FEASIBILITY STUDY FOR A PATIENT "IN BED" WEIGHING SYSTEM

by

Timothy Kane McFarlan

A Thesis Submitted to the Faculty of the

DEPARTMENT OF ELECTRICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

1 9 7 9

STATEMENT BY AUTHOR

This thesis has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: Timothy McFarlan

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

Kenneth C. Mylrea — Nov 30, 1979

KENNETH C. MYLREA — Date

Associate Professor of Electrical Engineering

PREFACE

The purpose of the research presented in this thesis is to investigate an alternative method of obtaining patient weight for use in hospitals. Presently used techniques are cumbersome, traumatic to patients, or are too expensive for widespread hospital use. The concepts that promoted this research were provided by Warren Jewett, of the Engineering Experiment Station and Department of Anesthesiology. Special acknowledgment is given to Richard Harris, M. D., for the opportunity provided me to spend time in the Special Care Nursery of the Health Sciences Center which provided insight to the needs of the neonatal patient.

TABLE OF CONTENTS

TABLE OF CONTENTS, Continued.

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS, Continued.

LIST OF TABLES

ABSTRACT

The purpose of this thesis is to investigate the feasibility of using the pressure produced inside a water mattress as a means of determining applied weight. This technique would be applied in hospitals where continuous weight monitoring would best benefit neonates, dialysis patients, burn victims, or any other bedridden patients whose weight balance is critical.

Existing patient weighing systems suffer from several drawbacks, including lack of ability to continuously monitor patient weight without spending $4000 to $6000, and the popular mechanical scales are awkward to use for both the nurse and the patient.

Analysis of the hydrostatics of a water mattress reveals pressure is a function of applied force and contact area. To experimentally evaluate the proposed weight measurement system, a transducer survey was conducted and two pressure transducer systems were implemented to evaluate four mattress designs.

Each subsequent mattress design represented the next step in the development of an ideal mattress. The final mattress was able to meet all specifications except sensitivity to location of applied weight and variations in contact area. Failure to design a mattress insensitive to these effects resulted in the recommended abandonment of this weight measuring approach.

CHAPTER 1

INTRODUCTION

Present techniques used in hospitals for obtaining patient weight information range from cumbersome mechanical balance scales to expensive electronic load cell systems. Weight monitoring is of primary importance for patients whose fluid and nutritional balance is critical. Patients in this category include burn victims, neonates and dialysis patients. No convenient, inexpensive and accurate means of obtaining patient weight information is available for hospital use.

For burn victims, fluid balance is very critical due to their susceptibility to loss of large volumes of fluid. This fluid loss is compensated for through intravenous infusion. The ability to monitor weight on a continuous basis would provide the necessary information for adjusting infusion rates to maintain constant body weight. Present weighing techniques involve movement of the patient with or without his bed on a mechanical scale, or a very expensive load cell system is used. These techniques are not readily acceptable and oftentimes fluid infusion is administered by guesswork without any weight information.

Neonates represent another example of the type of patient that would benefit from an improved weight monitoring system. Most hospitals presently weigh their premature babies every 24 hours by removing the baby from its controlled incubated environment, placing the child on a mechanical scale platform and taking a weight reading. Unfortunately, premature babies are susceptible to congestive heart failure and without a very observant nurse, a 100-200 gram weight increase due to retention of fluids over a few hours can go unnoticed. The ability to continuously monitor the weight of these babies without removing them from their incubator is a very attractive alternative. Such a weighing system would eliminate the trauma involved in taking weight measurements and would enable nursing personnel to easily identify weight trends characteristic to congestive heart failure and other diseases.

The type of weighing system used on dialysis patients is a function of the patient's disability. Outpatient dialysis is performed while the patient sits in a chair and is intubated to the dialysis machine. An optimum weight determination is made by a physician for each patient prior to the dialysis procedure. Nurses and dialysis technicians are then responsible for determining the patient's initial weight, and over a four-hour period they must adjust the transmembrane pressure and other artificial kidney variables to accomplish the appropriate weight loss. Invariably, the weight of each patient has to be spot-checked over the four-hour period to assure a reasonable rate of weight loss. This is accomplished using a stand-up mechanical scale which is wheeled over to the patient's chair. The patient with blood pump tubing attached is asked to stand on the scale to verify a normal change in weight. This is typically repeated six times over a four-hour period for each patient.

Acute dialysis patients are dialized in bed. Here weight measurements are taken using a large mechanical scale that lifts the patient and his bed up off the floor. Initial readings without the patient are necessary and more than one person is required to operate the scale. The only other alternative is to use an electronic system where a summation of forces is accomplished by load cells located beneath each leg of the patient's bed to give weight readings. Although the load cell system overcomes some of the patient trauma associated with movement, it is very expensive (i.e., typically $6,000 per bed).

For both outpatient and acute dialysis patients, weighing systems are not meeting the needs of the patient or the medical personnel providing care for the patients. Among patient trauma, expense, convenience of use and lack of continuous weight information, improvements in hospital weighing systems are needed.

The drawbacks cited here with existing weighing systems prompted the research in this thesis. The approach suggested here involves determining weight by monitoring pressure in a special mattress. Water mattresses are used presently in some hospitals strictly for the increased patient comfort and relief from bed sores. Water mattresses also offer the potential of controlling patient temperature by varying the temperature of the water in the mattress. In addition to weight information, sensitive pressure monitoring provides the potential for determination of heart rate and respiration rate through ballistic pressure variations. To accomplish this by merely having a patient lie on a mattress is truly an exciting concept.

CHAPTER 2

GOALS AND OBJECTIVES

The concept proposed in this thesis involves measuring the pressure produced in a deformable water-filled isovolumetric container (i.e., a water bag) and correlating the pressure measurements to the force applied to the container. The goal of this research is to investigate a means of monitoring patient weight on a continuous basis using water pressure measurements in a closed mattress. Because of problems encountered in obtaining weight information during the research stages, the original goals of this thesis had to be altered. Original goals included the design, construction, and clinical testing of a patient weighing system. The weighing system was to also include development of a means of obtaining respiration rate, heart rate, and temperature control as part of the overall system. The intention was to stress the electrical engineering aspects of the transducer and signal conditioning electronics in the development of this monitoring device. The mattress design was conceived to be a small part of the overall investigation. However, well into the experimental stages, considerable difficulty was encountered in the development of a mattress system that accomplished the desired weight/pressure response.

The goals and objectives had to be redefined such that the thesis was devoted to a feasibility study of obtaining weight information using the pressure produced within a deformable water bag.

CHAPTER 3

PRINCIPLES OF HYDROSTATIC PRESSURE MEASUREMENT

Manometric Measurements

This section involves a discussion of the hydrostatic considerations involved in obtaining weight information using water pressure. According to Pascal's Law, "pressure (P) exerted at any point upon a confined liquid is transmitted undiminished in all directions". If an area (A) is specified in a liquid, the pressure acting on that surface produces a force (F), which is a vector of magnitude (P) in a direction normal to the surface (A), as shown in Fig. 1. Pressure (P) is defined as a force (F) applied over a given area (A) [see equation (3.1) below].

$$P = F/A = \frac{whA}{A} = wh \tag{3.1}$$

P = pressure produced as shown in Fig. 1 . . . . (N/m^2)

F = force produced by weight of the water . . . (N)

A = area of the column of water . . . . (m^2)

h = height of the column of water . . . . (m)

w = density of water . . . . (kg/m^3)

In the case of a water column, a pressure is generated by the weight of the water acting as the force applied over an area determined by the container. Notice that according to our definition, the cross sectional area of the container cancels out of Eq. (3.1) and the only

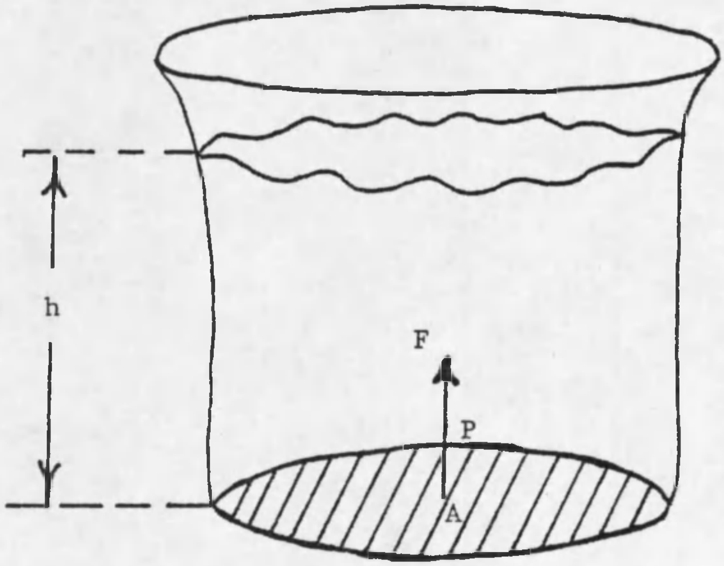


Fig. 1. Hydrostatic Pressure Provided by a Water Column.

With the area (A) defined as shown, the pressure acting on that surface produces a force (F), which is a vector of magnitude (P) in a direction normal to the surface (A).

factors determining pressure are the density of the fluid (w) and the height (h) of the water column. This is why pressure is often expressed as the height of a column of fluid (e.g., mm of mercury or cm of water).

A piston can be used to demonstrate the linear relationship between force and pressure that can be achieved using water pressure (see Fig. 2).

Here we have an additional externally applied force (F') involved in the pressure computation. The resulting pressure can be calculated as given in Eq. (3.2) below.

$$P = \underbrace{wh}_{P_1} + \underbrace{F'/A}_{P_2} \tag{3.2}$$

P = Total resultant pressure . . . . (kg/m^2)

F'= force produced by weight of the water . . . . (N)

A = area of the column of water . . . . (m^2)

h = height of the column of water . . . . (m)

w = density of fluid . . . . (kg/m^3)

The specific density of water multiplied by the fluid is the pressure caused by the column of water and the second term (F'/A) is the pressure resulting from the externally applied force. In the calculations used here, water will be considered an incompressible fluid, thus the column height (h) remains a constant as does the resulting pressure (wh). Defining a given pressure (P) as reference zero is accomplished by nulling out the pressure due to the column of water. This allows us to focus our attention on the pressure caused by the

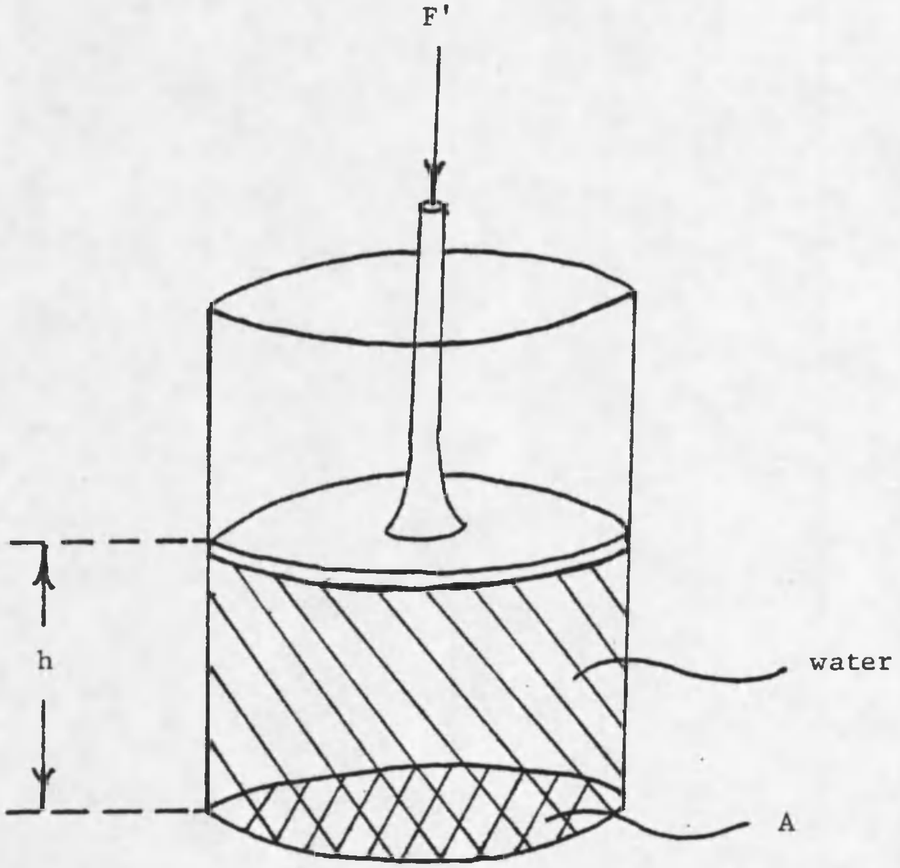


Fig. 2. Piston Pressure System.

An external force (F') applied to a piston creates a pressure which is a function of height (h) of the water column and contact area (A).

externally applied force (F'). Figure 3 shows a typical force-pressure relationship for a piston as a function of cross sectional area (A). Note that the point of origin is the initial pressure of a piston with zero external force applied, therefore only the pressure of the water column, which we have defined as zero, remains. The above analysis can be applied to the proposed weighing system. If a patient's weight is used as the external force as depicted in Fig. 4, the pressure in the piston can be used to determine the applied external force knowing the cross sectional area of the piston.

Incorporating a piston system into a practical patient weighing system involves several drawbacks attributed to the platform that would support the patient. To accommodate patient comfort, a mattress or cushion would have to be placed under the patient which would effectively dampen out any high frequency motility information such as ballistic pressure responses from heart and respiration rate. The loss of temperature control would also be sacrificed for patient comfort. Other problems with this technique would include mechanical friction losses and stability of the platform. Similar problems would be encountered if bellows or spring constant devices were used. These problems can be overcome through the use of a water mattress. Weight determination through direct monitoring of the pressure in the mattress is much more desirable, and the potential for monitoring respiration rate, heart rate, and regulating body temperature is maintained.

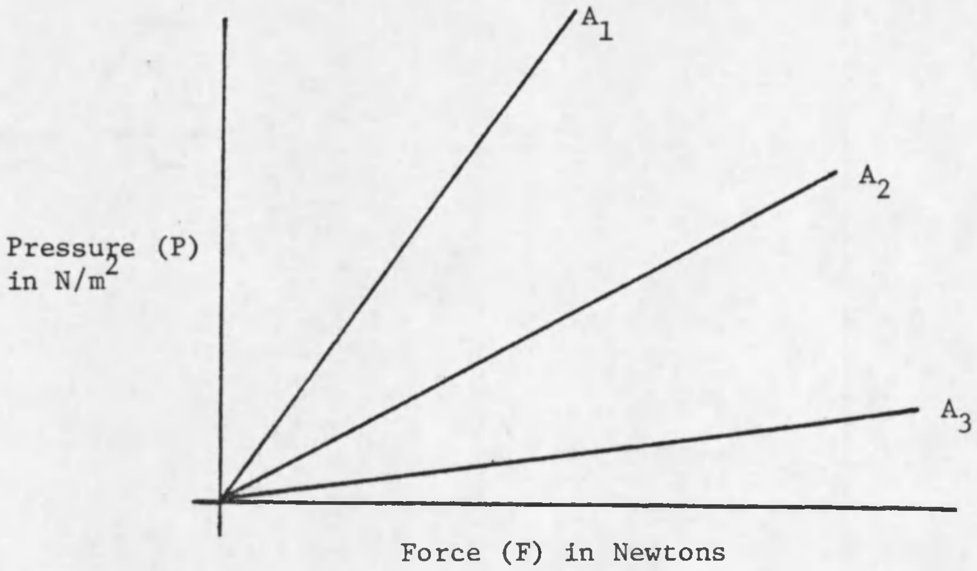


Fig. 3. Force/Pressure Relationship for a Piston.

Graph of a typical force/pressure relationship for a piston as a function of cross sectional area where $A_1 < A_2 < A_3$.

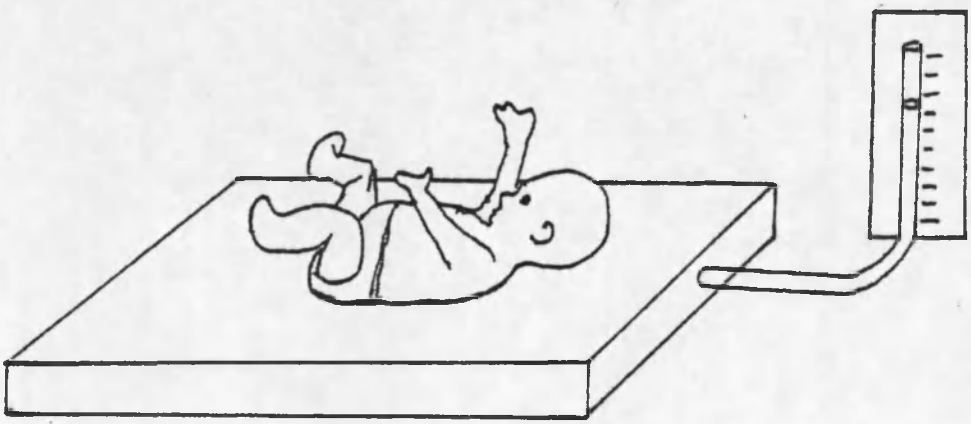


Fig. 4. Conceptual Diagram of Weighing System.

Illustration of proposed weighing technique. Child produces a force on the water mattress which produces a pressure increase reflected by the manometer.

Hydrostatics of a Water Bag

The hydrostatic pressure found in a water mattress can be considered equivalent to that within a spherical water bag. In order to discuss the hydrostatic principles of a spherical water bag, the assumption that we are working with an isovolumetric container (i.e., the bag doesn't stretch) must be made. However, in this case the walls of the container are deformable. The deformability of the walls of the water bag introduces a significant modification to the force-pressure relationship given earlier for the piston in Eq. (3.2). The cross sectional area (A) of the rigid cylinder wall in the piston system is easily defined. However, with the spherical water bag we encounter a more complicated cross section. In the free standing water bag, the hydrostatic pressure of the water causes the bottom of the water bag to flatten out forming a contact area with the supporting surface. The magnitude of this contact area is a function of the size of the water bag membrane and the amount of water in it. The pressure in the free-standing water bag is proportional to the height of the water column as shown in Fig. 5. As more water is forced into the water bag, a smaller bottom contact area and a higher initial pressure results. This is reflected in the relative heights (h_1 and h_2) of the two water bags.

To use a spherical water bag as part of a patient weighing system the hydrostatic effects of applying a force to the surface of the water bag must be determined. To do this, it is necessary to start with a system model depicting the measurement technique proposed here. This is shown in Fig. 6.

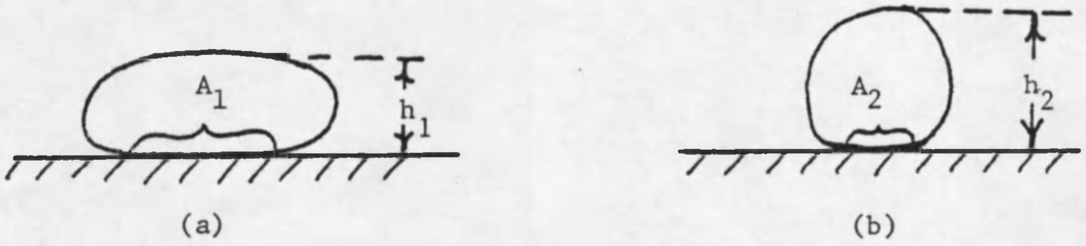


Fig. 5. Initial Pressure of a Water Bag.

The water mattresses in (a) and (b) demonstrate that relative initial pressures are depicted by the height and contact area of the mattress. Here (b) has a higher initial pressure than the mattress in (a).

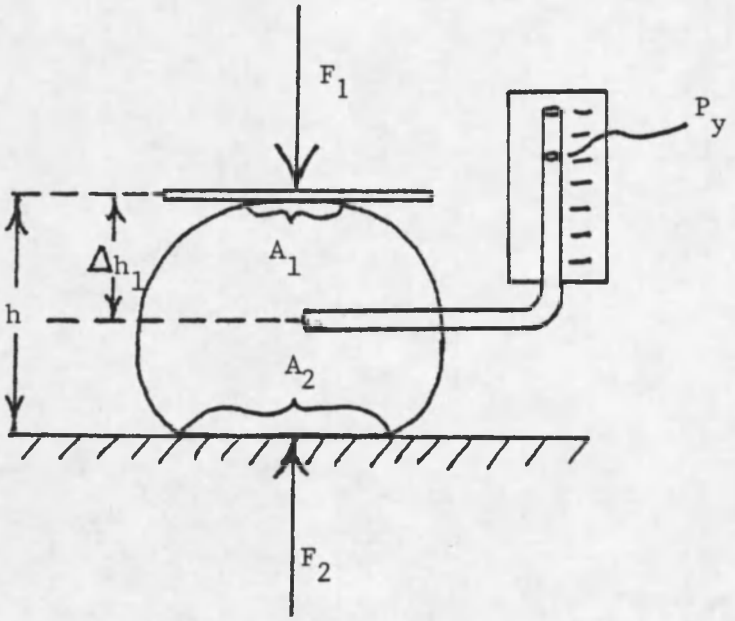


Fig. 6. Model Depicting Forces and Pressures Involved in Weighing System.

Pressure model depicting the proposed weighing system with weight F_1 applied.

With the application of force F_1, a contact area A_1 is formed with the mattress surface. Pressure P_y is measured here using a manometer through a port in the water bag located a distance Δh_1 from the top of the mattress. The manometer remains in a fixed position relative to the mattress so that no hydrostatic effects of vertical displacement are encountered.

The relationship between F_1, A_1, and F_2, A_2 is given in Eq. (3.3) below.

$$F_2/A_2 = F_1/A_1 + wh \tag{3.3}$$

F_1, F_2 = Forces acting in the direction shown in Fig. 6 (N)

A_1, A_2 = Contact areas as shown in Fig. 6 (m^2)

w = Specific density of the fluid in the mattress of Fig. 6 (kg/m^3)

h = Height of the mattress of Fig. 6 (m)

This relationship holds due to the continuous properties of water as was stated earlier in Pascal's Law. This means that the pressure at the bottom of the mattress is equal to the pressure at the top of the mattress plus the hydrostatic pressure caused by the height of the water mattress.

The pressure P_y on the system shown in Fig. 6 is derived in Eq. (3.4) below.

$$P_y = F_1/A_1 + w\,\Delta h_1 = F_2/A_2 - w(h-\Delta h_1) \tag{3.4}$$

The effect of changes in F_1 and A_1 on P_y can be determined by applying partial differentiation to P_y as given in Eqs. (3.5) through (3.7).

$$d\,P_y = \left(\frac{\partial\,P_y}{\partial\,F_1}\right)_{A_1} d\,F_1 + \left(\frac{\partial\,P_y}{\partial\,A_1}\right)_{F_1} d\,A_1 \tag{3.5}$$

$$= (1/A_1)\,d\,F_1 + -\,(F_1/A_1^{\,2})d\,A_1 \tag{3.6}$$

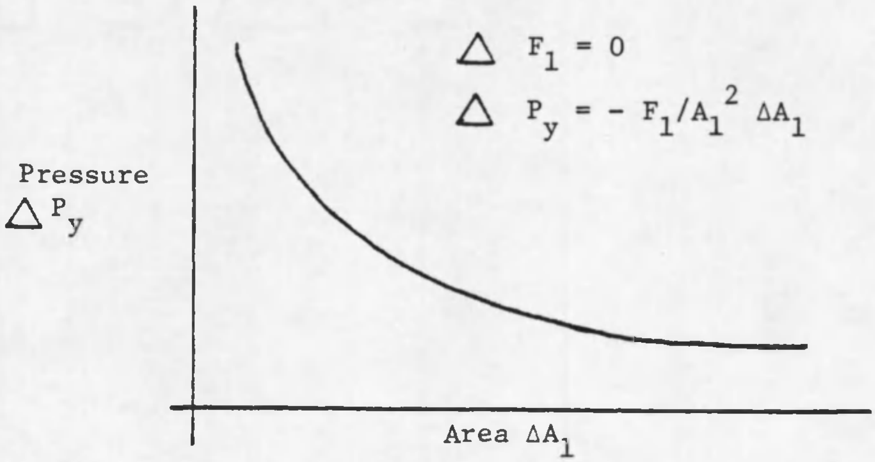
$$\int d\,P_y = \Delta\,P_y = (1/A_1)\Delta\,F_1 - (F_1/A_1^{\,2})\Delta\,A_1 \tag{3.7}$$

$$= (1/A_1)\Delta\,F_1 - [(F_1/A_1)(\Delta A_1/A_1)] \tag{3.8}$$

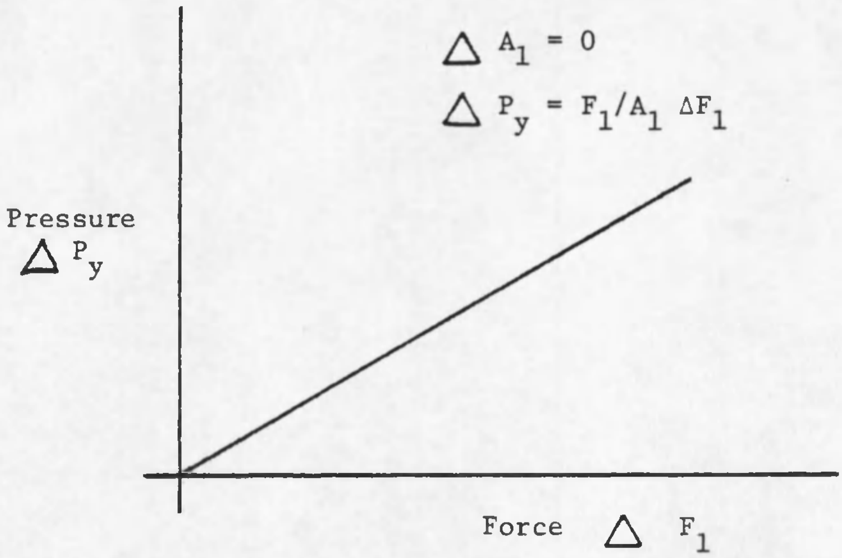
If force F_1 changes (by an amount ΔF_1), the effects of ΔF_1 on pressure P_y is determined by differentiating P_y. The relationship given in Eq. (3.7) demonstrates the combined effect of both ΔF_1 and ΔA_1 on ΔP_y. If we isolate the two terms by letting ΔF_1 be zero and varying ΔA_1, the relationship shown in Fig. 7a results. Furthermore, for the second case, if we let ΔA_1 be zero and vary ΔF_1, the relationship shown in Fig. 7b results.

Figure 7a demonstrates an exponential relationship between the change in pressure ΔP_y and a change in area ΔA_1. Equation (3.7) is rewritten in the form given in Eq. (3.8), it can be seen that for an incremental increase in area A_1, the pressure P_y will be reduced by an amount proportional to the fractional change in area (i.e., $\Delta A_1/A_1$).

Figure 7b demonstrates a linear correlation between pressure P_y and force F_1 provided contact area A_1 remains constant. This type of pressure response is illustrated by the piston as previously discussed. A direct correlation between pressure and applied force as suggested



(a) With a constant applied force (F_1) any change in contact area A_1 causes a change in pressure P_y as illustrated.



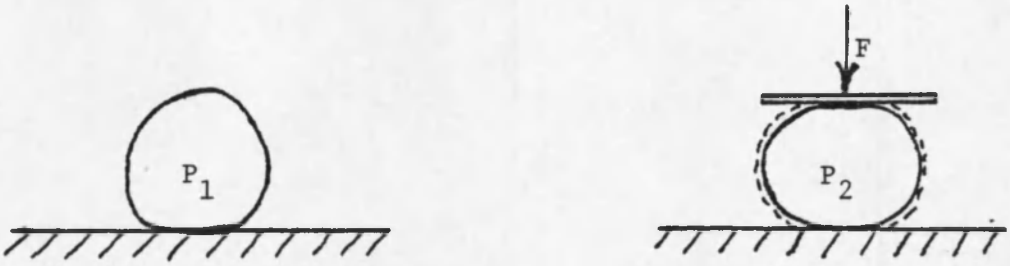
(b) With a constant contact area A_1, changes in force F_1 cause a change in pressure P_y as shown.

Fig. 7. Graphical Illustration of the Two Components of Eq. (3.4).

by Fig. 7b to the mattress would prove that the proposed weighing technique is viable. The effect of variations in contact area that the patient has with the mattress would prove that the proposed weighing technique is viable. The effect of variations in contact area that the patient has with the mattress must be evaluated experimentally. The significance of this error is discussed later in Sections VI and VII of the text.

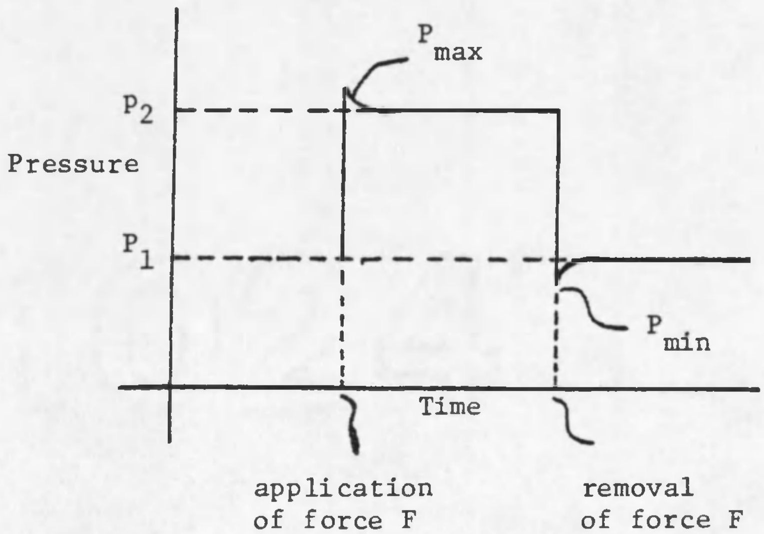
Materials

Up to this point we have assumed our sphere is isovolumetric. Using real life materials it is impossible to find a membrane that has a modulus of elasticity of zero. This implies that there is always some wall stretching associated with the changes in wall stress that occur with variations in pressure inside the sphere. The effects of membrane elasticity can be demonstrated by observing the cross section of our sphere shown in Fig. 8. In Fig. 8a, we have a water bag with initial pressure P_1. Monitoring pressure P_1 demonstrates a constant stable pressure indicating that the forces in the membrane are equilibrated and the volume remains unchanged (see Fig. 8c). When an external force is applied as shown in Fig. 8b, we have an immediate increase in pressure to P_{max}. This increase in pressure causes an increased tension in the walls of our water bag. As a result, the walls stretch, increasing the volume of the bag causing a reduction in pressure as shown in Fig. 8c. With time the tension in the wall equilibrates with the pressure P_2.



(a) Water bag with initial pressure P_1.

(b) Application of force F_1 to the mattress causes stretching of the mattress membrane.



(c) Pressure measurements resulting from the application and removal of force F_1 show the elastic effect of the mattress material.

Fig. 8. Illustration of the Effects of Elasticity of the Mattress Membrane.

For the materials used in this study, pressure stabilization times were found to take several seconds. Because of this problem, pressure readings were taken approximately 10 seconds after weight was applied. It was found that waiting longer time periods resulted in random drifting of pressure readings due to system sensitivity limitations unrelated to the characteristic exponential decay in pressure caused by stretch. When the weight was removed, the mattress exhibited a shrinking or recovery as a result of the reduced stress on the walls of the materials. The material exhibits this hysteresis effect consistently providing the elastic limit of the material is not exceed (i.e., Hooke's Law). The elastic limit is a function of the type of material used and the thickness of the material. For a given mattress, the volume capacity of the bag determines the maximum filling pressure or the maximum height of the water column. Referring to Fig. 8, if too much pressure is applied to the mattress and the elastic limit exceeded, the mattress will recover to a pressure lower than P_1 and remain there with the removal of weight. This indicates a permanent increase in the initial volume of the mattress due to excessive stretching. To avoid the problem of exceeding the elastic limit of the mattress, a preliminary pressure study was conducted on each mattress to determine whether or not the maximum weight to be used (typically 10kg) would exceed the elastic limit.

It was found there are essentially two kinds of drift. The first has already been discussed (referred to as short term drift) and is due to the stretching of the mattress, which takes time to stabilize after weight is added or removed from the mattress. The second form of drift is referred to as long-term drift and encompasses the effects of thermal expansion of materials and thermal electronic drift of the measurement system. The effects of short-term error can be compensated for by taking pressure measurements ten seconds after the weight is applied or removed to allow for stabilization. Long-term drifting ultimately determines the system's maximum achievable degree of resolution. The transducer-pressure measurement system was designed for minimum drift and maximum resolution and will be discussed in Chapter 5.

In summary, the principles of measurement discussed to this point describe the force-pressure relationship of water bags. The extent that these principles affect actual weight measurements is impossible to determine from a theoretical standpoint and the feasibility of the proposed weighing system must be evaluated further using experimental means. Before any preliminary tests can be conducted or mattresses be designed, test criteria has to be developed and a pressure transducer system must be chosen.

CHAPTER 4

DEFINITION OF MATTRESS DESIGN CRITERIA

The mattresses used here must exhibit the necessary force-pressure relationship and provide patient comfort and safety to meet the requirements demanded in a hospital environment. Each mattress design will be subject to the test criteria listed below.

Test Criteria

1. Patients can naturally be expected to move while lying in bed. Patient movement causes shifts in weight distribution over different portions of the mattress. For this reason, the pressure-weight characteristics of the mattress must exhibit immunity to the location of applied weight. For example, if 4 kgs is placed in the geometric center of the mattress, ideally the same pressure should be produced when 4 kgs is placed in one corner of the mattress. For this reason, a maximum allowable error of 10 percent over 75 percent of the full mattress area was established.
2. To accurately correlate weight applied to a mattress with the pressure produced and avoid the complication of curve fitting and nonlinear compensation, a linear relationship is required. To measure significant weight changes in neonates, a 10 gram resolution is necessary. For neonatal mattresses, a 10k gram maximum weight will be used. This 10 gram accuracy requirement demands a linearity specification of ±0.1% of full scale.

3. Limitations must also be placed on system drift. Ten seconds after weight application or removal, the pressure reading taken should be within 10 grams of the actual weight and not drift out of this range for a four-hour period.
4. Shifting of patient weight on a water mattress can cause an undesirable rocking effect which jeopardizes patient comfort and safety (particularly for neonates). Limitations on the amount of rocking must be specified so as not to exceed a level that would cause patient distress.
5. In addition to changes in location of applied weight, normal patient movement and variations in patient size can be expected to cause changes in contact area which will occur even when weight remains the same (i.e., a patient lying on his back versus on his side). For this reason, a 10 percent change in contact area should not cause a pressure variation resulting in a weight change larger than ±10 grams.

Table 1 is a summary of mattress design criteria.

Table 1. Summary of Mattress Design Criteria.

Parameter	Maximum Allowable Limits
1. Sensitivity to location	10% Pressure change over 75% of mattress total area
2. Linearity	±0.1% F.S.
3. Long-term drift	± 10 grams excursion over four-hour period
4. Rocking	Must be less than that considered uncomfortable and hazardous.
5. Effects of Contact Area	±10 grams for a 10% change in contact area

CHAPTER 5

TRANSDUCER DEVELOPMENT

Development of General Transducer Specifications

Using a pediatric size mattress (i.e., $0.25m^2$) filled with water and connected to a monometer, it was found that a 10kg weight with a contact area of $0.052m^2$ (i.e., 80 square inches), produced a 980 N/m^2 (10cm H_2O) pressure change when applied to the mattress. To resolve 10 grams, a pressure resolution of 0.98 N/m^2 (0.01cm H_2O) is necessary. Based on a full-scale pressure of 980 N/m^2, this translates to a 0.1% resolution. Linearity, hysteresis and temperature stability should ideally be a factor of ten better than 0.1% so that the accumulative error would not affect the 0.1% of full-scale resolution.

Frequency response should be sufficient to track normal changes in body weight with time. For weight information a frequency response of DC to 1Hz is required.

Transducer cost will represent a significant fraction of the total weighing system cost. To maintain total system cost below $1,000 (final consumer price), the transducer, including electronic signal, conditioning and display, must be kept below a commercially available price of $700. The remaining cost is dedicated to the mattress. A summary of transducer specifications is given in Table 2.

Table 2. Summary of Transducer Specifications.

Parameter	Specification
Range	0 - 980 N/m^2
Linearity	0.01% deviation from a straight line from zero to full scale
Frequency Response	DC to 1 Hz
Retail Cost	Less than $700
Hysteresis	Less than 0.01% of full scale

Review of Available Measurement Methodologies

Pressure measurements have been accomplished by employing transducers in many forms. The specifications described here cannot fully be met by any available pressure measurement system and will require tradeoffs in the selection.

This section is devoted to giving an overview and evaluation of available pressure transducer systems for the application described here. Selection and construction of a transducer system follows in the next section.

All pressure measurement systems must start with a pressure force summing device which converts gas or liquid energy into a physical displacement. Among the pressure sensing devices available are diaphrams, bellows and bourdon tubes. Force summing devices can be coupled to a variety of electrical devices for converting mechanical displacement into an electrical signal. The available methods of transduction used in pressure transducers are capacitive, differential transformer, inductive, force balance, peizoelectric, potentiometric, strain gage and vibrating wire. The capacitive transducers use a diaphram positioned between two fixed plates which deflects with pressure, causing a capacitive change in two circuits. This change in capacitance can then be used to change the frequency of an oscillator or null a capacitance bridge. Capacitive transducers can offer up to 0.05 percent accuracy for pressure ranges 69 to 1.38×10^5 N/m^2 (0.01 to 200 psi) with a frequency response from 0Hz to 100Hz. The major drawbacks to capacitive sensors include their susceptability to stray capacitance in lead wires, extreme temperature sensitivity which requires a controlled environment and extreme expense (i.e., $2,000 and up).

The linear variable differential transformer (LVDT) is a displacement transducer that produces an electrical output proportional to the displacement of a separate movable core. AC excitation is applied to the primary and two identical secondaries, symmetrically spaced from the primary, are connected in a series-opposing circuit (see Fig. 9). Moving the transformer core varies the mutual inductance of each secondary to the primary, which determines the voltage induced from the primary to each secondary. If the core is centered between the secondary windings, the voltage induced in each secondary is identical and 180 degrees out-of-phase so the net output voltage is zero. If the core is moved off center, the mutual inductance of the primary with one secondary will increase while the other decreases resulting in a differential voltage appearing across the secondaries. For displacements within the range

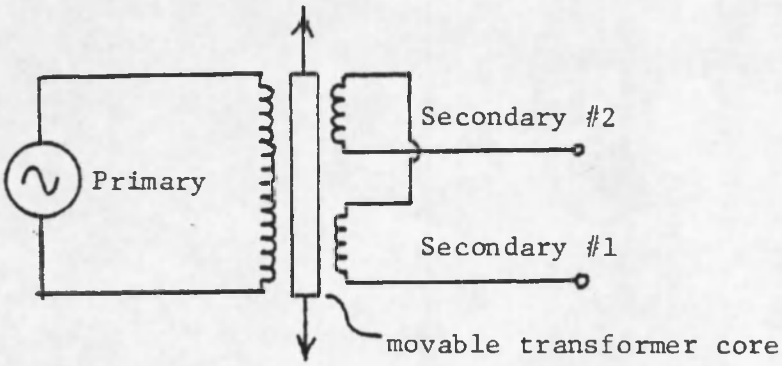


Fig. 9. Equivalent Electric Circuit of an LVDT.

of the LVDT, this voltage is linear. The construction of the LVDT is such that no physical contact between the core and coil occurs so that no mechanical deterioration occurs due to friction. Absence of friction results in infinite resolution, excellent stability, and no hysteresis. Temperature effects on zero drift and sensitivity are low enough that they are immeasurable. Frequency response is poor relative to other transducers due to the mass of the core, but frequencies up to 10Hz can be tracked (depending on the mass of the core). Also, AC excitation is required as is demodulation for DC voltage readout applications.

The variable reluctance/inductive sensors are comprised of a pressure displaced diaphram positioned between two "E" cores, which alters the inductive loop between the coil on each "E" core (see Fig. 10). The ratio of reluctance of magnetic flux path between the two coils produces output levels as high as 40mV/Vac. Full scale pressure ranges can be chosen by selecting the proper diaphram (i.e., 0.01 - 10,000 psi). This type of transducer offers low internal volumetric displacement (typically 0.003 cubic inches), high natural frequency (up to 1kHz), extreme ruggedness and high accuracy (±0.25%). These sensors require AC excitation and DC to AC to DC electronics. They offer linearity of 0.01% of full scale and are susceptible to stray magnetic fields.

Another type of transducer is the force balance transducer, which employs a capacitive or LVDT sensor within their design. A signal generated by the sensor is amplified and used to produce a feedback restoring force equal to the force which displaces the force summing element, returning the system to a null condition. The force restoring

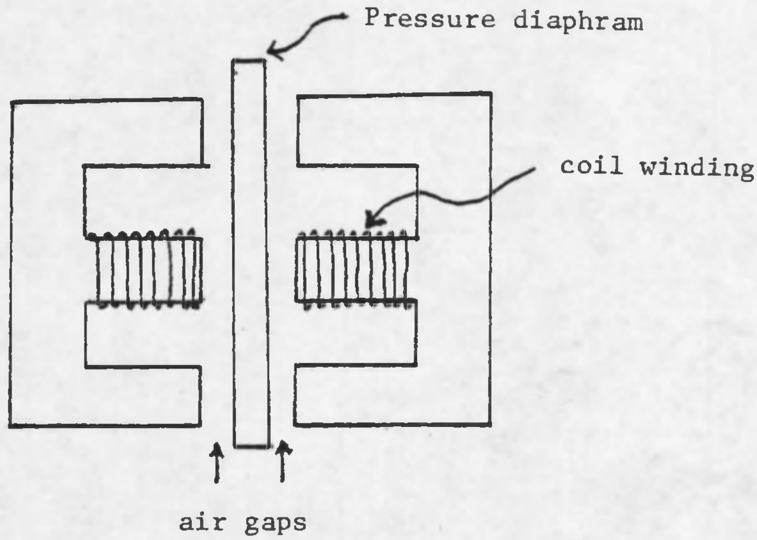


Fig. 10. Cross Section of a Variable Reluctance Transducer.

element typically consists of an electromagnetic coil or servo motor. The magnitude of power required to restore the force balance to null condition is measured across a series dropping resistor in the feedback loop. This kind of transducer system offers extreme accuracy (i.e., better than 0.05% full scale), high output (0-10Vdc), static and low frequency dynamic pressure measurements 0-5Hz, good stability and wide pressure range selection. Unfortunately, the force balance pressure transducer is reserved for use in laboratory instrumentation due to expense ($3,000), shock and vibration sensitivity, large size and extensive electronics.

Piezoelectric transducers can be coupled to a force summing diaphram which induces strain when pressure is applied. The strain applied to asymmetrical crystalline materials through a pressure sensing diaphram generates an electric charge. Piezoelectric transducers provide extremely high frequency response for dynamic pressures (i.e., over 100kHz), small size, rugged construction and a self-generating signal. Unfortunately, piezoelectric materials are very temperature sensitive, they provide no static information, so only AC pressure measurements can be determined, a high input impedance amplifier is required near the sensor and low noise cabling to an amplifier is required.

Potentiometric transducers can typically provide pressure measurements by bonding the wiper of the potentiometer to the force summing device as shown in Fig. 11. The potentiometer acts as a voltage ratio device where either AC or DC excitation can be used. Potentiometric transducers can provide high output without amplifiers and depending on

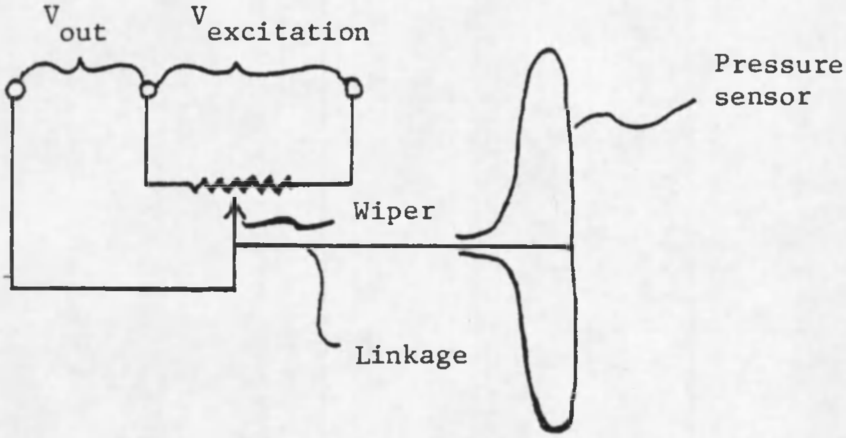


Fig. 11. Potentiometric Transducer.

Changes in pressure causes a change in output voltage through movement of the potentiometer wiper.

the potentiometer, special shaped output curves are available. Unfortunately, potentiometers only offer limited life, course resolution (i.e., 1%), and large hysteresis, due to the excessive mechanical friction associated with their mechanism. Also only low frequency response is possible due to the mass and friction of the wiper arm (i.e., 0-50Hz) and large volume changes are required in the pressure sensors to provide the necessary wiper excursion.

Transducers utilizing vibrating wire and tubes use a force summing diaphram to change the tension of a fine wire or cylindrically configured element. A magnetic field is used to excite the wire or cylinder so that it resonates. As pressure changes, so does the tension on the wire or cylinder and the resonant frequency changes. Frequency deviation is electronically measured and is proportional to pressure. This transduction process can provide extreme accuracy (i.e., 0.02%) and good frequency response (i.e., as much as 100Hz). However, the system is very sensitive to temperature, shock and vibration, and provides a very nonlinear output.

Three kinds of resistive strain gages are available, the unbonded metallic filament strain gage, the bonded metallic foil gage, and the bonded piezoresistive (or semi-conductor) strain gage. Resistance changes in metallic gages result from dimensional changes that occur with strain, whereas semiconductor gages change resistance because their resistivity changes with strain. Unbonded strain gages are made of one or more filaments of resistive wire stretched between supporting insulators. The displacement of the sensing element causes a change in length which results in a change in resistance. Unbonded strain gages provide low level outputs (i.e., typically 4mV/Vin for commercially available pressure units), and can offer reasonably high accuracy (0.1%) for ranges $1.45 \times 10^{-3} N/m^2$ to $1.45 N/m^2$ (10 to 10,000 psi). Foil strain gages consist of a foil of resistance alloy bonded to an epoxy film. The backing film is an adhesive bonded to an elastic member to sense the strain of member due to applied stress. Foil gages typically exhibit gage factors of 2 and provide a low level output (i.e., 2mV/Vin).

Semiconductor strain gages are constructed of a thin filament, either p or n doped silicon which is provided with ohmic contacts and bonded to an epoxy backing film. Bonding is achieved the same as was described for foil strain gages. Semiconductor strain gages provide gage factors ranging from 80 to 150 depending on the impurity content of the doped silicon. As a whole, strain gages have excellent dynamic response (over 1kHz frequency response), long life, low hysteresis and good repeatability. Strain gages generally do not provide good linearity (i.e., 1% typically) and tend to exhibit high temperature sensitivity and require compensation (i.e., 0.01%/°F after compensation is achievable).

A summary of specifications for the commercially available pressure measurement systems discussed here is presented in Table 3.

Table 3. Comparison of Transducer Types.

Transducer Type	Accuracy	Full Scale Pressure* Range (N/m^2)	Freq. Response (Hz)	Typical Output* Levels
Capacitive	0.05%	$70-1.4x10^6$	0-100	>5V
LVDT	0.5%	$2x10^3-7x10^7$	0-100	>5V
Force Balance	0.05%	$7.0x10^3-3.5x10^7$	0-5	>5V
Piezoelectric	1.0%	$7.0x10^2-7x10^7$	1-100	1V
Potentiometer	1.0%	$3.5x10^2-7x10^7$	0-50	5V
Unbonded Metallic Strain Gage	0.25%	$3.5x10^3x7x10^7$	0-2k	4mV/Vin
Bonded Metallic Foil Strain Gage	0.5%	$3.5x10^4-7x10^7$	0-1k	2mV/Vin
Semiconductor Strain Gage	0.25%	$3.5x10^4-7x10^7$	0-1k	20mV/Vin
Variable Reluctance	0.25%	$7x10^1-7x10^7$	0-1k	20mV/Vin
Vibrating Wire and Tube	0.02%	$7x10^3-7x10^5$	0-100	5V

*Typical values for commercially available pressure transducers.

Selected Pressure Transducer Systems

Reviewing the specifications for the pressure measurement system discussed earlier, the most important specifications are full scale pressure range [i.e., 980.6 N/m^2 (10cm H_2O)] and accuracy (0.1%). Examining range first (using Table 3), only three commercially available transducer systems have full scale ranges as low as 980 N/m^2. These include capacitive, piezoelectric and variable reluctance transducers. The capacitive transducer requires a controlled temperature environment which is not a realistic constraint in a hospital environment. In addition, the cost for capacitive transducers is in excess of $2,000, which eliminates its possible application in a low-cost patient weighing system. The piezoelectric transducer offers only 1% accuracy which is a factor of ten lower than that specified, and an additional problem that eliminates the piezoelectric transducer from consideration is its inability to reflect static levels of pressure. Considering the slow changes that take place with body weight, a low frequency cutoff of 1Hz, as demonstrated by piezoelectric transducers, will not reflect body weight. Variable reluctance transducers provide a good compromise between piezoelectric and capacitive transducers. Accuracy is slightly lower than specified (0.25% vs. 0.1%) and frequency response covers DC as required. These transducers require a carrier/demodulator amplifier which typically provides a 10Volt DC output for full scale pressure. No other off-the-shelf transducer systems cover the lower pressure range of 980.6N/m^2 so the possibility of constructing a custom transducer system was investigated.

Reviewing Table 3 for possible candidate transducer systems indicates two that will provide better than specified accuracies, they are the Force-Balance and Vibrating wire and tube systems. As one might expect, the cost of construction and environmental constraints make their use unpractical. The next most likely candidates, judging from accuracies, are the strain gage and LVDT transducers. Strain gages initially looked attractive with the slightly higher accuracy but constructional complications associated with bonding, thermal compensation requirements, low output levels and non-linearity caused the decision to go in favor of constructing an LVDT pressure measurement system.

The accuracy of an LVDT pressure transducer system is primarily dependent on selection of a good force summing device to convert pressure into displacement. Available force summing devices are bourdon tubes, metal bellows, convoluted diaphragms and pressure capsules. Bourdon tubes are available in four configurations (C-tube, spiral, helical, and twisted) and are primarily used for high pressure ($1.0345 \times 10^5 N/m^2$, 15psi) applications. For low range pressure measurements, bellows, diaphrams and capsules are used. The basic difference between these devices is the spring constant values available (i.e., relative deflection for a given force). For the very low pressures of interest, bellows offer the largest deflection for a given pressure and element size. Selecting the metal used to construct the bellows determines the bellow's thermal coefficient of elastic modulus. Thermally induced changes in modulus of the material can produce scale factor changes (m deflection/N/m^2) in the output. Because of the mechanical imperfections inherent in bellows, typical linearity is 0.5% of full scale and hysteresis is less than 0.1% of full scale. A slight trade-off in accuracy had to be made because of the 0.5% typical linearity within the deflection range. This specification plays a major role in determining the system's overall accuracy. In order to choose the correct spring constant and area for the bellows needed here, an LVDT with an appropriate linear range must be selected concurrently.

The basic LVDT-bellows configuration is shown in Fig. 12. As mentioned previously, the LVDT is a mutual inductance element that produces an electrical output proportional to the displacement of a separate movable core (see Fig. 13).

Two identical secondaries, symmetrically spaced from the primary, are connected in a series-opposing circuit. Nominal linear displacements range from less than ±0.05" to ±0.25" and linearity of better than 0.05% full scale are typical. Resolution of an LVDT based on its principle of magnetic coupling (i.e., no friction) is effectively infinite. In practice the limitation on system resolution is dependent on the electronic equipment used to sense the LVDT output. Better than microinch resolution is not uncommon. LVDT's, being transformers, work using AC excitation and output voltages. Accurate AC voltage levels are often inconvenient to provide and measure. Because this is true, DC-LVDT's are available that offer the convenience of DC excitation and output voltages. A microcircuit carrier generator, passive demodulator and DC amplifier was developed and incorporated commercially in the LVDT housing to overcome this problem. With the additional electronics,

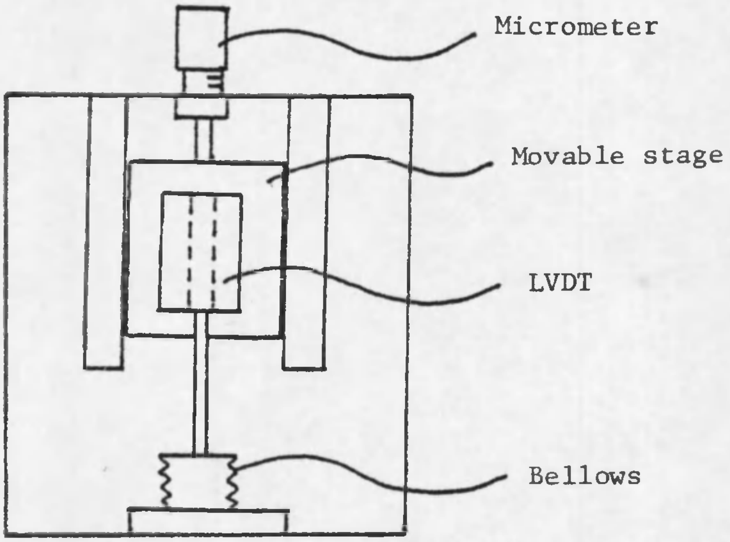


Fig. 12. LVDT Transducer Design.

This LVDT transducer design involves a force summing bellows which is coupled to the movable core within the LVDT. The micrometer is used to move the stage for adjusting zero offset.

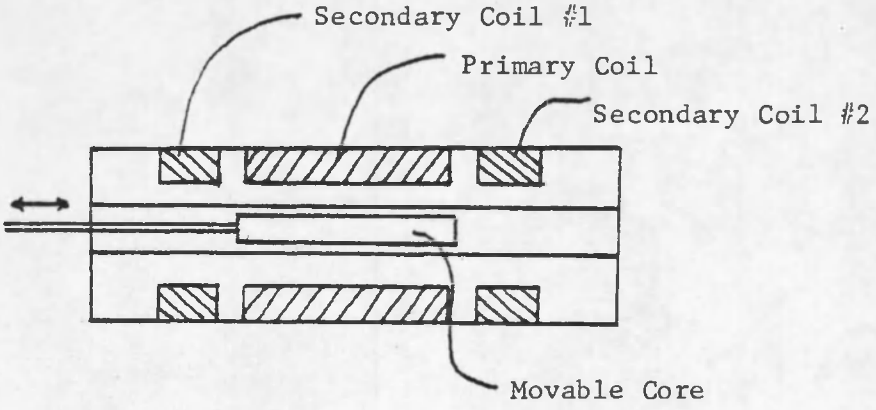


Fig. 13. Cross Section of an LVDT.

This cross section of an LVDT illustrates the coil configuration used in an LVDT. The movable core determines mutual coupling from the primary to the two secondaries.

input voltages must be chosen to give good temperature stability and reasonable output levels. Output impedance considerations must also be considered to avoid loading the DC amplifier and degrading linear performance. Hospital environments should not present any extreme environmental conditions, regarding temperature and humidity, and no extreme magnetic field environments should be encountered.

LVDT Transducer Design

A DC-LVDT model 100HR was obtained courtesy of Schaevitz Engineering, Camden, New Jersey. Specifications included ±0.1" full scale displacement capability of the LVDT, it was necessary to calculate the spring constant and area of the desired bellows. Using $980.6 N/m^2$ ($10 cmH_2O$) as full scale, it was determined that for a $2.58 \times 10^{-4} m^2$ ($0.4 in^2$) cross sectional area bellows, the required spring constant to cause a $2.54 \times 10^{-3} m$ (0.1") deflection is determined below:

Given: Full Scale Pressure = $980.6 N/m^2$

Bellows cross sectional area = $2.58 \times 10^{-4} m^2$

Full Scale LVDT displacement = $2.54 \times 10^{-3} m$

Max Force: Full Scale Pressure x Bellows Cross Sectional Area

$= (980.6 N/m^2) \times (2.58 \times 10^{-4} m^2)$

$= 2.53 \times 10^{-1} N$

Ideal Spring Constant: = Maximum Force/Maximum Displacement

$= 2.53 \times 10^{-1} N / 2.54 \times 10^{-3} m$

$= 99.61 N/m$

After several attempts failed of finding a bellows locally that would meet these requirements, Metal Bellows Corporation donated a bellows specified with a $2.58 \times 10^{-4} m^2$ cross sectional area and 350N/m spring constant. With this bellows, full scale displacement is specified as $2.53 \times 10^{-1} N/350 N/m = 7.25 \times 10^{-4}$ (or 0.74 mm). Using a quick calculation, one can determine the displacement of the bellows caused by a ten gram change in weight. To do this, recall that 10kgs produces an approximate pressure change of $980.6 N/m^2$. Assuming linearity, 10 grams will produce a $0.9806 N/m^2$ ($0.01 cmH_2O$) pressure change. The equivalent force and displacement is calculated below:

Given = 10 grams will produce a $0.9806 N/m^2$ pressure change

Bellows cross sectional area = $2.58 \times 10^{4} m^2$

Bellows spring constant = 350.7N/m

Force = (Pressure) x (Bellows cross sectional area)

$= (9.806 \times 10^{-1} N/m^2)(2.58 \times 10^{-4} m^2)$

$= 2.53 \times 10^{-4} N$

Displacement = Force/Bellows spring constant

$= 2.53 \times 10^{-4} N/350.7 N/m$

$= 7.2 \times 10^{-7} m$

$= 0.72\ \mu m$

The ability of an LVDT to measure a displacement this small is possible due to its infinite resolution capability. Using 21.26V/cm (or 54 Volt/inch) as the LVDT sensitivity, the voltage change for a 10 gram change in weight can be calculated.

Given: LVDT Sensitivity = 21.26V/cm = 2126V/m

Bellows displacement for a 10 gram force = 7.2×10^{-7}m

Voltage change for a 10 gram force = (LVDT sensitivity) x (Bellows displacement for a 10 gram force)

$$= (2126\text{V/m})(7.2 \times 10^{-7}\text{m})$$

$$= 1.54 \times 10^{-3}\text{V} = 1.54\text{mV}$$

This voltage change can be measured with reasonable ease using a good digital voltmeter. Considering the resolution capability of the LVDT (i.e., typically 2.54×10^{-8}m), it can be expected that the major source of error in the system will result from the nonlinearity and hysteresis of the bellows. A theoretical accuracy of 0.5% of full scale can therefore be anticipated.

Transducer Construction

The transducer system was built on a 12" long x 8½" wide x ¼" thick aluminum base (see Fig. 14). Two guides were used to allow movement of a smaller base on which the LVDT was mounted using a clamp. A spring was used to hold the LVDT base against a micrometer which provided a zeroing mechanism. Two brass rods were threaded into each end of the LVDT core and supported by two knife edged teflon rings to minimize the friction associated with movement of the core within the LVDT. A platform on the end of one of the brass rods was provided to rest on the bellows surface. The bellows was glued to an aluminum tube and mounted on an aluminum support. The other end of the aluminum tube was threaded with ¼" standard pipe thread used for the water mattress connection.

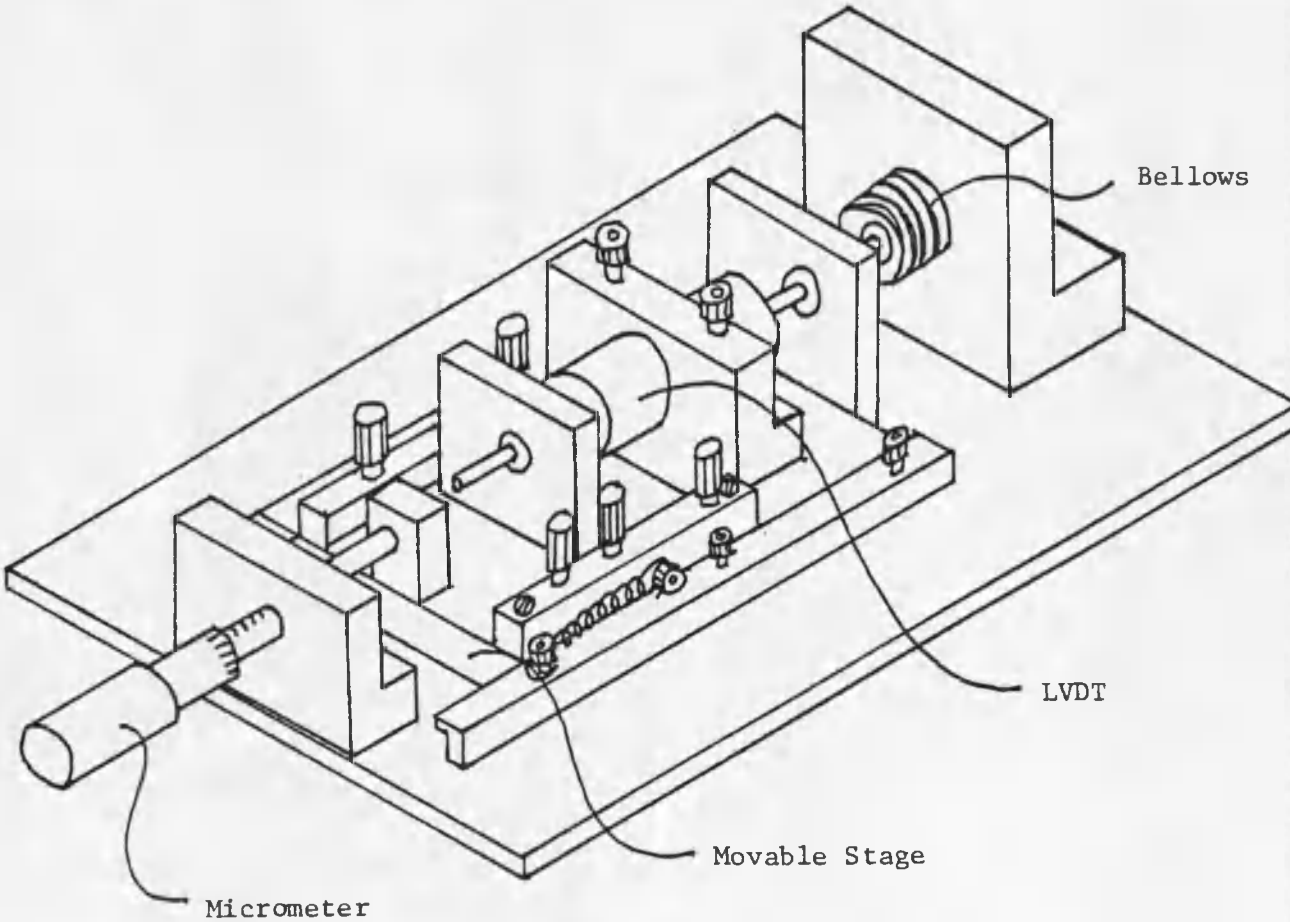


Fig. 14. Constructed LVDT Transducer.

Diagram of the LVDT/Bellows Pressure Transducer. Deflection of the bellows causes movement of the LVDT core.

A Tektronix PS503A power supply was used to provide the 24V DC required by the LVDT for operation and a DM501 4½ digit digital multimeter was used to monitor LVDT outputs. Specifications for each are listed in Table 4.

A study was conducted to accurately determine characteristics of the transducer system. The bellows was examined first. Pertinent measured data is listed in Table 5.

The spring constant of the bellows was determined by standing the transducer system vertically and measuring the deflection of the bellows with added weight. Deflection was measured using the LVDT (this test was actually conducted after the LVDT sensitivity was measured). A 10 gram weight was applied to the bellows four times and an average deflection was measured.

A test for linearity was also conducted by using a water manometer connected to the same water circuit as the pressure transducer as shown in Fig. 15. Three different initial pressures were used by varying the amount of water in the circuit and weights were applied to the tubing to increase the pressure in the system. The results are shown in Fig. 16. Notice that for the two smaller initial pressures, a linear bellows deflection for increased pressure followed nicely. The highest initial pressure ($3.82 \times 10^3 N/m^2$ or $39 cmH_2O$) exceeded the linear range of the bellows to the point where it was causing a higher nonlinear spring constant to result. During actual pressure measurements, it was necessary to keep the system pressure less than $3.43 \times 10^3 N/m^2$ (or $35 cmH_2O$) to stay within the bellows linear displacement range of $6.35 \times 10^{-3} m$.

Table 4. Performance Specifications for DC Excitation Voltage and DC Voltmeter Used for the LVDT Transducer.

PS 503A Dual Power Supply

Characteristics	Performance
Outputs	0 to at least 40 DC across the plus and minus terminals
Minimum Resolution	50mV
Load Regulation	Within 1mV
Line Regulation	Within 5mV for a ±10% line voltage charge
Ripple and Noise	1mV peak-to-peak or less
Stability	0.1% +5mV of drift in 8 hours

DM 501 Digital Multimeter

Characteristics	Performance
Ranges	2V, 20V, 200V, 1,000V
Accuracy	±0.1% of reading ±2 counts
Common Mode Rejection	≥100db at DC, 80db at 60Hz with 1kΩ imbalance
Normal Mode Rejection	≥30db at 60Hz increasing 20db per decade
Input Resistance	10MΩ

Table 5. Experimental Values Found for Bellows Parameters.

Outer Diameter	Cross Sectional Area	Spring Constant
2.2×10^{-2}m	3.8×10^{-4}m^2 (0.4 sq. inches)	553 gram/inch 1.22 lbs/inch 21.8kg/meter 213.4 Newtons/meter

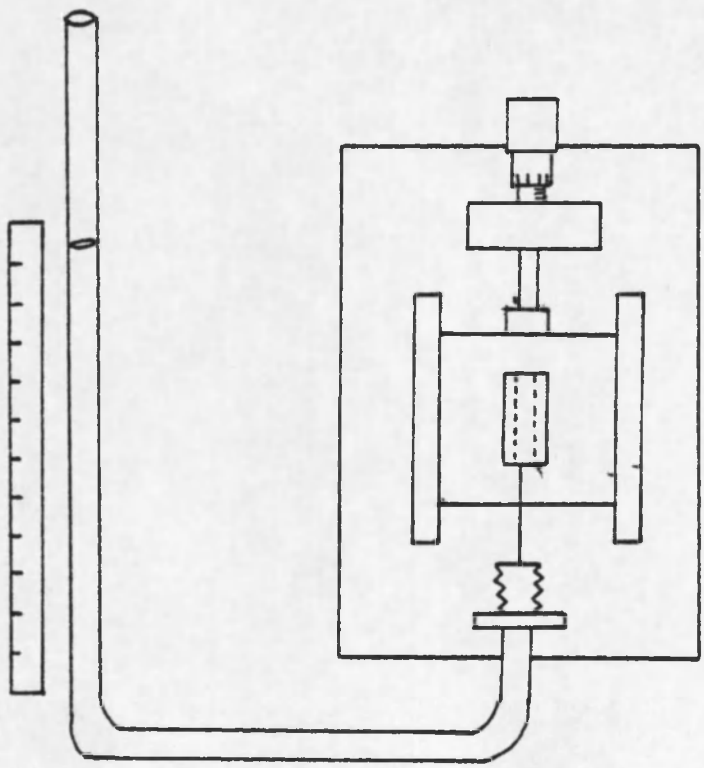


Fig. 15. Experimental Setup Used for Bellows Linearity Study.

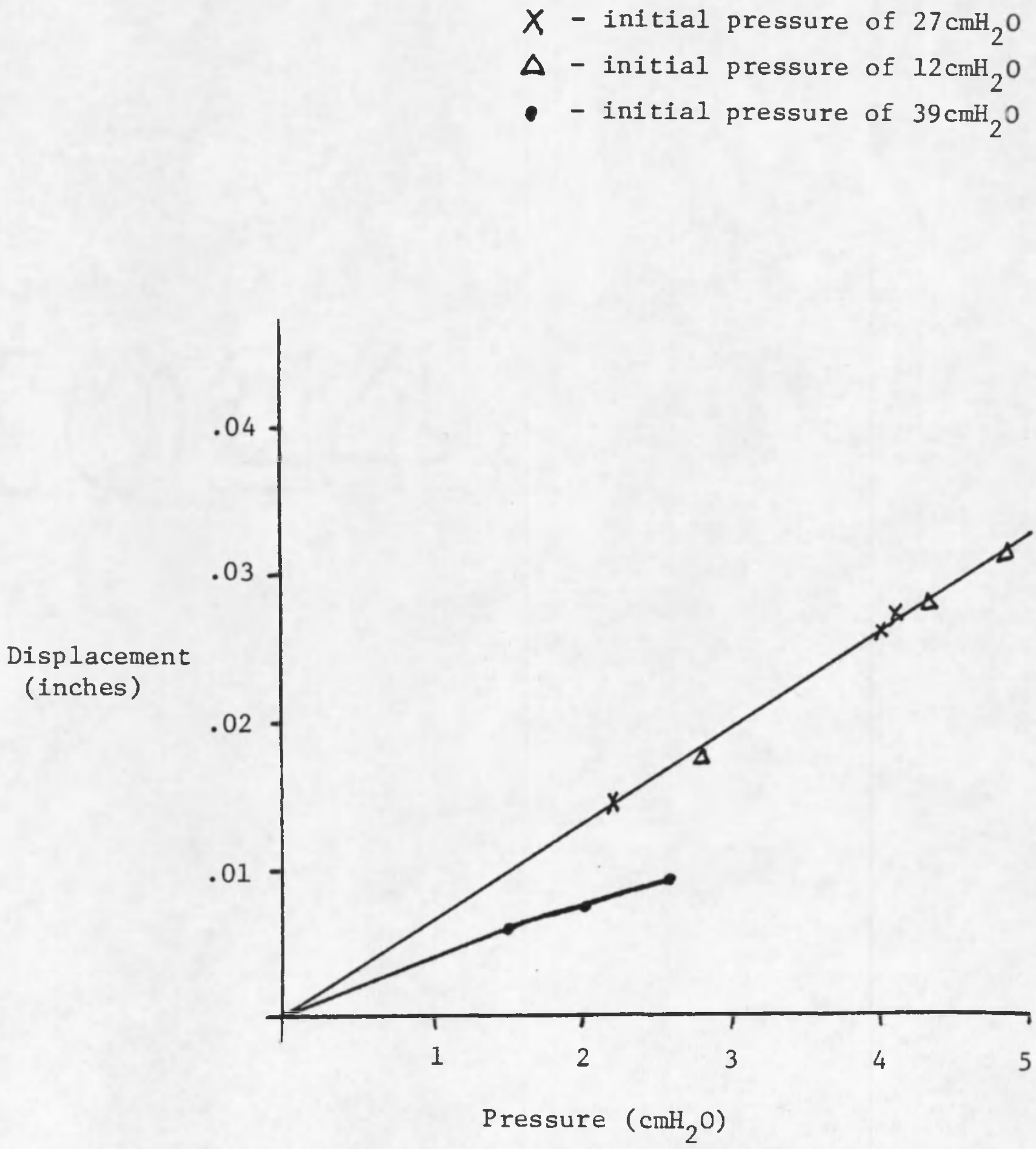


Fig. 16. Effect of Initial Pressure on Linearity of Bellows.

For higher pressures, the height of the bellows could be adjusted with respect to the mattress or pressure source to stay within a linear pressure range.

It became difficult to measure an exact linearity specification for the bellows due to the human error involved in reading the water manometer pressure. Perfect linearity is based on the 213.4N/m deflection (given in Table 5), and Table 6 indicates the correlation of the manometer pressure readings to the effective pressure indicated by the bellows. These correlations are quite good when one considers the accumulative human error involved in measurement. For example, the average human can expect to resolve a manometer reading to within two mmH_2O (based on meniscus error and consistency). This already introduces a 5-10 percent correlation error. The reflected bellow pressures also represent an accumulation of some measurement errors in the diameter and spring constant measurements. The conclusion must be that the correlation of these pressure valves is good within human resolution, however, measurement accuracy is not sufficient to determine the true linearity of the bellows. It became necessary to use the manufacturer's specification of linearity, namely 0.5% of a straight line, over the full linear displacement range.

To further evaluate the performance of the transducer, a sensitivity and stability test was conducted for the LVDT using the Tektronic equipment. The zeroing micrometer was used to accomplish a known core displacement. Several individual 6.35×10^{-4}m (or 0.025 inch) movements were produced with the 2.54×10^{3}V/m (or 45.064 Volts/inch)

Table 6. Correlation of Manometer Pressure Readings to the Effective Pressure Indicated by the Bellows.

Manometer Pressure Reading (cm H_2O)	Transduced Bellows Pressure (cm H_2O)	Percent Error
2.19	2.0986	4.17
2.80	2.5468	9.0
3.97	3.7402	5.8
4.13	3.9003	5.6
4.3	4.0022	6.9
4.85	4.5116	7.0

sensitivity using a 24V DC transducer excitation as recommended by the manufacturer.

Stability was measured by holding the core of the LVDT in a fixed position over a 24-hour period and observing voltage fluctuations. For the purpose of this test, the Tektronix power supply and DVM were allowed to warm up for the manufacturer's suggested warm-up time of 20 minutes. The 24V DC excitation was adjusted and connected to the LVDT. The DVM was connected to the LVDT and set on a 2 Volt full scale setting. The micrometer was used to adjust the transducer's output to as close to zero as possible (the LVDT's resolution exceeds one's ability to adjust the micrometer, ±1mV was the best that could be achieved). 1mV corresponds to 5.64×10^{-7}m or 0.564 μm. The result of a 24-hour stability study is shown in Fig. 17. This study indicates a worst case system drift of 2.1mV and -1.2mV from the initial voltage. From the theoretical voltage to weight conversion calculations discussed previously, these drifts would correspond to a +16.4g and a -6.6g weight change using a 0.12825mV/g sensitivity. A requirement to rezero the transducer system more often than once every 24 hours would reduce this error, however, this error indicates that if we assume perfect bellows performance, a resolving power of approximately 10-15 grams can be achieved. Combining this error with the error introduced by the linearity of the bellows, a worst case error of 65 grams is accomplished or 0.65% of full scale. For the purpose of developing a prototype system, this was decided to be adequate for preliminary measurements. However, concurrent with the research conducted with this transducer system,

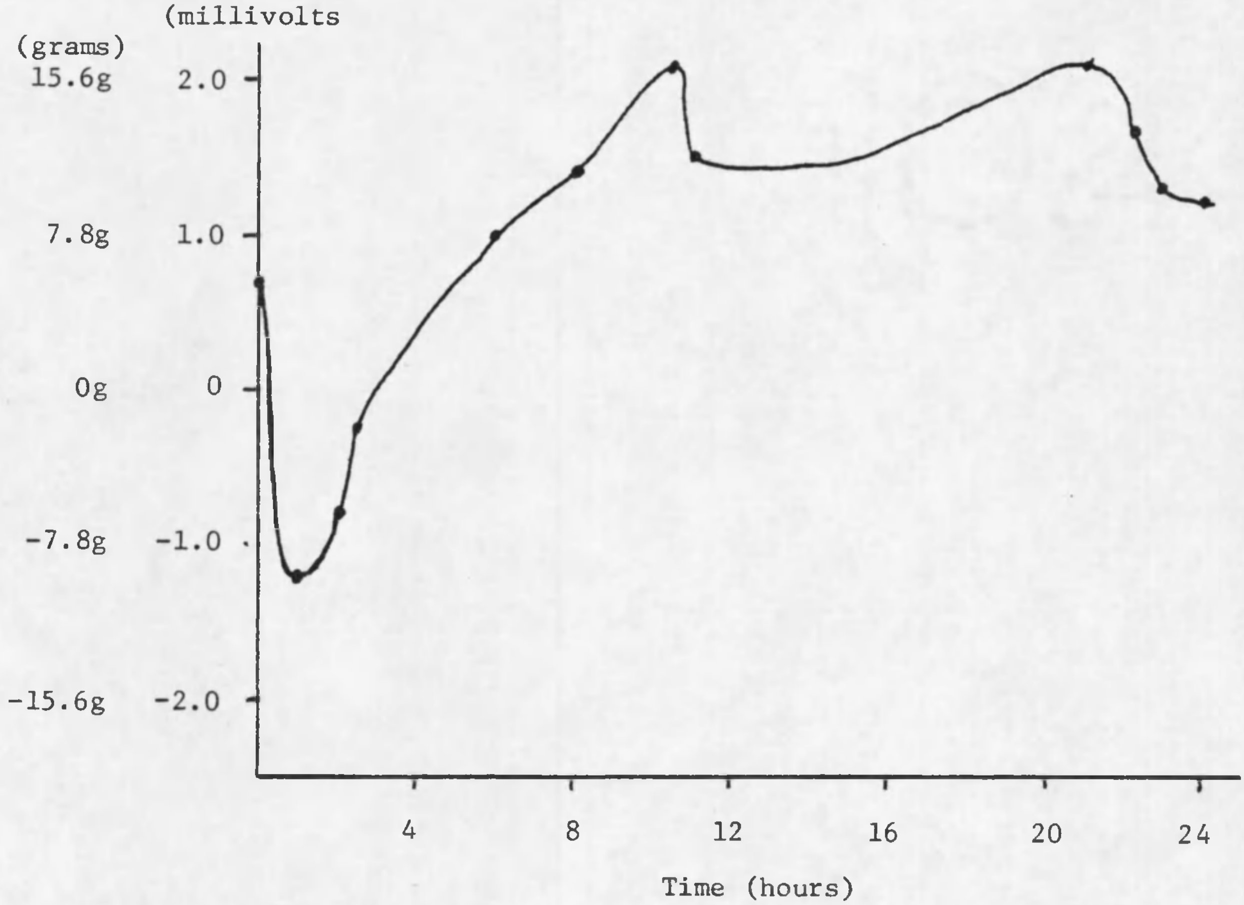


Fig. 17. Transducer Stability Study.

Transducer stability conducted over a 24-hour period. Units of drift are given in both voltage (volts) and weight (grams).

a second transducer system was being investigated. A summary of the total transducer system performance specifications are listed in Table 7.

Further experimentation with the transducer revealed an additional source of error due to friction associated with the movement of the LVDT core and bellows. In order to compensate for this friction, the transducer assembly was used in a vertical position and a small vibrating motor was attached to the transducer platform to overcome the effects of static friction forces. The combination of repositioning and vibrating the platform was found to reduce frictional errors so that resolving weights smaller than 10 grams was possible. The summed total effect of all errors was not considered to change due to friction and an estimated accuracy of 0.5% for typical measurements was maintained.

Selection of a Second Transducer

As mentioned in the Selected Pressure Transducer System Section of this thesis, variable reluctance transducers were mentioned as a desirable candidate. Consulting Table 1 reveals a 0.25% achievable accuracy with full scale ranges from $68.94 N/m^2$ to $6.894 \times 10^7 N/m^2$ reasonable temperature effects (i.e., 0.02%/°F), output levels at 40mV/Vin and DC frequency response. An additional feature is the commercial availability and differential configuration. Variable reluctance transducers cost approximately $600 including everything except a DVM for output measurements. A survey of available variable reluctance transducers resulted in choosing a Validyne DP103 differential pressure transducer with a $\pm 980.6 N/m^2$ full scale (± 10cm H_2O) pressure range. This transducer is a diaphram type magnetic reluctance transducer and is shown

Table 7. Summary of LVDT/Bellows Transducer System Specifications.

Specification	Performance	Assumptions
Pressure Range	0-14.5cmH_2O	Based on 0.1" linear range of LVDT
Sensitivity	310mV/cmH_2O	Using 24Vdc LVDT excitation voltage
Linearity	0.5-1.0% of full scale	Worst case based on combined manufacturer's specifications
Drift	0.125% full scale	Worst case for bellows and LVDT for a 65°F to 80°F temperature range

in simplified form in Fig. 10. Transducer operation is based on a diaphram of magnetically permeable material, supported between two symmetrical assemblies which completes a magnetic circuit with each "E" core. Diaphram deflection with application of pressure increases the gap in the magnetic flux path of one core and decreases the gap symmetrically in the other. The magnetic reluctance varies with the gap, determining an inductance ratio. This inductance ratio is measured in an AC bridge circuit in which an output voltage proportional to pressure is obtained. Demodulation following the suppressed carrier bridge output is required for a DC signal. A Validyne CD15 Carrier Demodulator was required to produce the 15kHz sine wave excitation and resulting output demodulation. The DC output is obtained from an active filter circuit and gives a uniform response from steady state to 1kHz. Other manufacturer specifications are listed in Table 8.

Table 8. Validyne CD15 Sine Wave Carrier Demodulator Specifications.

Parameter	Specification
Input Sensitivity	15mV/V excitation
Transducer Excitation	5Vrms, 5kHz sine wave
Output Voltage	±10Vdc @ 10ma for ±10cmH_2O input
Output Impedance	10 ohms nominal
Frequency Response	DC to 1kHz

The variable reluctance transducer was reserved for use with the final mattress design due to its differential pressure configuration. The LVDT transducer was used for all other pressure measurements taken in this study.

CHAPTER 6

MATTRESS DEVELOPMENT

The design specifications from Chapter 4, Definition of Mattress Design criteria is listed here again for reference in Table 9.

In order to determine the feasibility of developing a mattress that would meet these functional specifications, some preliminary tests were conducted to provide insight on the effects of mattress design criteria. To do this, two readily available mattress designs were tested. Small pediatric size mattresses were chosen initially for ease in handling.

The first mattress tested, Mattress I, was obtained by cutting off the head support pillow of a swimming pool air mattress. Construction was of two canvas reinforced rubber sheets, approximately $0.2m^2$, heat sealed together. The mattress was filled with water that had been boiled and cooled in a sealed container to aid in evacuation of as much air as possible from the water. Air bubbles were manipulated out of the mattress and pressure tubing was used to connect the LVDT bellows transducer and mattress filling tube. The same transducer mattress configuration was used for preliminary tests of mattresses I and II. A diagram is shown in Fig. 18.

Mattress I was first subjected to a pressure/weight study to reveal the pressure characteristics of the mattress. The test procedure involved zeroing the transducer using the micrometer adjustment and

Table 9. Mattress Design Criteria.

Mattress Design Criteria	Maximum Allowable Error
1. Sensitivity to location of applied weight	10% pressure variation over 75% of total mattress area
2. Linearity	±0.1% of full scale pressure
3. Drift	±10 grams after ten seconds of setting and remain for at least four hours.
4. Sensitivity to effects of contact area with	±10 grams for a 10% change in contact area
5. Rocking	Must be low enough so as not to cause patient discomfort

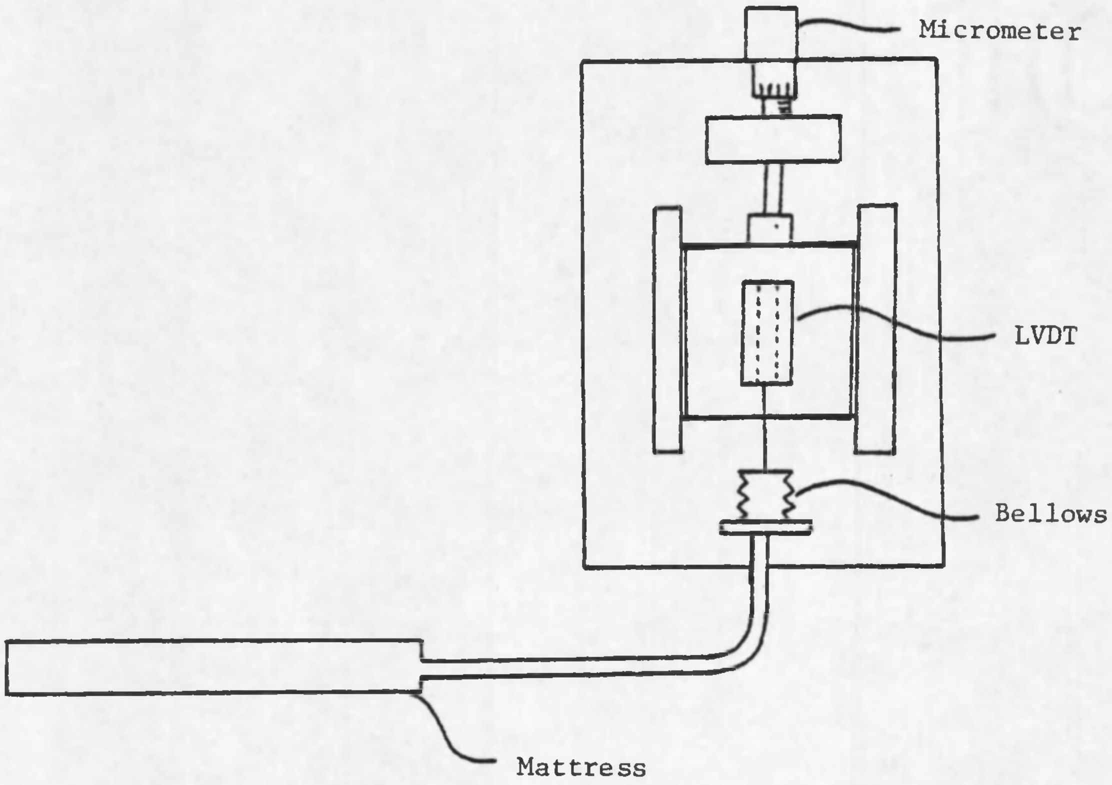


Fig. 18. Diagram of Mattress-transducer System.

applying weights one at a time to a $5.2 \times 10^{-2} m^2$ base which were placed on the center of the mattress surface. The $5.2 \times 10^{-2} m^2$ inch base was used to simulate the body surface area of a typical infant (based on the body surface area equation). With each applied weight, a reading was taken on the DVM. The results of this study are shown in the graph of Fig. 19. This graph demonstrates a reasonably linear weight measurement with an average sensitivity of 146μVolts/gram. During the course of the study, voltage readings were found to decay with time after the application of each weight. If readings were not taken at consistent time intervals after each weight was applied (i.e., 10 seconds), the readings would have varied significantly. To determine the extent of this error, a hysteresis study was conducted by measuring the range of readings resulting from the addition of a 4kg weight over a five-minute time period. The results are shown in Fig. 20 and indicate that as much as a 300 gram weight change was falsely indicated. Observation of the exponential change in weight readings lead to the conclusion that the error is due to stretching of the mattress. This stretching is reflected in the slight nonlinearity of the pressure-weight curve of Fig. 19. This potential source of error was discussed earlier and based on the required resolution of 10 grams, this stretching error is unacceptable. The only way to minimize this error is to choose a mattress material that offers a low modulus of elasticity (i.e., less stretch).

The second mattress, Mattress II, was constructed of two 2.3×10^{-4}m (0.009 inches), thick polycarbonate sheets sealed together at the edges using methylene chloride applied with a syringe. A hole

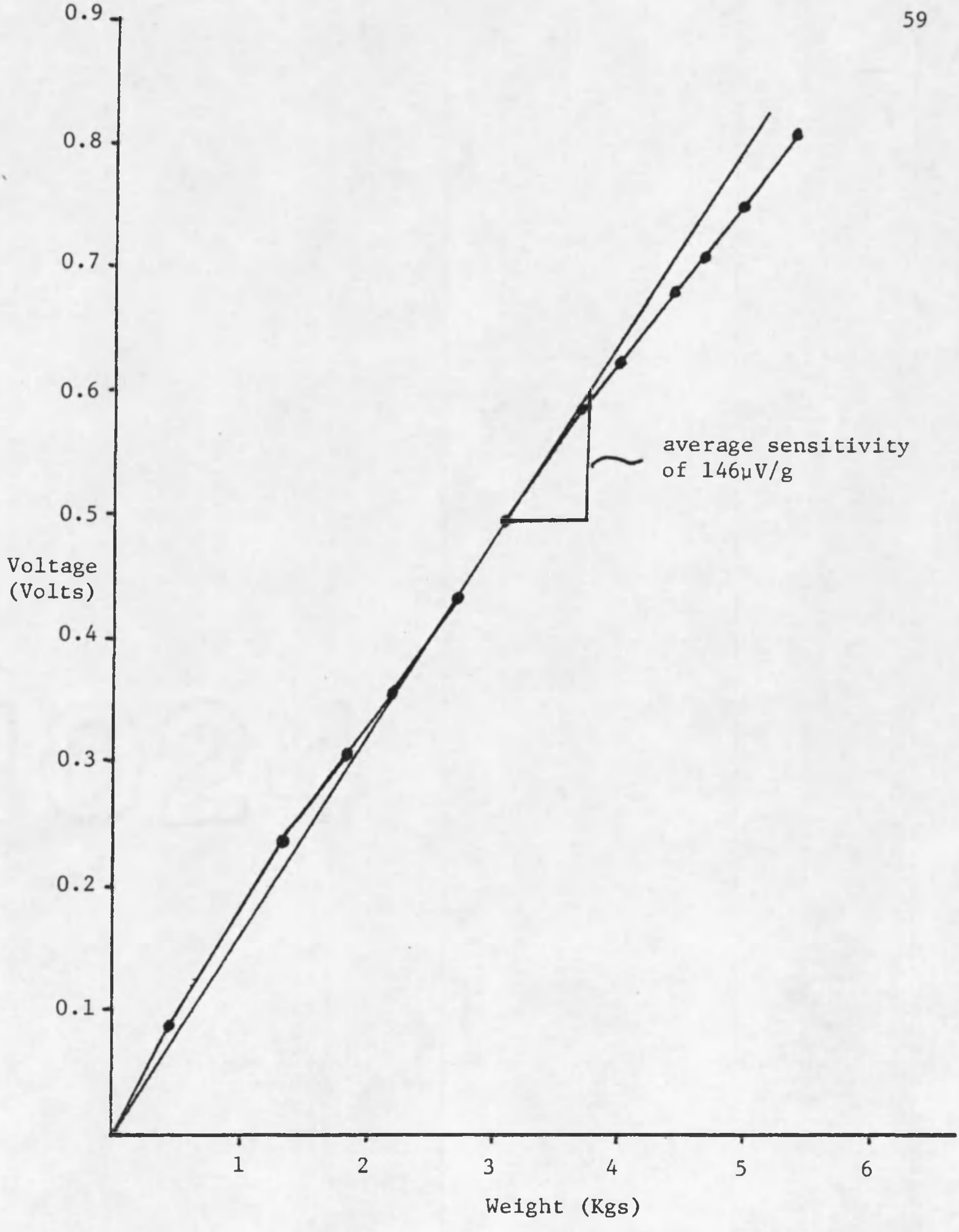


Fig. 19. Pressure/Weight Curve for Mattress I.

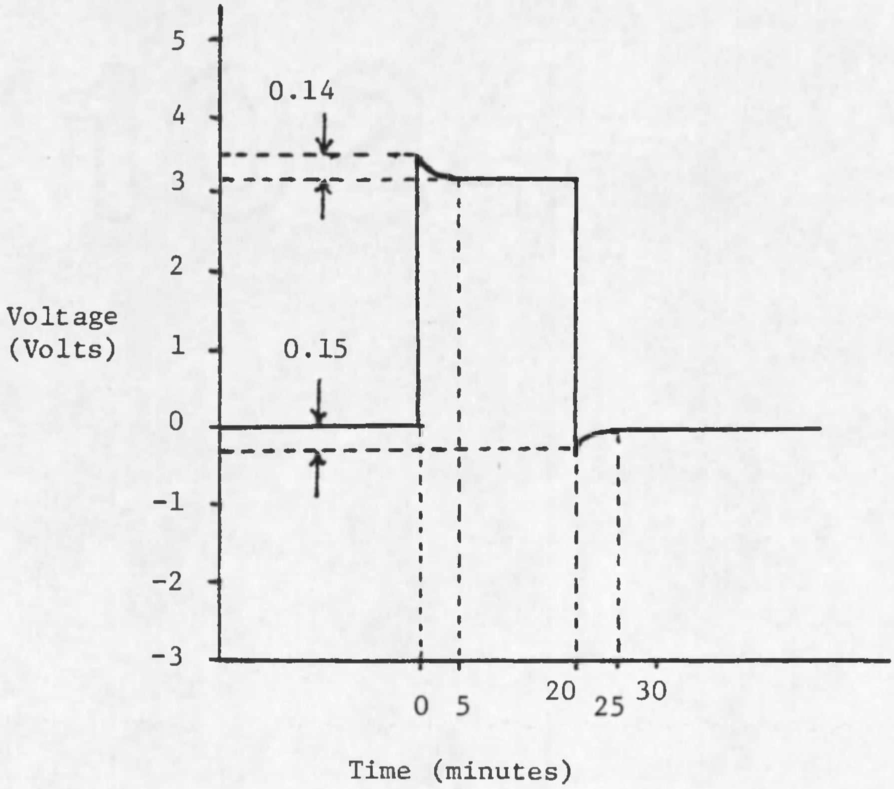


Fig. 20. Effects of Stretch on Mattress I.

Application and removal of a 4kg weight to Mattress I illustrates the error due to stretching. This error is as high as ±300 grams.

was cut in one of the sheets to allow for a filling tube which was also connected using methylene chloride. A pressure/weight study was conducted using the same test procedure as described for Mattress I. Results are shown in Fig. 21. Sensitivity for this mattress was calculated to be 250μV/gram. In comparison to Mattress I, Mattress II exhibits a higher sensitivity due to the smaller mattress area (i.e., $0.0645 m^2$ for Mattress II vs. $0.2 m^2$ for Mattress I). Because of a higher modulus of elasticity, the polycarbonate mattress exhibited a better linear pressure/weight response compared to Mattress I.

The second test performed on Mattress II determined the repeatability of the system response to the application of a 4kg weight. In this test the system was zeroed using the transducer micrometer adjustment and then a 4kg weight reading was taken using the $5.2 \times 10^{-2} m^2$ base. The system was rezeroed after each removal of the 4kg weight, and this was repeated six times. The results are given in Table 10. The mean reading was 0.9584V with a standard deviation of 0.0085V. The maximum deviation from the mean includes +0.01V and -0.0114V. Using the system sensitivity, this accounts for an error in weight of +40 grams and -46 grams, which is outside the ±10 gram requirement.

System drift was then measured by applying a 4kg weight to the mattress surface and taking voltage measurements over a one-hour period. Resulting voltage measurements reflect total system drift (i.e., mattress stretch, pressure drift, mechanical bellows drift, electronic LVDT, power supply and DVM drift). Results are shown in the graph of Fig. 22. Results indicate a worst case drift error of 6.8 grams for a one-hour period, which is within the ±10 gram specification.

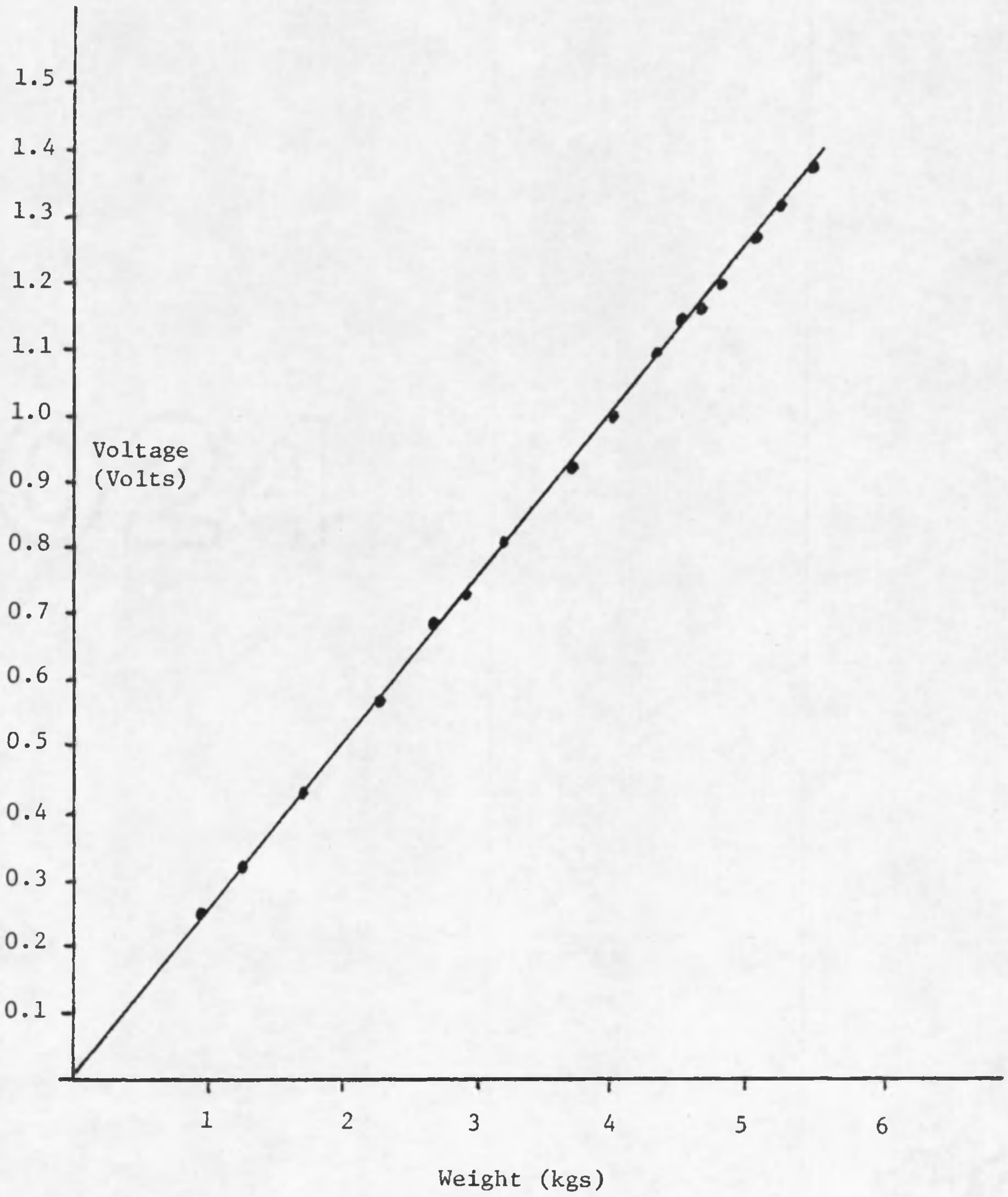


Fig. 21. Pressure/Weight Characteristics for Mattress II.

Table 10. Results of Repeatability Studies Conducted on Mattress II.

Sample Number	Zero Reading (V)	4kg Reading (V)	Total* Change (V)
1	0.0003	0.9719	0.9716
2	0.0016	0.9636	0.9620
3	0.0022	0.9591	0.9569
4	0.0000	0.9602	0.9602
5	0.0036	0.9720	0.9684
6	0.0039	0.9539	0.9500
7	0.0024	0.9415	0.9491
8	0.0019	0.9489	0.9470
9	0.0062	0.9663	0.9601

*Avg. Voltage Change = 0.9584
Standard deviation = 0.0085 or 35 grams

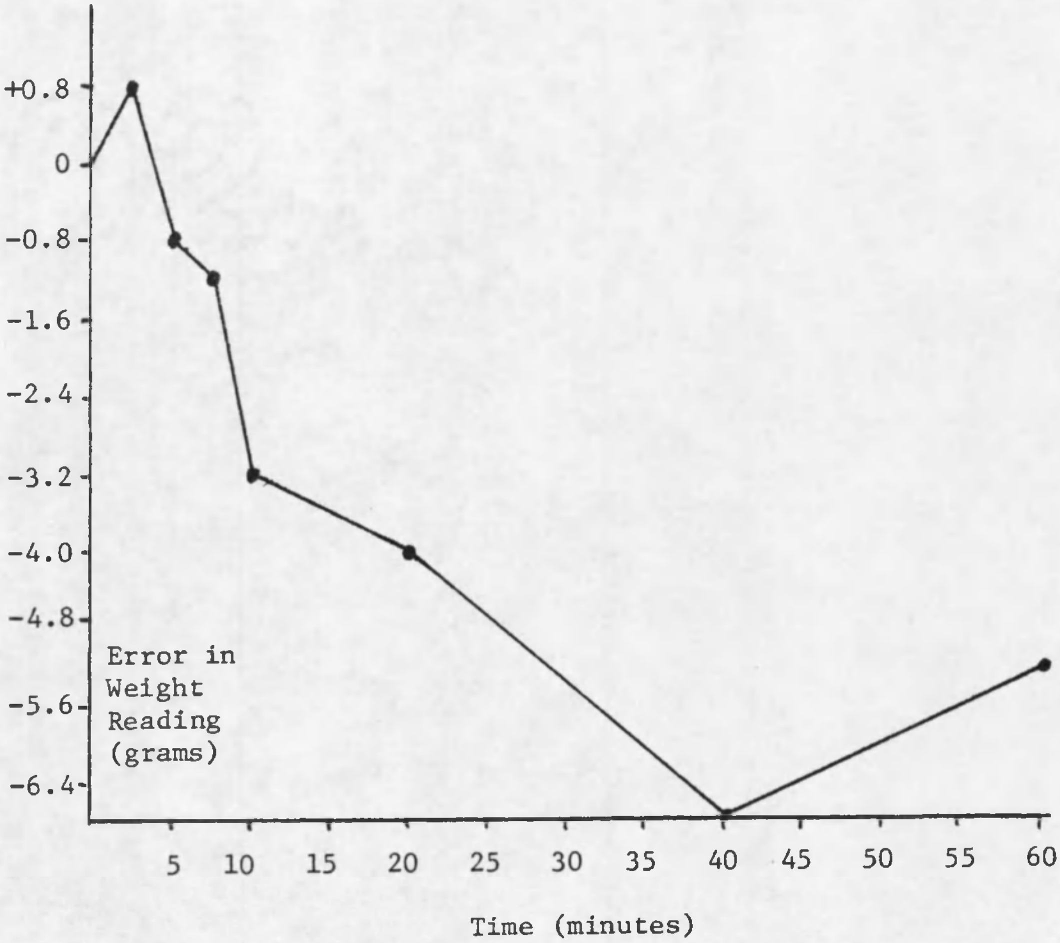


Fig. 22. Time Stability Study of Weight Measurement System for Mattress II.

To determine the resolving power of the weighing system a study was conducted to determine the voltage changes produced by small changes in weight. To more accurately simulate weight changes of a baby, the system was loaded with a 4kg weight before small weights were added and subtracted. Ten weights ranging from 100g down to 10g increments were first added and then removed from the surface of the mattress in random order. This process was performed twice with the largest error corresponding to an erroneous weight reading of 30 grams.

Preliminary Indications

Based on the preliminary tests conducted on both mattresses the importance of mattress material selection is demonstrated through drift and resolution tests. The two pillow type mattresses both suffered some configurational problems including serious rocking problems which in a clinical setting would threaten the patient's comfort and safety. Both of these pillow mattresses did not lend themselves to evaluation of sensitivity to the location of applied weight or contact area of applied weight due to their instability. Results of the tests conducted did reveal significant indications regarding the effects of stretch, resolution, linearity, and stability.

Mattress I demonstrated as much as a 300 gram weight error due to the effects of stretch in the mattress material. Because of this unacceptable error, the final mattress must be constructed of a material that offers a very low modulus of elasticity to minimize the error due to stretch. Mattress II, made of a more stiff material (i.e., polycarbonate vs. canvas reinforced rubber) was much less susceptible to the effects of stretch (refer to Fig. 22). However, one is limited regarding the extent of rigidity that the mattress material can provide because deformability of the material is just as important so that the mattress material does not provide structual support that causes a loss of pressure as a result of the applied weight. The polycarbonate mattress exhibited a worst case stretch-drift weight of 7 grams which is within the mattress design criteria of ±10 grams. The resolution of the polycarbonate mattress exhibited a worst case variation of 30 grams. It was speculated that the polycarbonate did not offer sufficient deformability to accomplish the resolving power of ±10 grams and that a thinner polycarbonate material might provide a better tradeoff. Problems with the rigidity of the polycarbonate mattress were further demonstrated after one month when in the areas where kinks developed, leaks formed in the polycarbonate.

In summary, the design features of the final mattress have to overcome the problem of rocking, exhibited by both Mattress I and II. The mattress material will have to exhibit a modulus of elasticity lower than Mattress I to overcome the drifting problems due to stretch. Linearity and resolution will have to be improved to accomplish the 0.1% and ±10 grams specified by reconfiguring the mattress and proper choice of mattress material. Determining sensitivity to the location of the applied weight was not possible with Mattress I and II due to the instability of the pillow configuration, and will have to be tested in the next mattress.

CHAPTER 7

FINAL RESULTS

Two mattresses were constructed in an attempt to overcome the shortcomings of Mattresses I and II. The first mattress, Mattress III, was made of 6.35×10^{-3}m (¼") plexiglas and a 2.3×10^{-4}m polycarbonate sheet. The base and sides were made of plexiglas and were milled to size 63.5cm x 33.02cm x 5.03cm (25 inch x 13 inch x 2 inch), see Fig. 23. Methyl methacrylate was used to seal the joints and make them water tight. A single 63.5cm x 33.0cm sheet of polycarbonate was sealed over the top using methyl methacrylate. A hole was tapped in one of the sides to accept a ¼ inch standard pipe thread for connection to the bellows-LVDT transducer.

The plexiglas portion of the mattress was used to provide a more rigid foundation to minimize the effects of stretch and rocking. With this mattress configuration, it is possible to determine the effects of contact area and sensitivity to location of the applied weight.

The same filling process as described for previous mattresses was used for this mattress to evacuate as much air as possible. The water was boiled, sealed, cooled and emptied into the mattress through filter paper to eliminate debris. Very rigid polyethylene tubing was used to inter-connect the mattress and transducer. The first test conducted determined the mattress' sensitivity to the contact area of the applied weight.

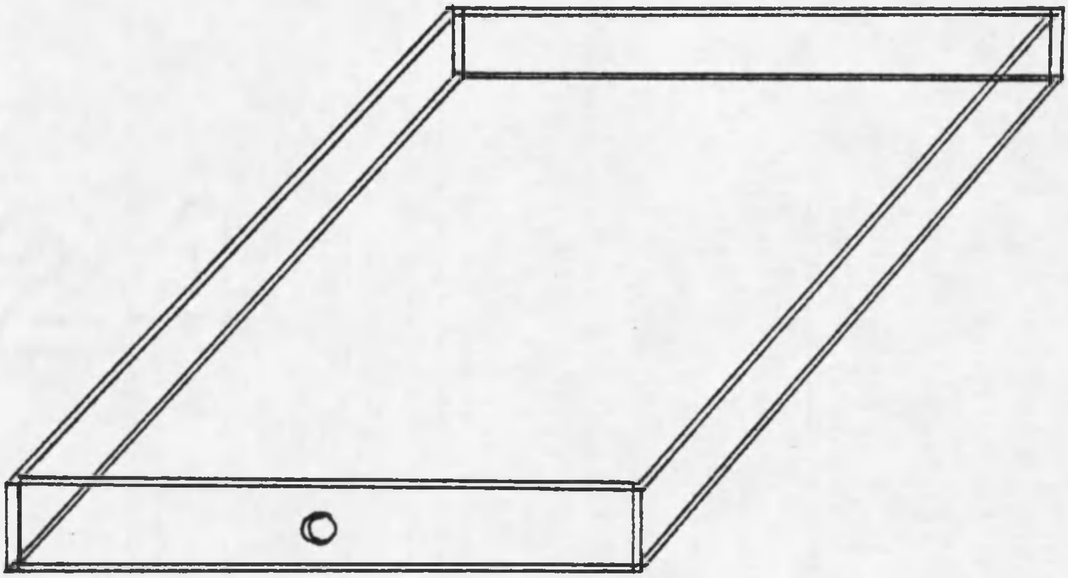


Fig. 23. Illustration of Mattress III.

Top covering is polycarbonate sheet with bottom and sides made of plexiglas. Connection to transducer pressure measurement system is through the $\frac{1}{4}$" pipe fitting tapped in one of the plexiglas ends as shown.

Effects of Contact Area

To determine the effect of changes in contact area for equal applied weights, three boards with different areas were cut. To simulate an infant's contact area an 81 inch square inch wood base was constructed. The second base represented a 10% reduction in area at 72.9 square inches and the third base was 29 square inches (i.e., approximately a 60% reduction in area). With each test, one of the bases was placed in the center of the mattress and the transducer was zeroed. A weight of 906.7 grams was then placed on the base and a voltage reading was taken. Any variation in readings would be due to changes in the contact area. The contact area test was repeated three times for each base and Table 11 reflects average values.

Using the voltage change for the 81 square inch base as the control value, we find a sensitivity of 176μV/g. The pressure increase induced by the 10% reduction in contact area reflects a 15 gram increase in weight. This error is 5 grams higher than the maximum allowable ± grams. The 29 square inch base reflects a 64% reduction in contact area. Here the erroneous pressure increase amounts to an unacceptable 146 grams.

Sensitivity to Location of the Applied Weight

A 906.7 gram weight was placed on an 81 square inch base and moved to five locations on the mattress surface. For test purposes the mattress surface was measured down to the central 75% of the total mattress area. Weights were then applied to the corners of this 75%

Table 11. Effects of Contact Area on Weight Readings for Mattress III.

Base Area	Average Voltage Change (Volts	Equivalent Weight Error
29 sq. in.	0.185	146 grams
73 sq. in.	0.162	15 grams
81 sq. in.	0.159	0 "Control"*

*Test weight = 906.7 grams

effective area and to the center. Variation in readings from one location to the next illustrate a fault in the mattress' ability to translate applied weight to a pressure independent of the location of applied weight. The results of each of the five readings are given in Table 12.

The center weight reading was used as a control and weight errors for the four corner readings are indicated in Table 12. The average weight error for the four corner readings amounted to 214 grams or a 24% reduction in measured weight. This is unacceptable when considering the specified 10% maximum allowable change in weight under these circumstances.

Results of other tests include good linearity in pressure/weight response when contact area and location remain constant. As shown in the two previous tests, a slope of 5.68kg/Volt results when applying weight to an 81 square inch base located on the geometric center of the mattress. Worst case deviation off of a straight line contributed a maximum error of less than 20 grams. For a 4kg test, this amounts to

Table 12. Effect of Location of Applied Weight on Weight Readings.

	Center	Corner 1	Corner 2	Corner 3	Corner 4
Voltage Change (Volts)	0.156	0.107	0.120	0.124	0.122
Reflected Weight Error (grams)	*0 "Control"	278.3	204.5	181.8	193.1

*Test weight = 906.7g

a 0.5% linearity. Drift was measured to be no more than that of the transducer alone (i.e., 10 grams measured over a four-hour period).

Patient comfort for this mattress was improved due to the stability of the plexiglas foundation. The plexiglas increased the damping of any net water motion so that rocking was minimized.

Discussion

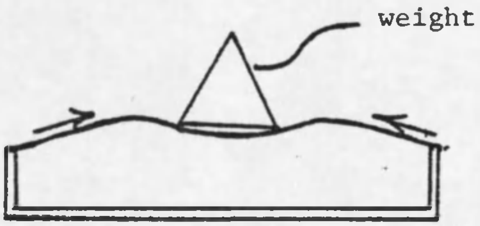
The results of tests conducted on Mattress III indicate some solutions to old problems and introduction of some new ones. Linearity, drift and rocking all indicate a good choice of mattress materials and configuration. Greatly reduced rocking problem is perhaps the most significant improvement and both linearity and drift factors were improved. This configuration of mattress allowed the study of two untested criteria, namely the effect of contact area and sensitivity to location of the applied weight. The results of these tests indicate unacceptable results according to our original specifications repeated in Table 13.

Table 13. Design Criteria Not Met by Mattress III.

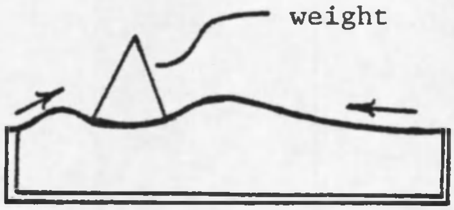
Test Criteria	Design Specification	Measured Value for Mattress III
Sensitivity to location of	10% maximum allowable change in reading over 75% effective mattress area	24% variation over 75% effective area
Sensitivity to changes in contact area	10% change in contact area should not change weight reading more than 0.10%	10% change in contact area resulted in a 1.6% error

The sensitivity to location of the applied weight was observed to be due to the support offered by the plexiglas sides of the mattress. Maximum sensitivity was realized in the geometric center and motion towards the sides caused a reduction in sensitivity. The boundaries of the 75% effective area caused a 24% reduction in sensitivity compared to the center. This can be demonstrated by observing the vertical forces involved with a weight placed in the center (as shown in Fig. 24a) vs. a weight placed towards the side (as in Fig. 24b). This error can only be corrected by eliminating the side supports and designing a different configuration.

Contact area was also found to cause a significant error. For a given weight, a 10% reduction in contact area resulted in a 1.6% increase in pressure. For an average size baby (i.e., 4kg), this amounts to an error of 64 grams which is unacceptable.



(a) Illustration of the mild vertical force components experienced by the plexiglas sides when a weight is placed in the geometric center of the mattress.



(b) Illustration of the more severe vertical force components experienced by the plexiglas sides when a weight is placed towards one side.

Fig. 24. Illustration of Vertical Forces Involved in Supporting the Applied Weight by the Plexiglas Walls.

Mattress IV

In a final attempt to overcome the errors in weight measurement found in the first three mattresses, a drastic reconfiguration was necessary in design. Since Mattress III was successful in overcoming problems with stretch, drift, and rocking, the remaining problems included sensitivity to the location of applied weight and sensitivity to variation in contact area.

Analyzing the problem of sensitivity to location of applied weight and relating it to the configuration of Mattress III, suggests the need for a uniform water surface where no side supports or variation in curvature of the mattress material occur (variations in radius of curvature reflect changes in tension).

Sensitivity to changes in contact area by Mattress III also suggests the need for a uniform surface with no rigid supports. It indicates the need for a configuration that provides maximum volume and minimum area so that any applied force will be translated into pressure. The tendency for previous mattresses to take the shape of a sphere was observed with application of weights with small contact areas. Starting with a spherical configuration would give the water no room to be displaced and should result in a direct pressure translation independent of contact area. Obviously a single sphere would produce tremendous rocking problems so Mattress IV was constructed of several small tubes. The mattress was 25" x 13" x 1.5" in dimension and was made using ¼" plexiglas. Three-fourths inch square brass stock was used to mount 16 brass sleeves 11/16" in diameter (see Fig. 25). Sweat soldering techniques were used to seal brass end pieces and attach the brass sleeves to the

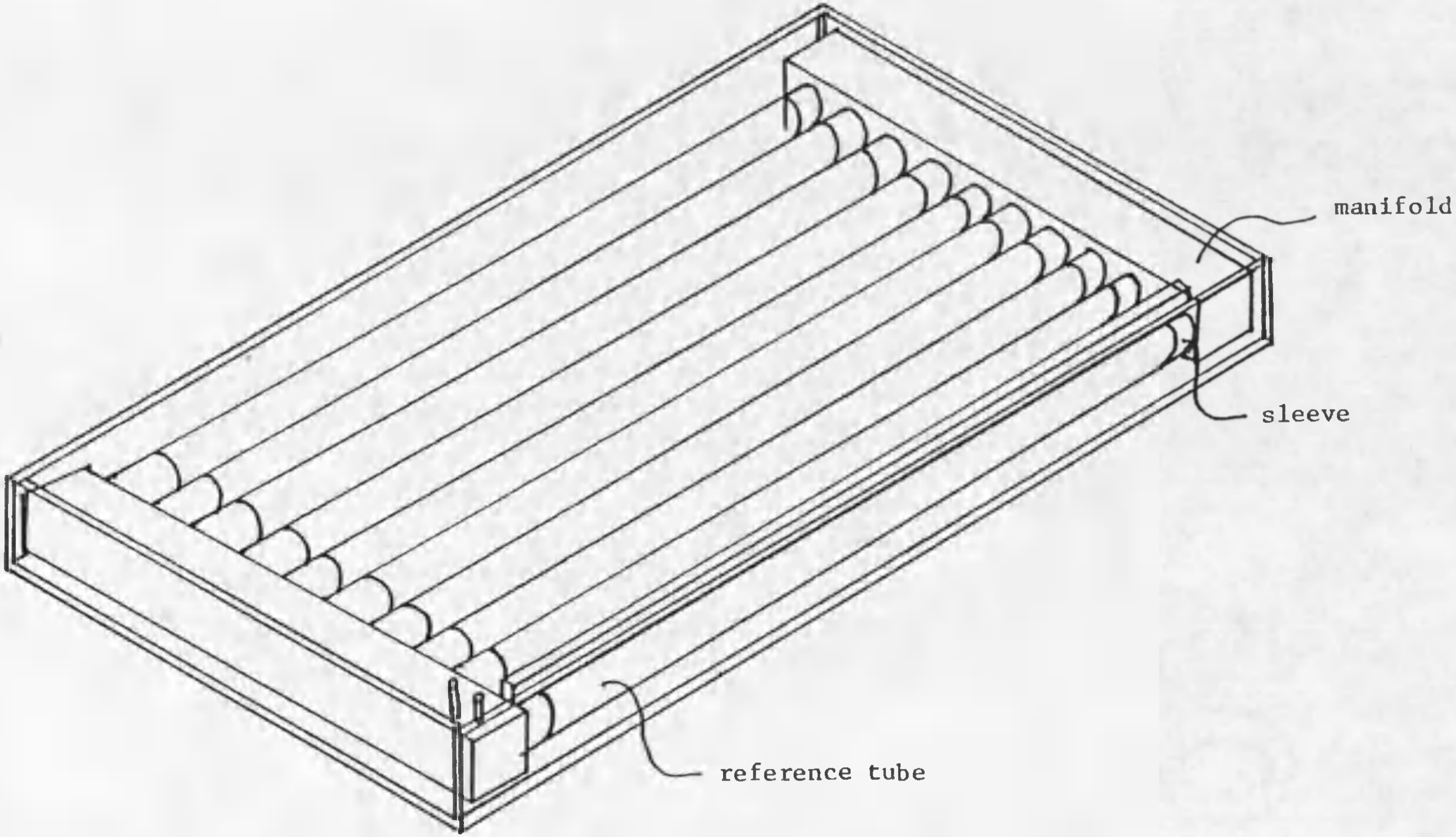


Fig. 25. Diagram of Mattress IV.

square brass stock as shown to form two manifolds. "O" rings were fitted to the brass sleeves after a groove was cut in each sleeve to improve the integrity of the water-tight seal.

The tubing used with this mattress determined the success of its pressure/weight characteristics. Tubing characteristics had to include the following:

1. provide a minimum amount of stretch,
2. provide a maximum amount of elasticity (offer no support to the applied weight),
3. provide a uniform consistent surface throughout its length,
4. provide no irritation to the patient's skin.

After testing several kinds of tubing (including polycarbonate, polyethelyne, reinforced rubber, vinyl, and Teflon), Teflon tubing was found to best meet these criteria. Samples of several thicknesses ranging from 0.001" to 0.01" were tested and tubing with 0.004" thickness and 5/8" O.D. Korves TFE tubing provided by Chemplast, Inc., Wayne, New Jersey, was chosen as providing optimal characteristics. Tubing was purchased from Chemplast Corporation. Because the chosen material was teflon, construction of the brass sleeves and "O" rings was critical for obtaining a water-tight seal. Fortunately, it was possible to shrink the diameter of the Teflon tubing by applying heat using a heat gun (type used for shrink tubing).

The technique used to connect the tubing to the brass manifolds involved holding the Teflon over the 3/4" long sleeves and by applying heat it was possible to shrink the tubing tightly over the "O" rings and brass sleeve. This was done for both ends of all 16 tubes. The actual effective mattress area is 22" x 13" due to the manifolds.

One tube was separate from the others so that it could be used as a reference pressure tube (see Fig. 25). This tube allowed the use of a differential pressure transducer. One part of the differential pressure transducer was connected to the main mattress pressure system and the other part was connected to the independent pressure tube. This allowed for temperature compensation through the controlled use of a stopcock connected between the two pressure systems.

The sensitivity of the Teflon mattress was found to be 1.25" H_2O/kg using an 81 square inch contact area. In order to determine the effects of changes in contact area, a constant weight was applied using several bases with different areas. For each base, pressure changes were recorded. The graph of Fig. 26 shows that the pressure in the mattress is dramatically effected by the changes in contact area and is not linearly proportional to the applied force. The errors introduced for contact areas ranging from six square inches to 140 square inches as shown in Fig. 26, range from +11.75kg to -1.15kg. Although it is very unlikely, a patient would change his contact area with the mattress this severely by turning on his side or back. It is reasonable to expect the patient to prop himself up on his elbows or hands which would certainly result in dramatic contact area variations.

If 81 square inches is used as the initial contact area (typical for a small baby), with 1.25 square inches/kg sensitivity, a 10%

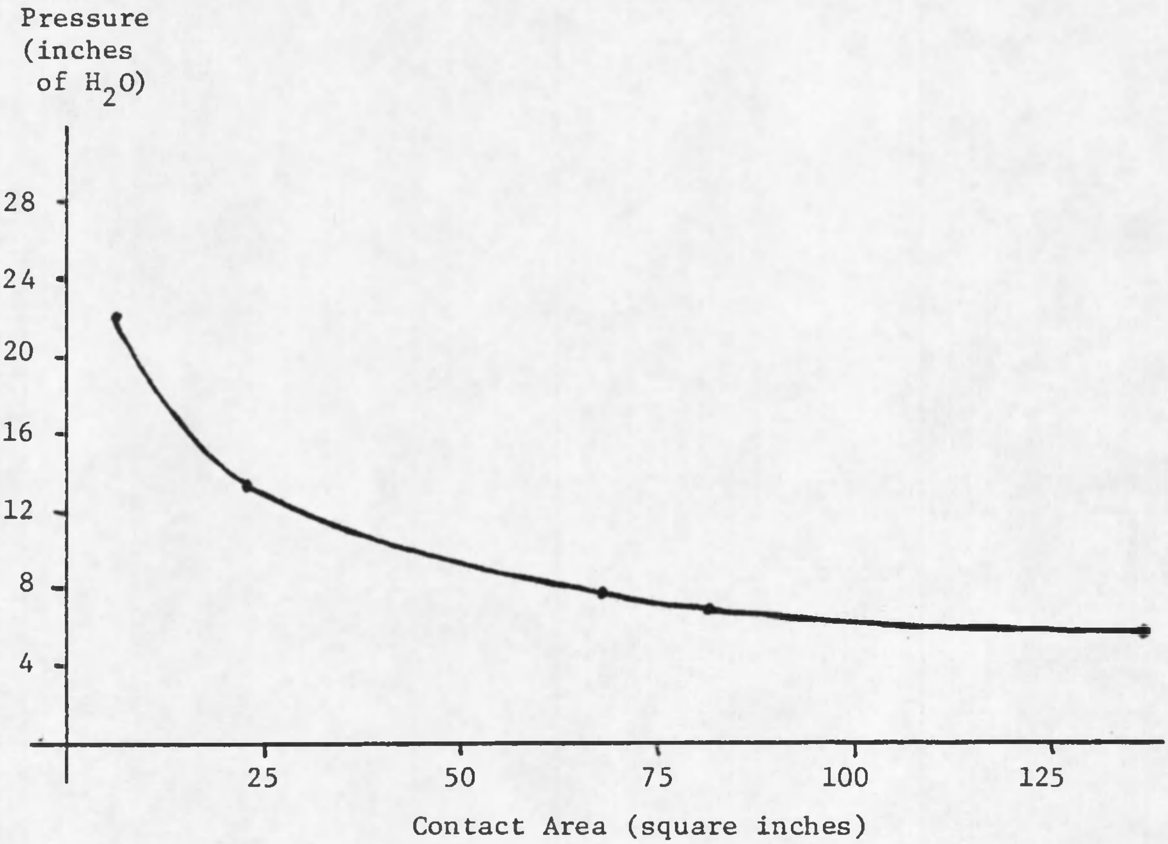


Fig. 26. Plot Illustrating Pressure as a Function of Contact Area.

change in contact area results in an error of 9% in weight reading. This figure far exceeds the 0.25% required specifications.

Sensitivity to location of applied weight was tested by dividing the total effective mattress area into 18 sections (see Fig. 27). A weight with a 7 square inch base was applied to each of the 18 positions and voltage readings were taken for each position. The mean voltage for the 18 readings was 4.562 Volts and standard deviation was calculated to be ±0.22 Volts (or 4.8%). Because of the absence of walls in Mattress IV, the specification for sensitivity to location of applied weight was defined such that less than a 10% error over the full width and 90% of the effective length was required. Data reveals that as much as a 31% error due to location within these specifications was observed, which is unacceptable.

It was observed that this error was a direct result of a failure on the part of the Teflon tubing to meet characteristic Number 3 (see page 75). The Teflon tubing is manufactured through a process of melt extrusion where only loose diameter tolerances are possible. Any given tube was found to vary in diameter as much as 0.125 inches over its length. This variation in diameter resulted in variations in pressure readings as a function of position due to variations in contact area from one position to the next.

The failure of Mattress IV to meet design criteria can be isolated to two items:

1. Variations in the diameter of Teflon tubing caused in inconsistent surface which provided different contact areas for the

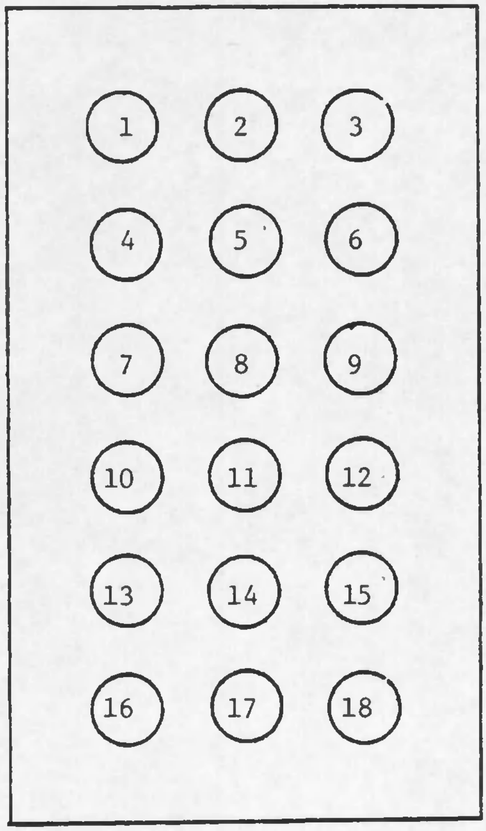


Fig. 27. Determining Sensitivity to Location of Contact Area.

Eighteen locations as indicated above were used to apply a weight and test the mattresses' ability to provide pressure responses independent of the location of applied weight.

same applied weight. This resulted in variations in output as a function of position of applied weight.

2. The tubular mattress configuration failed to overcome the contact area problem. The Teflon tubing, despite its maximum volume/minimum area design, did not offer the necessary pressure/weight response needed for the weight measurements required here.

Both of these errors are due to inconsistant contact areas between the applied weight and the mattress surface. This phenomenon was discussed earlier in Chapter 3 where a change in pressure (ΔP) was mathematically described as a function of applied force (F), and contact area (A). This equation is repeated here for further discussion.

$$\Delta P = (1/A_1)\ \Delta F - ((F/A^2)\ \Delta A)$$

It was also mentioned that with force held constant this relationship could be reduced to:

$$\Delta P = -(F/A^2)\ \Delta A$$

or

$$\Delta P/\Delta A = -F/A^2$$

The efficiency of this relationship can be examined by correlating the experimental data of Fig. 26 with theoretical values extracted from this equation. The results are shown in Fig. 28. Experimental values were obtained by determining the slope of straight lines drawn between the experimental values obtained in Fig. 26. Scaled values were determined by calculating a constant for F equal to 229.85 (this number was

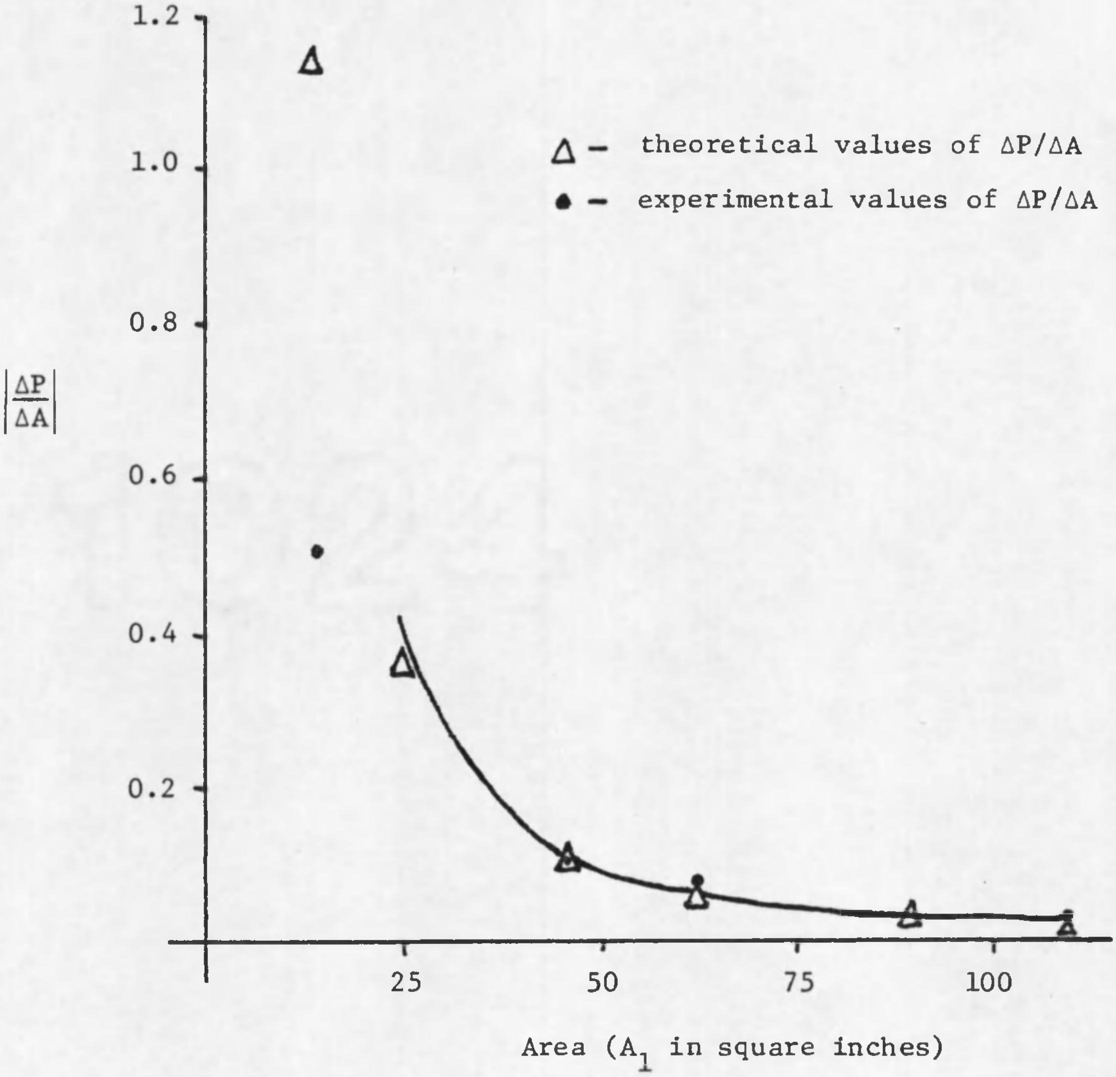


Fig. 28. Plot Illustrating the Correlation between Theoretical and Experimental Values of $\Delta P/\Delta A$.

arbitrarily used to scale the theoretical $1/A^2$ curve to match the experimental value of $\Delta P/\Delta A$ at an area equal to 45 square inches).

Examination of the two curves demonstrated a good fit for areas above 62 square inches and a poor fit for the one value taken at an area equal to 14 square inches. This experimental value was rejected due to bottoming out of the weight that was applied to the mattress which dramatically reduced the resulting pressure. This correlation illustrates the reason for the failure of water mattresses to faithfully produce a linear relationship between the pressure produced by an applied force.

CHAPTER 8

CONCLUSIONS

The concept of using a water mattress to accomplish measurement of patient weight and simultaneously provide potential body temperature regulation, heart rate and respiratory rate information along with improved patient comfort is truly an attractive one. The evolution throughout this thesis of an effective and practical patient weighing system resulted in a drastic change in the areas of anticipated research emphasis. It was originally intended that the applied engineering skills would focus on signal conditioning and the application of electronic design. Failure to identify the hydrostatic complications associated with mattress design resulted in major modification of the research emphasis.

The purpose of this thesis was redefined to involve the design of a mattress which would provide a pressure/weight response conducive to the development of a patient weighing system. After the design and construction of four mattresses, the conclusion of this thesis is that a patient weighing system meeting specified design criteria cannot be accomplished with a deformable water mattress. Mattress IV represents the culmination of acquired knowledge in mattress design. Problems left unresolved by Mattress IV included failure to meet specifications for sensitivity to variations in contact area and location of applied weight. To accomplish immunity to variations in location of applied weight, it would be necessary to obtain teflon tubing of a consistent diameter. Improvements in the manufacturing process of the tubing would have to take place to solve this problem.

A solution to the effect of variations in contact area was never resolved. The tubular mattress design was an attempt to overcome this problem through providing a maximum volume/minimum area mattress. Despite these attempts, the fact remains that the hydrostatic pressure produced in a water mattress is proportional to the applied pressure (as was discussed with Fig. 26) and cannot be directly correlated to applied force unless contact area is maintained constant.

APPENDIX A

TABLE OF UNITS AND ABBREVIATIONS

Units	Abbreviation	
Length	m	meter
	cm	centimeter
	mm	millimeter
	μm	micrometer
Force	g	gram
	kg	kilogram
	N	Newton
Time	s	second
	min	minute
	hr	hour
Voltage	V	Volt
	mV	millivolt
	μV	microvolt
Resistance	Ω	ohm
Pressure	N/m^2	Newton per meter squared
	mmHg	millimeters of mercury
	cmH_2O	centimeters of water
Temperature	°C	degree centigrade
Frequency	Hz	Hertz

SELECTED BIBLIOGRAPHY

Barna, P., (Ed.), Fluid Mechanics for Engineers, 2nd Ed., Chapter 1, London, Butterworths, 1964.

Bell and Howell, Pressure Transducer Handbook, 1974.

Cromwell, L., F. Weibell, Biomedical Instrumentation and Measurements, Prentice Hall, New York, 1973.

Currie, I. G., Fundamental Mechanics of Fluids, McGraw-Hill, New York, 1974.

DeLoatch, E. M., Design of an Automatic Weight Scale for an Isolette, Final Report for the Summer Institute in Biomedical Engineering, Goddard Space Flight Center, 1973.

Health Devices, "Patient Weighing Systems," Emergency Care Research Institute, Vol. 4, #1, Nov. 1974.

Hercig, E. E., Handbook of Measurement and Control, Schaevitz Engineering, Pennsauken, New Jersey, 1972.

Malvern, Lawrence E., Introduction to the Mechanics of a Continuous Medium, Prentice Hall, New York, 1969.

Millman, J., C. Halkias, Integrated Electronics, McGraw-Hill, New York, p. 550, 1972.

Prager, W., Introduction to the Mechanics of Continia, Dover Publications, New York, 1973.

Wait, J. V., L. P. Huelsman, and G. A. Korn, Introduction to Op Amp Theory and Application, McGraw-Hill, New York, p. 28, 1975.

Weast, R. C. (Ed.), Handbook of Chemistry and Physics, CRC Press, New York, 53rd Edition.