ELECTROMAGNETIC PENETRATION THROUGH A GRATING
OF INFINITY CYLINDERS

by
Jeffrey Paul Quintenz

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SIGNED: Jeffrey Paul Quintero

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

[Signature]
Donald G. Dudley
Associate Professor of Electrical Engineering

12/8/72 Date
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF ILLUSTRATIONS</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. SOLUTION FOR THE E FIELD</td>
<td>3</td>
</tr>
<tr>
<td>3. THE CASE OF NORMAL INCIDENCE</td>
<td>10</td>
</tr>
<tr>
<td>4. THE CASE OF OBLIQUE INCIDENCE</td>
<td>24</td>
</tr>
<tr>
<td>5. CONCLUSIONS</td>
<td>31</td>
</tr>
<tr>
<td>APPENDIX A: LISTING OF COMPUTER PROGRAM FOR NORMAL INCIDENCE</td>
<td>32</td>
</tr>
<tr>
<td>APPENDIX B: LISTING OF COMPUTER PROGRAM FOR OBLIQUE INCIDENCE</td>
<td>39</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>47</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Geometry of the grating</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Phase relation diagram</td>
<td>5</td>
</tr>
<tr>
<td>3.</td>
<td>Time response with cylinder spacing as parameter</td>
<td>15</td>
</tr>
<tr>
<td>4.</td>
<td>Transfer function with cylinder spacing as parameter</td>
<td>17</td>
</tr>
<tr>
<td>5.</td>
<td>Effects of rise time and cylinder spacing on transmitted field peak intensity</td>
<td>19</td>
</tr>
<tr>
<td>6.</td>
<td>Effects of rise time and cylinder spacing on transmitted field peak intensity (db)</td>
<td>20</td>
</tr>
<tr>
<td>7.</td>
<td>Equivalence of grating to magnetic walled wave guide</td>
<td>22</td>
</tr>
<tr>
<td>8.</td>
<td>Time response with incidence angle as parameter</td>
<td>27</td>
</tr>
<tr>
<td>9.</td>
<td>Effects of incidence angle and cylinder spacing on peak intensity of transmitted field</td>
<td>29</td>
</tr>
<tr>
<td>10.</td>
<td>Effects of incidence angle and cylinder spacing on peak intensity of transmitted field (db)</td>
<td>30</td>
</tr>
</tbody>
</table>
ABSTRACT

The problem of a plane wave incident at an oblique angle upon a grating of perfectly conducting cylinders is considered. Emphasis is placed upon numerically obtaining the transmitted field in the time domain. Plots of field penetration verses cylinder spacing, incidence angle, and input pulse rise times are presented. Characteristic time response plots are also given. Listing of the computer programs which produce these results are included.
CHAPTER 1

INTRODUCTION

The purpose of this paper is to obtain time domain results for the problem of a plane wave incident upon a grating of infinite cylinders. This problem has been considered in the frequency domain by several authors [1, 2, 3, 4, 5]. The uniqueness of this presentation lies in the fact that the time response is obtained for various grating dimensions and angles of incidence.

The method of solution used here is very similar to that given by Groves [6]. Figure 1 shows the geometry of the parallel wire structure. An initial assumption made about the cylinders is that their diameters are small compared to a wavelength at all frequencies of interest in the input pulse. This assumption allows the cylinders to be treated as filaments and hence simplifies the expression for the current densities. The cylinders are also assumed to be perfectly conducting. Finally, the plane wave is polarized with the E field parallel to the axes of the cylinders.
Figure 1. Geometry of the grating.
CHAPTER 2

SOLUTION FOR THE E FIELD

An integral equation for the currents in the fila­ments can be found by considering the solution for the vector potential \( \mathbf{A} \) in free space, viz.:

\[
\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int_{V'} \frac{j(\mathbf{r}') e^{-ikR}}{R} dV'
\]

where

\[
R = \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{\frac{1}{2}}
\]  

(2)

and the integral is over the volume of the cylinders. Since the cylinders are assumed filamentary, the current density can be expressed as follows:

\[
j(\mathbf{r}') = \delta(x') \sum_{n=-\infty}^{\infty} J_n(z') \delta(y' - nd) \hat{A}_z
\]

(3)

Substituting (3) into (1) gives

\[
\mathbf{A}(\mathbf{r}) = \hat{A}_z \frac{\mu}{4\pi} \sum_{n=-\infty}^{\infty} \int_{V'} \frac{J_n(z') \delta(x') \delta(y' - nd) e^{-ikR}}{R} dV'
\]

(4)

or

\[
A_z(\mathbf{r}) = \frac{\mu}{4\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{J_n(z') e^{-ikR_0}}{R_0} dz'
\]

(5)
where

\[ R_0 = \left[ x^2 + (y - nd)^2 + (z - z')^2 \right]^{1/4} \]  

(6)

Since the cylinders are of infinite extent in z and the excitation (the plane wave) is z independent, there can be no z variation of \( J \), hence:

\[ A_z(\hat{r}) = \frac{\mu}{4\pi} \sum_{n=-\infty}^{\infty} J_{nz} \int_{-\infty}^{\infty} \frac{e^{-ikR_0}}{R_0} \, dz' \]  

(7)

Because there is no z variation, any value of z can be used in \( R_0 \) in the above equation. A convenient choice is \( z = 0 \). Then the problem becomes two dimensional, viz.:

\[ A_z(x,y) = \frac{\mu}{4\pi} \sum_{n=-\infty}^{\infty} J_{nz} \int_{-\infty}^{\infty} \exp \left[ -ik \frac{x^2 + (y - nd)^2 + z'^2}{[x^2 + (y - nd)^2 + z'^2]^{1/2}} \right] \, dz' \]  

(8)

Consider the \( J_{nz} \) terms. By the use of figure 2 it can be shown how the current in the \( n^{th} \) filament is related to the current in the \( 0^{th} \) filament. The incident plane wave can be written \( \exp(-ikx \cos \theta_0 - iky \sin \theta_0) \), and it can readily be seen that only its phase changes as it travels and that it changes by the amount \( \Delta \phi = -k \Delta r \) where \( \Delta r \) is the distance traveled. Since the currents in the filaments are generated by the incident field, a phase change in this field causes the same phase change in the filament current. If the \( 0^{th} \) filament is chosen as the reference of zero phase, then the current in the \( n^{th} \) cylinder will differ from it only in phase.
Figure 2. Phase relation diagram.
and that phase difference is equal to $\Delta \phi = -k\Delta r$. Figure 2 shows that $\Delta r = nd \sin \theta_0$. Therefore,

$$J_{nz} = J_{0z} e^{-ikn d \sin \theta_0}$$

(9)

Now (8) can be written

$$J_{nz} = J_{0z} \sum_{n=-\infty}^{\infty} e^{-ikn d \sin \theta_0}$$

Now (8) can be written

$$A_z(x, y) = \frac{\mu}{4\pi} J_0 z \sum_{n=-\infty}^{\infty} e^{-ikn d \sin \theta_0}$$

The above integral can be evaluated by Gradshteyn and Ryzhik [7] No. 3.876.1 and 3.876.2. Substituting the result into (10) yields

$$A_z(x, y) = \frac{\mu}{4\pi} J_0 z \sum_{n=-\infty}^{\infty} e^{-ikn d \sin \theta_0}$$

$$\int_{-\infty}^{\infty} \frac{\exp[-ik |x^2 + (y - nd)^2 + z^2|^{1/2}]}{|x^2 + (y - nd)^2 + z^2|^{1/2}} dz$$

(10)

$$A_z(x, y) = \frac{\mu}{4\pi} J_0 z \sum_{n=-\infty}^{\infty} e^{-ikn d \sin \theta_0}$$

$$H_0^2(k(x^2 + (y - nd)^2)^{1/2})$$

(11)

The E field resulting from this potential is the scattered E field, viz.:

$$\hat{E}_s = -i\omega \hat{A} + \frac{\nabla (\nabla \cdot \hat{A})}{i\omega \mu \varepsilon}$$

(12)

but since

$$A_x = A_y = 0 \quad \text{and} \quad \frac{\partial A_z}{\partial z} = 0, \quad \nabla \cdot \hat{A} = 0$$

(13)

Then
\[ E_s = -i\omega A \]  

or

\[ E_{zs}(x, y) = -\frac{\omega \mu J_0 z}{4} \sum_{n=-\infty}^{\infty} e^{-iknd \sin\theta_0} \]

\[ H_0^{(2)}\left[ k\left(x^2 + (y - nd)^2\right)^{1/2}\right] \]  

The total field is the sum of the incident and scattered fields.

\[ E_z = E_{z\text{inc}} + E_{zs} \]  

Applying the boundary condition \( E_z = 0 \) at each conductor results in

\[ E_{z\text{inc}} = -E_{zs} = -\frac{\omega \mu J_0 z}{4} \sum_{n=-\infty}^{\infty} e^{-iknd \sin\theta_0} \]

\[ H_0^{(2)}\left[ k\left(x_c^2 + (y_c - nd)^2\right)^{1/2}\right] \]  

where \((x_c, y_c)\) are the coordinates at the conductor surface.

Consider the \( m \)th cylinder. For filamentary scatters, we must have

\[ \lim_{x \to 0} \exp\left[-ik(x_c \cos\theta_0 + y_c \sin\theta_0)\right] = \frac{\omega \mu}{4} J_0 z \lim_{x \to 0} \sum_{n=-\infty}^{\infty} \sum_{y_c = m \lambda}^{y_c + m \lambda} \]

\[ H_0^{(2)}\left[ k\left(x_c^2 + (y_c - nd)^2\right)^{1/2}\right] e^{-iknd \sin\theta_0} \]  

\[ \lim_{x \to 0} \lim_{y_c \to 0} \]
\[ e^{-ikmd \sin \theta_0} = \]
\[ \omega \mu J_{0z} \lim_{x \to 0, y_c = md} \left\{ e^{-ikmd \sin \theta_0} H_0(2) \left[ k \left( x_c^2 + (y_c - md)^2 \right)^{\frac{1}{2}} \right] \right\} \]
\[ + \sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0(2) \left[ k \left( x_c^2 + (y_c - nd)^2 \right)^{\frac{1}{2}} \right] \]  
(19)

where \( \sum \) is the sum excluding \( n=m \).

Then
\[ e^{-ikmd \sin \theta_0} = \frac{\omega \mu}{4} J_{0z} \lim_{a \to 0} e^{-ikmd \sin \theta_0} H_0(2) \left[ k a \right] \]
\[ + \sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0(2) \left[ k d \left( m - n \right)^{\frac{1}{2}} \right] \]  
(20)

This can be written
\[ 1 = \frac{\omega \mu}{4} J_{0z} \left\{ \lim_{a \to 0} H_0(2) \left[ k a \right] + \sum_{n=-\infty}^{\infty} e^{-inkd \sin \theta_0} H_0(2) \left[ \left| n \right| k d \right] \right\} \]
(21)

where now \( \sum \) is the sum excluding \( n=0 \). Solving (21) for \( J_{0z} \) yields:
\[ J_{0z} = \frac{4}{\omega \mu \left[ \lim_{a \to 0} H_0(2) \left( ka \right) + \sum_{n=-\infty}^{\infty} H_0(2) \left( \left| n \right| k d \right) e^{-inkd \sin \theta_0} \right]} \]  
(22)
Substituting (22) into (15) gives

$$E_{zs}(x,y) = \sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0^{(2)} \left[ k \left( x^2 + (y - nd)^2 \right)^{\frac{1}{2}} \right]$$

$$\lim_{a \to 0} H_0^{(2)}(ka) + \sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd)e^{-inkd \sin \theta_0}$$

but

$$H_0^{(2)}(ka) \longrightarrow 1 - \frac{2i}{\pi} \ln \frac{yka}{2}$$

for small values of ka. In (24) γ is Euler's constant γ = 1.781. Inserting (24) into (23) gives

$$E_{zs}(x,y) = \sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0^{(2)} \left[ k \left( x^2 + (y - nd)^2 \right)^{\frac{1}{2}} \right]$$

$$\frac{2i}{\pi} \ln \frac{yka}{2} - 1 - \sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd)e^{-inkd \sin \theta_0}$$

Now the expression for the total E field can be written as

$$E_z(x,y) = \exp\left[ -ik(x \cos \theta_0 + y \sin \theta_0) \right]$$

$$\sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0^{(2)} \left[ k \left( x^2 + (y - nd)^2 \right)^{\frac{1}{2}} \right]$$

$$\frac{2i}{\pi} \ln \frac{yka}{2} - 1 - \sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd)e^{-inkd \sin \theta_0}$$
CHAPTER 3

THE CASE OF NORMAL INCIDENCE

When the incoming plane wave is normally incident upon the grating, equation (26) becomes

\[ E_z(x, y) = e^{-ikx} + \sum_{n=-\infty}^{\infty} \frac{H_0^{(2)}(nkd)}{2i \pi \ln \frac{\gamma ka}{2} - 1 - 2} \sum_{n=1}^{\infty} H_0^{(2)}(nkd) \]

(27)

For the values of kd of interest in this paper the summations in (27) converge much too slowly to be of practical use. This expression is practical only when kd << 1. It is possible, however, to replace these summations with equivalent ones which converge very rapidly in the range of interest. Morse and Feshbach [8] prove that

\[ \sum_{n=-\infty}^{\infty} H_0^{(2)}(nkd) = \exp \left( -k \left( \frac{2\pi n}{d} \right)^2 - k^2 \right) \]

(28)
where

\[ R = -2 \ln \left[ 1 - 2e^{-\frac{2\pi|x|}{d}} \cos \left( \frac{2\pi y}{d} \right) + e^{-\frac{4\pi|x|}{d}} \right] \]

Further,Infeld,Smith and Chien [9] show that

\[
\sum_{n=1}^{\infty} H_0^{(2)}(nk\delta) = \frac{1}{kd} - \frac{1}{2} + \frac{i\left(\gamma + \ln \frac{kd}{4\pi}\right)}{\pi} + \frac{1}{\pi} \sum_{n=1}^{n_0} \left\{ \frac{1}{\sqrt{(kd/2\pi)^2 - n^2}} - \frac{i}{n} \right\}
\]

\[
+ \frac{i}{\pi} \sum_{n=n_0+1}^{\infty} \left( \frac{1}{\sqrt{n^2 - (kd/2\pi)^2}} - \frac{1}{n} \right)
\]

(29)

where \( n_0 \) is the greatest integer less than \( \frac{kd}{2\pi} \). Using (29) and (28) in (27) gives an expression for \( E_z(x,y,\omega) \) which converges rapidly enough to use in a numerical analysis.
$$E_z(x,y,\omega) = e^{-ikx}$$

$$\left[R + \frac{4\pi i}{kd} e^{ik|\mathbf{x}|} + 4 \sum_{n=1}^{\infty} \cos\left(\frac{2\pi ny}{d}\right) \left\{ \frac{2\pi}{d} \frac{\exp\left(-\sqrt{\left(\frac{2\pi n}{d}\right)^2 - k^2} |x| \right)}{\sqrt{\left(\frac{2\pi n}{d}\right)^2 - k^2}} - \frac{\exp\left(-\frac{2\pi n}{d} |x| \right)}{n} \right\} \right] \star$$

$$+ \frac{2i}{\pi} \ln \frac{\gamma ka}{2} - 1 - 2 \left[ \frac{1}{kd} - \frac{1}{2} + \frac{1}{\pi} \sum_{n=1}^{n_0} \frac{1}{\sqrt{(kd/2\pi)^2 - n^2}} + \frac{i}{\pi} \left( \gamma + \ln \frac{kd}{4\pi} \right) - \sum_{n=1}^{n_0} \frac{1}{n} + \sum_{n=n_0+1}^{\infty} \left( \frac{1}{\sqrt{n^2 - (kd/2\pi)^2}} - \frac{1}{n} \right) \right]$$

(30)
This expression is the impulse response of the grating or the grating transfer function. The time response of the grating to an input pulse $f(t)$ can now be found, viz.:

$$e_z(x, y, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E_z(x, y, \omega) F(\omega) e^{i\omega t} \, d\omega$$  \hspace{1cm} (31)

Throughout this paper the input pulse is of the double exponential type which is described mathematically as

$$f(t) = A \left[ e^{-\alpha t} - e^{-\beta t} \right]$$  \hspace{1cm} (32)

where $A$ is the normalization constant. It is easily shown then, that

$$F(\omega) = A \left[ \frac{\alpha - i\omega}{\alpha^2 + \omega^2} - \frac{\beta - i\omega}{\beta^2 + \omega^2} \right]$$  \hspace{1cm} (33)

A computer program has been written to produce the frequency spectrum of the impulse response using (30). This program (FSCLDR) also multiplies the system function by the spectrum of the double exponential pulse (33) and performs the numerical inverse Fourier transform of the resulting product. The final result is then the time response of the grating when excited by the double exponential input pulse. A listing of FSCLDR appears in Appendix A.

As a matter of convenience the time plot scales presented have been shifted by the amount $x/c$ so that the
zero time reference on each plot is actually the time required for the pulse to propagate the distance to $x$. This time shift facilitates comparisons between different parameters.

The plots presented display the effects of varying the cylinder spacing, $d$, and the input pulse rise time. It should be noted that, in the analysis, $a$ was assumed much smaller than a wavelength and the numerical work, in accordance with this assumption, becomes $a$ independent.

The first parameter variation considered is that of changing the cylinder spacing. Figure 3 displays plots of the grating response for different values of $d$. These plots represent the transmitted E field at the point $x = 1.22$ m, $y = 0$ m for a unit magnitude input pulse which has a 10 ns rise time and a 350 ns fall time. The plots show clearly that a wider cylinder spacing allows more of the pulse to pass through the grating. This is expected since a wider spacing lowers the cutoff frequency and hence, more of the input pulse spectrum penetrates the grating. If comparison is made between the grating and very short sections of parallel plate wave guide placed side by side, this cutoff frequency can be better understood. In the case of the parallel plate wave guide with plates spaced $d$ apart, a well known result is that $f_{co} = \frac{nc}{2d}$ for the $TE_n$ or $TM_n$ modes. Modes with frequencies below the cutoff frequency are evanescent,
Figure 3. Time response with cylinder spacing as parameter.
and those frequency components below cutoff are attenuated as distance down the guide is increased. The wider the plate spacing is made the lower the cutoff frequency and the greater the amount of the input spectrum which propagates down the wave guide.

These plots also demonstrate something about pulses transmitted through the grating in general. It can be shown that in the limit as \( \omega \to 0 \) (27) approaches zero. Therefore, there is no d.c. component of the transmitted field. The time response of the grating then must time average to zero. This is indeed the case with all of the plots obtained. The initial large pulse with a positive area is countered by a small magnitude "tail" which persists for a much longer time below the zero axis. The result is the predicted zero time average. Figure 4 displays the transfer functions of the grating for the above cases.

As stated above, the input pulse used in these examples has a rise time of 10 ns. Figure 3 shows that the rise time of the transmitted pulse is enhanced as the spacing of the cylinders narrows. This is explained by the fact that the transmitted pulse is composed of the incident pulse plus the sum of reradiated pulses from the individual cylinders which reach the point \((x,y)\) delayed in time by an amount dependent upon the cylinder spacing. These reradiated pulses are also reduced in magnitude by the usual \( r^{-\frac{1}{2}} \) amount for two
Figure 4. Transfer function with cylinder spacing as parameter.
dimensional radiation, when they arrive at the field point. These two effects, time delay and magnitude decrease, together in effect fold the input pulse over before it can reach its maximum. The result is best explained by visualizing the transmitted pulse beginning to rise in a manner identical to the input pulse but reaching its peak and falling off sooner than the input pulse. The rise time is enhanced because the peak of the pulse is lowered while its leading edge is maintained.

Another aspect of interest is the effect that the rise time of the input pulse has on the peak value of the transmitted E field. Figure 5 displays this effect. Here the field point is again $x = 1.22 \text{ m}$, $y = 0 \text{ m}$, and all input pulses have 350 ns fall times. It can be see from this figure that the peak value of the transmitted field increases as the input pulse rise time decreases. This is expected since decreasing the rise time accentuates the higher frequencies of the pulse spectrum, and these higher frequencies pass through the grating more easily. Figure 6 displays the same information as figure 5, however, here the peak field is expressed in decibels.

The effects of varying the field point are best explained by considering an equivalent problem. By the method of images the grating problem can be shown to be equivalent to the case of a parallel plate wave guide with
Effects of rise time and cylinder spacing on transmitted field peak intensity.
Figure 6. Effects of rise time and cylinder spacing on transmitted field peak intensity (db).
magnetic walls and a cylindrical boss on one of the plates. Figure 7 shows this equivalence. Figure 7-a is the original grating. Figure 7-b shows the equivalent case valid for the region \(-\frac{d}{2} \leq y \leq \frac{d}{2}\). This figure is included to demonstrate that if the field is known for \(0 \leq y \leq \frac{d}{2}\) then it is determined for any value of \(y\) by symmetry considerations and the method of images. This very useful observation justifies restricting any numerical work to the region of \(y\) for \(0 \leq y \leq \frac{d}{2}\).

The parallel plate equivalent case helps to explain the numerical results obtained for \(x\) variation. In all cases considered there is no noticeable \(x\) variation. Values of \(x\) considered ranged from .15 m to over 30 m. The analogy can be made to a pulse propagating down a perfect magnetic walled wave guide. Pulse degradation on a wave guide of this type is due to dispersion. Since the frequency spectrums of these transmitted pulses have accentuated high frequencies and cutoff low frequencies, the major portion of the spectrum lies in the linear region of the \(\omega-\beta\) curve. Thus, the effects of dispersion are reduced because of the nearly constant phase velocity for all frequency components of substantial magnitude.

As in the results of \(x\) variation, changing \(y\) seemed to have little effect. It can be reasoned that since the cylinder radius is very small and spaced periodically and the
Figure 7. Equivalence of grating to magnetic walled wave guide.
input pulse is \( y \) independent that little \( y \) variation should be expected except at a very close distance behind the grating.
THE CASE OF OBLIQUE INCIDENCE

The feasibility of producing numerical results for oblique angles of incidence depends upon the ability to reduce the general expression of (26) to a useable form similar to that derived for the case of normal incidence. The summations in (26) must be replaced by equivalent summations which converge more rapidly.

Consider first the summation in the denominator of (26). This summation can be written

$$\sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd) \ e^{-inkd \sin \theta_0} =$$

$$\sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd) \left[ \cos(nkd \sin \theta_0) - i \sin(nkd \sin \theta_0) \right] \ (34)$$

The right hand summation in (34) can be simplified by canceling respective sine terms for +n and -n, and the result is

$$\sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd) \ e^{-inkd \sin \theta_0} =$$

$$\sum_{n=1}^{\infty} H_0^{(2)}(nk) \cos(nkd \sin \theta_0) \ (35)$$
An expression given by Wait [10] can now be applied to the right hand side of (35) to yield the final result

$$\sum_{n=-\infty}^{\infty} H_0^{(2)}(|n|kd) e^{-inkd \sin \theta_0} =$$

$$-1 + \frac{2i}{\pi} \ln \frac{kd\gamma}{4\pi} + \frac{i}{\pi \sqrt{\left(\frac{kd}{2\pi} \sin \theta_0\right)^2 - \left(\frac{kd}{2\pi}\right)^2}} + \frac{i}{\pi} \sum_{n=1}^{\infty}$$

$$\left[ \frac{1}{\sqrt{(n+kd \sin \theta_0)^2 - \left(\frac{kd}{2\pi}\right)^2}} + \frac{1}{\sqrt{(n-kd \sin \theta_0)^2 - \left(\frac{kd}{2\pi}\right)^2}} - \frac{2}{m} \right]$$

The remaining summation in (26) can be manipulated into a form which is shown, again by Wait [10] to be equivalent to a more rapidly converging series. The resulting expression is

$$\sum_{n=-\infty}^{\infty} e^{-iknd \sin \theta_0} H_0^{(2)} \left[ k\sqrt{x^2 + (y - nd)^2} \right] = \frac{i}{\pi} e^{-iky \sin \theta_0}$$

$$\sum_{n=-\infty}^{\infty} \exp \left[ \frac{2\pi}{d} \left( iny - x \sqrt{(n - \frac{kd \sin \theta_0}{2\pi})^2 - \left(\frac{kd}{2\pi}\right)^2} \right) \right]$$

The final expression for numerically computing $E_z(x,y,\omega)$ for
the case of oblique incidence is obtained by substituting
the results of (36) and (37) into (26), viz.:

\[
E_z(x,y,\omega) = e^{-ik(x \cos \theta_0 + y \sin \theta_0)}
\]

\[
e^{-iky \sin \theta_0} \sum_{n=-\infty}^{\infty} \exp \left[ \frac{2\pi}{d} \left( iny - x \sqrt{(n - \frac{k \sin \theta_0}{2\pi})^2 - \left( \frac{k d}{2\pi} \right)^2} \right) \right]
\]

\[
+ \frac{2 \ln \frac{2\pi a}{d} - \frac{1}{\sqrt{(\frac{k d}{2\pi} \sin \theta_0)^2 - \left( \frac{k d}{2\pi} \right)^2}}}{\frac{1}{\sqrt{(n + \frac{k d}{2\pi} \sin \theta_0)^2 - \left( \frac{k d}{2\pi} \right)^2}} - \sum_{n=1}^{\infty} \left[ \frac{1}{\sqrt{(n - \frac{k d}{2\pi} \sin \theta_0)^2 - \left( \frac{k d}{2\pi} \right)^2}} - \frac{2}{n} \right]
\]

Using methods similar to those described for normal incidence, the time domain response of the grating can be obtained numerically for various angles of incidence of the plane wave excitation. The computer program ARBANG, listed in Appendix B, performs this operation. The first case presented is shown in figure 8. In this figure three time responses have been superimposed displaying the effects of varying the angle of incidence. The field point is again \(x = 1.22\, \text{m}, \, y = 0\, \text{m}\), and the input pulse is of unit magnitude with a 10 ns rise time and a 350 ns fall time. As
Figure 8. Time response with incidence angle as parameter.
expected the pulse amplitude decreases as the angle increases away from the normal.

In practical applications it is of interest to know the peak magnitude of the transmitted pulse relative to that of the input pulse. Figure 9 presents the peak magnitude of the transmitted pulse for various angles of incidence and cylinder spacings. The peak value appears to fall off roughly as the cosine of the incidence angle, especially for the smaller cylinder spacings. Figure 10 gives the above information with the peak field value expressed in decibels.
Figure 9. Effects of incidence angle and cylinder spacing on peak intensity of transmitted field.
Figure 10. Effects of incidence angle and cylinder spacing on peak intensity of transmitted field (db).
CHAPTER 5

CONCLUSIONS

From the results presented here, it is seen that an electromagnetic pulse penetrating a grating of cylinders undergoes three characteristic changes. The peak amplitude of the transmitted pulse is reduced, the rise time of the pulse is reduced as it passes through the grating, and the resulting transmitted pulse time averages to zero. The extent to which the first two of these changes occur depends almost solely upon the cylinder spacing.

All of the above results are derived for small cylinder radius and E field polarized parallel to the axes of the cylinders. Future work should extend the results obtained here to include arbitrary polarization and cylinder radius.
APPENDIX A

LISTING OF COMPUTER PROGRAM FOR NORMAL INCIDENCE

The computer program for computing the frequency and time domain results for the case of normal incidence is listed here. The program is written in Fortran IV for use on a CDC 6400 computer.

```
PROGRAM FSCLDR(INPUT,OUTPUT,TAPE2)
COMMON /CONST/ INOCATE, TWOP1, INDF, INDOUT
DIMENSION F(660), ZP(660), Z(660), LN(10), DEL(10),
IFF(660), SY(100), SYY(100), T(205), GT(205), GTE(205),
2GTO(205)
COMPLEX DENT, EI, GK, G(660), 11, FW, PST, HNKD
REAL K, JO, K2
CALL INITIAL (AH3950283K, 2, 0, 3, 0, 0)

CONSTANTS
C = 299792500.0, PI = 3.141592654, TWOPI = 6.283185308
GAMMA = EXP(0.577215665)
GA = 0.577215665

FOURIER BLOCK

LN(1) = 1, LN(2) = 11, LN(3) = 21, LN(4) = 31, LN(5) = 41, LN(6) = 51
LN(7) = 61, LN(8) = 457, DEL(1) = 1, DEL(2) = 9, DEL(3) = 90,
DEL(4) = 900, DEL(5) = 9000, DEL(6) = 90000, DEL(7) = 250000,
NH = 8
DEL(8) = 10.0, 7 LN(9) = 537
INDF = LN(NH+1)
T(1) = 0.5, INC = 5.0, E = 10, INDT = 201
READ 19, INOCATE

F(1) = 0.
```

32
DO 1 L=1,NB
NS=LN(L+1)-1
NA=LN(L)
DO 1 N=NA,NS
1 F(N+1)=F(N)*DEL(L)
DO 2 I=2*INDT
2 T(I)=T(I-1)*TINC
T(INDT+1)=0.
T(INDT+2)=12.5*10.*-9

C
READ 20, DI, AI, XF, YF, THETA, ALV, IALE, BEV, IREE
ALE=IALE*BEV=IREE*ALV*10.*IALE*BE=BEV*10.*IBEE
AL2=AL*AL%BE2=BE%BE
TM=ALOG(AL/BE)/(AL-BE)
CMN=1./(EXP(-AL*TM)-EXP(-BE*TM))
D=DI*0.254%A=AI*0.254%X=XF*3.048%Y=YF*3.048
XM=AHS(X)
YM=AHS(Y)
I1=CMPLX(0., 1.)
PRINT 21, DI, AI, XF, YF, D, A, X, Y, THETA, AL, BE
A=A/2.8PRINT7$PRINT8
TPD=TwoPI/D
TPDX=TPD*X
TPDY=TPD*Y
FPD=2.*TPD
R=-2.*ALOG(1.-2.*EXP(-TPDX)*COS(TPDY)+EXP(-2.*TPDX))
DEN=ALOG(GAMMA*AL/2.)
F(1)=1.
DO 7 I=1,1INDF
w=Twopi/F(1)
K=K/C
K2=K*K
w2=w*w$DNA=AL2+w2%DN=BE2+w2%CR=CMN*(AL/DNA- BE/DN)
CI=-CMN*(w/DNA-w/DN)
FW=CMPLX(CR, CI)
XK=X*K
E1=CMPLX(COS(XK), -SIN(XK))
DENT=CMPLX(-1., 2.*(DEN+ALOG(K))/PI)
ZS=K/TPU
ZS2=ZS*ZS

C
CALCULATING SUM OF H(NKD)
C
N=0
SUT=1000.
SUM=1./SQRT(1.-ZS2)-1.
YO=GA*ALOG(ZS/2.)
IF (ABS(SUM), LT, 10.*ABS(YO)) GO TO 4
DO 3 N=2.5
FN=N
FM2 = FN*FN
SUM = SUM + FN*SUM
IF (ABS((SUT-SUM)/SUM) .LT. 0.000001) GO TO 4
SUT = SUM
Y0 = Y0*SUM/PI
IT1 = N
J0 = 1.0*(K*D)*0.5
HNKD = CMPLX(J0, Y0)

EXEC = EXEC - 2.0*HNKD

CALCULATING SUM OF H(X*N*Y*K*D)

SUM = 0.0
SUT = 10.00
DO 5 N = 1, 50
FN = N
FNN = FN*TPD
FNN2 = FNN*FNN
SRF = SQRT(FNN2 - K2)
SUM = SUM + COS(FNN*Y)*((EXP(-SRF*XM)*TPO/SRF - EXP(-FNN*XM))/1FN)
IF (ABS((SUM-SUT)/SUM) .LT. 0.000001) GO TO 6
SUT = SUM
5 GK = R*FPD*II*(COS(K*XM) - II*SIN(K*XM))/K + 4.0*SUM
IT2 = N
GK = GK/((T*N)*PI*II)
GK = CONJG(GK)

E = E1 + GK/EXEC
G(I) = CONJG(E*FW)
PST = CMPLX(COS(XK), -SIN(XK))
G(I) = G(I)*PST
EM = CAHS(E)
FF(I) = 20.*ALOG10(EM)
IF (FF(I) .LT. -120.) FF(I) = -120.
GMAG = CAHS(G(I))
FWM = 20.*ALOG10(CABS(FW))
PRINT 22, F(I), E, FF(I), G(I), GMAG, IT1, IT2, FW
F(I) = 0.0, G(I) = (0.0, 0.0)
CALL FOURIER (F, G, LND, NM, TG, GTE, GTO)
PRINT 23
PRINT 24, (T(I), G1(I), GTE(I), GTO(I), I = 1, INDT)

PLOT BLOCK

INPT = 457
DO 8 M = 1, 2, 457
ZP(I) = ALOG10(F(I))
8 Z(I) = F(I)/1000000.
Z(I) = Z(2)
$ZP(1) = ZP(2)$
CALL PLOT (0.,-1.,-3)
CALL PLOT (0.,2.,-3)
ICY=9
ICYM=ICY-1
ITOT=9*ICYM
DO 10 L=1,ICYM
LC=L-1
DO 9 M=1,9
LM2=M+LC+9
SY(LM2)=ALOG10(1. + M) + LC
9 CONTINUE
10 CONTINUE
DO 11 M=1,6
SYY(M)=1.
11 CONTINUE
IP=INPT+1
IPP=IP+1
ZP(IP)=0.
ZP(IPP)=1.
Z(IP)=0.
Z(IPP)=12.5
IPLT=1
FF(IP)=-120.
FF(IPP)=20.
INDF=INF
CALL LINE (ZP*FF*INDF*1.*0.2)
CALL SYMBOL (1..1..1..13H) IN INCHES = 0..13
CALL NUMBER (2.2,1.,1,01,0.2)
CALL SYMBOL (1..0..84..1..5HAL = 0..5)
CALL NUMBER (1.44,0..84..1..ALV..0..2)
CALL SYMBOL (1.81,0..84..1..1HE..0..1)
CALL NUMBER (1.91,0..84..1..ALE..0..1)
CALL SYMBOL (2.01,0..84..1..5HRE = 0..5)
CALL NUMBER (2.45,0..84..1..BEV..0..2)
CALL SYMBOL (2.82,0..84..1..1HE..0..1)
CALL NUMBER (2.92,0..84..1..BEF..0..1)
CALL SYMBOL (1..68..1..11MX IN FEET = 0..11)
CALL NUMBER (2.0,0..68..1..XF..0..2)
CALL SYMBOL (1..52..1..11HY IN FEET = 0..11)
CALL NUMBER (2.0,0..52..1..YF..0..2)
CALL SYMBOL (1..36..1..17HINCIDENCE ANGLE = 0..17)
CALL NUMBER (2.6,0..36..1..THETA..0..1)
CALL PLOT (0..99..3)
IF (IPLT.EQ.3) GO TO 16
DU 13 J=1,6
CALL SYMBOL (0..SYY(J)..1..13,90..-2)
13 CONTINUE
CALL SYMBOL (-.38..0..1..4M-120..0..4)
CALL SYMBOL (-.38..1..4M-100..0..4)
CALL SYMBOL (-.38,2..1,4H-80..0..4)
CALL SYMBOL (-.38,3..1,4H-60..0..4)
CALL SYMBOL (-.38,4..1,4H-40..0..4)
CALL SYMBOL (-.38,5..1,4H-20..0..4)
CALL SYMBOL (-.38,6..1,4H-0..0..4)
CALL SYMBOL (-.50,2..1,23HFREQUENCY RESPONSE (DR),90.
1*23)
CALL PLOT (0.,0.,3)
IF (IPLT.EQ.2) GO TO 15
DO 14 J=1,ITOT
CALL SYMBOL (SY(J),0.,1,1J,0.,-2)
CONTINUE
CALL SYMBOL (0.,-.225,1,1H0,0.,1)
CALL SYMBOL (1.,-.225,1,1H1,0.,1)
CALL SYMBOL (2.,-.225,1,1H2,0.,1)
CALL SYMBOL (3.,-.225,1,1H3,0.,1)
CALL SYMBOL (4.,-.225,1,1H4,0.,1)
CALL SYMBOL (5.,-.225,1,1H5,0.,1)
CALL SYMBOL (6.,-.225,1,1H6,0.,1)
CALL SYMBOL (7.,-.225,1,1H7,0.,1)
CALL SYMBOL (8.,-.225,1,1H8,0.,1)
CALL SYMBOL (25HFREQUENCY, HZ (POWERS OF
14H TEN),0.,29)
CALL PLOT (11.,0.,-3)
IPLT=2
GO TO 12
15 CALL LINE (2FF,INDF,1,0.2)
CALL AXIS (0.,0.,14HFREQUENCY, MHZ,-14.8.,0.,0.,12.5.
112.5)
CALL PLOT (11.,0.,-3)
IPLT=3
CALL SYMBOL (6.,6.,1,13HD IN INCHES =0..13)
CALL NUMBER (7.2,6.,1,DI,0.,2)
CALL SYMBOL (6.,5.,A4.,1,HAL =0.,5)
CALL NUMBER (6.4,5.,84.,1,ALV,0.,2)
CALL SYMBOL (6.,81,5.,84.,1,1HE,0.,1)
CALL NUMBER (6.,91,5.,94.,1,ALE,0.,-1)
CALL SYMBOL (7.,21,5.,84.,1,SHBE =0.,5)
CALL NUMBER (7.,65,5.,84.,1,BEV,0.,2)
CALL SYMBOL (8.,32,5.,84.,1,1HE,0.,1)
CALL NUMBER (8.,12,5.,84.,1,BEV,0.,-1)
CALL SYMBOL (6.,5.,68.,1,11HX IN FEET =0.,11)
CALL NUMBER (7.,0,5.,68.,1,1XF,0.,2)
CALL SYMBOL (6.,5.,52.,1,11HY IN FEET =0.,11)
CALL NUMBER (7.,5,52.,1,1YF,0.,2)
CALL SYMBOL (6.,5.,36.,1,17INCIDENCE ANGLE =0.,17)
CALL NUMBER (7.,6,5.,36.,1,1META,0.,1)
16 CALL SCALE (GT,6.,INDT,1,10.)
AS=-GT(INDT+1)/GT(INDT+2)
CALL AXIS (0.,0.,13M TIME RESPONSE,13,6.,90.,GT(INDT+1)
1*GT(INDT+2),10.)
XAS=0.
DO 17 J=1,10
   XAS=XAS+.8
17  CALL SYMBOL (XAS,AS,1,13,0.,-1)
   CALL SYMBOL (3.5,-5.,1.,9HTIME (NS),0.,9)
   CALL PLOT (U,AS,3)
   CALL PLOT (H,AS,2)
   CALL PLOT (0.,0.,3)
   FPN=0.
   XAS=-.1
   DO 18 J=1,10
      XAS=.8+XAS
      FPN=FPN+10.
   18  CALL NUMBER (XAS,ASM,1,FPN,0.,-1)
      CALL PLOT (0.,0.,3)
      CALL LINE (T,GT,INDT,1,0,0)
      CALL PLOT (0.,0.,999)
      C = STOP
      C
      19  FORMAT (A7)
      20  FORMAT (F3.0,F5.3,2F6.2,F3.0,F4.2,11,F4.2,11)
      21  FORMAT (1H1,25X,7HRATING,25X,*FIELD POINT*/,9X,
               1CYLINDER SEPARATION CYLINDER DIAMETER*,14X,1HX,
               27X*IH*/1H *INCHES*,9X,F5.2,17X,F5.4,9X,4FEET,4X,
               3F6.2,3X,F5.3,//1H *METERS*,9X,F5.4,16X,F7.6,AX,
               4*METERS*,2X,F6.3,3X,F6.4,5X,*THEITA = *F4.0//,5X,
               5*AL = *E9.2)
      22  FORMAT (1H,7E15.6,3X,12,3X,12,3X,E15.6)
      23  FORMAT (1H1,1H,GT,GTE,GT0,7//)
      24  FORMAT (1H,4E15.6)
END
SUBROUTINE FOURIER (X,Y,LIN,DELTX,NBLOK1,UL,V,CABL,PHI)
C
---FOURIER TRANSFORM
COMMON /CONST/ INDCATE,TWOPI,INDT,INDF
DIMENSION X(1),Y(1),LIN(1),DELTX(1),U(1),V(1),
CAVL(1),PHI(1)
K=0$KPT=INDT
IF (INDICATE.EQ.7HF0FFREQ) GO TO 1
   K=1$KPT=INDF
   DO 7 K=1,KPT
      ACOMP=0.$HCOMP=C.$TWOP1UK=TWOPI*U(K)
      DO 5 L=1,NBLOK1
         ADDD=0.$M0DD=0.$SAVEN=0.$REVEN=0.
         M1=LIN(L)$MN=L1N(L+1)$M02=2*M0$MN2=2*MN
         M1=M02+2$M2=MN-2$DETLX=DELTX(L)
         CA=COS(TWOP1UK*X(M1))$SA=SIN(TWOP1UK*X(M1))
         CP=COS(TWOP1UK*2.$DETLX)$SB=SIN(TWOP1UK*2.$DETLX)
         C7=CA*SSZ=SA
      DO 2 M=M1,M2,2
        

A0DD = A0DD + Y(2*M-1) * CZ
H0DD = H0DD + Y(2*M-KB) * SZ
CA = CZ $ SA = SZ = CZ = CA * CB = SA * SB
S7 = SA * CH + CA * SB
M1 = M1 - 1
CA = COS(TWOPIUK * X(M1)) * SA = SIN(TWOPIUK * X(M1))
CZ = CA $ SZ = SA
DU = 3 M = M1 * M2 * 2
AEVEN = AEVEN + Y(2*M-1) * CZ
BEVEN = BEVEN + Y(2*M-KB) * SZ
CA = CZ $ SA = SZ $ CZ = CA * CB = SA * SB
SZ = SA * CH + CA * SB
ZETAMO = TWOPIUK * X(MO) $ ZETAMN = TWOPIUK * X(MN)
C70 = COS(ZETAMO) $ CZN = COS(ZETAMN)
S70 = SIN(ZETAMO) $ SZN = SIN(ZETAMN)
A0DD = A0DD + 5 * Y(MU2 - 1) * CZ0 = Y(MN2 - 1) * CZN
B0DD = B0DD + 5 * Y(MU2 - KB) * SZ0 + Y(MN2 - KB) * SZN
ASUM = Y(MN2 - 1) * SZN - Y(M02 - 1) * SZ0
BSUM = Y(MN2 - KB) * CZN - Y(M02 - KB) * CZ0
TH = TWOPIUK * DELTXL * TH2 = TH * TH
IF (AHSF(TH) LT .1) GO TO 4
STHDTM = SIN(TH) / TH5 CTHM = COS(TH)
ALPHA = 1 + STHDTM * CTH = 2 * STHDTM * STHDTM / TH
BETA = 2 * (1 + CTH * CTH = 2 * STHDTM * CTH) / TH2
GAMMA = 4 * STHDTM * CTH / TH2 * GOTO 75
4
TH = TH2 * TH2
ALPHA = 2 * TH2 / 45 * (1. - 1 TH2 / 7. + TH4 / 105.)
BETA = 2 / 3. + 2 * TH2 / (1. / 15. - 2 * TH2 / 105. + TH4 / 567.)
ACOMP = ACOMP + DELTXL * (ALPHA * ASUM + BETA * A0DD + GAMMA * AEVEN)
5
BCOMP = BCOMP + DELTXL * (-ALPHA * ASUM + BETA * B0DD + GAMMA * BEVEN)
IF (INDICATE .EQ. 7 .FOFTIME) GO TO 6
CA6L(K) = 4 * ACOMP & PHI(K) = 4 * BCOMP
V(K) = 2 * (ACOMP + BCOMP) * GOTO 90
6
V(2*K-1) = ACOMP & V(2*K) = BCOMP
7
CONTINUE
RETURN
END
APPENDIX B

LISTING OF COMPUTER PROGRAM FOR OBLIQUE INCIDENCE

The computer program for computing the frequency and time domain results for the case of oblique incidence is listed here. The program is written in Fortran IV for use on a CDC 6400 computer.

```
PROGRAM AR9ANG(INPUT, OUTPUT, TAPE2)
COMMON /CONST/ INDCATE, TWOPI, INDT, INDF
DIMENSION F(660), ZP(660), Z(660), LN(10), DEL(10),
IFF(660), SY(100), SYY(100), T(205), GT(205), GTE(205),
2GTO(205)
COMPLEX SUMTC, SUMT, SUM, SMT, SMCT, C1, C2, E, EI, G, G(660),
1II, FW, PST, SM, FTMB
REAL K, JO, K2, KSTAY
CALL INITIAL (8H3950283K, 2, 0.3, 0, 0)

CONSTANTS
C
C C = 299792500, SPI = 3.141592654, TWOP = 6.283185308
C
C FOURIER BLOCK
C
LN(1) = 1, LN(2) = 11, LN(3) = 21, LN(4) = 31, LN(5) = 41, LN(6) = 51,
LN(7) = 61, LN(8) = 457, DEL(1) = 1, DEL(2) = 9, DEL(3) = 90,
DEL(4) = 900, DEL(5) = 9000, DEL(6) = 90000, DEL(7) = 250000,
N3 = 8
DEL(8) = 10.**7, LN(9) = 537
INDF = LN(N3 + 1)
T(1) = 0, TINC = 5.E-10, INDT = 201
READ 29, INDCATE

FREQUENCY AND TIME BLOCK

F(1) = J,
DO 1 L = 1, N3
```

39
NS = LN(L+1) - 1
NA = LN(L)
OI 1 N=NA, NS
F(N+1) = F(N) + DEL(L)
OI 2 I = 2, INOT
T(I) = T(I-1) + TING
T(INOT+1) = 0.
T(INOT+2) = 12.5 * 10. ** -9

C
READ 30, DI, AI, XF, YF, THETA, ALV, IALE, BEV, IBEE, IPRNT
AL = IALE ** BE = IBEE$AL = ALV + 1$. ** IALE$BE = BEV + 10$. ** IBEE
AL2 = AL*AL$BE2 = BE*BE
T(1) = ALOG(AL/BE)/(AL-BE)$CNM=1./EXP(-AL*TM)-EXP(-BE*TM))
D = DI*254$A = AI*0.254$X = XF*3048$Y = YF*3048
I1 = CMPLX(0., 1.)
PRINT 31, DI, AI, XF, YF, D, A, X, Y, THETA, AL, BE
A = A/2.
PRINT 32
IF (IPRNT.EQ.0) GO TO 3
PRINT 33
3 TPO = TWOPID
ATP0L = 2.*Aalog(A*TP0)
THETR = THETA*PI/180.
STA = SIN(THETR)
STB = COS(THETR)
STAY = STA*Y
XSTB = X*STB
AGTM = XSTR*Y*STA
TPOX = TPO*X
TPOY = TPO*Y
F(1) = 1.
OI 17 I = 1, INDF
W = TWOPIF(I)
K = W/C
W2 = W*W$DNA = AL2+W2$DNB = BE2+W2$CR = CNM*(AL/DNA- BE/DNB)
CI = -CNM*(W/DNA-W/DNB)
FW = CMPLX(CR, CI)
XK = K/AGTM
EI = CMPLX(COS(XK),-SIN(XK))
ZS = K/TPO
FTM9 = CMPLX(0., 1./ZS*STB))
ZS2 = ZS*ZS
ZSSN = ZS*STA
KSTAY = K*STAY
C
X<STB = XSTB*K
ZSSTB = STB*7S
V1 = -SIN(X<STB)/ZSSTB
V2 = -COS(X<STB)/ZSSTB
SJMT = CMPLX(V1, V2)
SJMT=CMPLX(1000.,0.)
DJ 6 NNT=1,100
FNNT=NNT
TPDY=TPDY*FNNT
ARSR=(FNNT-ZSSN)**2-ZS2
IF (ARSR.LT.0.) GO TO 15
ROOT=SQRT(ARSR)
SJ=CMPLX(COS(TPDYM),SIN(TPDYM))
SJM=SUM*EXP(-TPD*XROOT)/ROOT+SUMT
FNNT=-NNT
TPDY=TPDY*FNNT
ARSR=(FNNT-ZSSN)**2-ZS2
IF (ARSR.LT.0.) GO TO 15
ROOT=SQRT(ARSR)
SJ=CMPLX(COS(TPDYM),SIN(TPDYM))
SJM=SUM*EXP(-TPD*XROOT)/ROOT+SUMT
IF (CABS((SUMT-SUMTC)/SUMT).LT.00001.AND.NNT.GE.3) GO TO 7
SJMT=SUMT
CONTINUE
SJCT=CMPLX(0.,0.)
DJ 11 NNF=1,100
FNDF=NNF
ARSP=(FNDF+ZSSN)**2-ZS2
ARSM=(FNDF-ZSSN)**2-ZS2
IF (ARSP.LT.0.) GO TO 13
FR1=1./SQRT(ARSP)
C1=CMPLX(FR1,0.)
IF (ARSM.LT.0.) GO TO 14
FR2=1./SQRT(ARSM)
C2=CMPLX(FR2,0.)
SM=C1+C2-2./FNDF
SMT=SM+SMT
IF (CABS(SMT).EQ.0.) GO TO 10
IF (CABS((SMT-SMTC)/SMT).LT.00001) GO TO 12
SMT=SMT
CONTINUE
C1=CMPLX(COS(KSTAY),-SIN(KSTAY))
E=E+C1*SUMT/(ATPOL+FTM3-SMT)
GO TO 16
FR1=1./SQRT(-ARSP)
C1=CMPLX(0.,-FR1)
GO TO 8
FR2=1./SQRT(-ARSM)
C2=CMPLX(0.,-FR2)
GO TO 9
ROOT=SQRT(-ARSR)
BIGA=TPDY-TPDX*ROOT
SJ=CMPLX(SIN(BIGA),-COS(BIGA))
SJM=SUM/ROOT+SUMT
IF (FNNT.LT.0.) GO TO 5
GO TO 4

C
16 G(I)=CONJG(E*FW)
PST=CMPLX(COS(XK),-SIN(XK))
G(I)=G(I)*PST
EM=CABS(E)
FF(I)=20.*ALOG10(EM)
IF (FF(I).LT.-120.) FF(I)=-120.
GMAG=CABS(G(I))
FWM=20.*ALOG10(CABS(FW))
IF (IPRINT.NE.0) PRINT 34, F(I),E,FF(I),G(I),GMAG,NNT,
1NNF,FWM

17 CONTINUE
F(I)=0.,G(I)=(0.,0.)
CALL FOURIER (F,G,LN,DEL,N3,T,GT,GTE,GTO)
PRINT 35
PRINT 36, (T(I),GT(I),GTE(I),GTO(I),I=1,INDT)

C
C PLOT BLOCK
C
INPT=457
DO 18 I=2,457
ZP(I)=ALOG10(F(I))
18 Z(I)=F(I)/1000000.
Z(1)=Z(2)
ZP(1)=ZP(2)
CALL PLOT (0.,-11.,-3)
CALP PLOT (0.,-2.,-3)
ICY=9
ICYM=ICY-1
ITOT=9*ICYM
DO 20 L=1,ICYM
LC=L-1
DO 19 M=1,9
LM2=M+LC*9
SY(LM2)=ALOG10(1.+M)+LC
19 CONTINUE
20 CONTINUE
DO 21 N=1,6
SYY(M)=1.+M
21 CONTINUE
IP=INPT+1
IPP=IP+1
ZP(IP)=0.
ZP(IPP)=1.
Z(IP)=0.
Z(IPP)=12.5
IPLT=1
FF(IP)=-120.
FF(IPP)=20.
INOF=INPT
CALL LINE (ZP,FF,INOF,1,0,2)
CALL SYMBOL (1.,1.,1,13HD IN INCHES =,0.,13)
CALL NUMBER (2.2,1.,0,1,DI,0.,2)
CALL SYMBOL (1.8,0.84,.1,5HAL =,0.,5)
CALL NUMBER (1.45,0.84,.1,ALV,0.,2)
CALL SYMBOL (1.91,0.84,.1,1HE,0.,1)
CALL NUMBER (2.45,0.84,.1,BEV,0.,2)
CALL SYMBOL (2.01,0.84,.1,5HBE =,0.,5)
CALL NUMBER (2.82,0.84,.1,1HE,0.,1)
CALL NUMBER (2.92,0.84,.1,1BEV,0.,1)
CALL SYMBOL (1.,.68,.1,11HX IN FEET =,0.,11)
CALL NUMBER (2.0,.68,.1,XF,0.,2)
CALL SYMBOL (1.,.52,.1,11HY IN FEET =,0.,11)
CALL NUMBER (2.0,.52,.1,YF,0.,2)
CALL SYMBOL (1.,.36,.1,17HINCIDENCE ANGLE =,0.,17)
CALL NUMBER (2.6,.36,.1,THETA,0.,1)
CALL PLOT (0.,0.,3)
IF (IPLT.EQ.3) GO TO 26
DO 2 3 0=1,6
CALL SYMBOL (0.,SYY(J>,.1,13,90.,-2)
CONTINUE
CALL SYMBOL (.38,0.,1,4H-120,0.,4)
CALL SYMBOL (.38,1.,1,4H-100,0.,4)
CALL SYMBOL (.38,2.,1,4H- 80,0.,4)
CALL SYMBOL (.38,3.,1,4H- 60,0.,4)
CALL SYMBOL (.38,4.,1,4H- 40,0.,4)
CALL SYMBOL (.38,5.,1,4H- 20,0.,4)
CALL SYMBOL (.38,6.,1,4H- 0,0.,4)
CALL SYMBOL (.5,2.,1,123HFREQUENCY RESPONSE (DB),19.,23)
CALL PLOT (0.,0.,3)
IF (IPLT.EQ.2) GO TO 25
DO 24 J=1,ITOT
CALL SYMBOL (SY(J),0.,1,13,0,-2)
CONTINUE
CALL SYMBOL (0.,.225,.1,1H0,0.,1)
CALL SYMBOL (1.,.225,.1,1H1,0.,1)
CALL SYMBOL (2.,.225,.1,1H2,.0.1)
CALL SYMBOL (3.,.225,.1,1H3,0.,1)
CALL SYMBOL (4.,.225,.1,1H4,0.,1)
CALL SYMBOL (5.,.225,.1,1H5,0.,1)
CALL SYMBOL (6.,.225,.1,1H6,.0.1)
CALL SYMBOL (7.,.225,.1,1H7,.0.1)
CALL SYMBOL (8.,.225,.1,1H8,0.,1)
CALL SYMBOL (2.5,38.,1,25HFREQUENCY, HZ (POWERS OF,14TEN),1.,29)
CALL PLOT (11.,0.,-3)
IPLT=2
GO TO 22
CALL LINE (Z,FF,INDF,1,0,2)
CALL AXIS (0.,0.,14HFREQUENCY, MHZ,-14,8.,.,0.,12.5, 112.5)
CALL PLOT (11.,0.,-3)
I=LT=3
CALL SYMBOL (6.,6.,1,13HD IN INCHES =,6.,13)
CALL NUMBER (7.2,6.,1,0I,0.,2)
CALL SYMBOL (6.44,5.84,1,ALV,0.,2)
CALL SYMBOL (6.81,5.84,1,1HE,0.,1)
CALL NUMBER (6.91,5.84,1,ALE,0.,-1)
CALL SYMBOL (7.21,5.84,1,5HBE =,0.,5)
CALL NUMBER (7.65,5.84,1,9EV,0.,2)
CALL SYMBOL (8.02,5.84,1,1HE,0.,1)
CALL NUMBER (8.12,5.84,1,9EE,0.,-1)
CALL SYMBOL (6.5,6.8,1,11HX IN FEET =,0.,11)
CALL NUMBER (7.0,5.68,1,11XF,0.,2)
CALL SYMBOL (6.5,5.52,1,11HY IN FEET =,0.,11)
CALL NUMBER (7.5,5.52,1,11YF,0.,2)
CALL SYMBOL (6.5,5.36,1,17HINCIDENCE ANGLE =,0.,17)
CALL NUMBER (7.6,5.36,1,THETA,0.,1)
CALL SCALE (GT,6.,INDT,1,10.)
AS=-GT(INDT+1)/GT(INDT+2)
CALL AXIS (0.,0.,13HTIME RESPONSE,13,6.,90.,GT(INDT+1) 1,GT(INDT+2),10.)
XAS=y.
DO 27 J=1,10
XAS=XAS+.8
27 CALL SYMBOL (XAS,AS,1,13,*,1,-1)
CALL SYMBOL (3.5,-.5,1,9HTIME (NS),0.,9)
CALL PLOT (0.,AS,3)
CALL PLOT (8.,AS,2)
CALL PLOT (0.,0.,3)
FPN=y.
XAS=-.1
DO 28 J=1,10
XAS=.8*XAS
FPN=FPN+1)
ASM=AS-.35
28 CALL NUMBER (XAS,ASM,1,FPN,0.,-1)
CALL PLOT (0.,0.,3)
CALL LINE (T,GT,INDT,1,0,0)
CALL PLOT (0.,0.,999)
C
STOP
C
29 FORMAT (A7)
30 FORMAT (F3.0,F5.3,2F6.2,F3.0,F4.2,I1,F4.2,2I1)
31 FORMAT(1H1,25X,7HGRATING,25X,*FIELD POINT*,/.9X, 1*CYLINDER SEPARATION CYLINDER DIAMETER*,14X,1HX, 27X,1HY,/.1H *INCHES*,9X,F5.2,17X,F5.4,9X,4HFEET,4X,
END

SUBROUTINE FOURIER (X, Y, LIN, DELTX, NBLOKI, U, V, CARL, PHI)

C COMMON /CONST/ INDCATE, TWOPI, INDT, INDF
1CARL(1), PHI(1)
DIMENSION X(1), Y(1), LIN(1), DELTX(1), U(1), V(1)
K3=0 $KPT=INDT
IF (INDCATE.EQ.7HF0FFREQ) GO TO 1
K3=1 $KPT=INDF

1 DO 7 K=1,KPT
ACOMP=0. $3COMP=0. $2WOPIUK=TWOPI*U(K)
DO 5 L=1,NBLOKI
AODD=0. $3ODD=0. $2EVEN=0. $3EVEN=0.
M0=LIN(L) *MN=LIN(L+1) *M02=2*M0 *MN2=2*MN
M1=M0+2 $M2=MN-2 $DELTXL=DELTX(L)
CA=COS(TWOPIUK*X(M1)) $SA=SIN(TWOPIUK*X(M1))
C3=COS(TWOPIUK*2.*DELTXL) $S3=SIN(TWOPIUK*2.*DELTXL)
C7=CA $SZ=SA
DO 2 M=M1,M2,2
AADO=AADO+Y(2*M-1)*CZ
3ODD=3ODD+Y(2*M-K3)*SZ
CA=CZ*SA=SZ*CZ=CA*CB-SA*SB
2 DO 3 M=M1,MN,2
AODD=AODD+Y(2*M-1)*CZ
3EVEN=3EVEN+Y(2*M-K8)*SZ
CA=CZ*SA=SZ*CZ=CA*CB-SA*SB
3 DO 3 M=M1,MN,2
ZETAMG=TWOPIUK*X(M0) $ZETAMN=TWOPIUK*X(MN)
CZ=COS(ZETAMG) $CZN=COS(ZETAMN)
SZ0=SZ=COS(ZETAMG) $SZN=SIN(ZETAMN)
A3DD=A3DD+*Y(M0-2-1)*CZ+Y(MN2-1)*CZN
93DD=93DD+*Y(M02-KB)*SZ0+Y(MN2-KB)*SZN
ASUM=Y(MN2-1)*SZN-Y(M02-1)*SZ0
BSUM=Y(MN2-KB)*CZN-Y(M02-KB)*CZ0
T4=TWOPIUK*DELTXL $TH2=TH*TH
IF (ABSF(TH).LT..1) GO TO 4
STHDTH=SIN(TH)/TH $CTH=COS(TH)
ALPHA=(1.+STHDTH*CTH-2.)*STHDTH*STHDTH)/TH
\begin{verbatim}
BETA=2.*((1.+CTH*CTH-2.*STH*CTH)/TH2
GAMMA=4.*((STH*CTH)/TH2)$GOT075
4
TH4=TH2*TH2
ALPHA=2.*TH2/45.*((1.-TH2/7.+TH4/105.)
BETA=2./3.+2.*TH2*(1./15.-2.*TH2/105.+TH4/567.)
GAMMA=4./3.+TH2/15.*(-2.+TH2/14.-TH4/756.)
ACOMP=ACOMP+DELTXL*(ALPHA*ASUM+BETA*AODD+GAMMA*AEVEN)
B3 OMP=B3 OMP+DELTXL*(-ALPHA*ASUM+BETA*BODD+GAMMA*B3 EVEN)
IF (INDCATE.EQ.7HF0TIME) GO TO 6
CABL (K)=4.*ACOMP$PHI(K)=4.*BCOMP
V(K)=2.*ACOMP$GOT090
V(2*K-1)=ACOMP$V(2*K)=BCOMP
7 CONTINUE
RETURN
END
\end{verbatim}
REFERENCES


