OPTIMAL CONTROL OF A VALVE TO AVOID COLUMN SEPARATION AND MINIMIZE WATERHAMMER PRESSURES IN A PIPELINE

by

FAIQ HUSSAIN PASHA

A Thesis submitted to the Faculty of the

DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE
WITH A MAJOR IN CIVIL ENGINEERING

In the Graduate College

THE UNIVERSITY OF ARIZONA

1 9 8 9

STATEMENT BY AUTHOR

This thesis has been submitted in partial fulfillment of requirements for an advanced degree at the University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this thesis are allowable without special permission, provided that accurate acknowledgement of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate college when in his or her judgment the proposed use of the material is in the interest of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: ______________________

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

Dinshaw N. Contractor

Dr. Dinshaw N. Contractor
Professor of Civil Engineering
And
Engineering Mechanics

6/23/89.
Date

ACKNOWLEDGEMENTS

The author wishes to express his thanks to his advisor Professor Dinshaw Contractor for his patience and great technical assistance during the course of study. Special thanks to Professor M. S. Petersen for academic advice and all the help and support. Thanks to the member of committee Dr. S. Sen for his interest and cooperation during this work. Thanks to my friends Muhammad Usman Ghani and Dr. A. S. Elansary for their help and interest.

Finally thanks to all the members of my family (parents, sisters and brother, who are waiting in a small city of Pakistan) for their patience and moral support.

TABLE OF CONTENTS

Page

LIST OF ILLUSTRATIONS 6

LIST OF TABLES 9

NOMENCLATURE 10

ABSTRACT 12

CHAPTER

1. INTRODUCTION 13

2. LITERATURE REVIEW FOR LIMITING PRESSURE TRANSIENTS . . . 15

3. WATERHAMMER ANALYSIS 17

3 - 1 Partial Differential Equations for Waterhammer 17

3-1-1 Equation of Motion 18

3-1-2 Equation of Continuity 20

3 - 2 Solution of Partial Differential Equations 20

3-2-1 The Method of Characteristics 22

3 - 3 The Finite Difference Equations 23

3 - 4 Boundary Conditions 26

4. OPTIMIZATION 28

4 - 1 The Simplex Method 29

4 - 2 Methodology 31

4 - 3 Model Description 35

4 - 4 Spline Interpolation 36

5. MODEL VERIFICATION 40

TABLE OF CONTENTS - Continued

Chapter		Page
6.	COMPUTER IMPLEMENTATION AND RESULTS	50
	6 - 1 Case I	51
	6 - 2 Case II	59
	6 - 3 Case III	68
	6 - 4 Case IV	73
7.	DISCUSSION AND CONCLUSION	90
APPENDIX A	LISTING OF THE COMPUTER PROGRAM	93
APPENDIX B	COMPUTER INPUT AND OUTPUT FOR CASE II, tc =4.4 L/a	115
APPENDIX C	COMPUTER INPUT AND OUTPUT FOR CASE IV, tc =4.4 L/a	119
REFERENCES		123

LIST OF ILLUSTRATIONS

Figure		Page
3.1	Freebody diagram for application of equation of motion.	19
3.2	Control volume for continuity equation.	21
3.3	Characteristic lines in the x-t plane.	24
3.4	Characteristic grid.	24
4.1	Typical simplex search in two-dimensions.	30
4.2	Three dimensional simplex (tetrahedron) and possible outcome, 1) reflection, 2) reflection and expansion, 3) contraction along one dimension, 4) contraction along all dimensions.	33
4.3	The flow chart of the Nelder and Mead simplex.	37
4.4	The flow chart of the Computer Model.	38
5.1	Optimal valve closure policies for step-closure and minimizing the maximum pressure, tc =3.319 sec (2.1L/a), model verification.	42
5.2	Pressure variation at the valve for optimal and linear valve closures, tc =3.319 sec (2.1L/a), model verification.	43
5.3	Discharge variation at the valve for optimal valve closure, tc =3.319sec (2.1L/a), model verification.	44
5.4	Optimal valve closure policies for step-closure and minimizing the maximum pressure, tc =6.480 sec (4.1L/a), model verification.	45
5.5	Pressure variation at the valve for optimal and linear valve closures, tc =6.480 sec (4.1L/a), model verification.	46
5.6	Discharge variation at the valve for optimal valve closure, tc =6.480 sec (4.1L/a), model verification.	48
5.7	Percentage reduction in dynamic pressure head as a function of valve closure time, model verification.	49

LIST OF ILLUSTRATIONS - Continued

Figure Page

6.1 Percentage reduction in dynamic pressure head as a function of valve closure time, case I. 52

6.2 Optimal valve closure policy, tc =.984 sec (2.2L/a), case I. 53

6.3 Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec (2.2L/a), case I. 54

6.4 Discharge variation at the valve for optimal valve closure, tc =.984 sec (2.2L/a), case I. 56

6.5 Optimal valve closure policy, tc =1.968 sec (4.4L/a), case I. 57

6.6 Pressure variation at the valve for optimal and linear valve closures tc =1.968 sec (4.4L/a), case I. 58

6.7 Discharge variation at the valve for optimal valve closure, tc =1.968 sec (4.4L/a), case I. 60

6.8 Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case II. 61

6.9 Percentage reduction in the dynamic pressure head as a function of valve closure time, case II. 62

6.10 Optimal valve closure policy, tc =.984 sec (2.2L/a), case II. 63

6.11 Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec (2.2L/a), case II. 65

6.12 Discharge variation at the valve for optimal valve closure, tc =.984 sec (2.2L/a), case II. 66

6.13 Optimal valve closure policy, tc =1.968 sec (4.4L/a), case II. 67

6.14 Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec (4.4L/a), case II. 69

6.15 Discharge variation at the valve for optimal valve closure, tc =1.968 sec (4.4L/a), case II. 70

6.16 Optimal valve closure policy, tc =.984 sec (2.2L/a), case III. 71

LIST OF ILLUSTRATIONS - Continued

Figure Page

6.17 Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec (2.2L/a), case III. 72

6.18 Discharge variation at the valve for optimal valve closure, tc =.984 sec (2.2L/a), case III. 74

6.19 Optimal valve closure policy, tc =1.968 sec (4.4L/a), case III. 75

6.20 Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec (4.4L/a), case III. 76

6.21 Discharge variation at the valve for optimal valve closure, tc =1.968 sec (4.4L/a), case III. 77

6.22 Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case III. 78

6.23 Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case IV. 80

6.24 Percentage reduction in the dynamic pressure head as a function of valve closure time, case IV. 81

6.25 Optimal valve closure policy, tc =.984 sec (2.2L/a), case IV. 82

6.26 Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec (2.2L/a), case IV. 83

6.27 Discharge variation at the valve for optimal valve closure, tc =.984 sec (2.2L/a), case IV.. 85

6.28 Optimal valve closure policy, tc =1.968 sec (4.4L/a), case IV. 86

6.29 Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec (4.4L/a), case IV. 87

6.30 Discharge variation at the valve for optimal valve closure, tc =1.968 sec (4.4L/a), case IV. 89

LIST OF TABLES

Table Page

5.1 Comparison of Maximum Pressures Obtained by Step Valve Closure and Optimal Valve Closure. 47

NOMENCLATURE

A	Cross-sectional area
A_g	Opening area of valve
a	Speed of wave in the fluid
b	side length of simplex
C_d	Orifice discharge coefficient
D	Diameter of pipe
F	Objective function value
Fh	The highest value of objective function
Fl	The lowest value of objective function
Fm	The second highest value of objective function
f	Darcy Weisbach friction factor
g	Gravitational acceleration
H	Piezometric height
$Hmax$	Maximum pressure head
$Hmin$	Minimum pressure head
$Hres$	Reservoir head
H_t	$\partial H/\partial t$
H_x	$\partial H/\partial x$
Hv	Vapor pressure
L	Length of pipe
n	number of variables used in simplex

n1	number of reaches in pipe
Q	discharge
t	time
tc	Time of closure of valve
V	Velocity
V_x	$\partial V/\partial x$
V_t	$\partial V/\partial t$
x	distance
Δx	a small segment of distance
∂x	thickness of fluid element
α	angle of inclination of pipe with x-axis
$\alpha 1$	reflection coefficient
$\beta 1$	contraction coefficient
$\gamma 1$	expansion coefficient
ξ	parameter used in establishing initial τ-time curves
γ	specific weight of fluid
λ	multiplier parameter for partial differential equations
τ	dimensionless valve opening
τh	τ-time curve which gives the highest value of objective function
τm	τ-time curve which gives the second highest value of objective function
τl	τ-time curve which gives the lowest value of objective function
τo	shear stress

ABSTRACT

Fluid transients (maximum pressure) in a pipeline can be reduced significantly by using an optimal valve closure. In some situations, the optimal valve closure policy may result in minimum pressures below the vapor pressure of the fluid. Consequently, column separation will occur with high pressures in the pipeline that were not taken into account in analysis of the optimal valve closure policy. Thus it is desirable to minimize the maximum pressure (Hmax) in a pipeline, subject to the constraint that the minimum pressure (Hmin) be greater than or equal to a pre-selected pressure, e.g. the vapor pressure of the fluid (Hv). This is accomplished by changing the objective function to a minimization of (Hmax-Hmin), with the stipulation that when Hmin < Hv, the objective function becomes (Hmax-Hv).

The method of characteristics is used to simulate fluid transients in a pipe, and the simplex method is used to optimize the objective function. A numerical example is provided to illustrate the valve-closure policies obtained when different objective functions are used, e.g. minimizing Hmax, maximizing Hmin, and minimizing (Hmax-Hmin). It is shown that a valve-closure policy can be found that minimizes Hmax while avoiding water column separation.

CHAPTER 1

INTRODUCTION

In steady flow, the velocity at a point in a fluid system does not change with time whereas in unsteady flow, the velocity will change. Transition flow is the state when flow conditions are changing from one steady state to another steady state. In pipeline flow, transient flow and waterhammer are used to describe the unsteady flow.

Sudden disturbances in the flow system, such as changes in valve settings, pump operation, and changes in reservoir elevation, create changes in pressure head and velocity. The changes can be positive or negative depending upon the operation of valve or pump and parameters of pipeline. If the pressure in a closed conduit drops below vapor pressure of the fluid, cavities referred to as column separation are formed in the water column.

Severe waterhammer and column separation in a flow system are undesirable as these can cause damage to the operating system or pipes. Negative pressures should be increased to slightly higher than the vapor pressure of the fluid to avoid column separation. It is impossible to completely avoid waterhammer, but it is desirable to decrease waterhammer effects so that they will not result in damage to the system.

A piping system can be designed to withstand any maximum and minimum pressures expected to occur during the life of the system but the cost may be uneconomical. Therefore, various devices are used to avoid undesirable pressure rise and drop or column separation.

The devices commonly used to reduce excessive pressures and avoid column separation are:

1- Surge tanks

2- Air chambers

3- Valve operating criteria

Transient flow can be controlled by the following operations of a valve.

1- The valve closes or opens in a certain manner to reduce positive and increase negative pressures.

2- When pressure exceeds a certain level the valve opens to allow rapid outflow, e.g. a pressure-relief valve.

3- To prevent the pressure from dropping to vapor pressure, the valve opens to admit air, e.g. air-inlet valves.

Valves that can be operated easily (according to specific instructions) are worthwhile, considering the savings that would result from improved pipeline design or control system operation. No extra equipment is required, when a valve operating policy is adopted to limit the fluid transients. The simplex method is used to optimize the valve closure for a simple flow system including a reservoir, a pipeline, and a valve. A computer model is developed which gives the optimal valve closure policy (for a certain time of closure) to avoid column separation and minimize waterhammer pressure.

CHAPTER 2

LITERATURE REVIEW FOR LIMITING PRESSURE TRANSIENTS

In the past several decades, investigators have developed methods to compute numerically the magnitude of waterhammer transients in pipelines. One of their aims was to reduce the waterhammer pressure by any appropriate means. In the beginning, system designers explored how waterhammer pressures changed when arbitrarily changing the valve-operating policy. Of the policies investigated, they could choose the policy which gave the least waterhammer pressure.

Streeter (1963) proposed a technique called "valve stroking" to control waterhammer pressures, i.e. a valve operating policy which limits the pressure rise to a pre-set maximum without flow reversal and without separation of the fluid column. He developed equations for closing a valve at the downstream end of a pipeline leading from a reservoir so that the minimum pressure remains above the steady state values at all times.

Driels (1975) studied minimizing waterhammer pressure by closing the valve in two linear stages, the mode of closure being characterized by the point at which the valve actuator speed changes. This point was determined by using a simplex search algorithm, so that for given initial flow conditions and valve closure time, the waterhammer pressure can be minimized.

Contractor (1983) suggested that waterhammer pressures can be minimized by operating the valve in an optimally prescribed manner, in a given time of closure. He used dynamic programming to determine the valve closure policy so that the pressure rise at the valve can be minimized. He also concluded that the benefits of optimal gate closure are greatest when the time of closure is small ($2L/a < t_c < 3L/a$).

Contractor (1985) studied optimal valve closure in two or three stages to minimize waterhammer pressure rises. The Box-complex method was used to optimize a valve closure policy for a given time. He also introduced the step valve closure policy. In this policy, the valve is closed instantaneously to a certain opening and is kept at that opening for $2L/a$ seconds, then it is closed to another opening and so on until the valve is completely closed. The intermediate openings are determined by using the Box-complex method.

Sen and Contractor (1986) applied minimax optimization procedure to minimize the maximum pressure anywhere along the pipeline. Conway (1986) applied the dynamic programming technique to a series pipeline to determine the optimal valve-closure policy and concluded that the dynamic programming scheme is sensitive to pipe configuration, junction location, friction factor, and closure period. Azoury et al. (1986) studied, for a simple pipeline discharging into free air, the effect of valve closure policy on waterhammer pressure under friction conditions and presented a chart that can be used to determine valve schedule for minimum waterhammer pressure.

Goldberg (1987) proposed a time-optimal valve closure procedure called Quick Stroking. El-Ansary and Contractor (1988) proposed an optimal valve closure to minimize waterhammer pressures and stresses in a pipeline. They used the simplex method for optimizing the valve closure policy.

CHAPTER 3

WATERHAMMER ANALYSIS

The distribution of pressure and velocity in a closed conduit, with flowing water, depends upon the conditions under which the flow occurs. If the water is considered incompressible and the flow is steady, the velocity of water remains constant and Bernoulli's energy equation can be applied at any two sections of the pipe. However, when the flow is unsteady, i.e. when velocity at each section varies with time, abrupt pressure changes occur inside the pipe, and Bernoulli's equation is no longer applicable. These pressure fluctuations are referred to as waterhammer because of the hammering sound which often accompanies the phenomenon. The flow is called transient flow and is analysed as follows:

3-1 Partial Differential Equations for Waterhammer

Two basic differential equations are applied to describe the unsteady flow through a closed conduit: the equation of motion and the continuity equation. The following assumptions are made in the derivation of the equations (Chaudhry, 1979).

1. Flow is one-dimensional and the velocity distribution is uniform over the cross-section of the conduit.
2. Formulae for computing the steady state friction losses in the conduit are valid during the transient state.

3-1-1 Equation of Motion

The derivation applies Newton's second law of motion to a short segment of fluid flowing in a pipe. The following notations are used; distance x, time t, discharge Q, flow velocity V, and piezometric head H at the centerline of the conduit above the specified datum. H and Q are dependent variables, and x and t are independent variables.

In Figure 3.1 a fluid element of thickness δx has cross-sectional area A. The area is a function of distance x. The tube consisting of liquid is inclined to the horizontal at an angle of α such that elevation increases with increasing +x. The forces on the free body in the positive x direction are, surface contact pressure component on the transverse face and the pressure component on the periphery. The forces acting in the negative x direction are shear force, weight component in the x direction, and the surface contact normal pressure on the transverse face. Streeter and Wyile (1978) derived the equation of motion as

$$gH_x + VV_x + V_t + \frac{f\,V\,|V|}{2D} = 0 \qquad (3.1)$$

where

g - gravitational acceleration

H_x- partial derivative of piezometric head with respect to x

V - velocity

V_t - partial derivative of velocity with respect to time

V_x- partial derivative of velocity with respect to distance

f - Darcy Weisbach friction factor

D - diameter of pipe

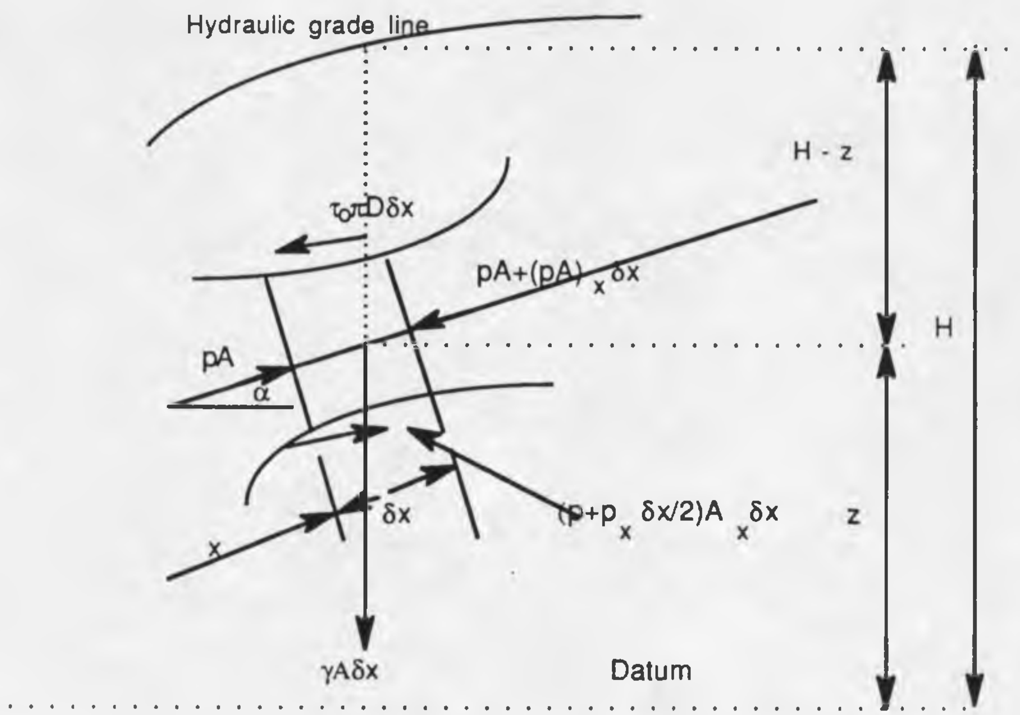

Figure 3.1 Freebody diagram for application of equation of motion.

3-1-2 Equation of Continuity

The control volume of length δx, in Figure 3.2, is fixed relative to the pipe. At time t, the net mass inflow into the control volume will be equal to the rate of change of mass within the control volume. This is required by the law of conservation of mass. Wylie and Streeter (1978) give the continuity equation in which H and V are dependent variables and x and t are independent variables as

$$VH_x + H_t - V \sin \alpha + \frac{a^2}{g} V_x = 0 \qquad (3.2)$$

where

V - velocity

H_x - partial derivative of piezometric head with respect to distance x

H_t - partial derivative of piezometric head with respect to time t

V_x - partial derivative of velocity with respect to distance x

a - wave speed in the pipe

3-2 Solution of Partial Differential Equations

The dynamic and continuity equations need to be solved simultaneously for waterhammer or transient flow in a pipeline. They are quasi-linear, hyperbolic, and partial differential equations. While closed form solution of these equations is impossible (Chaudhry , 1979), there are numerical techniques to solve these equations by using computer analysis, such as the characteristics method. In this method the partial differential equations are transformed into total differential equations and these latter equations are then solved by a finite difference technique.

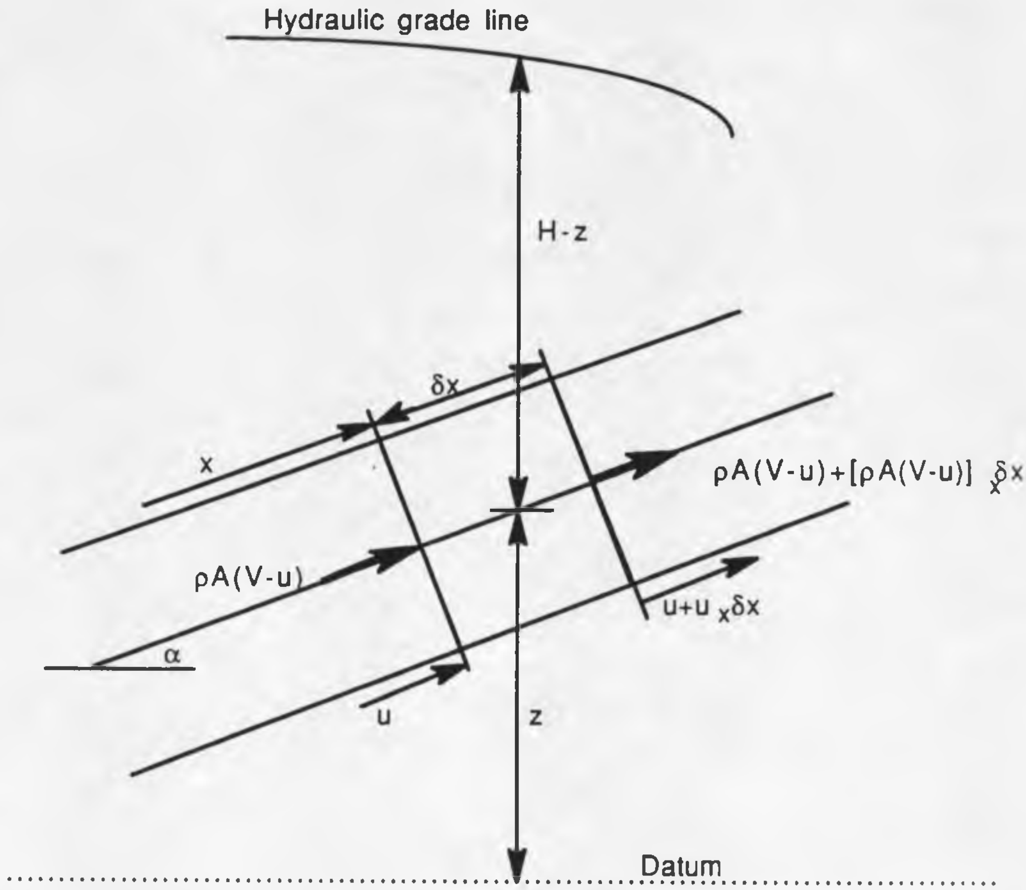

Figure 3.2 Control volume for continuity equation.

3-2-1 The Method of Characteristics

The two quasi-linear partial differential equations are transformed into four ordinary differential equations by the method of characteristics. In the derivation, terms of lower magnitude are dropped. The simplified dynamic equation and equation of continuity are identified below (Streeter and Wylie , 1979).

$$L_1 = gH_x + V_t + f\frac{V|V|}{2D} = 0 \tag{3.3}$$

$$L_2 = H_t + a^2 \frac{V_x}{g} = 0 \tag{3.4}$$

The pair of Equations (3.3) and (3.4) is replaced by a linear combination of equations. Using λ as a linear factor, the equations can be combined as

$$L_1 + \lambda L_2 = 0 \tag{3.5}$$

$$gH_x + V_t + f\frac{V|V|}{2D} + \lambda\left[H_t + a^2 V_x\right] = 0 \tag{3.6}$$

Any two real and distinct values of λ give two equations in terms of two dependent variables H and V. If V and H are dependent on x and t,and independent variable x is permitted to be a function of t, then the derivative of V and H can be written as

$$\frac{dH}{dt} = H_x\frac{dx}{dt} + H_t \tag{3.7}$$

$$\frac{dV}{dt} = V_x\frac{dx}{dt} + V_t \tag{3.8}$$

Equation (3.6) can be replaced by the ordinary differential equation

$$\lambda\frac{dH}{dt} + \frac{dV}{dt} + f\frac{V|V|}{2D} = 0 \tag{3.9}$$

if

$$\frac{dx}{dt} = \frac{g}{\lambda} = \frac{\lambda a^2}{g} \tag{3.10}$$

The solution of Equation (3.10) gives

$$\lambda = \pm g/a \tag{3.11}$$

and

$$\frac{dx}{dt} = \pm a \tag{3.12}$$

Substitution of these values of λ in Equation (3.9) gives two pairs of equations which are identified as C^+ and C^- equations.

$$\frac{g}{a}\frac{dH}{dt} + \frac{dV}{dt} + f\frac{V|V|}{2D} = 0 \tag{3.13}$$

$$\frac{dx}{dt} = + a \tag{3.14}$$

$$-\frac{g}{a}\frac{dH}{dt} + \frac{dV}{dt} + f\frac{V|V|}{2D} = 0 \tag{3.15}$$

$$\frac{dx}{dt} = - a \tag{3.16}$$

In the x-t plane, these Equations (3.14) and (3.16) are represented by straight lines. C^+ and C^- are called characteristic lines along which Equations (3.13) and (3.15) are valid, as shown in Figure 3.3. This pair of equations now can be written in finite difference form and solved conveniently with the digital computer.

3-3 The Finite Difference Equations

A pipeline is divided into n1 equal reaches of length Δx. The time step Δt is the time required by the wave to travel Δx distance and is computed by

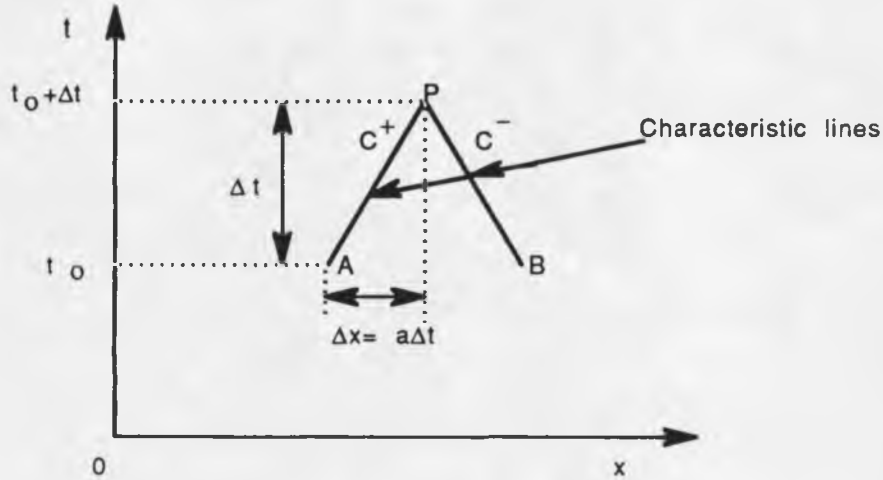

Figure 3.3 Characteristic lines in the x-t plane.

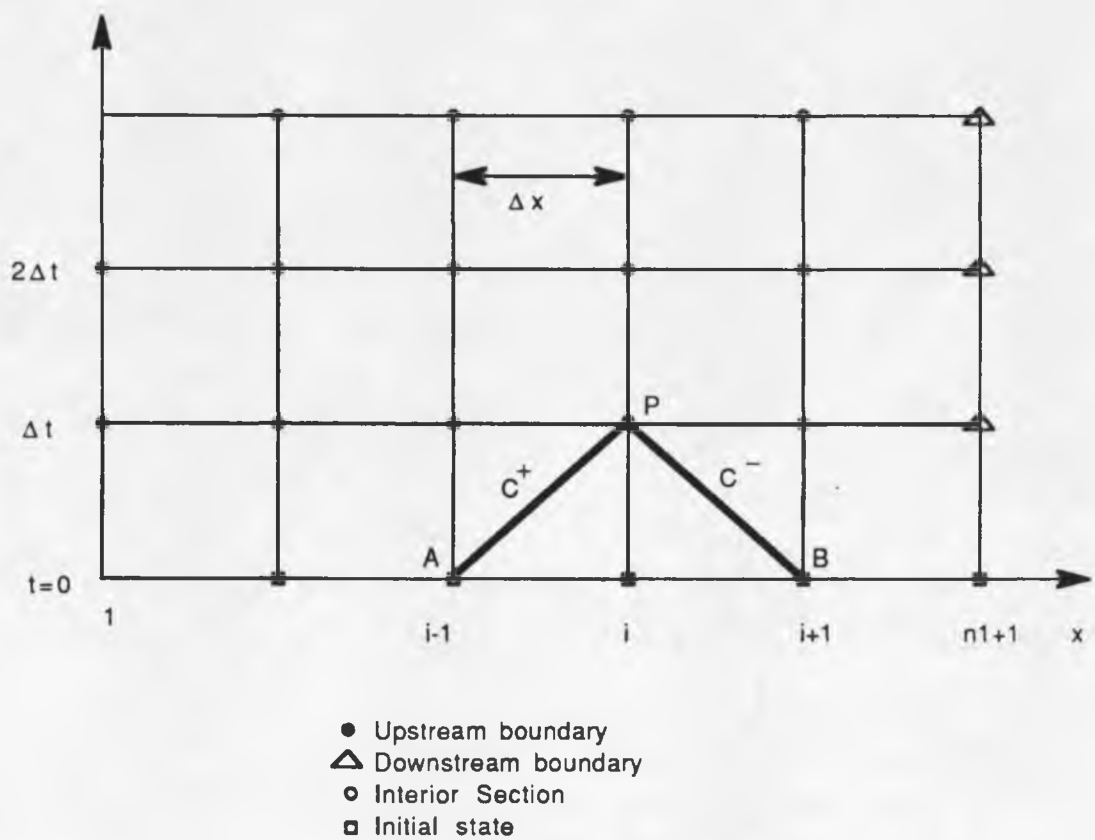

Figure 3.4 Characteristic Grid.

$$\Delta t = \frac{\Delta x}{a} \tag{3.17}$$

At any point on x-t plane in Figure 3.4, say point P, the values of H and V are unique i.e. the H and V are independent of which characteristic they were approached from (Watters, 1984). If we construct C^+ and C^- characteristics through the point, we have two ordinary differential equations which apply along their respective characteristics. The differential equations can now be expressed in finite difference form. Equations (3.13) and (3.15) become (Steeter and Wylie ,1979)

$$H_P - H_A + \frac{a}{gA}(Q_P - Q_A) + \frac{f\,\Delta x\,Q_A|Q_A|}{2g\,DA^2} = 0 \tag{3.18}$$

$$H_P - H_B - \frac{a}{gA}(Q_P - Q_B) - \frac{f\,\Delta x\,Q_B|Q_B|}{2g\,DA^2} = 0 \tag{3.19}$$

solving equations for Hp

$$C^+: \quad H_P = H_A - B(Q_P - Q_A) - R(Q_A|Q_A|) \tag{3.20}$$

$$C^-: \quad H_P = H_B + B(Q_P - Q_B) + R(Q_B|Q_B|) \tag{3.21}$$

where

$$B = \frac{a}{gA} \tag{3.22}$$

$$R = \frac{f\,\Delta x}{2g\,DA^2} \tag{3.23}$$

Initially at time t=0, the conditions are steady and the values of H and Q are known. The problem occurs from t= Δt onward, H and Q are computed for each grid point along t= Δt, and then proceeding to t=2 Δt etc., until a desired maximum time has been reached. At any interior grid point i, the two equations are solved simultaneously for the known Hpi and Qpi. Equations (3.20) and (3.21) can be written in terms of grid points as

$$C^+: \quad H_{Pi} = C_P - BQ_{Pi} \tag{3.24}$$

$$C^-: \quad H_{Pi} = C_M + BQ_{Pi} \tag{3.25}$$

where Cp and Cm are known constants

$$C_P = H_{i-1} + BQ_{i-1} - RQ_{i-1}\,|Q_{i-1}| \tag{3.26}$$

$$C_M = H_{i+1} - BQ_{i+1} + RQ_{i+1}\,|Q_{i+1}| \tag{3.27}$$

Figure 3.4 shows that the end points affect the interior points after the first time step. So some boundary conditions need to be introduced.

3-4 Boundary Conditions

At the ends of the pipeline either the C^+ or the C^- characteristic provides one equation for two unknowns, as illustrated in Figure 3.4.

The boundary condition at the upstream end of the pipeline is assumed to be a constant head reservoir, Hres, Hp=Hres and the C^- equation are two equations for solving two unknowns Hp and Qp.

At the downstream end of the pipeline, the valve opening area provides the necessary boundary condition. The orifice equation for steady state through the valve can be written as

$$Q_0 = (C_d\,A_g)_0\,\sqrt{(2gH_0)} \tag{3.28}$$

in which

Qo - steady state flow

Ho - steady state head at the valve

(CdAg)o - the area of the valve opening times the discharge coefficient at steady state

An equation similar to Equation (3.28) may be written for the transient state as

$$Q_P = (C_d A_g) \sqrt{(2g \Delta H)} \tag{3.29}$$

in which ΔH is the instantaneous drop in hydraulic grade line across the valve. A dimensionless parameter τ can be defined as

$$\tau = \frac{(Cd\,Ag)}{(Cd\,Ag)o}$$

Thus, we get

$$Q_P = \frac{Q_0\,\tau}{\sqrt{(H_0)}} \sqrt{(\Delta H)} \tag{3.30}$$

τ is 1 for steady state flow and 0 for a fully closed valve. By solving Equations (3.24) and (3.30) for Qp

$$Q_P = -BC_V + \sqrt{\{(BC_V)^2 + 2C_V C_P\}} \tag{3.31}$$

where

$$C_V = \frac{(Q_0\,\tau)^2}{2H_0} \tag{3.32}$$

Hp can be found from Equation (3.24) or (3.30).

CHAPTER 4

OPTIMIZATION

Optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function. There is no single method available for solving all optimization problems efficiently; a number of optimization techniques are available for solving different types of problems.

Optimization techniques, also known as mathematical programming techniques, are useful in determining the maximum or minimum of a given function of several variables for the given set of constraints.

All optimization methods are classified into two general categories as,

1- derivative-free methods

2- gradient methods

The gradient or descent methods require both function and derivative evaluations while derivative-free, or direct search, methods require function evaluations only. In general, gradient methods seem to be more effective, due to the added information provided. If analytical derivatives are available, there is no doubt that a gradient technique should be used. However, if numerical derivative approximations are utilized, the efficiency of gradient methods would be approximately the same as that of the derivative-free methods. The simplex method is one of several widely-used techniques of derivative-free methods.

4-1 The simplex method

The geometric figure formed by a set of n+1 points in n-dimensional space is called the simplex. When the points are at unequal distances the simplex is called irregular. In two dimensional space, the number of points are three, and the simplex is a triangle; in three dimensional space, it is a tetrahedron.

The currently accepted simplex technique is due to Nelder and Mead (1965). The procedure is an extension of the simplex method by Spendly, et.al., (1962). This technique accelerates the simplex method and makes it more general. This simplex method adapts itself to the local landscape, using reflected, expanded, and contracted points to locate the minimum. Unimodality is assumed, and thus several sets of starting points should be considered. Derivatives are not required.

This method is clearly applicable to the problem of minimizing a mathematical function of several variables, having constraints. In the method the simplex adapts itself to the local landscape, elongating down along inclined planes, changing direction on encountering a valley at an angle, and contracting in the neighborhood of a minimum, as shown in Figure 4.1. The criterion for stopping the process depends upon the accuracy of results needed.

The method can be applied to several kinds of problems unless the function is discontinuous. It does not require unidirectional searches or any line search techniques. The method can be applied to non-differentiable functions and when the first partial derivatives are discontinous. This method contains very simple mathematical operations and each step is computed using the previous step so that a lot of computer memory is not required. As the number of the variables increases, the mean number of evaluations for convergence increases rapidly (Nelder and Mead 1965). Rapid and safe convergence can

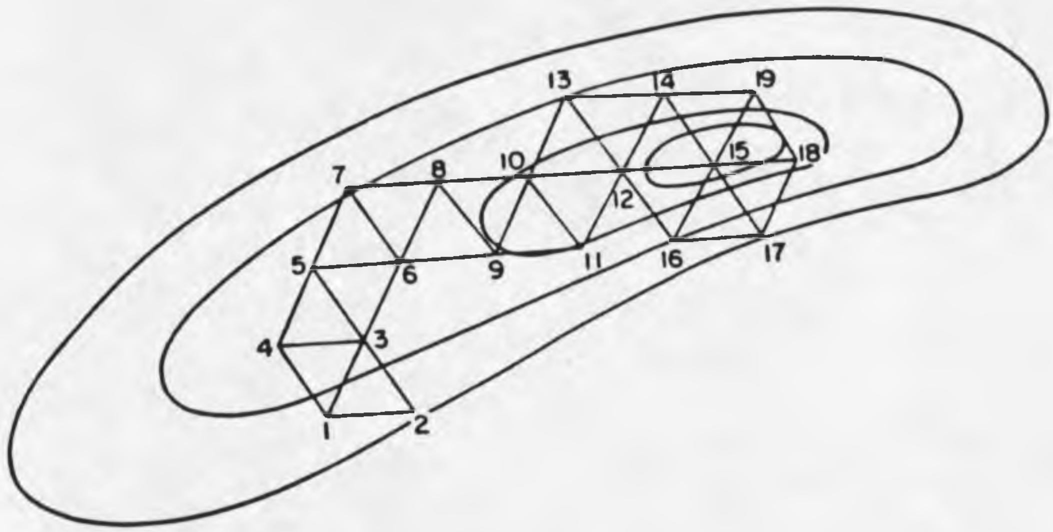

Figure 4.1 -Typical simplex search in two-dimensions.

be achieved by using a maximum number of variables of ten, resulting in efficient use of computer time.

4-2 Methodology

In n-dimensional space, n+1 vertices make a simplex. These vertices are denoted by τ_i (i=1,2,.....,n+1). An initial point τ_1 is selected, with coordinates in n-dimensional space denoted as $(\tau_{1,1},\tau_{1,2},.....,\tau_{1,n})$, and the point must satisfy all the constraints.

An initial simplex is constructed consisting of the starting point and the following additional points (Kuester and Mize, 1973).

$$\tau_j = \tau_1 + \zeta_j, \qquad j = 2,3,.......,n+1$$

where ζ_j is determined from the following table

j	$\zeta_{1,j}$	$\zeta_{2,j}$	.	.	$\zeta_{n-1,j}$	$\zeta_{n,j}$
2	p	q	.	.	q	q
3	q	p	.	.	q	q
.	.	.	.	.	.	.
.	.	.	.	.	.	.
n	q	q	.	.	p	q
n+1	q	q	.	.	q	p

where

$$p = \frac{b}{n\sqrt{2}} \left[\sqrt{(n+1)} + n - 1\right] \qquad (4.2)$$

$$q = \frac{b}{n\sqrt{2}} \left[\sqrt{(n+1)} - 1\right] \qquad (4.3)$$

where

n - total number of variables

b - side length of simplex, it determines the area of search; as b decreases, the area of search decreases

After establishing the initial simplex, the function is evaluated at each vertex of the simplex. Let τ and F denote the point and the objective function value respectively. τh, τm and τl are the points of the simplex which give the highest, the second highest and the lowest values of the objective function. Fh, Fm and Fl are the objective function values respectively. $\bar{\tau}$ is the centroid of these points excluding the worst point τh and is found using the following relation:

$$\bar{\tau} = 1/n \left\{ \sum_{j=1}^{n+1} (\tau_j) - \tau_h \right\} \tag{4.4}$$

The point, having the highest value of the objective function, is called the worst point, and is replaced by a new point. The new point is found by the following four operations. These operations are shown graphically in Figure 4.2.

I - Reflection

The reflected point of τh is located as follows:

$$\tau^* = (1 + \alpha_1)\bar{\tau} - \alpha_1 \tau_h \tag{4.5}$$

where α_1 is a positive reflection coefficient and is equal to 1.0. The function value at the new reflected point is given as F^*. If F^* lies between Fh and Fl, then τh is replaced by τ^* and a new simplex is established.

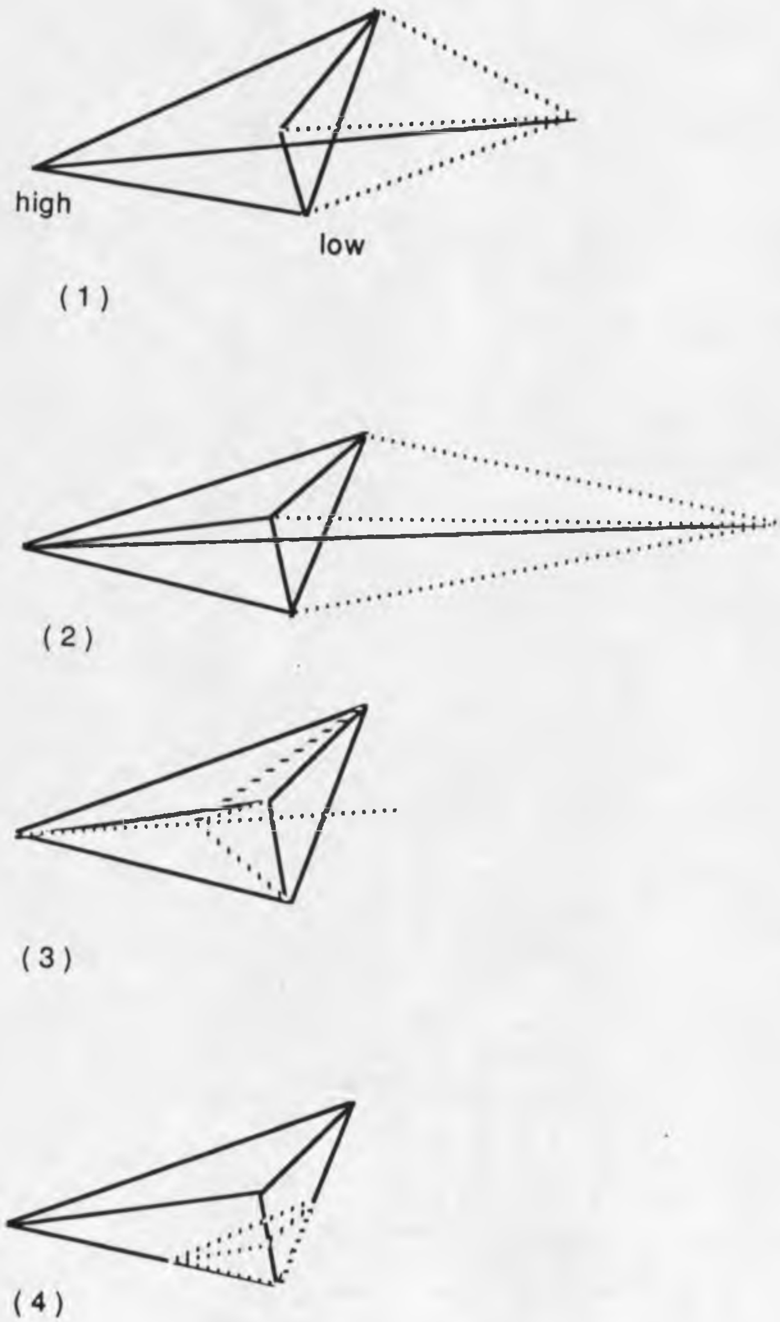

Figure 4.2 - Three dimensional simplex (tetrahedron) and possible outcome, 1) reflection, 2) reflection and expansion, 3) contraction along one dimension, 4) contraction along all dimensions.

II- Expansion

If F^* is less than Fl, then τ^* is a new best (minimum) point, and the simplex is expanded in the same direction by using the following relation:

$$\tau^{**} = (1 - \gamma_1)\,\bar{\tau} + \gamma\,\tau^* \qquad (4.6)$$

where γ_1 is the expansion coefficient and is equal to 2.0. If $F^{**} < Fl$, expansion is successful, and τh is replaced by τ^{**}, but if $F^{**} > Fl$, expansion has failed, and τh is replaced by τ^*. In either case we start computations with the new simplex.

III - Contraction

If on reflection we find that $F^* > Fi$ for all $i \neq h$, i.e. F^* is greater than Fm, a contracted point is located as follows:

$$\tau^{**} = (1-\beta_1)\,\bar{\tau} + \beta_1\,\tau_h \qquad \text{if } F_h < F^* \qquad (4.7)$$

$$\tau^{**} = (1-\beta_1)\,\bar{\tau} + \beta_1\,\tau^* \qquad \text{if } F^* < F_h \qquad (4.8)$$

where β_1 is the contraction coefficient and is equal to 0.5. If F^{**} < minimum of F^* and Fh, τh is replaced by τ^{**}.

IV - Contraction in All The Dimensions Towards The Lowest Point

If on contraction F^{**} > minimum of Fh and F^*, we have a failed contraction. For such a failed contraction, each point in the simplex is replaced by the the following relation:

$$\tau_j = (\tau_j + \tau_l)/2 \qquad j = 1,2,3,\ldots\ldots,n+1 \qquad (4.9)$$

and restart with the new simplex.

The best values of α_1, β_1 and γ_1 are those, which converge the simplex fast towards the minimum. Nelder and Mead (1965) found experimentally that the best values are 1.0, 0.5 and 2.0 respectively.

4-3 Model Description

τ is a dimensionless valve opening (area) used in waterhammer computations.

$$\tau = \frac{(C_d A_g)}{(C_d A_g)_o} \tag{4.10}$$

where

$(C_d A_g)_o$ -at steady state

$(C_d A_g)$ -intermediate state

Commonly, 10 points in τ-time space are chosen for optimization as these give rapid convergence (Nelder and Mead ,1965). For very precise results, the number of points should be the same as the number of computational time steps Δt in time of closure of valve. τ is the position of the valve opening at different times. For optimization purposes, the ten positions of τ are chosen at equal time intervals within the time of valve closure, tc. The boundary conditions are that $\tau = 1$ at $t = 0$ and $\tau = 0$ at $t = tc$. During the optimization procedure, if τ is calculated to be greater than 1.0 or less than 0.0, τ is assigned a value of 0.99999 or 0.00001, respectively.

Waterhammer computations use τ at time intervals of Δt (time step), and these values of τ are interpolated from the optimized values of τ, which are generally at larger time intervals, using the natural cubic spline method (SPLINE subroutine).

Δt = length of pipe/(wave speed$\cdot$divisions of pipe)

The flow chart for the simplex method is given in Figure 4.3, and the flow chart for the computer model is given in Figure 4.4.

4-4 Spline Interpolation

A smooth interpolation can be obtained graphically by using some mechanical means such as french curves or a flexible elastic bar to pass through the desired points. Cubic spline functions are mathematical functions that analog the flexible elastic bar. Polynomial equations are used to represent the curves, requiring a smooth transition at junction points. The construction of cubic spline interpolation function for valve opening τ as a function of time t is given as follows (Hornbeck,1975):

A number of points t_i, i = 1,....,n, which may not necessarily be evenly spaced, with their functional value τ_i, i = 1,....,n are given. These points make n nodes and n-1 intervals. For each interval the following cubic equation is used as the interpolating function between the two nodes.

$$F(t_j) = \tau_j = a_0 + a_1 t_j + a_2 t_j^2 + a_3 t_j^3 \qquad (t_i \le t_j \le t_{i+1}) \qquad (4.11)$$

There are four unknowns in the Equation (4.11). The (n-1) intervals result in 4(n-1) unknowns, and 4(n-1) equations are required to solve for the unknowns. These equations are summarized as follows:

- The function F(t) is continous and has continous first and second derivatives at each of the (n-2) interior nodes resulting in 3(n-2) equations.
- For each node $F(t_i) = \tau_i$ i =1,2,...,n will give n equations, making the total number of equations so far, equal to 4n-6.
- Two more equations are needed to determine completely the spline function. These conditions are achieved by setting $F''(t_1) = F''(t_n)=0$. These

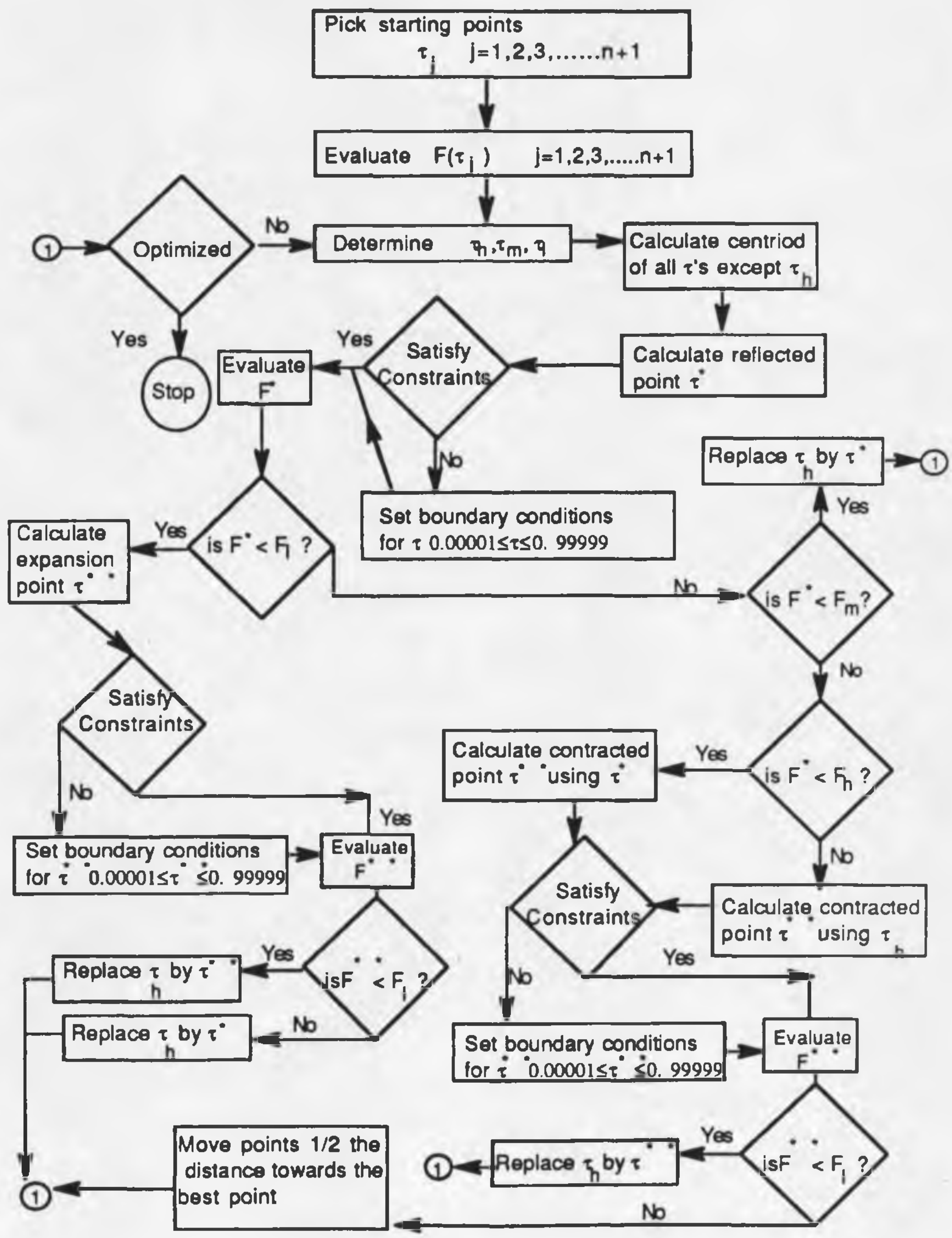

Figure 4.3 The flow chart of the Nelder and Mead simplex.

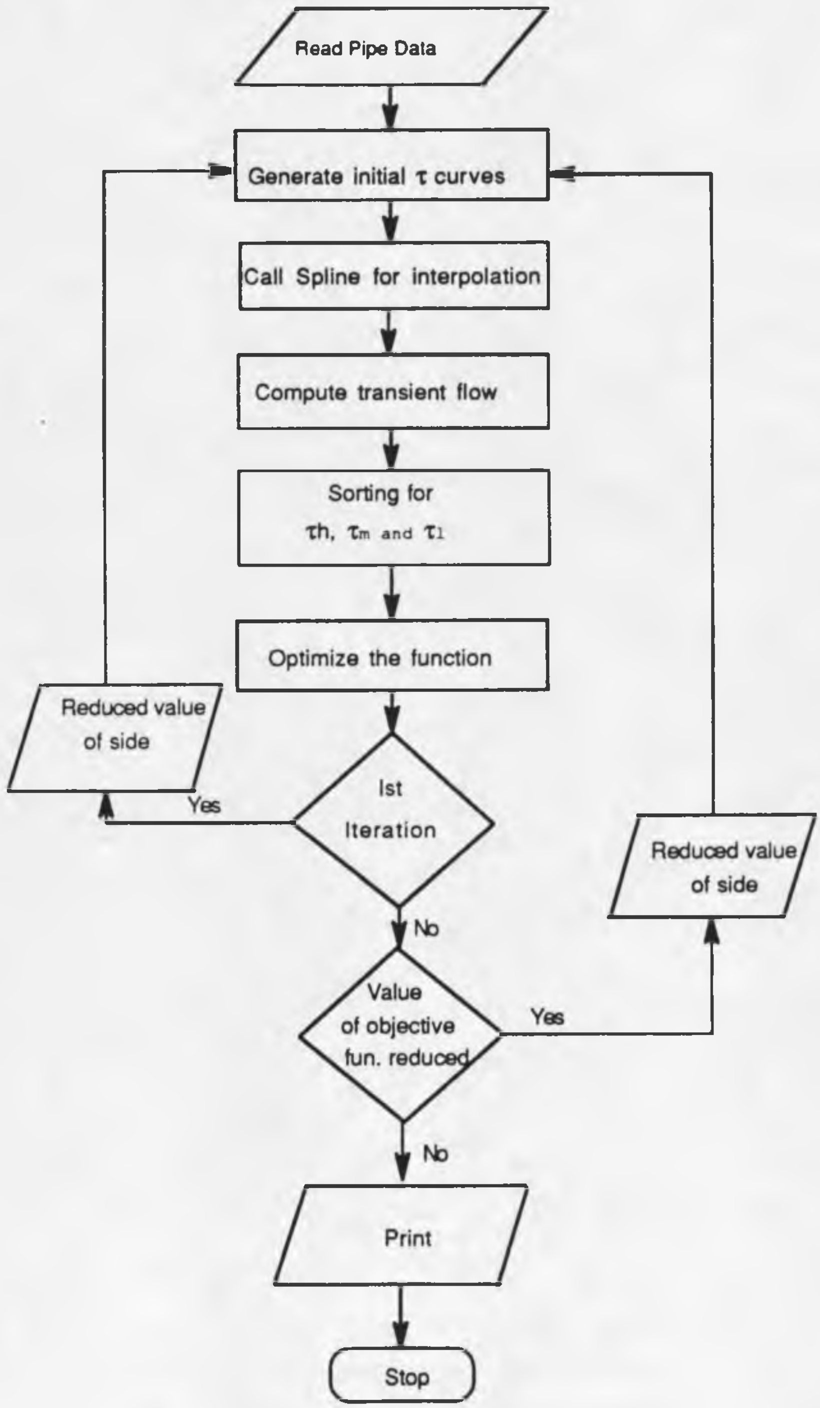

Figure 4.4 The flow chart of the Computer Model.

conditions make the so-called natural spline and the number of equations equals 4(n-1).

CHAPTER 5

MODEL VERIFICATION

The results obtained from the present model are compared with results obtained by the step-closure policy given by Contractor, (1985). The same basic data are used for this model, except that the transient computations are made at six equidistant points along the pipe, whereas in the step-closure policy there are eleven equidistant points. The time step used in the step-closure program is one half of the time step used in this model.

The following data are used to verify the model.

pipe length	L	=	4030.0	ft
pipe diameter	D	=	0.0833	ft
Darcy Weisbach friction factor	f	=	0.036	
initial velocity	Vo	=	1.10	ft/sec
$(C_d A_g)_o$		=	0.000074	
fluid wave speed	a	=	2550.00	ft/sec
reservoir head	Hres	=	140.00	ft

<u>Problem #1</u>

The objective of the problem is to determine the optimum valve-closure policy for a given time of closure; i.e. such that the maximum pressure head anywhere in the pipeline is reduced to a minimum. As a ten-dimensional space is used in the simplex model, eleven τ curves are needed to start the simplex procedure. Time of closure, tc, is taken to be slightly greater than 2L/a i.e. 3.319 sec. The optimum valve closure policy obtained is shown in Figure 5.1 and is very close to that given by Contractor (1985). The pressure variation, at the valve, with linear and optimum valve closures, shown in Figure 5.2, indicates that the maximum pressure is reduced from 206.3 ft to 165.3 ft and minimum pressure rises from 85.7 ft to 105.9 ft with the optimal valve closure policy. The percentage reduction in dynamic pressure is computed as follows:

The steady state pressure at the valve	=	105 94	ft
The maximum pressure for linear valve closure	=	206.36	ft
The maximum pressure for optimal valve closure	=	165.33	ft

$$\text{Percentage reduction in dynamic pressure} = \frac{206.36-165.33}{206.36-105.94} = 40.86\ \%$$

The variation in discharge, when the valve is closed optimally, is shown in Figure 5.3.

Problem #2

In this problem, the time of closure of the valve is 4.1L/a sec. i.e. 6.480 sec. Other input data are the same as used in problem #1, except that a twenty one-dimensional space is used for the simplex procedure. The ten and the twenty one dimensional spaces give more or less the same pressure but there is a small difference in valve closure policy. The optimum valve closure policy is shown in Figure 5.4. The maximum pressure at the valve is reduced from 169.7 ft to 153.3 ft when the valve is closed optimally as compared to linear closure, as shown in Figure 5.5. The percentage reduction in dynamic pressure head is computed as follows:

The steady state pressure at the valve	=	105.94	ft
The maximum pressure for linear valve closure	=	169.74	ft
The maximum pressure for optimal valve closure	=	153.32	ft

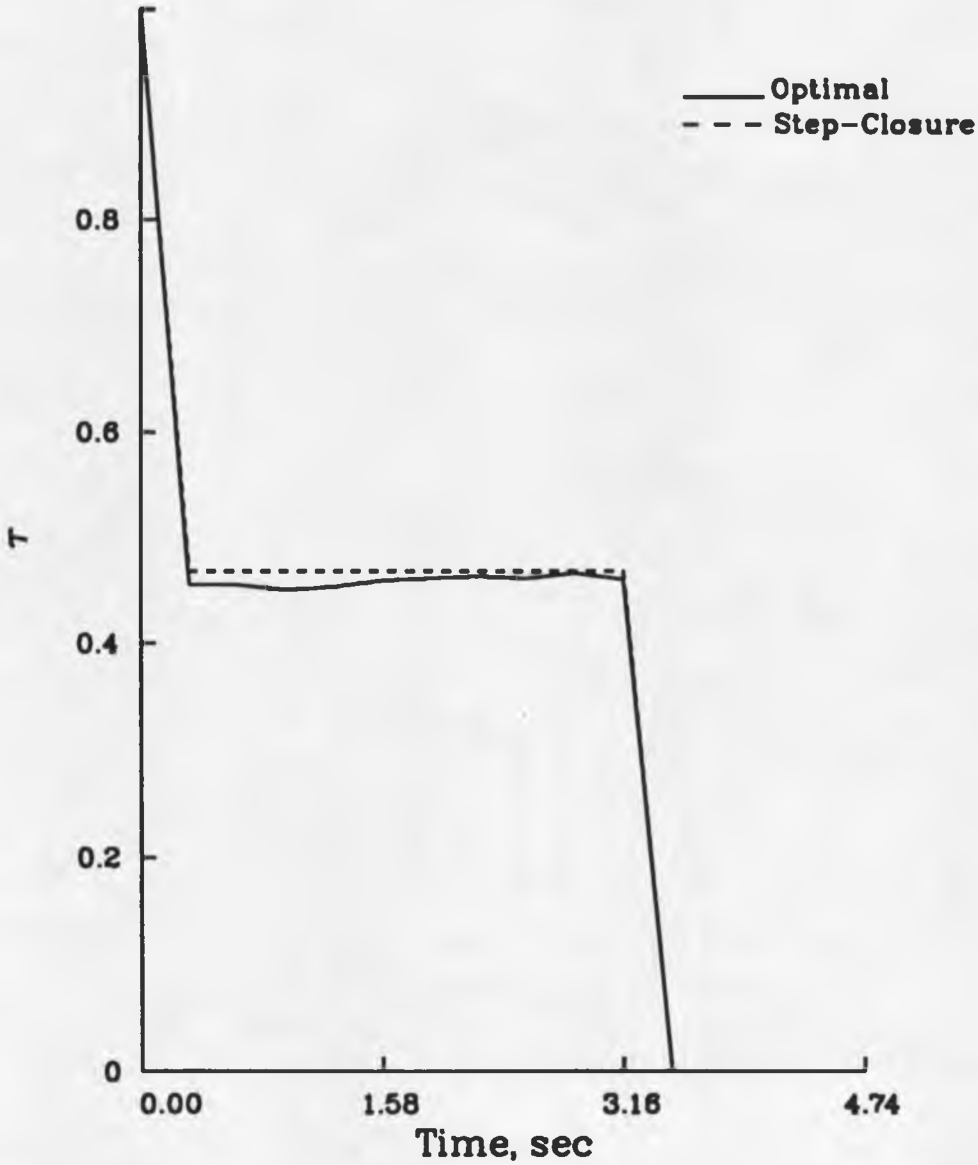

Figure 5.1 –Optimal valve closure policies for step–closure and minimizing the maximum pressure, tc =3.319 sec (2.1L/a), model verification.

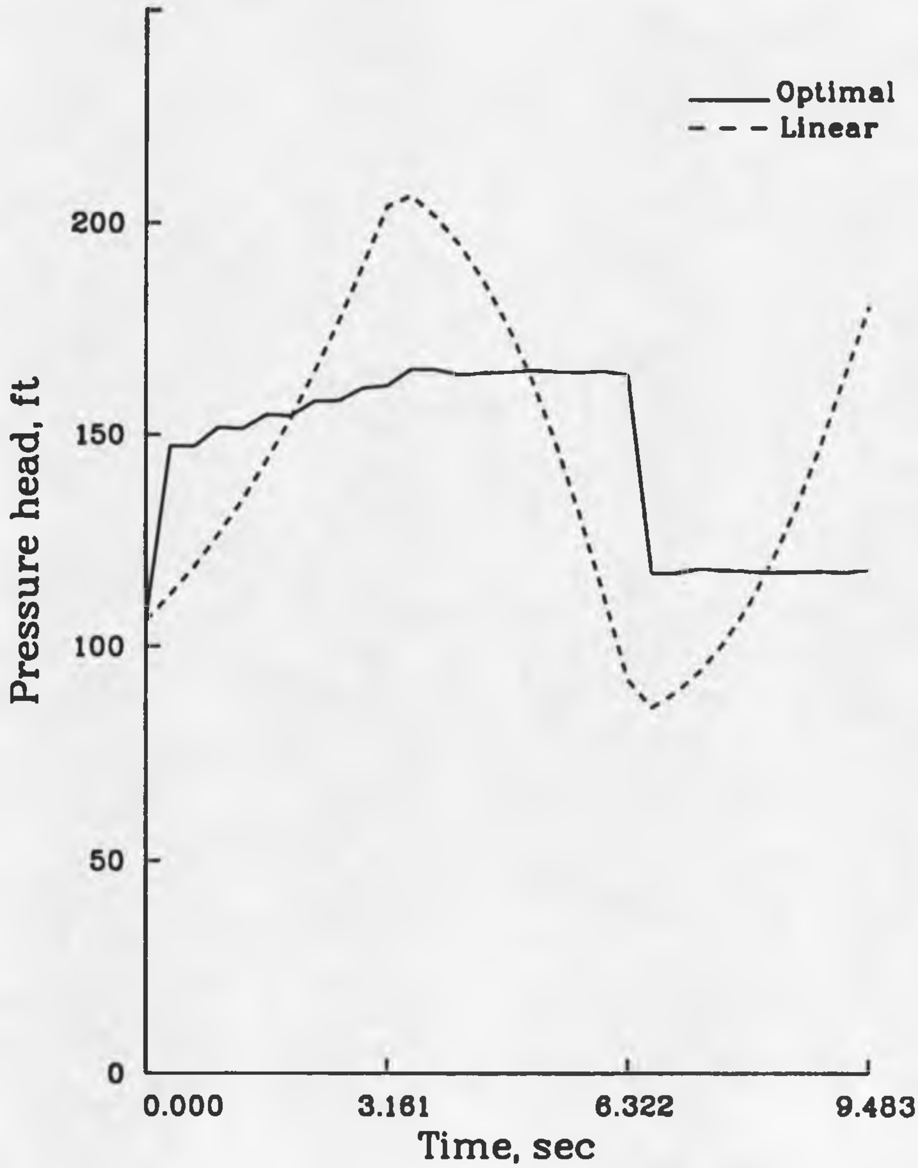

Figure 5.2 –Pressure variation at the valve for optimal and linear valve closures, tc =3.319 sec, (2.1L/a), model verification.

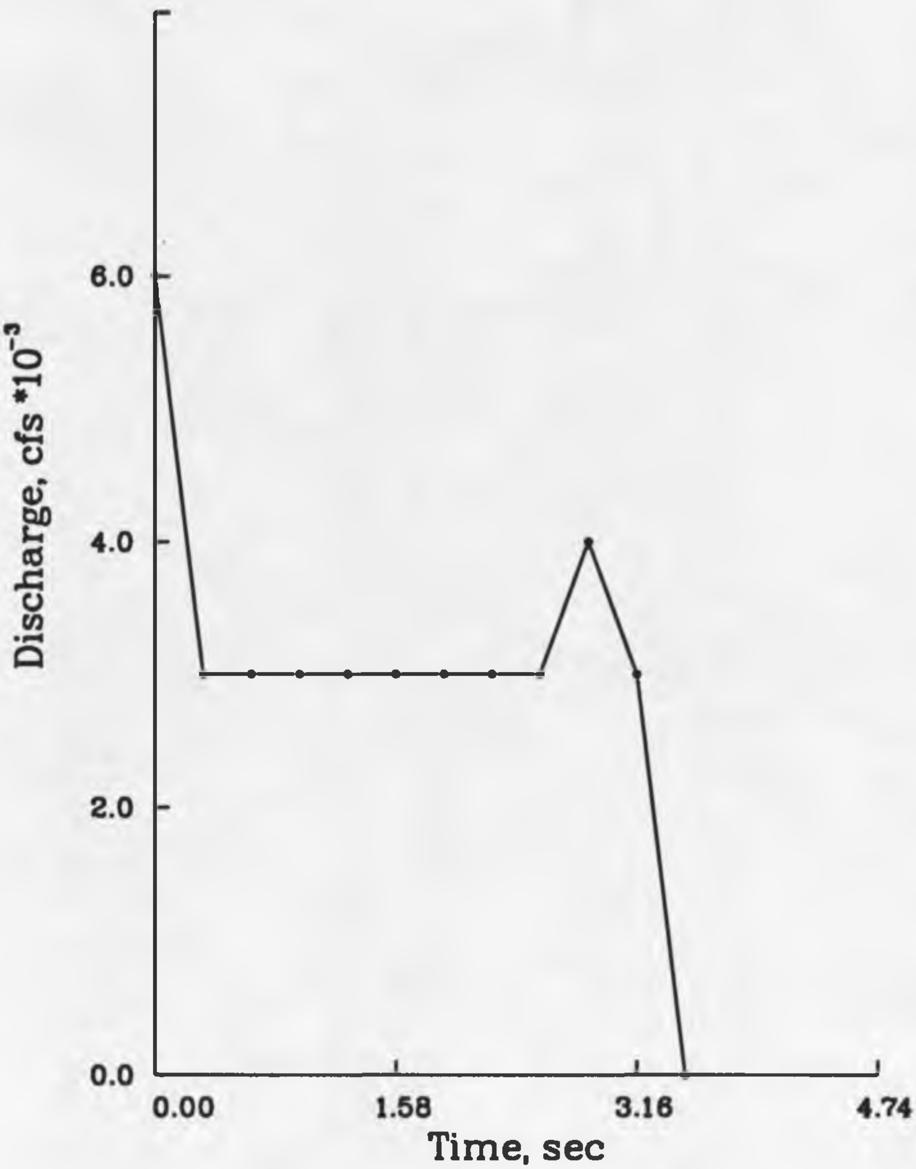

Figure 5.3 —Discharge variation at the valve for optimal valve closure, tc =3.319 sec, (2.1L/a), model verification.

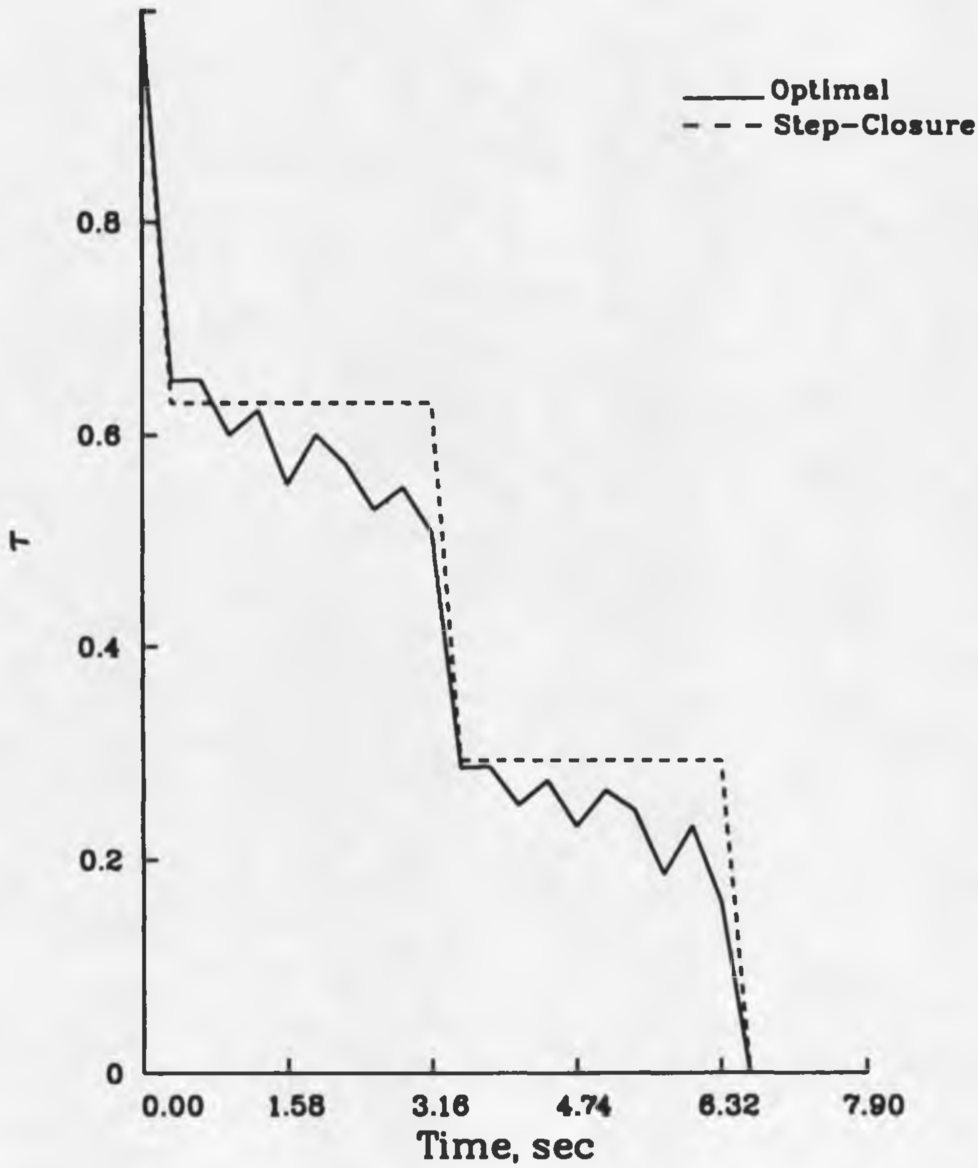

Figure 5.4 -Optimal valve closure policies for step-closure and minimizing the maximum pressure, tc =6.480 sec (4.1L/a), model verification.

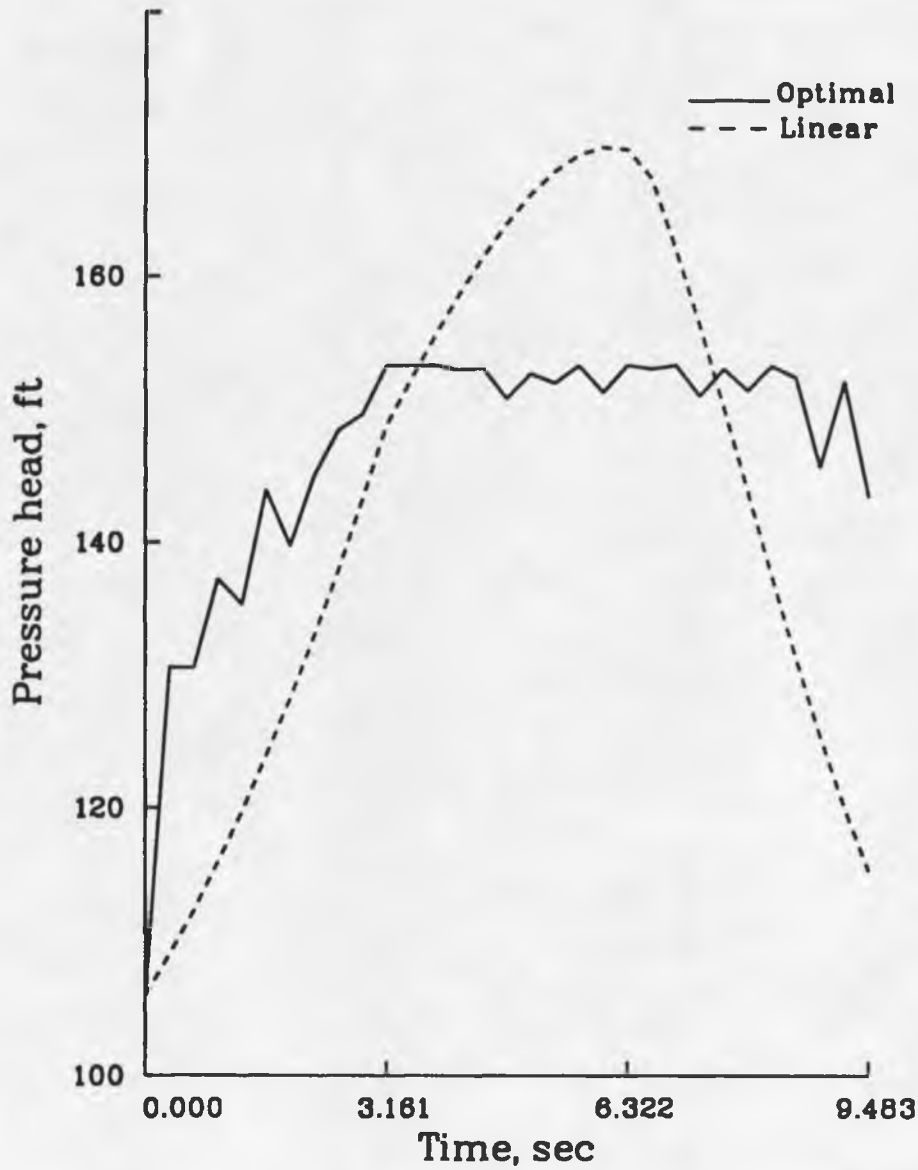

Figure 5.5 –Pressure variation at the valve for optimal and linear valve closures, tc =6.480 sec, (4.1L/a), model verification.

$$\text{Percentage reduction in the dynamic head} = \frac{169.74-153.32}{169.74-105.94} = 25.74\ \%$$

Figure 5.6 shows the discharge variation at the valve for optimal valve-closure.

Computations were made for different times of closures of the valve. Data in Table 5.1 show a comparison of the maximum head obtained by the present model and produced by Contractor (1985).

Table 5.1. Comparision of Maximum Pressures Obtained by Step Valve Closure and Optimal Valve Closure.

Valve Closure Time		Maximum Pressure Head (ft)		
sec	L/a	Linear Valve Closure	Optimal Step Valve Closure	Optimal Valve Closure
3.319	2.1	208.2	166.3	165.3
6.480	4.1	169.5	155.6	153.3
9.640	6.1	159.5	147.6	149.1
12.801	8.1	153.8	146.3	146.3

Small differences in the results of the two optimal policies may be attributed to the different time steps used in the two methods. Computations with a smaller time step should be more accurate than computations with a larger time step.

The percentage reduction in dynamic pressure is plotted against valve closure time in Figure 5.7. As is clear from Figure 5.7, the highest peak of reduction of 41% occurs with a valve closure of about 2.2L/a sec. and the second highest peak of reduction of 25 % occurs at about 4.4L/a sec.

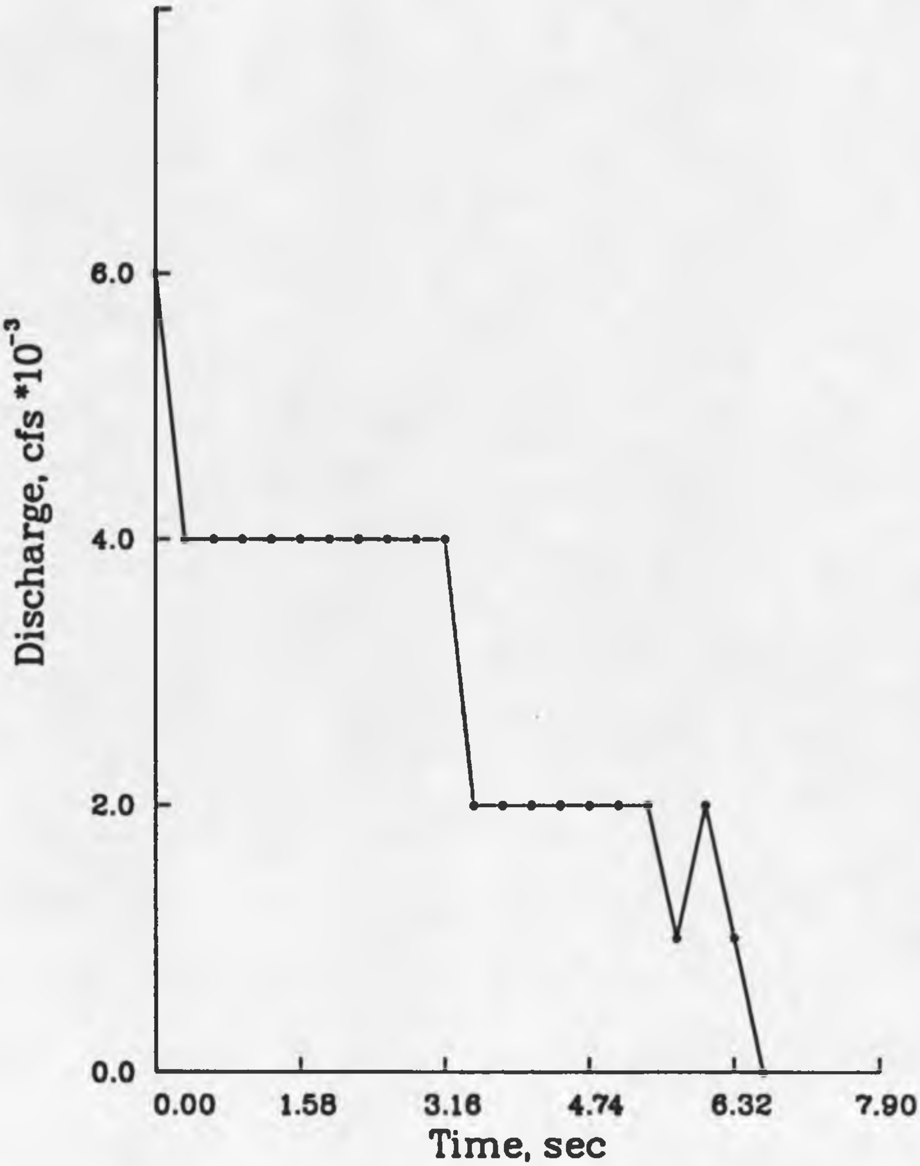

Figure 5.6 —Discharge variation at the valve for optimal valve closure, tc =6.480 sec, (4.1L/a), model verification.

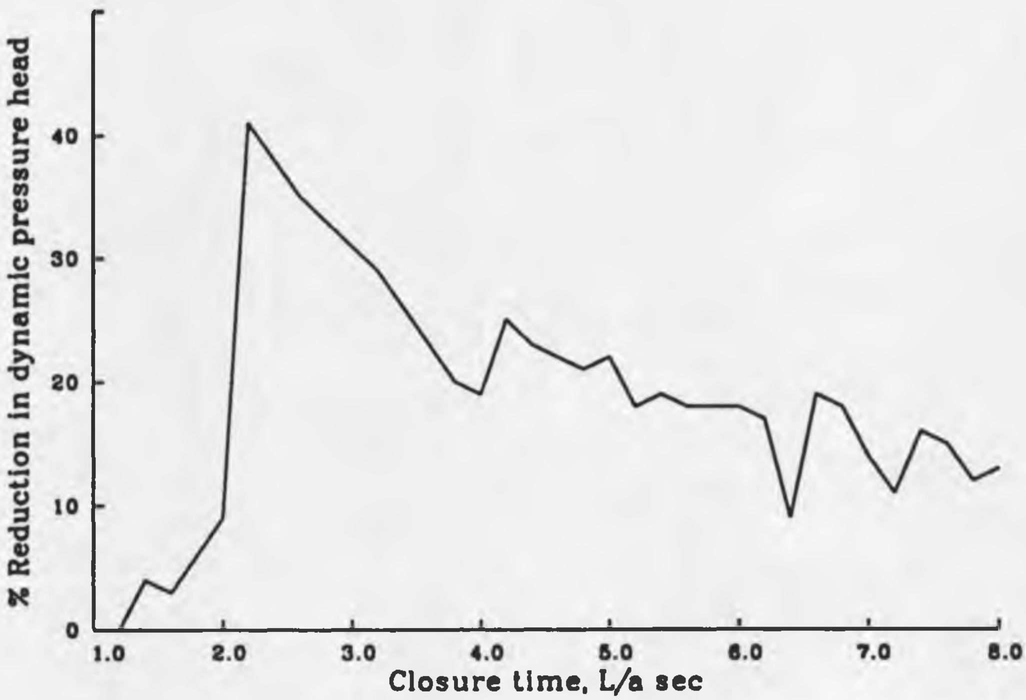

Figure 5.7 –Percentage reduction in dynamic pressure head as a function of valve closure time, model verification.

CHAPTER 6

COMPUTER IMPLEMENTATION AND RESULTS

A simple pipeline with a constant head reservoir at the upstream end and a valve at the downstream end is chosen for application of the model. Different cases are studied, with different flow rates and objective functions. The valve operation, with higher discharge through it, creates higher pressure variation. These pressure variations are very often undesirable. The aim is operate the valve so as to reduce the maximum pressure and to increase the minimum pressure, if it is below the vapor pressure of the fluid, or to reduce the maximum pressure while keeping the minimum pressure at some limiting value.

The objective function, which is to be optimized by the simplex, is the key factor in determining the solution of the problem. It can control only one of the limiting values at a time; either minimum pressure or maximum pressure in a specified time of valve closure. Four cases are studied and presented to show how selection of the objective function changes the nature of the solution.

All four cases that are studied, use the same basic data which are given below:

length of pipe	=	600.00	meters
diameter of pipe	=	0.50	meters
reservoir head	=	150.00	meters
Darcy Weisback friction factor	=	0.018	
gravity acceleration g	=	9.806	meters/sec^2
fluid wave speed	=	1341.13	meters/sec

6-1 Case I

This is the simplest case in which the steady state velocity of the flow is very low. The product of discharge coefficient and cross-sectional area of the valve at steady state ($(C_dA_g)_o$) is 0.009. The discharge and velocity through the pipe are 0.5 cu.m/sec and 2.55 m/sec respectively. The objective in this case is to minimize the maximum pressure head obtained from waterhammer computations. The optimum valve closure policy obtained from the above computations, does not result in a minimum pressure lower than the vapor pressure of the fluid.

The program is executed for different times of closure, and the percentage reduction in dynamic pressure is plotted against valve closure time of in Figure 6.1. The maximum reduction of 62 % in dynamic pressure is achieved at tc= 2.2L/a sec. The second highest peak of reduction of 37 % is achieved at 4.4L/a sec. These two conditions with tc equal to 2.2L/a and 4.4L/a are studied in greater detail as follows:

Problem #1

The optimum valve closure policy for tc= 2.2L/a is shown in Figure 6.2. This is a one-step valve closure, where the valve is closed to τ =0.48 in a very short time, is then held constant for some time, and finally is closed fully in a very short time.

A comparison of pressures at the valve, between linear and optimum valve closures, is given in Figure 6.3. This figure shows that the maximum pressure head decreases from 450 m to 260 m and the minimum pressure increses from -147 m to 41 m when using the optimal valve-closure policy as compared to linear valve closure. In this computation, only the maximum pressure is minimized. It can be seen from Figure 6.3 that minimization of the maximum pressure also results in an increase of the

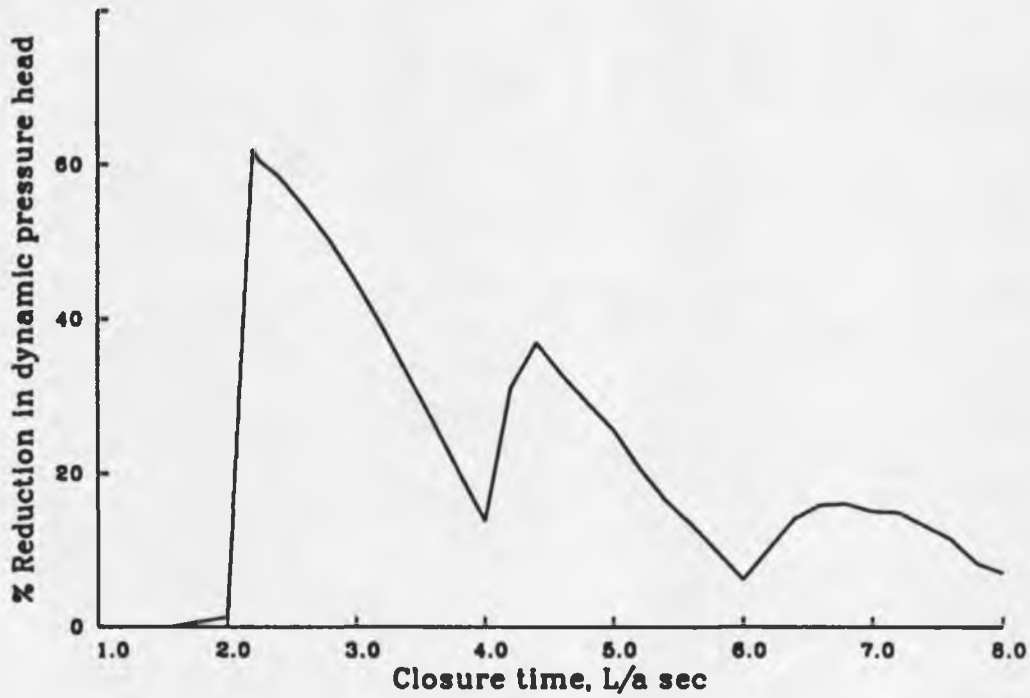

Figure 6.1 –Percentage reduction in dynamic pressure head as a function of valve closure time, case I.

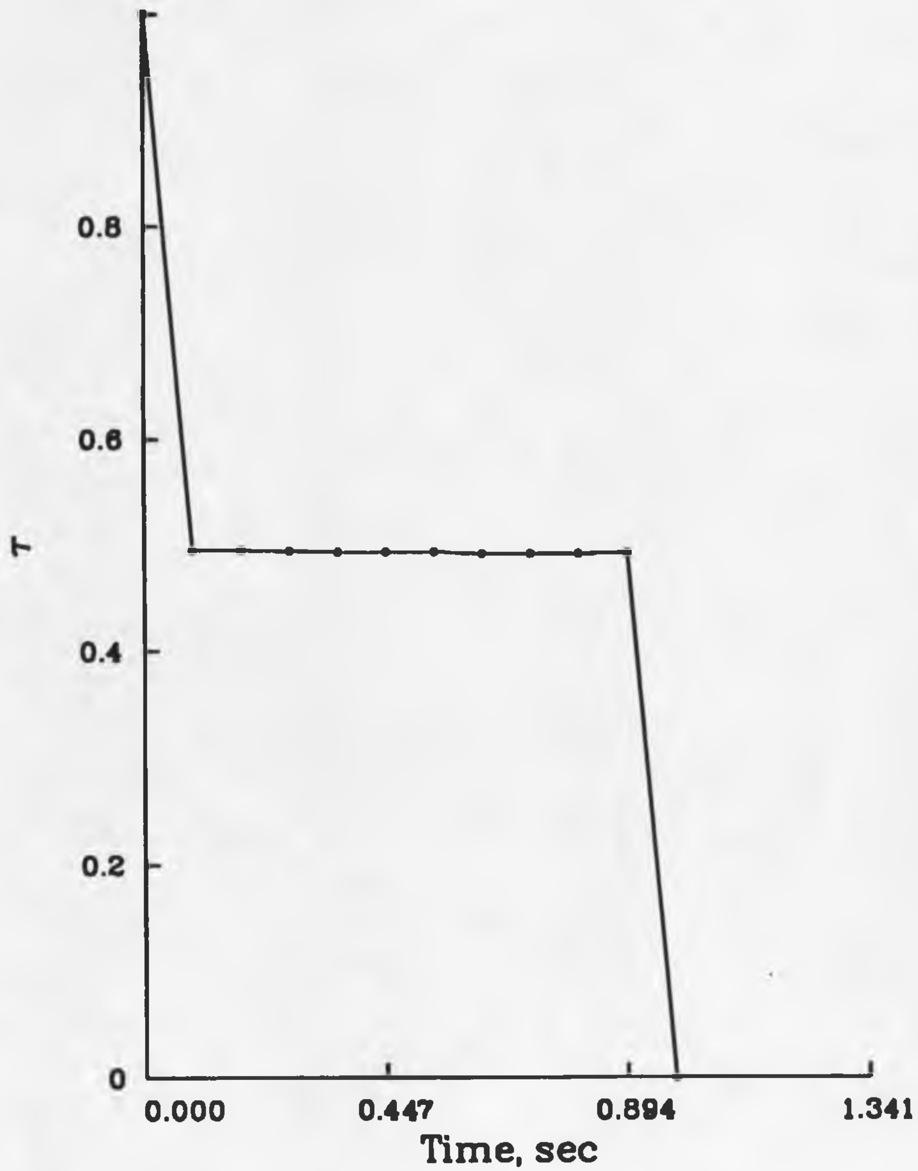

Figure 6.2 –Optimal valve closure policy, tc =.984 sec, (2.2L/a), case I.

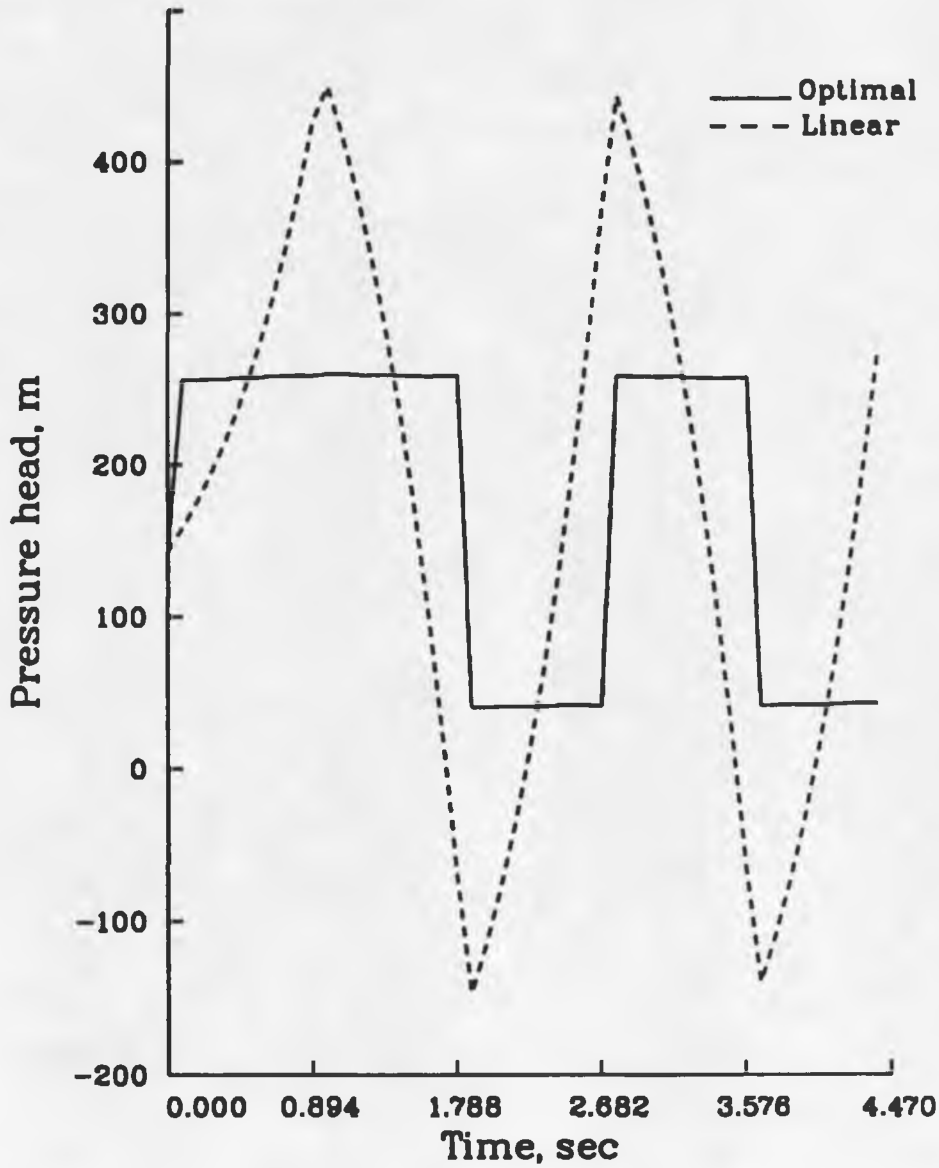

Figure 6.3 –Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec, (2.2L/a), case I.

minimum pressure from -146 m to 40 m. Thus, column separation is avoided simultaneously with reduction of the maximum pressure.

The steady state pressure at the valve	=	143.49	m
The maximum pressure resulting linear valve closure	=	450.39	m
The maximum pressue resulting optimal valve closure	=	259.99	m

$$\text{Percentage reduction in dynamic pressure} = \frac{450.39-259.99}{450.39-143.49} = 62.04\ \%$$

The discharge variation at the valve, shown in Figure 6.4, indicates discharge is reduced only 36 % when the valve is closed to $\tau= 0.52$.

Problem #2

The optimum valve-closure policy for time of closure of 4.4L/a sec (1.968 sec) is shown in Figure 6.5. This is a two step valve closure. In the first step the valve is closed to $\tau=0.65$ in a very short time and is held constant to about 2L/a sec. In the second step, it is closed to $\tau=0.32$ and is kept constant to about 4L/a sec then finally closed fully in a very short time.

A comparison of pressure head variation at the valve, with linear and optimal valve closures, is shown in Figure 6.6. This shows that the maximum pressure is reduced 42.6 m and the minimum pressure is increased 31.4 m when closing the valve optimally as compared to linear closing.

The steady state pressure at the valve	=	143.49	m
The maximum pressure while using linear closure	=	258.83	m
The maximum pressure using optimal valve closure	=	216.23	m

$$\text{Percentage reduction in dynamic pressure} = \frac{258.83-216.23}{258.83-143.49} = 36.93\ \%$$

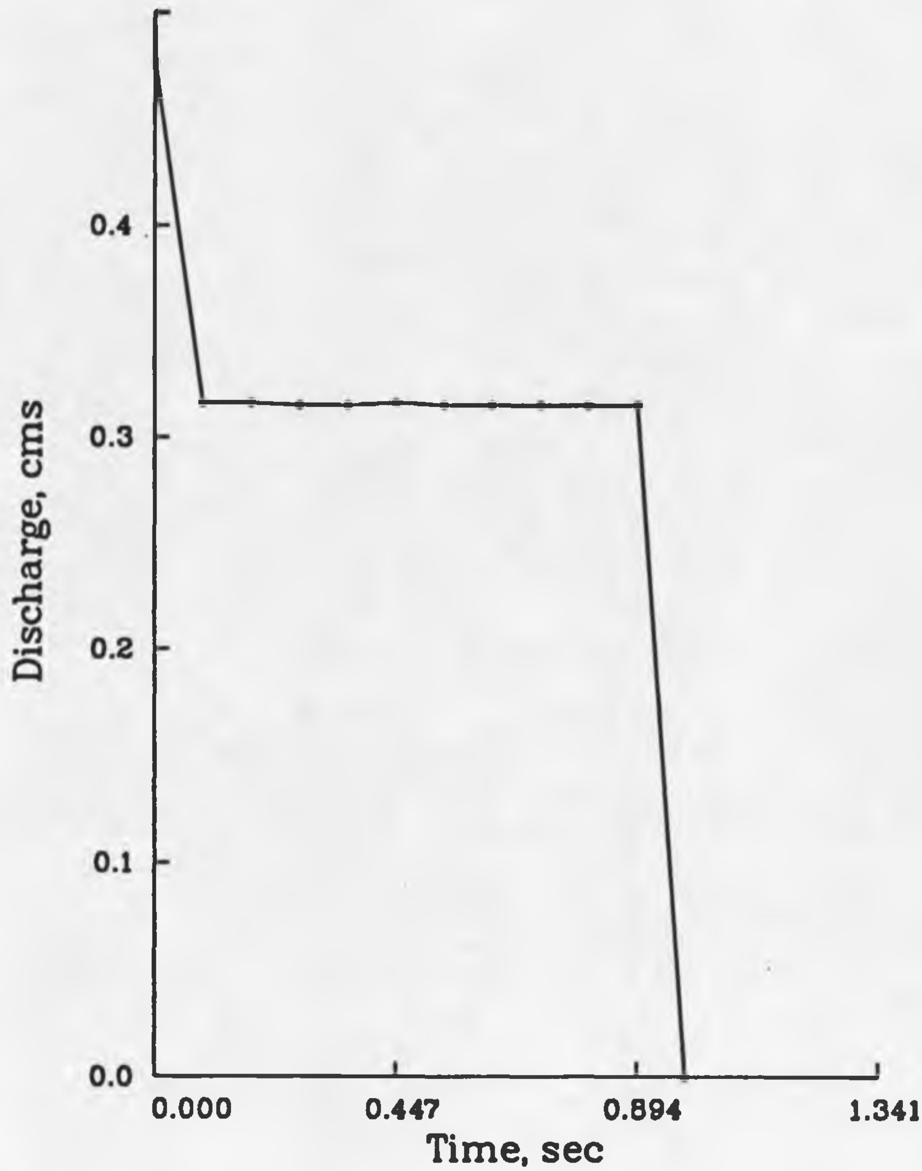

Figure 6.4 —Discharge variation at the valve for optimal valve closure, tc =.984 sec, (2.2L/a), case I.

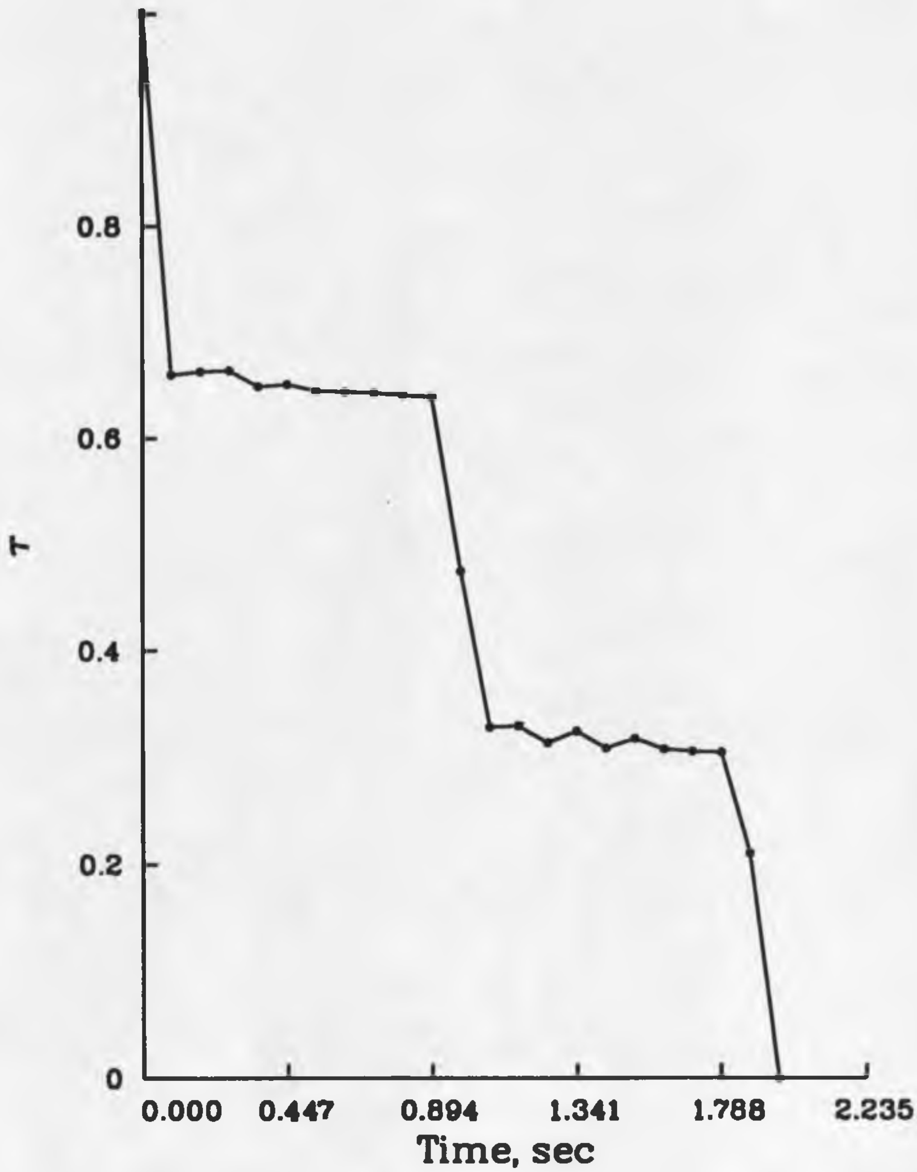

Figure 6.5 –Optimal valve closure policy, tc =1.968 sec, (4.4L/a), case I.

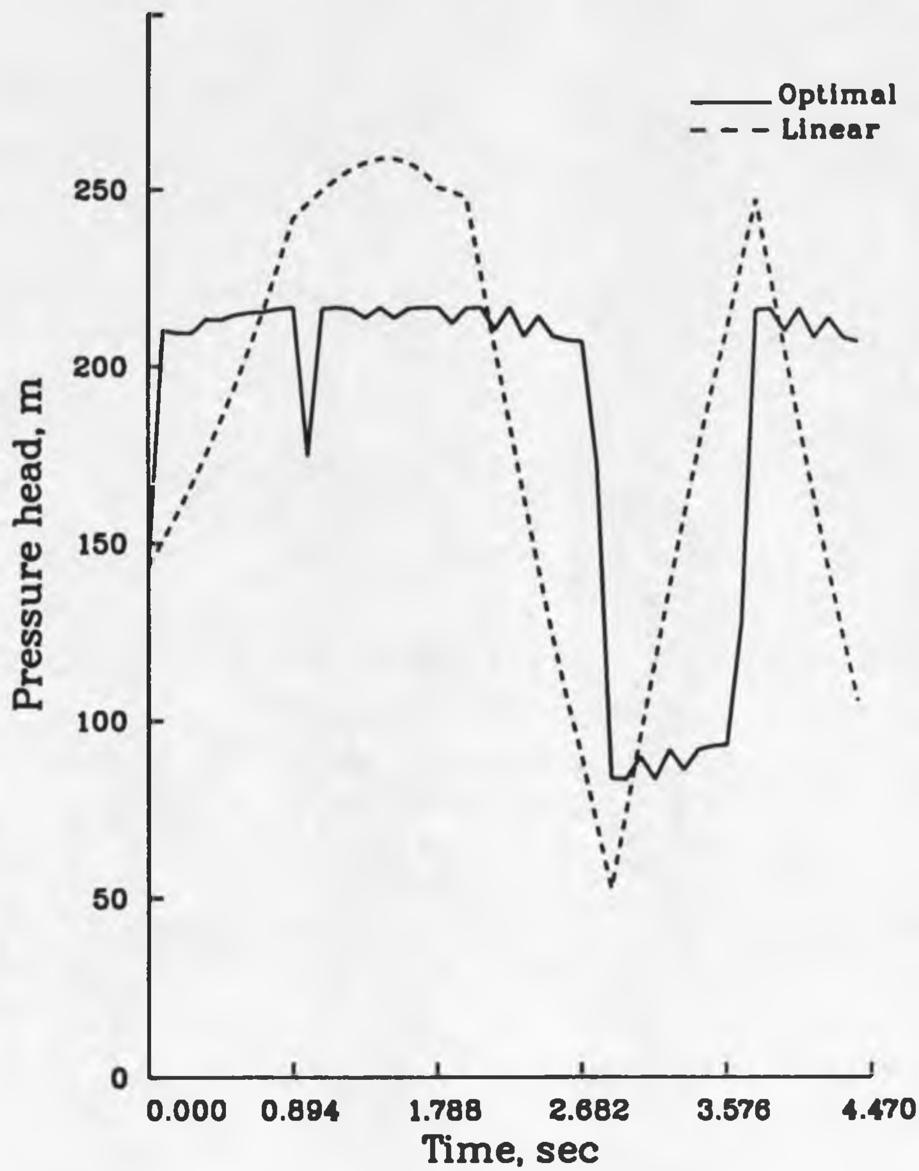

Figure 6.6 –Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec, (4.4L/a), case I.

The discharge variation at the valve is shown in Figure 6.7. In the first step, when the valve is closed to τ= 0.65, discharge reduces to 79 %, and in the second step, when the valve is at τ= 0.30, the dischrage is 37 %.

6-2 Case II

In this case, the steady state area of the valve opening is increased so that $(C_dA_g)_o$ =0.038, resulting in a steady state velocity of 7.80 m/sec and a discharge of 1.532 cu.m/sec. The objective function that was specified was simply minimization of the maximum pressure. Figure 6.8 shows that as the time of valve closure increases, the maximum pressure decreases and the minimum pressure increases. This figure also shows that abrupt changes in pressure occur at specific closure times. These time of closures are slightly larger than 2L/a, 4L/a and 6L/a sec. As the time of closure increases, the change in pressure decreases, as shown in Figure 6.8. It is seen that minimum pressure is higher than the vapor pressure of the fluid, only when time of valve closure is greater than 6L/a sec.

This program is executed for different times of closure and a graph is plotted of percentage reduction in dynamic maximum pressure versus valve closure time of Figure 6.9. The maximum reduction of 65 % is achieved at 4.4L/a sec, and the second highest peak of reduction of 61 % is occured at 2.2L/a sec. For a better understanding and comparison, of the different cases, problems are studied in detail for tc equal to 2.2L/a sec and 4.4L/a sec.

Problem #1

The optimum valve closure policy for tc= 2.2L/a sec (0.984 sec) is shown in Figure 6.10. This is a one-step valve closure. In cases I and II, the same data are used, except that in case II the initial valve opening is bigger than in case I. In case I, the

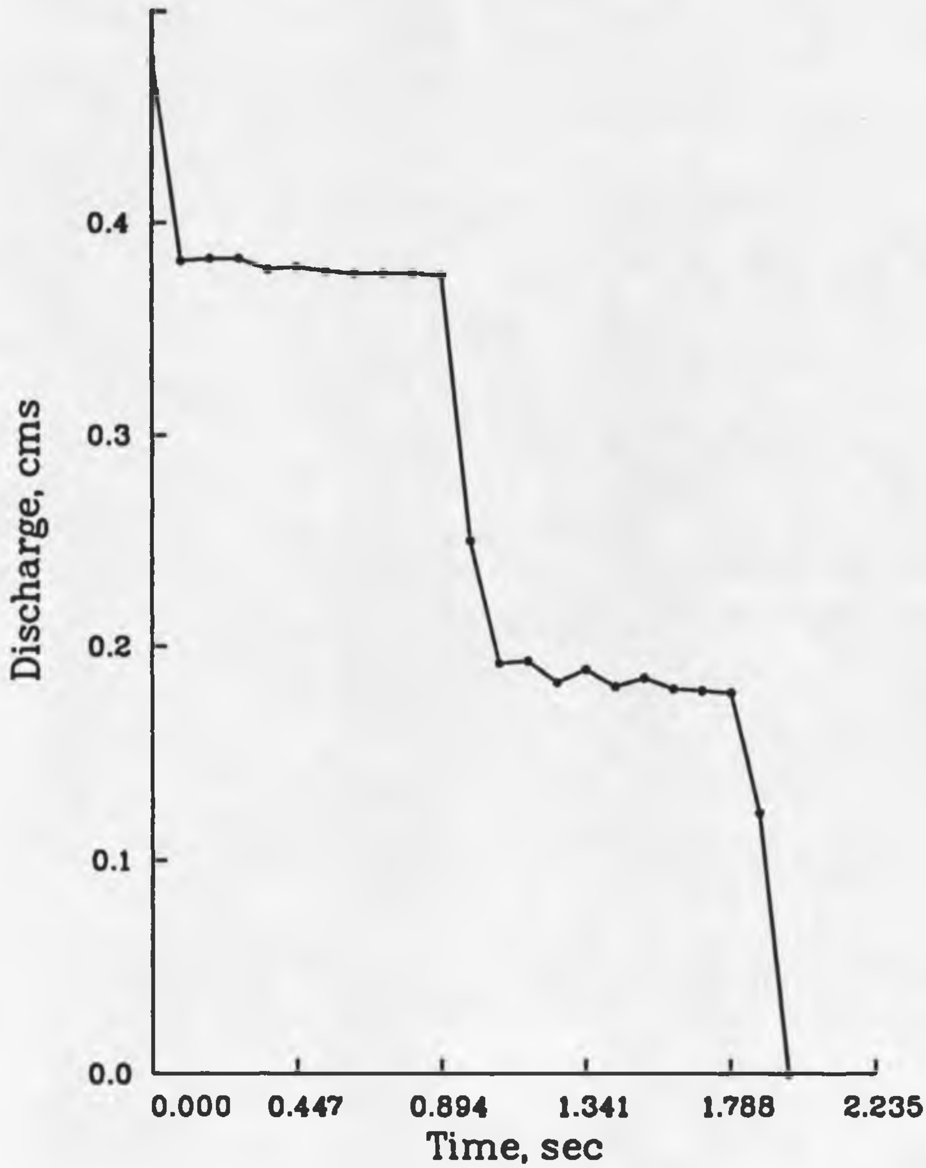

Figure 6.7 –Discharge variation at the valve for optimal valve closure, tc =1.968 sec. (4.4L/a), case I.

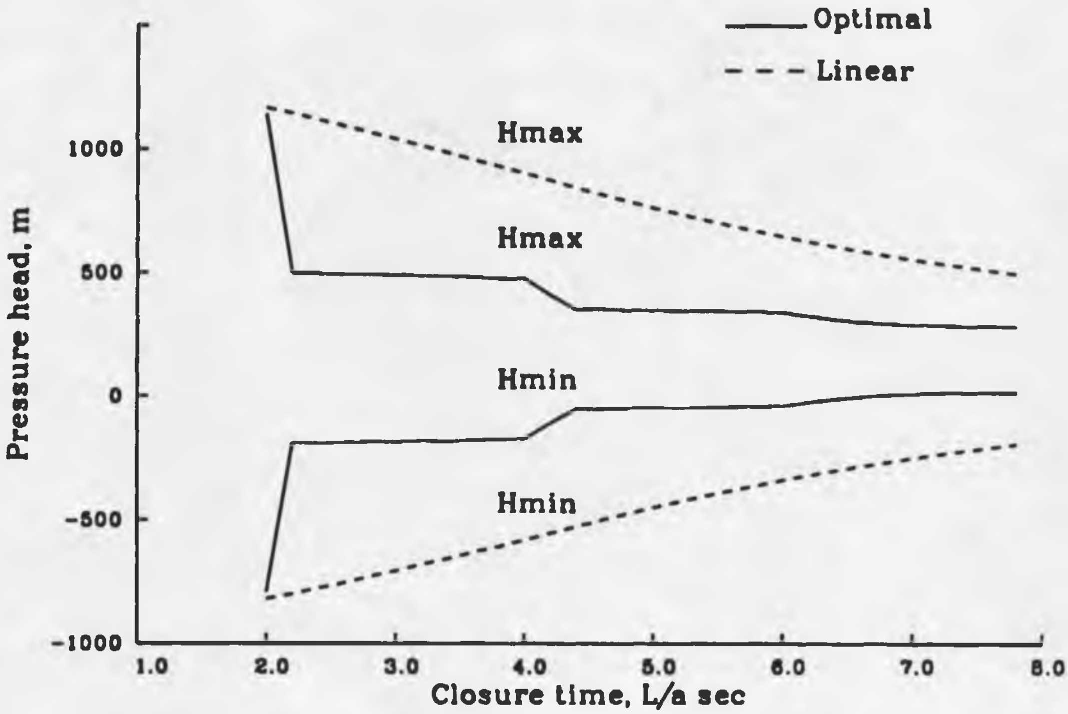

Figure 6.8 –Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case II.

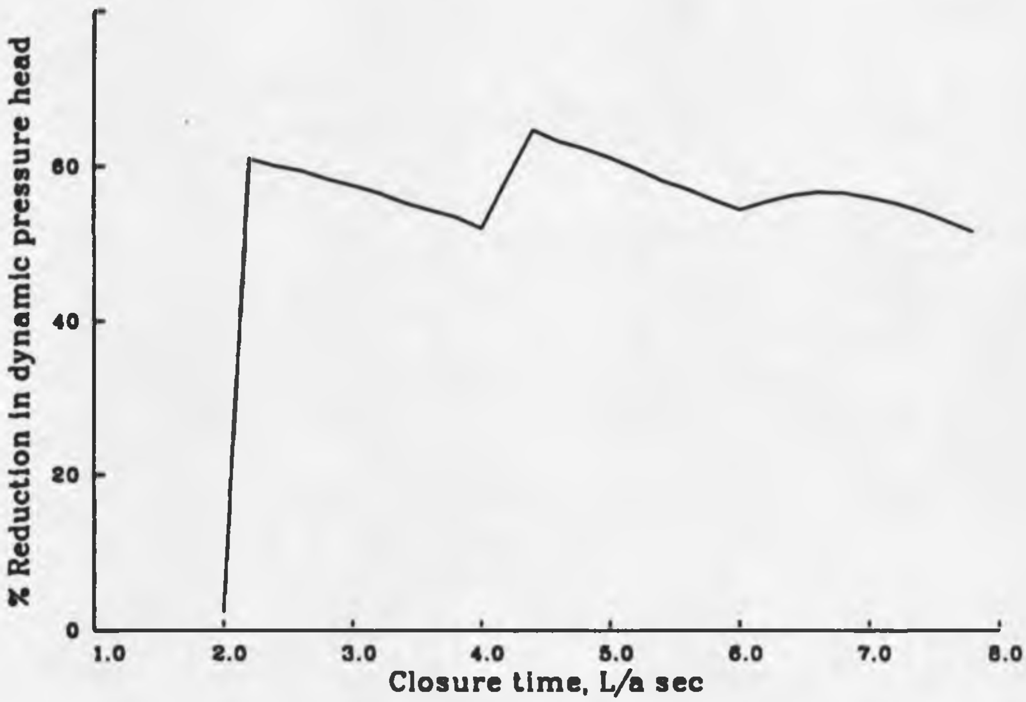

Figure 6.9 –Percentage reduction in dynamic pressure head as a function of valve closure time, case II.

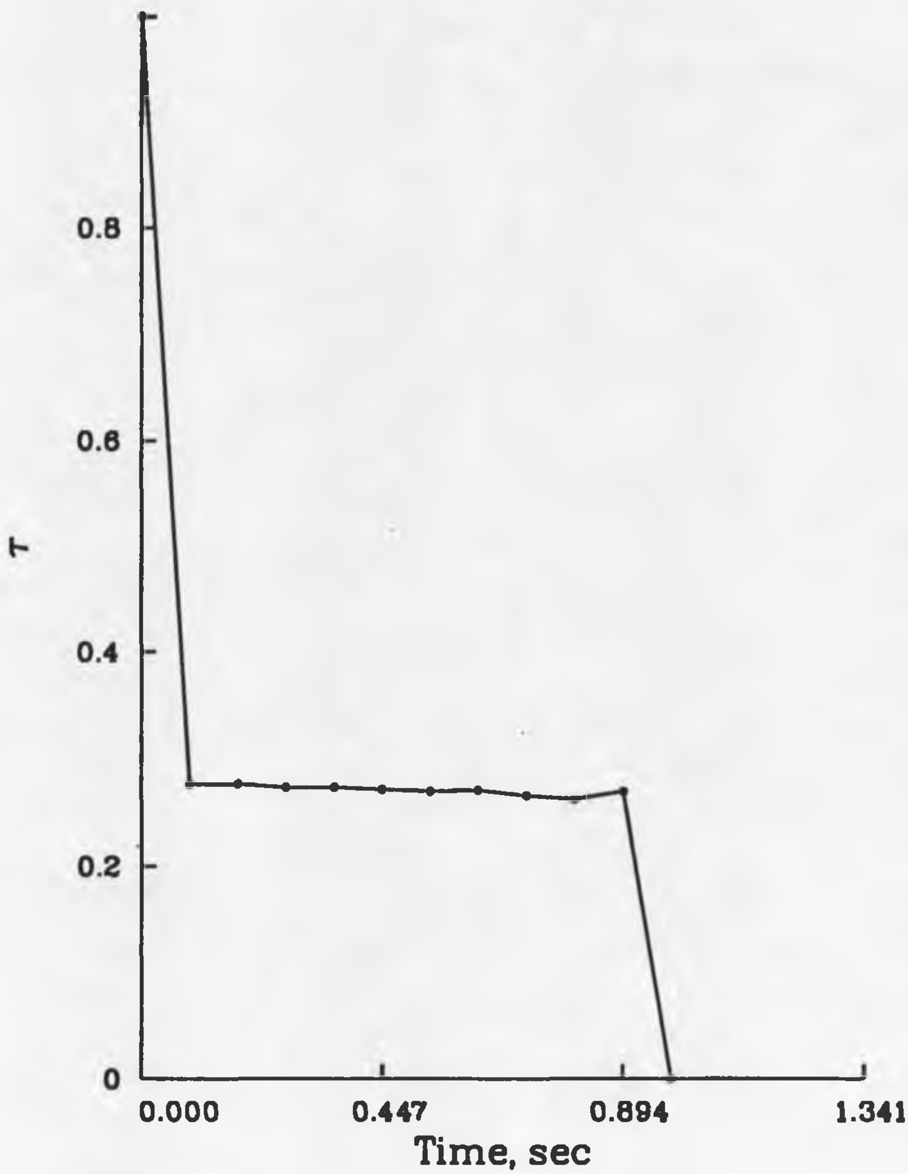

Figure 6.10 –Optimal valve closure policy, tc =.984 sec, (2.2L/a), case II.

optimum valve closure is at τ=0.48 whereas in case II the optimum valve closure is at about τ =0.28.

A comparison of pressures at the valve with linear and optimal valve closures, Figure 6.11, shows the maximum pressure is reduced from 1144 m to 496 m and the minimum pressure is increased from -800 m to -190 m.

The steady state pressure at the valve = 82.92 m

For linear valve closure

The maximum pressure at the valve = 1144.50 m

The minimum pressure at the valve = -799.89 m

For optimum valve closure

The maximum pressure at the valve = 496.97 m

The minimum pressure at the valve = -190.05 m

Percentage reduction in dynamic pressure (max) $= \dfrac{1144.50-496.97}{1144.50-82.92} = 61.00\ \%$

Percentage increase in dynamic pressure (min) $= \dfrac{-799.89-(-190.05)}{-799.89\ -\ (82.92)} = 69.08\ \%$

Figure 6.12 shows the variation in the discharge at the valve. When the valve is closed to an opening of τ= 0.28, the discharge reduces to only 66 %.

Problem #2

In this problem the time of closure of the valve is 4.4L/a sec and the optimum valve-closure is given in Figure 6.13. This is also a two-step valve closure, except that the valve is closed to τ=0.40 in the first step as compared to τ=0.65 in case I. In the second step, the valve is closed to τ=0.20 as compared to τ=0.32 in case I. The computer output is given in Appendix B.

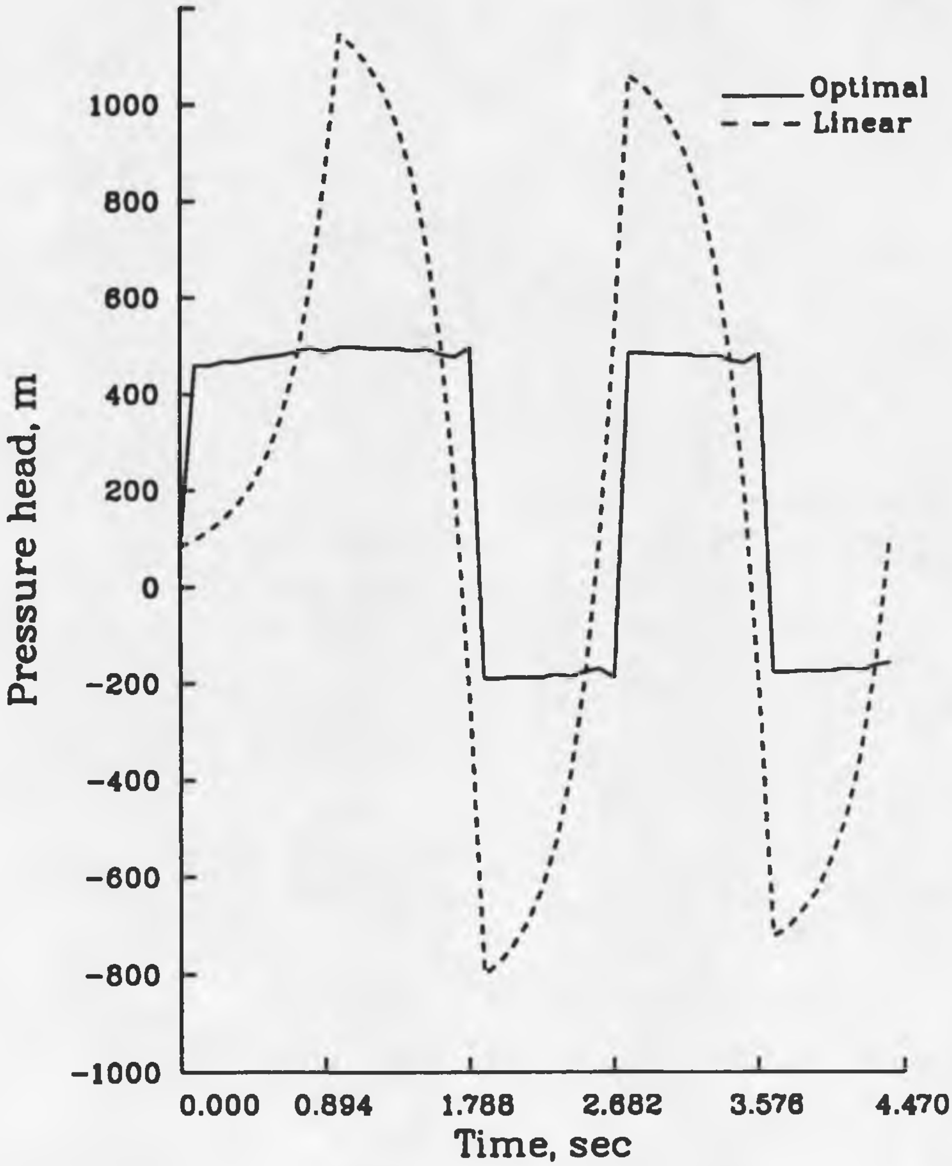

Figure 6.11 –Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec, (2.2L/a), case II.

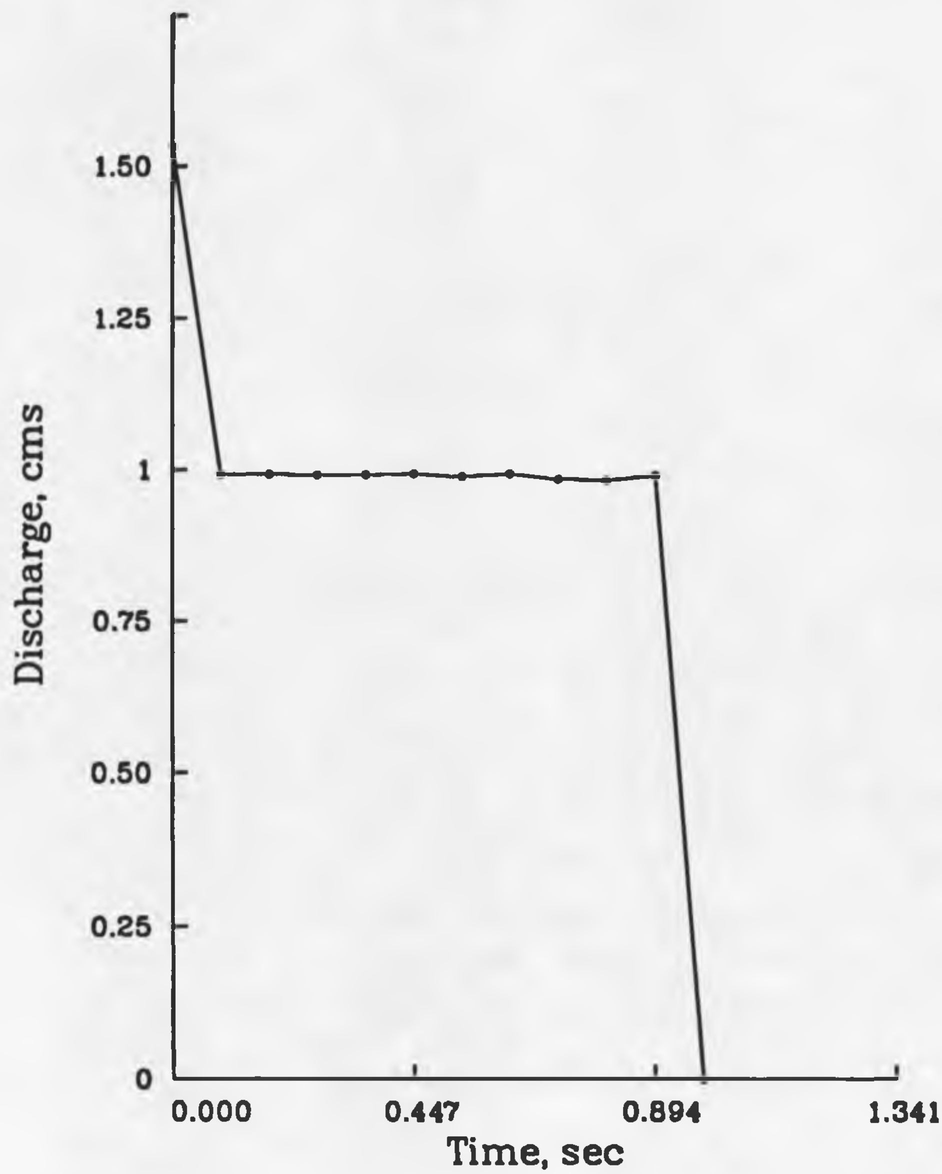

Figure 6.12 –Discharge variation at the valve for optimal valve closure, tc =.984 sec, (2.2L/a), case II.

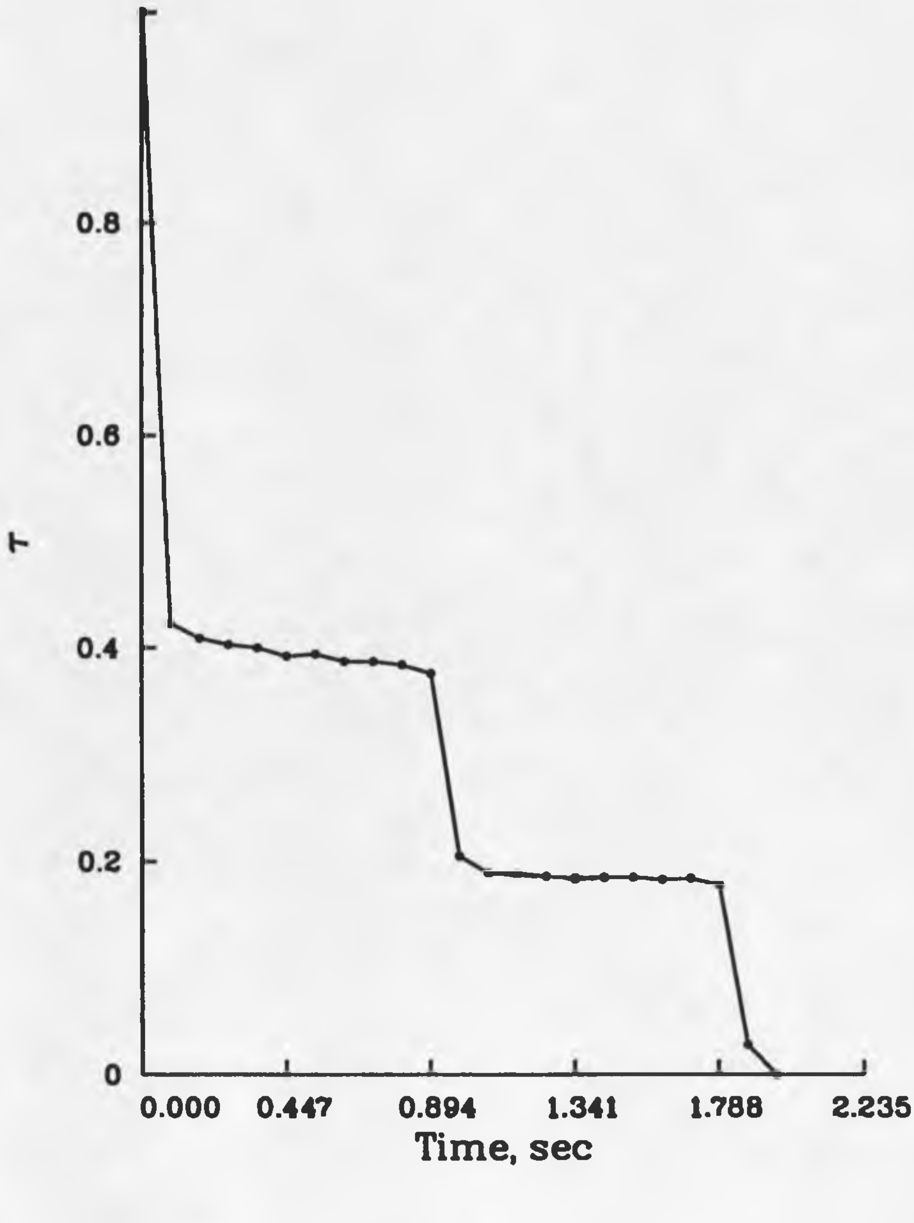

Figure 6.13 –Optimal valve closure policy, tc =1.968 sec, (4.4L/a), case II.

A comparison of pressures at the valve with linear valve closure and optimal valve closure, Figure 6.14, shows that the maximum pressure is reduced from 845 m to 352 m and the minimum pressure increased from -525 m to -50 m.

The steady state pressure at the valve = 82.92 m

For linear valve closure

The maximum pressure at the valve = 845.24 m

The minimum pressure at the valve = -525.51 m

For optimum valve closure

The maximum pressure at the valve = 351.92 m

The minimum pressure at the valve = -49.57 m

$$\text{Percentage reduction in maximum pressure} = \frac{845.24-351.92}{845.24-82.92} = 64.71\ \%$$

$$\text{Percentage increase in minimum pressure} = \frac{-525.51-(-49.57)}{-525.51-(82.92)} = 78.22\ \%$$

Figure 6.15 shows that the discharge is 83 % when the valve opening is $\tau = 0.40$ in the first step of valve closure, and in the second step, when the valve opening is $\tau = 0.20$, the discharge is 37 %.

6-3 Case II

The same data are used as in case II except that the objective function is changed to maximization of the minimum pressure. The simplex converges towards maximum value of the minimum pressure, which is the steady state pressure at the valve. The valve closure policy, for tc =2.2L/a sec, and the pressure variation at the valve for linear and optimal valve closures, are shown in Figures 6.16 and 6.17, respectively. The maximum pressure in Figure 6.17 is much higher than the maximum

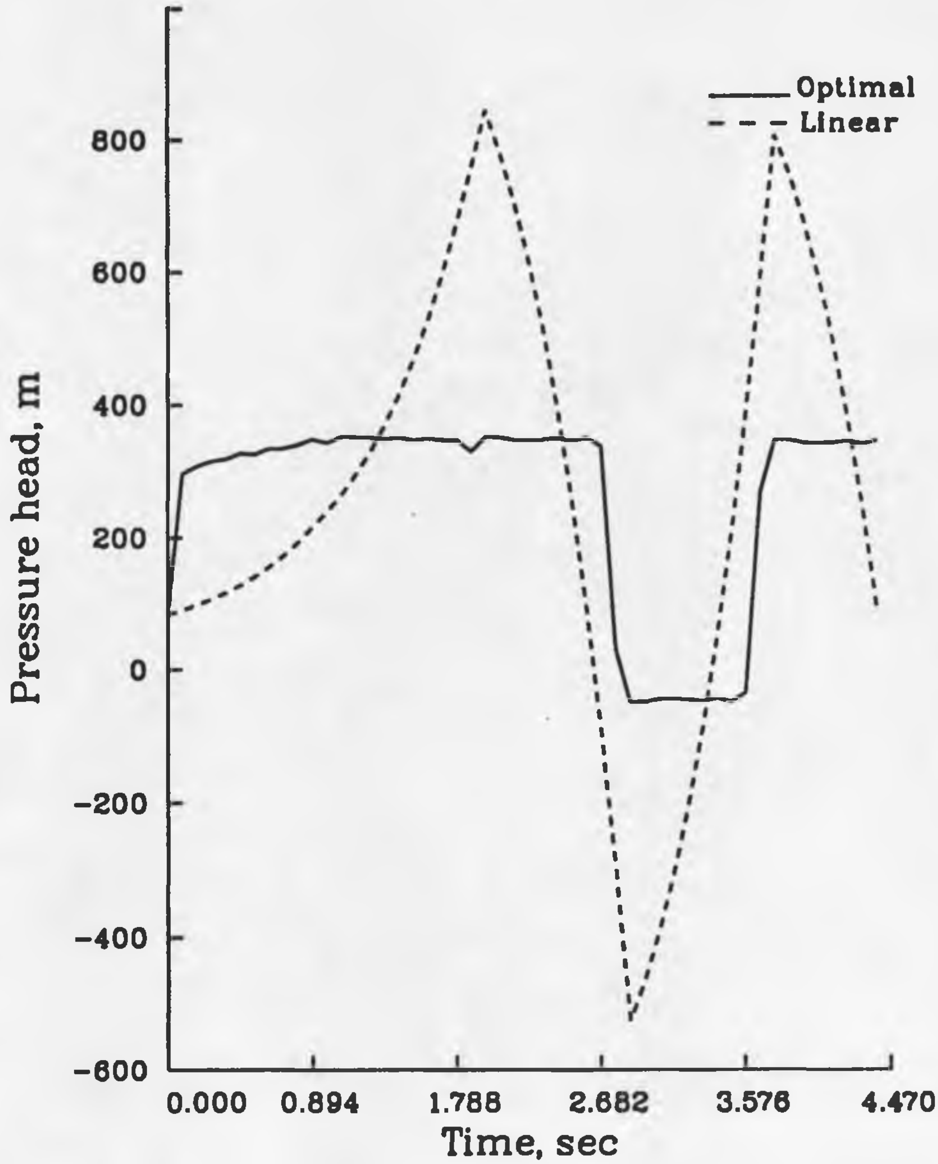

Figure 6.14 –Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec, (4.4L/a), case II.

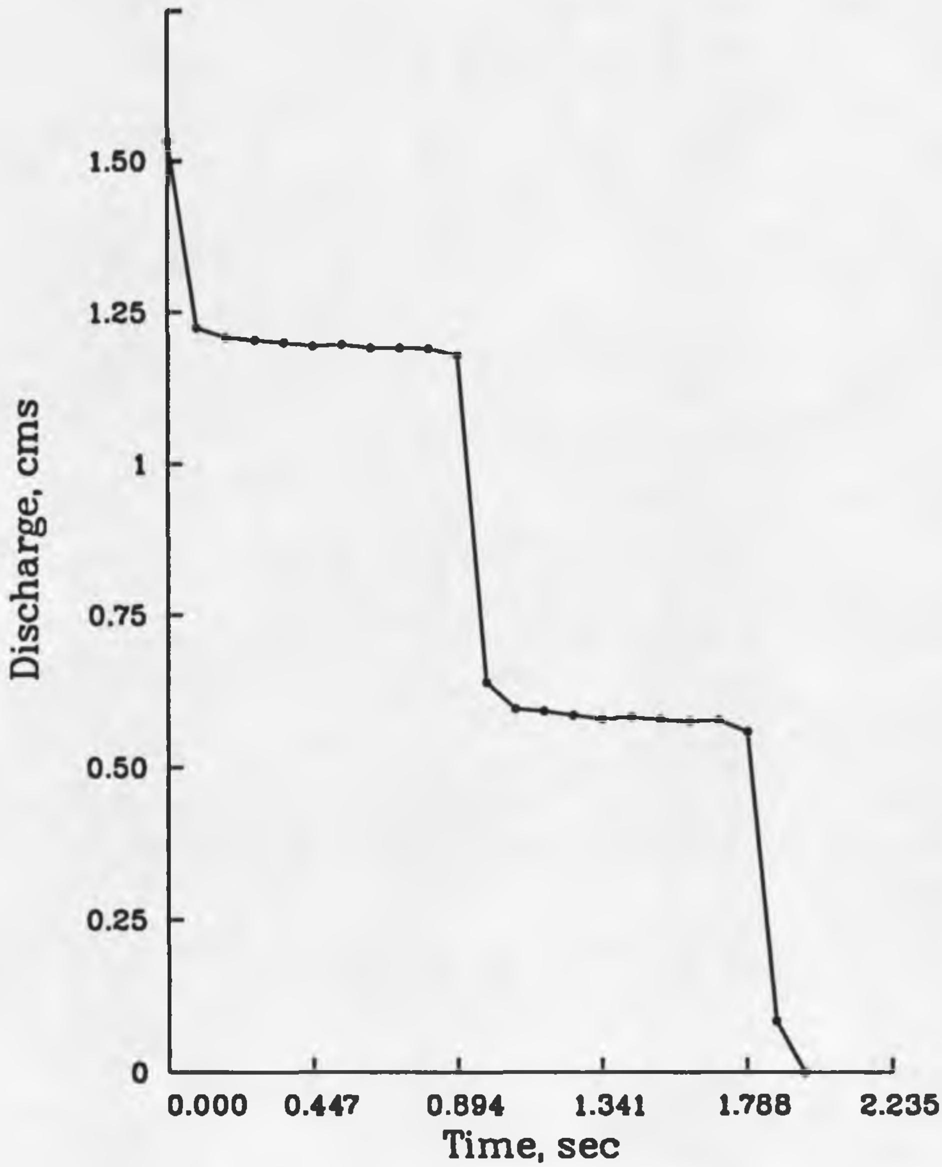

Figure 6.15 –Discharge variation at the valve for optimal valve closure, tc =1.968 sec, (4.4L/a), case II.

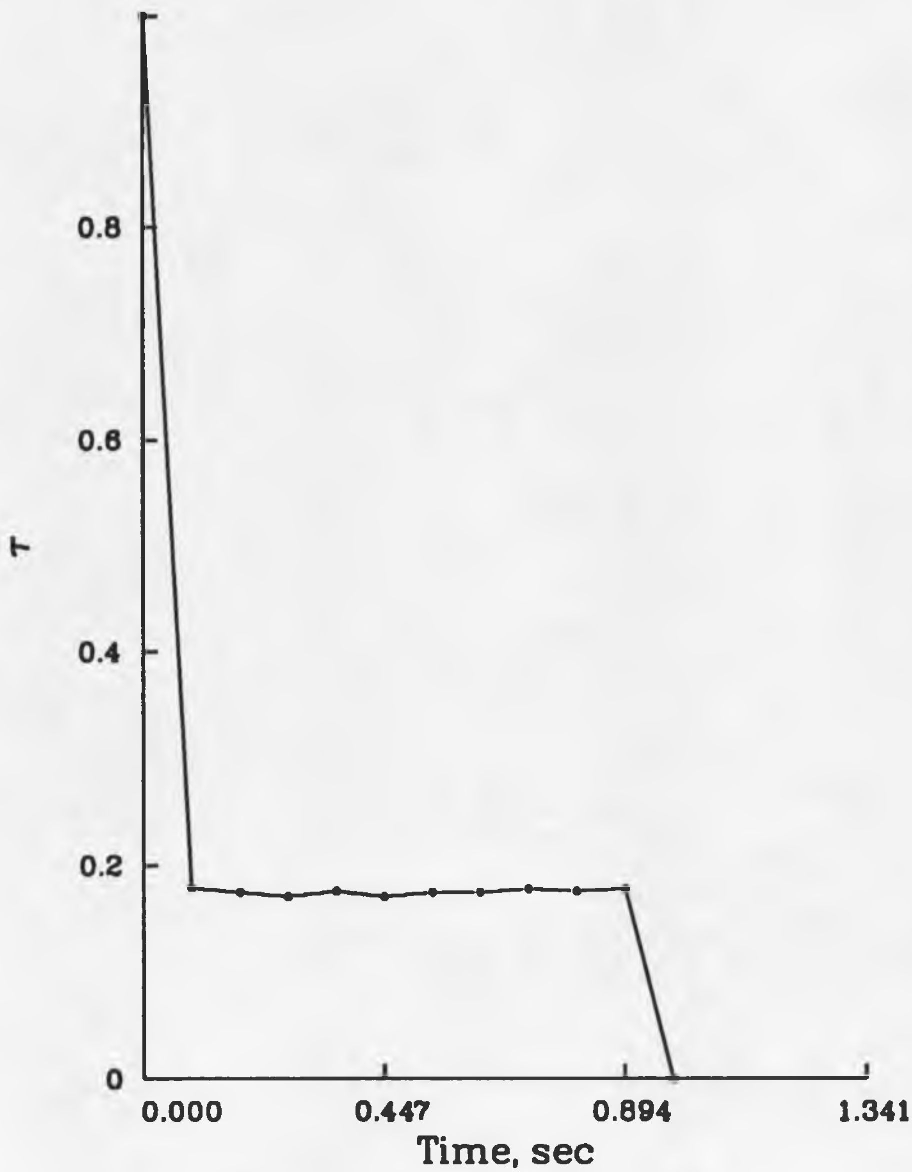

Figure 6.16 –Optimal valve closure policy, tc =.984 sec, (2.2L/a), case III.

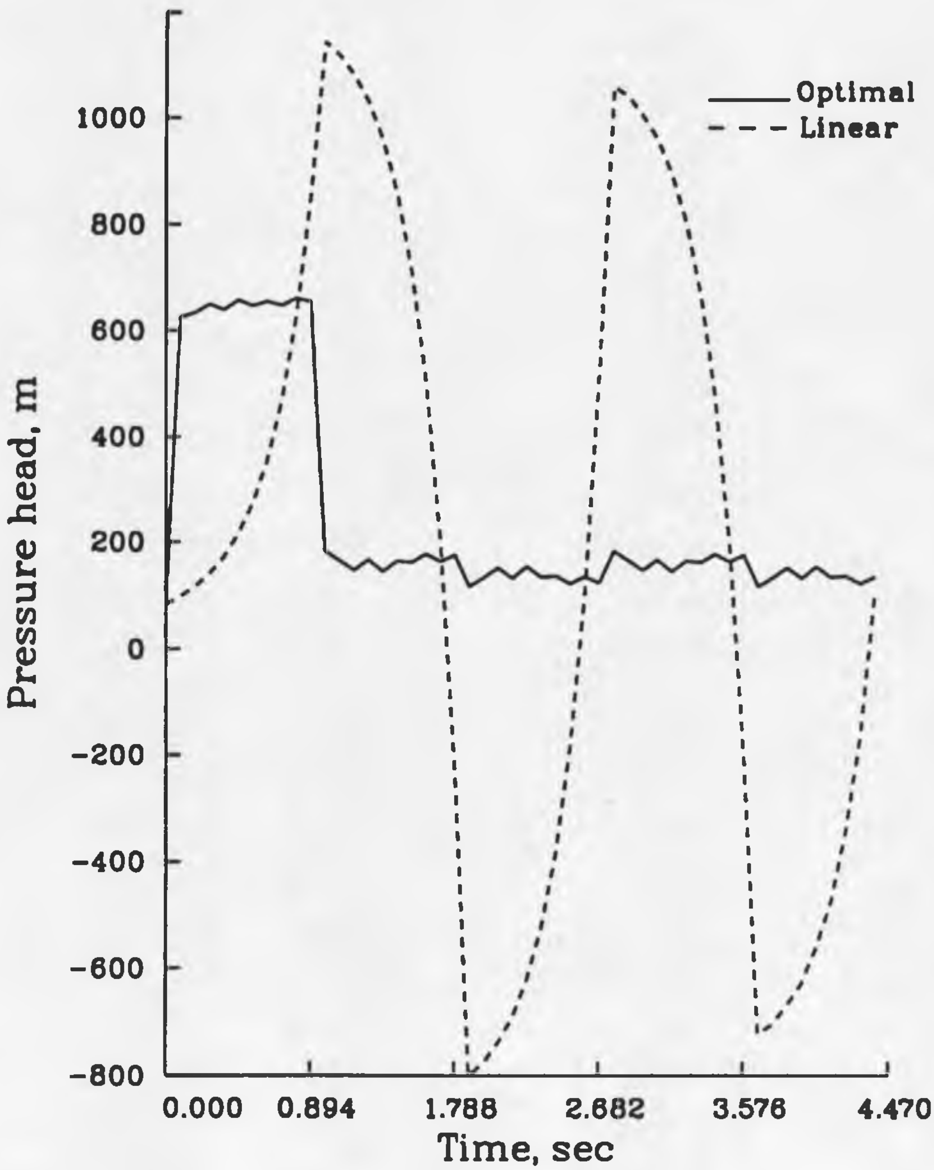

Figure 6.17 –Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec, (2.2L/a), case III.

pressure in Figure 6.11. Figure 6.18 shows the discharge variation at the valve. Figures 6.19 and 6.20 show the valve closure for tc =4.4L/a sec and pressure variations for linear and optimal valve closures, respectively. The maximum pressure in Figure 6.20 is much greater than the maximum pressure in Figure. 6.14. Discharge variation is shown in Figure 6.21. The maximum and minimum pressures for linear and optimum valve closures are plotted versus valve closure time in Figure 6.22. Figure 6.22 shows that optimal closure always raises the minimum pressure to the steady state pressure at the valve, i.e. 82.9 m, and it is shown as a straight line. The pattern of change of maximum pressure in Figure 6.22 is strange. As the results are undesirable the details are not discussed further.

6-4 Case IV

The same data are used in this case as in case II and case III, but the objective function is different. In case II the objective function used for optimization is minimization of the maximum pressure irrespective of any value of the minimum pressure (which, most of the time, was below -10 m, the vapor pressure head). In case IV the objective function is specified as the minimization of the difference of maximum and minimum pressures with the constraint that minimum pressure should be equal to or greater than some limiting value, e.g. vapor pressure at -10 m.

When the simplex tries to minimize the value of the objective function, in the absence of some limiting value of the minimum pressure, the simplex gives very high values of maximum pressure and minimum presssure i.e. steady state pressure at the valve, because at these values their difference is minimum. By introducing the limiting value of minimum pressure the simplex tries to minimize the difference between maximum and minimum pressures when the minimum pressure is less than the limiting

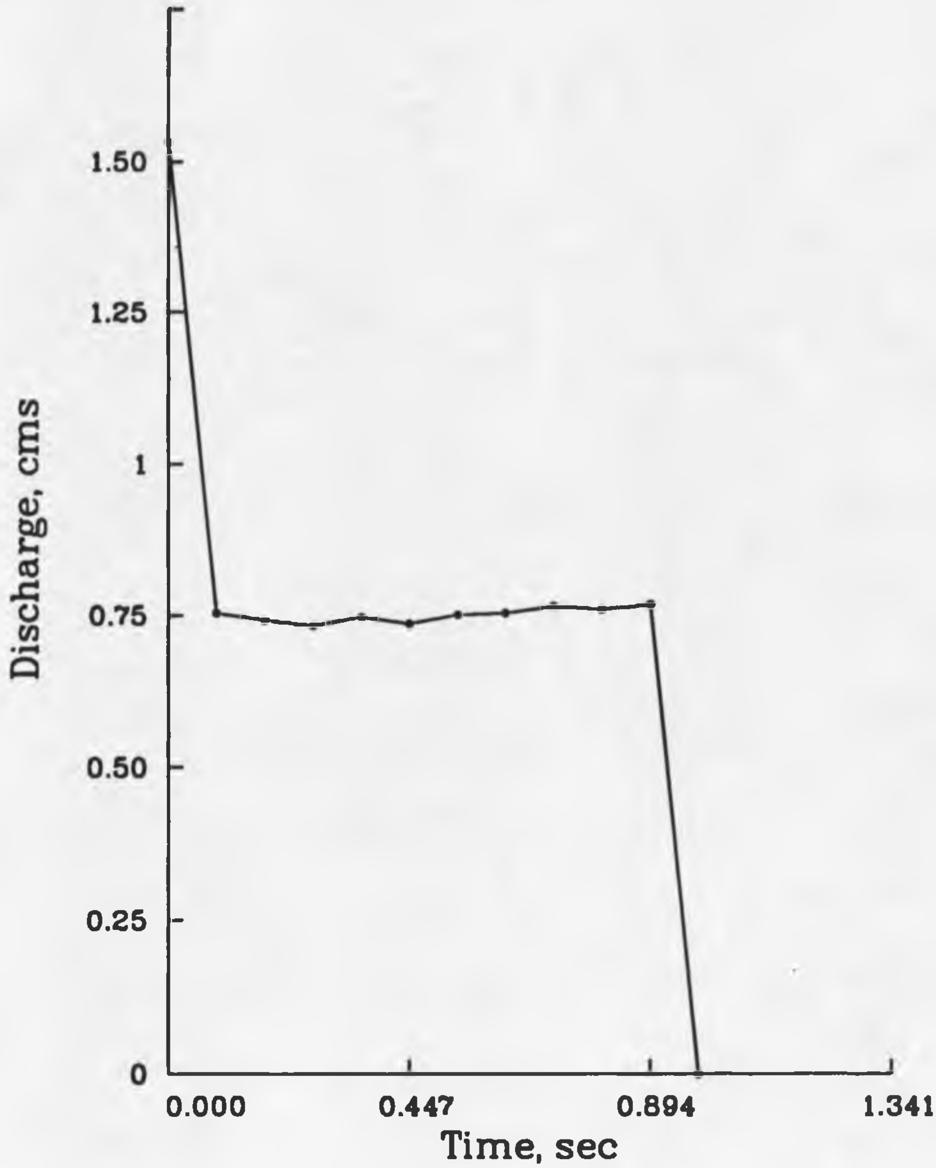

Figure 6.18 –Discharge variation at the valve for optimal valve closure, tc =.984 sec, (2.2L/a), case III.

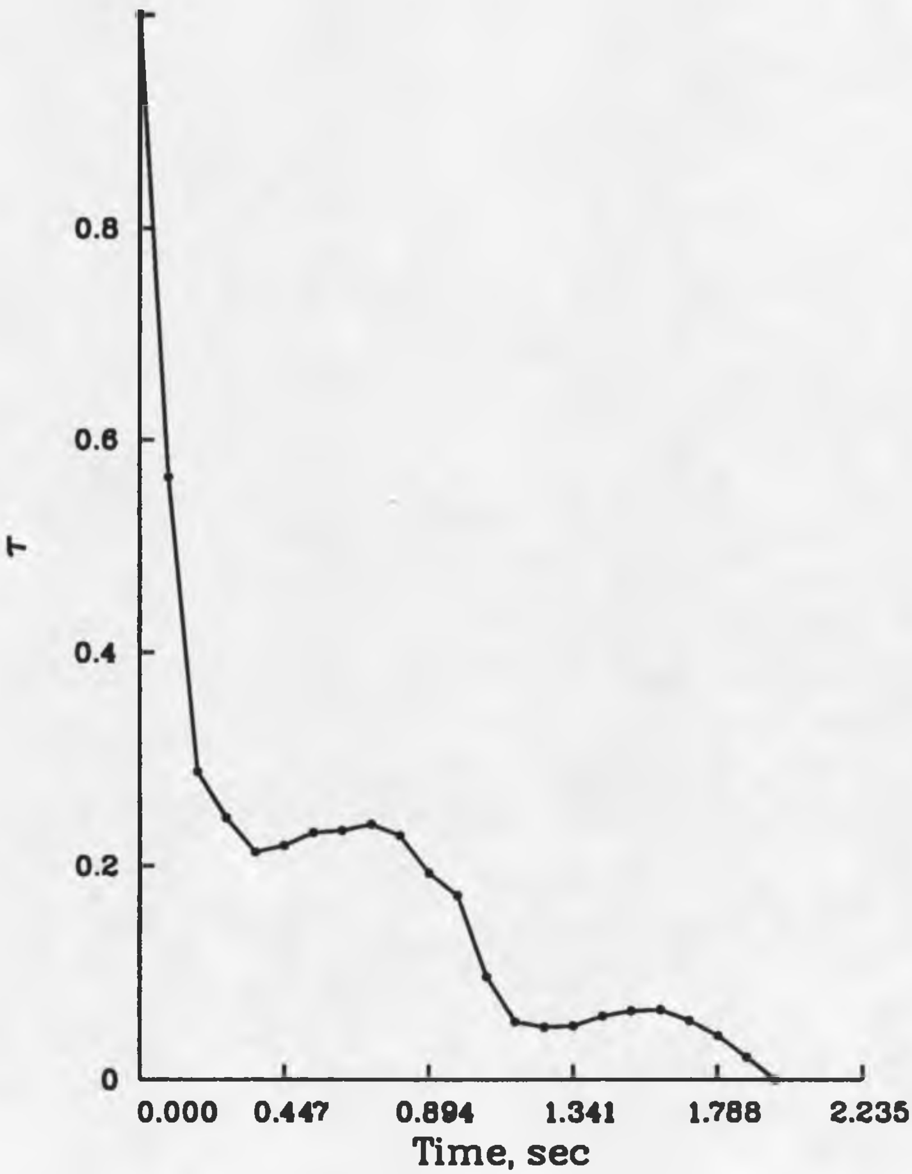

Figure 6.19 –Optimal valve closure policy, tc =1.968 sec, (4.4L/a), case III.

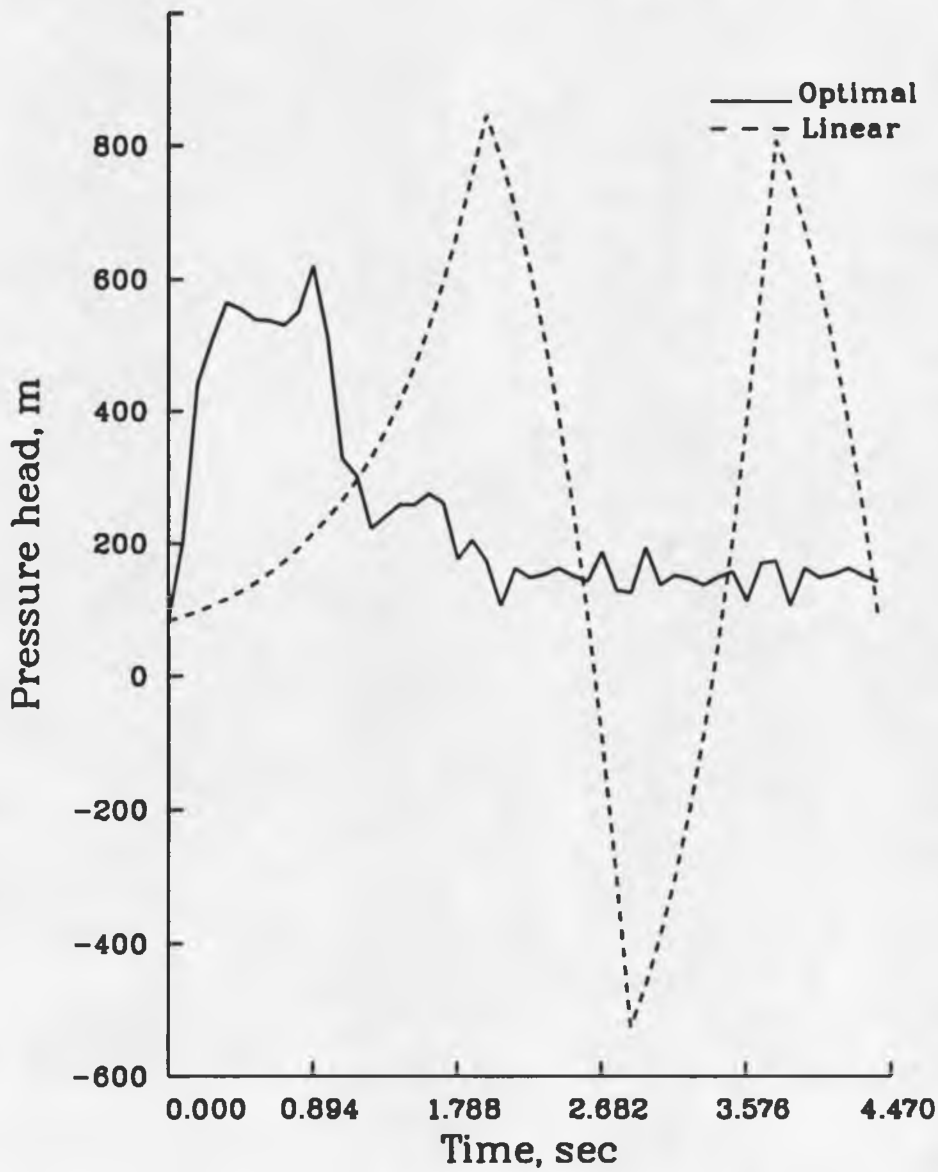

Figure 6.20 –Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec, (4.4L/a), case III.

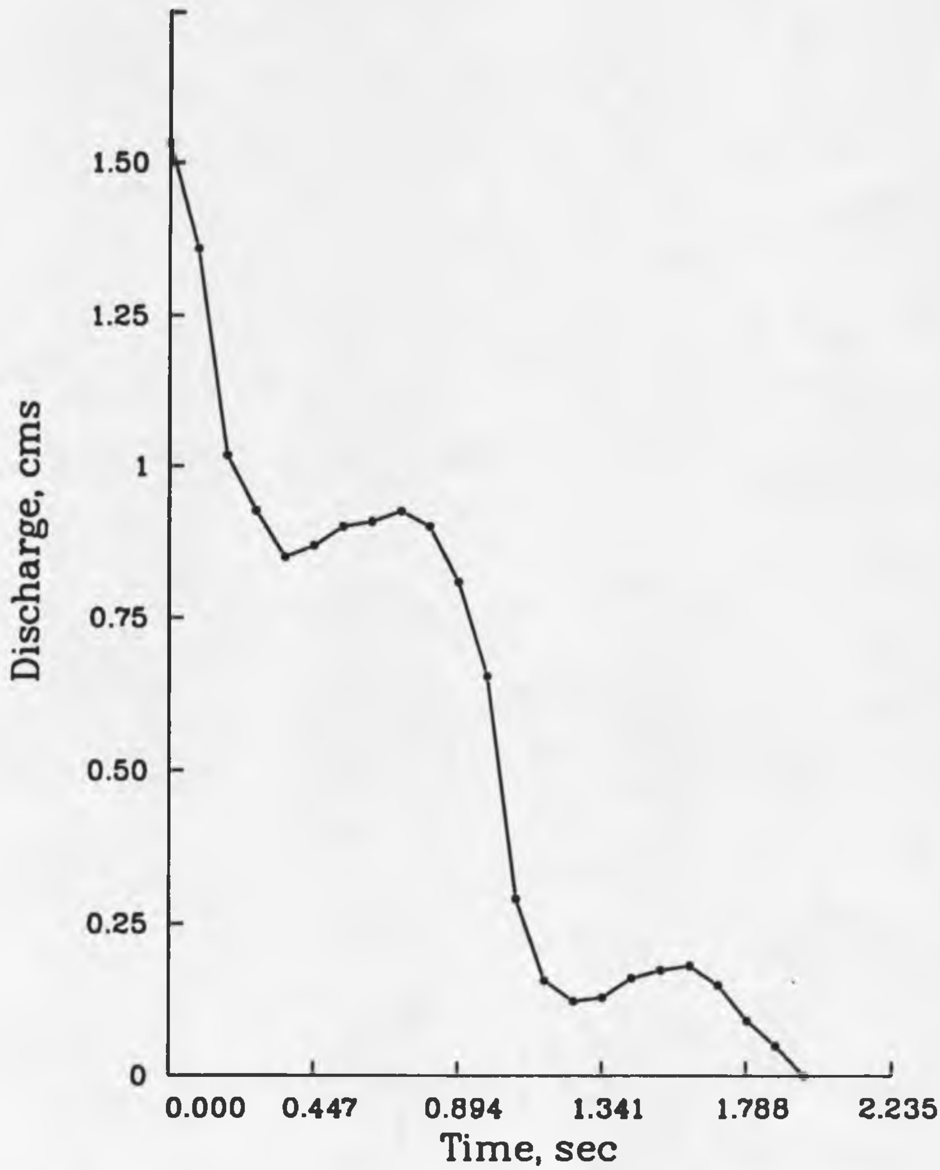

Figure 6.21 –Discharge variation at the valve for optimal valve closure, tc =1.968 sec, (4.4L/a), case III.

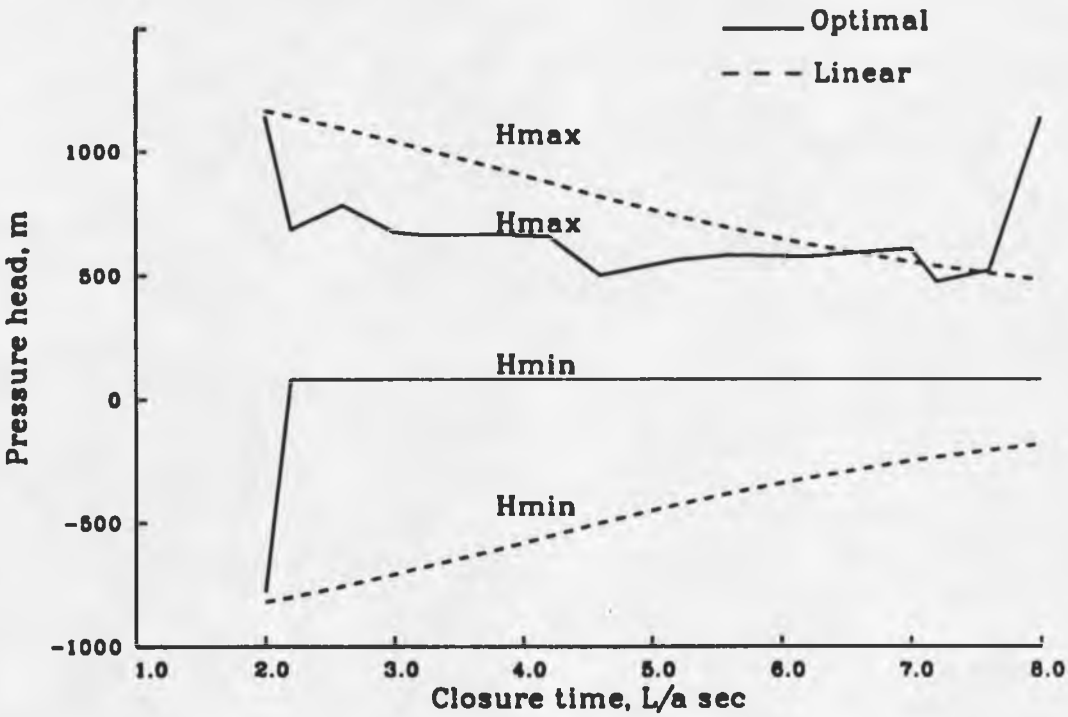

Figure 6.22 –Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case III.

value. When the actual minimum pressure is equal to or greater than the limiting value, the simplex tries to minimize the difference of the maximum pressure and the limiting value of minimum pressure. By this technique the simplex first tries to increase the minimum pressure to some limiting value, then it tries to minimize the maximum pressure while keeping the minimum pressure equal to or greater than the limiting value.

Figure 6.23 shows the variation of maximum and minimum pressures at the valve for linear and optimal valve closures. In case II the minimum pressure was less than the vapor pressure (-10. m) until the time of closure was 6.4L/a sec (shown in Figure 6.8). Figure 6.23 shows that optimization increases the minimum pressure to -10. m (limiting value) when the minimum pressure was less than the limiting value (Figure 6.8). But when the minimum pressure is greater than the limiting value (-10 m), this case acts like case II.

The program was executed for different times of closure, and a graph of percent reduction in maximum pressure as a function of valve closure time is shown in Figure 6.24. This figure shows the maximum pressure reduction of 63 % occurs at about 4.4L/a sec, and the second highest peak of pressure reduction of 57 % occurs at about 6.6L/a sec. The following problems are studied in detail for time of closures of 2.2L/a and 4.4L/a sec.

<u>Problem #1</u>

The optimum valve closure for tc =2.2L/a sec shown in Figure 6.25 is a one-step valve closure. The valve is closed down to τ=0.2, as compared to case II where the valve was closed to τ=0.28. The pressure variation at the valve for linear and optimum valve closures, given in Figure 6.26, indicates the maximum pressure is

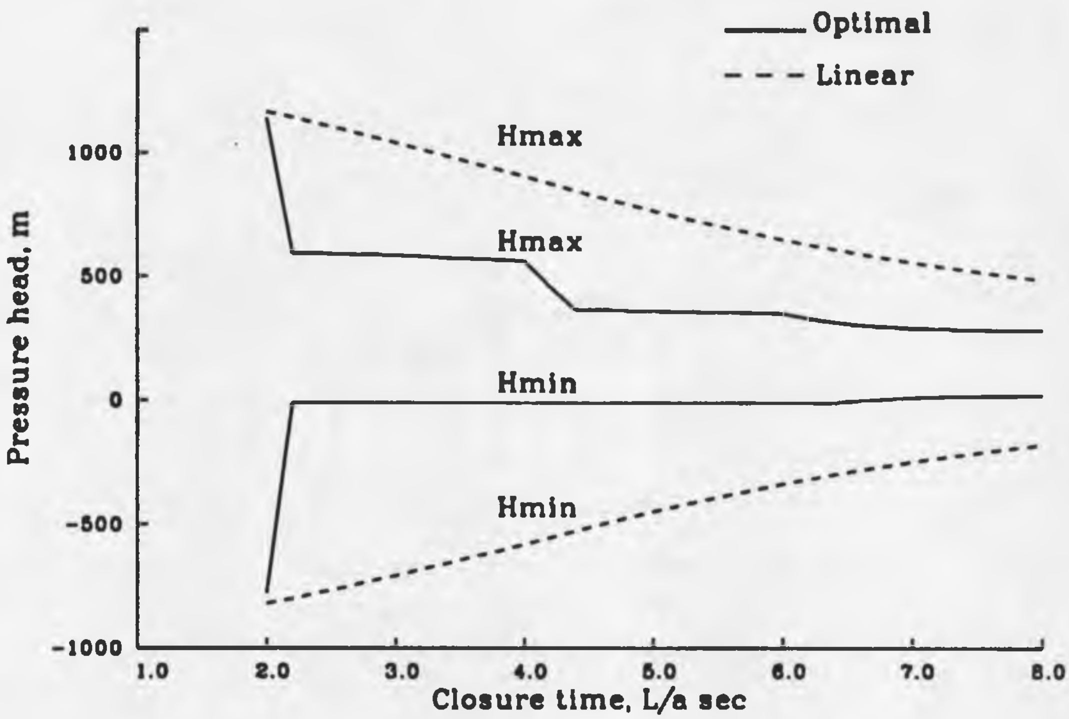

Figure 6.23 –Maximum and minimum pressures for linear and optimal valve closures as a function of valve closure time, case IV.

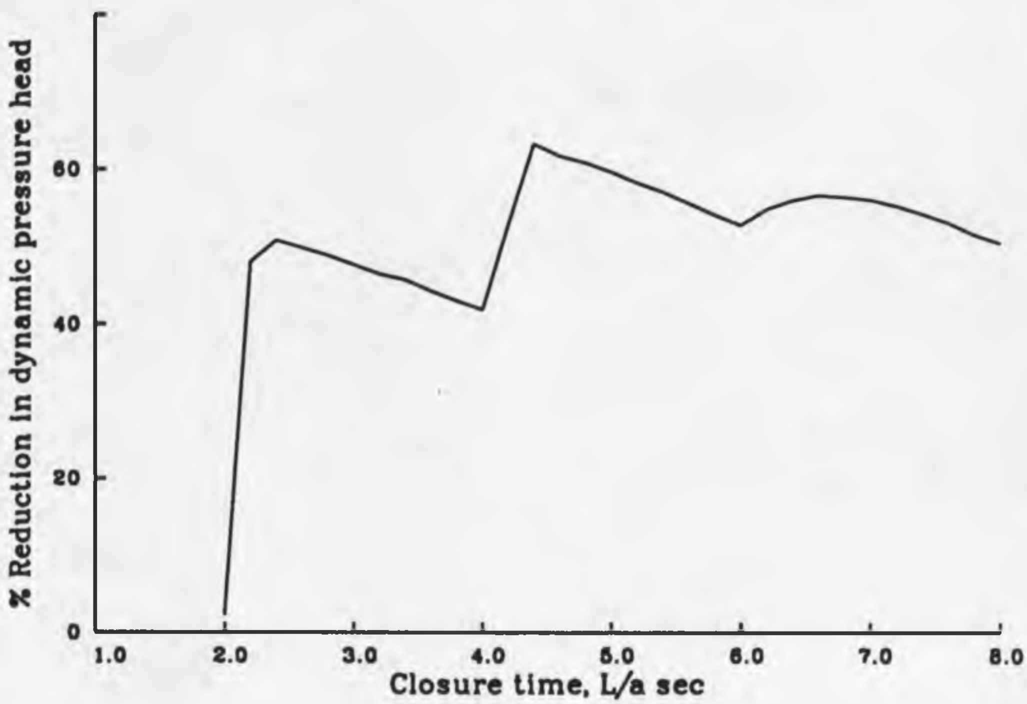

Figure 6.24 –Percentage reduction in dynamic pressure head as a function of valve closure time, case IV.

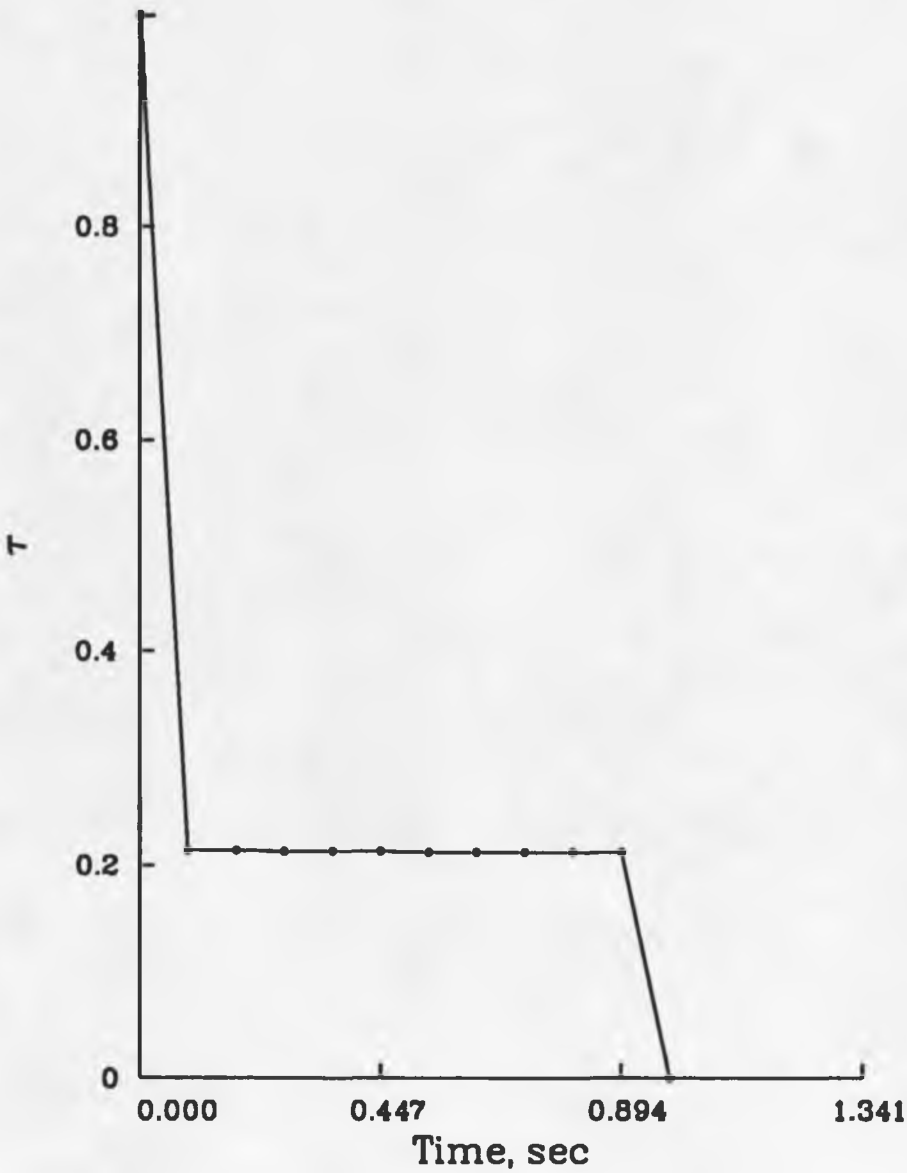

Figure 6.25 –Optimal valve closure policy, tc =.984 sec, (2.2L/a), case IV.

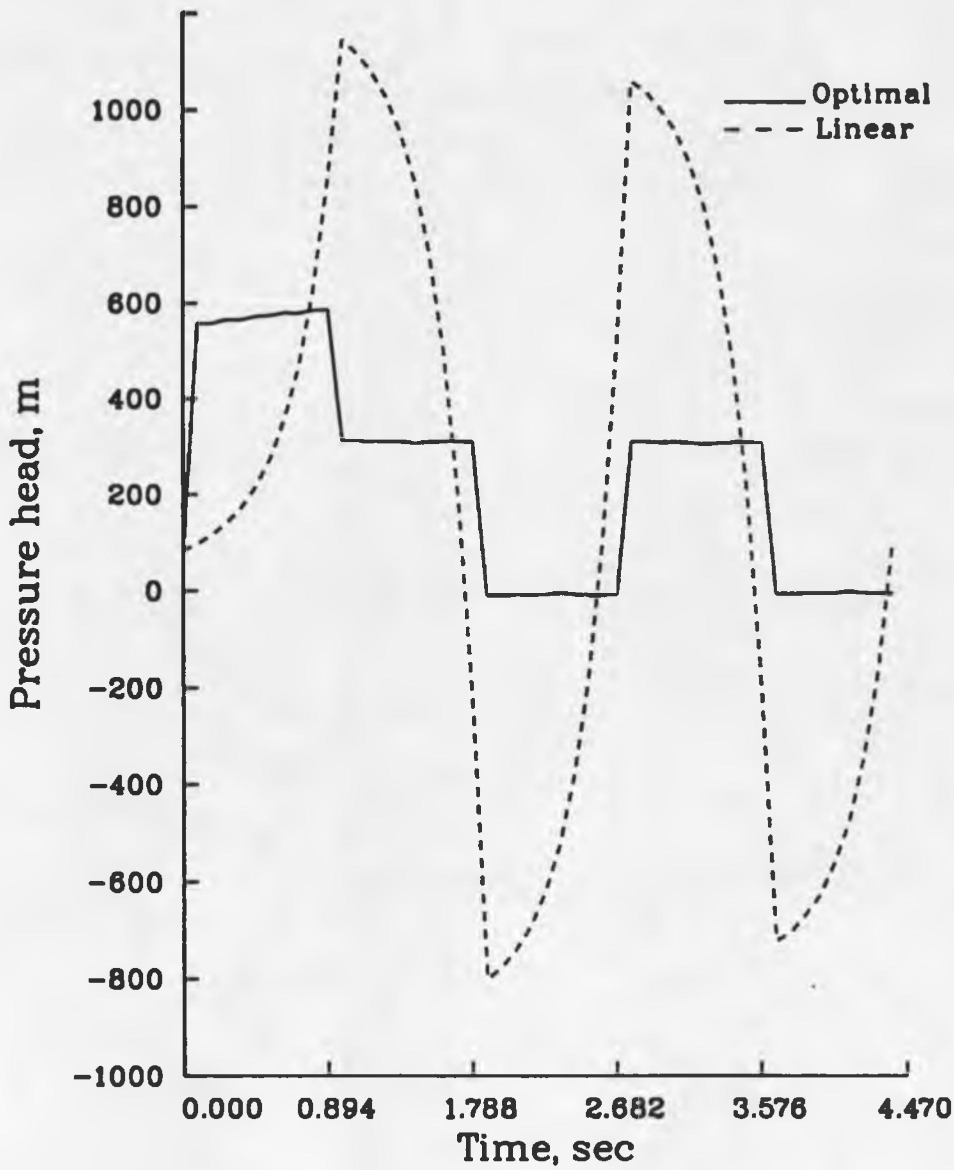

Figure 6.26 –Pressure variation at the valve for optimal and linear valve closures, tc =.984 sec, (2.2L/a), case IV.

reduced from 1144 m to 586 m and the minimum pressure raised from -800 m to -10 m.

The steady state pressure at the valve = 82.92 m

For linear valve closure

The maximum pressure at the valve = 1144.50 m

The minimum pressure at the valve = -799.89 m

For optimal valve closure

The maximum pressure at the valve = 593.97 m

The minimum pressure at the valve = -9.99 m

$$\text{Percentage reduction in dynamic pressure (Hmax)} = \frac{1144.50-593.97}{1144.50-\ \ 82.92} = 51.86\ \%$$

$$\text{Percentage increase in dynamic pressure (Hmin)} = \frac{-799.89-(-9.99)}{-799.89-\ \ (82.92)} = 89.4\ \%$$

The computation shows that in case IV the percent reduction in maximum pressure is about 9.11 % less than in case II, but the percent increase in minimum pressure in case IV is about 20.32 % greater than in case II.

Figure 6.27 shows that the flow at τ= 0.20 is 55 % of the steady state flow.

Problem #2

Time of closure used in this problem is 4.4L/a sec, and the optimal valve closure policy is shown in Figure 6.28. Valve closure policy in case IV differs from that in case II is that at the end of the first step in case IV, the valve is slightly opened from τ= 0.36 to τ= 0.42 for a short time. Details of valve closure policies in case II and case IV are given in Appendices B and C, respectively. The pressure variation for linear and optimal valve closures, given in Figure 6.29, shows that maximum pressure

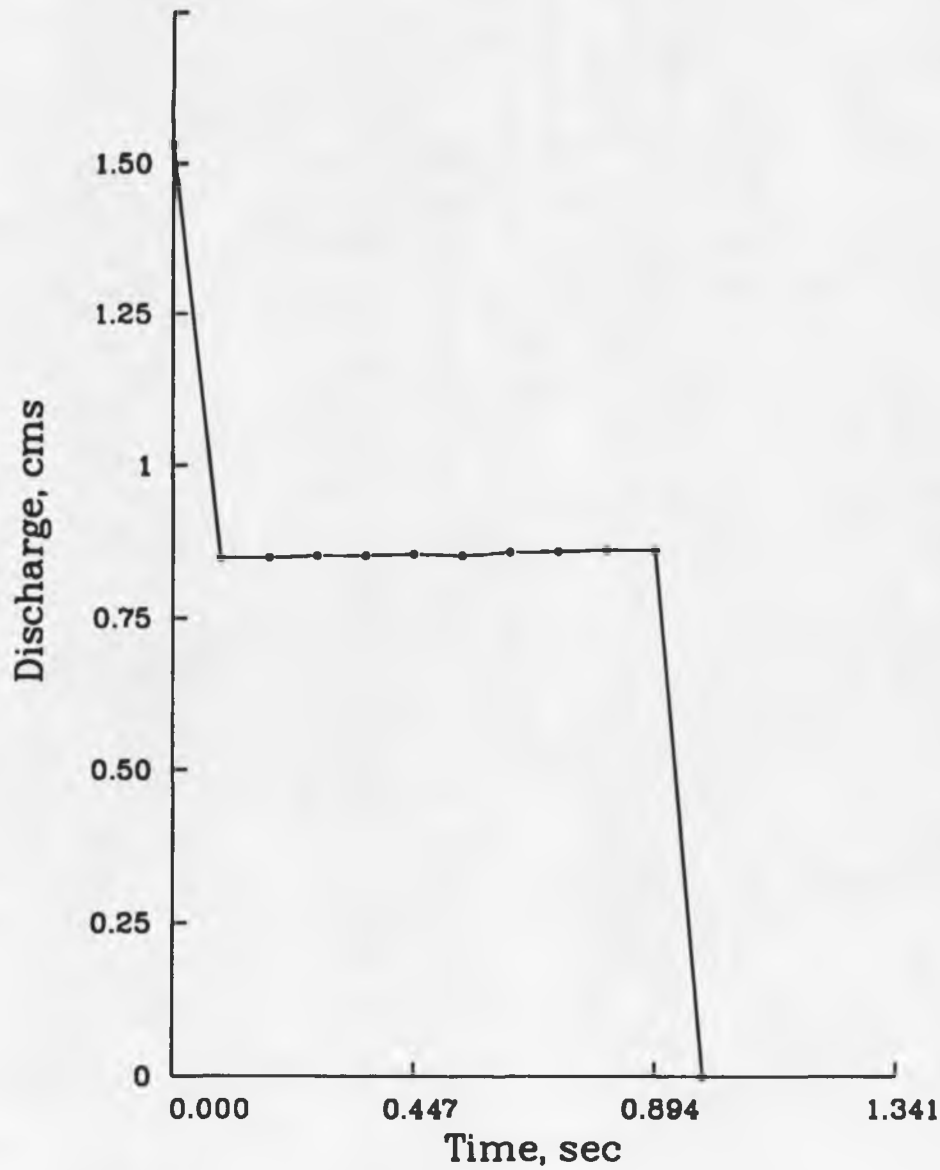

Figure 6.27 –Discharge variation at the valve for optimal valve closure, tc =.984 sec, (2.2L/a), case IV.

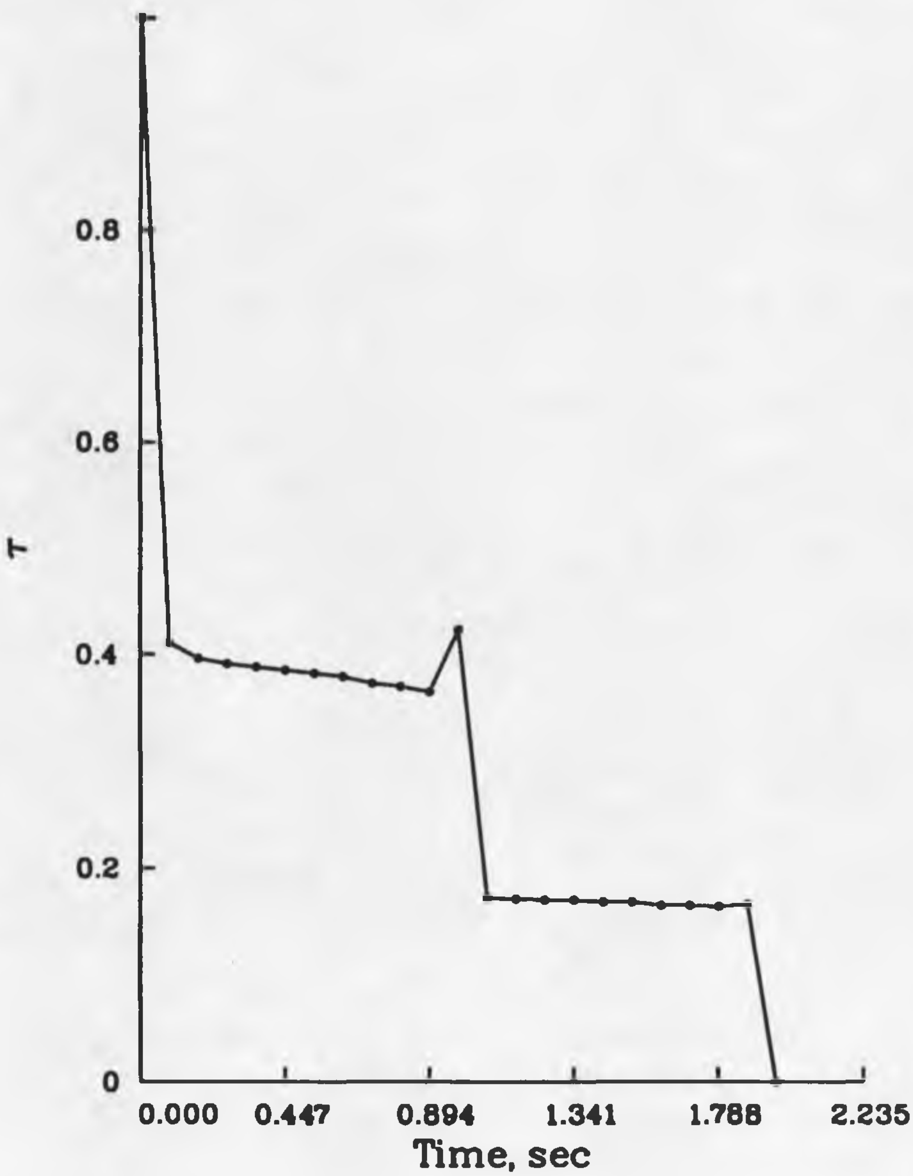

Figure 6.28 –Optimal valve closure policy, tc =1.968 sec, (4.4L/a), case IV.

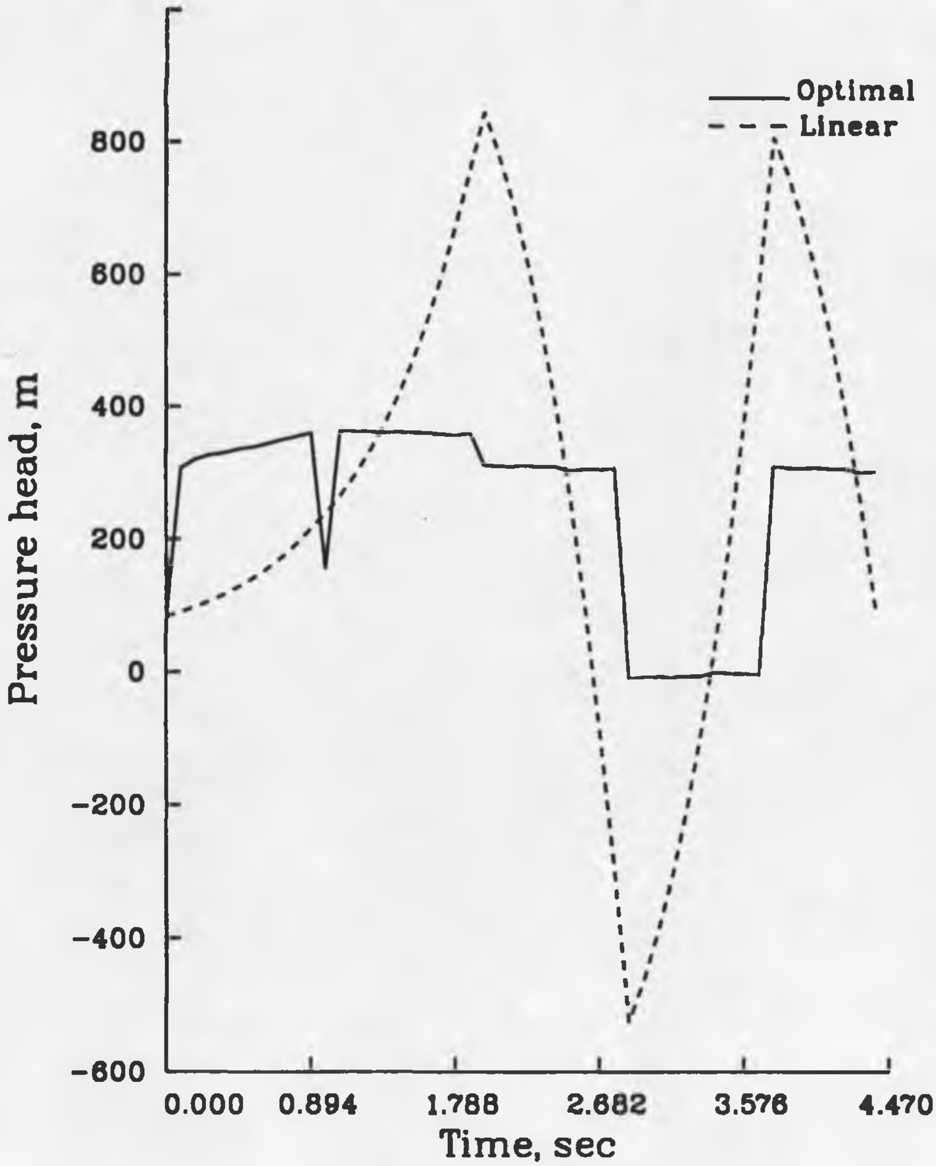

Figure 6.29 —Pressure variation at the valve for optimal and linear valve closures, tc =1.968 sec, (4.4L/a), case IV.

decreases from 845 m to 363 m and minimum pressure increases from -525 m to -10. m.

The steady state pressure at the valve	=	82.92	m
For linear valve closure			
The maximum pressure at the valve	=	845.24	m
The minimum pressure at the valve	=	-525.51	m
For optimum valve closure			
The maximum pressure at the valve	=	362.97	m
The minimum pressure at the valve	=	-10.00	m

$$\text{Percentage reduction in dynamic pressure(Hmax)} = \frac{845.24-362.97}{845.24-\ 82.92} = 63.26\ \%$$

$$\text{Percentage increase in dynamic pressure (Hmin)} = \frac{-525.51-(-10.34)}{-525.51\ -\ (82.92)} = 84.26\ \%$$

The above computations show that percentage reduction in maximum pressure is about 1.45 % less in case IV than in case II, but that the increase in minimum pressure is about 6.45 % more in case IV than in case II.

Figure 6.30 shows the variation in flow through the valve during valve closure time.

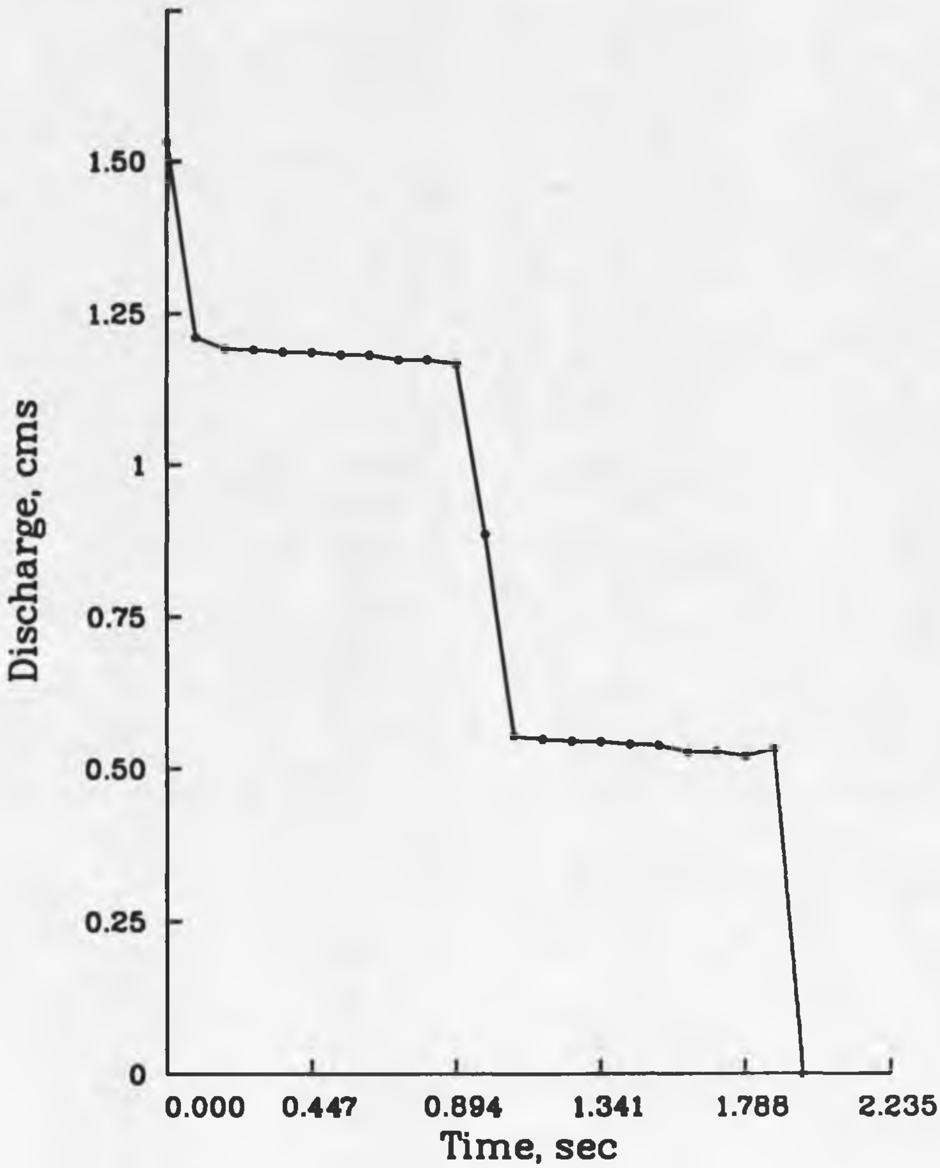

Figure 6.30 –Discharge variation at the valve for optimal valve closure, tc =1.968 sec, (4.4L/a), case IV.

CHAPTER 7

DISCUSSION AND CONCLUSION

The change in pressure head during transient flow depends largely on the steady state velocity of flow; the higher the velocity, the greater the fluctuation in pressure head. When the maximum pressure is minimized, the minimum pressure also increases, as shown in Figure 6.8 (case II). Sometimes the steady state velocity in the pipe is such that when minimizing the maximum pressure, the minimum pressure is higher than the vapor pressure of the fluid and in such situations there is no concern about column separation. At other times, the velocity through the pipe is high enough so that the minimum peressure rises when minimizing the maximum pressure but it is still lower than the limiting value (i.e vapor pressure). This will cause column separation, as shown in Figure 6.8 (case II). Column separation is undesirable and should be avoided.

There are several ways to avoid column separation. One way is to maximize minimum pressure, but in this case the maximum pressure increases to very high values which is not desirable. The straight line in Figure 6.22, shows the upper limit of minimum pressure, that is the steady state pressure at the valve.

A second way is to minimize the difference between maximum and minimum pressures; this case gives higher values of both maximum and minimum pressures. Thus, while column separation is avoided by this technique the maximum pressure rises to very large values.

The purpose of this study was to avoid column separation while keeping the maximum pressure to a minimium. After studying all the cases, it was realized that optimization should be carried out in two phases. In the first phase, minimize the difference between the maximum and minimum pressures until the minimum pressure is equal to the some limiting value (e.g. vapor pressure of the fluid). After the minimum pressure reaches the limit, a second phase of optimization should minimize the maximum pressure. The first phase is achieved by minimizing the objective function {Hmax-Hmin}, when Hmin is < the limiting value (Hv). When Hmin rises to the limiting value (Hv) the second phase minimizes {Hmax-Hv}. During the second phase of minimization, Hmin may be $\geq$ Hv. This objective function is used in case IV. Figure 6.23 shows that, when compared with case II in Figure 6.8, this technique increases the minimum pressure to the limiting value and at the same time also increases the maximum pressure. But the increase in maximum pressure is less than the increase in the minimum pressure as is clear from the computations in problems #1 and #2 in case IV.

There are many settings of valve closure, with small differences, which give the same maximum and minimum pressures. Sometimes we get a wavy pattern of optimal valve closure which is difficult to apply, practically. When a smaller number of variables are used for the simplex, the natural cubic spline function gives fluctuations in the closure policy. Most of the time this can be handled by using the number of variables equal to the number of time steps in the time of valve closure, as is clear from valve closure policies discussed in chapter 6. But, sometimes, it is not controlled by this technique. Then, for practical purposes, the mean line passing through the wavy line should be used and checked for the transient pressures produced.

For future work, more constraints should be given to the τ values so that the optimal valve closure policy will be practicable. In low resistance flows, a constraint can be given so that τ value for the next time step should be equal to or less than the τ value at the previous time step. But for optimal valve closure at high friction flows, sometimes, the valve must be opened slightly after closing to reduce the maximum pressure as shown in Figure 6.28.

APPENDIX A

LISTING OF THE COMPUTER PROGRAM

This program is written in Fortran language and computes transient pressures and dischareges at six equidistant locations along the pipe. By default it optimizes the difference between maximum and minimum pressures when the limiting minimum pressure is given. The objective function can be changed by changing the lines of statement number 99 in subroutines HAMMER and HAMMER1 which are (P(I)=HHP-HLP) and (PSTR=HHP-HLP), respectively. For minimizing the maximum pressure use PSTR and P(I) equal to HHP. In this case the value of the limiting pressure will not affect the objective function even it is given as an input. For maximizing the minimum pressure use P(I) and PSTR equal to -HLP in the lines of statement nuber 99 in subroutines HAMMER and HAMMER1 and give the value of limiting pressure equal to very high say 100000 because it helps to sort out the minimum pressure (program default).

During execution of the program, the highest, the second highest, and the lowest values of the objective function are shown on the console. When the difference between the highest and lowest values becomes equal to or less than .005 the execution stops and asks for another value of side (should be less than .5) for better search or to prepare the output file. The value of side should be continuously given till the end results do not vary much.

```
*****************************************************************
*****************************************************************

**                    PROGRAM  WHM  V 1.0                      **
**           WRITTEN BY FAIQ HUSSAIN PASHA,  1989              **
**                 UNIVERSITY OF ARIZONA                       **
**              CIVIL ENGINEERING DEPARTMENT                    **
**                 TUCSON ARIZONA 85721                        **

**       TO FIND THE OPTIMUM VALVE CLOSURE THAT MINIMIZE       **
**      THE MAX PRESSURE AND AVOID THE COLUMN SEPERATION       **
*****************************************************************

************************************************************************
*       THE PROGRAM IS SUBDIVIDED INTO THE FOLLOWIN SECTIONS           *
*                                                                      *
*       1.WHM          MAIN PROGRAM  CONTROL SUBROUTINES,GENERATE      *
*                                    INITIAL TAU CURVES                *
*       2.INPUT        SUB           INPUT DATA                        *
*       3.SPLINE1      SUB           FOR INTERPOLATION                 *
*       4.HAMMER       SUB           COMPUTE WATERHAMMER               *
*                                    BY STREETER & WYILE               *
*       5.SORT         SUB           SORTING OBJECTIVE FUNCTION        *
*       6.OPTIMIZE     SUB           USING SIMPLEX BY NELDER & MEAD
*
*       7.SPLINE2      SUB           FOR INTERPOLATION                 *
*       8.HAMMER1      SUB           COMPUT WATERHAMMER                *
*       9.HAMMER2      SUB           COMPUTE & WRITE DOWN WATERHAMMER*
************************************************************************

**********************************************************************
*                 CONSTANTS AND VARIABLES                            *
*                                                                    *
*       NUM       - NO. OF EQUAL PIPE SEGMENTS                       *
*       SIDE      - ALLOCATE SEARCH AREA FOR SIMPLEX                 *
*       ALPHA     - REFLECTION CO-EFFICIENT FOR SIMPLEX              *
*       BETTA     - CONTRACTION CO-EFFICIENT FOR SIMPLEX             *
*       GAMMA     - EXPANSION CO-EFFICIENT FOR SIMPLEX               *
*       NO        - ROW DIMENSION OF MATRIX                          *
*       NOO       - COLUMN DIMENSION OF MATRIX                       *
*       DT        - TIME STEP                                        *
*       TC        - TIME OF CLOSURE OF VALVE                         *
*       N         - NO. OF STEPS IN TC                               *
*       NOS       - NO. OF N-DIMENSION IN TAU-TIME SPACE             *
```

```
*       PP         - POINT IN TAU-TIME SPACE USED FOR OPTIMIZATION  *
*********************************************************************

        REAL TAU(30,50),P(30),PL,PH,PM,TAUSTR(50),TAUDSTR(50),
     *  PP(30,30),DN,PPDSTR(50)

        INTEGER J,K,L,M,N,KM,KL,KH,NOO,NO,NOS

        COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP

        CALL INPUT (NOS)

        IPR         =1
        NUM         =5
        SIDE        =.5
        ALPHA       =1.
        BETTA       =.5
        GAMMA       =2.

        DT   =ALENGTH/(NUM*C)
        N    =INT(TC/DT)+1
        N    =N+1
        NO   =30
        NOO  =50

C NO. OF POINTS USING IN SIMPLEX SHOULD BE =OR<TAU POINTS
        DN   =TC/(NOS+1)
        IF (DN.LT.DT) NOS=TC/DT

C INITIAL CURVES ARE GENERATING
        PP(1,1) =1.0
        PP(1,NOS+2)=0.0

        DO JJ=2,NOS+1
        PP(1,JJ)=.0
        END DO

C CUE1 AND CUE2 ALLOCATE THE SEARCH AREA FOR THE SIMPLEX.

  13    CUE1 =SIDE*((NOS+1)**.5+NOS-1)/(NOS*2**.5)
        CUE2 =SIDE*((NOS+1)**.5-1)/(NOS*2**.5)

        DO II =2,NOS+1
```

```
            PP(II,1)=1.0
            PP(II,NOS+2)=0.0
            DO JJ = 2,NOS+1
            IF(II.EQ.JJ) THEN
            PP(II,JJ)=PP(1,JJ)+CUE1
            ELSE
            PP(II,JJ)=PP(1,JJ)+CUE2
            END IF
            END DO
      END DO

C TAU POINTS ON THE INITIAL CURVES ARE FOUND OUT USING THE SPLINE
C SUBROUTINE

      DO INI =1,NOS+1
      CALL SPLINE1(DN,DT,N,NO,NOO,PP,TAU,INI,NOS)
      END DO

C HAMMER SUBROUTINE IS CALLED TO FIND OUT THE WATERHAMMER TRANSIENT

      DO I=1,NOS+1
      CALL HAMMER(TAU,NO,NOO,N,DT,I,P)
      END DO

C TAU CURVES ARE SORTED OUT WHICH GIVE THE HIGHEST, SECOND HIGHET
C AND THE LOWEST FUNCTION VALUES.
 17   CALL SORT(P,NO,NOO,N,KH,KM,KL,PH,PM,PL,NOS)
      PRINT *,PH,PM,PL


      CALL OPTIMIZE(N,NO,NOO,KH,KM,KL,PP,ALPHA,DT,PL,PH,
     * PM,P,GAMMA,BETTA,TAUSTR,TAUDSTR,DN,PPDSTR,NOS)


      IF (PH-PL.LE..005) GO TO 18
      GO TO 17


 18   PP(1,1) =PPDSTR(1)

      DO JJ =2,NOS+2
      PP(1,JJ)=PPDSTR(JJ)
      END DO
      WRITE (*,*) '    Hmax              Hmid              Hmin '
```

```
      WRITE (*,*) 'GIVE EITHER THE REDUCED VALUE OF SIDE (<.5) OR  ZERO'

C  SIDE VALUE ALLOCATE THE SEARCH AREA FOR OPTIMUM VALUE. KEEP ON REDUCING
C  THE SEARCH AREA FOR BETTER RESULTS UNTIL THERE IS NO CHANGE IN THE
C  OPTIMUM VALUE.

      READ (*,*) SIDE
      IF (SIDE.EQ.0) GOTO 19
      GOTO 13

C   OUTPUT FILE IS PREPARING

 19   CALL HAMMER1(TAUSTR,NOO,N,DT,P1)
      CALL HAMMER1(TAUDSTR,NOO,N,DT,P2)
      IF (P1.GE.P2) THEN
      CALL HAMMER2(TAUDSTR,NOO,N,DT)
      ELSE
      CALL HAMMER2(TAUSTR,NOO,N,DT)
      END IF
      STOP
      END


C ******************** SUBROUTINE INPUT *********************************
C    THIS SUBROUTINE IS USED TO PREPARE INPUT DATA FOR THE PROGRAM.
C       EACH TIME IT REQUIRES NEW DATA SET.
C***********************************************************************

      SUBROUTINE INPUT (NOS)

      COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP
      REAL C,ALENGTH,TC,TMAX,DIA,DF,G,HR,CDA,HLPP
      INTEGER NOS

      WRITE (*,10)
 10   FORMAT('  WAVE SPEED IN FPS OR MPS =  ')
      READ (*,*) C

      WRITE (*,11)
 11   FORMAT ('  LENGTH OF THE PIPE IN FT OR M = ')
      READ (*,*) ALENGTH

      WRITE (*,12)
 12   FORMAT ('  DIAMETER OF THE PIPE (FT OR M)  = ')
```

```
      READ (*,*) DIA

      WRITE (*,13)
13    FORMAT ('  DARCEY WEISBACH FRICTION FACTOR =')
      READ (*,*) DF

      WRITE (*,14)
14    FORMAT (' HEAD IN RESERVOIR      (FT OR M) =')
      READ (*,*) HR

      WRITE (*,15)
15    FORMAT (' PRODUCT OF DISCHARGE AND AREA (CdA) =')
      READ (*,*) CDA

      WRITE (*,16)
16    FORMAT (' GRAVITATIONAL ACCELERATION g (FT/SS OR M/SS)=')
      READ (*,*) G

      WRITE (*,17)
17    FORMAT ('  TIME OF CLOSURE OF VALVE (SEC)      =')
      READ (*,*) TC

      WRITE (*,18)
18    FORMAT ('  MAXIMUM TIME TO WHICH COMPUTATIONS NEED (SEC)=')
      READ (*,*) TMAX

      WRITE (*,19)
19    FORMAT (' NO.OF n-DIMENSIONS IN SIMPLEX (PREFERABLY 10)=')
      READ (*,*) NOS

      WRITE (*,20)
20    FORMAT ('  MINIMUM PRESSURE HEAD YOU WANT (FT OR M) =')
      READ (*,*) HLPP


32    WRITE (*,30)C,ALENGTH,DIA,DF,HR,CDA,G,TC,TMAX,NOS,HLPP
30    FORMAT (' THE FOLLOWING VARIABLES ARE USED IN THIS PROGRAM:'/
     *        ' 1. WAVE SPEED                  =',F8.2/
     *        ' 2. LENGTH OF PIPE              =',F8.2/
     *        ' 3. DIAMETER OF PIPE            =',F8.4/
     *        ' 4. DARCEY WEISBACH FACTOR      =',F8.6/
     *        ' 5. HEAD IN RESERVOIR           =',F8.2/
     *        ' 6. CdA                         =',F8.6/
     *        ' 7. g                           =',F8.3/
```

```
     *        '  8. TIME OF CLOSURE OF VALVE     =',F8.3/
     *        '  9. MAXIMUM TIME                 =',F8.3/
     *        '  10. n-DIMENSIONS                =',I4/
     *        '  11. MINIMUM PRESSURE HEAD       =',F8.3/
     * '  ENTER THE NUMBER OF THE VALUE TO CHANGE OR 0 TO CONTINUE'/)

      READ (*,65) IANS
65    FORMAT (I2)
      IF (IANS.EQ.0) GOTO 60
      WRITE (*,31)
31    FORMAT (' ENTER THE VALUE '/)
      GOTO (41,42,43,44,45,46,47,48,49,50,51) IANS

41    READ (*,*) C
      GOTO 32

42    READ (*,*) ALENGTH
      GOTO 32

43    READ (*,*) DIA
      GOTO 32

44    READ (*,*) DF
      GOTO 32

45    READ(*,*) HR
      GOTO 32

46    READ (*,*) CDA
      GOTO 32

47    READ (*,*) G
      GOTO 32

48    READ (*,*) TC
      GOTO 32

49    READ (*,*) TMAX
      GOTO 32

50    READ (*,*) NOS
      GOTO 32

51    READ (*,*) HLPP
```

```
      GOTO 32

60    WRITE (*,30)C,ALENGTH,DIA,DF,HR,CDA,G,TC,TMAX,NOS,HLPP
      RETURN
      END

*************************************************************************
*       *****        SUBROUTINE SPLINE1        ******                   *
*       THIS SUBROUTINE IS USED TO FIND OUT THE TAU POINTS FROM         *
*       THE GIVEN CURVES. EQUATIONS ARE SOLVED BY GUASS ELIMINA-        *
*                          TION METHOD.                                 *
*************************************************************************


      SUBROUTINE SPLINE1(DN,DT,N,NO,NOO,PP,TAU,INI,NOS)
      REAL MAT(30,30),MATR(30),TAU(NO,NOO),G(30),PP(NO,NO)
     * ,DN,DT,T,COFF,PO1,PO2,PO3
      INTEGER J,L,I,N,M,NO,NOO,INI,NOS


      PP(INI,1)=1.0
      PP(INI,NOS+2)=0.0
      J =1
      DO I = 1,NOS
           IF (J.NE.1) THEN
           MAT(I,J) =1.0
           ELSE
           MAT(I,J) =0.0
           END IF
           MAT(I,J+1)  =4.
           IF (J.NE.NOS)THEN
           MAT(I,J+2)=1.0
           ELSE
           MAT(I,J+2) =0.0
           END IF
           MATR(I)  =6.0*(PP(INI,J+2)-2.0*PP(INI,J+1)+
     *                     PP(INI,J))/(DN**2.0)
      J =J+1

      END DO


      M  =1
      DO I =1,NOS
```

```
            COFF=MAT(I,M+1)
            MAT(I,M)  =MAT(I,M)/COFF
            MAT(I,M+1)=MAT(I,M+1)/COFF
            MAT(I,M+2)=MAT(I,M+2)/COFF

            MATR(I)   =MATR(I)/COFF
            IF (I.EQ.NOS) GO TO 101

            MAT(I+1,M+1)  =MAT(I+1,M+1)-MAT(I,M+1)
            MAT(I+1,M+2)  =MAT(I+1,M+2)-MAT(I,M+2)
            MAT(I+1,M+3)  =MAT(I+1,M+3)-MAT(I,M+3)

            MATR(I+1)     =MATR(I+1)-MATR(I)
      M =M+1
101   END DO


      DO J =1,NOS
           G(NOS+2-J) =MATR(NOS+1-J)/MAT(NOS+1-J,NOS+2-J)
     * -G(NOS+3-J)*MAT(NOS+1-J,NOS+3-J)
      END DO


      T  =0
      L  =1
      TAU(INI,1) =1.0
      TAU(INI,N) =0.0

      DO J =2,N-1
      T=T+DT
      LL=L
      IF (T.GT.L*DN) LL=L+1
      IF (T.GT.(L+1)*DN) LL=L+2
      IF (T.GT.(L+3)*DN) LL=L+3

      L=LL

      PO1=G(L)/6.0*((L*DN-T)**3/DN-DN*(L*DN-T))
      PO2=G(L+1)/6.0*((T-(L-1)*DN)**3/DN-DN*(T-(L-1)*DN))
      PO3=PP(INI,L)*(L*DN-T)/DN
      PO4=PP(INI,L+1)*(T-(L-1)*DN)/DN
      TAU(INI,J)=PO1+PO2+PO3+PO4
```

```
      IF (TAU(INI,J).GT.1.0) TAU(INI,J)=.9999999
      IF (TAU(INI,J).LE.0.0) TAU(INI,J)=.0000001


      END DO
      RETURN
      END


*********************************************************************
*        *************  SUBROUTINE HAMMER      ************          *
*        THIS IS TAKEN FROM STREETER & WYLIE AND IT COMPUTES THE     *
*        WATERHAMMER TRANSIENTS AT EACH POINT OF THE PIPE AFTER EVERY*
*        TIME STEP.                                                  *
*********************************************************************

      SUBROUTINE HAMMER(TAU,NO,NOO,N,DT,I,P)

      INTEGER L,M,NS,N,I
      DIMENSION HP(11),QP(11),H(11),Q(11),TAU(NO,NOO),P(NO)
      COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP


      NS=NUM+1
      R=DF*ALENGTH/(2.*G*DIA**5*.7854**2*FLOAT(NUM))
      B=C/(G*.7854*DIA*DIA)

C FIND STEADY STATE FLOW AND STORE INITIALVARIABLES

      QO=SQRT(2.*G*CDA*CDA*HR/(R*FLOAT(NUM)*2.*G*CDA*CDA+1.))
      HHP=0
      HLP=HLPP
      DO 20 M=1,NS
      H(M)=HR-(M-1)*R*QO*QO
      IF (H(M).GT.HHP) THEN
      HHP = H(M)
      END IF
      IF (H(M).LT.HLP) HLP=H(M)
 20   Q(M)=QO
      CVP=.5*QO*QO/H(NS)
      T=0.
      K=0
      L=1
```

```
 30     GO TO 40
 40     T=T+DT
        L=L+1
        IF (T.GT.TMAX) GO TO 99
C COMPUTATION OF INTERIOR POINTS
        DO 50 M=2,NUM
        CP=H(M-1)+Q(M-1)*(B-R*ABS(Q(M-1)))
        CM=H(M+1)-Q(M+1)*(B-R*ABS(Q(M+1)))
        HP(M)=.5*(CP+CM)
        IF (HP(M).GT.HHP) THEN
        HHP=HP(M)
        END IF
        IF (HP(M).LT.HLP) THEN
        HLP=HP(M)
        END IF
 50     QP(M)=(HP(M)-CM)/B
C BOUNDARY CONDITIONS
        HP(1)=HR
        QP(1)=Q(2)+(HP(1)-H(2)-R*Q(2)*ABS(Q(2)))/B
        IF (T-TC) 55,60,60
 55     CV=TAU(I,L)*TAU(I,L)*CVP
        IF (CV.LT.0.0)THEN
        CV=0.
        END IF
        GO TO 70
 60     TAU(I,L)=0.
        CV=0.
 70     CP=H(NUM) +Q(NUM)*(B-R*ABS(Q(NUM)))

        AA =CV**2*B**2+CV*CP*2.
        IF (AA.LT.0.0) CV =0.

        QP(NS)=-CV*B+SQRT(CV*CV*B*B+CV*CP*2.)
        HP(NS)=CP-B*QP(NS)
        IF (HP(NS).GT.HHP) THEN
        HHP=HP(NS)
        END IF
        IF (HP(NS).LT.HLP) THEN
        HLP=HP(NS)
        END IF
        DO 80 M=1,NS
        H(M)=HP(M)
        IF (H(M).GT.HHP) THEN
        HHP=H(M)
```

```
      END IF
      IF (H(M).LT.HLP) THEN
      HLP=H(M)
      END IF
80    Q(M)=QP(M)
      K=K+1
      IF(K/IPR*IPR-K) 40,30,40
99    P(I) =HHP-HLP
      RETURN
      END


******************************************************************
*        ************    SUBROUTINE SORT    *******              *
*        THIS FIND OUT THE CURVE OF THE HIGHEST OBJECTIVE FUNCTION*
*        VALUE AND SIMILLARLY THE SECOND HIGHEST AND THE MINIMUM. *
******************************************************************


       SUBROUTINE SORT(P,NO,NOO,N,KH,KM,KL,PH,PM,PL,NOS)
      REAL P(NO)
      PH  =P(1)
      KH  =1


      DO I = 2,NOS+1
      IF (PH.GE.P(I))THEN
      PH=PH
      ELSE
      PH=P(I)
      KH =I
      END IF
      END DO

      PL=100000000

       DO I = 1,NOS+1
       IF (PL.LE.P(I)) THEN
        PL = PL
        ELSE
        PL =P(I)
        KL=I
        END IF
```

```
      END DO
      PM =-100000000
      DO I =1,NOS+1
         IF (I.EQ.KH) THEN
         GOTO 16
         END IF
         IF (P(I).GE.PM) THEN
         PM =P(I)
         KM =I
         END IF
16     END DO
      RETURN
      END

*******************************************************************
*          ********  SUBROUTINE OPTIMIZE   **********             *
*       THIS CONVERGES THE OBJECTIVE FUNCTION TO A MINIMUM POINT *
*       IT USES THE SIMPLEX BY NELDER & MEAD.                     *
*******************************************************************

      SUBROUTINE OPTIMIZE(N,NO,NOO,KH,KM,KL,PP,ALPHA,DT,
     * PL,PH,PM,P,GAMMA,BETTA,TAUSTR,TAUDSTR,DN,PPDSTR,NOS)

      INTEGER N,KM,KL,IPR,KH,NO,NOO,NOS

      REAL TAUBAR(50),TAUSTR(NOO),TAUDSTR(NOO),PP(NO,NOO)
     *  ,PL,PH,DT,ALPHA,GAMMA,BETTA,C,ALENGTH,DIA,DF,G,HR,CDA
     *  ,PPDSTR(50),PPSTR(50),PPBAR(50),TAU(30,50),
     *  P(NO),PM,TC,TMAX,HLPP

      COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP



      DO JJ = 2,NOS+1
                SUM=0.0
                DO I =1,NOS+1
                 SUM =PP(I,JJ)+SUM
                END DO
                PPBAR(JJ)=(SUM-PP(KH,JJ))/(NOS)
      END DO

      DO JJ =2,NOS+1
       PPSTR(JJ)=PPBAR(JJ)-ALPHA*(PP(KH,JJ)-PPBAR(JJ))
```

```
          IF (PPSTR(JJ).LE.0.0)THEN
          PPSTR(JJ)=0.000001
          END IF
          IF (PPSTR(JJ).GE.1.0) THEN
          PPSTR(JJ)=.99999999
          END IF
     END DO

     PPSTR(1)=1.0
     PPSTR(NOS+2)=0.0


     CALL SPLINE2 (DN,N,PPSTR,TAUSTR,DT,NO,NOO,NOS)

     CALL HAMMER1(TAUSTR,NOO,N,DT,PSTR)


     IF (PSTR.LT.PL) THEN

     DO JJ =2,NOS+1
     PPDSTR(JJ)=PPBAR(JJ)+GAMMA*(PPSTR(JJ)-PPBAR(JJ))
     IF (PPDSTR(JJ).LE.0.0)THEN
     PPDSTR(JJ)=0.000001
     END IF
          IF (PPDSTR(JJ).GE.1.0) THEN
          PPDSTR(JJ)=.99999
          END IF
     END DO
C
     PPDSTR(1)=1.0
     PPDSTR(NOS+2)=0.0

     CALL SPLINE2 (DN,N,PPDSTR,TAUDSTR,DT,NO,NOO,NOS)

     CALL HAMMER1(TAUDSTR,NOO,N,DT,PDSTR)


     IF (PDSTR.LT.PL) THEN
     DO JJ =2,NOS+1
        PP(KH,JJ)=PPDSTR(JJ)
     END DO
     P(KH)=PDSTR
     ELSE
     DO JJ =2,NOS+1
```

```
      PP(KH,JJ)=PPSTR(JJ)
      END DO
      P(KH)=PSTR
      END IF
      ELSE
      IF (PSTR.GE.PM) THEN
      IF (PSTR.GE.PH) THEN
      DO JJ=2,NOS+1
      PPDSTR(JJ)=PPBAR(JJ)+BETTA*(PP(KH,JJ)-PPBAR(JJ))
      IF (PPDSTR(JJ).LE.0.0) THEN
      PPDSTR(JJ)=0.000001
      END IF
                IF (PPDSTR(JJ).GE.1.0) THEN
                PPDSTR(JJ)=.99999
           END IF
      END DO
      ELSE
      DO JJ =2,NOS+1
      PPDSTR(JJ)=PPBAR(JJ)+BETTA*(PPSTR(JJ)-PPBAR(JJ))
      IF (PPDSTR(JJ).LE.0.0)THEN
      PPDSTR(JJ)=.000001
      END IF
                  IF (PPDSTR(JJ).GE.1.0) THEN
                  PPDSTR(JJ)=.99999
                  END IF
      END DO
      END IF
C
      PPDSTR(1)=1.0
      PPDSTR(NOS+2)=0.0
C
      CALL SPLINE2 (DN,N,PPDSTR,TAUDSTR,DT,NO,NOO,NOS)

      CALL HAMMER1(TAUDSTR,NOO,N,DT,PDSTR)


      IF (PSTR.GE.PH) THEN
      PHH =PH
      ELSE
      PHH =PSTR
      END IF
      IF (PDSTR.LE.PHH) THEN
      DO JJ = 2,NOS+1
      PP(KH,JJ)=PPDSTR(JJ)
```

```
      END DO
      P(KH)=PDSTR
      ELSE

          DO I = 1,NOS+1
          IF (I.EQ.KL) THEN
          GOTO 20
          ELSE
          DO JJ=2,NOS+1
          PP(I,JJ)=(PP(I,JJ)+PP(KL,JJ))/2
          END DO
        END IF

          CALL SPLINE1 (DN,DT,N,NO,NOO,PP,TAU,I,NOS)
          CALL HAMMER (TAU,NO,NOO,N,DT,I,P)
20        END DO
      END IF
      ELSE
      DO JJ = 2,NOS+1
      PP(KH,JJ)=PPSTR(JJ)
      END DO
      PP(KH,1)=1.0
      PP(KH,NOS+2)=0.0
      P(KH)=PSTR
      END IF
      END IF
      RETURN
      END

************************************************************

      SUBROUTINE SPLINE2 (DN,N,PP,TAU,DT,NO,NOO,NOS)

      REAL MAT(30,30),MATR(30),TAU(NOO),G(30),PP(NO),
     *  DN,DT,T,COFF,PO1,PO2,PO3

      INTEGER J,L,I,N,M,NOS

      J =1
      DO I = 1,NOS
              IF (J.NE.1) THEN
```

```
                  MAT(I,J) =1.0
                  ELSE
                  MAT(I,J) =0.0
                  END IF
                  MAT(I,J+1)  =4.
                  IF (J.NE.NOS)THEN
                  MAT(I,J+2)=1.0
                  ELSE
                  MAT(I,J+2) =0.0
                  END IF
                  MATR(I)  =6.0*(PP(J+2)-2.0*PP(J+1)+
     *                      PP(J))/(DN**2.0)
        J =J+1

        END DO


        M  =1
        DO I =1,NOS
             COFF=MAT(I,M+1)
             MAT(I,M)  =MAT(I,M)/COFF
             MAT(I,M+1)=MAT(I,M+1)/COFF
             MAT(I,M+2)=MAT(I,M+2)/COFF

             MATR(I)   =MATR(I)/COFF
             IF (I.EQ.NOS) GO TO 101

             MAT(I+1,M+1)  =MAT(I+1,M+1)-MAT(I,M+1)
             MAT(I+1,M+2)  =MAT(I+1,M+2)-MAT(I,M+2)
             MAT(I+1,M+3)  =MAT(I+1,M+3)-MAT(I,M+3)

             MATR(I+1)     =MATR(I+1)-MATR(I)
        M =M+1
101         END DO


        DO J =1,NOS
         G(12-J) =MATR(NOS+1-J)/MAT(NOS+1-J,NOS+2-J)
     *   -G(NOS+3-J)*MAT(NOS+1-J,NOS+3-J)
        END DO
```

```
      T  =0
      L  =1
      TAU(1) =1.0
      TAU(N) =0.0

      DO J =2,N-1
      T=T+DT
      LL=L
      IF (T.GT.L*DN) LL=L+1
      IF (T.GT.(L+1)*DN) LL=L+2
      IF (T.GT.(L+2)*DN) LL=L+3
      L=LL

      PO1=G(L)/6.0*((L*DN-T)**3/DN-DN*(L*DN-T))
      PO2=G(L+1)/6.0*((T-(L-1)*DN)**3/DN-DN*(T-(L-1)*DN))
      PO3=PP(L)*(L*DN-T)/DN
      PO4=PP(L+1)*(T-(L-1)*DN)/DN
      TAU(J)=PO1+PO2+PO3+PO4

      IF (TAU(J).GT.1.0) TAU(J)=.9999999
      IF (TAU(J).LE.0.0) TAU(J)=.0000001


      END DO
      RETURN
      END


****************************************************************


      SUBROUTINE HAMMER1(TAUSTR,NOO,N,DT,PSTR)

      INTEGER L,M,NS,N,I,NUM
      REAL HP(11),QP(11),H(11),Q(11),PSTR,TAUSTR(NOO),HLP
      COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP

      NS=NUM+1
      R=DF*ALENGTH/(2.*G*DIA**5*.7854**2*FLOAT(NUM))
      B=C/(G*.7854*DIA*DIA)
C FIND STEADY STATE FLOW AND STORE INITIALVARIABLES
      QO=SQRT(2.*G*CDA*CDA*HR/(R*FLOAT(NUM)*2.*G*CDA*CDA+1.))
      HHP=0.
```

```
      HLP=HLPP
       DO 20 M=1,NS
         H(M)=HR-(M-1)*R*QO*QO
         IF (H(M).GT.HHP) THEN
         HHP = H(M)
       END IF
      IF (H(M).LT.HLP) HLP=H(M)
20    Q(M)=QO
      CVP=.5*QO*QO/H(NS)
      T=0.
      K=0
      L=1
30    GO TO 40
40    T=T+DT
      L=L+1
      IF (T.GT.TMAX) GO TO 99
C COMPUTATION OF INTERIOR POINTS
      DO 50 M=2,NUM
      CP=H(M-1)+Q(M-1)*(B-R*ABS(Q(M-1)))
      CM=H(M+1)-Q(M+1)*(B-R*ABS(Q(M+1)))
      HP(M)=.5*(CP+CM)
      IF (HP(M).GT.HHP) THEN
      HHP=HP(M)
      END IF
      IF (HP(M).LT.HLP)THEN
      HLP=HP(M)
      END IF
50    QP(M)=(HP(M)-CM)/B
C BOUNDARY CONDITIONS
      HP(1)=HR
      QP(1)=Q(2)+(HP(1)-H(2)-R*Q(2)*ABS(Q(2)))/B
      IF (T-TC) 55,60,60
55    CV=TAUSTR(L)*TAUSTR(L)*CVP
      IF (CV.LT.0.0) THEN
      CV=0.
      END IF
      GO TO 70
60    TAUSTR(L)=0.
      CV=0.
70    CP=H(NUM) +Q(NUM)*(B-R*ABS(Q(NUM)))

      AA =CV**2.*B**2.+CV*CP*2.
      IF (AA.LT.0.0) CV=0.
      QP(NS)=-CV*B+SQRT(CV*CV*B*B+CV*CP*2.)
```

```
      HP(NS)=CP-B*QP(NS)
      IF (HP(NS).GT.HHP) THEN
      HHP=HP(NS)
      END IF
      IF (HP(NS).LT.HLP) THEN
      HLP=HP(NS)
      END IF
      DO 80 M=1,NS
      H(M)=HP(M)
      IF (H(M).GT.HHP) THEN
      HHP=H(M)
      END IF
      IF (H(M).LT.HLP) THEN
      HLP=H(M)
      END IF
80    Q(M)=QP(M)
      K=K+1
      IF(K/IPR*IPR-K) 40,30,40
99       PSTR =HHP-HLP
      RETURN
      END


****************************************************************

      SUBROUTINE HAMMER2(TAUSTR,NOO,N,DT)

      INTEGER L,M,NS,N,I,NUM

      REAL HP(11),QP(11),H(11),Q(11),PSTR,TAUSTR(NOO),HLP

      COMMON C,ALENGTH,DIA,DF,G,HR,CDA,TC,TMAX,NUM,IPR,HLPP

      CHARACTER*15 OUTFILE

      WRITE (*,*) 'WRITE THE NAME OF THE OUTPUT FILE'
      READ (*,'(A)') OUTFILE
      OPEN(31,STATUS='NEW',FILE=OUTFILE)


      TAUSTR(1) =1.
      NS=NUM+1
      R=DF*ALENGTH/(2.*G*DIA**5*.7854**2*FLOAT(NUM))
```

```
      B=C/(G*.7854*DIA*DIA)
C FIND STEADY STATE FLOW AND STORE INITIALVARIABLES
      QO=SQRT(2.*G*CDA*CDA*HR/(R*FLOAT(NUM)*2.*G*CDA*CDA+1.))
      HHP=0.
      HLP=10000000
       DO 20 M=1,NS
      H(M)=HR-(M-1)*R*QO*QO
      IF (H(M).GT.HHP) THEN
      HHP = H(M)
      END IF
      IF (H(M).LT.HLP) HLP=H(M)
 20   Q(M)=QO
      CVP=.5*QO*QO/H(NS)
      T=0.
      K=0
      L=1
      WRITE(32,26)
 26   FORMAT(///'     TIME  PRESSURE     TAU   ')
      WRITE(31,25) C,ALENGTH,DIA,DF,HR,H(NS),QO,CDA,TC,G,TMAX,DT
     * ,B,NUM,IPR
 25   FORMAT(' C,ALENGTH, DIA,DF=',2F8.1,2F8.4/' HR, HO, QO ='
     *         ,2F8.2,F8.3/
     * ' CDA, TC',F8.7,F8.3/'G,TMAX,DT,B=',F8.3,F8.1,F8.5,2X,F10.2/'
     *   NUM, IPR='
     * ,2I3//'  HEADS AND DISCHARGES ALONG THE PIPE'//'    TIME X/L=
     *     0.     .2      .4      .6      .8      1.     TAU')
 30   WRITE (31,35)T,(H(M),M=1,NS),TAUSTR(L),(Q(M),M=1,NS)
 35   FORMAT (1H F7.3,5H   H=,6F8.2,F7.3/10X,3H Q=,6F8.3)
 40   T=T+DT
      L=L+1
      IF (T.GT.TMAX) GO TO 99
C COMPUTATION OF INTERIOR POINTS
      DO 50 M=2,NUM
      CP=H(M-1)+Q(M-1)*(B-R*ABS(Q(M-1)))
      CM=H(M+1)-Q(M+1)*(B-R*ABS(Q(M+1)))
      HP(M)=.5*(CP+CM)
      IF (HP(M).GT.HHP) THEN
      HHP=HP(M)
      END IF
      IF (HP(M).LT.HLP) HLP=HP(M)
 50   QP(M)=(HP(M)-CM)/B
C BOUNDARY CONDITIONS
      HP(1)=HR
      QP(1)=Q(2)+(HP(1)-H(2)-R*Q(2)*ABS(Q(2)))/B
```

```
      IF (HP(1).GT.HHP) HHP=HP(1)
      IF (HP(1).LT.HLP) HLP =HP(1)
      IF (T-TC) 55,60,60
55    CV=TAUSTR(L)*TAUSTR(L)*CVP
      IF (CV.LT.0.0) THEN
      CV=0.
      END IF
      GO TO 70
60    TAUSTR(L)=0.
      CV=0.
70    CP=H(NUM) +Q(NUM)*(B-R*ABS(Q(NUM)))

      AA =CV**2.*B**2.+CV*CP*2.
      IF (AA.LT.0.0)  CV=0.
      QP(NS)=-CV*B+SQRT(CV*CV*B*B+CV*CP*2.)
      HP(NS)=CP-B*QP(NS)
      IF (HP(NS).GT.HHP) THEN
      HHP=HP(NS)
      END IF
      IF (HP(NS).LT.HLP) HLP =HP(NS)
      DO 80 M=1,NS
      H(M)=HP(M)
      IF (H(M).GT.HHP) THEN
      HHP=H(M)
      END IF
      IF (H(M).LT.HLP) HLP=H(M)
80    Q(M)=QP(M)
      K=K+1
      IF(K/IPR*IPR-K) 40,30,40
99    PSTR =HHP-HLP
      PRINT *,HHP,HLP
      CLOSE (31)
      RETURN
      END

C*****************************************************************
```

APPENDIX B

COMPUTER INPUT AND OUTPUT FOR CASE II, tc=4.4L/a

The objective function in this case is to minimize the maximum pressure. The following input data are used.

wave speed	=	1341.14	m/sec
length of pipe	=	600.00	m
diameter of pipe	=	0.50	m
Darcy Weisbach friction factor	=	0.018	
head in reservoir	=	150.00	m
C_dA_g	=	0.038	
g	=	9.806	m/sec^2
time of closure of valve	=	1.968	sec
maximum time	=	4.5	sec
n-dimensions	=	19	
limiting minimum pressure head	=	-10.00	m

Limiting minimum pressure head does not play any role in the optimization because of the objectve function. The following is the computer output.

```
C,ALENGTH, DIA,DF=  1341.1   600.0  0.5000  0.0180
HR, HO, QO =  150.00   82.92   1.532
CDA, TC.0380000   1.968
G,TMAX,DT,B=   9.806      4.5 0.08948      696.54
  NUM, IPR=  5  1
```

HEADS AND DISCHARGES ALONG THE PIPE

TIME	X/L=	0.	.2	.4	.6	.8	1.	TAU
0.000	H=	150.00	136.58	123.17	109.75	96.33	82.92	0.000
	Q=	1.532	1.532	1.532	1.532	1.532	1.532	
0.089	H=	150.00	136.58	123.17	109.75	96.33	296.78	0.423
	Q=	1.532	1.532	1.532	1.532	1.532	1.225	
0.179	H=	150.00	136.58	123.17	109.75	307.78	308.27	0.409
	Q=	1.532	1.532	1.532	1.532	1.229	1.209	
0.268	H=	150.00	136.58	123.17	318.80	319.15	315.70	0.403
	Q=	1.532	1.532	1.532	1.232	1.213	1.205	
0.358	H=	150.00	136.58	329.85	330.06	326.53	318.90	0.400
	Q=	1.532	1.532	1.236	1.216	1.209	1.201	
0.447	H=	150.00	340.92	340.99	337.39	329.70	327.78	0.392
	Q=	1.532	1.239	1.220	1.212	1.204	1.195	
0.537	H=	150.00	351.95	348.27	340.53	338.52	326.26	0.394
	Q=	0.952	1.223	1.216	1.208	1.199	1.198	
0.626	H=	150.00	158.99	351.38	349.28	337.01	334.94	0.387
	Q=	0.921	0.932	1.212	1.202	1.201	1.192	
0.716	H=	150.00	151.21	161.65	347.79	345.63	335.19	0.387
	Q=	0.912	0.912	0.921	1.205	1.196	1.192	
0.805	H=	150.00	152.71	149.39	159.66	345.88	340.60	0.384
	Q=	0.904	0.901	0.908	0.917	1.196	1.191	
0.895	H=	150.00	148.20	150.79	149.20	156.27	348.46	0.376
	Q=	0.891	0.900	0.897	0.902	0.915	1.180	
0.984	H=	150.00	148.12	148.03	147.50	153.44	343.09	0.205
	Q=	0.896	0.887	0.893	0.895	0.889	0.640	
1.074	H=	150.00	149.82	144.88	152.25	333.20	351.94	0.189
	Q=	0.883	0.889	0.885	0.881	0.622	0.597	
1.163	H=	150.00	146.76	153.99	329.44	349.55	351.40	0.188
	Q=	0.883	0.881	0.877	0.614	0.591	0.593	
1.253	H=	150.00	154.14	330.19	350.09	347.57	351.00	0.186
	Q=	0.880	0.871	0.612	0.589	0.585	0.586	
1.342	H=	150.00	332.28	349.07	348.22	351.53	348.91	0.184
	Q=	0.859	0.612	0.585	0.583	0.584	0.580	
1.432	H=	150.00	343.80	350.22	350.51	349.56	350.42	0.185
	Q=	0.347	0.574	0.583	0.580	0.578	0.583	
1.521	H=	150.00	168.56	345.25	351.54	349.41	347.03	0.185

	Q=	0.293	0.319	0.569	0.578	0.579	0.579	
1.611	H=	150.00	152.14	170.55	344.18	349.02	349.37	0.183
	Q=	0.292	0.290	0.316	0.568	0.579	0.576	
1.700	H=	150.00	152.02	151.75	168.70	344.17	347.67	0.184
	Q=	0.286	0.288	0.289	0.317	0.566	0.578	
1.790	H=	150.00	149.62	150.23	152.42	168.02	347.34	0.178
	Q=	0.285	0.286	0.290	0.288	0.318	0.559	
1.879	H=	150.00	148.21	150.29	149.59	156.24	329.98	0.028
	Q=	0.286	0.286	0.284	0.290	0.282	0.084	
1.968	H=	150.00	150.67	147.59	154.11	311.33	351.94	0.000
	Q=	0.288	0.284	0.287	0.278	0.057	0.000	
2.058	H=	150.00	149.37	154.48	309.10	349.59	351.27	0.000
	Q=	0.282	0.289	0.278	0.054	-0.003	0.000	
2.147	H=	150.00	153.80	310.66	349.73	349.03	347.23	0.000
	Q=	0.289	0.276	0.056	-0.003	-0.003	0.000	
2.237	H=	150.00	311.06	348.84	350.57	347.38	346.79	0.000
	Q=	0.270	0.057	-0.004	-0.001	0.000	0.000	
2.326	H=	150.00	344.83	350.96	346.49	348.33	347.53	0.000
	Q=	-0.174	-0.010	0.000	-0.001	0.002	0.000	
2.416	H=	150.00	190.00	342.48	348.72	346.64	349.88	0.000
	Q=	-0.290	-0.232	-0.007	0.003	-0.001	0.000	
2.505	H=	150.00	147.89	187.91	342.63	350.27	345.75	0.000
	Q=	-0.289	-0.286	-0.228	-0.007	0.001	0.000	
2.595	H=	150.00	148.00	148.27	189.60	341.74	350.66	0.000
	Q=	-0.282	-0.285	-0.286	-0.230	-0.006	0.000	
2.684	H=	150.00	150.38	149.78	147.61	190.14	337.72	0.000
	Q=	-0.281	-0.282	-0.287	-0.284	-0.230	0.000	
2.774	H=	150.00	151.77	149.72	150.40	143.83	29.93	0.000
	Q=	-0.282	-0.283	-0.281	-0.287	-0.278	0.000	
2.863	H=	150.00	149.34	152.38	145.94	-9.58	-49.62	0.000
	Q=	-0.285	-0.281	-0.284	-0.275	-0.057	0.000	
2.953	H=	150.00	150.62	145.58	-7.37	-47.29	-49.06	0.000
	Q=	-0.279	-0.285	-0.275	-0.054	0.003	0.000	
3.042	H=	150.00	146.24	-8.91	-47.44	-46.85	-44.97	0.000
	Q=	-0.286	-0.273	-0.056	0.003	0.003	0.000	
3.132	H=	150.00	-9.31	-46.56	-48.38	-45.12	-44.64	0.000
	Q=	-0.267	-0.056	0.004	0.001	0.000	0.000	
3.221	H=	150.00	-42.60	-48.77	-44.24	-46.17	-45.27	0.000
	Q=	0.172	0.010	0.000	0.001	-0.002	0.000	
3.311	H=	150.00	110.45	-40.28	-46.56	-44.39	-47.70	0.000
	Q=	0.287	0.229	0.007	-0.003	0.001	0.000	
3.400	H=	150.00	152.09	112.51	-40.43	-48.09	-43.51	0.000
	Q=	0.285	0.283	0.226	0.007	-0.001	0.000	
3.490	H=	150.00	151.97	151.71	110.84	-39.55	-48.48	0.000

	Q=	0.279	0.282	0.283	0.228	0.006	0.000	
3.579	H=	150.00	149.63	150.22	152.36	110.30	-35.59	0.000
	Q=	0.278	0.279	0.284	0.281	0.228	0.000	
3.669	H=	150.00	148.26	150.28	149.60	156.10	268.79	0.000
	Q=	0.279	0.280	0.278	0.284	0.275	0.000	
3.758	H=	150.00	150.65	147.64	154.01	307.86	347.35	0.000
	Q=	0.282	0.277	0.280	0.272	0.056	0.000	
3.848	H=	150.00	149.38	154.37	305.68	345.05	346.91	0.000
	Q=	0.276	0.282	0.271	0.053	-0.003	0.000	
3.937	H=	150.00	153.72	307.20	345.20	344.72	342.76	0.000
	Q=	0.283	0.270	0.055	-0.003	-0.003	0.000	
4.026	H=	150.00	307.60	344.34	346.24	342.91	342.54	0.000
	Q=	0.264	0.056	-0.004	-0.001	0.000	0.000	
4.116	H=	150.00	340.42	346.62	342.05	344.05	343.06	0.000
	Q=	-0.171	-0.010	0.000	-0.001	0.002	0.000	
4.205	H=	150.00	189.11	338.13	344.44	342.20	345.57	0.000
	Q=	-0.283	-0.227	-0.007	0.003	-0.001	0.000	
4.295	H=	150.00	147.94	187.07	338.28	345.95	341.33	0.000
	Q=	-0.282	-0.280	-0.223	-0.007	0.001	0.000	
4.384	H=	150.00	148.05	148.31	188.73	337.42	346.34	0.000
	Q=	-0.276	-0.279	-0.280	-0.225	-0.006	0.000	
4.474	H=	150.00	150.37	149.78	147.67	189.26	333.50	0.000
	Q=	-0.275	-0.276	-0.281	-0.278	-0.226	0.000	

APPENDIX C

COMPUTER INPUT AND OUTPUT FOR CASE IV, tc=4.4L/a

The same data, as used in Appendix B, are used for case IV (minimizing the difference between maximum and minimum pressures while limiting the minimum value). But in this case the limiting minimum pressure head (input) plays an important role. Following is the computer output.

C,ALENGTH, DIA,DF= 1341.1 600.0 0.5000 0.0180
HR, HO, QO = 150.00 82.92 1.532
CDA, TC.0380000 1.968
G,TMAX,DT,B= 9.806 4.5 0.08948 696.54
NUM, IPR= 5 1

HEADS AND DISCHARGES ALONG THE PIPE

TIME	X/L=	0.	.2	.4	.6	.8	1.	TAU
0.000	H=	150.00	136.58	123.17	109.75	96.33	82.92	0.000
	Q=	1.532	1.532	1.532	1.532	1.532	1.532	
0.089	H=	150.00	136.58	123.17	109.75	96.33	306.89	0.411
	Q=	1.532	1.532	1.532	1.532	1.532	1.211	
0.179	H=	150.00	136.58	123.17	109.75	317.79	319.87	0.396
	Q=	1.532	1.532	1.532	1.532	1.214	1.192	
0.268	H=	150.00	136.58	123.17	328.71	330.63	326.62	0.391
	Q=	1.532	1.532	1.532	1.218	1.196	1.190	
0.358	H=	150.00	136.58	339.66	341.43	337.34	329.52	0.388
	Q=	1.532	1.532	1.222	1.200	1.193	1.186	
0.447	H=	150.00	350.63	352.25	348.10	340.22	335.09	0.385
	Q=	1.532	1.225	1.204	1.197	1.190	1.185	
0.537	H=	150.00	363.10	358.87	350.95	345.76	337.69	0.382
	Q=	0.925	1.207	1.201	1.193	1.189	1.182	
0.626	H=	150.00	159.92	361.70	356.46	348.34	343.27	0.379
	Q=	0.889	0.903	1.197	1.193	1.186	1.181	
0.716	H=	150.00	150.44	159.24	359.01	353.88	349.14	0.373
	Q=	0.883	0.882	0.898	1.189	1.185	1.173	
0.805	H=	150.00	149.39	149.57	158.37	359.70	354.20	0.370
	Q=	0.875	0.877	0.877	0.892	1.177	1.173	
0.895	H=	150.00	149.14	148.60	152.02	160.34	359.52	0.365
	Q=	0.872	0.870	0.872	0.867	0.883	1.166	
0.984	H=	150.00	149.21	151.58	150.62	153.57	154.70	0.423
	Q=	0.865	0.866	0.860	0.863	0.859	0.885	
1.074	H=	150.00	152.41	151.22	153.12	145.10	363.09	0.172
	Q=	0.861	0.856	0.857	0.852	0.865	0.552	
1.163	H=	150.00	151.98	153.94	145.73	361.43	361.86	0.171
	Q=	0.846	0.853	0.847	0.859	0.547	0.547	
1.253	H=	150.00	151.53	146.53	361.06	361.23	361.79	0.170
	Q=	0.844	0.838	0.854	0.544	0.544	0.544	
1.342	H=	150.00	144.59	357.49	360.79	361.41	360.07	0.170
	Q=	0.830	0.846	0.537	0.541	0.541	0.543	
1.432	H=	150.00	354.81	357.65	357.86	359.63	361.29	0.168
	Q=	0.848	0.530	0.534	0.534	0.540	0.539	
1.521	H=	150.00	361.82	355.19	356.51	357.75	360.10	0.168

	Q=	0.234	0.538	0.528	0.533	0.531	0.537	
1.611	H=	150.00	151.02	360.67	355.10	356.99	359.97	0.165
	Q=	0.231	0.232	0.537	0.525	0.530	0.526	
1.700	H=	150.00	149.52	151.56	361.12	357.32	357.81	0.165
	Q=	0.230	0.231	0.231	0.534	0.520	0.527	
1.790	H=	150.00	150.54	150.63	154.40	361.92	355.44	0.164
	Q=	0.232	0.229	0.229	0.226	0.530	0.520	
1.879	H=	150.00	151.11	153.38	152.09	153.14	359.94	0.166
	Q=	0.228	0.230	0.224	0.227	0.228	0.531	
1.968	H=	150.00	152.84	152.56	152.12	150.76	311.49	0.000
	Q=	0.228	0.223	0.227	0.226	0.228	0.000	
2.058	H=	150.00	151.45	151.58	151.24	310.32	309.45	0.000
	Q=	0.219	0.225	0.225	0.228	-0.002	0.000	
2.147	H=	150.00	148.76	150.13	309.64	309.78	309.16	0.000
	Q=	0.223	0.220	0.226	-0.003	0.000	0.000	
2.237	H=	150.00	148.69	306.67	308.52	308.48	310.10	0.000
	Q=	0.222	0.224	-0.007	-0.001	-0.001	0.000	
2.326	H=	150.00	307.77	306.93	305.51	308.85	307.79	0.000
	Q=	0.225	-0.005	-0.004	-0.005	-0.002	0.000	
2.416	H=	150.00	308.10	306.61	307.26	304.82	307.59	0.000
	Q=	-0.232	-0.002	-0.004	-0.004	-0.004	0.000	
2.505	H=	150.00	148.99	308.43	305.93	306.00	301.86	0.000
	Q=	-0.229	-0.230	-0.002	-0.003	-0.002	0.000	
2.595	H=	150.00	150.48	148.46	307.17	302.96	304.42	0.000
	Q=	-0.228	-0.229	-0.229	-0.001	0.002	0.000	
2.684	H=	150.00	149.47	149.37	145.64	305.58	304.06	0.000
	Q=	-0.229	-0.227	-0.227	-0.224	0.002	0.000	
2.774	H=	150.00	148.90	146.65	147.93	146.89	306.75	0.000
	Q=	-0.226	-0.227	-0.222	-0.225	-0.226	0.000	
2.863	H=	150.00	147.19	147.46	147.90	149.24	-10.00	0.000
	Q=	-0.225	-0.221	-0.225	-0.224	-0.226	0.000	
2.953	H=	150.00	148.56	148.43	148.77	-8.85	-7.97	0.000
	Q=	-0.217	-0.223	-0.223	-0.226	0.002	0.000	
3.042	H=	150.00	151.23	149.87	-8.17	-8.30	-7.70	0.000
	Q=	-0.220	-0.218	-0.224	0.003	0.000	0.000	
3.132	H=	150.00	151.30	-5.23	-7.06	-7.02	-8.63	0.000
	Q=	-0.220	-0.222	0.007	0.001	0.001	0.000	
3.221	H=	150.00	-6.33	-5.48	-4.08	-7.38	-6.34	0.000
	Q=	-0.223	0.005	0.004	0.005	0.002	0.000	
3.311	H=	150.00	-6.64	-5.18	-5.81	-3.41	-6.14	0.000
	Q=	0.230	0.002	0.004	0.004	0.004	0.000	
3.400	H=	150.00	151.00	-6.97	-4.50	-4.57	-0.47	0.000
	Q=	0.227	0.228	0.002	0.003	0.002	0.000	
3.490	H=	150.00	149.53	151.53	-5.73	-1.56	-3.00	0.000

	Q=	0.226	0.227	0.227	0.001	-0.002	0.000	
3.579	H=	150.00	150.53	150.62	154.32	-4.15	-2.66	0.000
	Q=	0.227	0.225	0.225	0.222	-0.002	0.000	
3.669	H=	150.00	151.09	153.32	152.05	153.08	-5.31	0.000
	Q=	0.224	0.225	0.220	0.223	0.224	0.000	
3.758	H=	150.00	152.78	152.52	152.09	150.75	308.54	0.000
	Q=	0.223	0.219	0.223	0.222	0.224	0.000	
3.848	H=	150.00	151.42	151.56	151.22	307.40	306.52	0.000
	Q=	0.215	0.221	0.221	0.224	-0.002	0.000	
3.93	H=	150.00	148.78	150.13	306.73	306.85	306.26	0.000
	Q=	0.218	0.216	0.222	-0.003	0.000	0.000	
4.026	H=	150.00	148.71	303.82	305.62	305.59	307.17	0.000
	Q=	0.218	0.220	-0.007	-0.001	-0.001	0.000	
4.116	H=	150.00	304.91	304.06	302.68	305.94	304.92	0.000
	Q=	0.221	-0.005	-0.004	-0.005	-0.002	0.000	
4.205	H=	150.00	305.21	303.77	304.39	302.01	304.71	0.000
	Q=	-0.228	-0.002	-0.004	-0.004	-0.004	0.000	
4.295	H=	150.00	149.01	305.54	303.10	303.16	299.10	0.000
	Q=	-0.225	-0.226	-0.002	-0.003	-0.002	0.000	
4.384	H=	150.00	150.47	148.49	304.31	300.19	301.60	0.000
	Q=	-0.224	-0.225	-0.225	-0.001	0.002	0.000	
4.474	H=	150.00	149.48	149.38	145.72	302.75	301.28	0.000
	Q=	-0.225	-0.223	-0.223	-0.220	0.002	0.000	

REFERENCES

1. Azoury, P.H., Bassiri, A. M. and Najm, H., " Effect of Valve-Closure Schedule on Waterhammer," ASCE, Journal of Hydraulic Engineering, Vol. 112, No. 10, October 1986.

2. Chaudhry, M. H., Applied Hydraulic Transients, Van Nostrand Reinhold Co., New York, 1979.

3. Contractor, D. N., "Two and Three Stage Valve-Closures to Minimize Waterhammer Transients." ASME, Vol. 98-7, 1985.

4. Contractor, D. N., " Valve Stroking to Control Waterhammer Transient using Dynamic Programming," ASME Symp. Num. Methods in Fluid Transient Anal.,Fluids Engineering Division, Houston Texas, Jun. 1983.

5. Conway, A. B., "Minimizing Waterhammer Transients in a Series Pipeline," M.S. thesis, Department of Civil Engineering and Engineering Mechanics, University of Arizona, 1986.

6. Driels, M. R., " Predicting Optimum Two-Stage Valve Closure," ASME, WInter Annual Meeting, Houston, Texas, Nov. 1975. WA-75/FE-2.

7. El-Ansary, A. S. and Contractor, D. N.,"Minimization of Axial Stresses and Pressure Surges in Pipes using Nonlinear Optimization," ASME Vol. 140, Pressure Vessels and Piping Conference Pittsburgh,Pennsylvania, Jun. 1988.

8. Forsythe, G. E., Malcolm, M. A. and Moler, C. B., Computer Methods for Mathematical Computations, Prentice-Hall, Inc., 1977.

9. Goldberg, D. E. and Kerr, C. L., "Quick Stroking: Design of Time-Optimal Valve Motions," ASCE, Journal of Hydraulic Engineering, Vol. 113, No. 6, June 1987.

10. Hornbeck, R. W., Numerical Methods, Quantum Publishers, Inc., 1975.

11. Kuester, J. L. and Mize, J. H., Optimization Techniques with Fortran, McGraw-Hill, New York, 1973.

12. Murray, W., Numerical Methods for Unconstrained Optimization, Academic Press Inc., 1972.

13. Nelder, J. A. and Mead, R., " A Simplex Method for Function Minimization," Computer Journal, Vol. 7, No. 4, 1965.

14. Sen, S. and Contractor, D. N., "Reduction of Pressure Surges by Minimax Optimization," Jour. of Applied Mathematics Modeling, Vol. 10, No. 4, August, 1986.

15. Spendley, W., Hext, G. R. and Himswort, F. R., " Sequential Application of Simplex Design in Optimization and Evolutionary Design," Technometrics, Vol. 4, No. 4, 1962.

16 Streeter, V. L., " Valve Stroking to Control Water Hammer," Journal of the Hydraulics Division, ASCE, Vol. 89, No. HY2, Proc. Paper 3452, March 1963.

17. Streeter, V. L. and Wylie, E. B., Fluid Mechanics, McGraw-Hill, Inc., 1979.

18. Watters, G. Z., Analysis and Control of Unsteady Flow in Pipelines, Butterworth Publishers, 1984.

19. Wiely, E. B., and Streeter, V. L., Fluid Transients, McGraw-Hill Inc., 1978.