves since the equations only consider the maximum acceleration and do not allow for asperities on the sliding surface.

The block 2 tests at this frequency compare more favorably. At slopes of 25, 24, and 23°, the Hendron curves fall on the high end of the measured range of displacement, and the curves obtained from Newmark's equations fall near the low end of this range (fig. 39 and 40). As the slopes are lowered to 22 and 21°, the comparison is still reasonably good; however, more departure from the measured range is observed (fig. 41). The test curves again flatten out after a few seconds, and the steady portion shows a slower rate of descent than the calculated rates. Therefore, even though the blocks start out more rapidly than estimated, due to the push of the transducer spring, the predicted rates are faster than the observed rates.

Series M, Amax = 0.26 g, Freq. = 5.17 Hz

The block 1 tests run at 5.17 Hz showed the same results as before, at slopes of 24, 23, and 22° (fig. 42 through 44). The predictions are conservative, but compare fairly well with the measured curves from the sliding block tests. There is some flattening of the curve slopes noticeable in these test curves even at steeper angles which causes them to depart from the calculated curves with time. As before, the tests at flatter slopes show a considerably pronounced flattening after the first 1 or 2 seconds, and demonstrate a much slower rate of sliding than predicted by Newmark's and Hendron's equations.

Tests on block 2 run with the same table motions showed a slight change in the pattern. At 23°, Hendron's equation predicted a slower rate of sliding than what was measured and Newmark's prediction was slightly slow, but very close. At a 22° slope the model test results appeared to fall into two groups. Newmark's equation produced a line which followed one group quite closely, while the other group, showing a slower sliding rate, was predicted by Hendron's technique. The curves produced at 21° fall into a wide range with the Hendron line on the slow side of the midrange and Newmark's on the fast side. A severe change takes place when the slope is lowered to 20°. The test curves flatten out dramatically to nearly horizontal, and although the predicted lines follow the initial sliding rates fairly well, the break in slope causes them to separate dramatically. The difference in sliding rates from 21 to 20° is more than was predicted.

Low Frequency Tests

Series N, Amax = 0.076 g, Freq. = 3.67 Hz

The curves recorded from the tests run at the slowest frequencies were impossible to predict using any of the available analytical techniques as can be seen in figures 45 and 46. At slopes less than 26°

the blocks were inconsistent and would occasionally move very, very slowly, or at times would not move at all. The test curves for 26° show very slow, fairly steady downslope movement, but this is very near the estimated angle of sliding friction of 26.4° used in the analysis. The predicted motions using this angle are very fast as can be observed on the graphs. This demonstrates the importance of knowing the "real" angle of friction which should be used in the analyses. At slopes greater than 26°, the sliding rates increased a small amount, but the Newmark and Hendron equations cannot be applied since the slope angle exceeds the sliding friction angle.

Series P, Amax = 0.12 g, Freq. = 3.60 Hz

For test series P the table stroke was increased to 0.18 inch and although the blocks slid at a faster rate, it was not enough to bring the curves substantially closer to the calculated lines. The blocks still slid very slowly at a slope of 26° even though both analytical techniques predicted rapid downslope movement (figs. 47 and 48). At 27 and 28°, the rates of sliding were faster, as expected, but block 1 did not show as great an increase in velocity as block 2. Block 2 moved very rapidly downslope at 28°.

Midrange Frequency Tests

Series V, Amax = 0.194, Freq. = 4.7 Hz

The equations derived by Newmark and by Hendron, et al., provided reasonably good, though conservative, estimates of the displacements measured for these middle frequency tests. As observed from the earlier tests, the prediction again become less accurate and more conservative for flatter slopes. Also, block 2, which is heavier and has a smaller sliding surface than block 1, slides at a faster rate, making the calculated sliding curves fall closer to the measured curves than for block 1.

Block 1 generally slid at a steadier pace than block 2 during these tests. At 26°, the test curves fell into a tight range, and the Newmark and Hendron curves were on the fast side of this range. Lowering the slope to 25° widened the observed range considerably, but all but three of these curves remained fairly tightly grouped. The predicted rates fall well to the left of this group. As might be expected, at 24° the curves are spread out even more. Newmark and Hendron's equations produce lines which are slightly left of the fastest observed curves, but are far from the slowest curves plotted from the model tests (figs. 49 through 51).

The model tests from block 2 at slopes of 26, 25, and 24° consistently resulted in a sharp decrease in the rate of sliding after a fast start. At 26°, this break occurs after the first second, but is not quite as pronounced as for the flatter slopes. Hendron's method produces a line which is very close to the test curves for this case. The tests at 25° all started sliding at about the same rate, which is within the range of Newmark and Hendron's predictions, but the curves flatten out drastically to give very confusing results. At the end of the transducer's measurable range the curves from the sliding block lie way on the slow side of the calculated lines and demonstrate a very slow rate of displacement for the sliding block. This same phenomenon is also repeated for the curves measured at a 24° slope.

CHAPTER 6

CONCLUSIONS AND APPLICATIONS

If applied using the proper physical parameters, the equations developed by Newmark and by Hendron et al., can provide a conservative estimate of the amount of displacement to be expected as a result of dynamic ground motions. However, since these estimates are consistently on the conservative side, if relied on too heavily they may result in expensive overdesign of the slope in question. As demonstrated by the laboratory tests discussed earlier, the estimates become more conservative and less reliable as the potential sliding slope becomes flatter or as the frequency of the ground motion decreases. Therefore, care must be exercised in using either of these methods, and they should not be considered more than rough estimates. The pseudostatic method can be used to compute a factor of safety for the slope, but not to estimate displacements.

An important consideration in any stability analysis is the accurate determination of the physical properties of the materials being analyzed. Frequently, separate methods used to find properties give conflicting results, and engineering judgment must be used to draw conclusions. Although a sliding friction angle of 26.4° was used in this study, it was chosen as a best estimate based on static sliding tests and direct shear tests. The use of direct-shear tests alone would have resulted in unreasonable values for Ø and made the analysis more difficult. In practice, the analysis should probably be carried out using more than one friction angle selected from within the range of values determined in the testing program.

The motions of the shaking table used for the model tests were constantly checked for irregularities using the linear displacement transducer. The checks consistently showed a smooth, harmonic sinusoidal motion, but this kind of motion does not often occur in nature. Before these equations can be used for predicting earthquake or blasting effects, the number and magnitude of critical pulses to be considered must be decided upon. For blasting, this can be done using experimental data from blast tests which are easily obtainable. However, for earthquakes this is not as simple, but involves selecting a design earthquake for the site and then deciding how to apply it to the analysis.

One method for doing this is to select an equivalent number of cycles which will produce the same effect as the entire time-history of the earthquake. This is normally done by applying the maximum acceleration value of the design earthquake to the system several times over a period of time equal to the quake's duration. However, the amplitude of the pulses and the number of cycles may be varied to give a better approximation of the earthquake's effect. This is discussed in limited detail by Newmark (1965).

Although these model tests should be of help in deciding whether any of these methods should be used in designing earth slopes, it should be taken into consideration that they are greatly oversimplified in several ways. The table motions are simple harmonic motions, the slopes are flat and relatively smooth, and the blocks are only in contact on the bottom sliding surface. Tests which more accurately found in nature could be designed as a followup to the tests performed in this investigation. Also, if data from actual earthquake-induced slope displacements become available, they should be checked with the techniques discussed here.

Further tests and more comprehensive analyses would be of great benefit to those interested in applying these techniques. As mentioned earlier, the Hendron and Newmark equations should be used with three or four different sliding friction angles covering a range of reasonable values. Wedge failure could be modeled and tested on the shaking table. Also, wetting the sliding surface and placing clay on the contact would simulate moist gouge-filled faults. It is believed that the results from this study can be a start for many more dynamic rock slope stability testing programs.

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