ELECTROMAGNETIC DIFFRACTION THROUGH A SQUARE SLOT INTO A CAVITY
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## ABSTRACT

This thesis deals with the problem of diffraction of a plane wave through a square slot into a cavity. The cavity is formed by two infinite planes of conducting material, and the plane wave is normally incident. Propagation is in the z-direction and the initial polarization is in the $x$-direction. Fields are broken into $T E_{\text {to }} y$ and $T M_{\text {to }} y$ modes. The problem is then met numerically. Coupled mode solutions are found. Edge boundary conditions are set up as linear equations in which the coefficients are coefficients of the coupled mode solutions. The solutions for these coefficients indicate that the total $\mathrm{TM}_{\text {to }} \mathrm{y}$ solution is negligible at frequencies less than the half-wavelength resonance of the aperture. At frequencies away from the resonances of the cavity, a change from a 20 to 1 to a 10 to 1 ratio between the aperture dimension and the cavity dimension produced little change in the fields. Magnitudes of some resulting fields are presented.

## CHAPTER I

## INTRODUCTION

Suppose the response inside a building to an external electromagnetic pulse (EMP) is desired. Mathematically, such a problem would be very complex. "Maxwell's equations would have to be solved under very unwieldy boundary conditions, a formidable task. The problem this thesis sets out to solve numerically is a simplification of such a problem. The building is replaced with a cavity that is infinite in two directions. Instead of many windows of various shapes and sizes as there would be in a building, this cavity has one window of square shape. Even with such a simplified geometry the problem has not been solved anaIytically. Suzuki (1956) gives an approximate analytical solution to the problem of electromagnetic diffraction through a thin rectangular aperture into free space; however, his results cannot readily be reapplied to the problem at hand because of the nature of his approximations. The approach of this thesis is therefore of a basically numerical nature, search for an analytical solution being waived because of the complexities involved.

The diffraction problem is as follows: The problem seeks a solution to Maxwell's equations on the geometry in Fig. 1 assuming a plane wave impinges normally to a sheet which is infinitely conductive and infinitely extended and in which there is a square slot. The wave then diffracts through the slot into a cavity formed by another infinitely extended, infinitely conductive sheet a distance wrom the sheet with the slot in it. The two sheets are parallel to each other. The z-direction of the coordinate system is set in the direction of propagation. The conducting sheets then lie in constant-z planes. The x-axis and the $y$-axis are set parallel to the sides of the siot. The origin of the coordinate system is set in the middle of the slot. Thus, the points ( $\pm \mathrm{s} / 2, \pm s / 2,0$ ) would represent the vertices of the square slot. $z<0$ implies the free space region from which the plane wave impinges, and. $0<z<W$ implies the cavity region. We denote the former as region 1 and the latter as region 2. The plane $z=0$ we will denote as sheet 1 and the plane $z=W$ as sheet 2 . On sheet 1 the area of the aperture will be called surface $A$, and the area outside $\mathbb{A}$ on sheet I will be called surface S. The plane wave polarization is taken such that the electric field vector is in the $x$-direction.

In chapter 2 the mathematics of the problem is set up. Use of a breakdown of the fields into $T E_{\text {to }} y$ and


Side view of cavity region


Direction of incident electric field

$\sigma=\infty$


View of aperture from free space region

Figure l. Two views of the geometry of the problem.
$T_{t o} y$ modes allows solution of the problem with a set of integral equations for two aperture fields, $E_{x}^{A}$ and $A_{y}^{A}$. Some familiarity with the use of green's functions is assumed as they are used extensively in the setting up of the machinery of the problem.

In chapter 3 the numerical details of the problem are examined. Şuch things as the numerical solution of the integral equations and the removal of singularities from the kernels of these equations are discussed. A numerical solution to the diffraction problem is then presented.

In chapter 4 results are briefly discussed and suggestions for future work are presented. The appendix gives the computer program used in the research. It is based on the method presented in chapter 3 and is included for those who may want to duplicate these results.

## CHAPTER 2

## THEORETICAL REDUCTIONS

Lack of a closed form solution to this problem is unfortunate but not catastrophic. With the aid of a computer it is possible to do many things, in particular, solve many problems, and this problem is no exception. As in many numerical solutions, the sampling theorem and the storage of the computer together will place limitations on the generality of the solution. For the purpose of EMP analysis, this limitation is not severe because the frequency spectrum of the pulse is mostly within the limitation. Outright programming of Maxwell's equations, however, is unpractical, and it will be necessary to cast the problem into a more tractable form.

## Green's Functions <br> For Region One and Two

As in most diffraction problems, the Dirichlet green's functions of the Helmholtz equation for the two regions under the condition that the slot is shorted out are needed.

Let $\nabla^{2}=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}$. In region 1 we have the problem of expressing $G_{1}(x, y, z, \xi, \eta, \zeta)$ where:

$$
\left(\nabla^{2}+k^{2}\right) G_{1}(x, y, z, \xi, \eta, \zeta)=\delta(x-\xi) \delta(y-\eta) \delta(z-\zeta)
$$

with:

$$
-\infty<z \leq 0 ;-\infty<\zeta<0 ;-\infty<x, y, \xi, \eta<\infty ;
$$

$k$ some constant parameter; and with $G_{1}(x, y, 0, \xi, \eta, \zeta)=0$. The solution of this problem is seen to be:

$$
\begin{aligned}
& G_{1}(x, y, z, \xi, \eta, \zeta)=\frac{e^{-i k r+}}{4 \pi r+}-\frac{e^{-i k r-}}{4 \pi r-} ; \operatorname{Im}(k)<0 ; \\
& r+^{2}=(x-\xi)^{2}+(y-\eta)^{2}+(z-\zeta)^{2} ; \text { and } \\
& r^{2}=(x-\xi)^{2}+(y-\eta)^{2}+(z+\zeta)^{2} \quad .
\end{aligned}
$$

The first term of $G_{1}$ takes care of the singular point at $(\xi, \eta, \zeta)$; the second term forces $G_{1}$ to zero on sheet 1 , since at $\mathrm{z}=0 \quad \mathrm{r}+=\mathrm{r}-$.

Similarly in region 2 we have the problem:
$\left(\nabla^{2}+k^{2}\right) G_{2}(x, y, z, \xi, \eta, \zeta)=\delta(x-\xi) \delta(y-\eta) \delta(z-\zeta) ;$
$0 \leq z \leq W$; $0<\zeta<W$; $-\infty<x, y, \xi, \eta<\infty$; and with

$$
G_{2}(x, y, 0, \xi, \eta, \zeta)=G_{2}(x, y, w, \xi, \eta, \zeta)=0 .
$$

Let $z$ be the set of all integers. The solution to this problem is:

$$
\begin{aligned}
& G_{2}(x, y, z, \xi, \eta, \zeta)=\sum_{n \varepsilon z} \frac{e^{-i k r_{n}+}}{4 \pi r_{n}+}-\sum_{n \varepsilon Z} \frac{e^{-i k r_{n}}}{4 \pi r_{n}^{-}} \\
& \left(r_{n}+\right)^{2}=(x-\xi)^{2}+(y-n)^{2}+(z-\zeta+2 n W)^{2} \\
& \left(r_{n}-\right)^{2}=(x-\xi)^{2}+(y-n)^{2}+(z+\zeta+2 n W)^{2}
\end{aligned}
$$

The $n=0$ term of the first sum takes care of the singular point, and the rest of the terms meet boundary conditions on sheet 1 and sheet 2 .

## Useful Identities

We will now use Green's second identity to obtain some useful formulae. Green's second identity is:

$$
\iiint_{\mathrm{V}}\left(\psi \nabla^{2} \phi-\phi \nabla^{2} \psi\right) d V=\iint_{S}\left(\psi \frac{\partial \phi}{\partial \mathrm{n}}-\phi \frac{\partial \psi}{\partial \mathrm{n}}\right) \mathrm{dS}
$$

Let V be region 1 . Let $\phi$ be some solution to the Helmholtz equation in region 1 with boundary conditions on $\phi$ chosen such that $\phi=0$ on surface $S$ of sheet 1 . No stipulation is made on $\phi$ over the aperture surface, $A$. Let $\psi(x, y, z)=$ $G_{1}(x, y, z, \xi, \eta, \zeta)$. Resulting is:

Region 1
$\iiint_{\text {gion }}\left[G_{1}(x, y, z, \xi, \eta, \zeta) \nabla^{2} \phi(x, y, z)-\right.$

$$
\left.\phi(x, y, z) \nabla^{2} G_{1}(x, y, z, \xi, \eta, \zeta)\right] d x d y d z
$$

$=\int_{\text {Sheet }}\left[G_{1}(x, y, z, \xi, \eta, \zeta) \frac{\partial \phi(x, y, z)}{\partial z}-\right.$
Sheet 1 $\left.\phi(x, y, z) \frac{\partial G_{1}(x, y, z, \xi, \eta, \zeta)}{\partial z}\right] d S$
The right-hand side of the equation simplifies because of boundary conditions on $G_{1}$ and on $p:$

$$
\begin{aligned}
& \int_{\text {Sheet }}\left[G_{1}(x, y, z, \xi, \eta, \zeta) \frac{\partial \phi(x, y, z)}{\partial z}-\right. \\
& \left.\qquad \phi(x, y, z) \frac{\partial G_{1}(x, y, z, \xi, \eta, \zeta)}{\partial z}\right] d S \\
& =-\iint_{A} \phi(x, y, 0)\left[\left.\frac{\partial G_{1}(x, y, z, \xi, \eta, \zeta)}{\partial z}\right|_{z=0}\right] d x d y
\end{aligned}
$$

The left-hand side simplifies upon substitutions made from the Helmholtz equation:
$\underset{\text { Region }}{\iiint_{I} G_{1}(x, y, z, \xi, \eta, \zeta) \nabla^{2} \phi(x, y, z)-}$

$$
\begin{aligned}
& \left.\phi(x, y, z) \nabla^{2} G_{1}(x, y, z, \xi, \eta, \zeta)\right] d x d y d z \\
=\iiint\left\{G_{1}(x, y, z, \xi, \eta, \zeta) \quad\right. & \left(-k^{2} \phi(x, y, z)\right)-\phi(x, y, z)[ \\
& -k^{2} G_{1}(x, y, z, \xi, \eta, \zeta)+ \\
& \delta(x-\xi) \delta(y-\eta) \delta(z-\zeta)]\} d x d y d z \\
= & \iiint-\phi(x, y, z)[\delta(x-\xi) \delta(y-\eta) \delta(z-\zeta)] d x d y d z \\
= & -\phi(\xi, \eta, \zeta)
\end{aligned}
$$

The result:

$$
\phi(\xi, \eta, \zeta)=\iint_{A} \phi(x, y, 0)\left[\left.\frac{\partial G_{1}(x, y, z, \xi, \eta, \zeta)}{\partial z}\right|_{z=0}\right] d x d y
$$

Now $\quad \frac{\partial}{\partial z} \frac{e^{-i k r+}}{r+}=\left(-i k-\frac{1}{r+}\right) \frac{z-\zeta_{t}}{r+} \frac{e^{-i k r+}}{r+}$
and $\quad \frac{\partial}{\partial z} \frac{e^{-i k r-}}{r^{-}}=\left(-i k-\frac{1}{r-}\right) \frac{z+\zeta}{r-} \frac{e^{-i k r-}}{r_{-}^{-}}$

Thus

$$
\begin{gathered}
\frac{\partial}{\partial z} G_{1}(x, y, z, \xi, \eta, \zeta)=\frac{1}{4 \pi}\left[\left(-i k-\frac{1}{r+}\right) \frac{z-\zeta}{r+} \frac{e^{-i k r+}}{r+}-\right. \\
\left.\left(-i k-\frac{1}{r-}\right) \frac{z+\zeta}{r-} \frac{e^{-i k r-}}{r-}\right]
\end{gathered}
$$

As noted before, $r+=r-$ at $z=0$, therefore, define $r$ :

$$
r^{2}=(x-\xi)^{2}+(y-\eta)^{2}+\zeta^{2}=r+\left.^{2}\right|_{z=0}=r-\left.^{2}\right|_{z=0}
$$

Thus

$$
\begin{aligned}
{\left.\left[\frac{\partial}{\partial z} G_{1}(x, y, z, \xi, \eta, \zeta)\right]\right|_{z} } & =0 \frac{-2}{4 \pi}\left(-i k-\frac{1}{r}\right) \frac{\zeta}{r} \frac{e^{-i k r}}{r} \\
& =\frac{-2}{4 \pi} \frac{\partial}{\partial \zeta} \frac{e^{-i k r}}{r}
\end{aligned}
$$

Substituting back into our integral identity:

$$
\phi(\xi, \eta, \zeta)=-\iint_{A} \phi(x, y, 0)\left[\frac{\partial}{\partial \zeta} \frac{2}{4 \pi} \frac{e^{-i k r}}{r}\right] d x d y
$$

The differentiation is independent of the integration. Removal yields the final result:

$$
\phi(\xi, \eta, \zeta)=-\frac{\partial}{\partial \zeta} \iint_{A} \phi(x, y, 0) \frac{2}{4 \pi} \frac{e^{-i k r}}{r} d x d y
$$

The corresponding result in region 2 can be obtained by letting $V$ be the volume of region 2 , and setting $\psi(x, y, z)$ equal to $G_{2}(x, y, z, \xi, \eta, \zeta)$. $\phi$ is again a solution to the Helmholtz equation which we require to vanish on surface $S$ of sheet 1. We also require that $\phi$ vanish on sheet 2 . The left-hand side of Green's identity again becomes - $\phi(x, y, z)$. The right-hand side becomes:

$$
\iint\left(\psi \frac{\partial \phi}{\partial n}-\phi \frac{\partial \psi}{\partial n}\right) d S=\iint_{A}-\phi(x, y, 0)\left[-\frac{\partial G_{2}(x, y, z, \xi, \eta, \zeta)}{\partial z}\right] d x d y
$$

The minus sign in the bracketed term arises because the outward normal is in the minus z-direction. As before:

$$
\begin{aligned}
& \frac{\partial}{\partial z} \sum_{n \varepsilon Z} \frac{e^{-i k r_{n}^{+}}}{r_{n}^{+}}=\sum_{n \in Z}\left(-i k-\frac{I}{r_{n}^{+}}\right) \frac{e^{-i k r_{n}^{+}}}{r_{n}^{+}} \frac{z-\zeta+2 n W}{r_{n}^{+}} \\
& \frac{\partial}{\partial z} \sum_{n \in Z} \frac{e^{-i k r_{n}^{-}}}{r_{n}^{-}}=\sum_{n \varepsilon Z}\left(-i k-\frac{1}{r_{n}^{-}}\right) \frac{e^{-i k r_{n}-}}{r_{n}^{-}} \frac{z+\zeta+2 n W}{r_{n}^{-}}
\end{aligned}
$$

At $z=0$ :

$$
\begin{aligned}
& \left(r_{n}+\right)^{2}=(x-\xi)^{2}+(y-n)^{2}+(-\zeta+2 n W)^{2} \\
& \left(r_{n}-\right)^{2}=(x-\xi)^{2}+(y-n)^{2}+(\zeta+2 n W)^{2}
\end{aligned}
$$

Define:

$$
\begin{aligned}
r_{n}^{2} & =(x-\xi)^{2}+(y-\eta)^{2}+(\zeta+2 n W)^{2} \\
& =(x-\xi)^{2}+(y-\eta)^{2}+(-\zeta-2 n W)^{2}=\left.\left(r_{n}-\right)^{2}\right|_{z=0}=\left.\left(r_{-n}+\right)^{2}\right|_{z=0}
\end{aligned}
$$

The summations are carried out over all the integers, therefore, the index $n$ can be replaced with $-n$ in either series without affecting the value of it. Performing such a replacement in the $r_{n}+$ series yields:

$$
\begin{aligned}
\left.\frac{\partial G_{2}}{\partial z}\right|_{z=0}= & \sum_{n \varepsilon Z}\left(-i k-\frac{1}{r_{-n^{+}}}\right) \frac{e^{-i k r_{-n^{+}}}}{4 \pi r_{-n^{+}}} \frac{-\zeta-2 n W}{r_{-n^{+}}} \\
& -\sum_{n \varepsilon Z}\left(-i k-\frac{1}{r_{n}^{-}}\right) \frac{e^{-i k r_{n}^{-}}}{4 \pi r_{n}^{-}} \frac{\zeta+2 n W}{r_{n}^{-}}
\end{aligned}
$$

Substituting $r_{n}$ and factoring a minus sign from the first series:

$$
\begin{aligned}
\left.\frac{\partial G_{2}}{\partial z}\right|_{z=0} & =\frac{-2}{4 \pi} \sum_{n \varepsilon Z}\left(-i k-\frac{1}{r_{n}}\right) \frac{e^{-i k r_{n}}}{r_{n}} \frac{\zeta+2 n W}{r_{n}} \\
& =\frac{-2}{4 \pi} \frac{\partial}{\partial \zeta} \sum_{n \varepsilon Z} \frac{e^{-i k r_{n}}}{r_{n}}
\end{aligned}
$$

Green's second identity becomes:

$$
\begin{aligned}
\phi(\xi, \eta, \zeta) & =\iint_{A} \phi(x, y, 0)\left[-\left.\frac{\partial G_{2}}{\partial z}\right|_{z=0}\right] d x d y \\
& =\frac{2}{4 \pi} \frac{\partial}{\partial \zeta} \iint_{A} \phi(x, y, 0)\left[\sum_{n \varepsilon Z} \frac{e^{-i k r_{n}}}{r_{n}}\right] d x d y
\end{aligned}
$$

Definition of $\hat{g}_{1}$ and $\hat{g}_{2}$, and their restrictions.

Let

$$
g_{1}(x, y, z, \xi, \eta)=\frac{e^{-i k r}}{2 \pi r} ; \text { where } r^{2}=(x-\xi)^{2}+(y-\eta)^{2}+z^{2}
$$

and

$$
\hat{g}_{2}(x, y, z, \xi, n)=\frac{e^{-i k r_{n}}}{2 \pi r_{n}} ; r_{n}^{2}=(x-\xi)^{2}+(y-n)^{2}+(z+2 n W)^{2}
$$

Note that $r=r_{0}$. The two identities become:

$$
\begin{aligned}
& \phi(x, y, z)=-\frac{\partial}{\partial z} \iint_{A} \phi(x, y, 0) \hat{g}_{1}(x, y, z, \xi, \eta) d \xi d \eta \\
& \phi(x, y, z)=\frac{\partial}{\partial z} \iint_{A} \phi(x, y, 0) \hat{g}_{2}(x, y, z, \xi, \eta) d \xi d \eta
\end{aligned}
$$

the former holding in region $l$ and the latter holding in region 2. Frequent use will be made of them in what follows. The functions $\hat{g}_{1}(x, y, 0, \xi, \eta)$ and $\hat{g}_{2}(x, y, 0, \xi, \eta)$ will also find frequent use. Define:

$$
\begin{aligned}
& g_{1}(x, y, \xi, \eta)=\hat{g}_{1}(x, y, 0, \xi, \eta) \\
& g_{2}(x, y, \xi, \eta)=\hat{g}_{2}(x, y, 0, \xi, \eta)
\end{aligned}
$$

Note that:

$$
g_{1}+g_{2}=\frac{e^{-i k r}}{2 \pi r}+\sum_{n \varepsilon Z} \frac{e^{-i k r_{n}}}{2 \pi r_{n}}
$$

At $z=0$, we have $r_{n}=r_{-n}$ which makes it possible to double over the second series. Let I be the set of nonnegative integers. We find:

$$
\begin{aligned}
g_{1}+g_{2} & =\frac{e^{-i k r}}{2 \pi r}+\frac{e^{-i k r_{0}}}{2 \pi r_{0}}+2 \sum_{n \varepsilon I-\{0\}} \frac{e^{-i k r_{n}}}{2 \pi r_{n}} \\
& =2 \sum_{n \varepsilon I} \frac{e^{-i k r_{n}}}{2 \pi r_{n}} ; \quad r_{n}^{2}=(x-\xi)^{2}+(y-n)^{2}+(2 n W)^{2}
\end{aligned}
$$

To reduce the tedium of writing out the independent variables and to avoid the possible confusion that might arise from so doing, a set of conventions is adopted for working with $\hat{g}_{1,2}$ and with $g_{1,2}$. Let $C$ represent some arbitrary three-dimensional function, and let $Y$ represent some arbitrary two-dimensional function. Also let $\mathrm{B}^{\mathrm{A}}$ be an arbitrary two-dimensional function non-trivial only in the region $-s / 2 \leq x, y \leq s / 2$. Superscript $A$ will always
imply triviality outside this region. Then, when we write:

$$
\mathrm{c}=\iint \mathrm{B}^{\mathrm{A}} \hat{\mathrm{~g}}_{1,2} \mathrm{dA}
$$

we mean:

$$
C(x, y, z)=\iint_{A} B^{A}(\xi, \eta) \hat{g}_{I, 2}(x, y, z, \xi, \eta) d \xi d \eta
$$

Similarly, when we write:

$$
Y=\iint B^{A} g_{I, 2} d A
$$

we mean:

$$
Y(x, y)=\iint_{A} B^{A}(\xi, \eta) g_{1,2}(x, y, \xi, \eta) d \xi d \eta
$$

Thus a phrase like $\left(\partial^{2} / \partial y^{2}+k^{2}\right) \iint B^{A} g_{1,2} d A$ carries meaning because of our agreement on the use of the independent variable $y$.

## Maxwell's Equations

Consider Maxwell's equations in the frequency domain. In a source-free homogeneous region we have:

$$
\nabla \times \overline{\mathrm{E}}=-2 \overline{\mathrm{H}} \quad \nabla \times \overline{\mathrm{H}}=\hat{\mathrm{Y}} \overline{\mathrm{E}} \quad \nabla \cdot \overline{\mathrm{E}}=0 \quad \nabla \cdot \overline{\mathrm{H}}=0
$$

The notation is as in Harrington (1961) with $\hat{z}=i \omega \mu$ and $\hat{y}=i \omega \varepsilon$. These equations imply the two vector wave equations:

$$
\left(\nabla^{2}+\mathrm{k}^{2}\right) \overline{\mathrm{E}}=0 ;\left(\nabla^{2}+\mathrm{k}^{2}\right) \overline{\mathrm{H}}=0 ; \text { where } \mathrm{k}^{2}=-\hat{\mathrm{z}} \hat{\mathrm{y}} .
$$

The derivation is well-known; the important result is that the rectangular components of the $\bar{E}$ and $\bar{H}$ fields must satisfy the Helmholtz equation.

Because of the divergenceless nature of the $\bar{E}$ and $\bar{H}$ fields, it is possible to make use of vector potentials. In our problem it is convenient to let:

$$
\begin{aligned}
& \bar{E}=-\nabla \times\left(F_{y} \bar{Y}\right)+\frac{1}{\hat{Y}} \nabla \times \nabla \times\left(A_{y} \bar{y}\right) \\
& \bar{H}=\nabla \times\left(A_{y} \bar{y}\right)+\frac{1}{\hat{z}} \nabla \times \nabla \times\left(F_{y} \bar{y}\right),
\end{aligned}
$$

where $\bar{Y}$ is the unit vector in the $y$-direction. This is equivalent to considering $\bar{E}$ and $\bar{H}$ as the superposition of $T E_{\text {to }} y$ and $\mathrm{TM}_{\text {to }} \mathrm{y}$ fields. We shall require that $\mathrm{A}_{\mathrm{y}}$ and $F_{y}$ satisfy the Helmholtz equation. Here are the modal decompositions:

$$
\begin{aligned}
& \text { TM }_{\text {to }} Y \quad E_{X}=\frac{1}{\hat{y}} \frac{\partial^{2} A_{y}}{\partial x \partial y} \quad H_{x}=\frac{\partial A}{\partial z} \\
& E_{Y}=\frac{1}{\hat{X}}\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) A_{Y} \quad H_{Y}=0 \\
& E_{z}=\frac{1}{\hat{Y}} \frac{\partial^{2} A}{\partial z \partial y} \\
& H_{z}=-\frac{\partial A y}{\partial x} \\
& T E_{\text {to }} y \quad E_{x}=-\frac{\partial F y}{\partial z} \\
& H_{x}=\frac{1}{\frac{1}{2}} \frac{\partial^{2} F y}{\partial x \partial y} \\
& E_{y}=0 \\
& H_{y}=\frac{1}{\hat{Z}}\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) F_{y} \\
& E_{z}=\frac{\partial F_{y}}{\partial x} \\
& H_{z}=\frac{1}{\hat{z}} \frac{\partial^{2} F y}{\partial z \hat{\partial} y}
\end{aligned}
$$

These decompositions are also shown in Fig. 2 for convenience.

## and $\frac{\text { Boundary Conditions }}{\text { Continuity Requirements }}$

The boundary conditions on $A_{y}$ and $F_{y}$ are those dictated by the boundary conditions on $\bar{E}$ and $\bar{H}$. The boundary conditions on $\bar{E}$ and $\bar{H}$ for the problem are that the tangential components of $\bar{E}$, namely $E_{x}$ and $E_{y}$, vanish on

$$
\begin{array}{rlrl}
T M_{\text {to }} y & E_{x} & =\frac{1}{\hat{y}} \frac{\partial^{2} A y}{\partial x \partial y} & H_{x}=\frac{\partial A}{\partial z} \\
E_{y} & \left.=\frac{1}{\hat{Y}} \frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) A_{y} & H_{y}=0 \\
E_{z} & =\frac{1}{\hat{Y}} \frac{\partial^{2} A_{y}}{\partial z \partial y} & H_{z}=-\frac{\partial A}{\partial x}
\end{array}
$$

$T E_{\text {to }} y$

$$
\begin{array}{ll}
E_{x}=-\frac{\partial F_{y}}{\partial z} & H_{x}=\frac{1}{2} \frac{\partial^{2} F_{y}}{\partial x \partial y} \\
E_{y}=0 & H_{y}=\frac{1}{\delta}\left(\frac{\partial^{2}}{\partial y^{z}}+k^{2}\right) F_{y} \\
E_{z}=\frac{\partial F_{y}}{\partial x} & H_{z}=\frac{1}{\hat{z}} \frac{\partial^{2} F y}{\partial z \partial y}
\end{array}
$$

Fig. 2. Reproduction of the modal decomposition as given on page 13 of text.
area $S$ of sheet 1 and on sheet 2 . Also required is that the normal component of $\overline{\mathrm{H}}$, namely $\mathrm{H}_{z}$, be zero on the same regions.

The fields must be continuous throughout the two regions, and they must have continuous first derivatives. There is also a continuity requirement on $E_{x}$ and $E_{y}$ on sheet 1 . $E_{x}$ should be continuous in $y$ on sheet $I$, and $E_{y}$ should be continuous in $x$ on sheet $1_{0} \because$ This fact implies the conditions:

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{X}}(\mathrm{x}, \pm \mathrm{s} / 2)=0 ; \text { for all } \mathrm{x} \\
& \mathrm{E}_{\mathrm{Y}}( \pm s / 2, y)=0 ; \text { for all } y
\end{aligned}
$$

In diffraction problems one frequently assumes one field in the region of incidence and another in the shadow region. Continuity requires a matching of these two solum tions over the aperture region for all components of the fields.

Finally, we shall require that $\mathrm{E}_{\mathrm{X}}(\mathrm{x}, \mathrm{y}, 0)$, the $\mathrm{E}_{\mathrm{X}}$ field on the aperture, be symmetric in both $x$ and $y$. The geometry of the problem is completely symmetric in $y$, therefore, the condition of $y$ symmetry. The condition of symmetry in $x$ arises because, physically, $E_{x}$ as a function of $x$ should be independent of the initial direction of polarization.

## Resonance Solution

A plane wave incident on sheet 1 will be reflected from all but surface A. Without loss of generality, then, we can assume that the incident field is :

$$
\left.\begin{array}{l}
E_{x}^{i \cdot}=E_{0}\left(e^{-i k z}-e^{i k z}\right) \\
H_{y}^{i}=\hat{n} E_{0}\left(e^{-i k z}+e^{i k z}\right)
\end{array}\right) \quad \text { region } l
$$

where $\hat{f}$ is the intrinsic impedance of the medium. If we extend this solution into region 2 , we see that all boundary conditions and continuity requirements are met, if and only if, $k W$ is an integer multiple of $\pi$. Under this condition $g_{2}, \hat{g}_{2}$, and $G_{2}$ fail to converge, and it is only under this condition that they fail to converge. By the uniqueness theorem the solution :

$$
\left.\begin{array}{l}
E_{x}=E_{o}\left(e^{-i k z}-e^{i k z}\right) \\
H_{y}=\hat{\eta} E_{o}\left(e^{-i k z}+e^{i k z}\right)
\end{array}\right)
$$

both regions
is thus the solution when the cavity becomes resonant.

## Coupled Mode Solution

Assume that the wavenumber, $k$, is such that the cavity is not resonant. Under these conditions $g_{2}, \hat{g}_{2}$, and $G_{2}$ all converge. Moreover, the resonant solution no longer meets the boundary conditions on sheet 2. The field in region 2 cannot be trivial for $H_{y}$ would then suffer a discontinuity across the aperture. More complex solutions must be looked for.

## Let:

$$
\begin{aligned}
& F_{Y^{1}}=+\iint E_{x}^{A} \hat{g}_{1} d A+E_{o}\left(e^{-i k z}+e^{i k z}\right) / i k \\
& F_{y^{2}}=-\iint E_{x}^{A} \hat{g}_{2} d A
\end{aligned}
$$

For the moment, let $A_{Y l}=A_{Y 2}=0$.
Using our identities we see that:

$$
\begin{aligned}
E_{x}(x, y, 0) & =\lim _{z}-\frac{\partial}{\partial z} \iint E_{x}^{A} \hat{g}_{1} d A=\lim _{z} \Re_{0} \frac{\partial}{\partial z} \iint E_{x}^{A} \hat{g}_{2} d A \\
& =E_{x}^{A}(x, y)
\end{aligned}
$$

Thus, $E_{x}^{A}$ is appropiately named. A similar use of the idemtities shows that $H_{z}$ is continuous across the aperture and has the value:

$$
H_{z}(x, y, 0)=-\frac{1}{\hat{z}} \frac{\partial}{\partial y} E_{x}^{A}(x, y)
$$

Thus $E_{x}$ and $H_{z}$ meet continuity requirements and boundary conditions on $S$. They are solutions to the Helmholtz equation and meet boundary conditions on sheet 2 because $\frac{\partial \hat{g}_{2}}{\partial z}$ meets these requirements. Continuity of $H_{Y}$ on $A$ implies:

$$
\begin{aligned}
& \lim _{z \uparrow 0} \frac{1}{\hat{z}}\left(\frac{\partial^{2}}{\partial y^{z}}+k^{2}\right) F_{y I}=\lim _{z \downarrow 0} \frac{1}{\hat{z}}\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) F_{y 2} \\
& \begin{aligned}
\lim _{z \uparrow 0}\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint E_{x}^{A} \hat{g}_{1} d A & -i k E_{0}\left(e^{-i k z}+e^{i k z}\right) \\
& =\lim _{z \downarrow 0}\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right)-\iint E_{x}^{A} g_{2} d A
\end{aligned}
\end{aligned}
$$

Now

$$
\lim _{z \uparrow 0} \iint E_{x}^{A} \hat{g}_{1} d A=\iint E_{x}^{A} g_{1} d A
$$

and,

$$
\lim _{\mathrm{z} \uparrow 0} \iint \mathrm{E}_{\mathrm{x}}^{\mathrm{A}} \hat{g}_{2} \mathrm{dA}=\iint \mathrm{E}_{\mathrm{x}}^{\mathrm{A}} \mathrm{~g}_{2} \mathrm{dA}
$$

Thus the equation for continuity of $H_{y}$ becomes:

$$
\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint E_{x}^{A}\left(g_{1}+g_{2}\right) d A=2 i k E_{o} \quad(o n A)
$$

Throughout $E_{x}^{A}$ has been some unknown field of the aperture. The above equation is the first equation that allows us to solve for $\mathrm{E}_{\mathrm{X}}^{\mathrm{A}}$. 'The above equation can be written:

$$
\iint E_{x}^{A}\left(g_{1}+g_{2}\right) d A=\frac{-2 E}{i k}+A(x) \cos k y+B(x) \text { sink }
$$

Symmetry of $E_{x}(x, y, 0)=E_{x}^{A}$ implies that the $B(x)$ inky term must be zero. We now check the continuity of $E_{z}$ and $H_{x}$ :

$$
\begin{align*}
\lim _{z \uparrow 0} H_{x 1}-\lim _{z \downarrow 0} H_{x 2} & =\frac{1}{\hat{z}} \frac{\partial^{2}}{\partial x \partial y} \iint E_{x}^{A}\left(g_{1}+g_{2}\right) d A \\
& =\frac{1}{\hat{z}} \frac{\partial^{2}}{\partial x \partial y}\left[\frac{-2}{i k} E_{0}+A(x) \cos (k y)\right] \\
& \left.=\frac{-1}{\hat{z}} \frac{\partial A(x)}{\partial x} k \sin (k y) \quad \text { (on } A\right) \\
\lim _{z \uparrow 0} E_{z 1}-\lim _{z \nmid 0} E_{z 2} & =\frac{\partial}{\partial x} \iint E_{x}^{A}\left(g_{1}+g_{2}\right) d A \\
& =\frac{\partial A(x)}{\partial x} \cos (k y) \tag{onA}
\end{align*}
$$

If we let $A(x)=0$ or a constant we would have met all continuity requirements. The only unmatched requirement is the requirement on $E_{x}$ along the $y= \pm s / 2$ edges of the aperture. It would be fortuitous, indeed, if inversion of the integral equation for $E_{X}^{A}$ yielded a solution that met these conditions; and, in fact, such a solution is not
obtained. Thus, the edge requirements make necessary the addition of $\mathrm{TM}_{\text {to }} \mathrm{y}$ modes.

## Let:

$$
A_{y l}=-\hat{y} \frac{\partial}{\partial z} \iint A_{y}^{A} \hat{g}_{1} d A ; \quad A_{y 2}=\hat{y} \frac{\partial}{\partial z} \iint A_{y}^{A} \hat{g}_{2} d A
$$

Note that:

$$
\lim _{z \uparrow 0} A_{Y I}=\hat{y} A_{y}^{A}=\lim _{z \downarrow 0} A_{Y}
$$

Since $E_{y}(x, y, 0)^{\circ}=\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) A_{y}^{A}$, it meets the boundary conditions on $S$. On sheet $1, E_{x}$ and $H_{z}$ are non-trivial on the aperture only, for similar reasons. $E_{y}, E_{x}$, and $H_{z}$ are all continuous across the aperture.

$$
E_{x}(x, y, 0)=\frac{\partial^{2}}{\partial x \partial y} A_{y}^{A} ; \quad H_{z}(x, y, 0)=-\hat{y} \frac{\partial}{\partial x} A_{y}^{A}
$$

To analyze continuity of $E_{z}$ and $H_{x}$ we first note:

$$
\begin{aligned}
& \lim _{z \uparrow 0} \frac{\partial^{2}}{\partial z^{2}} \iint A_{y}^{A} \hat{g}_{1} d A=-\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A} g_{1} d A \\
& \lim _{z \downarrow 0} \frac{\partial^{2}}{\partial z^{2}} \iint A_{y}^{A} \hat{g}_{2} d A=-\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A} g_{2} d A
\end{aligned}
$$

Then,

$$
\begin{aligned}
& \lim _{z \uparrow 0} E_{z 1}-\lim _{z \downarrow 0} E_{z 2}=\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \frac{\partial}{\partial y} \iint A_{y}^{A}\left(g_{1}+g_{2}\right) d A \\
& \lim _{z \uparrow 0} H_{x 1}-\lim _{z \downarrow 0} H_{x 2}=\hat{y}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A}\left(g_{1}+g_{2}\right) d A
\end{aligned}
$$

The above four equations are valid only on $A$.

Intermode Coupling to Match
Continuity Requirements
Remembering:

$$
\left.\iint E_{x}^{A}\left(g_{1}+g_{2}\right) d A=\frac{-2 E_{0}}{i k}+A(x) \cos (k y) \quad \text { (on } A\right)
$$

we can combine the two mode solutions to find a more general solution. $H_{X}$ and $E_{z}$ continuity conditions imply: $\mathrm{H}_{\mathrm{x}}$ :

$$
\hat{y}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A}\left(g_{1}+g_{2}\right) d A=\frac{1}{\hat{z}} \frac{\partial A(x)}{\partial x} k \sin (k y)
$$

$\mathrm{E}_{\mathrm{z}}:$

$$
\frac{\partial}{\partial y}\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A}\left(g_{1}+g_{2}\right) d A=-\frac{\partial A(x)}{\partial x} \cos (k y)
$$

Both of the above are valid on $A$. Noting $\hat{y}^{2}=-k^{2}$, we have that the above two equations are satisfied if:

$$
\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) \iint A_{y}^{A}\left(g_{1}+g_{2}\right) d A=-\frac{\partial A(x)}{\partial x} \frac{\sin (k y)}{k}
$$

Equivalently:

$$
\begin{aligned}
& \iint A_{Y}^{A}\left(g_{1}+g_{2}\right) d A=- \iint_{A} \frac{\partial A(\xi)}{\partial \xi} \frac{\sin (k \eta)}{k} \frac{i}{4} H_{0}(k r) d \xi d \eta+ \\
& h(x, y)
\end{aligned}
$$

where $r^{2}=(x-\xi)^{2}+(y-\eta)^{2}, h(x, y)$ is any solution to the homogeneous two-dimensional Helmholtz equation, and $H_{0}$ stands for the zeroth order Hankel function of the second kind (Stakgold 1968, section 7.12 in vol. II). A purely theoretical solution would be to try to find the proper $A(x)$ and homogeneous solutions so that $E_{x}$ and $E_{y}$ meet the boundary conditions on A at the appropriate edges. This is not our path; instead, a numerical solution is presented.

## CHAPTER 3

## NUMERICAL REDUCTIONS

In Chapter 2 solution of Maxwell's equations in the regions of the diffraction problem was reduced to the solution of some integral equations for $E_{x}^{A}$ and $A_{y}^{A} \cdot E_{x}^{A}$ and $A_{Y}^{A}$ were coupled through boundary conditions on $A$ and continuity requirements across A .

A key problem to the solution of these integral equations is the following: Given $f(x, y)$ on $A$, find $h^{A}(x, y)$ with:

$$
\begin{aligned}
& \iint h^{A}\left(g_{1}+g_{2}\right) d A=f(x, y) \text {; where } \\
& g_{1}+g_{2}=\sum_{n \varepsilon I} \frac{e^{-i k r} n}{\pi r_{n}}, \text { and } r_{n}^{2}=(x-\xi)+(y-n)+(2 n W) .
\end{aligned}
$$

Assuming that $h^{A}(x, y)$ has some degree of uniformity, we can approximate $h^{A}(x, y)$ with two-dimensional pulse functions. That is, divide $A$ into a grid of squares with $h^{A}$ constant over each square. Now, if $f(x, y)$ and the integral are sampled at the center of each of these grid squares, the integral equation breaks down into a set of solvable linear equations. Knowing the integral of the kernel over each of the squares for all of the sampling points makes it possible for the linear equations to be inverted by matrix methods.

## Kernel Evaluation

- The integral of the kernel over a grid square is approximated by the value of the kernel at the centerpoint of the grid square with the exception of the grid square that is sampled. At the sampled grid square the integral of the kernel would go singular if this method were applied because $r_{0}=0$ and the term in $r_{0}$ would, consequently, blow up. On this square the integral is approximated as follows:

$$
I=\int_{\frac{-D S N}{2}}^{\frac{D S N}{2}} \int_{\frac{-D S N}{2}}^{\frac{D S N}{2}} \frac{e^{-i k r_{0}}}{r_{0}} d x d y \simeq \frac{4}{\pi} \int_{0}^{2 \pi} \int_{0}^{D S N / 2} \frac{e^{-i k r_{0}}}{r_{0}} r d r d \theta
$$

where DSN is the length of the grid square, $r_{0}^{2}=r^{2}=x^{2}+y^{2}$, and the factor of $4 / \pi$ arises from the ratio of the areas of the square and the circle. Thus:

$$
I=\left.8 i \frac{e^{-i k r}}{k} 0\right|_{0} ^{D S N / 2}
$$

This term is finite. The integral of the rest of the terms of the kernel for this grid square are evaluated by the centerpoint method.

The kernel series converges rather slowly. Also, when $n$ is large the $(x-\xi)$ term and $(y-n)$ term become insignificant. For these reasons, the value of

$$
\sum_{N G I}^{\infty} \frac{e^{-i k r} n}{r_{n}} ; r_{n}=2 n W
$$

is evaluated separately, NGI,being large, and is then added
onto:

$$
\sum_{n=0}^{N G I-1} \frac{e^{-i k r} n}{r_{n}} ; \quad r_{n}^{2}=(x-\xi)^{2}+(y-n)^{2}+(2 n W)^{2}
$$

The savings in computational time more than make up for the small loss of accuracy when this technique is used.

$$
\text { Solution for } \mathrm{E}_{\mathrm{x}}^{\mathrm{A}} \text { and } \mathrm{A}_{\mathrm{A}}^{\mathrm{A}}
$$

The integral equations can now be inverted, and a numerical solution can be attempted. Let $\mathrm{E}_{\mathrm{xp}}^{\mathrm{A}}$, " the particular $T E_{\text {to }} y$ solution", be the solution to:

$$
\begin{equation*}
\iint E_{\mathrm{xp}}^{\mathrm{A}}\left(g_{1}+g_{2}\right) d A=\frac{-2 \mathrm{E}_{\mathrm{o}}}{i \mathrm{k}} \tag{onA}
\end{equation*}
$$

As noted before, $\mathrm{E}_{\mathrm{xp}}^{\mathrm{A}}$ does not meet the boundary conditions at the edge of the slot. Suppose we found $\mathrm{E}_{\mathrm{xh}}^{\mathrm{A}}$, "homogeneous $\mathrm{TE}_{\text {to }} \mathrm{y}$ solution", the solution to:

$$
\begin{equation*}
\iint E_{x h}^{A}\left(g_{1}+g_{2}\right) d A=\cos (k y) \tag{onA}
\end{equation*}
$$

Since any constant factor times this meets all the boundary conditions and continuity requirements, excepting, of course, the jump requirement in $H_{y}$ which is met by $E_{x p}^{A}$, we might try to find $B$ such that:

$$
E_{x p}^{A}+B E_{x h}^{A}
$$

meets the edge conditions. This approach is the classical diffraction approach when working with a single mode solution, e.g., see Suzuki (1956). For this problem, however, a proper $B$ cannot be found to meet all the boundary
conditions. Let $A_{y h}^{A}$, "homogeneous $T M_{\text {to }}^{y}$ solution", be the solution to:

$$
\begin{equation*}
\iint A_{y h}^{A}\left(g_{1}+g_{2}\right) d A=x \sin (k y) \tag{onA}
\end{equation*}
$$

Note:

$$
\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) x \sin (k y)=0
$$

Thus any multiple of $A_{y h}^{A}$ meets all the boundary and continuity requirements excepting the jump in $H_{Y}$. Note also that $A_{y h}^{A}$ is made odd so that $\left(\partial^{2} / \partial x \partial y\right) A_{y h}^{A}$ will be even, and therefore meet symmetry requirements on $E_{x}$. We are now tempted to try to find $B$ and $C$ such that:

$$
E_{x}(x, y, 0)=E_{x p}^{A}+B E_{x h}^{A}+C \frac{\partial^{2}}{\partial x \partial y} A_{y h}^{A}
$$

meets the $E_{x}$ edge conditions, and

$$
E_{y}(x, y, 0)=\left(\frac{\partial^{2}}{\partial y^{2}}+k^{2}\right) A_{y}^{A}
$$

meets the $E_{y}$ edge conditions. This solution will not work in general again, however, through all these blind alleys we are leading up to a method of solution.

Suppose that there are $2 \mathrm{n} \times 2 \mathrm{n}$ grid squares. Symmetry will reduce the $E_{x}$ edge conditions to $n$ equations and the $\mathrm{E}_{\mathrm{y}}$ edge conditions to n more equations. We thus have 2 n equations to meet with only three solutions. If n is not trivially small, then we need more solutions. Solutions with coupled continuity across A provide a source of these needed solutions.

Let $E_{x k_{x}}^{A}$ be the solution to:

$$
\begin{equation*}
\iint E_{x k_{x}}^{A}\left(g_{1}+g_{2}\right) d A=\cos \left(k_{x} x\right) \cos (k y) \tag{onA}
\end{equation*}
$$

and let $A_{y}^{A} k_{x}$ be the solution to:

$$
\iint A_{y}^{A} k_{x}\left(g_{1}+g_{2}\right) d A=\iint_{A} \frac{k_{x}}{k} \sin \left(k_{x} \xi\right) \sin (k \eta) \frac{i}{4} H_{O}(k r) d A
$$

where, $r^{2}=(x-\xi)^{2}+(y-\eta)^{2}$, and $d A=d \xi$ d $\eta$. Together $\left(E_{x}^{A} k_{x}, A_{y}^{A} k_{x}\right)$, " a coupled solution" , meet continuity and boundary conditions, excepting the jump in $H_{y}$. The solutions have been chosen so that $\mathrm{E}_{\mathrm{x}}(\mathrm{x}, \mathrm{y}, 0)$ is symmetric. The number of solutions obtained in this manner is unlimited because of the ability to change $k_{x}$ and obtain a new solution. On a $2 n \times 2 n$ grid we need $2 n-2$ coupled solutions. Suppose that $2 \mathrm{n}=6$. Solution to the diffraction goes as follows: Define $4=2 n-2$ values of $k_{x} ; k_{x 1}, k_{x 2}, k_{x 3}, k_{x 4}$. Find associated $A_{y}^{A} k_{x j}$ and $E_{x k_{x j}}^{A} \quad$ solutions. Also, find $A_{y h}^{A}, E_{x h}^{A}$, and $E_{x p}^{A}$. Then:

$$
\begin{array}{r}
B E_{x h}^{A}+C \frac{\partial^{2}}{\partial x \partial y} A_{y h}^{A}+\sum_{j=1}^{4} D_{j}\left(E_{x k_{x j}}^{A}+{\left.\frac{\partial^{2}}{\partial x \partial y^{A}} A^{A} k_{x j}\right)=-E_{x p}^{A}}_{(\text {at } y=s / 2)}\right.
\end{array}
$$

$B(0)+C A_{y h}^{A}+\sum_{j=1}^{4} D_{j} A_{y k_{x j}^{A}}=0$

$$
\text { ( at } x=s / 2)
$$

represent six equations with six unknowns. Note that we
have set $E_{y}$ to zero at $x=s / 2$ by setting the total $A_{y}$ field to zero there. This procedure eliminates any numerical instability in taking $\left(\partial^{2} / \partial y^{2}+k^{2}\right) A_{y}$. The unknown coefficients can be solved for by using matrix routines. The result is a set of fields that meet all boundary conditions and continuity requirements, and hence, an approximation of the actual solution of the diffraction problem.

Removal of the singularity of $H_{0}(k r)$ The integral:

$$
\iint_{A} \frac{i}{4} H_{o}(k r) \sin \left(k_{x} \xi\right) \sin (k \eta) d \xi d \eta
$$

is performed by a Riemann sum. Center point values of $\sin \left(k_{x} \xi\right) \sin (k \eta)$ were used as were all but the sampled grid square values of $H_{o}(k r)$. The excepted value would be singular on the sampled grid if such a technique were used. The desired value is found in a manner similar to the removal of the singularity of $\exp \left(-i k r_{0}\right) / r_{0}:$

$$
\begin{aligned}
\iint_{A} \frac{i}{4} H_{0}(k r) d x d y & \simeq \frac{4}{\pi} \int_{0}^{2 \pi} \int_{0}^{D S N / 2} \frac{i}{4} H_{0}(k r) r d r d \\
& =\frac{2 i}{k^{2}}\left[\left.x H_{1}(x)\right|_{0} ^{D S N / 2}\right]
\end{aligned}
$$

where $r^{2}=x^{2}+y^{2}$, and $H_{1}$ is the first order Hankel function of the second kind. The diffraction problem is now suitable for programming.

## CHAPTER 4

## RESULTS AND FUTURE WORK

Results have recently been obtained using the method discussed in chapter 3. Most significant was that the magnitude of the effect of the total $\mathrm{TM}_{\text {to }} \mathrm{y}$ field was three or four orders less than the magnitude of the effect of the total $T E_{\text {to }}$ field on the resultant $E_{x}$ aperture field. Also, the coefficient of the homogeneous ${ }^{7}{ }^{\prime}{ }_{t o n} y$ field, $A_{y h}^{A}$, was essentially zero. Based on these results we might conclude that a pure $\mathrm{TE}_{\text {to }} \mathrm{y}$ solution would yield valid results at lower frequencies if the edge conditions could be met.

Results are shown in Figs. 4-11. At low frequencies the magnitude of the electric field goes linearly with $k$. Near the value of $k$ where the slot length is a half-wavelength, however, the fields increase in a resonance effect; see Fig. 12. The effect of changing $W$ seems small away from cavity resonance; see Figs. 9 and 10.

Suggested for future work is a time domain analysis of the EMP problem. This task will require a fairly large computer run as far as computational time is concerned. The program runs approximately 40 decimal seconds for one value of $k$ with a 12 by 12 grid over the aperture, and it requires 60 K of core. A recommendation: in such an analysis $k$ values


Fig. 3. The sampling points of the program.

Figures 4 through ll: Calculated fields for various values of $k$ and $W(k=.01-3.0 ; W=10$ or 20 )

The numbers in the arrays, Figures 4 through 11 , represent the magnitudes of their respective fields.. The numbers are arrayed to give the effect of a two-dimensional plot. They represent values taken over the grid squares of the fourth quadrant. These sampling points are indicated in Fig。3. Thus, reading down an array means reading in the minus-y direction, and reading across from left to right means reading in the direction of increasing $x$. The origin is in the upper left corner of the array.

It is important to note that the values of the $y$-component of the vector A are magnified by a factor of 100,000. The magnitude of the incident electric field is taken to be unity, and the initial direction of this field is in the $x$ - direction. Both electric fields are symmetric in both $x$ and $y$, while the $y$-component of $A$ is antisymmetric in the two variables.

| Total Ay Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . 0000 | . 0.000 | . 0000 | .0000 | . 0000 | .0000 |
| .0000 | . 0000 | . 0000 | .0000 | . 0000 | .0000 |
| . 0000 | .0000 | .0000 | .0000 | . 0000 | .0000 |
| . 0000 | .0000 | . 0000 | .0000 | . 0000 | . 0000 |
| . 0000 | . 0000 | .0000 | . 0000 | . 0000 | . 0000 |
| .0000 | .0000 | .0000 | .0000 | . 0000 | .0000 |
|  | 4 |  |  |  |  |
| $\mathrm{E}_{\mathrm{x}}$ Component of $\mathrm{F}_{\mathrm{y}}$ Field |  |  |  |  |  |
| . 0032 | . 0032 | . 0032 | . 0033 | . 00036 | . 0041 |
| . 0030 | . 0030 | . 0031 | . 0032 | . 0034 | . 0039 |
| . 0027 | .0027 | . 0027 | . 0028 | . 0030 | . 0034 |
| . 0021 | . 0021 | .0022 | . 0022 | . 0023 | . 0027 |
| . 0013 | . 0013 | .0013 | .0013 | . 0014 | .0016 |
| .0000 | .0000 | . 0000 | . 0000 | . 0000 | . 0000 |
| Total Ex Field |  |  |  |  |  |
| .0032 | . 0032 | . 0032 | . 0033 | . 0036 | . 0041 |
| .0030 | . 0030 | . 0031 | . 0032 | . 0034 | . 0039 |
| .0027 | . 0027 | . 0027 | .0028 | . 0030 | . 0034 |
| . 0021 | . 0021 | . 0022 | . 0022 | . 0023 | . 0027 |
| . 0013 | . 0013 | . 0013 | .0013 | . 0014 | . 0016 |
| .0000 | .0000 | . 0000 | .0000 | . 0000 | .0000 |

Fig. 4. Calculated fields for wavenumber of .01 and a cavity dimension of 20.0

| Total $A_{y}$ Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| .0001 | .0000 | . 0000 | .0000 | . 0000 | . 0000 |
| . 0001 | .0000 | . 0000 | . 0000 | . 0000 | . 0000 |
| .0001 | . 0000 | . 0000 | .0000 | .0000 | .0000 |
| . 0000 | .0000 | .0000 | . 0000 | . 0000 | . 0000 |
| - 00.00 | .0000 | .0000 | . 0000 | . 0000 | . 0000 |
| .0000 | . 0000 | . 0000 | .0000 | .0000 | .0000 |
|  | * |  |  |  |  |
| $\mathrm{E}_{\mathrm{x}}$ Component of $\mathrm{F}_{\mathrm{y}}$ Field |  |  |  |  |  |
| . 0317 | . 0320 | . 0325 | . 0335 | . 0357 | . 0415 |
| .0301 | . 0303 | . 0308 | . 0317 | . 0337 | . 0391 |
| . 0267 | . 0269 | . 0273 | . 0281 | . 0297 | . 0342 |
| . 0213 | . 0214 | . 0217 | . 0222 | . 0234 | . 0266 |
| . 0130 | . 0131 | . 0132 | .0135 | . 0141 | .0157 |
| . 0000 | .0000 | . 0000 | .0000 | .0000 | . 0000 |
| Total E $\mathrm{x}^{\text {Field }}$ |  |  |  |  |  |
| . 0317 | . 0320 | . 0325 | . 0335 | . 0357 | . 0415 |
| . 0301 | . 0303 | . 0308 | . 0317 | .0337 | . 0391 |
| . 0267 | . 0269 | . 0273 | . 0281 | . 0297 | . 0342 |
| . 0213 | . 0214 | . 0217 | . 0222 | . 0234 | . 0266 |
| . 0130 | . 0131 | . 0132 | . 0135 | . 0141 | . 0157 |
| . 0000 | .0000 | . 0000 | . 0000 | .0000 | .0000 |

Fig. 5. Calculated fields for wavenumber of .l and a cavity dimension of 20.0.

See legend of Fig. 4.

| Total A Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| .0018 | .0014 | .0003 | .0001 | . 00.01 | . 0000 |
| . 0015 | . 0011 | .0003 | .0001 | .0001 | . 0000 |
| .0012 | .0008 | . 0002 | .0001 | . 0000 | . 0000 |
| . 0009 | . 0006 | . 0002 | . 0001 | .0000 | . 0000 |
| .0005 | .0004 | . 0001 | .0000 | .0000 | . 0000 |
| .0002 | .0001 | .0000 | .0000 | .0000 | .0000 |
|  | 4 |  |  |  |  |
| $\mathrm{E}_{\mathrm{x}}$ Component of $\mathrm{F}_{\mathrm{y}}$ Field |  |  |  |  |  |
| . 2758 | . 2775 | . 2817 | . 2902 | . 3084 | . 3575 |
| . 2615 | . 2631 | . 2669 | .2746 | .2914 | . 3367 |
| . 2317 | . 2330 | . 2362 | . 2426 | . 2565 | . 2943 |
| . 1839 | . 1849 | . 1872 | .1917 | . 2015 | . 2284 |
| . 1121 | . 1126 | .1139 | .1162 | . 1212 | . 1346 |
| . 0000 | .0000 | . 0000 | . 0000 | . 0000 | .0000 |
| $\cdots$ |  | - 1 |  | . |  |
| Total $\mathrm{E}_{\mathrm{X}}$ Field |  |  |  |  |  |
| . 2758 | . 2775 | . 2817 | . 2902 | .3084 | . 3575 |
| . 2615 | . 2631 | . 2669 | .2745 | . 2914 | . 3367 |
| . 2317 | . 2330 | . 2362 | . 2426 | . 2565 | . 2943 |
| . 1839 | . 1849 | . 1872 | .1917 | . 2015 | . 2284 |
| .1121 | . 1126 | . 1139 | .1162 | . 1212 | . 1346 |
| . 0000 | .0000 | . 0000 | . 0000 | .0000 | .0000 |

Fig. 6. Calculated fields for wavenumber of .8 and a cavity dimension of 20.0 。

See legend of Fig. 4.

Total A. Field

| .0059 | .0030 | .0005 | .0001 | .0002 | .0000 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .0050 | .0024 | .0004 | .0001 | .0002 | .0000 |
| .0040 | .0018 | .0004 | .0001 | .0001 | .0000 |
| .0030 | .0013 | .0003 | .0001 | .0001 | .0000 |
| .0018 | .0008 | .0002 | .0000 | .0001 | .0000 |
| .0006 | .0003 | .0001 | .0000 | .0000 | .0000 |

$E_{x}$ Component of $F_{y}$ Field.

| .3615 | .3637 | .3688 | .3795 | .4028 | .4658 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .3425 | .3445 | .3493 | .3591 | .3804 | .4385 |
| .3033 | .3049 | .3089 | .3170 | .3345 | .3830 |
| .2404 | .2416 | .2444 | .2501 | .2624 | .2968 |
| .1463 | .1469 | .1484 | .1514 | .1575 | .1746 |
| .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |

Total $E_{x}$ Field

| .3615 | .3637 | .3688 | .3795 | .4028 | .4658 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .3425 | .3445 | .3493 | .3591 | .3804 | .4385 |
| .3033 | .3049 | .3089 | .3170 | .3345 | .3830 |
| .2404 | .2416 | .2444 | .2501 | .2624 | .2958 |
| .1463 | .1469 | .1484 | .1514 | .1575 | .1746 |
| .0000 | .0000 | .0000 | .0000 | .0000 | .0000 |

Fig. 7. Calculated fields for wavenumber of 1.0 and a cavity dimension of 20.0.

See legend of Fig. 4.

| Total Ay Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| .0043 . | . 0049 | . 0013 | . 00006 | . 0002 | . 0000 |
| .0037 | . 0040 | .0011 | .0004 | .0002 | . 0000 |
| .0030 | . 0031 | .0009 | . 0004 | . 0001 | . 0000 |
| . 0022 | . 0022 | . 0006 | . 0003 | .0001 | . 0000 |
| .0013 | . 0013 | .0004 | . 0002 | .0001 | . 0000 |
| .0004 | .0004 | . 0001 | . 0001 | .0000 | . 0000 |
|  | . |  |  |  |  |
| $\mathrm{E}_{\mathrm{x}}$ Component of $\mathrm{F}_{\mathrm{y}}$ Field |  |  |  |  |  |
| . 4896 | . 4923 | . 4986 | . 5120 | . 5416 | . 623.6 |
| . 4636 | . 4660 | .4718 | . 4840 | . 5111 | . 5867 |
| . 4099 | . 4119 | . 4166 | . 4266 | .4489 | . 5118 |
| . 3242 | . 3256 | . 3289 | . 3359 | .3514 | . 3959 |
| .1967 | . 1974 | . 1992 | . 2027 | . 2104 | . 2322 |
| .0000 | .0000 | . 0000 | . 0000 | . 0000 | . 0000 |
| Total Ex Field |  |  |  |  |  |
| . 4896 | . 4923 | . 4986 | . 5120 | . 5416 | . 6236 |
| .4636 | . 4660 | . 4718 | . 4840 | . 5111 | . 5867 |
| . 4099 | . 4119 | . 4166 | . 4266 | . 4489 | . 5118 |
| . 3242 | . 3256 | . 3289 | . 3359 | . 3514 | . 3959 |
| . 1967 | . 1974 | . 1992 | . 2027 | . 2104 | . 2322 |
| .0000 | . 0000 | .0000 | . 0000 | . 0000 | . 0000 |

Fig. 8. Calculated fields for wavenumber of $I .25$ and a cavity dimension of 20.0.

See legend of Fig. 4.

| Total Ay Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . 0203 | .0161 | . 0028 | . 0015 | . 0007 | . 0000 |
| . 0178 | . 0136 | . 0024 | . 0012 | .0006 | . 0000 |
| . 0147 | . 0109 | .0019 | . 0009 | . 0005 | . 0000 |
| .0109 | . 0079 | . 0014 | . 0007 | . 0004 | . 0000 |
| . 0067 | . 0048 | . 0009 | . 0004 | . 0002 | . 0000 |
| .0022 | .0016 | . 0003 | . 0002 | .0001 | .0000 |
|  |  |  |  | - |  |
| $E_{x}$ Component of $F_{y}$ Field |  |  |  |  |  |
| 1.1078 | 1.1078 | 1. 1100 | 1.1205 | 1.1568 | 1. 2889 |
| 1. 0452 | 1.0451 | 1. 0467 | 1.0558 | 1.0884 | 1.2092 |
| . 9176 | . 9171 | . 9179 | . 9244 | . 9500 | 1.0487 |
| . 7181 | . 7174 | . 7173 | . 7207 | . 7368 | . 8044 |
| . 4297 | . 4291 | . 4284 | . 4292 | . 4357 | .4667 |
| . 0000 | . 0000 | . 0000 | . 0000 | .0000 | .0000 |
| Total Ex Field |  |  |  |  |  |
| 1.1078 | 1.1078 | 1.1100 | 1.1205 | 1.1568 | 1.2889 |
| 1.0452 | 1.0451 | 1.0467 | 1.0558 | 1.0884 | 1.2092 |
| .9176 | . 9171 | . 9179 | . 9244 | . 9499 | 1.0487 |
| . 7181 | . 7174 | . 7173 | . 7207 | .7368 | . 8044 |
| . 4297 | . 4291 | . 4284 | . 4292 | .4357 | .4667 |
| . 0000 | .0000 | .0000 | . 0000 | .0000 | . 0000 |

Fig. 9. Calculated fields for wavenumber of 2.0 and a cavity dimension of 20.0

See legend of Fig. 4.

| Total Ay Field |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| . 0201 | . 0159 | . 0028 | . 0014 | . 0007 | .0000 |
| . 0177 | . 0134 | . 0023 | . 0011 | .0006 | .0000 |
| . 0145 | . 0107 | . 0019 | . 0009 | . 0005 | . 0000 |
| .0108 | . 0078 | . 0014 | . 0007 | . 0004 | .0000 |
| . 0066 | . 00.48 | . 0009 | .0004 | . 0002 | . 0000 |
| . 0022 | . 0015 | . 0003 | . 0002 | . 0001 | . 0000 |
| $\mathrm{E}_{\mathrm{X}}$ Component of $\mathrm{F}_{\mathrm{Y}}$ Field |  |  |  |  |  |
| 1.1004 | 1.1004 | 1.1025 | 1.1129 | 1.1490 | 1.2801 |
| 1.0382 | 1. 0382 | 1.0397 | 1.0487 | 1.0810 | 1. 2009 |
| . 9114 | . 9110 | . 9117 | . 9182 | . 9435 | 1.0416 |
| . 7132 | . 7126 | . 7125 | . 7158 | . 7318 | . 7989 |
| .4268 | . 4262 | . 4255 | . 4263 | . 4327 | . 4635 |
| .0000 | . 0000 | . 0000 | .0000 | . 0000 | . 0000 |
| Total E $\mathrm{X}_{\mathrm{x}}$ Field |  |  |  |  |  |
| 1.1004 | 1.1004 | 1.1025 | 1.1129 | 1.1490 | 1. 2801 |
| 1.0382 | 1.0381 | I. 0297 | 1.0487 | 1.0810 | 1.2009 |
| . 9114 | . 9110 | . 9117 | . 9182 | . 9435 | 1.0416 |
| . 7132 | . 7126 | . 7125 | . 7158 | . 7318 | . 7989 |
| . 4268 | . 4262 | . 4256 | . 4263 | .4327 | . 4635 |
| .0000 | .0000 | . 0000 | . 0000 | . 0000 | .0000 |

Fig. 10. Calculated fields for wavenumber of 2.0 and a cavity dimension of 10.0.

See legend of Fig. 4.


Fig. 11. Calculated fields for wavenumber of 3.0 and a cavity dimension of 20.0

See legend of Fig. . 4.
1.

| － |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | ＋ | ， |  |  |  | 㘍 | ＋1 |  |  |
| － | $\cdots$ |  |  |  | ， |  |  |  | $\cdots$ |  |  |
|  |  |  |  |  |  | ， |  |  |  |  |  |
| 析 |  | ， |  |  |  |  |  | － |  |  |  |
|  |  |  |  |  |  |  |  | ， |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| － | $+$ |  |  |  |  |  |  |  | 霍 |  |  |
|  |  |  |  |  |  |  |  |  | $7$ |  |  |
| 1 | $7 \pm$ | － |  | － | $\cdots$ | － | － | ＋+ | $1+$ |  |  |
|  |  |  |  | Tm： |  |  |  |  |  |  |  |
| 電 |  |  |  |  |  |  |  | 1 F | ， |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
| － |  | ＋ |  | $\cdots$ | 1 | $1+$ |  | ， | ， |  |  |
| $+$ |  | ， | ， | 1 | － | － | ． | 1 | ＋ |  |  |
| \％ | T＋．7 |  |  |  | T | ＋ |  | － | ， |  |  |
|  |  |  |  |  |  | T． |  |  |  |  |  |
| 1 |  |  |  | ＋ |  | ＋1． | 1 | ， |  |  |  |
|  |  |  |  | ， |  | T | － | ， |  |  |  |
|  |  |  |  |  |  | $\pm$ | － | ， | $\pm$ | $\pm$ |  |
| $F$ | $\bigcirc$ |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | ＋1 |  |  |  |  |  |  |  |
| ETY |  |  |  | ． | I | $\pm$ | － | m | － |  |  |
|  |  | ＋ |  |  |  |  |  |  |  |  |  |
| W， |  |  | $\cdots$ | 1 | 4． | 1 | T | $\cdots$ |  |  |  |
| $\cdots$ | ＋ | ． |  | 1 | ＋1． | $1+1$ | 17 | ＋ | ： 1 | \％ |  |
| － | ＋ | ＋ | － | 1－7 | 1 | ＋1＋ | ＋ | 13＋1 |  | ＋ | ， |
| T | $\stackrel{ }{ }$ |  | ＋ |  | $\underline{\square}$ | $1+$ | ＋ |  | ＋ | It | ＋ |
|  |  |  |  |  | ＋ | ＋1－ | $\pm$ |  | S | ＋1－ |  |
|  |  |  |  |  |  | $\cdots$ | ＋17 |  |  |  | $\cdots$ |
| $\pm$ |  |  |  |  |  |  |  |  |  |  |  |
|  | $\pm$ | － |  |  |  |  |  |  |  |  |  |
| T－ | 1 |  |  | Tre $=$ |  | ＋ | － | $\square+$ |  | 4 |  |
| ＋ |  | ． |  | 1 | T |  |  |  |  | 4． | ＋ |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | ＋ | $\pm$ | $\underline{+1}$ | ＋ | － | 4 | ： |  |  |  |
| $\cdots$ |  |  |  |  |  |  |  |  |  |  |  |
| － |  |  |  |  |  |  |  | $\cdots$ |  |  |  |
|  |  |  |  | U |  |  |  | It |  |  |  |
|  | $\pm$ |  |  | $\underline{-1}$ | 1 | İ | $\underline{T}$ | L | ＋1． |  |  |
|  |  |  |  |  |  |  |  | T |  |  |  |
| $\pm$ |  | $\underline{+}$ | \＃ | TH＋7 |  |  | H | $+$ | $4 \pm$ | $1+1$ | $\pm$ |
| ． 01 |  |  |  | ． 1 |  |  |  |  |  |  |  |

Fig．12．Graph of field versus wave number．
The field is evaluated at the point nearest the center of $A$ ．
that give cavity resonances should be used as much as possible. Resonances of the cavity will play a dominant role in the response of the system. Also, corresponding fields are found relatively easily.

Also. suggested is a closer look at the theoretical approach. Even the more basic theorems, such as, a proof of the uniqueness of the integral equation inversion, seem to be lacking. Also, through a theoretical approach, the validity of the given numerical solution might be corroborated.

In conclusion, we submit that another interesting and fruitful area would be generalizations of the problem to, for example, propagation through a square aperture into various types of cavities. Such a problem would require two things: a changing of the kernel of the integral equations by the addition of a different green's function and a reexamination of the resonance solution. Eventually, through enough such generalizations, the problem of diffraction into an actual structure might become in some sense solvable.

## APPENDIX

## PROGRAM FOR COUPLED MODE SOLUTION

The numerical scheme set forth in chapter 3 for finding the solution to the diffraction problem was programmed. This program is presented here preceded by a list of the more important symbols:

AK the wavenumber, $k$
AM(I,J) initially the storage for anti-symmetric-fieldmatrix inversion, later storage for finding coefficients of field solutions

AN $(I, J)$ storage for symmetric-field-matrix inversion
$B C(I, J, K)$ - storage for field solutions; in $K$ dimension, first 2 (NPS-2) for coupled fields; the two homogeneous fields, particular field, and two total fields are next in $K$; $J$ gives $y$-value; and $I$ gives x-value

CI the square root of minus one
DSN the grid square dimension
$F R \quad$ storage for $g_{1}+g_{2}$
$F R R \quad$ storage for zeroth order Hankel function of second kind

NBK parameter in finding TAII, roughly, the number of half-cycles of the series before a 2-cycle averaging routine starts

NGI starting point for the calculation of TAIL
NN the number of grid points along a side of a quadrant

NPS the number of grid points along a side of the aperture which equals $2 *$ NN

NT: the maximum number of terms per half-cycle allowed in the calculation of TAIL

SS the aperture side length
SUMO the value of the singularity of $g_{I}+g_{2}$; see page of text

TAIL, the result of summation from NGI to infinity of $\left(\exp \left(-i k r_{n}\right)\right) / r_{n}$, where $r_{n}=2 n W$; see pp. and of text

W the distance between the plates of the cavity

```
            PROGRAM CHARLY(INPUT,OUTPUT)
C THE APERTURE IS DIVIDED INTO FOUR QUADRANTS AND SYMMETRY IS
C ADPLIED. NN IS THE NUMHER OF GRIN SQUARES ALONG THE EUGE OF A SIDE
C SUCH A QUAORANT. NPS IS THE NUMBEM ALONG THE EDGE OF THE APERTURE
C AK IS THE WAVE NUMBER.
COMPLEX TAIL
COMPLEX ZHAT
COMPLEX SUMO
COMPLEX AMI
COMPLEX BC
COMPLEX FRR
CUMPLEX TEMP,BETA,SUM
    COMPLEX FR,C,AM,B,AEX, CI
COMPLEX AN, ASUM
DIMEIVSION STO(6.4)
DIMENSION BC (6,6,26)
DIMENSION FR(78), FRR(78)
    DIMENSION AN (36,36),B(36),C(36)
OIMENSION AN(36,36)
DIMENSION AM1(36,36)
OIMENSION AAA(10)
NN=4
NN=3
NN=5
NN=6
NN1 = NN-1
NPS = 2*NN
    NPS1 = NPS-1
NPTS = NN**2
NV = NPS
MVI = NV -2
FNV = NV
```

```
    CI = CMPLX{0.0.1:!
    PI = 3.141592653589
    W=200000.
    W=5.
    W=10.
    W=20.
    SS. = 1.
    S2 = 55/2.
    AK = .005
    AK =.05
    AK = 6.
    AK = 3.
    AK = 1.
\DeltaK =.1
    AK = 2.5
    AK = 3.5
    ZHAT = CMPL X(0.,2.)
PRINT 170.AK,NN
170 FORMAT(* WAVENUMBER*,F10.5.* INDEX*.I10)
FNPS = NPS $ DSN = SS/FNPG
AOSN = AK*OSN
DSN2 = 2.#DSN $ O = OSN$$2
AA = DSN/?. S D2 = D2/PI
DO 400N=2.NPS S FN = N
FN=((FH-1.)#OSN)##2 S N N = = (N-1)*N/2
OO401 M = 1,N S FM = M
    NM = N1 +M
FM=((FM-1.)*DSN)*42 $ F FP = AK*SQRT(FN*FM)
FRR(NM) =.25#O2#CMPLX(BESGY(FO,0..0),BESGJ(FP,0.00))
CONTINIJE
CONTINUE
FP = AK * AA
```

```
    FRR(1)=(DSNOCMPLX(BESGY(FP,1,.0),BESGJ(FP,1.,0)))/AK
    1 * CMPLX(4..0.)/(PI*AK*AK)
    DO 402 N = 1,6 S NL = N*(N-1)/2
    DO 403 M = 1,N S NM = Nl. M
    AAA(n)= CABS( FRR(NM))*100.
403 CUNTINUE
    PKINT 160.(AAA(MA), MA = 1.N)
402. CONTINUE
    W2 = 2.*W
    PEACH = AK#W/(2.*PI)
    SUMO = 2.*PI*CI*(CEXP(-AK*AA*CI)-1.)/AK
    SUMO = SUMO & 4./PI
    PRINT 2:SUMO
    FORMAT( 4E18.8,I10)
    PRINT }14
145 FORMAT(* GREEN#S MATRIX*)
    NT = 40
    OPA = 500.
    NEK = 40
        NGI = 15
    LHS = LPC = LMS = LMC = MLMS = MLMC = MLPS = MLPC = NBLS =NBLC =O
    J=NGI -1
    RPSS = RMSS = RPSC = RMSC = SUMI = SUMZ = 0.
    NHK1 = NAK +1 & NHK2 = NHK +2
    NHK3 = NRK * 4
    CSM=CSP = CCM = CCP = 0.
120 J = J+1
    FJ=J
    IF((NALC.GE.NBK3) .AND. (NBLS .GE. NBK3)) GO TO 130
    RJ =W2\DeltaFJ
    ARJ = AK#QJJ
    IF( NBLC .GE. NAK3I GO TO l23
    CJ}=\operatorname{cos}(ARJ)/R
```

```
    ACJ = ABS(CJ)
    SUM: = SUMI + CJ
    IF( CJ.LT. ACJ) GO TO 1?2
    LPC=LPCC+1
    IF( LPC.GT. NT) GO TO l25
    IF(LPC.NE. I) GO TO 126
    IF{ MLMC .LT. LMC) MLMC = LMC
    LMC = 0
    NBLC = NALC + 1
GO TO 126
122 LMC = LMC +1
    IF( LMC.GT. NT) GO TO 125
IF( LMC.NE. 1) GO TO 126
IF( MLPC .LT. LPC) MLPC = LPC
LPC = 0
    NBLC = NALC + 1
126 IF( VALC .LT. NBK) GO TO 123
    IF( RJ..LT. DPA) NBLC = NBLC -a
    IF( QJ .LT. DPA) GO TO 123
    RPSC = RPSC + SUMI
    CCP = CCP * 1.
    IF( NBLC .LT. NBKI) GO TO 123
    IF( NALC..GT. NRK2) GOTO123
RMSC = RMSC + SUMI
CCM = CCM +1.
    IF( NBL.S .GE. NBK3) GO TO I2N
    SJ = SIN(ARJ)/RJ
ASJ=ABS(SJ)
SUMR = SIJM2 + SJ
    IF( SJ.LY. ASJ) GO TO 12?
LPS # LPS +l
IF( LPS.GT. NT) GO TO 129
    IF( LPS .NE. 1) GOTO 1l26
IF( nLMS .LT. LMS ) MLMS = LMS
```

```
    LMS = 0
    NBLS = NHLS & 1
    GO TO11?.6
127 LMS = LMS +1
    IF( LMS .GT. NT) GO TO 129
    IF( LMS .NE, 1) GO TO 1126
    IF( MLPS \cdotLT. LPS) MLPS = LPS
    LPS = 0
    NBLS = NBLS + 1
1126 IF( NBLS .LT. NBK) GO TO 120
    IF( RJ LLT, DPA) NBLS = NBLS - 4
    IF( RJ .LT. DPA) GO TO 120
    RPSS = RPSS + SUMZ
    CSP = CSP +1.
    IF( NALS .LT. NBKI) GO TO 120
    IF( NHLS .GT. NBK2) GO TO 120
    RMSS = RMSS + SUM2
    CSM = CSM +1.
    GO TO 120
130 CSUM = ROSC/( 4.*CCP) + RMSC/( 2.*CCM)
    SSUM = RPSS/ ( 4.* CSP) * RMSS/( 2.*CSM)
    TAIL = ( CMPLX(CSUM,-SSUM))"D?
    FORMAT!* SINE POSITIVE NEG. COSINE, BLOCK LENGTHS*,4I10)
    IJ = 2
    GO TO 12
        PRINT 131,LMC,LPC
    FORMAT(* NON CONVERGENCE COSINE*, 2IIO)
    NBLC = NHK3
    GO TO 120
    PRINT 132, LMS, LPS
    FORMAT1 * NON CONVERGENCE STNE*, 2I101
    NELS = NBK3
    GO TO 120
12 DO 135 N = 1,NPS $ FN = N
```

```
    Nl = (N-1) * N/2
    FN=((FN-1.)*DSN)**2
        DO 136 M = 1,N & FM = M
        NM = NL + M
    FM = ((FM-1.) #DSN)*#2 & FP = FN + FM
    SUM = CMPL.X(0.,0.)
    DO 155J=IJ,NGI $ FJ =J
    FJ = ((FJ-1.)*W2)**2
    RJ = SQRT (FJ+FP) S ARJ = AK*RJ
    SUM = SUM + CEXP(-ARJ#CI)/RJ
155 CONTINUE
    FR(NN) = SUMND2 + TAIL
    IJ = 1
136 CONTINUE
135 CONTINIJE
    FR(1) = FR(1) + SUMO
    DO 163 N = 1,6 S N1 = N*(N-1)/2
    DO 164 M = 1,N $ NM = N1 +M
    AAA(M) = CABS(FR(NM))*100.
    COVTINUE
    PRINT 160,(AAA(MA), MA = 1.N)
        contINUE
D2 = DSN##2
    FNN=NN F FNN=FNN & .5
    DO 100 N = 1,NN S Nl = (N-1)#NN
    00 101 M = 2,NN S NM =N1+M
    DO 102L=1.NJPS $ FL mL
    CSP = 1.
    IF( L .L.E. NN) CSP = -1.
    FLA =ABS(FNN-FL)-5 S S L, LA =FLA
    DO 103 K =1,NPS $ FK = K
```

```
    CCP = 1.
    IF( K .LE.NN) CCP = -1.
    FKA =ABS(FNN-FK) +.5 $ KA =FKA
    LK = LA*NN+KA
    NL = IABS( N-L+NN) + 1
    MK = IABS(M-K +NN) +1
    IF( NL .GT. MK) NLMK = NL*(NL-1)/2 * MK
    IF( NL .LE. MK) NLMK = MK*(MK-1) /2 +NL
    AML (NHT,LK) = AMI (NM,LK) *FR(NLMK)*CSP*CCP
    AN(NM,LK) = AN(NM:LK) + FR(NLMK)
CONTINUE
CONTINUE
101 CONTINUE
100 CONTINUE
    CSP = PI #FNPS$SS/5.
    IF( CSP.LTT. AK) CSP = AK
        IF( CSP .LT, AK) PRINT 273,CSН
273 FORYAT1 ///, WARNING .... WAVE NUMBER EXCEEDING SAMPLING
    IMAXIMUM*.F12.51
    CSP = CSP / 1.5
    SJ = CSP/(FNV-2.)
    DO 203 N = 1,NN $ FN = N
    STO(N.1) = COS((FN-.5)&ADSN) $ STO(N,2) = - SIN((FN-.5)*ADSN)
203 CONTINUE
    DO 201 NLO = 1,NV1 $ FL = NLO
    XK = FL*SJ $ CCM = XK#OSN
    DO 253 N = 1,NN S FN = N
    STO(N,3)=COS((FN-.5)*CCM)
    STO(N,4) = - SIN((FN-.5)*CCM)
263
    cONTINUE
    MK = 2*NLO $ NL = MK-1
    FP = XK/AK
    DO 204 N=1,NN $ S = STO(N,1)
    T = STO(N.2)
```

```
        DO 265 M = 1,NN $ D = STO(M.3)
        P = STO(M.4) $ BC(M,N,MK) = CMPLX(P&T,O.)
        AC(M,N,NL)=CMPLX(S*O,0.)
    265 CUNTINUE
    264 CONTINUE
    201 CONTINUE
C INTEGRAION OF ODD FUNCTION WITH HANKEL FUNCTION
            NBK1 = 24NV-4
            DO 294 NLO = 2,NBK\,2
    DO 280 N = 1.NN
    DO 2H1 M = 1.NIN S SUM = CMPLX(0.,0.)
    DO 2H2L=1,NPS 5 NL = IABS(N-L) +1
    CSM=1. s IF(L.LE. NN) CSM = - 1.
    FL=L $ FLA = ABS(FNN-FL) *.5
    LA = FLA
    DO 2H3 K=1,NPS S MK = InBS(M-K) &1
    CCM = 1. $ IF(K.LE.NN) CCM = - 1.
    FK=K S FKA = ABS (FNN - FK) +. S
    KA = FKA
    IF( NL.GT. MK) NLMK = NL*(NL-1)/2 * MK
    IF( NL -LE. MK) NLMK = MK*(MK-1)/2 + NL
    SUM = SUM * FRR(NLMK) * RC(KA,LA,NLO)*CSM*CCM
283 CONTINUE
282 CONTINUE
    AM(M,N) = SUM
281 CONTINUE
280 CONTINUE
    OO 285 N = 1,NN
    DO 206 M = 1,NN
    BC(M,N,NLO) = AM(M,N)
286 CONTINUE
285 CONTINUE
284 CONTINUE
    NLO = NBKI *I
```

```
    DO 2RT N=1,NN S S = STO(N,1)
    DO 289 M = 1,NN
    GC(M,N,NLO)=CMPLX(S,O.)
288 CONTINUE
287 CONTINUE
    NLO = NHKI +2
    DO 289 N=1,NN S S = DSN*STO(N&2)
    nO 290 M = 1,NN 5 FM = M
    FM = S* (FM=.5)
    BC(M,N,NLD) = CMPLX(FM:O.)
290 CONTINUE
289 CONTINUE
    NLO = NHK! +3
        DO 291 N = 1,NN
    DO 2y2 M = 1.NN
    BC(M,N,NLO) = '2,*CI/AK
292 CONTINUE
291 CONTINUE
    NBK1 = NLO
    NPTS = NN##Z
    LPS = 1
    DO 2.3 NLO = 1.NGK1
    PRINT 234,NLO,LPS
234 FORVAT (//,* ODD NLO IMPLIES FX FIELO,LPS CHECKS IF 1, EVEN NLO
    IIMPLIES AY FIELD, LPS CHECKS IF O*,/,* NLO IS %., I5,* LPS IS*.
    2I5./1)
    IF( LPS .EQ. O) GO TO 294
    DO 2.95 N = 1.NPTS
    DO 296 M = 1.NPTS
    AM(M,N) = AN(M,N)
296 CONTINUE
295 CONTINUE
    LPS = O $ GO TO 303
```

```
294 CONTINUE
    00 360 N = 1.NPTS
    00361 M = 1,NPTS
    AM(N,M) = AM1(NOM)
361 CONTINUE
360 COVIINUE
        LPS = 1
303 CONTINUE
    00 298 N = 1,NN S Nl = NN:(N-1)
    CO 299 M = 1,NN $ NM = NI+M
    C(NM) = BC(M,N,NLO)
    AAA(M) = CABS(C(NM))
299 CONTINUE
    PKINT 160,(AAA(MA),MA=1,NN)
298 CONTINUE
    PKINT 340
    340 FORMAT(//,'* ABOVE IS THE OBSEPVATION MATRIX BELOW THE RESULTB.//)
C AM MATRIX SOLUTION .. GAUSS- JORDAN
        NMI = NPTS - 1
    DO 693 KK = 1,NM1
        KKPI = KK +1
        L = KK
C COLUMN SEARCH FOR THE LARGEST mAgNITUDE VALUE L
    UO 500 I = KKPI,NPTS
    D = REAL (AM(I,KK))
    P= AIMAG (AN(I,KK))
    S = REAL (AM (LOKK))
    T=AIMAG( AM( LOKK))
600 IF ( (0#O +P&P) .GT. (S*S +T$T) )L=I
    IF ( L .FQ. KK) GO TO 620
    DO 610 J = KK,NOTS
    TEMP = AM(KK,J)
    AM(KK,J) = AM(L.J)
```

```
    610 AM(L.J) = TEMP
                TEMP = C(KK)
        C(KK)=C(L)
        C(L) = TEMP
    620 DO 693 I = KKP1,NPTS
        BETA = AM(I,KK) / AM(KK,KK)
        DO 650 J = KKPI, NPTS
    650 AM(I,J) = AM (I,J) - BETA* AMP KK,J)
    693 C(I) = C(I) - BETA * C(KK)
C BACK SOLUTION
    B (NPTS) = C(NPTS) / AM(NPTS,NPTS)
    I = NMI
710 IP1=I +1
    SUM = (0.,0.)
        DO 700 J = IPI, NPTS
700 SUM = SUM + AM(I,J) * B(J)
        B(I)=(C(I)-SUM)/AM(I.I)
        I=I-1
    IF (I .GE. 1) GO TO 710
C END OF MATRIX SOLUTION
    DO 215N=1,NN $ N1 = (N-1)$NN
    DO 216 M = 1,NN $ N NM = N1+M
    HC(M,N,NLO)= B(NM)
    AAA(M)=CABS(B(NM))
    216 CONTINUE
    PKINT 16\cap,( AAA(MA), MA = 1,NN)
    215 CONTINUE
    293 CONTINUE
        P = USN*OSN2
            DO 362 NLO = 2,NHK1,2
CALCULATE THE EX FIELD COMPONENT OF THE TM TO Y SOLN.
    DO 363 N = 1.NN
    363 AM(NLO,N) = (HGC(N,NNL,NLO) - HC(N,NN,NLO))
```

```
    DO 354 N = 2,NN1 $ NL = N+1
    LA = N-1
    364 AM1(N,NLO) = (AM(NLO,NL)-AM(NLO,LA))/P
    AM1(1,NLO) = (AM(NLO,2) * AM(NLO,1))/P
    AMl(NN,NLO) = (AM(NLO,NN) - AM(NLO,NN1))/D2
    362 CONTINUE
    NBK2 = 2$NPS - 4
    DO 365 NLO = 2,NBK2,2 S NL = NLO -1
        MK = NLO/2
    DO 356 N=1,NN
    AMI (N,MK) = AMl(N,NLO) + BC (N,NN,NL)
    CONTINUE
    NBK3 = 2*NPS-2
            DO 367 N = l,NN
    AML (V,NPS) = AMI (N,NBK3)
    367 AM1 (N,NPSI) = BC(N,NN,NBK3-1)
C THE AY FIELD IS SET TO ZERO ON thE EDGE
    DO 136G NLO = 1,NVI S NL =2*NLO
        DO1367 N = 1,NN $ N1 = NN+N
1367 AM1(N1.NLO) = BC(NN.N,NL)
1366 CONTINUE
    NBK2 = 2&NPS-1
    DO 369 N = 1,NN $ Nl = NN+N
        C(N) = BC(N,NN:NBKZ)
        AM\(NL,NPS) = BC(NN,N,NBK3)
    AMI(N1,NPS1) = CMPLX(0.,O.)
    369 C(N1) = CMPLX( 0.,0.1
    NPTS = NPS
        DO 370 N = 1,NPTS
        DO 371 M = 1.NPTS
        AM(M,N) = AMI (M,N)
        COVTINUE
    CONTINUE
```

```
C AM MATRIX SOLUTION .. GAUSS- JORDAN
        NM1 = NPTS - 1
        D01593 KK = 2,NM1
        KKP1 = KK +1
        L = KK
C COLUMN SEARCH FOR THE LARGEST MAGNITUDE VALUE L
            DO1600 I = KKP1,NPTS
            D = REAL (AM(I,KK))
            P=AIMAG (AM(I,KK))
            S = REAL (AM (L,KK))
            T = AIMAG( AM( L,KK))
2600 IF ( (D*D +P#P).GT, (S*S +T*T) )L=I
    IF (L ,EQ.KK) GO TO1620
    D01010 J = KK,NPTS
    TEMP = AM(KK,J)
    AM(KK,J) = AM(L,J)
1610 AM(L,J) = TEMP
            TEYP = C(KK)
        C(KK)=C(L)
    C(L) = TEMP
        001693 I = KKP1,NPTS
        BETA = AM(I,KK) / AM(KK,KK)
        DO2650 J = KKP1, NPTS
    AM(I,J) = AM (I,J) - BETA* AM; KK,J)
            C(I) =C(I) - BETA *C(KK)
1693 C(I) = C(I)
    B (NPTS) = C(NPTS) / AM(NPTS,NPTS)
    I = NMI
1710 IPI = I +1
    SUM = (0.,0.)
    001700 J = IP1, NPTS
1700 SUM = SUM + AM(I,J) B(J)
    B(I) = C(I)-SUM)/AM(I,T)
    I = I - I
```

```
    IF ( I .GE. &) GO TO\710
C END OF MATRIX SOLUTION
        HRIVT 235,(B(MA),MA,MA = 1,NOTS)
    N&K2 = NAK3 +2
        PRINT 236
    236 FORMATI//,* TOTAL AY FIELD*://I
    NHK1 = NPS1 - 1
    UO 373 N = 1,NN
        DO 374 M = 1,NN
        SUM = E(NPTS) #BC(M,N,NBK3)
    0O 3r2 NLO = 1.NBK1 N NL = 28NLO
    SUM = SUMM* H(NLO)*BC(M,N,NL)
    covTINUE
    BC(M.N.NBK2) = SUM
    AAA(M) = CABS(SUM) * 100n00.
    CONTINUE
    PHINT 160,(AAA(MA),MA=1,NN)
373 CONTINUE
    PKINTI38?
1382 FORMAT( //,* EX COMPONENT OF FY FIELO*,//)
    NHK? = NHK3.3
    00 375 N = 1,NN
    DO 376 M = 1,NN
    SUM = B(NPS1) BC(M,N,NBK3-1) - BC (M,N,NBK3+1)
    DO 377 NLO = 1,NBK1 S NL. = 20NLO-1
    SUM = SUM + B(NLO)*RC(M,N,NL)
377
    ONTINUE
    BC(M,N,NBK2) = SUM
    AAA(Y) = CABS(SUM)
376 CONTINUE
    PRINT 160,(AAA(MA),MA=1,NN)
    CONTINUE
    PRINT 237
    FORMAT(//,# TOTAL EX FIFLD*,//)
```

```
        P = DSN2*OSN2
    NBK3 = 24PPS
        NBK1 = NHK3 +1
        NBKZ = NAK3 +2
    00 368 N = 1.NN
    DO 378 M = 2.NND $ NL x Mal
    MK = M+1
378 AM(M,N) = BC(N,MK,NBK3) - BC(N.NL,NBK3)
    AM(1,N)=HC(N.2.NGK3) + QC(N.1,NBK3)
    368 AM(VN,N) = ?.*(BC(N,NN,NBK3) - RC(N,NN1,NBK3)) -
    00 379 N = 1.NN
    OO 3uO M = 2,NN1 $ NL = M-1
    MK = M+1
380 8C(M,N,NQK2) = HC(M,N.NAK1) + (AM(N,MK) - AM(N,NL))/P
    BC(1,N,NBK2)=BC(1,N,NBK1) + (AM(N,2) + AM(N,1))/P
379 AC(NV,N,NAK2) = BC(NN,N,NHK1) * 2."(AM(N,NN) - AM(N,NN1))/P
    DO 341 N = 1,NN
    DO 382 M = 1.NN
382 AAA(M) = CABS(BC(M,N,NBK2))
    PRINT 16O!(AAA(MA)!MA = 1,NN)
    cONTINUE
        PRINT 19?. AK, W
        FORMAT(//, * AK IS * FIZ.5.* WIS* FIZ.5)
        PEACH = AK*W / ( 2.*PI)
        PRINT 2OO,NGI,NBK,MLMC,MLMS,PFACH
    200 FORMAT(///%* TAIL ROUTINE WITH STARTING POINT NGI *,IIO./, * NUM
    1RER OF &LOCKS*,I1O,* BLOCK LENGTH REGISTERS*,2IIO,* INTEGRAL'PAR
    2AMETEH #, F12.5)
    PRINT 194
    FORMATP * RUNNING WITH NO CONSTRAINT EQUATIONS*:
    FORMAT( lOF12.4)
        FORMAT(2E18.8.110)
        STOP
    FND
```

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