BUCKLING OF PORTAL FRAMES
UNDER COMBINED LOADS

by

Amir Behrooz Ghazvinian

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APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

Donald A. DaDeppo ______________________ Nov. 20, 1981 ______________

DONALD A. DADEPPO — Date

Professor of Civil Engineering
and Engineering Mechanics

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ABSTRACT

This thesis deals with the stability of pin connected rectangular frames. Loading on a frame consists of a uniformly distributed load on the beam and two equal concentrated loads on the columns. The basic slope-deflection equations are used to derive the necessary expressions for determining the critical load and type of buckling of frames with varied physical properties. A comparison is made between traditional methods that neglect the effect of axial forces and results obtained from this analysis. A design guide utilizing the critical load values of the frames is provided.

CHAPTER 1

INTRODUCTION

This study is concerned with determining the buckling mode and load of the rectangular frame shown in Figure 1.

There are two recognized modes of instability associated with such a frame (Tall 1974).

1. The symmetrical mode (Figure 2).
2. The anti-symmetrical mode (Figure 3).

In the first mode, the frame fails by snap-thru buckling, keeping its symmetrical shape before and after reaching the critical load. It is said that this type of failure induces no sudden change of the deformed configuration and can occur only when positive bracing is provided to prevent the frame from swaying side ways.

In the second mode, the frame fails by side-sway buckling, shifting from its symmetrical shape suddenly to an anti-symmetrical configuration at the instant when the critical load is reached. It is claimed that failure by side-sway is always produced under smaller loads than that required for a snap-thru failure, for any given frame.

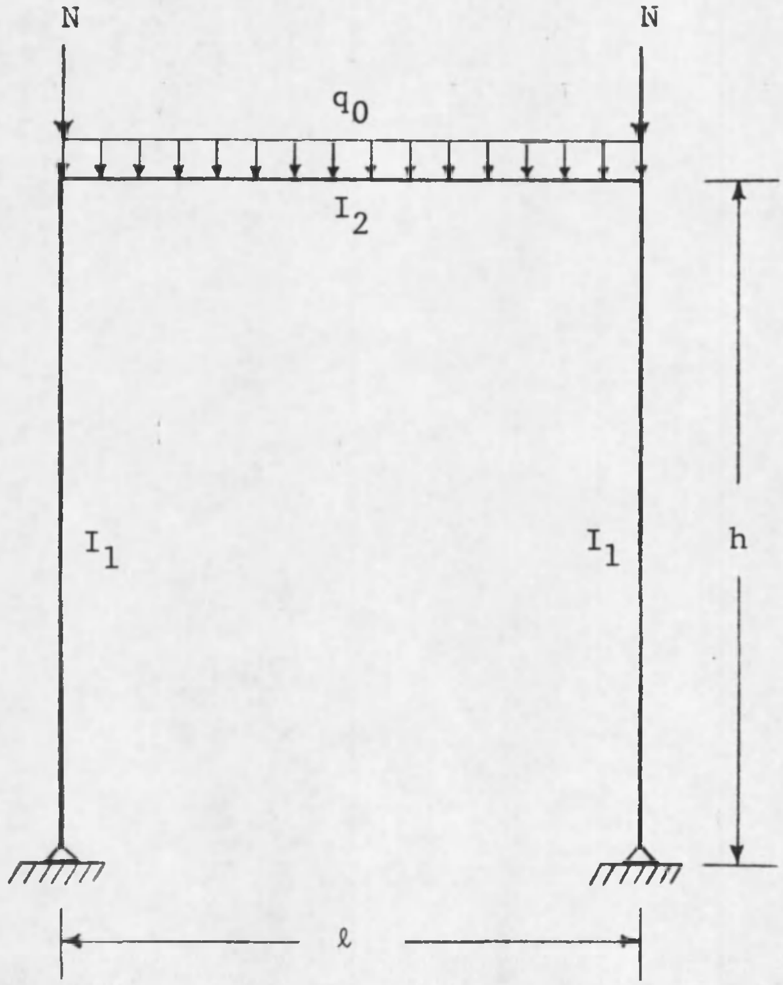

Figure 1. Loading condition

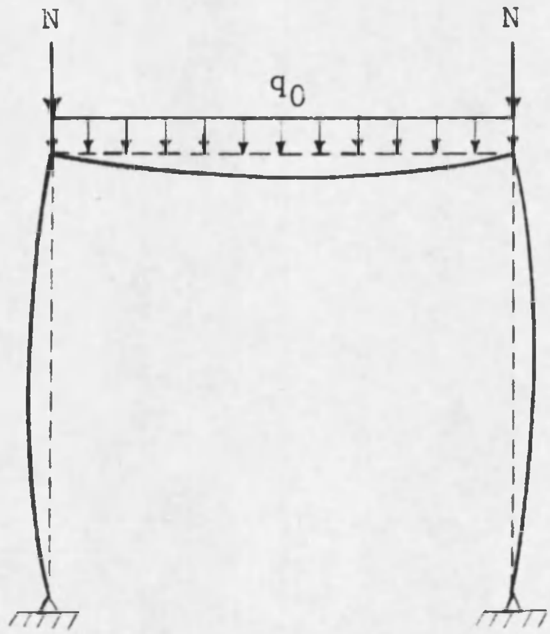

Figure 2. Symmetric Instability of a Portal Frame

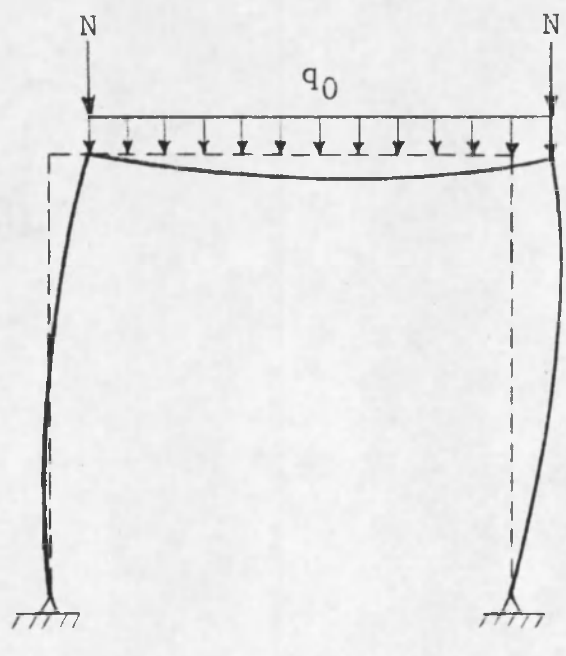

Figure 3. Anti-symmetric Instability of a Portal Frame

The tradition in analyzing the critical strength of the frame, loaded as shown in Figure 1, is to replace the distributed load by two equivalent concentrated loads on top of the columns. This leads to a great deal of simplification in solving the problem and intuitively it seems to yield results of acceptable degree of accuracy. However, it should be recognized that there is a basic difference between these problems. For the original problem, the frame members are subjected to combined bending moment and axial force and the analysis requires that one solve a nonlinear boundary value problem to determine the equilibrium configuration from which buckling occurs. In the simplified loading case, axial forces only are present in the members at the instant of failure and the equilibrium configuration, namely the undeformed configuration, from which buckling occurs is known.

Traditional design methods used to estimate the effects of compressive axial loads, employ and "effective length factor," K. The factor, K, can be estimated by:

1. Interpolating between the K values found for columns with idealized end conditions.
2. Using charts or tables which consider varying end conditions.
3. Performing a stability analysis of the entire structure.

The first procedure yields only approximate K factors, which must be used with engineering judgment. The second method results in more accurate values. The third provides exact solutions; however, it often requires time-consuming and excessive numerical calculations.

The usual methods that are used to determine the force distribution in frame members such as moment-distribution and slope-deflection, yield results that are satisfactory for most problems. However they simplify the problem by not considering the effects of axial forces on the members and secondary bending moments caused by eccentricity of axial forces. Current A.I.S.C. specifications (American Institute of Steel Construction 1970) are conservative to allow for the effect of additional moments caused by lateral displacements of columns. It should also be noted that current methods being used assume that a linear relationship exists between the moment at a joint and its rotation. Application of this assumption can be clearly seen from moment-area and conjugate-beam methods, where loads may also be superimposed to solve a problem.

Not until recently has it been practical to carry out the time-consuming calculations involved in a more exact method. These calculations can now be done with the aid of large computers in a short time and cheaply. In fact, until results of more exact analyses have been widely substantiated, accepted and made available to designers, simplified methods of analysis of uncertain accuracy will continue to be used.

It is, therefore, the purpose of this investigation to examine the results of a more accurate analysis performed on enough frames of varied physical properties to establish charts and guidelines that can be used for practical design applications.

CHAPTER 2

INVESTIGATION

This investigation is limited to a simple rectangular frame with general dimensions and loads shown in Figure 4. However, the same method of analysis can be applied to other frames as well.

The modulus of elasticity, E, is assumed to be constant for all members. Also the frame is assumed to be hinged at the column bases. The physical properties of the frame such as moment of inertia, I, and the length of the members will be considered as parameters.

By varying the parameters involved in this problem, there can be an infinite number of frames to be solved. Therefore, to keep this study within reasonable bounds, a number of frames will be chosen in the range considered practical for most design applications as well as to establish a pattern for frames that are not represented in the charts.

The frame can reach its critical buckling load under three possible loading conditions.

1. Uniform load alone.
2. A combination of uniform and concentrated loads.
3. Concentrated load alone.

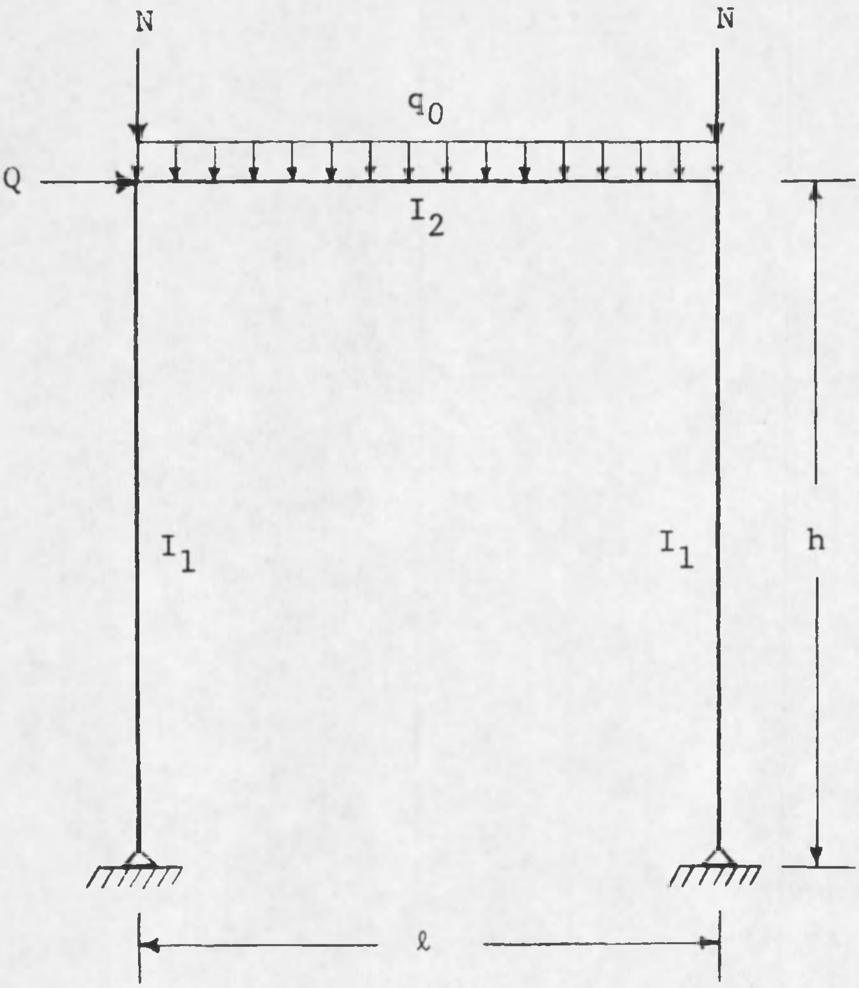

Figure 4. Loading Condition of the Rectangular Frame.

Critical loads for loading conditions 1 and 3 will be obtained first. Then to find the critical values of loading condition 2, a value for concentrated load, N, will be assumed and distributed load, q_o, will be incremented from zero until the critical buckling load of the frame is reached.

When the loading on the frame reaches the critical buckling load, it may fail only in one of two possible modes mentioned previously. It will be assumed that the frame is braced against the possibility of out of plane buckling. Also it is possible for the girder to be in tension when the frame buckles. Since this is not very likely, equations will be set up to solve for the condition that, all members are in compression. However, should any member go into tension, it will be detected.

As mentioned before, use of this method involves time-consuming calculations. Therefore, a computer program will be used to handle the arithmetic and to allow for a sufficient number of frames to be analyzed.

CHAPTER 3

DISCUSSION OF ANALYSIS

The equations in this analysis will be derived from the basic slope-deflection equations for a uniform member in axial compression and statics. It will be assumed that angles between joining members will not change as a result of loading the frame.

End moments of members, as given by slope-deflection equations and shown in Figure 5 are the sum result of the following:

1. Fixed-end moments.
2. Moments caused by the rotations of the joints.
3. Moments due to chord rotation when translation of joints with respect to one another is possible.

The stiffness factors are designated as C and S which are functions of the axial force P. In the usual form of the slope-deflection equations the effects of axial forces are neglected and the values $C = 4$ and $S = 2$ are used. However, in this analysis, axial forces will be taken into account, making C and S variables to be evaluated in the course of the analysis. The chord rotation resulting from joint translation is designated

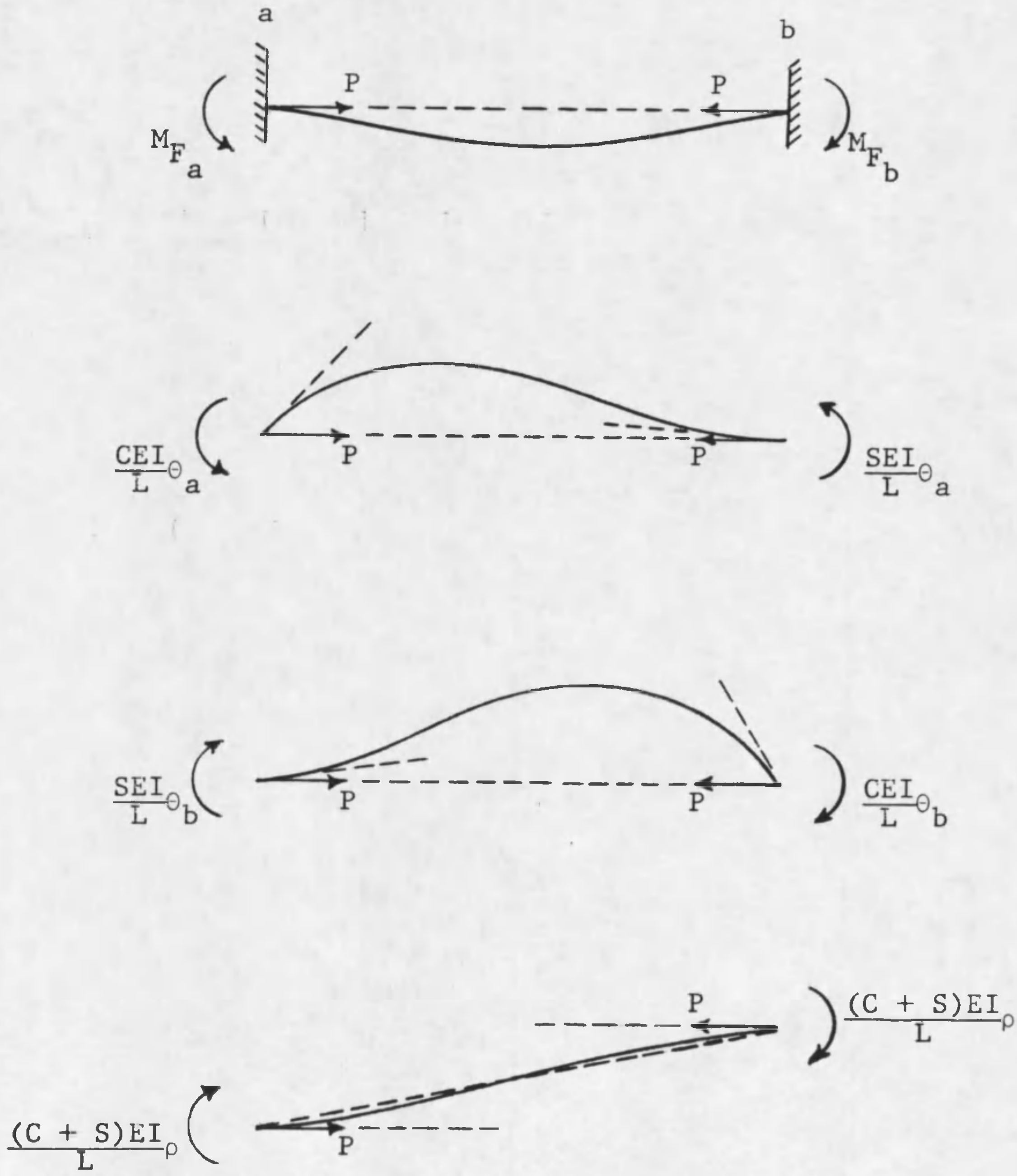

Figure 5. Components of the End Moments in Slope-Deflection Equations.

as ρ. Given $M_{12} = M_{43} = 0$, $\rho_1 = \rho_3 = \rho$ and $\rho_2 = 0$ the slope-deflection equations will become:

$$M_{12} = \frac{EI_1}{h}[C_1\Theta_1 + S_1\Theta_2 - (C_1 + S_1)\rho] = 0 \qquad (1)$$

$$M_{21} = \frac{EI_1}{h}[C_1\Theta_2 + S_1\Theta_1 - (C_1 + S_1)\rho] \qquad (2)$$

$$M_{23} = \frac{EI_2}{\ell}[C_2\Theta_2 + S_2\Theta_3] + M_{F_{23}} \qquad (3)$$

$$M_{32} = \frac{EI_2}{\ell}[C_2\Theta_3 + S_2\Theta_2] + M_{F_{32}} \qquad (4)$$

$$M_{34} = \frac{EI_1}{h}[C_3\Theta_3 + S_3\Theta_4 - (C_3 + S_3)\rho] \qquad (5)$$

$$M_{43} = \frac{EI_1}{h}[C_3\Theta_4 + S_3\Theta_3 - (C_3 + S_3)\rho] = 0 \qquad (6)$$

Figure 6 shows the internal forces and moments acting on the frame members and at the joints. The forces are resolved into horizontal and vertical components. Now, using the principles of statics equations of equilibrium for the joints and members can be written.

First, considering the three members of the frame, the following equations are obtained:

$$M_{21} + P_1h\rho - hV_{21} = 0 \qquad (7)$$

$$M_{34} + P_3h\rho - hV_{34} = 0 \qquad (8)$$

$$M_{32} + M_{23} + \ell V_{32} + q_0\frac{\ell^2}{2} = 0 \qquad (9)$$

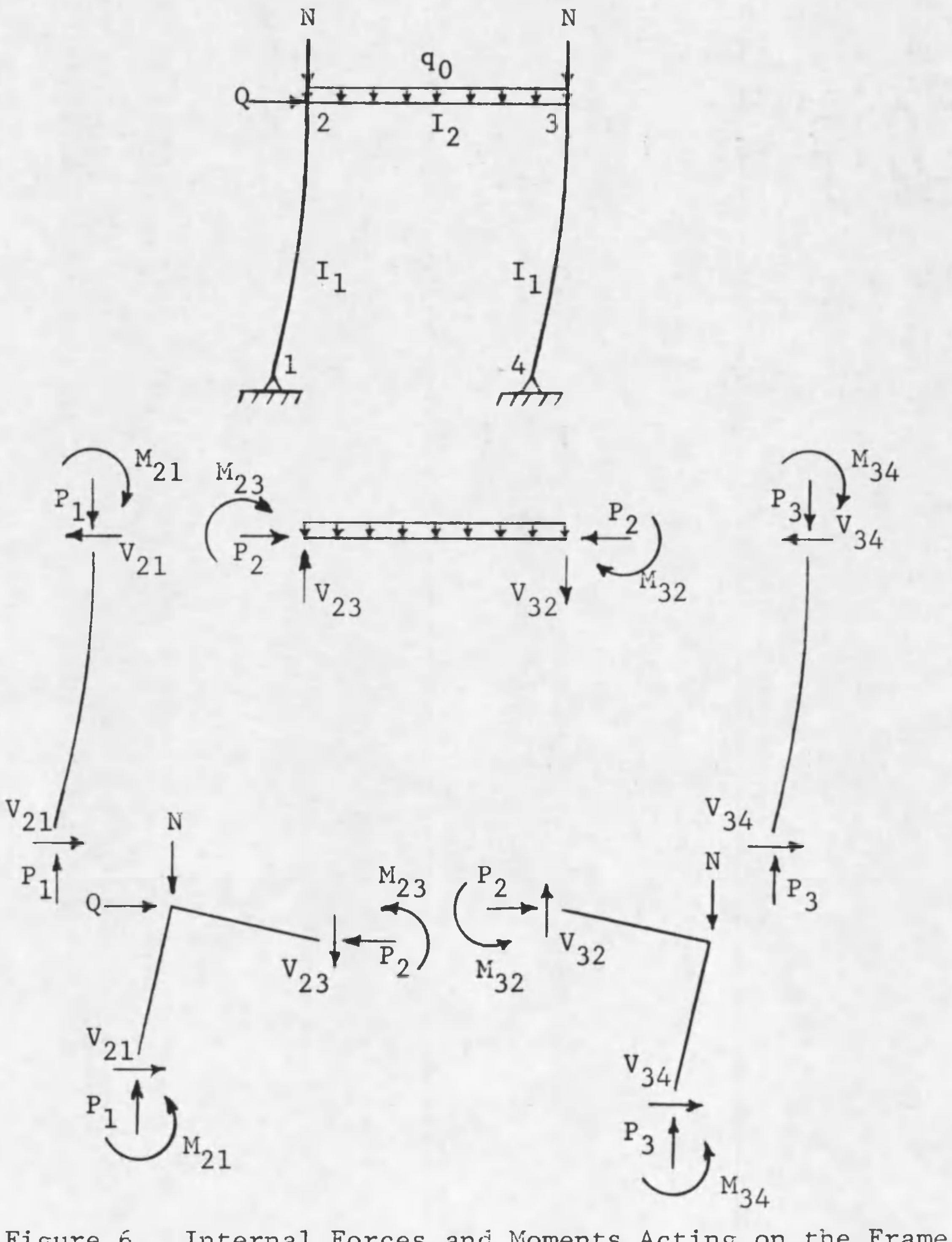

Figure 6. Internal Forces and Moments Acting on the Frame

$$M_{23} + M_{32} + \ell V_{23} - q_0\frac{\ell^2}{2} = 0 \tag{10}$$

The equations of equilibrium for joints 2 and 3 are as follows:

$$M_{21} + M_{23} = 0 \tag{11}$$

$$Q + V_{21} - P_2 = 0 \tag{12}$$

$$N + V_{23} - P_1 = 0 \tag{13}$$

$$M_{32} + M_{34} = 0 \tag{14}$$

$$P_2 + V_{34} = 0 \tag{15}$$

$$N - V_{32} - P_3 = 0 \tag{16}$$

Now, using values of the end moments obtained by the slope-deflection method in equations 2 through 5 and also taking the equivalent expressions for the internal shear forces from equation 7 through 10 and substituting these values into equations 11 through 16 will result in the following equations:

$$\frac{EI_1}{h}\left[C_1\Theta_2 + S_1\Theta_1 - (C_1 + S_1)\rho\right] + \frac{EI_2}{\ell}\left[C_2\Theta_2 + S_2\Theta_3\right] + M_{F_{23}} = 0 \tag{11a}$$

$$\frac{EI_1}{h^2}\left[C_1\Theta_2 + S_1\Theta_1 - (C_1 + S_1)\rho\right] + P_1\rho - P_2 + Q = 0 \tag{12a}$$

$$\frac{EI_2}{\ell^2}\left[C_2\Theta_2 + S_2\Theta_3\right] + \frac{EI_2}{\ell^2}\left[C_2\Theta_3 + S_2\Theta_2\right] + P_1 - N$$

$$- q_0\frac{\ell}{2} = 0 \qquad (13a)$$

$$\frac{EI_2}{\ell}\left[C_2\Theta_3 + S_2\Theta_2\right] + \frac{EI_1}{h}\left[C_3\Theta_3 + S_3\Theta_4 - (C_3 + S_3)\rho\right]$$

$$- M_{F_{23}} = 0 \qquad (14a)$$

$$\frac{EI_1}{h^2}\left[C_3\Theta_3 + S_3\Theta_4 - (C_3 + S_3)\rho\right] + P_3\rho + P_2 = 0 \qquad (15a)$$

$$\frac{EI_2}{\ell^2}\left[C_2\Theta_2 + S_2\Theta_3\right] + \frac{EI_2}{\ell^2}\left[C_2\Theta_3 + S_2\Theta_2\right] - P_3 + N$$

$$+ q_0\frac{\ell}{2} = 0 \qquad (16a)$$

As a means of simplifying these expressions and avoiding the necessity of keeping track of units, quantities will be non-dimensionalized as follows:

$$\overline{Q} = \frac{Q\ell^2}{EI_2}, \qquad \overline{q}_0 = \frac{q_0\ell^3}{EI_2}, \qquad \overline{N} = \frac{N\ell^2}{EI_2}$$

$$\overline{P}_1 = \frac{P_1\ell^2}{EI_2}, \qquad \overline{P}_2 = \frac{P_2\ell^2}{EI_2}, \qquad \overline{P}_3 = \frac{P_3\ell^2}{EI_2}$$

$$\overline{M}_{F_{23}} = \frac{M_{F_{23}}\ell}{EI_2}, \qquad \overline{M}_{F_{32}} = \frac{M_{F_{32}}\ell}{EI_2}$$

To further simplify the equations obtained previously, two variables, α and λ, are introduced to represent the following ratios:

$$\frac{h}{\ell} = \lambda$$

$$\frac{I_2 h}{\ell I_1} = \alpha$$

The fixed-end moments may be expressed in the following form (Timoshenko and Gere 1961):

$$M_{F_{23}} = \frac{-q_0 \ell^2}{12} F(U_2)$$

$$M_{F_{32}} = \frac{q_0 \ell^2}{12} F(U_2)$$

Where U_2 is such that $\overline{P}_2 = 4(U_2)^2$. Non-dimensionalized values of these fixed-end moments can be obtained as follows:

$$\overline{M}_{F_{23}} = \frac{M_{F_{23}} \ell}{EI_2} = -\frac{q_0 \ell^3}{12EI_2} F(U_2) = -\frac{\overline{q}_0}{12} F(U_2)$$

$$\overline{M}_{F_{32}} = \frac{M_{F_{32}} \ell}{EI_2} = \frac{q_0 \ell^3}{12EI} F(U_2) = \frac{\overline{q}_0}{12} F(U_2)$$

Resulting equations from non-dimensionalizing equations 11a through 16a are:

$$\left(\frac{S_1}{\alpha}\right)\Theta_1 + \left(\frac{C_1}{\alpha} + C_2\right)\Theta_2 + (S_2)\Theta_3 - \frac{1}{\alpha}(C_1 + S_1)\rho$$

$$+ \overline{M}_{F_{23}} = 0 \qquad (11b)$$

$$\left(\frac{S_1}{\alpha\lambda}\right)\Theta_1 + \left(\frac{C_1}{\alpha\lambda}\right)\Theta_2 - \left(\frac{C_1}{\alpha\lambda} + \frac{S_1}{\alpha\lambda} - \overline{P}_1\right)\rho - \overline{P}_2 + \overline{Q} = 0 \qquad (12b)$$

$$(C_2 + S_2)\Theta_2 + (C_2 + S_2)\Theta_3 + \overline{P}_1 - \overline{N} - \frac{\overline{q}_o}{2} = 0 \qquad (13b)$$

$$(S_2)\Theta_2 + \left(C_2 + \frac{C_3}{\alpha}\right)\Theta_3 + \left(\frac{S_3}{\alpha}\right)\Theta_4 - \left(\frac{C_3}{\alpha} + \frac{S_3}{\alpha}\right)\rho$$

$$- \overline{M}_{F_{23}} = 0 \qquad (14b)$$

$$\left(\frac{C_3}{\alpha\lambda}\right)\Theta_3 + \left(\frac{S_3}{\alpha\lambda}\right)\Theta_4 - \left(\frac{C_3}{\alpha\lambda} + \frac{S_3}{\alpha\lambda} - \overline{P}_3\right)\rho + \overline{P}_2 = 0 \qquad (15b)$$

$$(C_2 + S_2)\Theta_2 + (C_2 + S_2)\Theta_3 + \overline{N} - \overline{P}_3 + \frac{\overline{q}_0}{2} = 0 \qquad (16b)$$

Since E, I_1 and h, can not have a zero value, equations 1 and 6 can be rewritten as follows:

$$(C_1)\Theta_1 + (S_1)\Theta_2 - (C_1 + S_1)\rho = 0 \qquad (1b)$$

$$(C_3)\Theta_4 + (S_3)\Theta_3 - (C_3 + S_3)\rho = 0 \qquad (6b)$$

For purposes of this analysis and to facilitate the solution of equations, $\overline{Q}$ will be set equal to zero. The equations can then be satisfied with $\Theta_2 = -\Theta_3$, $\overline{P}_1 = \overline{P}_3$, and $\rho = 0$. This leads to:

$$\overline{P}_1 = \overline{N} + \frac{\overline{q}_0}{2}$$

Now, from equations 1b and 6b, values of Θ_1 and Θ_4 can be calculated in terms of Θ_2 and Θ_3:

$$\Theta_1 = -\frac{S_1}{C_1}\Theta_2 \tag{17}$$

$$\Theta_4 = -\frac{S_3}{C_3}\Theta_3 \tag{18}$$

From equation 12b the value of Θ_2 can be obtained.

$$\Theta_2 = \frac{\alpha\lambda C_1(4U_2^{\,2})}{C_1^{\,2} - S_1^{\,2}} \tag{19}$$

Using equation 17 and the non-dimensionalized value of $\overline{M}_{F_{23}}$, equation 11b can be rewritten in the following form:

$$\left(\frac{C_1}{\alpha} + C_2 - \frac{S_1^{\,2}}{\alpha C_1} - S_2\right)\Theta_2 - \frac{\overline{q}_0}{12}F(U_2) = 0$$

After substituting the expression for Θ_2 into this equation and rearranging terms, it becomes:

$$f = \left(\frac{C_1}{\alpha} + C_2 - \frac{S_1^{\,2}}{\alpha C_1} - S_2\right)4U^2{}_2 - \frac{\overline{q}_0}{12}\left(\frac{C_1}{\alpha\lambda} - \frac{S_1^{\,2}}{\alpha\lambda C_1}\right)F(U_2)$$

$$= 0 \tag{20}$$

Later, this equation will be used to find U_2, which in turn will allow calculation of the axial force $\overline{P}_2$. It has been shown that $\overline{P}_1 = \overline{N}_1 + \overline{q}_0/2$; therefore, for a given frame and load, the axial force, $\overline{P}_1$, can be calculated.

In order to solve equation 20 for U_2, first the stiffness factors C and S must be evaluated. Expressions have been derived to compute these stiffness factors in terms of the compressive axial force in the member (Salmon and Johnson 1971). Given that the axial force $P = 4U^2EI/\ell^2$, where I and ℓ are the moment of inertia and length of the member under consideration, these expressions are:

$$h^\circ = U \sin 2U - 2U^2 \cos 2U \tag{21}$$

$$k^\circ = 2U^2 - U \sin 2U \tag{22}$$

$$\ell^\circ = 1 - \cos 2U - U \sin 2U \tag{23}$$

Where U is in radians, then

$$C = \frac{h^\circ}{\ell^\circ} \tag{24}$$

$$S = \frac{k^\circ}{\ell^\circ} \tag{25}$$

Also, the value of $F(U_2)$ can be obtained from the following expression:

$$F(U) = \frac{3}{U^2}(1-U \cot U) \tag{26}$$

It is now possible to determine C_1 and S_1. The only unknowns that remain to be evaluated in equation 20 are C_2 and S_2. However C_2 and S_2 are functions of U_2, which is the unknown being solved for.

To solve for U_2, the Newton-Raphson method of root finding will be used. This is an iterative process by which a trial value for U_2 is applied to equation 20. Then this value is improved upon until the degree of precision required is achieved. The formula used by this method is:

$$U_2^{(S+1)} = U_2^{(S)} - \frac{f^{(S)}}{f'^{(S)}} \qquad (27)$$

Where $U_2^{(S)}$ is the trial value of U_2, $f^{(S)} = f(U_2^{(S)})$, and $f'^{(S)} = f'(U_2^{(S)})$. It should be noted that all primes used in this text represent derivatives with respect to U.

Differentiating equation 20 with respect to U_2 will provide the expression for $f'(U_2^{(S)})$.

$$f'(U_2) = \left(\frac{C_1}{\alpha} + C_2 - \frac{S_1^2}{\alpha C_1} - S_2\right)8U_2 + (C'_2 - S'_2)4U_2^2$$

$$- \frac{\overline{q}_0}{12}\left(\frac{C_1}{\alpha\lambda} - \frac{S_1^2}{\alpha\lambda C_1}\right)F'(U_2) = 0 \qquad (28)$$

In equation 28

$$C' = \frac{C(1-C)}{U} + \frac{4U^2}{\ell^\circ}\,\text{Sin } 2U \qquad (29)$$

$$S' = \frac{S(1-C)}{U} + \frac{2U}{\ell^\circ}(1 - \text{Cos } 2U) \qquad (30)$$

$$F'(U) = \frac{1}{U^3}(-6 + 3U \text{ Cot } U + 3 U^2 \text{ CSC}^2 U) \qquad (31)$$

So, by taking a trial value for $U_2^{(1)}$, there exists all the expressions needed to use equation 27, as many times as needed, to calculate the value of $U_2^{(S)}$ that will satisfy equation 20 to the degree that is desired. Once such a value for U_2 is obtained, it will be possible to calculate the end moments and the remaining unknown forces of the frame using equations 2 through 16.

CHAPTER 4

LOADING

It was previously shown that the axial forces are dependent upon the existing loads on the frame, $\bar{q}_0$ and $\bar{N}$. As mentioned before, a value for $\bar{N}$ will be assumed first, and then the value of $\bar{q}_0$ starting from zero will be incremented until the critical total load of the frame is obtained. Therefore, an incremental change in loading, $\Delta\bar{q}_0$, will be introduced. This may be thought of as the change in loading with respect to time at a rate $\dot{\bar{q}}_0$, where $\Delta\bar{q}_0 = \dot{\bar{q}}_0\Delta T$. So the axial force $\bar{P}_1$ which was equal to $\bar{N} + \bar{q}_0/2 = 4U_1^2/\alpha\lambda$, after taking this change in loading into account, will be as follows:

$$\bar{P}_1 + \Delta\bar{P}_1 = \bar{N} + \frac{\bar{q}_0 + \Delta\bar{q}_0}{2} = \frac{4(U_1 + \Delta U_1)^2}{\alpha\lambda} \qquad (32)$$

From this equation the corresponding value for the change in U_1, designated as ΔU_1 can be obtained. To reduce the amount of guessing and iterations necessary to determine U_2, an expression will be derived to provide an estimate of change in the value of U_2 each time the load is increased by the increment $\Delta\bar{q}_0$. Since by equation 20, $f(U_1, U_2) = 0$.

$$df = \left(\frac{\partial f}{\partial U_1}\right)\Delta U_1 + \left(\frac{\partial f}{\partial U_2}\right)\Delta U_2 = 0$$

from which

$$\Delta U_2 = - \frac{\left(\frac{\partial f}{\partial U_1}\right)}{\left(\frac{\partial f}{\partial U_2}\right)}\Delta U_1$$

So now the first trial value for U_2 will be

$$U_2^{(1)} = U_2 + \Delta U_2$$

An appropriate point to start the calculations will be for the unloaded frame when $\overline{P}_1 = \overline{P}_2 = \overline{P}_3 = 0$. The correct values for the variables are:

$$C_1 = C_2 = 4, \; S_1 = S_2 = 2$$

$$F(0) = 1, \; F'(0) = 0$$

which are substituted into equation 20 to obtain the initial trial value of U_2 at the time when the first increment of $\Delta\overline{q}_0$ is applied to the unloaded frame.

CHAPTER 5

STABILITY

As mentioned before, the frames are solved for values of $\bar{q}_o$ starting from zero and increasing until the critical load $\bar{q}_{ocr}$ is reached. Any attempt to increase the applied load beyond this point will cause the frame to fail by buckling. Equations for determination of a critical state of loading are derived from rate equations of equilibrium obtained by differentiating equations 11b through 16b, 1b, and 6b with respect to time as follows:

$$(\dot{C}_1)\Theta_1 + (C_1)\dot{\Theta}_1 + (\dot{S}_1)\Theta_2 + (S_1)\dot{\Theta}_2 - (\dot{C}_1 + \dot{S}_1)\rho - (C_1 + S_1)\dot{\rho} = 0 \qquad (1c)$$

$$(\dot{C}_3)\Theta_4 + (C_3)\dot{\Theta}_4 + (\dot{S}_3)\Theta_3 + (S_3)\dot{\Theta}_3 - (\dot{C}_3 + \dot{S}_3)\rho - (C_3 + S_3)\dot{\rho} = 0 \qquad (6c)$$

$$\left(\frac{\dot{S}_1}{\alpha}\right)\Theta_1 + \left(\frac{S_1}{\alpha}\right)\dot{\Theta}_1 + \left(\frac{\dot{C}_1}{\alpha} + \dot{C}_2\right)\Theta_2 + \left(\frac{C_1}{\alpha} + C_2\right)\dot{\Theta}_2 + (\dot{S}_2)\Theta_3 + (S_2)\dot{\Theta}_3 - \frac{1}{\alpha}(\dot{C}_1 + \dot{S}_1)\rho - \frac{1}{\alpha}(C_1 + S_1)\dot{\rho} + \dot{\bar{M}}_{F_{23}} = 0 \qquad (11c)$$

$$\left(\frac{\dot{S}_1}{\alpha\lambda}\right)\Theta_1 + \left(\frac{S_1}{\alpha\lambda}\right)\dot{\Theta}_1 + \left(\frac{\dot{C}_1}{\alpha\lambda}\right)\Theta_2 + \left(\frac{C_1}{\alpha\lambda}\right)\dot{\Theta}_2 - \left(\frac{\dot{C}_1}{\alpha\lambda} + \frac{\dot{S}_1}{\alpha\lambda} - \dot{\bar{p}}_1\right)\rho$$

$$- \dot{\bar{p}}_2 - \left(\frac{C_1}{\alpha\lambda} + \frac{S_1}{\alpha\lambda} - \bar{p}_1\right)\dot{\rho} + \dot{\bar{Q}} = 0 \qquad (12c)$$

$$(\dot{C}_2 + \dot{S}_2)\Theta_2 + (C_2 + S_2)\dot{\Theta}_2 + (\dot{C}_2 + \dot{S}_2)\Theta_3 + (C_2 + S_2)\dot{\Theta}_3$$

$$+ \dot{\bar{p}}_1 - \dot{\bar{N}} - \frac{\dot{\bar{q}}_0}{2} = 0 \qquad (13c)$$

$$(\dot{S}_2)\Theta_2 + (S_2)\dot{\Theta}_2 + \left(\dot{C}_2 + \frac{\dot{C}_3}{\alpha}\right)\Theta_3 + \left(C_2 + \frac{C_3}{\alpha}\right)\dot{\Theta}_3 + \left(\frac{\dot{S}_3}{\alpha}\right)\Theta_4$$

$$+ \left(\frac{S_3}{\alpha}\right)\dot{\Theta}_4 - \left(\frac{\dot{C}_3}{\alpha} + \frac{\dot{S}_3}{\alpha}\right)\rho - \left(\frac{C_3}{\alpha} + \frac{S_3}{\alpha}\right)\dot{\rho} - \dot{\bar{M}}_{F_{23}} = 0 \qquad (14c)$$

$$\left(\frac{\dot{C}_3}{\alpha\lambda}\right)\Theta_3 + \left(\frac{C_3}{\alpha\lambda}\right)\dot{\Theta}_3 + \left(\frac{\dot{S}_3}{\alpha\lambda}\right)\Theta_4 + \left(\frac{S_3}{\alpha\lambda}\right)\dot{\Theta}_4 - \left(\frac{\dot{C}_3}{\alpha\lambda} + \frac{\dot{S}_3}{\alpha\lambda} - \dot{\bar{p}}_3\right)\rho$$

$$- \left(\frac{C_3}{\alpha\lambda} + \frac{S_3}{\alpha\lambda} - \bar{p}_3\right)\dot{\rho} + \dot{\bar{p}}_2 = 0 \qquad (15c)$$

$$(\dot{C}_2 + \dot{S}_2)\Theta_2 + (C_2 + S_2)\dot{\Theta}_2 + (\dot{S}_2 + \dot{C}_2)\Theta_3 + (S_2 + C_2)\dot{\Theta}_3$$

$$- \dot{\bar{p}}_3 + \dot{\bar{N}} + \frac{\dot{\bar{q}}_0}{2} = 0 \qquad (16c)$$

where

$$\dot{C}_1 = \frac{dC_1}{d\bar{p}_1}\dot{\bar{p}}_1 = \frac{dC_1}{dU_1}\frac{dU_1}{d\bar{p}_1}\dot{\bar{p}}_1 = C_1' \dot{U}_1$$

similarly $\dot{S}_2 = S_2' \dot{U}_2$ and so on.

The condition to be satisfied at critical loading is that the rate equations have a non-trivial solution for a zero rate of loading. So the new conditions are $\dot{\bar{N}} = \dot{\bar{Q}} = \dot{q}_o = 0$. As before, the following conditions will also be used again: $\theta_3 = -\theta_2$, $\rho = 0$ and $\bar{p}_1 = \bar{p}_3 = \bar{N} + \frac{\bar{q}_0}{2}$

Using these relations and equations 17 and 18 in equations 11c through 16c, 1c, and 6c, the following relations can be obtained:

$$\dot{\theta}_1 = \left(\frac{-\dot{S}_1 C_1 + S_1 \dot{C}_1}{C_1^2}\right)\theta_2 + \left(-\frac{S_1}{C_1}\right)\dot{\theta}_2 + \left(\frac{C_1 + S_1}{C_1}\right)\dot{\rho}$$

$$\dot{\theta}_4 = \left(\frac{-\dot{S}_3 C_3 + S_3 \dot{C}_3}{C_3^2}\right)\theta_3 + \left(\frac{-S_3}{C_3}\right)\dot{\theta}_3 + \left(\frac{C_3 + S_3}{C_3}\right)\dot{\rho}$$

Using these relations also and continuing,

$$\left[-\frac{\dot{S}_1 S_1}{\alpha C_1} + \frac{-S_1 \dot{S}_1 C_1 + S_1^2 \dot{C}_1}{\alpha C_1^2} + \frac{\dot{C}_1}{\alpha} + \dot{C}_2 - \dot{S}_2\right]\theta_2$$

$$+ \left[\frac{C_1}{\alpha} + C_2 - \frac{S_1^2}{\alpha C_1}\right]\dot{\theta}_2 + (S_2)\dot{\theta}_3 + (C_1 + S_1)\left[\frac{S_1}{\alpha C_1} - \frac{1}{\alpha}\right]\dot{\rho}$$

$$+ \dot{\bar{M}}_{F_{23}} = 0 \qquad (11d)$$

$$\left(\dot{S}_2 - \dot{C}_2 - \frac{\dot{C}_3}{\alpha} + \frac{S_3\ \dot{S}_3}{\alpha C_3} - \frac{-S_3\ \dot{S}_3\ C_3 + S_3^2\ \dot{C}_3}{\alpha C_3^2}\right)\Theta_2 + (S_2)\dot{\Theta}_2$$

$$+ \left(C_2 + \frac{C_3}{\alpha} - \frac{S_3^2}{\alpha C_3}\right)\dot{\Theta}_3 + (C_3 + S_3)\left(\frac{S_3}{\alpha C_3} - \frac{1}{\alpha}\right)\dot{\rho} - \dot{\bar{M}}_{F_{23}} = 0 \qquad (14d)$$

$$\left(- \frac{S_1\ \dot{S}_1}{\alpha\lambda C_1} + \frac{-S_1\ \dot{S}_1\ C_1 + S_1^2\ \dot{C}_1}{\alpha\lambda C_1^2} + \frac{\dot{C}_1}{\alpha\lambda}\right)\Theta_2 + \left(\frac{-S_1^2}{\alpha\lambda C_1} + \frac{C_1}{\alpha\lambda}\right)\dot{\Theta}_2$$

$$+ \left[(C_1 + S_1)\left(\frac{S_1}{\alpha\lambda C_1} - \frac{1}{\alpha\lambda}\right) + \bar{p}_1\right]\dot{\rho} - \dot{\bar{p}}_2 = 0 \qquad (12d)$$

$$\left(\frac{S_3\ \dot{S}_3}{\alpha\lambda C_3} - \frac{\dot{C}_3}{\alpha\lambda} - \frac{-S_3\ \dot{S}_3\ C_3 + S_3^2\ \dot{C}_3}{\alpha\lambda C_3^2}\right)\Theta_2 + \left(\frac{C_3}{\alpha\lambda} - \frac{S_3^2}{\alpha\lambda C_3}\right)\dot{\Theta}_3$$

$$+ \left[(C_3 + S_3)\left(\frac{S_3}{\alpha\lambda C_3} - \frac{1}{\alpha\lambda}\right) + \bar{p}_3\right]\dot{\rho} + \dot{\bar{p}}_2 = 0 \qquad (15d)$$

$$(C_2 + S_2)\ (\dot{\Theta}_2 + \dot{\Theta}_3) + \dot{\bar{p}}_1 = 0 \qquad (13d)$$

$$(C_2 + S_2)\ (\dot{\Theta}_2 + \dot{\Theta}_3) - \dot{\bar{p}}_3 = 0 \qquad (16d)$$

From equations 13d and 16d, it can be seen that $\dot{\bar{p}}_3 = -\dot{\bar{p}}_1$, therefore, $\dot{C}_3 = -\dot{C}_1$, and $\dot{S}_3 = -\dot{S}_1$, also since $\bar{p}_1 = \bar{p}_3$, relations $C_1 = C_3$ and $S_1 = S_3$, hold true. Using these relations in the process, add equations 11d to 14d and 12d to 15d; also subtract equations 11d from 14d and 12d from 15d, to obtain

$$\frac{2}{\alpha}\left(\dot{C}_1 - \frac{\dot{S}_1\ S_1}{C_1} + \frac{-S_1\ \dot{S}_1\ C_1 + S_1^2\ \dot{C}_1}{C_1^2}\right)\Theta_2 + \left(C_2 + \frac{C_1}{\alpha} - \frac{S_1^2}{\alpha C_1}\right.$$

$$\left. + S_2\right)(\dot{\Theta}_2 + \dot{\Theta}_3) + \frac{2}{\alpha}\left(\frac{S_1}{C_1} - 1\right)(C_1 + S_1)\dot{\rho} = 0 \qquad (34)$$

$$\frac{2}{\alpha\lambda}\left(\dot{C}_1 + \frac{-S_1\ \dot{S}_1\ C_1 + S_1^2\ \dot{C}_1}{C_1^2} - \frac{S_1\ \dot{S}_1}{C_1}\right)\Theta_2$$

$$+ \frac{1}{\alpha\lambda}\left(C_1 - \frac{S_1^2}{C_1}\right)(\dot{\Theta}_2 + \dot{\Theta}_3) + 2\left[\frac{1}{\alpha\lambda}(C_1 + S_1)\left(\frac{S_1}{C_1} - 1\right)\right.$$

$$\left. + \bar{P}_1\right]\dot{\rho} = 0 \qquad (35)$$

$$2(-\dot{S}_2 + \dot{C}_2)\Theta_2 + \left(\frac{C_1}{\alpha} + C_2 - \frac{S_1^2}{\alpha C_1} - S_2\right)(\dot{\Theta}_2 - \dot{\Theta}_3)$$

$$+ 2\ \dot{M}_{F_{23}} = 0 \qquad (36)$$

$$\frac{1}{\alpha\lambda}\left(C_1 - \frac{S_1^2}{C_1}\right)(\dot{\Theta}_2 - \dot{\Theta}_3) - 2\ \dot{\bar{P}}_2 = 0 \qquad (37)$$

By using the relation $\bar{p}_1 = 4U_1^2/\alpha\lambda$ and differentiating it with respect to time, the value of $\dot{\bar{P}}_1$, will be

$$\dot{\bar{P}}_1 = \frac{8U_1\ \dot{U}_1}{\alpha\lambda}$$

Substituting this expression into equation 13d, the expression for $\dot{U}_1$ can be found to be

$$\dot{U}_1 = - \frac{(C_2 + S_2)\ (\dot{\Theta}_2 + \dot{\Theta}_3)\ \alpha\lambda}{8U_1} \tag{38}$$

Also $\dot{\bar{M}}_{F_{23}}$ can be transformed as follows:

$$\dot{\bar{M}}_{F_{23}} = \frac{\partial \bar{M}_{F_{23}}}{\partial U_2}\ \dot{U}_2 = \frac{-\partial\left[\frac{\bar{q}_0}{12}F(U_2)\right]}{\partial U_2}\ \dot{U}_2$$

Using the expression for F(U) as given by equation 26 and proceeding with differentiation, one can obtain

$$\dot{\bar{M}}_{F_{23}} = \frac{\bar{q}_0}{4U_2}\left(\frac{2}{U_2^2} - \frac{1}{U_2\ \mathrm{Tan}(U_2)} - \frac{1}{\mathrm{Sin}^2\ (U_2)}\right)\dot{U}_2 \tag{39}$$

Employing equations 19, 38 and 39, also relations $\dot{C}_1 = C_1' \dot{U}_1$, $\dot{S}_1 = S_1' \dot{U}_1$, $\dot{C}_2 = C_2' \dot{U}_2$, $\dot{S}_2 = S_2' \dot{U}_2$, equation 34 through 37 become

$$\left[\frac{C_1\ U_2^2\ \alpha\lambda^2}{(C_1^2 - S_1^2)U_1}(C_2 + S_2)\left(\frac{2S_1\ S_1'\ C_1 - S_1^2\ C_1'}{C_1^2} - C_1'\right) + \left(C_2 + \frac{C_1}{\alpha} - \frac{S_1^2}{\alpha C_1} + S_2\right)\right]\cdot(\dot{\Theta}_2 + \dot{\Theta}_3)$$

$$+ \frac{2}{\alpha}(C_1 + S_1)\left(\frac{S_1}{C_1} - 1\right)\dot{\rho} = 0 \tag{34a}$$

$$\left[\frac{C_1 U_2^2 \alpha^2 \lambda^2}{(C_1^2 - S_1^2) U_1} (C_2 + S_2) \left(\frac{2S_1 S_1' C_1 - S_1^2 C_1'}{C_1^2} - C_1'\right)\right.$$

$$\left. + \left(C_1 - \frac{S_1^2}{C_1}\right)\right] (\dot{\Theta}_2 + \dot{\Theta}_3) + 2 \left[(C_1 + S_1) \left(\frac{S_1}{C_1} - 1\right)\right.$$

$$\left. + \alpha\lambda\bar{p}_1\right] \dot{\rho} = 0 \tag{35a}$$

$$\left(\frac{C_1}{\alpha} + C_2 - \frac{S_1^2}{\alpha C_1} - S_2\right) (\dot{\Theta}_2 - \dot{\Theta}_3) + 2 \left[\frac{\bar{q}_0}{4U_2} \left(\frac{2}{U_2^2} - \frac{1}{U_2 \text{ Tan } U_2}\right.\right.$$

$$\left.\left. - \frac{1}{\text{Sin}^2(U_2)}\right) + \frac{(C_2' - S_2')\alpha\lambda C_1 (4U_2^2)}{(C_1^2 - S_1^2)}\right] \dot{U}_2 = 0 \tag{36a}$$

$$\frac{1}{\alpha\lambda} \left(C_1 - \frac{S_1^2}{C_1}\right)(\dot{\Theta}_2 - \dot{\Theta}_3) - 16 U_2 \dot{U}_2 = 0 \tag{37a}$$

Equations 34a and 35a, also 36a and 37a may be represented in the following form:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} \dot{\Theta}_2 + \dot{\Theta}_3 \\ \dot{\rho} \end{Bmatrix} = 0 \tag{40}$$

and

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} \dot{\Theta}_2 - \dot{\Theta}_3 \\ \dot{U}_2 \end{Bmatrix} = 0 \tag{41}$$

So now the stability of the frame and the mode of its failure under critical loading can be examined by these equations. As noted before, the equilibrium of the frame is critical when a non-trivial solution to these equations can be found. The determinant of the coefficient matrix of equation 40 is designated as D_a and that of equation 41 as D_b. In order to find the critical loading of the frame, D_a and/or D_b have to be equal to zero. If at critical loading $D_a = 0$ and $D_b \neq 0$, then $\dot{\Theta}_2 - \dot{\Theta}_3 = 0$ and $\dot{U}_2 = 0$; therefore, $\dot{\Theta}_2 + \dot{\Theta}_3 \neq 0$ and $\dot{\rho} \neq 0$. Consequently, the frame fails by side-sway buckling under the critical load. If at critical loading $D_a \neq 0$ and $D_b = 0$, then $\dot{\Theta}_2 + \dot{\Theta}_3 = 0$ and $\dot{\rho} = 0$; therefore, $\dot{\Theta}_2 - \dot{\Theta}_3 \neq 0$ and $\dot{U}_2 \neq 0$, which means the frame fails by snap-thru buckling under the critical load. If D_a and D_b both should become zero, then the frame may fail in an arbitrary combination of the mentioned cases.

The foregoing relations are intended for the stability analysis of frames under combined loading that causes some bending deformation before buckling occurs. In the special case of pure axial loading, i.e., $\bar{q}_0 = 0$, the critical load of the frame can be written as;

$$2\bar{N}_{cr} = 2\bar{P}_{1cr} = \frac{8(U_1)^2{}_{cr}}{\alpha\lambda}$$

The value of $(U_1)_{cr}$ can be found from the following expression (Timoshenko and Gere 1961):

$$2(U_1)_{cr} \text{ Tan } (2U_1)_{cr} = 6 \frac{I_2 h}{I_1 \ell} = 6\alpha$$

By using the Newton-Raphson method on this equation, the value of $(U_1)_{cr}$ can be obtained for any given frame, which in turn will allow $2\bar{N}_{cr}$ to be calculated.

CHAPTER 6

RESULTS

All the frames that were analyzed failed by a side-sway buckling and none of the frames developed tension in the top member.

Figure 7 shows the relationship between the height to length ratio h/ℓ and $\bar{q}_{ocr}$ for different values of the inertia ratio I_2/I_1 when no concentrated loads are applied on the columns. Similarly Figure 8 represents the relationship between h/ℓ and $2\bar{N}_{cr}$ when $\bar{q}_o$ is absent. From these figures it can be seen that for increasing h/ℓ the load-carrying capacity of the frames decrease. Also for the frames of equal h/ℓ ratios, stability or load-carrying capacity increases as the inertia ratio I_2/I_1 is decreased.

Figures 9 and 10 show the relationship between $2\bar{N}$ and $\bar{q}_o$ for different values of I_2/I_1 and h/ℓ when one ratio is kept constant. An interesting observation can be made from these figures in that they show a practically straight line relationship between $2\bar{N}$ and $\bar{q}_o$. This may be expressed in the following form:

$$\frac{2\bar{N}}{2\bar{N}_{cr}} + \frac{\bar{q}_o}{\bar{q}_{ocr}} = 1$$

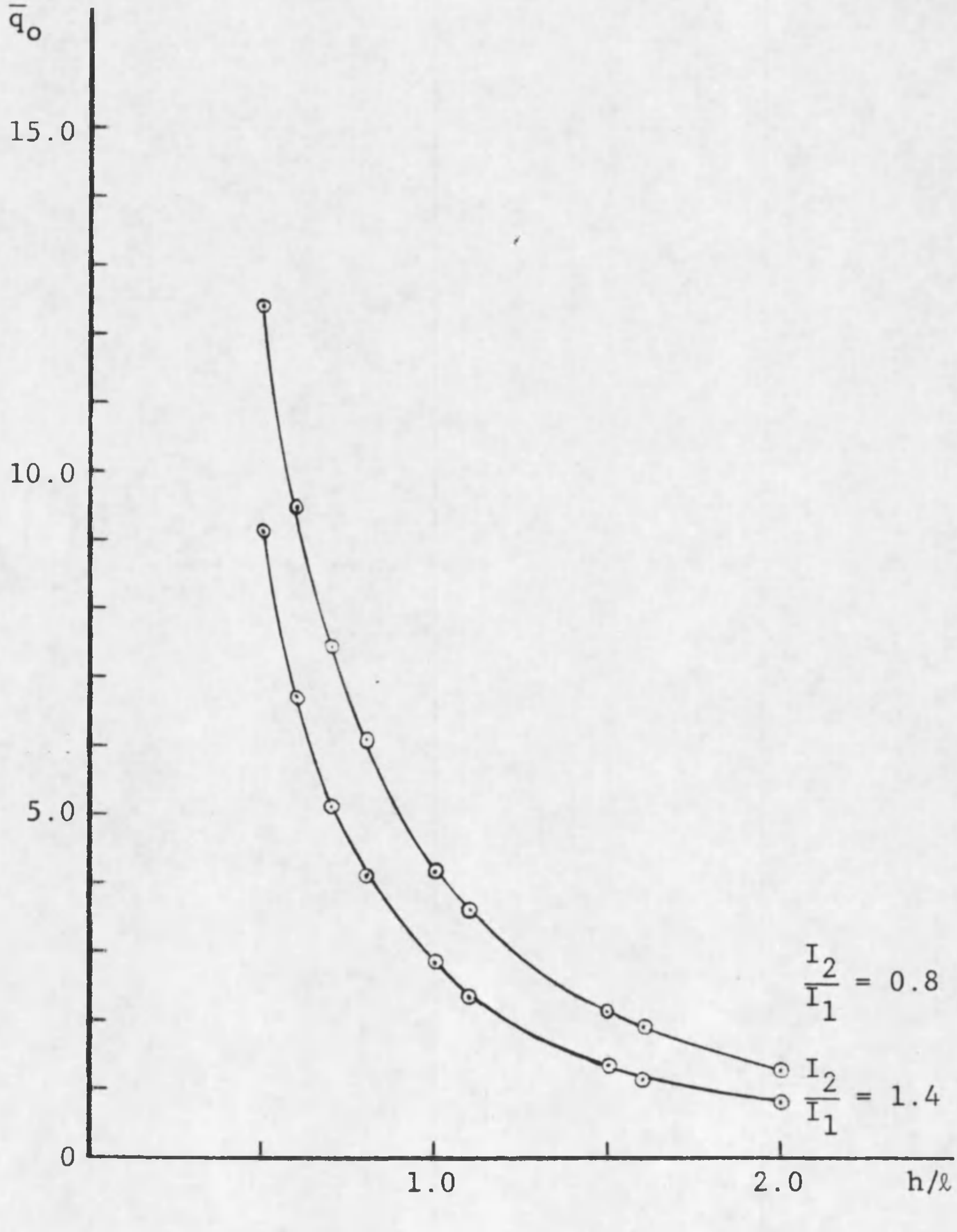

Figure 7. $\bar{q}_o$ vs. λ, for $\bar{N} = 0$.

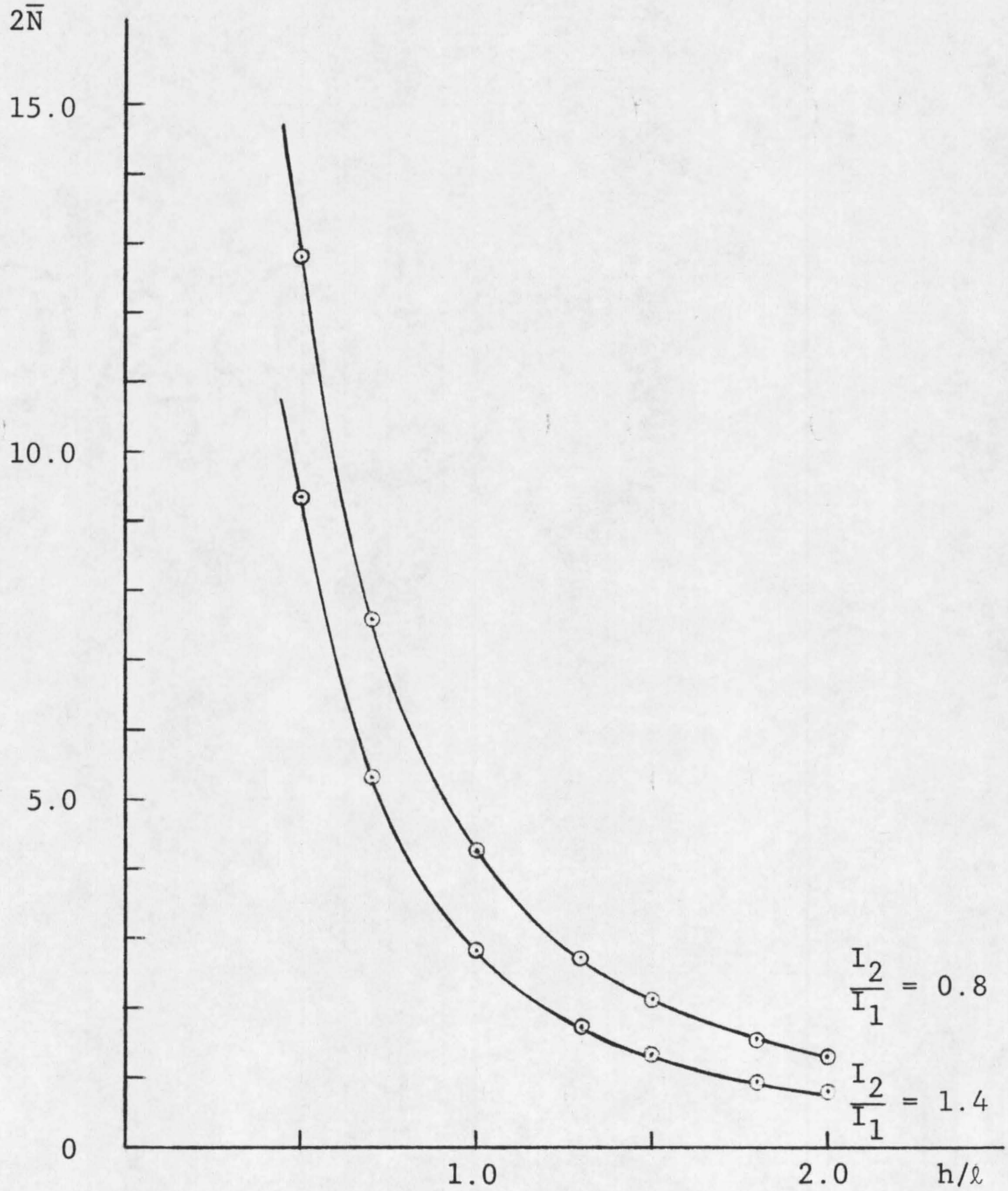

Figure 8. $2\overline{N}$ vs. λ, for $\overline{q}_o = 0$

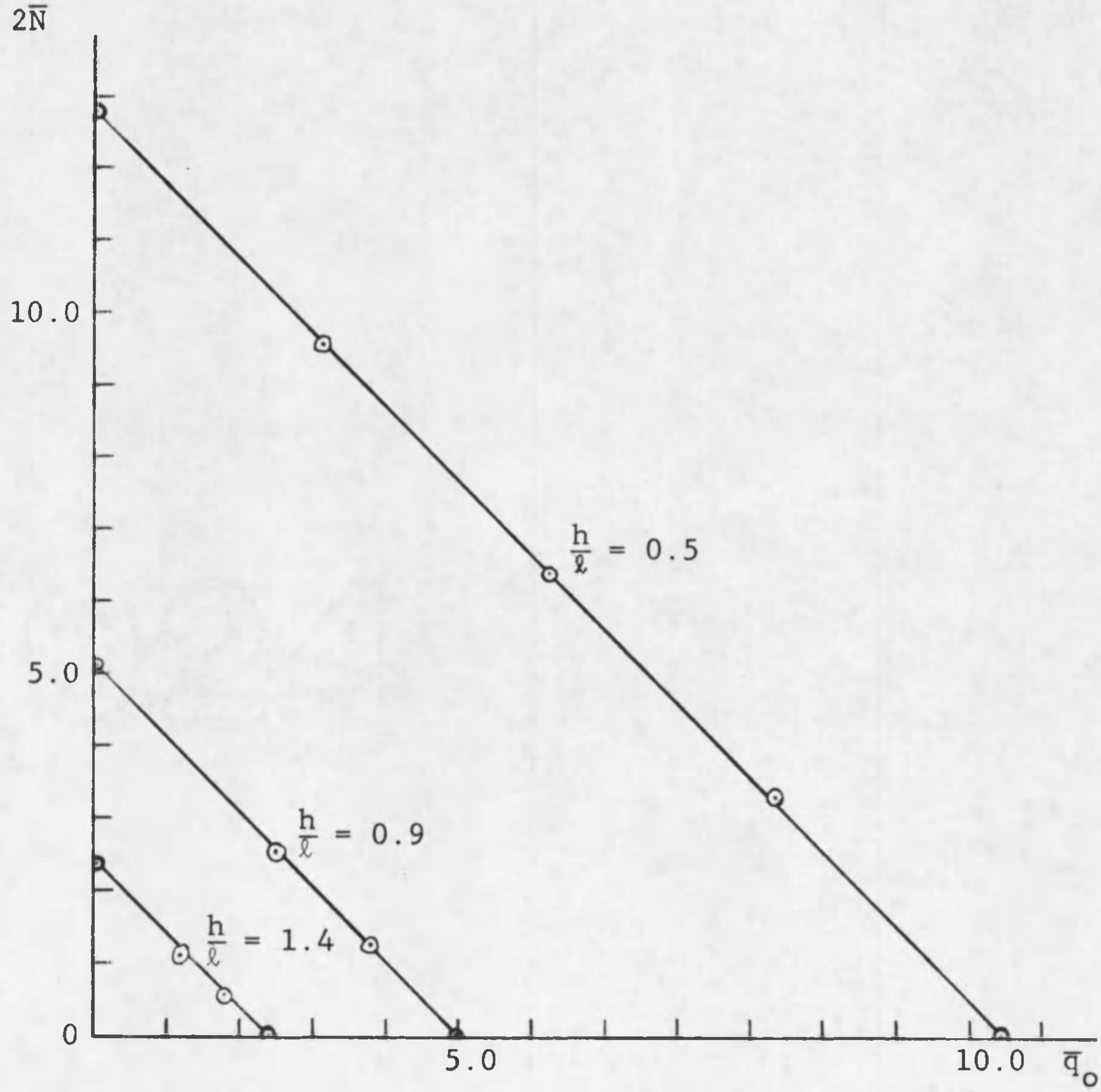

Figure 9. $2\bar{N}$ vs. $\bar{q}_o$ for $I_2/I_1 = 0.8$

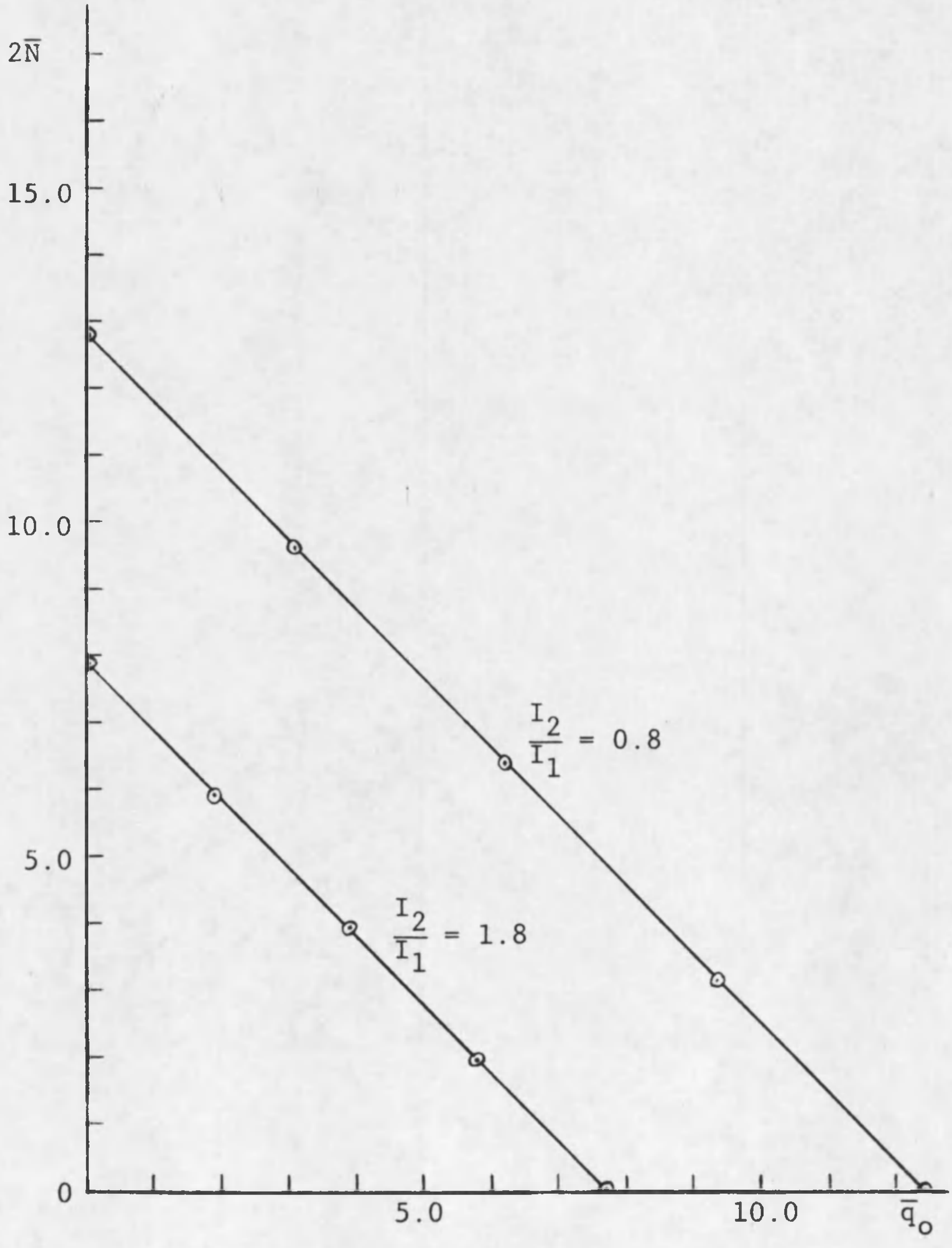

Figure 10. $2\overline{N}$ vs. $\overline{q}_o$ for $h/\ell = 0.5$

Values of $2\overline{N}_{cr}$ and $\overline{q}_{ocr}$ for many frames with different h/ℓ and I_2/I_1 ratios are provided in the Appendix B.

Figure 11 shows the relationship between the axial force, $\overline{P}_2$, and the transverse load, $\overline{q}_o$, for two different frames. The frame with ratios $h/\ell = 0.4$ and $I_2/I_1 = 0.2$, shows that the value of $\overline{P}_2$ is about 20% of $\overline{q}_o$. The other frame shows $\overline{P}_2$ to be 8% of $\overline{q}_o$ value. For some frames $\overline{P}_2$ is over 30% of $\overline{q}_o$. Which indicates the axial force, $\overline{P}_2$, could play a major role in a problem and should not be neglected. $\overline{P}_2$ values are also given in the appendix to provide information as to how important a part it will play in a given frame.

The relationship between the angle, Θ_2, and the loading, $\overline{q}_o$, is shown in Figure 12. The straight dashed line is the relationship that would have been obtained had a conventional beam theory that neglects the effects of axial forces been used. For some cases this could be acceptable, whereas in others it could lead to an underestimation of the joint rotation. Another note worthy point of this figure is that it shows a non-linear relationship between the joint rotation and the load.

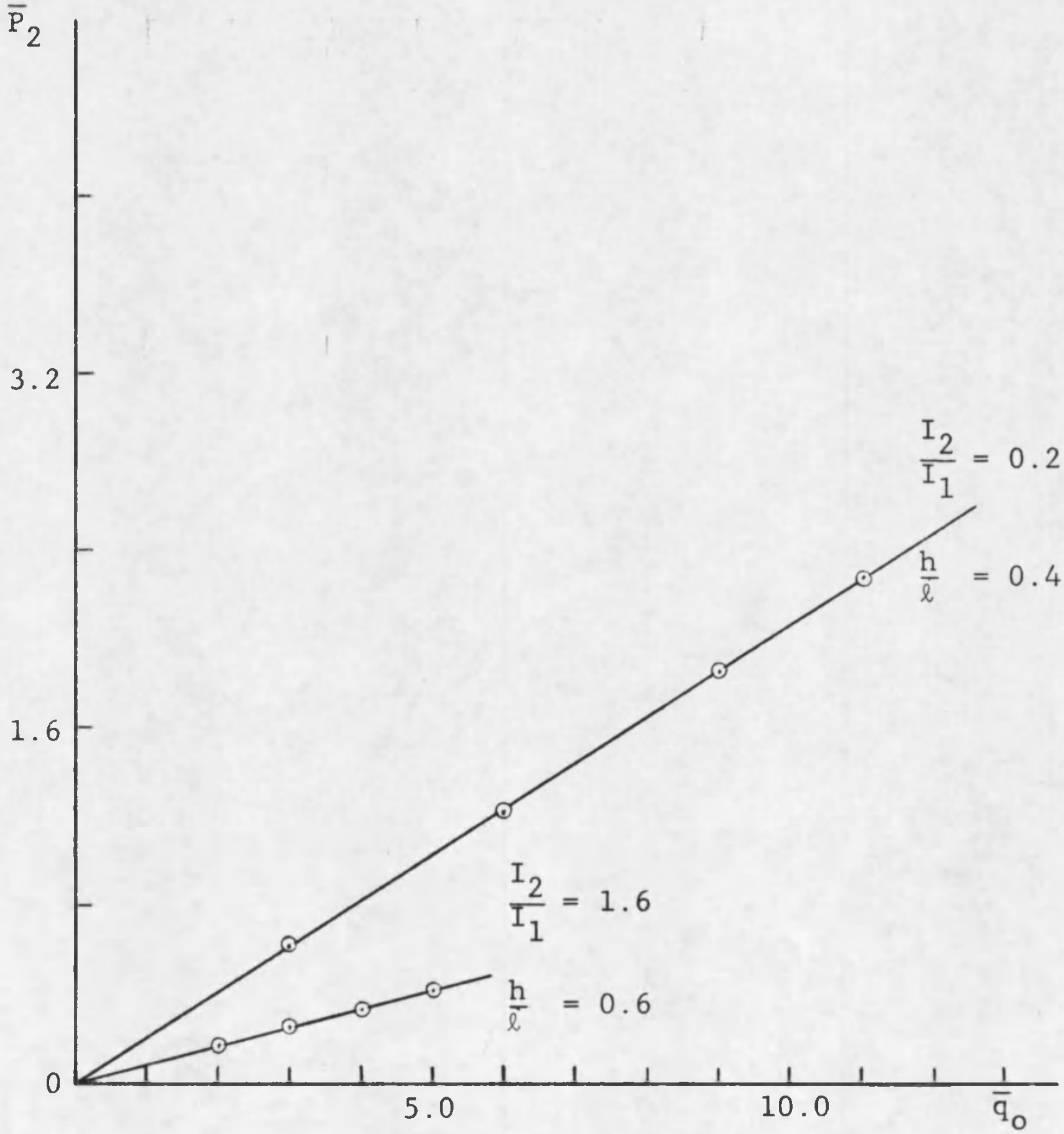

Figure 11. Axial Force, $\overline{P}_2$ vs. Transverse Load, $\overline{q}_o$ for $\overline{N} = 0$

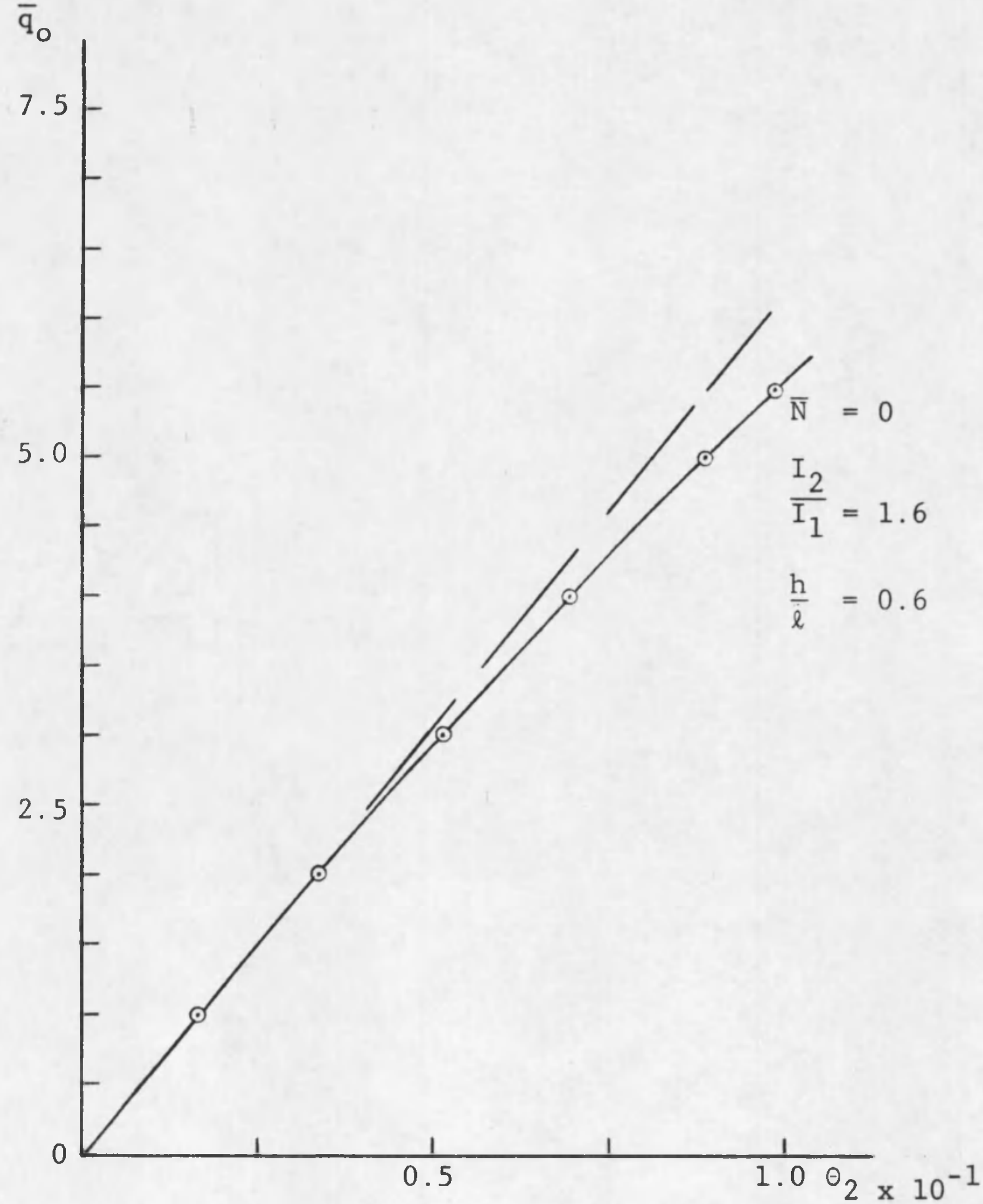

Figure 12. Transverse Load, $\overline{q}_o$ vs. Joint Rotation, Θ_2.

CHAPTER 7

CONCLUSION

From the results of this analysis, the most significant observation to be made is that all frames in the range considered fail by a side-sway type buckling. Therefore, bracing such frames would be essential to increase their load-carrying capacity, or when side-loading such as wind loads are involved. Also the lower values of h/ℓ and I_2/I_1, provide for more load-carrying capacity than the higher range.

The axial forces in the frame members do play a role in the frame analysis and they could have a profound impact upon the results when neglected.

In summation, it is indeed true that the stability of a frame is dependent upon the physical properties of that frame and it's loading. The axial forces could be a major factor involved in a problem and can not be neglected out of hand. Also the methods employing superposition could lead to erroneous results when used.

APPENDIX A

DESIGN GUIDE

For practical design purposes, the results from this analysis, which are provided in the appendix may be used as will follow. Keep in mind the following relations:

$$\bar{N} = \frac{N\ell^2}{EI_2}, \quad \bar{q}_o = \frac{q_o\ell^3}{EI_2}$$

For a given frame:

The critical values, $\bar{q}_{ocr}$ and $2\bar{N}_{cr}$, may be obtained directly or by interpolation using the tables of the Appendix B. Whenever the total critical load desired is comprised of both concentrated and distributed loads, two possible methods can be used to determine the value for each type of load.

1. By using a chart such as shown in Figure 8, one can assume a value for either $2\bar{N}$ or $\bar{q}_o$ and obtain the other. Construction of such a chart would involve drawing a straight line between points representing $\bar{q}_{ocr}$ and $2\bar{N}_{cr}$.
2. By using the expression $\dfrac{2\bar{N}}{2\bar{N}_{cr}} + \dfrac{\bar{q}_o}{\bar{q}_{ocr}} = 1$

For a given loading:

When the loading is given, there exists three possibilities

1. Ratio h/ℓ is given and I_2/I_1 is desired.
2. Ratio I_2/I_1 is known and h/ℓ is sought.
3. Both h/ℓ and I_2/I_1 ratios need to be determined.

For the first case, take the total critical load, $2\overline{N} + \overline{q}_o$, and treat it as $\overline{q}_{ocr}$. Then by using the tables in the appendix, a value for h/ℓ can be found, using interpolation when necessary. A similar procedure may be followed for the second case. As for the third possibility, using the total critical load and the tables, many frames may be chosen that can carry the load. Ultimately a decision will have to be made as to which frame would serve the total design purpose better. It is note worthy that using $2\overline{N} + \overline{q}_{ocr}$ as $\overline{q}_{ocr}$ in the tables, results in a slight under estimation of the load-carrying capacity of the frame.

APPENDIX B

TABLES OF RESULT

Table B1. Critical Loads of Frames With $I_2/I_1 = 0.1$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	30.804	10.611	37.711
0.4	25.154	5.750	27.747
0.5	20.559	3.567	21.781
0.6	17.153	2.406	17.815
0.7	14.596	1.719	14.990
0.8	12.626	1.282	12.880
0.9	11.071	0.986	11.245
1.0	9.817	0.779	9.942
1.1	8.787	0.628	8.881
1.2	7.927	0.514	8.002
1.3	7.200	0.428	7.261
1.4	6.578	0.360	6.629
1.5	6.040	0.306	6.085
1.6	5.572	0.263	5.612
1.7	5.160	0.227	5.197
1.8	4.796	0.198	4.830
1.9	4.472	0.174	4.504
2.0	4.183	0.153	4.212

Table B2. Critical Loads of Frames With $I_2/I_1 = 0.2$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	29.721	10.107	35.630
0.4	23.686	5.283	25.460
0.5	18.952	3.182	19.885
0.6	15.512	2.091	16.003
0.7	12.967	1.458	13.259
0.8	11.032	1.062	11.224
0.9	9.523	0.799	9.660
1.0	8.320	0.618	8.424
1.1	7.342	0.488	7.427
1.2	6.535	0.392	6.607
1.3	5.860	0.320	5.923
1.4	5.298	0.264	5.345
1.5	4.799	0.221	4.852
1.6	4.378	0.186	4.427
1.7	4.011	0.159	4.057
1.8	3.690	0.136	3.733
1.9	3.407	0.118	3.448
2.0	3.156	0.102	3.195

Table B3. Critical Loads of Frames With $I_2/I_1 = 0.4$.

h/ℓ	$\overline{q}_{ocr}$	$\overline{P}_2$	$2\overline{N}_{cr}$
0.3	27.658	9.106	32.006
0.4	21.066	4.456	22.447
0.5	16.252	2.551	16.849
0.6	12.894	1.604	13.214
0.7	10.488	1.075	10.691
0.8	8.708	0.755	8.854
0.9	7.352	0.550	7.467
1.0	6.296	0.412	6.391
1.1	5.455	0.316	5.537
1.2	4.774	0.247	4.847
1.3	4.215	0.197	4.280
1.4	3.750	0.159	3.809
1.5	3.359	0.130	3.413
1.6	3.026	0.107	3.076
1.7	2.741	0.089	2.787
1.8	2.495	0.075	2.537
1.9	2.281	0.064	2.320
2.0	2.093	0.055	2.130

Table B4. Critical Loads of Frames With $I_2/I_1 = 0.6$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	25.740	8.145	28.979
0.4	18.846	3.770	19.822
0.5	14.127	2.073	14.556
0.6	10.955	1.260	11.200
0.7	8.743	0.821	8.911
0.8	7.143	0.562	7.270
0.9	5.948	0.400	6.052
1.0	5.032	0.293	5.119
1.1	4.314	0.221	4.389
1.2	3.740	0.170	3.806
1.3	3.274	0.133	3.000
1.4	2.891	0.106	2.943
1.5	2.571	0.085	2.618
1.6	2.302	0.070	2.345
1.7	2.074	0.057	2.112
1.8	1.878	0.048	1.912
1.9	1.708	0.040	1.740
2.0	1.561	0.034	1.590

Table B5. Critical Loads of Frames With $I_2/I_1 = 0.8$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	23.974	7.256	26.429
0.4	16.976	3.211	17.707
0.5	12.444	1.710	12.782
0.6	9.489	1.013	9.694
0.7	7.473	0.646	7.619
0.8	6.039	0.434	6.152
0.9	4.892	0.304	5.075
1.0	4.182	0.220	4.259
1.1	3.561	0.163	3.627
1.2	3.069	0.124	3.126
1.3	2.673	0.096	2.723
1.4	2.349	0.076	2.393
1.5	2.081	0.060	2.120
1.6	1.857	0.049	1.891
1.7	1.667	0.040	1.697
1.8	1.505	0.033	1.532
1.9	1.365	0.028	1.390
2.0	1.244	0.023	1.267

Table B6. Critical Loads of Frames With $I_2/I_1 = 1.0$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	22.359	6.456	24.261
0.4	15.401	2.757	15.977
0.5	11.095	1.431	11.376
0.6	8.354	0.831	8.532
0.7	6.515	0.521	6.644
0.8	5.223	0.345	5.324
0.9	4.282	0.238	4.364
1.0	3.575	0.171	3.643
1.1	3.030	0.125	3.087
1.2	2.601	0.094	2.650
1.3	2.257	0.073	2.300
1.4	1.978	0.057	2.014
1.5	1.747	0.045	1.779
1.6	1.555	0.036	1.583
1.7	1.393	0.029	1.418
1.8	1.255	0.024	1.277
1.9	1.137	0.020	1.156
2.0	1.034	0.017	1.052

Table B7. Critical Loads of Frames With $I_2/I = 1.2$.

h/ℓ	$\overline{q}_{ocr}$	$\overline{P}_2$	$2\overline{N}_{cr}$
0.3	20.895	5.749	22.401
0.4	14.068	2.387	14.541
0.5	9.997	1.214	10.238
0.6	7.454	0.694	7.612
0.7	5.771	0.429	5.886
0.8	4.600	0.281	4.689
0.9	3.753	0.192	3.825
1.0	3.120	0.136	3.180
1.1	2.635	0.100	2.685
1.2	2.257	0.074	2.298
1.3	1.953	0.056	1.989
1.4	1.708	0.044	1.739
1.5	1.506	0.035	1.533
1.6	1.338	0.028	1.361
1.7	1.197	0.023	1.217
1.8	1.077	0.019	1.095
1.9	0.974	0.015	0.990
2.0	0.885	0.013	0.899

Table B8. Critical Loads of Frames With $I_2/I_1 = 1.4$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	19.571	5.133	20.791
0.4	12.932	2.083	13.333
0.5	9.089	1.043	9.302
0.6	6.726	0.588	6.867
0.7	5.177	0.360	5.280
0.8	4.108	0.233	4.188
0.9	3.339	0.158	3.403
1.0	2.768	0.112	2.820
1.1	2.332	0.081	2.376
1.2	1.992	0.060	2.028
1.3	1.722	0.046	1.752
1.4	1.503	0.035	1.529
1.5	1.323	0.028	1.346
1.6	1.174	0.022	1.193
1.7	1.049	0.018	1.066
1.8	0.943	0.015	0.956
1.9	0.852	0.012	0.865
2.0	0.774	0.010	0.785

Table B9. Critical Loads of Frames With $I_2/I_1 = 1.6$.

h/ℓ	$\overline{q}_{ocr}$	$\overline{P}_2$	$2\overline{N}_{cr}$
0.3	18.378	4.599	19.388
0.4	11.957	1.833	12.304
0.5	8.328	0.905	8.519
0.6	6.125	0.505	6.252
0.7	4.693	0.306	4.786
0.8	3.710	0.197	3.782
0.9	3.008	0.133	3.064
1.0	2.488	0.093	2.533
1.1	2.092	0.067	2.129
1.2	1.784	0.050	1.815
1.3	1.539	0.038	1.565
1.4	1.342	0.029	1.364
1.5	1.180	0.023	1.199
1.6	1.046	0.018	1.062
1.7	0.934	0.015	0.948
1.8	0.838	0.012	0.851
1.9	0.757	0.010	0.768
2.0	0.687	0.008	0.697

Table B10. Critical Loads of Frames With I_2/I_1 = 1.8.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	17.303	4.136	18.155
0.4	11.112	1.624	11.418
0.5	7.683	0.793	7.855
0.6	5.622	0.381	5.737
0.7	4.291	0.263	4.375
0.8	3.383	0.168	3.447
0.9	2.736	0.113	2.776
1.0	2.259	0.079	2.299
1.1	1.896	0.057	1.929
1.2	1.615	0.042	1.642
1.3	1.392	0.032	1.414
1.4	1.212	0.024	1.231
1.5	1.065	0.019	1.081
1.6	0.943	0.015	0.957
1.7	0.841	0.012	0.853
1.8	0.755	0.010	0.766
1.9	0.681	0.008	0.691
2.0	0.618	0.007	0.626

Table B11. Critical Loads of Frames With $I_2/I_1 = 2.0$.

h/ℓ	$\bar{q}_{ocr}$	$\bar{P}_2$	$2\bar{N}_{cr}$
0.3	16.333	3.736	17.064
0.4	10.375	1.449	10.649
0.5	7.129	0.700	7.285
0.6	5.194	0.384	5.300
0.7	3.953	0.229	4.029
0.8	3.109	0.146	3.167
0.9	2.509	0.097	2.554
1.0	2.068	0.068	2.104
1.1	1.734	0.048	1.763
1.2	1.475	0.036	1.499
1.3	1.270	0.027	1.290
1.4	1.105	0.021	1.122
1.5	0.970	0.016	0.984
1.6	0.859	0.013	0.871
1.7	0.766	0.010	0.776
1.8	0.687	0.008	0.696
1.9	0.619	0.007	0.627
2.0	0.562	0.006	0.569

REFERENCES

American Institute of Steel Construction, Inc. Manual of Steel Construction. Seventh Edition. New York, 1970.

Salmon, Charles G., Johnson, John E. Steel Structures Design and Behavior. Intext Educational Publishers. New York, 1971.

Tall, Lambert. Structural Steel Design. Second Edition. The Ronald Press Company. New York, 1974.

Timoshenko, Stephen P. Gere, James M. Theory of Elastic Stability. Second Edition. McGraw-Hill Book Company. New York, 1961.

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