RATIO TEMPERATURE THERMOGRAPHY

by

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I hereby recommend that this dissertation prepared under my
direction by Eustace Leonard Dereniak
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degree of Doctor of Philosophy

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SIGNED: Eustace L. Desenfle
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ABSTRACT

The mortality rate of breast cancer has not changed in 50 years despite extensive medical research. These discouraging results are due to not having an economical diagnostic instrument to screen the total population. It is now well known that early detection of small lesions, those that do not involve the axillary lymph nodes, is critical to improving survival rates.

Our present understanding of breast cancer indicates that increased metabolic activity occurs in the lesion and thus produces a local temperature increase. During the last 16 years medical thermographs have been used to provide diagnostic information by detecting these temperature changes. Present thermographic techniques accurately detect 3 out of 4 carcinomas; this is encouraging but not sufficient to be used alone in a clinical screening program.

This paper evaluates present thermographic techniques and examines the problem of skin emissivity variations which can produce erroneous temperature measurements. These false temperature variations are on the order of the decision level used by radiologists, and therefore cause significant confusion in the interpretation of the thermograms.

The ratio temperature thermograph is shown to reduce the effects of emissivity by measuring the spectral radiance at two prescribed wavelengths and ratioing the results. This minimizes the radiance's dependence on skin emissivity provided a correlation of the emissivity between spectral regions of 0.366 exists.
A dual channel ratio thermograph was built using state-of-the-art detectors and electronics to prove its feasibility. The instrument design and performance are extensively evaluated from a design as well as a hardware point of view.

Finally, the ratio temperature thermograph was quantitatively evaluated for small emissivity variations which demonstrated its capability of minimizing these effects. It was also evaluated for the detection of temperature changes.

This dissertation presents both technical and experimental evidence that ratio temperature thermography reduces the effects of skin emissivity variations which are potential sources for incorrect diagnosis of breast cancer thermograms.
CHAPTER 1

INTRODUCTION

For hundreds of years, physicians have used body temperature as a means of determining a patient's health. Recently they have sought to obtain further information about the patient's condition by measuring the areal distribution of surface temperature of a body with an optical, noncontacting instrument that does not affect the temperature being measured—a technique called thermography. A thermogram is a permanent recording, resembling a photograph, in which light and dark shades correspond to temperature patterns. Also, as in photographs, points on the recording bear a one-to-one correspondence with the respective points on the body, forming recognizable images.

Thermography has aroused considerable medical interest because of its ability to reveal and measure changes in static temperature patterns and variations caused by heat flow in the body. Moreover, thermography provides diagnostic information without exposing the patient to ionizing radiation.

All objects continuously emit electromagnetic radiation, the amount being dependent on the temperature and emissivity. The radiant power emitted by a human body is essentially all in the infrared region of the spectrum (approximately a kilowatt). The thermograph converts the received infrared radiation into an output, such as a photograph made from a cathode ray tube display. A thermograph consists of four
basic parts: the optical system, an infrared detector, processing electronics, and a display.

The optical system consists of a scanning mechanism and imaging optics. The scanning mechanism is a device that is used to selectively accept radiation emitted by each part of the area of interest on the object. The imaging system brings the radiation to a focus at the detector where it develops an electrical signal. The video signal output of the detector is amplified and converted into a suitable form for display by the processing electronics. This type of thermography is totally passive; that is, the radiation emitted naturally by the object is sensed, and no external infrared source of illumination is used. In contrast, an active system must illuminate the target and then detect the reflected radiation. Radar is a good example of an active system.

The thermograph may be described as being an electro-optical imaging device that operates as a spectral wavelength converter, translating spatial and thermal information about an object from the infrared spectrum to the visible spectrum. Electronic processing may be employed to enhance or identify certain signal characteristics as an aid to the radiologist or clinician evaluating the displayed image.

This thesis concerns the development of a new technique in thermography; that of differential thermography, which uses both active and passive ratio-temperature thermography, with our primary research emphasis being on passive ratio temperature thermography. It is hoped that this new concept will result in an earlier, more reliable detection of breast carcinoma. This results from avoidance of error
introduced by emissivity variations. Our research clearly reveals both in theory and experiment that such thermographic errors can be minimized if not eliminated, and that improved diagnosis can indeed be anticipated.

Problem of Breast Cancer

Dr. Irwin Freundlich (Freundlich, Wallace, and Dodd, 1968a) Past President of the American Thermographic Society, stated (p. 274):

The mortality rate of the breast carcinoma has not improved in almost four decades despite a longer survival time for the individual patient. In addition, there is evidence of an increase in the absolute number of new cases so that, in fact, the battle against breast cancer is being lost.

It is well known that patients, who do not have involvement of axillary lymph nodes at the time of surgery, have a much improved mortality rate when compared with those who do have metastatic spread to these lymph glands. In addition, the presence of axillary lymph node metastases can be related to the size of the primary lesion. The smaller the carcinoma in the breast, the more likely that the axillary lymph nodes will be free of metastatic carcinoma. Our objective, therefore, is to discover lesions earlier.

Despite considerable medical research effort in the control and cure of breast cancer, the mortality rate has not changed in the past half a century. These discouraging results are primarily due to the fact that the limits of palpability probably have been reached and better methods for finding lesions too small to be detected by palpation have not been widely utilized in population survey.

In contrast, the detection and cure of some other cancers, such as cervical cancer, has shown immense progress because of the simple and widely used "Pap test". Consequently the mortality rate has declined.
Breast cancer statistics are appalling when one considers that there are about 100,000 new breast carcinoma patients (about three out of each 1,000 women) projected for this year (Consumer Reports, 1974). Also, only 53% of the women treated for breast carcinoma by surgery will remain free of disease five years after surgery. However, 82% of women treated by surgery, when the cancer is still confined to the breast, are free of disease five years later, as compared to only 25% if the cancer has spread to the axillary lymph node. Therefore, it is well established that patients without axillary lymph node metastases have a greater chance of survival than patients whose cancer has spread beyond the breast (Fisher, Slack, and Bross, 1969; Freundlich, 1967). In addition, the number of patients with axillary lymph node involvement increases directly with the size of the primary lesion, and lesions too small to be palpable have a lower percentage of axillary lymph node metastases (Freundlich, Wallace, and Dodd, 1968b). These data indicate an earlier detection of breast cancer is mandatory for a lower mortality rate.

Research has shown that a malignancy causes increased metabolic activity, which in turn produces heat or temperature changes. If these temperature changes can be detected at an early stage of cancer, an early warning system could be developed. Thermograms have been used for several years to detect these temperature changes with discouraging results. Therefore, a new approach to detect these temperature changes is needed.
History of Temperature Measurements

For centuries temperature measurements have played an important role in medical diagnosis. The detection of general or local body temperature anomalies was recognized by the ancients as an indicator of abnormality or disease. As early as 400 B.C., Hippocrates diagnosed disease by using his hand as a crude thermometer to detect fever (Adams, 1849).

Nearly 2000 years later, in 1617, Galileo invented the first thermometer, called a thermoscope, and in 1625, Sanctorius of Padua devised an open thermometer for measuring the temperature of a person in fever (Mitchell, 1892). He was the first to claim to be able to determine the variation of the temperature of the human body during the course of certain fevers. Sanctorius describes a clinical thermometer in the first book of Canon of Avicenna in Venice in 1625 (Garrison, 1963). The expanding liquid for these early thermometers was water or alcohol. The alcohol was distilled wine spirits. These early thermometers are described in detail by Mitchell (1892). During the seventeenth and early eighteenth centuries, investigators such as Newton, Roemer, Halley, and Muschembrock were trying to define a calibration and standardization of the thermometer scale with no great success (Martine, 1740). Most investigators believed the freezing and boiling points of water should be used for calibration points, but the water thermometers were difficult to calibrate because of the change in the state of water at the boiling and freezing points. Alcohol thermometers, on the other hand, could be calibrated very nicely at the freezing point of
water; however, it is a gas at the boiling point of water. Since these were the only expanding liquids used for thermometers during these years, calibration and standardization were not established on a firm basis.

For a long time thereafter the thermometer simply existed without achieving a permanent place in medicine or science. In 1714, Gabriel Fahrenheit of Danzig invented a new method of cleaning mercury by distillation, so that it would not stick to the walls of tubes. This enabled him to substitute mercury for alcohol in thermometers. With this mercury thermometer he was able to calibrate both the freezing and boiling points of water without the technical problems of the previous instruments. He gave this mercury thermometer to his friend, Herman Boerhaave, a Dutch clinician, who used it to demonstrate the value of thermometry in the diagnosis of disease among patients at the University of Leiden. These experiments enabled clinicians to record absolute temperature rather than using only the subjective feeling of warmth. Fahrenheit chose for his temperature scale the interval between freezing salt water and a healthy person's blood temperature. The zero point was located on the thermometer's stem below the salt water's freezing point at half the interval distance between the two calibration points as shown in Fig. 1-1. He divided the thermometer interval by 12 and then later into 96 divisions; the number 96 being chosen because it had so many divisors. On this Fahrenheit scale, which is still used today (Worthington, 1940), the freezing and boiling points of water were 32 and 212, respectively.
Fahrenheit's success was a result of three developments:

(1) Establishment of a calibrated scale.

(2) Establishment of finer subdivisions than previously used.

(3) Use of capillary tubes of constant and controlled diameter so that all the thermometers were identical.

In 1742, a Swedish astronomer, Anders Celsius, recommended that the thermometer be calibrated in 100 parts (centigrade) between the freezing and boiling points of water (boiling point being zero in the original thermometer).

While a professor of medicine in Leipzig during the last part of the nineteenth century, Carl Wunderlich made more than 10,000 observations and published a book discussing the meaning of fever in diseases (Wunderlich, 1871). Before his book, fever was considered to be a disease in itself. Thereafter, it was recognized as only a symptom. His observations provided the foundation for modern clinical thermography. It also lead to Albutt's introduction of the clinical thermometer, and attempts to devise meaningful measurements of body surface temperature variations. These efforts failed primarily because of a
lack of knowledge about the complex thermodynamic balance of the human body and the fact that contact thermometers modified the temperature structure they were trying to measure.

The discovery of infrared radiation is credited to Sir William Herschel (1800). His discovery made remote sensing of body surface temperatures possible. Herschel discovered that the heating power of the sun's rays increased from the violet to the red end of the spectrum and that the maximum was not reached until the thermometer had been placed beyond the end of the visible red. Herschel found more heating in the infrared spectral region than at the radiation peak of 480 μm predicted by the Wien displacement law because of the prism dispersive character and the finite size (fixed slit width) of the thermometer used. The prism did not disperse the infrared as well as the visible spectrum, thus causing a spatial energy concentration. This pronounced heating effect was attributed to radiation, which he called infrared. The infrared region might well have been discovered by Landriani in 1777, who was the first to examine the energy distribution of the solar spectrum by passing a thermometer through it. Similar experiments were also conducted by Rochon in 1776 and Seniber in 1785 (Barr, 1960). In 1840, Sir John Herschel, son of Sir William, rendered the infrared spectrum visible by means of crude pictures, which he termed "thermographs" (Herschel, 1840). These were made by coating paper with lamp black and then soaking it in alcohol. When infrared radiation was focused on the prepared paper, the alcohol evaporated more rapidly from parts that received the stronger radiation, causing the paper to appear lighter
in those areas. It remained for Seebeck (1825) to discover the thermo-
couple, Nobili to refine Seebeck's thermocouple to a sensitive
thermopile, and Langley (1881) to develop the bolometer, before infra-
red began to be recognized as a quantitative tool.

Nearly a century and a half after the discovery of infrared
radiation, Professor Marianus Czerny (1929) developed a method under the
name of evaporography that provided the first practical system for
"seeing by heat waves." In the 1940's, however, military applications
of infrared techniques initiated a technological revolution and ex-
tended its practicality. The desire to see in the dark and for
infrared-seeking missiles produced the motivation to develop truly
sensitive equipment. Unfortunately, most of the instrumentation was
classified and not available for public use and medical application.
In 1956, a Baird Evaporograph was declassified and R. N. Lawson, MD
(1957), a Canadian surgeon, was able to use it to substantiate his
tory that most breast cancers are characterized by a rise in tempera-
ture. A synopsis of the historical development is shown in Table 1.1.

Infrared imaging technology has shown considerable progress
since the first evaporographic thermogram. Today, most thermographic
equipment uses photoconductive or photovoltaic detectors and scanning
optics. These modern thermographic instruments are extremely sensitive
to temperature changes; however, clinical results have been of limited
success. Although the early hopes for thermography have yet to be
realized, the potential value of these techniques as a diagnostic tool
is immeasurable.
Table 1.1. Historical Development of Temperature Measurement.

<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
<th>Discoverer</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 B.C.</td>
<td>Detected temperature by touch</td>
<td>Hippocrates</td>
</tr>
<tr>
<td>1617 A.D.</td>
<td>First thermoscope</td>
<td>Galileo</td>
</tr>
<tr>
<td>1625</td>
<td>Open thermometer</td>
<td>Sanctorius of Padua</td>
</tr>
<tr>
<td>1714</td>
<td>Mercury thermometer</td>
<td>Fahrenheit</td>
</tr>
<tr>
<td>1742</td>
<td>Centigrade thermometer</td>
<td>Celsius</td>
</tr>
<tr>
<td>1800</td>
<td>Discovered infrared</td>
<td>William Herschel</td>
</tr>
<tr>
<td>1825</td>
<td>Discovered thermocouple</td>
<td>Seebeck</td>
</tr>
<tr>
<td>1840</td>
<td>Imaged infrared</td>
<td>John Herschel</td>
</tr>
<tr>
<td>1871</td>
<td>Clinical thermometer</td>
<td>Wunderlich</td>
</tr>
<tr>
<td>1881</td>
<td>Discovered bolometer</td>
<td>Langley</td>
</tr>
<tr>
<td>1929</td>
<td>Evaporograph</td>
<td>Czerny</td>
</tr>
<tr>
<td>1956</td>
<td>Detected breast cancer with thermograph</td>
<td>Lawson</td>
</tr>
</tbody>
</table>
Typical Thermograph

Thermography is a method of forming pictorial representations of temperature patterns on the surface of an object.

A thermographic apparatus reacts to the radiation emitted by the body itself. Thus no special radiation source is necessary. Thermography must, therefore, clearly be distinguished from those methods in which one uses auxiliary infrared sources and detects the radiation reflected by the object (e.g., infrared photography, infrared image converters, sniperscopes, etc.) except in the special case called differential thermography.

The heart of the installation is the detector, normally a photodetector. The detector is responsible for the conversion of the incident radiation into an electrical signal, which can be amplified millions of times. The thermal radiation is focused on the detector using a mirror system having a large aperture. The picture of a part of the body is obtained by means of a mechanical scanning system that collects the radiation from the object point after point and line after line in rapid succession to form a raster. The electrical signal originating from the detector is, after amplification, used to feed the display unit. The building up of the display must be synchronous with the scanning of the patient. All modern systems use an image tube for representing the thermal picture, which also can be recorded on film for a permanent record.

The pictorial representation of the radiance pattern formed by the thermographic device is interpreted as a temperature pattern. It
is a measure of the change in temperature from point to point on the object. A common figure of merit for thermographic instruments is the noise equivalent temperature difference (NETD). It is derived and discussed in detail by Hudson (1968) and Loftis and Carwell (1971). The NETD is the change in temperature of an object required to produce a system signal-to-noise ratio of unity when the object fills the field of view. Modern commercially available thermographs have NETD values on the order of 0.01 to 0.5°K, depending on frame times.

Two important parameters of a thermograph are high spatial resolution and high frame rate, both of which tend to decrease the signal-to-noise ratio for a given temperature change. If the resolution is smaller, the sensitivity or frame rate must be larger. Correspondingly, if the frame rate is made shorter, resolution and/or sensitivity is lost. Therefore, a compromise must be made to optimize resolution, frame rate, and temperature sensitivity.

Typically, thermographs operate with a single-element detector. A schematic diagram of a thermographic system is shown in Fig. 1.2. Most thermographs operate either with InSb as the detector in the 3 to 5-μm region or with HgCdTe in the 8 to 12-μm region. Because the signal is derived from radiation emitted by the object, it is a passive technique. If an absolute temperature calibration is necessary, an internal reference blackbody is used for this purpose.
Current Status and Limitations

Although the breast cancer mortality rate has not improved in the last half century, progress has been made in categorizing women with a high risk of carcinoma. The high risk group consists of those women

(1) From a family with a history of breast carcinoma
(2) With previous breast carcinoma
(3) With extensive benign mastitis
(4) With a positive thermogram.

An evaluation of a thermogram is based on the fundamental premise that heat is generated by the increased metabolic activity of a tumor. This heat produces a temperature rise and/or increased vascularity.
A typical thermogram of a clinically asymptomatic woman is characterized by the similarity in the pattern of the right and left breast. Carcinoma shows itself by asymmetrical thermal patterns of the left and right breast.

A thermogram is shown in Fig. 1.3 relating to clinically asymptomatic women. Note that in general the thermogram is symmetric—the thermogram of the left breast is similar or even identical to the one of the right breast.

The abnormal breast is associated with the increased metabolic activity of the malignant tumor and the consequent elevated temperature (Lawson and Gaston, 1964; Lawson, 1956). This excess heat is mostly dissipated by venous convection through the skin. An asymmetry of the thermogram results, and the radiologist interprets the thermogram to be positive.

There are several variations or pattern asymmetries that a radiologist recognizes as a positive thermogram:

1. Diffuse unilateral increase in heat
2. Significant difference in the breast vascular patterns
3. Difference in areola temperatures
4. Radical changes in temperature across a single breast
5. Local area of heat increase or "hot spot."

A typical thermogram showing a hot spot as caused by a malignant tumor is given in Fig. 1.4.

A diagnostic thermogram can fall into four categories: those that were diagnosed correctly (true positives); those that were
Fig. 1.3. Negative Thermogram.
Fig. 1.4. Positive Thermogram.
diagnosed correctly (true positives); those that were actually negative but that the radiologist called positive (false positives); negative cases that were diagnosed correctly (true negatives); and finally, cases that were truly positive for carcinoma but were called negative (false negatives) (Lusted, 1969). These rates are discussed in detail in Appendix A.

Recent data (Pomerance, 1975) from the 27 screening centers throughout the United States indicate that only 43.8% of those women with cancer of the breast were correctly identified by thermography. With the addition of physical examination, this true positive rate increased to 74%.

Isard et al. (1972) report that 71% true positives of confirmed cancers were detected by thermography. Davey (1970) reports 73% of the known cancer cases were detected by thermography; however, his data were not taken with a large number of women. Dowdy et al. (1970) found 73% of the thermograms of patients with cancer proved to be correctly classified. Stark and Way (1970) discovered nine clinically unsuspected cancers, all nonpalpable and less than 1 cm in diameter. Dodd et al. (1973) report true positive rates of 87%. Of interest, however, is that 18.7% of the false positives developed into true positives several years later. This suggests that thermography may be a more sensitive procedure for detecting breast cancer than currently believed.
Based on the reported data, thermography is optimum for rapidly growing malignant lesions, whether they are detected early (small) or late. Presently, however, thermography is detecting three out of four cancerous breasts at best. Thermography is a simple, noninvasive, inexpensive technique. However, a more accurate instrument is needed, one that is easy and inexpensive to operate and that produces thermograms that can be interpreted by trained lay personnel. This is the hope of ratio thermography.

**Description of the Radiation Aspects of Thermography**

All objects continuously emit and absorb radiation. The amount of energy an object radiates is dependent on its temperature, $T$, and emissivity, $\varepsilon$. The ratio of actual rate of energy emission to the rate of energy emission of an ideal blackbody at the same temperature is called emissivity. Stefan (1879) found that the total power emitted by a blackbody ($\varepsilon = 1$) is proportional to the fourth power of temperature; actual surfaces emit and absorb less than an ideal black surface.

The emissive power (emittance) of an arbitrary surface at temperature $T$ is (Wolfe, 1965) (the notation is defined in Appendix B)

$$L_e = \frac{\varepsilon\sigma T^4}{\pi} \quad \text{(W/cm}^2\text{-sr)}$$

$$L_q = \frac{\varepsilon\sigma q T^3}{\pi} \quad \text{(photon/sec-cm}^2\text{-sr)}$$

(1.1)

(1.2)
In 1901, Max Planck (1901) developed a radiation law that shows how emitted radiation is spectrally distributed as a function of temperature.

\[ L_e(\lambda, T) d\lambda = \varepsilon_{\lambda} \frac{2c^2 \lambda^{-5}}{e^{hc/kT} - 1} \ d\lambda \quad \text{(W/cm}^2\text{-sr)} \quad (1.3) \]

\[ L_q(\lambda, T) d\lambda = \frac{2c^2 \lambda^{-4}}{e^{hc/kT} - 1} \ d\lambda \quad \text{(photons/sec-cm}^2\text{-sr)} . \quad (1.4) \]

If the emissivity is known, then the radiance is a unique measure of the temperature.

One of the basic limitations in using radiance as a measure of the temperature of the emitting body is knowing the emissivity. The characteristics of the human body surface are extremely important to the accuracy of temperature measurement. Although skin appears to be nearly a blackbody in the infrared region, its emissivity is not unity. Studies (Elam, Goodwin, and Lloyd-Williams, 1963; Lloyd-Williams, Cade, and Goodwin, 1963; Watmough and Oliver, 1968a, 1968b, 1969a, 1969b, 1969c; Patil and Lloyd-Williams, 1969; and Steketee, 1973b) have indicated that skin emissivity is about 0.95 to 0.99 in the 3 to 14-\(\mu\)m spectral region. There is a disturbing amount of disagreement among investigators. For example, Patil and
Lloyd-Williams (1969) reported an apparent emissivity greater than one. While it is desirable and also technically feasible to determine a temperature difference as low at 0.1°K, uncertainties in the emissivity of the human skin usually limit the accuracy to about 0.8°K. Possible variations in emissivity from one area to another and variations due to viewing angle (Lewis, Goller, and Teates, 1973; Watmough and Oliver, 1970; Watmough and Oliver, 1969b; and Irwin, Savara, and Rau, 1973) are not thoroughly understood and should be considered before final exact temperature conclusions are made. Otherwise, image structure in a thermogram due to spatial emissivity variations may be ascribed erroneously to temperature variations.

Reflection is another factor that must be considered when the emissivity is less than unity. From Kirchhoff's law, the reflectivity, \( \rho \), is given by

\[
\rho = 1 - \varepsilon.
\]  \hspace{1cm} (1.5)

Therefore, a body could reflect a detectable amount of energy from its environment.

Reflections of this type normally have an energy peak at wavelengths less than 4 or 5 \( \mu \text{m} \), which makes systems that operate in the 8 to 14-\( \mu \text{m} \) region insensitive to these influences.

Present thermograms can be misinterpreted due to emissivity differences across the surface of the skin. This would lead to a higher false positive rate. A second consideration is that present thermographs operate either in the 3 to 5-\( \mu \text{m} \) or 8 to 14-\( \mu \text{m} \) region.
Improvements have been made in the spatial and temporal resolution of infrared thermographs with concomitant improvements in diagnosis. There seems to be limits to the clinical improvements that these characteristics accomplish. However, other methods for the use of infrared radiation have not been extensively investigated, and these may prove to be significantly beneficial. These include selected spectral region, radiation ratios, differential thermography, and pattern recognition (Whitehouse, 1976).

Differential Thermography

Because present medical thermography did not attain the true positive rates that physicians feel are required for breast cancer detection, Dodd, Marsh, and Zermeno at M. D. Anderson Hospital introduced a new technique that they called differential thermography. The basic idea is the subtraction of an image obtained with illumination in the near infrared (active image) from a thermal image obtained in the far infrared with radiation emitted by the body (passive image). In particular, their technique of differential infrared scanning involves the instantaneous subtraction of a thermographic image (8 to 14 \( \mu m \)) from a near infrared image (0.9 \( \mu m \)).

Active imaging using illumination at 0.9 \( \mu m \) and infrared film has been used for many years to map vascular patterns to find irregularities resulting from the development of malignant tumors or from other changes in the female body (Gorman and Hirsheimer, 1939; Massopust, 1948; and Massopust and Gardner, 1950, 1954).
The image of any vessel seen on the breast in the near-infrared photograph (active) is a function of the size of the vessel and its depth beneath the skin. The image of a vessel seen on the breast in the far-infrared (passive), however, is a function of the size of the vessel, the depth beneath the skin, and its temperature. Consequently, if one subtracts the near-infrared from the far-infrared image, the resulting picture should be a function of the temperature only (assuming no emissivity variations). Zermano, Dodd, and Marsh (1974) showed this to be true using 0.9-μm spectral radiation. The 0.9-μm radiation was chosen purely due to the convenience of this instrumentation and the availability of a silicon detector. Their success was limited, and no positive conclusion concerning improved clinical performance was formed on their work since a clinical test would be required to show its validity (Johnson, 1974).
CHAPTER 2

THERMOGRAPHIC TECHNIQUES

The techniques applied most commonly in medical thermography are based on the thermal radiation emitted by the human skin. The majority of instruments available (Barnes, 1963; Cade, 1964; and Heerma Van Voss, 1969) are designed to meet industrial as well as medical requirements; therefore, their characteristics vary widely and are not specifically optimized for medical applications. These devices measure an approximation of radiation temperature or brightness temperature of the target object. The radiation temperature, $T_R$, of a surface is the temperature that a blackbody would have in order to produce an integrated radiance (summed over the entire spectrum) equal to that emitted by the surface. Unless the surface is a blackbody and in thermodynamic equilibrium with its surroundings, the radiation temperature will not be equal to the kinetic (or thermodynamic) temperature, $T$, of the surface material. The radiation temperature is defined as

$$T_R = \left( \frac{L}{\sigma} \right)^{\frac{1}{4}}. \quad (2.1)$$

The brightness temperature is the temperature at which a blackbody would have a spectral radiance equal to that of the given body in a narrow spectral interval centered usually at 0.6-μm. This temperature depends on the kinetic (thermodynamic) temperature of the body, the
emissivity in the band, and the wavelength of the measurement. Only for a perfect blackbody are the brightness and kinetic temperatures equal. The brightness temperature is defined as

\[ T_B = c_2 \left\{ \lambda \ln \left[ 1 + \frac{\varepsilon \lambda c_1(\pi L_q(\lambda, T)\lambda^5)^{-1}}{1 + \varepsilon \lambda c_1(\pi L_q(\lambda, T)\lambda^5)^{-1}} \right] \right\}^{-1} \]  \quad (2.2)

Commercial thermographic procedure usually assumes implicitly that the target is a blackbody; therefore, a small emissivity will produce apparent changes in the brightness temperatures, which are interpreted as temperature changes in the target. Figures 2.1 and 2.2 illustrate radiation and brightness temperatures, respectively.

To eliminate the shortcomings of present passive thermographic systems, our study investigates a new technique of ratio-temperature measurement. This concept has been used by Winch (1929), Harding (1952), Sweet (1940), Gibson (1951), and Russell and Lucks (1940) in the visual and near infrared. Bramson (1968) defines this as a color temperature, the temperature at which a blackbody would emit radiation with a spectral energy distribution whose slope matches that of the smoothed spectrum of the given body. Since color temperature has a different meaning for many researchers, we shall call the temperature which we obtain a ratio temperature, that obtained from the ratio of radiances in two spectral regions. The temperature obtained in any single band will be called the integral temperature. An example of how ratio temperature is determined is shown in Fig. 2.3. The slope of the spectrum is determined from measurements of the radiances at two predetermined wavelengths.
Fig. 2.1. Schematic Illustrating the Principle of Radiation Measurements.

Fig. 2.2. Schematic Illustrating the Principle of Brightness Measurements.
Fig. 2.3. Schematic Illustrating the Principle of Ratio Temperature Measurements.
and compared to the slope of the Planck radiation law. Using this technique, emissivity effects tend to ratio out of the final result.

Reiterating, passive thermography produces a self-emitted picture, determined by the thermal radiation of the skin. One problem with present thermographic equipment, such as Spectrotherm, Thermovision, Dynarad, and Thermiscope results from the inaccuracies in determining the skin temperature, which depends on the emissivity of the skin.

The emissivity has been treated in the literature extensively, but no satisfactory solution has been given (see Steketee, 1973b; Lewis et al., 1973; Elam et al., 1963; Watmough and Oliver, 1968a, 1968b; and Buchmüller, 1961). Appendices C and D discuss the physiological and emissive characteristics of skin.

**General Analysis**

As the image of the detector is scanned across the human body, it is exposed sequentially to different radiating elements of the body (see Fig. 2.4). Assume the system first views a radiating element of temperature, $T_a$, and emissivity, $\varepsilon_a$, and then a very short time later it views another element with temperature, $T_b$, and emissivity, $\varepsilon_b$. The differences are $\Delta T$ and $\Delta \varepsilon$, which we will assume to be small enough so they can be written $dT$ and $d\varepsilon$. In general, the difference $dM$ and relative difference $dM/M$ in photon emittance (exitance) from these two elemental areas can be written

$$dM = \frac{\partial M}{\partial T} dT + \frac{\partial M}{\partial \varepsilon} d\varepsilon$$

(2.3)
Fig. 2.4. Picture Elements.
\[ \frac{dM}{q} = \frac{1}{M} \frac{\partial M}{\partial q} \, dT + \frac{1}{M} \frac{\partial M}{\partial \varepsilon} \, d\varepsilon. \]  \hspace{1cm} (2.4)

If the emissivity is constant from one picture element to the next (i.e. \( d\varepsilon = 0 \)), the change and relative change in exitance received by the detector are given by

\[ \frac{dM}{q} = \frac{\partial M}{\partial q} \, dT \]  \hspace{1cm} (2.5)

\[ \frac{dM}{q} = \frac{1}{M} \frac{\partial M}{\partial q} \, dt. \]  \hspace{1cm} (2.6)

Under this assumption thermograms are then interpreted as a true pictorial representation of the temperature pattern of the skin. Every asymmetry appearing in the thermogram is then considered a positive result.

There is strong evidence (Elam et al., 1963; Lewis et al., 1973; and Watmough and Oliver, 1970) that the emissivity is not sufficiently constant spatially, spectrally, temporally, or thermally to permit its neglect. This means that \( d\varepsilon \neq 0 \). In addition to its variability, the emissivity is not accurately known from any of the measurements made to date. The result is that radiance difference—and consequently asymmetric patterns in the thermogram—can be caused by emissivity effects rather than temperature effects.

Hardy and Muschenheim (1936), Elam et al. (1963), Mitchell, Wyndham, and Hodgson (1967), Watmough and Oliver (1969b, 1969c), and most recently Steketee (1973a) have measured the emissivity as a function of the wavelength. The results of their experiments show great
disparity. Elam et al. find large variations in the 3 to 5-μm region, while Steketee finds an almost constant emissivity $\varepsilon = 0.98$ from 2-μm out to 14-μm (Fig. 2.5). The measurement uncertainty in both cases is given to be $d\varepsilon = 0.01$. These authors are aware of the uncertainty and in general conclude that as long as the uncertainty is not larger than 1% to 2%, there is no problem.

No data were found on the spatial variations of skin emissivity. Although we could find no discussion of the spatial variations of the emissivity of the skin in the literature, it seems highly unlikely that the emissivity should be spatially constant. Consider the problems of making a uniform large area blackbody. Variations of 1% in the emissivity are unavoidable when one uses a clean smooth metal surface and applies spray paint carefully. The human skin, however, is not very smooth, and despite all efforts to keep the skin clean and dry, minute layers of perspiration cannot be prevented completely. They will drastically influence the emissivity characteristics. One cannot expect the spatial uniformity to be better than about 1%, possibly worse, which implies $d\varepsilon/\varepsilon \geq 0.01$. The effects of this will depend somewhat on the spectral bandwidth of the thermograph, of which three cases can be considered. The first is when $M$ represents the entire spectrum ($\Delta \lambda = \infty$) of radiation so that the Stefan-Boltzmann law applies. This is directly applicable to the radiation temperature measurements. The second is monochromatic radiation ($\Delta \lambda = 0$), which is rather impractical in the instrumentation case, and the third is somewhere between ($\Delta \lambda = \text{finite}$), which is the case for all thermographs.
Fig. 2.5. Emissivity Measurements.

(a) Reflectivity of skin (---), transmission of skin (····), transmission of fluid (---) and emissivity E of skin (---) (after Elam et al., 1963).

(b) Spectral emissivity of the human skin (after Steketee, 1973a).
Total Radiation (Stefan-Boltzmann Law Applies)

All operational medical thermographs use photon detectors as sensors. Therefore, the analysis is carried out in terms of photon flux instead of the more common radiation unit of watts.

If the thermograph is spectrally sensitive to all wavelengths ($\Delta\lambda = \infty$), or at least has a large spectral bandwidth, a reasonable approximation to the radiant emittance (exitance) is the Stefan Boltzmann law

$$M_q = \varepsilon \sigma_q T^3,$$  

(2.7)

where $\sigma_q$ is a constant found by integration of the blackbody curve spectral response over the entire spectrum ($1.52 \times 10^{11}$ photons sec$^{-1}$ cm$^{-2}$$^3$K$^{-3}$). Then the difference and the relative difference can be expressed as

$$dM_q = 3 \varepsilon \sigma_q T^2 dT + \sigma_q T^3 d\varepsilon \text{ (photons/sec-cm$^2$)}$$  

(2.8)

$$\frac{dM_q}{M_q} = 3 \frac{dT}{T} + \frac{d\varepsilon}{\varepsilon}.$$  

(2.9)

The relative change is plotted in Fig. 2.6 for a 310°K blackbody (human body temperature). To examine the comparable effects of temperature and emissivity changes, consider the two terms on the right-hand side of the equation to have the same effect in changing the radiation.

A relative change of 0.01 in temperature causes a relative change of 0.03 in radiation, which could also be caused by a relative
Fig. 2.6. Total Radiation Contrast versus Relative Emissivity and Temperature Changes.
change of 0.03 in emissivity. The relative effect of temperature and emissivity demonstrates that the temperature is a factor of three or more influential. This is somewhat misleading since temperature ranges are typically of the order of 300°K where emissivity values are about 1; therefore, the factor of 3 is lost in the two orders of magnitude difference in absolute values. Substituting these values into Eq. (2.9) leads to

\[
\frac{dM_q}{M_q} = \frac{1}{100} \, dT + d\varepsilon. \tag{2.10}
\]

For this realistic situation a 1% emissivity change is equivalent to a change in temperature of 1.0°K.

**Monochromatic Radiation (Planck Radiation Law)**

The other extreme of the spectral response of a thermographic system is the quasi-monochromatic case. In this case the photon exitance is given by

\[
M_q(\lambda, T) = \varepsilon 2\pi\lambda^{-4} \cdot \left(\frac{\hbar c}{k\lambda T} - 1\right)^{-1}. \tag{2.11}
\]

The change in photon exitance is given by

\[
dM_q(\lambda, T) = \varepsilon 2\pi\lambda^{-4} \left(\frac{\hbar c}{k\lambda T} - 1\right)^{-2} \cdot \frac{\hbar c}{k\lambda T} \frac{\hbar c}{k\lambda T^2} \, dT \\
+ 2\pi\lambda^{-4} \left(\frac{\hbar c}{k\lambda T} - 1\right)^{-1} \, d\varepsilon. \tag{2.12}
\]
The relative change is

\[
\frac{dM_q(\lambda,T)}{M_q(\lambda,T)} = \frac{(hc/k\lambda T) \cdot e^{hc/k\lambda T}}{e^{hc/k\lambda T} - 1} \frac{dT}{T} + \frac{d\varepsilon}{\varepsilon} \tag{2.13}
\]

if we define \( x \) as follows

\[
x = \frac{hc}{kT}, \tag{2.14}
\]

then

\[
\frac{dM_q(\lambda,T)}{M_q} = \frac{x e^x}{e^x - 1} \frac{dT}{T} + \frac{d\varepsilon}{\varepsilon}. \tag{2.15}
\]

It is easily seen that this expression is different from Eq. (2.9) for relative change only in that \( xe^x/(e^x - 1) \) replaces the 3. The value for \( x \), which maximizes \( \partial M/\partial T \) with respect to wavelength, is very close to 5 (see Appendix E). Therefore, at maximum radiation contrast the relative exitance change is

\[
\frac{dM}{M_q} = 5 \frac{dT}{T} + \frac{d\varepsilon}{\varepsilon}. \tag{2.16}
\]

An error of 3% is introduced by using 5 instead of the precise factor \((x + 5)/2\). Just as for the entire spectrum, this is a linear function with a different slope. Figure 2.7 plots the monochromatic and full spectrum curves thus creating a boundary for any finite spectral region between them.
Fig. 2.7. Monochromatic and Total Radiation Contrast Change versus Relative Emissivity and Temperature Change.
Finite Spectral Band Radiation

Neither of the two previous examples is realistic for instrumentation, since all instruments are spectrally band-limited either by the detector, optics, or filters. For this situation Eq. (2.12) must be integrated over the spectral region of interest to obtain the exitance difference:

\[ \int_{\lambda_1}^{\lambda_2} dM_q(\lambda, T) d\lambda = \varepsilon \int_{\lambda_1}^{\lambda_2} c_1 \lambda^{-4} \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right]^2 \left[ \exp\left(\frac{c_2}{\lambda T}\right) \right] \]

\[ \cdot (c_2/\lambda T^2) d\lambda + \int_{\lambda_1}^{\lambda_2} c_1 \lambda^{-4} \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right] \cdot d\lambda d\varepsilon. \] (2.17)

The relative difference is

\[ \int_{x_1}^{x_2} M_q(x, T) dx \]

\[ \int_{x_1}^{x_2} M_1(x, T) dx \]

\[ = \int_{x_1}^{x_2} x^3 e^x (e^x - 1)^{-2} dx \]

\[ = \int_{x_1}^{x_2} x^2 (e^x - 1)^{-1} dx \]

\[ \int_{x_1}^{x_2} \frac{dT}{T} + \frac{dx}{\varepsilon}, \] (2.18)

which is derived in Appendix F. This is a rather awkward expression to evaluate.

The relative radiance differences of various spectral regions for a 300°K object will lie in the shaded area of Fig. 2.7. This leads to the conclusion that the coefficient of the dT/T term will be somewhere
between 3 and 5. Using these results and an emissivity $\varepsilon = 0.99 = 1.0$, the relative difference for total radiation is

$$\frac{dM}{M} = \frac{1}{100} \frac{dT + d\varepsilon}{q}.$$  \hspace{1cm} (2.19)

For the monochromatic case

$$\frac{dM}{q} = \frac{1}{60} \frac{dT + d\varepsilon}{q}.$$  \hspace{1cm} (2.20)

For the band-limited radiation, the factor is a function of bandwidth and spectral location. However, it is bounded by $1/60$ and $1/100$. Figure 2.7 is a graphical representation of the above equations. It permits one to determine the temperature difference equivalent of an emissivity variation.

The magnitude of the effect depends on whether a wideband detector, $\Delta \lambda = \infty$, or a narrowband detector, $\Delta \lambda = 0$, is used. For the wideband case, an emissivity change or uncertainty of 1% equals a temperature differential of $dT = 1.0^\circ K$ at $310^\circ K$ background, whereas for the narrowband case, a 1% emissivity change equals $dT = 0.6^\circ K$. Therefore, an asymmetry in the thermogram at a level of $dT = 1.0^\circ K$ could be caused by an emissivity asymmetry of 1%. Present clinicians consider $1^\circ K$ the criterion for differentiating between positive and negative readings.

Modern thermographic devices are capable of measuring temperature differences of $0.1^\circ K$ and less; therefore, emissivity effects should be of great concern. However, there are other effects, related to the skin
emissivity, that may be even more important. First of all there is the effect of the transmission of the skin. It is known that the skin is partly transparent between 3-μm and 5-μm. This has been shown by Elam et al. (1963) and Hardy and Muschenheim (1936). The consequence is that if one measures the skin temperature above a blood vessel with a thermometer one would measure for example, T = 310°K. However, a 3-μm to 5-μm radiometer would measure 313°K because it also receives radiation from the blood vessel, which is warmer than the skin, transmitted through the skin. These effects severely interfere with the determination of the temperature and emissivity in vivo and give rise to great uncertainty in large apparent variations of ε. Apparent emissivity values up to ε = 1.3 have been reported and are discussed by Steketee (1973a).

There is also the effect of the curvature of the human skin surface. Because of its ability to reflect radiation, the emissivity of the skin must be smaller than 1. Consequently, the apparent emissivity depends on the viewing angle. This has been discussed many times most recently by Lewis et al. (1973), Mitchell et al. (1967), and Watmough and Oliver (1970). It is pointed out by Lewis et al. (see Fig. 2.8) that a 75° angle of incidence, measured from the normal to the skin, can lead to an apparent 1.5°K temperature differential. As shown in Fig. 2.9, Watmough also finds that a 4°K hot spot can be masked or caused by an emissivity change resulting from a 70° viewing angle. Of course, one must be careful with his data as they assume the skin to be a dielectric, which is not really true. In practice, measures are taken to avoid this problem by taking oblique and lateral views. However, multiple views
Fig. 2.8. Temperature Difference due to Incidence Angle (after Lewis et al., 1973).
Fig. 2.9. Thermal Scanning of Curved Surfaces (after Watmough and Oliver, 1970).
introduce more chance of detector drift and increase the cost and length of clinical studies. Still another effect that must be considered in trying to determine the skin temperature with a radiometer is the influence of the background radiation. The total radiation from the skin includes a contribution from the background radiation from the walls of the room and equipment that is reflected by the skin into the radiometer. This effect warrants the following detailed analysis.

**Background Effects**

The optimum temperature of the room used for thermography has been debated for several years.

The room temperature determines the major part of the background radiation reflected from the subject. Other sources of background radiation include the thermographic instrument and the operator; however, these sources are less significant and will be neglected in this analysis.

The thermographic instrument measures a photon radiance, including the background, of

\[
L_q = \varepsilon q T_s^3 + (1 - \varepsilon) q T_b^3 \quad \text{(photon/sec-cm-sr)} \quad (2.21)
\]

if the background has uniform emissivity of 1 and the source is a Lambertian reflector.

The change in photon radiance \( L_q \) resulting from a change in source temperature \( T_s \) and emissivity \( \varepsilon \) can be expressed as

\[
\frac{dL_q}{q} = \varepsilon 3\sigma q T_s^2 dT_s + \sigma q (T_s^3 - T_b^3) d\varepsilon. \quad (2.22)
\]
The relative change, which can be called contrast, is found by normalizing to the total radiance, \( \sigma_q T_s^3 \) [Eq. (2.21)]. This is justified for thermography because \( (1 - \varepsilon)\varepsilon T_b^3 \) is small compared to \( \varepsilon\sigma T_s^3 \), the emissivity being about 0.99 and \( T_s \) and \( T_b \) being typically 310°K and 290°K, respectively.

The relative contrast expression

\[
\frac{dL_q}{L_q} = 3 \frac{dT_s}{T_s} + \left( 1 - \frac{T_b^3}{T_s^3} \right) \frac{d\varepsilon}{\varepsilon}
\]

shows that the effect of emissivity variations can be eliminated by making the room temperature \( T_b \) equal to the human body temperature \( T_s \). Then only skin temperature changes would be sensed. Under equilibrium conditions the patient would be part of a blackbody enclosure in which the radiation is everywhere homogeneous and isotropic. The optimum temperature for clinical examination from this analysis is still not clear (but not a subject of this investigation).

In a similar fashion to the total radiation case, one can write for the monochromatic radiance

\[
L_q(\lambda, T) = \varepsilon \left( \frac{2c}{\lambda^4} \right) \left[ \exp(x_s) - 1 \right]^{-1} + (1 - \varepsilon) \frac{2c}{\lambda^4} \left[ \exp(x_b) - 1 \right]^{-1}
\]

when

\[
x_s = \frac{hc}{k\lambda T_s}; \quad x_b = \frac{hc}{k\lambda T_b}.
\]
Using the same assumptions as stated for total radiation case,

\[
\frac{dL_q(\lambda, T)}{L_q(\lambda, T)} = \varepsilon \left( \frac{2c}{\lambda^4} \right) [\exp(x_s) - 1]^{-2} \frac{\exp(x_s) \cdot x_s dT_s}{T_s}
+ \left( \frac{2c}{\lambda^4} \right) \cdot \left\{ [\exp(x_s) - 1]^{-1} - [\exp(x_b) - 1]^{-1} \right\} d\varepsilon.
\]

(2.25)

Normalizing with the source radiance and assuming the reflected part, \((1 - \varepsilon)\), is small, we have

\[
\frac{dL_q(\lambda, T)}{L_q(\lambda, T)} = x_s e^{x_s} (e^{x_s} - 1)^{-1} \frac{dT_s}{T_s} + \left[ \frac{\exp(x_s) - 1}{\exp(x_b) - 1} \right] d\varepsilon.
\]

(2.26)

Again, when the background temperature equals the source temperature, the effect of emissivity is eliminated.

For spectral regions less than 15-\(\mu\)m and temperatures about 300°K

\(e^x \approx 23 \gg 1\); therefore Eq. (2.26) can be simplified to

\[
\frac{dL_q(\lambda, T)}{L_q(\lambda, T)} = x_s e^{x_s} \frac{dT_s}{T_s} + \left( 1 - \frac{x_s}{e^{x_s}} \frac{d\varepsilon}{\varepsilon} \right)
\]

(2.27)

or

\[
\frac{dL_q(\lambda, T)}{L_q(\lambda, T)} = \frac{hc}{k\lambda T} \frac{dT_s}{T_s} + \left\{ 1 - \exp \left[ x_s \left( 1 - \frac{T_s}{T_b} \right) \right] \right\} \frac{d\varepsilon}{\varepsilon}.
\]

(2.28)

This is the effect of background on a signal radiance measured monochromatically for a source with an emissivity less than one.
The maximum contrast occurs when Eq. (2.27) or (2.28) is maximized. The maximum temperature contrast is achieved when the wavelength of the radiation is as small as practical causing the first term of Eq. (2.28) to be maximized.

If the room temperature is less than the body temperature ($T_b < T_s$) then the second term also can become larger as the wavelength is decreased. In actual practice, radiologists cool the room to eliminate the temperature variations on the skin that are due to clothing. This cooling produces a natural temperature distribution of the body. By cooling the room (making $T_b$ lower) the second term of Eq. (2.28) is also increased, thus causing false emissivity contrast. The maximum contrast is achieved when the wavelength of the radiation is as small as practical and the background temperature is as low as practical. These results based on maximum contrast considerations are consistent with current practices in clinical thermography.

However, to eliminate surface emissivity effects, $T_b$ should be equal to $T_s$, thus making the second term zero. This is inconsistent with current practice. Radiologists find that the best empirical results occur when $T_b < T_s$. Although this practice is not supported by the previous analysis, the analysis does not consider the physiological effects of the cooling of the patient when the room temperature is reduced, and the uniformity of a blackbody room. Perhaps the increased metabolic activity of a chilled patient produces strong thermal effects that enhance the detection of carcinoma.
Differential Thermography Analysis

The differential thermography suggested by Dodd, Marsh and Zermeno at M. D. Anderson hospital is a step in the right direction for improving thermographic techniques. Their system used 0.9-μm as the near infrared picture (active picture); however, a better wavelength choice may well improve this technique further.

Figure 2.10 shows a simple model of the skin, subcutaneous tissue, and blood vessels. The illuminating radiation is transmitted and reflected at the epidermis and blood vessels. The difference in the reflection of the blood vessels, which is affected by the absorption of the oxyhemoglobin, and the reflection of the subcutaneous tissue is what we want to detect. Optimum imaging will prevail at a wavelength where transmission through the epidermis is maximum, reflection is minimal, absorption of the blood vessels is maximum, and reflection from subcutaneous tissue is optimized. All of these requirements do not fall at one wavelength; therefore, tradeoffs and compromises become essential.

The transmission of skin is shown in Fig. 2.11 as measured by Hardy and others and in Fig. 2.12 obtained in this laboratory. It is obvious that the optimum spectral region is not 0.9-μm. Hardy's data suggest 1.2-μm may be the optimum region, since this is the maximum of his transmission curve. Figure 2.12 shows 1.3 or 1.7-μm may be the choice from the transmission viewpoint. The general shape of the two curves is similar; however, Fig. 2.12 does not take into account the diffuse properties of the skin as did Hardy's. The improvement in transmission is about a factor of 2 using 1.7-μm or 1.2-μm as compared to
Fig. 2.10. Model of Skin.

Fig. 2.11. Spectral Transmission of Excised White Human Skin (after Hardy, Hammel, and Murgatroyd, 1956).
Fig. 2.12. Transmission Spectrum of Human Skin.
0.9-μm. However, another important factor to be considered is the reflection of the venous structure and the skin.

Figures 2.5a and 2.13 show the reflection of skin. Figure 2.13 is from Jacquez et al. (1955), while Fig. 2.5a is from Elam et al. (1963). The reflection curves show that the reflectivity is six times smaller at 1.7-μm than at 0.9-μm or 1.2-μm. This indicates the 1.7-μm region is optimal from the viewpoint of reflection.

Good imagery depends on reflection, transmission, and absorption of the epidermis, subcutaneous tissue, walls of blood vessels, and blood. Clearly, optimum imagery can be obtained only by detailed spectral studies of all these factors. Current best estimates are that the optimum wavelength will be centered about 1.7-μm. This is based on the fact that known transmission through the epidermis is best at these longer wavelengths (Figs. 2.11 and 2.12) and that reflection is correspondingly reduced (Figs. 2.5a and 2.13). The absorption band in the subcutaneous tissue, which seems to be centered at about 1.5-μm, is also avoided.

Present thermographic techniques require some new procedures and equipment to develop a better detection rate. Hopefully differential thermography, color temperature thermography, and/or differential color temperature thermography will provide these results in the future.
Fig. 2.13. Reflectance of Skin of Forearm of Very Fair Complexioned (——) and Very Dark Complexioned (---) Young White Males (Jacquez et al., 1955).
Integral temperature thermographs are useful instruments for deriving temperature changes or differences from radiance measurements under certain conditions. The principal requirement for accurate measurements is knowledge of the target's emissivity over the spectral range of the instrument. In addition, all types of thermographs are affected by the absorption in the intervening media between target and sensor. A ratio temperature thermograph substantially reduces these problems.

The ratio temperature of a body is the temperature a blackbody would have in order to emit continuum radiation having the same ratio of spectral radiance at two prescribed wavelengths. At any given temperature, every blackbody has the same wavelength distribution of radiance. This spectral distribution can be determined by measuring the radiance at two different wavelengths, thus yielding an accurate determination of the blackbody's temperature. The principle of ratio temperature measurement is shown in Fig. 3.1, which indicates the emission spectra of 310°K graybodies with the emissivities of 0.98 and 1.00.

A ratio temperature measurement is made by sampling the radiance of a spatial element at two points in the spectrum simultaneously, as indicated in the drawing. The wavelengths of the two passbands are
chosen to achieve sensitivity to the change in curvature of the black-body spectrum with temperature: typical values for the 300°K range are 5 µm and 13 µm. The ratioing technique eliminates the effect of wavelength-independent spatial and temporal emissivity variations.

Mathematically, the system output is

\[
D(T,\lambda_1) = \frac{\varepsilon_1 c_1 \lambda_1^{-4}[\exp(c_2/\lambda_1T) - 1]^{-1}}{\varepsilon_2 c_1 \lambda_2^{-4}[\exp(c_2/\lambda_2T) - 1]^{-1}}
\]

where \( T \) is the kinetic temperature of the object, \( D(T,\lambda_1) \) is the measured irradiance at \( \lambda_1 \), and \( D(T,\lambda_2) \) is the measured irradiance at \( \lambda_2 \).

As one can see, the ratio of the two radiances values at given wavelengths is dependent upon temperature except in the Rayleigh-Jeans region (\( c_2/\lambda T \ll 1 \)), which occurs at wavelengths much longer than those
used for thermography. For the case in which the emissivity is independent of wavelength \((\varepsilon_1 = \varepsilon_2)\), the ratio is independent of the emissivity, and the temperature one measures is the kinetic temperature.

If the emissivity is not constant with respect to wavelength, then the ratio temperature will be different from the kinetic temperature of the object. To determine the temperature this way, it is not necessary to know the actual emissivity in the two spectral regions, only their ratio. The measurement of the ratio of radiances is less dependent on surface conditions than the measurement of either one of them.

For the same reason, absorption by intervening media will have less effect on color temperature measurements than on brightness temperature measurements, provided the absorption is not highly wavelength dependent since radiance values at two wavelengths are being compared. Over the path length used in thermographic applications, the atmosphere will have minimal effect, provided the strong (and possibly time dependent) absorption bands of \(\text{H}_2\text{O}\) and \(\text{CO}_2\) are avoided. For a given instrumental design, the proper choice of \(\lambda_1\) and \(\lambda_2\) can almost eliminate ratio temperature errors due to the intervening media.

The ratio temperature is determined by direct measurement of the ratio of the spectral radiances at two wavelengths. In the region for which the Wien law is a good approximation, it can be shown (Bramson, 1968) that the ratio temperature \(T_C\) is given by

\[
T_C = \frac{c_2 [(1/\lambda_2) - (1/\lambda_1)]}{\ln[D(T,\lambda_1)/D(T,\lambda_2)] + 4 \ln(\lambda_1/\lambda_2)},
\]  

(3.2)
where \( D(T, \lambda_1) \) is a measured value of irradiance. Now let

\[
A = 4 \ln(\lambda_1/\lambda_2)
\]

\[
B = c_2[(1/\lambda_1) - (1/\lambda_2)],
\]

then

\[
T_c = -\frac{B}{A + \ln[D(T, \lambda_1)/D(T, \lambda_2)]}.
\] (3.3)

For this monochromatic case the relationship between the reciprocal of ratio temperature and the logarithm of the ratio of the spectral radiance can be expressed as a linear equation

\[
\frac{1}{T_c} = -\frac{A}{B} - \frac{1}{B} \ln \frac{D(T, \lambda_1)}{D(T, \lambda_2)}.
\] (3.4)

For the temperature region of interest, namely 300°K to 310°K, the equation can be considered linear in the spectral radiance ratio. In a similar fashion, solving for kinetic temperature in terms of ratio temperature from Eq. (3.2), one can show that

\[
T = \frac{T_c}{\ln(\varepsilon_1/\varepsilon_2)} \cdot \frac{\ln(\varepsilon_1/\varepsilon_2)}{1 - T_c c_2[(1/\lambda_1) - (1/\lambda_2)]}.
\] (3.5)

Clearly, when \( \varepsilon_1 = \varepsilon_2 \), the kinetic temperature and ratio temperatures are equal.

As in the case of integral temperature, the important parameter is not the absolute temperature but the difference in temperature measured at two different locations. The expression for ratio temperature (Eq. (3.4)) with the appropriate substitution can be shown to be
\[
\frac{1}{T_c} = \frac{1}{T} - \frac{\lambda_1 \lambda_2}{c_2 (\lambda_2 - \lambda_1)} \ln \frac{\varepsilon_1}{\varepsilon_2}.
\]  

(3.6)

In order to find the change in ratio temperature for a corresponding change in temperature and/or emissivity one can take the differential of Eq. (3.6). Since Eq. (3.6) is expressed in reciprocal temperature, the following relationship must be used.

\[
d\left(\frac{1}{T}\right) = -\frac{1}{T^2} \, dT
\]  

(3.7)

or

\[
dT = -T^2 \, d\left(\frac{1}{T}\right).
\]  

(3.8)

Therefore,

\[
dT_c = \frac{T_c^2}{T^2} \, dT + \frac{\lambda_1 \lambda_2 T_c^2}{c_2 (\lambda_1 - \lambda_2)} \left( \frac{d\varepsilon_1}{\varepsilon_1} - \frac{d\varepsilon_2}{\varepsilon_2} \right).
\]  

(3.9)

The ratio temperature difference is made up of two terms; one term is the true kinetic temperature change and a second term causes an error due to emissivity changes. The second term is the one of interest. Equation (3.9) indicates that the emissivity errors subtract, thus making the emissivity uncertainty have less influence on the measurement accuracy. If, for instance, \( \varepsilon_1 = 0.99 \pm 0.01 \) and \( \varepsilon_2 = 0.90 \pm 0.01 \), then \( (d\varepsilon_1/\varepsilon_1) = 0.0101 \) and \( (d\varepsilon_2/\varepsilon_2) = 0.111 \). The resulting error in the emissivities would then be \( d(\varepsilon_1/\varepsilon_2)/(\varepsilon_1/\varepsilon_2) = 0.001 \), which is an order of magnitude better than \( d\varepsilon_1/\varepsilon_1 \) or \( d\varepsilon_2/\varepsilon_2 \). This analysis is based on the emissivity uncertainties having a high degree of correlation.
The amount of correlation between $\varepsilon_1$ and $\varepsilon_2$ is a rather subjective argument. An analysis will be carried out in terms of a correlation coefficient, which determines the amount of correlation necessary to have a ratio temperature measurement with the same error as the integral temperature measurement.

If one assumes $d\varepsilon_1/\varepsilon_1$ and $d\varepsilon_2/\varepsilon_2$ are two random variables $\gamma_1$ and $\gamma_2$ with normal (Gaussian) distribution functions and identical deviations, their joint probability density function is (Papoulis, 1965)

$$f_{\gamma_1\gamma_2}(\gamma_1\gamma_2) = \frac{1}{2\pi\sigma^2(1-r^2)^{\frac{1}{2}}} \exp \left[ - \frac{1}{2(1-r^2)} \frac{1}{\sigma^2} \right]$$

$$\left( \gamma_1^2 - 2r\gamma_1\gamma_2 + \gamma_2^2 \right)$$  \hspace{1cm} (3.10)

where $r$ is the correlation coefficient between $\gamma_1$ and $\gamma_2$, and $\sigma$ is the standard deviation of either $\varepsilon_1$ or $\varepsilon_2$.

If $\gamma_1 - \gamma_2$ is written as $\gamma_c$, then the problem is to determine the variance of $\gamma_c$ as a function of the variances of $\gamma_1$ and $\gamma_2$ and the correlation coefficient $r$.

Following Papoulis (1965) one can write

$$\gamma_D = \gamma_1 + \gamma_2.$$  \hspace{1cm} (3.11)

Now solving for $\gamma_1$ and $\gamma_2$

$$\gamma_1 = \frac{1}{2}(\gamma_D + \gamma_c)$$  \hspace{1cm} (3.12)

$$\gamma_2 = \frac{1}{2}(\gamma_D - \gamma_c)$$  \hspace{1cm} (3.13)

where $\gamma_D$ is a coordinate transformation variable.
By direct substitution for \( \gamma_1 \) and \( \gamma_2 \) using Eqs. (3.12) and (3.13)

\[
\int_{\gamma_c \gamma_D}^{\gamma_D} \left( \frac{1}{2\pi\sigma^2(1-r^2)} \exp \left\{ -\frac{1}{4\sigma^2(1-r^2)} \right\} \right) \left[ \gamma_c^2(1+2r) + \gamma_D^2(1-2r) \right] \, d\gamma_c \quad , \quad (3.14)
\]

the density function is Gaussian as expected and is of the general form

\[
\exp\left(-x^2/2\sigma^2\right) , \quad (3.15)
\]

where the variance of \( \gamma_c \) is given by

\[
\sigma_c^2 = \frac{2(1-r^2)\sigma^2}{1 + 2r} . \quad (3.16)
\]

The important result is the value of the correlation when the variance of the ratio temperature is equal to that of the integral temperature or

\[
\sigma_c^2 = \sigma^2 = \frac{2(1-r^2)\sigma^2}{1 + 2r} , \quad (3.17)
\]

which reduces to a quadratic equation in \( r \) with a solution of \( r = 0.366 \). Therefore, if the correlation is equal to or greater than 0.366, ratio temperature will have less error than integral temperature.

The correlation between the emissivities is strongly related to the choice of the two spectral regions. Spectral emissivity changes should be smooth, so the resulting error in the ratio of the emissivities is expected to be very small. This means that the two spectral bands should be close together, favoring the 8 to 14-\( \mu \)m region, where no abrupt emissivity changes are reported. Caution is necessary in
dealing with the 3 to 5-µm region because of the apparent emission band reported by Lloyd-Williams, Case, and Goodwin (1963).

However, one expects the emissivity in both spectral regions to vary in a similar manner, so the ratio is independent over the spatial region being scanned. The situation in which the emissivity of a spatial element in one spectral region increases and in another spectral region decreases is highly unlikely.

**Error Analysis Monochromatic Case**

The ratio temperature and integral temperature thermographs are compared on the basis of errors involved in the measurements. The changes or difference in temperature can be expressed as follows for either type of system

\[ \Delta T = T_1 - T_2. \]  

The accuracy of \( \Delta T \) can be related to how well \( T_1 \) and \( T_2 \) are measured. Mathematically, the errors can be expressed as

\[ \sigma_{\Delta T}^2 = \sigma_{T1}^2 + \sigma_{T2}^2. \]  

If the errors in the measurement of \( T_1 \) and \( T_2 \) are less for one system, then the total error for that system would also be less for that system. Therefore, it remains to be shown that the errors of ratio temperature measurement are smaller than those of integral temperature (Herme, 1953; Pyatt, 1954).

Consider a source having a kinetic temperature, \( T \), with emissivities \( \epsilon_b \), \( \epsilon_1 \), and \( \epsilon_2 \) at wavelengths \( \lambda_b \), \( \lambda_1 \), and \( \lambda_2 \), respectively. Since
Wien's law will be used, a small error is introduced as discussed previously; however, both systems will be evaluated with the same approximations so that error should cancel out to the first order. Since temperature appears in the denominator, for mathematical convenience an error in $1/T$ will be found and then translated to an error in $T$.

For an integral temperature radiometer operating at $\lambda_b$, one can call the measured temperature $T_B$. Equating the measured and predicted radiance one finds from the emittance

$$L_{qb} = c_1 \lambda_b^{-4} \left[ \exp(-c_2/\lambda_b T_B) \right] = \epsilon_b c_1 \lambda_b^{-4} \left[ \exp(-c_2/\lambda_b T) \right];$$  \hspace{1cm} (3.20)

taking the logarithm of both sides

$$\ln \epsilon_b = \frac{c_2}{\lambda_b} \left( \frac{1}{T} - \frac{1}{T_B} \right).$$  \hspace{1cm} (3.21)

Defining the error

$$E_b = \frac{1}{T} - \frac{1}{T_B}.$$  \hspace{1cm} (3.22)

$$E_b = \frac{\lambda_b}{c_2} \ln \epsilon_b.$$  \hspace{1cm} (3.23)

$E_b$ is the systematic error of the integral temperature radiometer. If $T_C$ is the ratio temperature measured between $\lambda_1$ and $\lambda_2$, by recalling Eq. (3.1) with the appropriate substitutions:

$$\frac{\epsilon_1 c_1 \lambda_1^{-4} \left[ \exp(-c_2/\lambda_1 T) \right]}{\epsilon_2 c_1 \lambda_2^{-4} \left[ \exp(-c_2/\lambda_2 T) \right]} = \frac{c_1 \lambda_1^{-4} \left[ \exp(-c_2/\lambda_1 T_C) \right]}{c_1 \lambda_2^{-4} \left[ \exp(-c_2/\lambda_2 T_C) \right]}.$$  \hspace{1cm} (3.24)

Taking the logarithm of the equation, then
In \( \varepsilon_1 / \varepsilon_2 + \frac{c_2}{T} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) \) = \( \frac{c_2}{T_c} \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) \) (3.25)

and

\[ \ln \varepsilon_1 / \varepsilon_2 = -c_2 \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) \left( \frac{1}{T} - \frac{1}{T_c} \right). \] (3.26)

The reciprocal ratio temperature error can be written as

- \( E_c = \frac{1}{T} - \frac{1}{T_c} \) (3.27)

- \( E_c = -\frac{\lambda_1 \lambda_2}{c_2(\lambda_1 - \lambda_2)} \ln \varepsilon_1 / \varepsilon_2. \) (3.28)

As discussed previously, the ratio of the emissivities needs to be equal to one for a correct measurement. However, if the ratio is not unity, the error can be plus or minus unlike the case for integral temperature where it can only be negative.

\( E_b \) and \( E_c \), the systematic errors in the reciprocal temperature, were introduced as a mathematical convenience; the temperature error is of real interest. The corresponding error in the temperature can be expressed as

\[ e_b = T_B - T \] (3.29)

and

\[ e_c = T_c - T. \] (3.30)

With a minor amount of algebra one can obtain \( e_b \) and \( e_c \) from \( E_b \) and \( E_c \).

In order to make a direct comparison between integral and ratio temperature, one can assume that one of the wavelengths, \( \lambda_2 \), is common to both the integral radiometer and the ratio temperature radiometer \((\lambda_b = \lambda_2)\) and \((\varepsilon_b = \varepsilon_2)\).
From Eqs. (3.21) and (3.26)

\[ \frac{1}{T} - \frac{1}{T_B} = \frac{\lambda_2}{c_2} \ln \varepsilon_2. \]  \hspace{1cm} (3.31)

\[ \frac{1}{T} - \frac{1}{T_C} = \frac{-\lambda_1 \lambda_2}{c_2(\lambda_1 - \lambda_2)} \ln \varepsilon_2/\varepsilon_2. \]  \hspace{1cm} (3.32)

the solution of \( \ln \varepsilon_1 \) in terms of \( \ln \varepsilon_2 \) can be found (see Appendix F for details)

\[ \ln \varepsilon_1 = \ln \varepsilon_2 \left\{ 1 + \frac{\lambda_1 - \lambda_2}{\lambda_1} \left[ \frac{T \lambda_2}{c_2} \ln \varepsilon_2 + \frac{e_b}{e_c} \left( 1 - \frac{T \lambda_2}{c_2} \ln \varepsilon_2 \right) \right] \right\}. \]  \hspace{1cm} (3.33)

The error \( e_b \) can be negative only; however, \( e_c \) can be positive or negative depending on the selectivity of the surface emissivities versus wavelength, the worst case being \( \varepsilon_1 \) with a positive error and \( \varepsilon_2 \) with a negative error.

Therefore a range of emissivity values can be defined for any given ratio of errors in temperature

\[ \ln \varepsilon_2 \left\{ 1 + \frac{\lambda_2 - \lambda_1}{\lambda_1} \left[ \frac{T \lambda_2}{c_2} \ln \varepsilon_2 + \frac{e_b}{e_c} \left( 1 - \frac{T \lambda_2}{c_2} \ln \varepsilon_2 \right) \right] \right\} \]

\[ < \ln \varepsilon_1 \]

\[ < \ln \varepsilon_2 \left\{ 1 + \frac{\lambda_2 - \lambda_1}{\lambda_1} \left[ \frac{T \lambda_2}{c_2} \ln \varepsilon_2 - \frac{e_b}{e_c} \left( 1 - \frac{T \lambda_2}{c_2} \ln \varepsilon_2 \right) \right] \right\}. \]  \hspace{1cm} (3.34)
The ratio of errors in temperature can be less than, equal to, or greater than one, depending on whether the integral temperature has less error, the same error, or more error than the ratio temperature. A family of curves for $\varepsilon_1$ versus $\varepsilon_2$ can be plotted as a function of the ratio of the temperature errors for a given system ($\lambda$, $\lambda_2$, and $T$ defined). The case when the errors are equal ($e_b = e_c$) form a locus of points that defines two regions of minimum temperature error. A plot of $\varepsilon_1$ versus $\varepsilon_2$ is shown in Fig. 3.2 for $\lambda = 5$ $\mu$m and $\lambda = 13.5$ $\mu$m and an object temperature of 310°K.

The shaded area is the region in which the ratio temperature has less error than integral temperature ($e_b/e_c > 1$). Shown in the shaded area is a region representing the best guess as to the emissivity of skin (Skeketee, 1973a) at 5 $\mu$m and 13.5 $\mu$m. Clearly it is in this region that ratio temperature is a more accurate measurement.

**Error Analysis, Finite Bandwidths**

Real radiometers measure irradiances in finite spectral bands, and therefore for a real case the irradiance is integrated over finite spectral regions around $\lambda_1$ and $\lambda_2$. The monochromatic case treated previously never exists.

Instrumentally the following equation describes color temperature

\[
\frac{\int_{\Delta \lambda} L(T, \lambda_1) \tau(\lambda_1) \tau_f(\lambda_1) d\lambda}{\int_{\Delta \lambda_2} L(T, \lambda_2) \tau(\lambda_2) \tau_f(\lambda_2) d\lambda} = \frac{D(T, \Delta \lambda_1)}{D(T, \Delta \lambda_2)}, \tag{3.35}
\]

where
Fig. 3.2. Boundary Regions for Integral and Ratio Temperature Radiometers.
where \( D(T, \Delta \lambda_1) \) is the irradiance measured over a finite bandwidth \( \Delta \lambda_1 \), \( D(T, \Delta \lambda_2) \) is the irradiance measured over a finite bandwidth \( \Delta \lambda_2 \), \( \tau(\lambda) \) is the atmospheric transmission, and \( \tau_f(\lambda) \) is the filter transmission.

For a manageable calculation one assumes the emissivity to be nonselective, the atmospheric transmission constant over the spectral regions, and the filters to have equal transmission with rectangular spectral characteristics. Then

\[
\int_{\Delta \lambda_1} \frac{\lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T_C} \right) - 1 \right]^{-1} d\lambda}{\int_{\Delta \lambda_2} \lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T_C} \right) - 1 \right]^{-1} d\lambda} = \frac{D(T, \Delta \lambda_1)}{D(T, \Delta \lambda_2)} .
\]

This integral and the results for some representative systems are shown in Appendix H. The linearity with temperature is as expected from the monochromatic case. However, of most interest is that the width of the spectral bandwidth is not of great concern; in fact, from an instrumental standpoint the widest bandwidth is best. Therefore, by extension to the finite spectral region one still has a pseudo-linear relationship like that predicted by the monochromatic analysis.

The choice of \( \lambda_1 \) and \( \lambda_2 \) is governed by several qualitative factors. The slope of the curves in Appendix H for finite spectral regions is determined by the separation of \( \lambda_1 \) and \( \lambda_2 \); the greater the wavelength separation, the greater the slope and correspondingly, the temperature sensitivity.
In addition, the wavelength bands must be within the spectral response of the detectors and also within an atmospheric window.

Competing with the desire for wide spectral separation is the problem of emissivity variations, which tends to favor the condition that the two wavelengths be close together, and is a factor that favors the 8 to 14-\(\mu\)m region, where no emissivity changes are reported. Thus, it might prove best to pick two wavelengths in this spectral bandwidth. Caution is necessary in dealing with the 3 to 5-\(\mu\)m region because of the possible emission bands reported by Lloyd-Williams et al. (1963).
CHAPTER 4

SYSTEM DESCRIPTION OF A TWO-COLOR THERMOGRAPH

Instrument Design Approach

The thermographic system used for the experimental verification of the theory is a line scanner that produces a raster display of a thermal scene. The block diagram of the thermograph is shown in Fig. 4.1. The instrument measures the irradiance on its aperture from the scene in two spectral channels at 5 and 13 μm. The instrument has a 1-mrad instantaneous field of view. It can scan a raster of 60° by 10° using the Kennedy scanner (Kennedy, 1965). The noise equivalent temperature difference (NETD) for each channel used individually is about 0.15°K. The horizontal scan is accomplished by rotating a four-sided mirror; the vertical scan is accomplished by a mirror oscillating in a linear fashion. A total frame can be scanned in 15 sec. The energy is focused onto the focal plane assembly in a liquid-helium dewar flask by an off-axis paraboloid. A sapphire beamsplitter in the focal plane assembly reflects the radiation of wavelengths longer than 11.5 μm and transmits that of wavelengths shorter than 7 μm. The power of each beam is then passed through spectral filters before impinging upon the field stops. The field stops are aligned at the focal point, assuring picture registration during the ratioing process. Detectors are placed behind the field stops.

The signals are amplified and blanking is provided to display the scene of interest for each channel. The signals are then ratioed and/or
Fig. 4.1. Thermograph Block Diagram.
subtracted and displayed. Therefore, three pictures can be obtained simultaneously, 5 μm, 13 μm, and their ratio.

The salient features of the equipment design and construction were registration of the two fields of view by the use of a beamsplitter, safety and reliability, and cost effectiveness with the use of many available components. The largest drawback in this approach has been the use of a liquid-helium dewar flask. The main subassemblies are the focal plane components, optical scanner and images, calibration sources, signal-processing electronics, scan converter, and display. These are discussed below.

**Focal Plane Assembly**

The focal plane assembly is the heart of the entire system. It is a self-contained unit that can be considered a two-channel infrared radiometer. The unit shown in Figs. 4.2 and 4.3 consists of the dewar flask, infrared detectors, dichroic beamsplitter, spectral filters, preamplifiers, and power supply. The large flask is evacuated and contains the cryogenic fluids. The box on the side is the preamplifier electronics used to amplify and limit the bandwidth of the signal.

The dewar flask is a dual-cryogen type that uses both liquid helium and liquid nitrogen. A cross-section of it is shown in Fig. 4.4. The inner flask is filled with the liquid helium (4°K). The liquid nitrogen (77°K) upper flask has a radiation shield connected to it. This shield eliminates direct ambient-temperature radiation from causing a heat load on the helium flask. These subassemblies are made of stainless steel except for the heat sinks, which are copper, and the radiation
Fig. 4.2. Focal Plane Assembly.
Fig. 4.3. Focal Plane Assembly from Opposite Direction.
Fig. 4.4. Dewar Cross Section.
shield, which is aluminum. The outer shell is made of aluminum. It contains the vacuum pumping port, infrared windows, and hermetically sealed electrical feedthroughs. The hold time for the liquid nitrogen is about 24 hours. The liquid helium will hold for about 50 hours provided the liquid nitrogen is refilled. The window is potassium chloride (KCl), which has a uniform transmission over the spectral regions of interest (Fig. 4.5). This window is very hygroscopic, so care must be exercised in handling and protecting it. The baffle tube in the radiation shield is painted with 3M black paint (Pipher and Houck, 1971) to reduce scattered background into the inner cavity where the detectors are located.

All electrical wiring between the liquid-helium heat sink and the room-temperature connector is of 0.005-in.-diameter constantan to minimize conduction of heat into the flask. The detectors, filters, and beamsplitter are all mounted into a copper subassembly that mounts on the heat sink (Fig. 4.6). The advantage of this is that optical alignment and registration can be accomplished prior to fastening to the dewar and heat sink. The outer surface is buffed to a shine to reduce radiation losses.

An optical schematic is shown in Fig. 4.7, showing the F/13 beam focused on the detectors. The sapphire beamsplitter transmits the 5-μm (short) wavelength channel and reflects the 12-μm (long) wavelength channel. This materials provides a dichroic that is inexpensive and of high efficiency. The transmission and reflection curves of sapphire measured at room temperature are shown in Fig. 4.8. The reflection in
Fig. 4.5. Transmission of Potassium Chloride (KCl).
Fig. 4.6. Beamsplitter Detector Holder.
Fig. 4.7. Optical Layout of Focal Plane Assembly.
Fig. 4.8. Transmission and Reflection of Sapphire (Al₂O₃).
the 13-μm region is dependent on the reststrahlen bands. This caused some concern as to the effect of liquid-helium temperature operation; however, direct measurements showed that the wavelength shift was insignificant.

The spectral response of each channel was determined by an interference filter placed at the field stop for each channel. The 5-μm channel was determined by a single filter whose response is shown in Fig. 4.9. The 13-μm channel was determined by a long-pass filter (shown in Fig. 4.10) and barium fluoride (BaF₂) to define the long wavelength cutoff (shown in Fig. 4.11).

An indium antimonide photovoltaic detector was used for the 5-μm channel. A bismuth-doped silicon (Si:Bi), which is photoconductive, was used for the 13-μm channel. Their spectral responses are shown in Fig. 4.12 (Eisenman, 1975). Both detectors had a 0.040-in.-square sensitive area. An aperture of 0.030 in. was placed above the detector to act as the field stop. The 0.030-in. apertures were aligned optically and in co-registration by back illuminating each and assuring registration. This optical alignment assures picture registration since each picture element is viewed simultaneously by each channel. Background radiation is reduced by circular field stops extending out from the detector painted with black 3M paint. See Appendix I for the calculations of background flux \( (E_q) \) on the detector's surface.

The load resistor associated with each detector is mounted directly on the liquid-helium heat sink on sapphire standoffs. This procedure was developed to reduce the stray capacitance between the load
Fig. 4.9. 5-μm Channel Filter Transmission.

Fig. 10. 13-μm Channel Filter Transmission.
Fig. 4.11. Barium Fluoride Transmission vs Wavelength.

Fig. 4.12. Detector Spectral Response.
resistor and the ground plane, which causes oscillation and frequency bandpass reduction in the preamplifier.

13-\textmu m Channel Electronics

The 13-\textmu m channel uses a bismuth-doped silicon (Si:Bi) photodetector. To sense the resistance change a bias supply and load resistor are used as shown in Fig. 4.13. A change in detector resistance will produce a change in voltage, $V_d$, by the voltage divider network shown in Fig. 4.13. The mathematical representation of the voltage divider detector circuit is

$$V_d = \frac{R_d}{R_f + R_d} V_b$$

(4.1)

where an incremental change of $V_d$ with respect to a change in $R_d$ can be considered the signal output

$$\Delta V_d = \frac{R_f \Delta R_d}{(R_f + R_d)^2} V_b.$$  (4.2)

If the radiation is amplitude modulated, an alternating voltage will appear across the detector. The detector responsivity ($S_i$) is the signal current (or voltage) produced at the detector output from incident unit radiant power on the detector. Typically, the responsivity varies from 1 to 5 ampere/watt. Voltage responsivity is not significant because it is a function of the load resistor ($R_f$) as shown in Eq. (4.2). A typical low-noise voltage amplifier used to amplify these signal voltages is shown in Fig. 4.14.
Fig. 4.13. Photoconductor Biasing Circuit.
Fig. 4.14. Photoconductor Voltage Gain Preamplifier.
The resistance of these high-impedance photoconductive detectors is determined by the background photon flux on the detector (Dereniak and Wolfe, 1970). For very low backgrounds, the resistance can become extremely high, since the resistance is inversely proportional to the carrier concentration or the number of photons absorbed.

The capacitance of a detector can be found from

\[ C_d = \frac{D D_0 A_d}{d} = \frac{A_d D}{d} \cdot 8.84 \times 10^{-14}, \]  

with the capacitance of a typical detector being about 1 pf. This capacitance is insignificant when compared to the stray capacitance of the wiring, feedthroughs, etc.

The noise equivalent power (NEP) is the system noise \( i_n \) divided by the responsivity

\[ \text{NEP} = \frac{i_n}{S_1}. \]  

The current noises that are present in these detectors are Johnson noise, background photon noise, and \( 1/f \) noise (assuming a low noise preamplifier). Their respective expressions are

\[ i_j = \left( \frac{4KTA_f}{R_f} + \frac{4KTA_f}{d} \right)^{\frac{1}{2}}, \]  

\[ i_p = 2eQ(nA_dE_qA_f)^{\frac{1}{2}}, \]  

\[ i_f = (\beta f^{-\alpha})^{\frac{1}{2}}. \]
therefore
\[ \text{NEP} = \left( \frac{i_j^2 + i_p^2 + i_f^2}{S_i} \right)^{\frac{1}{2}} \]  
(4.8)

The 1/f noise is always present in a photoconductor since current \( i \) is always flowing through it, although the effect of this noise can be reduced by proper choice of detector and operating conditions.

If the limiting noise on the detector is the fluctuation in the arrival rate of the photons from the background, the detector is said to be a BLIP or background limited infrared photodetector. This condition is a fundamental limit for a system, being determined by the detector characteristics and the background level. The Johnson noise effect, on the other hand, can be made arbitrarily small compared to photon noise by increasing \( R_f \). It is desirable to select \( R_f \) to ensure background limited operation.

The expressions for the Johnson and photon noise voltages are, respectively,
\[ V_j = (4KTR_f \Delta f)^{\frac{1}{2}} \]  
(4.9)
\[ V_p = 2eQ(nE_q A_d \Delta f)^{\frac{1}{2}} R_f. \]  
(4.10)

Assuming for practical purposes the Johnson noise should be 1/5 the photon noise, one can write
\[ (4KTR_f \Delta f)^{\frac{1}{2}} = \frac{2eQ}{5} (nE_q A_d \Delta f)^{\frac{1}{2}} R_f. \]  
(4.11)

Rearranging terms,
\[ R_f E_q = (4KT/nA_d)(5/2eQ)^2, \]  
(4.12)
where

\[ E_q = \text{irradiance of the background (photon/sec-cm}^2) \]
\[ Q = \text{photoconductive gain} \]
\[ n = \text{quantum efficiency} \]
\[ e = \text{electron charge (}1.6 \times 10^{-19} \text{ J-}^\circ\text{K}) \]
\[ A_d = \text{detector area} \]
\[ k = \text{Boltzmann constant (}1.38 \times 10^{-23} \text{ J-}^\circ\text{K}) \]
\[ T_d = \text{detector temperature}. \]

This expression shows the desired relationship between background photon irradiance and load resistance. For any given detector, the right-hand side of the expression is constant, and therefore the minimum value for the load resistance is specified. Larger values are acceptable; however, they require a larger dynamic range for the bias voltage that must be applied to the detector and load resistor combination.

Dereniak and Wolfe (1970) discussed the product of the detector resistance and the background irradiance as a constant over a specified range of background flux. Typically, this constant is between \(10^{22}\) to \(5 \times 10^{22}\) ohm photon/sec-cm\(^2\). This expression can be used to relate load resistance to detector resistance on an order of magnitude basis

\[ R_d E_q = 5 \times 10^{22}. \quad (4.13) \]

This expression can be used to obtain a quantitative relation between the load resistor and detector resistor

\[ R_f/R_d \geq (4kT/nA_d) \frac{10^{-22}}{5} (5/2eQ)^2. \quad (4.14) \]
The inequality can be used since this expression is for the minimum load resistor value that achieves BLIP operation.

Note: For any detector where $A_d$, $n$, Q, and T are known (invariant) parameters, the $R_f/R_d$ ratio is a "constant" to maintain BLIP operation. For example, in the Si:Bi used, $n = 0.30$, $A_d = 1.6 \times 10^{-3}$ cm, $Q = 0.5$, and $T = 5^\circ K$ giving a ratio $R_f/R_d$ of 0.005. Therefore, if the load resistor $R_f$ is less than 0.005 times the detector resistance $R_d$, Johnson noise becomes predominant. This is an important criterion in determining the smallest load resistor one can use in a system to be BLIP operated.

**Preamplifier Noise Sources.** Feedback amplifiers have been used with photovoltaic infrared detectors for many years; however, they have not been used very extensively with cryogenically cooled photoconductive detectors. The circuit for the feedback amplifier used is shown in Fig. 4.15.

One can derive an ideal and noiseless feedback preamplifier as shown in Fig. 4.16 including the current and voltage noise generators at the input of the preamplifier.

Simple operational-amplifier theory (Wait, Huelsman, and Korn, 1975) can be used to show that the gain of the circuit in Fig. 4.16 is

$$G = \frac{R_f}{R_d}.$$

For dynamic range considerations $R_f$ must be much less than $R_d$ because the output voltage swing of the operational amplifier is limited. One
Fig. 4.15. Feedback Preamplifier.
Fig. 4.16. Feedback Preamplifier Noise Sources.
can also show that the input impedance to the feedback preamplifiers is \( R_f/A \).

The output noise can be found by taking the square root of the sum of the squares of each type of noise, since the noises are statistically independent random stochastic processes.

In order to be detector noise limited these noises must be much less than the detector noise. Considering the current noise first, the input noise voltage \( e_i' \) due to this noise current is

\[
e_i' = i_n \frac{R_d R_f}{R_d + R_f}.
\]  

However, due to negative feedback, part of the output voltage is at the input to the operational amplifier; this fraction can be called \( B \). Therefore, the output noise voltage appearing would be

\[
e_i = (e_i' + Be_i)A
\]

where \( A \) is the open loop gain;

\[
B = \frac{R_d}{R_f + R_d}.
\]

Then

\[
e_i = \frac{Ae_i'}{1 - AB} = \frac{1}{B} \frac{1}{(1/AB - 1)} i_n \frac{R_d R_f}{R_d + R_f}.
\]

Since \( R_d \gg R_f \),

\[
e_i = \frac{1}{(1/AB - 1)} R_f i_n.
\]
Since AB is large,

\[ e_i = -i_n R_f. \]  (4.21)

This is the noise voltage at the output due to noise current at the input of the preamplifier. The noise current is caused by the leakage current of the junction field-effect transistor (JFET) gate. To relate noise current to leakage current the following shot noise relationship holds:

\[ i_n = (2eI\Delta f)^{\frac{1}{2}} \]  (4.22)

where

- \( e \) = electron charge \((1.6 \times 10^{-19})\) (coulomb)
- \( I \) = leakage current (amperes)
- \( \Delta f \) = electrical bandwidth (Hz).

In similar fashion, the output noise voltage \( e_v \) due to a noise voltage source is

\[ e_v = A(e_n + Be_v) \]  (4.23)

or if \( AB \gg 1 \)

\[ e_v = \frac{-e_n}{B}. \]  (4.24)

Finally,

\[ e_v = -e_n \left( \frac{R_f}{R_d} + 1 \right). \]  (4.25)

This is a startling result! It says the noise voltage is amplified by the gain plus one. Since for these detectors, due to dynamic range
requirements, the gain is much less than one, $R_f/R_d \ll 1$, the noise voltage becomes a very significant factor. This implies the signal is amplified by a gain less than one, while the preamplifier noise is amplified by a factor larger than one. For this reason selection of a low-voltage noise FET is very important. Experience has shown that N-channel FETs have lower noise characteristics than P-channel types, perhaps because the majority of carriers are electrons, which have a higher mobility than holes.

The sum of the noises can be expressed as

$$e_T = \left[ (2eI\Delta fR_f)^2 + e_n^2 \left( \frac{R_f}{R_d} + 1 \right)^2 \Delta f \right]^{1/2} .$$

(4.26)

**Fixed Detector Bias.** By varying the bias across the photoconductor detector, one can find an optimum bias that will give the best NEP or $D^*$ (Wolfe, 1965). The feedback amplifiers maintain this optimum bias once it is set by means of feedback for different background flux values. In a servo-type arrangement, the feedback produces a virtual ground at the input to the amplifier. The actual input impedance is $Z_f/A$ as stated earlier, and is in parallel with the detector. The feedback amplifier by way of the virtual ground, assures constant bias across the detector at any background level.

**Microphonics.** The most important source of microphonics in this instrument is the sound waves generated by the rotating mirrors. These in turn move the leads, detector, or load resistor, thereby modulating $C_s$, the parallel combination of the detector capacitance and the stray capacitances from the load resistor and detector (Fig. 4.17).
Fig. 4.17. Detector-Load Resistor with Associated Capacitances.
The transfer function of the voltage amplifier can be written from inspection of Fig. 4.18 as

\[ e_0 = \frac{V_b}{R_d + \left(\frac{R_f}{1 + SR_fC_s}\right)} \cdot \frac{R_f}{1 + SR_fC_s} \]

\[ = \frac{V_b R_f}{R_d(1 + SR_fC_s) + R_f} \cdot \]  \hspace{1cm} (4.27)

Typically \( R_d > 10 R_f \), so

\[ e_0 = \frac{V_b R_f}{R_d} \cdot \frac{1}{1 + SR_fC_s} \cdot \]  \hspace{1cm} (4.28)

The microphonic signal is generated by the change in \( e_0 \) caused by a change in \( C_s \). The derivative gives the analytic expression for microphonic noise for the voltage amplifier

\[ de_0 = \frac{V_b R_f}{R_d} \cdot \frac{SR_f}{(1 + SR_fC_s)^2} \cdot dC_s. \]  \hspace{1cm} (4.29)

Similarly, one is able to consider the feedback amplifier shown in Fig. 4.19. The distributed capacitance along \( R_f \) has been evaluated by Hall et al. (1975). All the capacitances are again lumped into the \( C_s \) term, which appears at the input to the feedback preamplifier.

The transfer function for Fig. 4.19 is

\[ I_d = e_1 \left( \frac{1}{R_d} + SC_s \right) + (e_1 - e_0) \frac{1}{R_f} \]

\[ I_d = e_1 \left( \frac{1}{R_d} + \frac{1}{R_f} + SC_s \right) - \frac{e_0}{R_f} \]  \hspace{1cm} (4.30)
Fig. 4.18. Voltage Preamplifier Input with Lumped Capacitances.

Fig. 4.19. Feedback Preamplifier with Associated Capacitances.
\[ e_i = \frac{e_0}{A} \quad (4.31) \]

\[ I_d = \frac{e_0}{A} \left( \frac{1}{R_d} + \frac{1}{R_f} + SC_s \right) - \frac{e_0}{R_f}. \quad (4.32) \]

If \( R_d \gg R_f \), then \( e_0 \) is given by

\[ e_0 = \frac{V_b R_f}{R_d} \cdot \frac{1}{\left( A - SC_s R_f \right)} \quad (4.33) \]

where \( I_d = V_b/R_d \). Thus

\[ e_0 = \frac{V_b R_f}{R_d} \cdot \frac{1}{1 - (SC_s R_f/A)}. \quad (4.34) \]

The microphonic voltage \( \delta e_0 \) is given by

\[ \delta e_0 = \frac{V_b R_f}{R_d} \cdot \frac{SR_f/A}{\left(1 - SC_s R_f\right)^2} dC_s. \quad (4.35) \]

Comparison of equations (4.29) and (4.35) shows that the microphonic noise has been reduced by approximately the open loop gain of the circuit. Therefore, the feedback amplifier has reduced the effects of microphonics. This result, although it was a spinoff from the original design, became very significant in the final system using a rotating mirror for scanning (which introduced vibration into the system).

**Frequency Response.** It should be obvious that the frequency response of a feedback system is larger than the standard voltage preamplifier since the detector is shunted by an impedance of \( R_f/A \), as stated earlier. However, a more detailed analysis is required to give a quantitative estimate of this improvement.
A comparison of the feedback circuit in Fig. 4.19 and the more common voltage preamplifier in Fig. 4.14 can be made by evaluating the influence of the capacitances.

First consider Fig. 4.14 in a Norton equivalent circuit as shown in Fig. 4.20 where \( I_d \) is the signal current produced by a change in resistance (from Eq. (4.2))

\[
\Delta I_d = \frac{V_b}{R_d^2} \Delta R
\]  

(4.36)

where \( R_d \gg R_L \) and \( R_d \gg \Delta R_d \).

The output voltage is

\[
V_0 = \Delta I_d \left( \frac{1}{S(C_d + C_f) + \frac{1}{R_f} + \frac{1}{R_d}} \right)
\]  

(4.37)

A transfer expression \((V_0/\Delta R_d)\) can now be found. This transfer expression will be defined as the output voltage due to a change in detector resistance. This is unlike the normal transfer function in circuit theory. It is

\[
\frac{V_0}{\Delta R} = \frac{V_B}{R_d^2} \frac{1}{C_f + C_d} \frac{1}{S + \left( \frac{R_f R_d}{R_f + R_d} \right) (C_f + C_d)}^{-1}
\]  

(4.38)

For the typical situation, \( R_d \gg R_L \); therefore, the time constant associated with Eq. (4.38) is

\[
R_f (C_f + C_d).
\]  

(4.39)

This time constant determines the high frequency cutoff. Note that the detector capacitance adds to the feedback capacitance.
Fig. 4.20. Voltage Preamplifier Equivalent Circuit.
For the feedback preamplifier shown in Fig. 4.19 the output voltage can be shown to be

\[
\Delta V_0 = \frac{\Delta I_d}{S(C_d + C_f - AC_f)} + \frac{A}{1/R_d} + \left[\left(\frac{1 - A}{R_f}\right)\right] \cdot (4.40)
\]

The open loop gain A is very large, typically 100 dB. Assuming A >> 0, the transfer expression for Eq. (109) becomes

\[
\frac{V_0}{\Delta R} = \frac{V_B}{R_d^2} \frac{C_f}{S + \frac{1}{(1/R_f C_f)}} \cdot \cdot \cdot (4.41)
\]

The time constant is R_f C_f. This time constant is smaller than that in Eq. (4.39) for a voltage preamplifier, thus yielding a higher frequency response. The result is due to the effect of feedback, which causes an impedance of Z_f/A across the detector. It is interesting to note that the detector impedance, either resistive or capacitive, affects the frequency response. This indicates that load resistor selection and use (mounting) is very critical.

The second stage of Fig. 4.15 determines the gain and the bandwidth response of the 13-\(\mu\)m channel. The upper frequency cutoff is determined by the RC of the feedback resistor and capacitor, which are 100 k\(\Omega\) and 69 pf, respectively. However, because of the gain-bandwidth product of the 3542J operational amplifier, it in fact limits the frequency response to 16 kHz for a gain of 10.

The noise budget shows that the 13-\(\mu\)m channel is BLIP operated. The input noise voltage of the JFET is 6 nV/(Hz)\(^{1/2}\) max, the Johnson
of the load resistor (2 MΩ at LHe temperature) is 21 nV/(Hz)\(^{1/2}\). The background flux is \(1.1 \times 10^{14}\) photons/sec/cm\(^2\) calculated in Appendix I. This corresponds to a photon noise, from Eq. (4.10) of 270 nV/(Hz)\(^{1/2}\).

5-µm Channel Electronics

The 5-µm channel uses an indium antimonide InSb photovoltaic detector, which requires cryogenic temperature operation. This photovoltaic detector is a P-N junction diode constructed from an intrinsic semiconductor, in this case from p-type and n-type InSb. The photons produce hole-electron pairs in the P-N junction, which has an inherent electric field, thus causing a current to flow. Since recombination does not occur in photovoltaic detectors, they have an inherently higher detectivity by \((2)^{\frac{1}{2}}\) (unless there is sweepout in the photoconductor).

The typical V-I characteristic is shown in Fig. 4.21. When the detector is shielded to a very low background \((E_b = 0)\), curve 1 for the dark-current condition applies. When radiation is incident on the detector, curve 2 applies. The signal is produced by a change in current or voltage induced by a radiation change. The input impedance of the coupling circuit determines a load line in Fig. 4.21. A photovoltaic detector exhibits its highest detectivity when operated into a short circuit or when the load line is coincident with the current axis. The detector resistance is then the slope \((dV/dI)\) of the curve at \(V = 0\). The operational amplifier feedback circuit shown in Fig. 4.22 is normally used to produce a virtual ground or short circuit for these detectors. Actually, as discussed earlier, the input impedance is \(Z_f/A\).
Fig. 4.21. V-I Characteristic of Photovoltaic Detectors.

Fig. 4.22. Operational Amplifier Feedback Circuit.
The signal and background current generated by the detector flows through the feedback resistor and yields signal and background voltages at point A.

The preamplifier design that finally was used is shown in Fig. 4.23 (Hall et al., 1975). The detector and feedback resistor ($R_f$) were at liquid-helium temperature ($4.2^\circ K$). The low noise JFET was placed on the liquid nitrogen shield ($77^\circ K$). The components were placed at a cryogenic temperature near the detector in order to eliminate stray capacitance associated with long input leads. The feedback resistor was placed at liquid-helium temperature in order to reduce its Johnson noise. The JFET could not operate at $4^\circ K$ so it was mounted on the nearby $77^\circ K$ surface. The noise sources that must be considered for this detector are photon (shot current) noise, Johnson noise, 1/f noise, microphonic noise, and JFET noise.

The noise contribution by the JFET was analyzed for the 13-µm channel. The same analysis is applicable in this case since both preamplifiers use a feedback configuration. However, for this preamplifier the JFET noise is reduced because it is operated at liquid nitrogen temperature. The gate leakage current is also reduced because of the cooling, thus producing a very low current noise in the detector. As a result of this cooling, the magnitudes of these noises are negligible when compared to the other noise sources; therefore, they can be neglected.
Fig. 4.23. Photovoltaic Preamplifier Design.
The remaining significant noise contributors are listed below.

--Photon (shot current) noise
\[
\Delta V_p = \sqrt{\frac{2eI\Delta f}{R_f}} = \sqrt{(2e\Delta f V_0 R_f)} \quad (4.42)
\]

--Johnson noise
\[
\Delta V_J = \left(\frac{4K T \Delta f}{R_d} + \frac{2K T R_f \Delta f}{R_f}ight) \quad (4.43)
\]

--1/f noise
\[
\Delta V_F = (\beta I f^{-\alpha} \Delta f) \quad (4.44)
\]

Two of these noises depend on the current I flowing through the detector. By varying the offset adjustment (see Fig. 4.23) the voltage across the detector is varied. With no background flux present, the detector current follows curve 1 on the V-I curve in Fig. 4.21. By appropriate adjustment of the offset, the detector current can be set to zero, thereby eliminating the 1/f noise if photocurrent (background) is not present. This zero current point also corresponds to zero voltage across the detector inasmuch as curve 1 passes through the origin. This condition constitutes an optimum operating point for the photodetector. When detectable photons strike the active area of the photodetector, a photocurrent is generated, represented by curve 2. Inasmuch as there is no detector bias voltage, the operating current point falls on the current axis (I). A change in current without a corresponding change in voltage indicates that the input resistance is zero, which is the case for an operational amplifier in a feedback configuration.
Visual observations on an oscilloscope provided a qualitative verification of the reduction of 1/f noise. As the offset adjustment is varied, the peak-to-peak magnitude of the noise varies, passing through a minimum at a point corresponding to zero voltage across the detector. In addition, the noise appearing on the oscilloscope exhibits a reduction in lower frequency noise components.

The current flowing through the detector is a result of the incident background photon flux as shown in Fig. 4.21.

The remaining two noises that must be considered are photon noise and Johnson noise. Actually, the voltage across the detector is about -10 mV. This is due to the leakage current and offset voltages of the operational amplifier.

The detector resistance is determined by varying the voltage across the detector and measuring the output voltage swing ($\Delta E_0$) at point A (Fig. 4.22)

$$R_d = \frac{\Delta E}{\Delta E_0} R_f.$$  \hspace{1cm} (4.45)

The choice of load resistor is critical for four reasons: Johnson noise, dynamic range, responsivity, and frequency response. If the photon flux is low, the system will become Johnson noise limited. For the background flux derived in Appendix I, a value of load resistance can be found for BLIP operation.

Following a procedure similar to that discussed for the photoconductor, where $i_p \gg i_j$, one can find an expression for the ratio of feedback resistance to temperature ($R_f/T$) to maintain BLIP operation.
We can assume that a factor of 10 is sufficient for the ratio of photon noise to Johnson noise. Then

\[(2e^2\eta N)^{\frac{1}{2}} \gg \left(\frac{4KT}{R_f}\right)^{\frac{1}{2}}.\]  \hspace{1cm} (4.46)

For the background flux derived in Appendix H and a quantum efficiency of 0.63 quoted by the manufacturer, the resistance-to-temperature ratio is

\[
(2e^2\eta N)^{\frac{1}{2}} = 10 \left(\frac{4KT}{R_f}\right)^{\frac{1}{2}} \hspace{1cm} (4.47)
\]
or

\[
\frac{R_f}{T} = 100 \frac{4K}{2e^2\eta N}. \hspace{1cm} (4.48)
\]

Therefore, at room temperature the feedback resistance must be 30 M\(\Omega\) or larger. However, at 4°K it only has to be 400 k\(\Omega\). Thus, the cooling of the feedback resistor is extremely important.

A 10-M\(\Omega\) resistor was used at 4°K to make the responsivity high enough to couple the signal to the next stage and yet not impede the dynamic range of the output of the operational amplifier.

A special fixture was developed for mounting the feedback resistor to the liquid helium heat sink. The stray capacitance that was picked up along the resistor and the heat sink in the original mounting scheme was sufficient to make the feedback amplifier oscillate. This capacitance was shown in Fig. 4.17 at \(C_s'\). At high frequencies the gain
of the feedback amplifier is $Z_f/Z_d$ or $(C_d + C_s)/C_f$; if $C_d + C_s$ is large, the system will oscillate. To eliminate the large stray capacitance the resistor was mounted on sapphire standoffs as shown in Fig. 4.24. Sapphire is a good electrical insulator and a good thermal conductor, thus keeping the resistor cold.

The noise of this channel was also determined by the background flux, the Johnson noise contribution being $46 \text{nV}/(\text{Hz})^{\frac{1}{2}}$. The photon (shot) noise, using the background flux of $1.6 \times 10^{13}$ photons/sec-cm$^2$ determined in Appendix I, is $423 \text{nV}/(\text{Hz})^{\frac{1}{2}}$. This channel is also background noise limited.

The frequency response is determined by the second stage amplifier shown in Fig. 4.15. The bandwidth is 16 kHz with a gain of 10.

NEP and $D^*$ Calculations

The noise equivalent power (NEP) is the incident power at the detector required to produce an rms signal-to-noise ratio of one at the output for a specified frequency bandwidth

$$\text{NEP} = \phi_d/(S/N)$$

(4.50)

where

$$\phi_d = \frac{L_s A_c A_{bb}}{R^2} F_f v_0$$

(4.51)

$A_c = \text{collecting area}$

$A_{bb} = \text{blackbody area}$

$R = \text{range}$
Fig. 4.24. Liquid Helium Resistor Mounting.
\( F_p = \) form factor, peak-to-peak square wave to rms of fundamental component (0.45)

\( \tau_0 = \) optical transmission \((\tau_{KCl})(\tau_{\text{filter}})\)

\( L_s = \) radiance of the blackbody signal.

Appendix J provides an evaluation of \( \phi_d \) for the two spectral regions of the focal plane assembly.

Table 4.1 shows the data collected for each channel and Fig. 4.25 shows the laboratory test setup used for the data collection. The electrical bandwidth used for data collection was 1 Hz.

The NEP measured on the two channels are

- 5-μm channel
  \[
  \text{NEP} = 8 \times 10^{-14} \text{ W/(Hz)}^{\frac{1}{2}} \tag{4.52}
  \]

- 13-μm channel
  \[
  \text{NEP} = 1 \times 10^{-13} \text{ W/(Hz)}^{\frac{1}{2}}. \tag{4.53}
  \]

The corresponding D* values measured in each spectral region are

- 5-μm channel
  \[
  D_{\lambda}^* = 8.3 \times 10^{11} \text{ cm}^{\frac{1}{2}}/\text{W} \tag{4.54}
  \]

- 13-μm channel
  \[
  D_{\lambda}^* = 6.6 \times 10^{11} \text{ cm}^{\frac{1}{2}}/\text{W}. \tag{4.55}
  \]

The frequency response of the system is 16 kHz for each channel. The system is detector noise limited, and the noise is a function of background; however, the noise was higher than that predicted by the BLIP noise equation. This was shown by noting that the noise was less with no bias on the detector and also less when the field of view was pointed into a liquid nitrogen background. This excess noise was attributed
Table 4.1. Sample of a Data Sheet

<table>
<thead>
<tr>
<th>$V_D$</th>
<th>Signal</th>
<th>Noise</th>
<th>$\Delta f$</th>
<th>$f_0$</th>
<th>$R$</th>
<th>$A_{BB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4.25. Laboratory Test Setup.
to the current flow. However, a good quantitative explanation is not available.

**Signal Conditioning**

The processing of the infrared signal to an acceptable level in order to present it on a display is discussed for the ratio-temperature thermograph. However, as stated previously the two channels can be displayed independently. Figure 4.26 shows a block diagram of the system. The signals from each channel are amplified to a level to be compatible with the ratio circuit. The frequency bandwidth is limited to be compatible with frame time and resolution elements. The signal could not go below zero voltage because the display would be black; therefore, the signal level required a dc component. The dc level was present only during display so the signal was placed on a pedestal, so to speak, during display. The video signal then modulated the z axis of an X-Y-Z display scope. The x axis is the horizontal scan and the y axis is the vertical scan on the oscilloscope.

**X Scan**

The x axis on the video display is produced by a ramp function that varies from 0 to 5 volts. A synchronous pulse is picked off the mirror surface to determine the start of a scan line. The synchronous pulse is fed into a monostable (one-shot) multivibrator (74121) as shown in Fig. 4.27. This multivibrator is triggered on a negative transition. The signal is transistor-transistor logic (TTL) compatible, thus making the complementary output pulse levels either 0 or 5 volts.
Fig. 4.26. Schematic of a Ratio Temperature Thermograph.
Fig. 4.27. X-Scan Generator.
Once fired, the multivibrator's outputs are independent of further inputs and remain a function of timing components (a capacitor and resistor). The 20K variable resistor is used to determine the pulse duration. This pulse is used in a time delay, so the x scan will be centered in the optical field of view of the instrument. The output of the circuit is fed into a second monostable multivibrator. On the negative transition it triggers the second monostable multivibrator, which produces a pulse to correspond to a scan line. Figure 4.28 shows a time sequence for each scan line initiated by synchronous pulse. The output of the second monostable multivibrator is fed into an integrator, which integrates the pulse to a ramp function. Since it is necessary to reset the integrator to establish initial conditions, an FET is used across the integrator's capacitor. Resetting to zero is readily accomplished by the FET "turning-on" by the x-scan pulse, thus shorting the integrating capacitor. Since the integrator is inverting, the ramp pulse is going negative, an inverting amplifier is required on the output stage to be compatible with the display. The ramp pulse in time sync with the video is used to deflect the beam of the CRT in the x axis.

The x-scan pulse from the monostable multivibrator is also fed into the signal conditioning circuit and its complement is fed into the blanking circuit. These circuits will be discussed later.

Y Scan

The scan is accomplished by using a linear scanning mirror driven by a triangle function from a function generator. The function
Fig. 4.28. Timing Diagram.
generator is also used to drive the \( y \) deflection of the display. The \( x \)-scan pulse is summed to the video of each channel as shown in Fig. 4.29. This causes the desired information to be placed at a level that is above zero and that can be displayed directly on the display. The output gain control for each channel is used to fine adjust the signal levels for displaying either the 5-\( \mu \)m or 13-\( \mu \)m channel individually.

For the ratio-temperature display, each channel is processed through a video blanking circuit as shown in Fig. 4.30. Basically, the blanking circuit passes the video when the \( x \)-scan pulse is present and shorts it to ground otherwise.

The signal information from the two channels is ratioed to obtain the ratio temperature signal. The ratio is accomplished by taking the logarithm of each signal, subtracting the logarithms, and taking the anti-logarithm. The ratio is also displayed on the minitor. The ratio circuit is shown in Fig. 4.31.

**Optical Scanning System**

The optical system used to demonstrate the concept of color temperature was a modified forward looking infrared (FLIR) system. This system was built from available equipment. The optical system is shown in Fig. 4.32. The total system is reflective except for the vacuum window and the sapphire dichroic of the focal plane assembly.

The energy from the object is reflected by a four-sided rotating mirror, which produces the horizontal scan lines. An off-axis paraboloid is the only element of optical power in the entire system. It is
Fig. 4.29. Signal Conditioning Electronics.
Fig. 4.30. Video Blanking Circuit (Both Channels Identical).
Fig. 4.31. Ratio Circuit.
Fig. 4.32. Optical/Scanning System.
used at a 1:1 conjugate to image the resolution element on the detectors by means of a flat mirror, which wobbles to provide a vertical scan.

Four x-scan lines are produced for each revolution of the horizontal mirror. The mirror is driven at 200 rpm by a synchronous motor thereby obtaining uniform velocity. Therefore, the time of each x-scan line is 75 ms. A 200-line picture thus requires 15 seconds.

The collecting area of the optical system is not circular but is that shown in Fig. 4.33. The reason for this is that the horizontal scanning mirror is narrower than the off-axis paraboloid thus causing the entrance pupil position to be ambiguous. The collecting area is 20.3 cm² with a focal length of 75 cm. The effective F/No. is 15. The effective F/No. is what an unobscured clear aperture would produce for a given focal length. The actual F/No., the reciprocal of twice the numerical aperture, would be somewhat smaller.

The largest diameter of the diffraction spot (2.44λ F/No.) is 0.5 mm in the horizontal direction. The detector size is 0.75 mm, so some smearing of the edges of a resolution element is expected.

The aperture defining the illuminated surface of the detector is 0.75 mm. This is used with the optical system in a 1:1 conjugate plane; therefore, the spatial resolution on the object is also 0.75 mm, the object distance for this system is approximately 1.5 m, with refocusing required for each test. The depth of field is 2.0 cm, which is quite small for a thermographic unit.

The transmission of the system is defined by four aluminized mirrors, a sapphire dichroic, and spectral filters for each channel.
Fig. 4.33. Clear Aperture of Optical System.
The transmission of the 5- and 13-μm filters are 41% and 34%, respectively.

**Sensitivity Calculations**

The thermograph measures the spatial temperature profile of an object. How well the system does this thermal mapping is related to its thermal resolution or NETD. The NETD is the change in temperature of an adjacent resolution element in the object required to produce a change in the output of the system with a signal-to-noise ratio of one.

The object under consideration is a blackbody radiator with a spectral photon radiance of

\[
L_Q(\lambda,T) d\lambda = \frac{2c}{\lambda^4 \left[ \exp\left(\frac{hc}{kT}\right) - 1 \right]} \ d\lambda. \quad (4.56)
\]

The change in the object's radiance with respect to temperature can be found by differentiating Eq. (4.56) with respect to temperature:

\[
dL_Q(\lambda,T) d\lambda = \frac{2ec^2h}{KT^2} \cdot \frac{1}{\lambda^5} \cdot \frac{e^x}{e^x - 1} \ d\lambda dT, \quad (4.57)
\]

where \( x = \frac{hc}{kT} \). The corresponding change in photon flux, \( d\phi_Q \), at the entrance pupil of the instrument is

\[
d\phi_Q d\lambda = dL_Q(\lambda,T) d\lambda \cdot A_S \cdot \Omega \cdot \tau_a \quad (4.58)
\]

where \( A_S \) is the emitting area of the source, \( \Omega \) is the solid angle subtended by the entrance pupil at the source, and \( \tau_a \) is the atmospheric transmission.
The change in photon flux at the focal plane of the optical system (detector's sensitive surface) is determined by multiplying Eq. (4.58) by the optical transmission, $\tau_0$, and $hc/\lambda$ since further analysis requires the change to be in terms of radiant power

$$d\phi d\lambda = \frac{2\varepsilon c^3 h^2}{kT^2} \cdot \frac{1}{\lambda^6} \cdot \frac{e^X}{(e^X - 1)^2} \cdot A_S \cdot \Omega \cdot \tau_a \tau_0 t d\lambda. \quad (4.59)$$

If this change of radiant power incident on the detector is sufficient to produce a signal change equal to the noise, this radiant power change can be called the noise equivalent power (NEP) of the detector.

The NEP can be expressed as

$$\text{NEP} = \left( \frac{A_d \Delta f}{D^*} \right)^{\frac{1}{2}}$$

where $A_d =$ detector area and $\Delta f =$ system noise bandwidth. Then

$$dT = \frac{kT^2}{2\varepsilon c^3 h^2} \cdot \frac{(A_d \Delta f)^{\frac{1}{2}}}{A_S \tau_a \tau_0 \cdot \frac{\lambda^6 (e^X - 1)^2}{e^X (d\lambda)}}. \quad (4.61)$$

This equation for NETD is in terms of spectral power. In order to determine the NETD, an integration must be performed over each spectral region of interest (i.e., $\lambda_1$ to $\lambda_2$).

The system electrical bandwidth is determined by scan efficiency, number of lines per frame, and the number of resolution elements per line; the frame time, $t_F$, is the amount of time required to complete a full picture. The scan efficiency, $\xi$, is the ratio of the system horizontal field of view to the maximum horizontal field of view for constant scan velocity.
The time required to perform a single line scan of a frame is

\[ t = \frac{\zeta t_F}{N_L} \]  

(4.62)

where \( \zeta \) is the scan efficiency, \( t_F \) is the frame time, and \( N_L \) is the number of lines per frame.

The dwell time is defined as the time required to scan a resolution element, so

\[ t_d = \frac{t}{N_R} = \frac{\zeta t_F}{N_R N_L} \]  

(4.63)

The electrical bandwidth (3 dB points) required to reproduce a scene with good fidelity is the reciprocal of twice the dwell time

\[ \Delta f = \frac{1}{2t_d} \]  

(4.64)

or

\[ \Delta f = \frac{N_R N_L}{2\zeta t_F}. \]  

(4.65)

The equivalent noise bandwidth for the system can be found from

\[ \Delta f_S = \frac{1}{Y_0^2} \int_0^\infty |Y(f)|^2 df, \]  

(4.66)

where \( \Delta f_S \) is the filter noise bandwidth, \( Y(f) \) is the filter transmittance function, and \( Y_0 \) is the maximum absolute value of \( Y(f) \). The noise bandwidth of a filter is defined as the bandwidth of an ideal filter that has the same value of absolute transmittance in its passband as the maximum absolute transmittance of the filter and delivers the same mean square output voltage from a white noise source as the filter.
An ideal bandpass filter is defined as a filter with no attenuation in the passband and infinite attenuation outside the passband.

For a single-pole 3-dB bandwidth, the ratio of noise bandwidth to 3-dB bandwidth is $\pi/2$. Therefore, the system noise bandwidth is

$$f_s = \frac{\pi R_N L}{4\zeta f_F}.$$  \hspace{1cm} (4.67)

The areas of the detector and object are related by the optical magnification squared, or

$$M^2 = \frac{A_d}{A_s}.$$ \hspace{1cm} (4.68)

Substituting Eqs. (4.67) and (4.68) into (4.61) yields the NETD expression as a function of wavelength

$$\text{NETD} = \frac{kT^2}{2e\sigma^3 h^2} \frac{M^2}{(A_d)^{\frac{1}{2}}\tau_a\tau_0 D^*\Omega} \left( \frac{\pi R_N L}{4\zeta f_F} \right)^{3/4} \frac{\lambda^6(e^X - 1)^2}{e^X d\lambda}.$$ \hspace{1cm} (4.69)

Substituting the parameter for the system, the noise equivalent temperature differences for the two channels are

$$\text{NETD} = 0.145^\circ K \text{ for } 5-\mu m \text{ channel}$$ \hspace{1cm} (4.70)

$$\text{NETD} = 0.136^\circ K \text{ for } 13-\mu m \text{ channel}. \hspace{1cm} (4.71)$$

Table 4.2 summarizes the performance of the ratio-temperature thermograph channels.
Table 4.2. Summary of System Performance

<table>
<thead>
<tr>
<th></th>
<th>5-μm Channel</th>
<th>13-μm Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>InSb</td>
<td>Si:Bi</td>
</tr>
<tr>
<td>$D^*$</td>
<td>$8.3 \times 10^{11}$</td>
<td>$6.6 \times 10^{11}$</td>
</tr>
<tr>
<td>NEP</td>
<td>$8 \times 10^{-14}$</td>
<td>$1 \times 10^{-13}$</td>
</tr>
<tr>
<td>$\lambda_1-\lambda_2$</td>
<td>4.5-5.5 μm</td>
<td>12.9-1.4 μm</td>
</tr>
<tr>
<td>NETD</td>
<td>0.145 K</td>
<td>0.136 K</td>
</tr>
<tr>
<td>Optical resolution</td>
<td>0.75 mm</td>
<td>0.75 mm</td>
</tr>
<tr>
<td>Line time</td>
<td>75 ms</td>
<td>75 ms</td>
</tr>
<tr>
<td>Frame time (200 lines)</td>
<td>15 sec</td>
<td>15 sec</td>
</tr>
</tbody>
</table>
CHAPTER 5

DATA AND ANALYSIS

This chapter describes the various methodologies and approaches that were used to perform the testing of the ratio-temperature thermograph. These tests were conducted to establish the experimental verification of the conclusions arrived at in earlier chapters.

Three experimental studies were performed using the thermographic system described in Chapter 4. These studies consisted of (1) sensitivity calibration and coregistration experiments, (2) background effect experiments, and (3) integral and ratio temperature scans. The data from all three studies consist of single line scans of appropriate target objects. Line scans were chosen for ease of acquisition and quantitative evaluation. However, in order to make a diagnostic judgment, such as that made by a radiologist, two-dimensional thermograms are required.

Radiometric Sensitivity Calibration and Coregistration Experiments

In order to test the thermographic unit, a special calibration source was fabricated. This source was an infrared bar chart pattern consisting of a plate at ambient temperature that had slots machined into it and a back plate that was temperature controlled. It was capable of producing a thermal pattern that could be used for both radiometric sensitivity and spatial resolution measurements. A photograph of the calibration source is shown in Fig. 5.1. The spatial frequency
Fig. 5.1. Bar Chart Calibration Source.
of the test pattern varied from 0.12 cycles/mm to 0.5 cycles/mm. This range of spatial frequencies was chosen because it covers the expected spatial resolution necessary to detect carcinoma in patients.

The back plate was heated by an electric heating pad whose temperature was controlled by a variac. Both surfaces of the calibration source were prepared identically using 3M Nextel Black Velvet Coating (101-C10). Because the heating pad uses an electric coil, a uniform temperature distribution across the source was not attained. However, this did not affect the calibration because a predetermined position in the source was chosen for calibration and was monitored with a chromel/constantan thermocouple. This thermocouple has the largest responsivity of emf output for a given temperature change of any standard metallic thermocouple. Line scans of this bar target at both 5-μm and 13-μm channels are shown in Fig. 5.2. The temperature differences of the bar target was measured to be 9°K ± 1.5°K using the thermocouple.

The peak-to-peak signal and noise were measured on an oscilloscope and adjusted to rms by an appropriate peak factor. In the case of noise, a fixed peak factor (ratio of peak to rms) can be assigned only if the noise has been passed through a filter, which is the case for this instrument. This is because any value of noise can be expected to appear, given sufficient time, unless it is limited by system dynamic range. However, passing this noise through a filter puts a low-frequency envelope over the noise, thus changing it to follow a Rayleigh distribution instead of a Gaussian one (Bendat and Piersol, 1971). The peak factor for the square wave signal is taken as the relative rms amplitude of its
Fig. 5.2. Line Scans across Bar Chart.
fundamental sinusoidal component. The peak-to-peak signal and noise
values can be read on the scope and used directly for rms calculations
because the factors adjusting to rms cancel out. The ratio of the rms
signal to the peak-to-peak signal is \((2/\pi)^2\); for the noise it is \(1/3.68\);
and the effective bandwidth is \(\pi/2\). Therefore,

\[
\frac{S_{\text{rms}}}{N_{\text{rms}}} = \frac{S_p (2/\pi)^2}{N_p (1/3.68)(\pi/2)} \approx \frac{S_p}{N_p}
\]  

The peak-to-peak ratio of the signal to the noise observed
through the system was measured to be 40 and 30 for the 5-μm and 13-μm
channels, respectively. A 9°K temperature difference of the bar target
leads to a measured NETD of 0.22°K and 0.3°K for the 5-μm and 13-μm
channels, respectively. The error in this measurement is about 0.05°K.
The 5-μm channel is in good agreement with that predicted in Eq. (4.70).
However, the 13-μm channel was about a factor of two above the predicted
value.

By using the bar target in a manner similar to that discussed
above for sensitivity measurements, the spatial resolution of the thermo-
graphic system was also measured. The smallest resolvable bar pattern
was the 1-mm slots, which correspond to a 0.5 cycle/mm spatial frequency.
Figure 5.2 shows the line scans across the bar target and the detection
of the highest spatial frequency present.

The optical alignment of the two channels was checked during the
spatial resolution tests. Several alignment iterations were required
before coregistration was accomplished. The back-illumination technique
(discussed in Chapter 4) at room temperature was sufficient to establish
only approximate channel registration. However, once the package was cooled to 4°K, small changes in the aperture positions occurred. The time required for each registration adjustment was usually two days. The coregistration tests were done using an expanded oscilloscope scale to measure and compare the rise time of each channel on the same time base. The registration finally attained was about a 2-μsec difference between the two channels when the dwell time was 31-μsec, about 6%.

**Background Measurements**

A theoretical discussion of the background effects was given in Chapter 2. It was shown that the control of background temperature could be used either to eliminate emissivity variations or to improve contrast, and that the background flux reflected from the object could affect the measurement considerably. In this section, data are presented and discussed that verify those conclusions. The thermographic instrument was tested using the target shown in Fig. 5.3.

This target consisted of an aluminum plate with four black painted strips on its surface. The temperature of the aluminum plate was monitored using a chromel/constantan thermocouple rigidly affixed to the plate. This thermocouple was used for reasons stated earlier. Because the painted strips were very thin, typically 0.003-in. thick, it is assumed that they were at the same temperature as the aluminum plate. The black strips were made using 3M Nextel Velvet Coat, which has an emissivity of about 0.95 and a reflectivity of 0.05 (Breault, 1975). The lightly oxidized aluminum surface has an emissivity of 0.15 and a reflectivity of 0.85 (Bramson, 1968). Thus, the target will produce a
Fig. 5.3. Test Target for Background Effects Measurements.
bar pattern because of varying emissivity, which modulates a constant temperature emitter. The heated bar target was used as a background to enable measurement of the system's sensitivity during data collection periods.

These experiments were conducted in the Optical Sciences Center Infrared Laboratory. Therefore, no special provisions for controlling the room temperature were available. However, the target temperature could be controlled and was easily varied above and below room temperature. This is the complementary effect of varying the room temperature and was less of an implementation problem. Figure 5.4 shows a series of single line scans across the target for different target temperatures. It should be noted that an increase in radiation causes a downward going signal. Each photograph contains two line scans, the upper being the 5-μm channel and the lower being the 13.5-μm channel. The room temperature was 295 °K. Figure 5.4 shows that when the target temperature equals the room temperature, no emissivity changes are detected. This is exactly as expected from Eq. 2.28. When the target is above room temperature, the black paint emits more radiation than the aluminum reflects from the room background. Thus, as shown in Fig. 5.4c, the four black areas are brightest. In contrast, in Fig. 5.4b a reversal is observed because the target temperature is below the room temperature. In order to evaluate the signal levels on an absolute basis, one must find the change in radiance across the target. This can be found by substituting the parameters in Table 5.1 into Eq. (2.28) with dT = 0, and integrating over the spectral response of the system. If the target temperature is above the room temperature, the 5-μm channel senses
Fig. 5.4. Background Effects Measurements.

(a) At room temperature, (b) below room temperature, (c) above room temperature.
\[ dL_q = 9.8 \times 10^{14} \text{ photon/sec-cm}^2\text{-sr} \]  

or

\[ dL_e = 3.88 \times 10^{-5} \text{ W/cm}^2\text{-sr} \]  

Similarly for the 13-\( \mu \)m channel

\[ dL_q = 3.1 \times 10^{15} \text{ photon/sec-cm}^2\text{-sr} \]  

or

\[ dL_e = 4.58 \times 10^{-5} \text{ W/cm}^2\text{-sr} \]  

Table 5.1. Data from Background Effects Experiment

<table>
<thead>
<tr>
<th>Surface Type</th>
<th>( \varepsilon )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum surface</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td>3M black paint</td>
<td>0.95</td>
<td>0.05</td>
</tr>
<tr>
<td>room temperature</td>
<td>295°K</td>
<td></td>
</tr>
<tr>
<td>target (above room temp)</td>
<td>301°K</td>
<td></td>
</tr>
<tr>
<td>target (below room temp)</td>
<td>291°K</td>
<td></td>
</tr>
</tbody>
</table>

The change-in-radiance calculation from Eq. (2.28) is somewhat unmanageable at this point because system sensitivity was calculated on a temperature change basis, namely NETD. Therefore, a second figure of merit for the system must be derived: the system's noise equivalent radiance NEL.

The NEL is defined as the value of incident rms signal radiance required to produce an rms signal-to-noise ratio of unity. Stated
similarly, it is the change in rms signal radiance incident at the entrance pupil of an instrument that will cause the detector to produce an rms signal at its output terminal equal to the rms noise of the instrument referred to the detector output terminal. The NEL is often required to evaluate a system when a change in radiance at the entrance pupil is not temperature dependent.

The signal-to-noise ratio of an instrument is directly related to the figure of merit of the specific detector's noise equivalent power, NEP. The NEP is defined as the rms power change incident on the detector that will cause the detector to produce an rms signal at its output terminal equal to the rms noise at its output terminal for the instrument bandwidth. The rms power change incident on a detector due to a change in radiance of an extended source filling the entrance pupil of an instrument is given by

$$ \Delta \phi = \Delta L_e A_c \Omega T_0 F_F . $$

The change in radiance then is

$$ \Delta L_e = \frac{\Delta \phi}{A_c \Omega T_0 F_F} . \quad (5.7) $$

The NEL therefore is given by

$$ \text{NEL} = \frac{\text{NEP}}{A_c \Omega T_0 F_F} (\Delta f)^\frac{1}{2} K_\epsilon . \quad (5.8) $$

Using Eq. (5.8) and results of Chapter 4, one can solve for the NEL for each channel.
NEL = 6 x 10^{-6} \text{ W/cm}^2\text{-sr} \quad (5-\mu\text{m channel}) 
\quad (5.9)

NEL = 9 x 10^{-6} \text{ W/cm}^2\text{-sr} \quad (13.5-\mu\text{m channel}) \quad (5.10)

The predicted system signal-to-noise output can now be found by the use of the following equation.

$$S/N = \frac{dL_e}{NEL}.$$  \quad (5.11)

The results in Eqs. (5.3), (5.5), (5.9), and (5.10) lead to signal-to-noise ratios for these situations as follows:

$$S/N = 6.5 \quad (5-\mu\text{m channel}) \quad (5.12)$$

$$S/N = 5.0 \quad (13.5-\mu\text{m channel}). \quad (5.13)$$

The signal-to-noise ratio measured is about 7 for the 5-\mu m channel and 3 for the 13.5-\mu m channel. Thus, the predicted signal-to-noise performance is in good agreement with that measured.

In similar fashion when the target is below room temperature the radiance change calculated in Eq. (2.28) is

$$dL_e = -2.2 \times 10^{-5} \quad (5-\mu\text{m channel}) \quad (5.14)$$

and

$$dL_e = -3.65 \times 10^{-5} \quad (13.5-\mu\text{m channel}) \quad (5.15)$$

with a resulting predicted signal-to-noise ratio of

$$S/N = 3.7 \quad (5-\mu\text{m channel}) \quad (5.16)$$
and

\[ S/N = 4 \quad (13.5-\mu m \ channel). \quad (5.17) \]

The negative sign is absorbed by \( dL_e \), but in Fig. 5.4c one sees the phase reversal from Fig. 5.4b, which it indicates.

Because the measured signal-to-noise ratios are in good agreement with expected results, the instrument is functioning in a predictable manner. The analysis also shows that the emissivity effects are following the theory discussed in Chapter 2.

**Ratio Temperature Measurements**

The measurements used to illustrate the concept of the ratio-temperature technique were also carried out using single line scans. The single line scans are sufficient for quantitative analyses, demonstrate the concept, and are more quantitative than actual pictures for an engineering evaluation. However, a large number of clinical exams requiring two-dimensional displays will be necessary to assess its diagnostic usefulness.

In order to test the thermograph's capability to eliminate the emissivity effects on the skin, a test target was needed that had emissivity variations of about 1 or 2%. After trying several different approaches to develop a test target meeting these requirements, an approach of painting the forearms of a volunteer as illustrated in Fig. 5.5 was used. This type of target had two salient features: (1) actual skin (in vivo) was used, closely simulating the test conditions expected in actual practice and (2) emissivity changes on the
Fig. 5.5. Emissivity Changes on Skin.
order of those that are expected to cause inaccuracies in present-day thermographic units were produced. The emissivity changes shown on the forearms in Fig. 5.5 were produced by using a mixture of carbon black and carfusin (a skin marking dye used in medicine). An arm with this emissivity pattern was put in the object plane of the thermograph and 5-μm, 13-μm, and ratio-temperature line scans were taken of it. These line scans are shown in Fig. 5.6.

Although the exact change in emissivity is not known, the 5-μm and 13-μm line scans have a signal-to-noise ratio of about 2 or 3 for the three emissivity bumps, corresponding to about 0.75°K. This type of temperature change, as plotted in Fig. 2.7, indicates about a 1.2% emissivity change. The three variations seen in the 5-μm and 13-μm channels are suppressed in the ratio-temperature scan. Just as the theory of Chapter 3 indicated, the emissivity variation has largely been eliminated. Figure 5.6 definitely shows the value of ratio-temperature measurements when emissivity effects are of concern. However, these tests are not sufficiently conclusive to show that small temperature changes will not be washed out also.

In order to show that small temperature variations are measurable, a line scan was obtained across some obvious blood vessels. The back of the volunteer's hand was used as a source. Figure 5.7 shows the 5-μm and the ratio temperature scans, indicating that temperature changes do not wash out in the ratio-temperature thermogram. The top scan is the 5-μm channel scan that detects the temperature change of the blood
Fig. 5.6. 5-μm and 13-μm Integral Temperature and Ratio-Temperature Scans.
Fig. 5.7. Temperature Variations over Blood Vessels.
vessel. The lower scan is the ratio-temperature scan; it also detects the temperature change of the blood vessel, even with the combined noise of the two channels.

The indicated temperature change from the line scan is about one degree. The conclusion is that the color temperature is insensitive to emissivity variations whereas temperature change is detected in a manner similar to that of the integral temperature thermograph. This discussion can be extended to any infrared measurements where emissivity may be a problem, not just the application to medical thermographs.

**Summary and Conclusions**

The ratio-temperature thermograph has been shown to greatly eliminate the effects of emissivity variations of the skin. Both theoretical and experimental evidence has been presented to demonstrate this fact. Present systems sense only integral temperature that allow the emissivity effects corresponding to 1°K temperature variations to cause incorrect diagnoses by radiologists. If the ratio-temperature thermograph were in standard use, emissivity effects would no longer be of concern.

The ratio-temperature thermograph is somewhat more complex in that it requires additional detectors, electronics, and there is the difficulty of coregistration of channels. These requirements are expensive but they are not insurmountable.

The background effects of the room environment of the patient were also analyzed briefly. It was shown that if the background temperature is equal to the body temperature, emissivity effects are
eliminated for an integral temperature thermograph. This procedure is not desirable because it has been shown empirically by practicing physicians taking thermograms that the contrast of present thermograms is improved by reduced background temperature. However, with ratio-temperature techniques the contrast would depend only on system wavelengths and not on the temperature of the background. This would let the radiologist cool the room to any desirable level for metabolic rate considerations without affecting the contrast of the thermogram or enhancing false signals that are due to emissivity variations.

An additional advantage to cooling of the room is that the system is background-noise limited; therefore, the performance would increase by lowering the background temperature. Although ratio-temperature thermographs are more complex as far as instrumentation is concerned, they should prove better than present techniques used in clinical thermography.

By eliminating the emissivity effects that have been shown to be on the order of the radiologists' positive/negative decision (1°K), fewer false positives will result. This will protect healthy patients from ionizing radiation of mammograms and xerograms and the mental anguish associated with an indeterminant diagnosis.

Because ratio temperature techniques measure only temperature variations, it should improve true positive statistics and provide earlier detection of breast carcinoma. Thus more carcinomas should be detected when still confined to the breast.
APPENDIX A

RADIOLOGICAL PATTERN RECOGNITION

The evaluation of a diagnostic thermogram is carried out using a pattern recognition process that is based on the previous experience and training of the radiologist. Typically an asymmetrical pattern in the thermogram indicates carcinoma. Correct diagnosis depends upon the ability to distinguish relevant patterns in the midst of background shadows that are the result of irrelevant signals and noise. The radiologist makes his decision under these conditions of uncertainty.

A radiologist's diagnosis can be classified into four categories:
(1) Thermograms that are diagnosed correctly: these are true positives for patients who have disease and true negatives for healthy patients.
(2) Thermograms that are diagnosed incorrectly: healthy patients whom the radiologist diagnosed as diseased are false positives, and diseased patients who are diagnosed incorrectly are called false negatives. The four possible outcomes for the radiologist's diagnosis may be displayed in a matrix as shown in Figure A1.

The radiologist's true positive (TP) percentage is

\[ TP(\%) = \frac{\text{true positive he reads}}{\text{true positive he reads} + \text{false negatives he reads}} \times 100 \]

His false negative (FN) percentage is
\[ FN(\%) = \frac{\text{false negatives he reads}}{\text{true positives he reads} + \text{false negatives he reads}} \times 100 \]

Note that the sum of the percentage of true positives and the percentage of false negatives equals 100%.

The radiologist's true negative (TN) percentage is

\[ TN(\%) = \frac{\text{true negatives he reads}}{\text{true negatives he reads} + \text{false positives he reads}} \times 100 \]

His false positive (FP) percentage is

\[ FP(\%) = \frac{\text{false positives he reads}}{\text{true negatives he reads} + \text{false positives he reads}} \times 100 \]

Again the percentages of true negatives and false positives sum to 100%.

<table>
<thead>
<tr>
<th>Actual Disease Category</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>% True Positive</td>
<td>% False Negative</td>
</tr>
<tr>
<td>Negative</td>
<td>% False Positive</td>
<td>% True Negative</td>
</tr>
</tbody>
</table>

= 100% Patients with Carcinoma

= 100% Patients without carcinomas

Fig. A1. Radiologist's Diagnosis
In examining these outcomes, it is not immediately obvious that the distribution of true positives and false negatives is directly related to a radiologist's distribution of false positives and true negatives. However, an interesting reciprocal relationship does exist between these two groups. The total positive population and negative population can be plotted versus pattern asymmetry as shown in Fig. A2.

Since the pattern asymmetry varies considerably between positive and negatives, a region exists where similar patterns can be interpreted as positive or negative. This overlapping region is where the problems of false positive and false negative diagnosis occur. Each radiologist considers a certain, qualitative, degree of pattern asymmetry as indicative of possible carcinoma. Sometimes the radiologist would like further information, which is why mammograms and xerograms are taken. However, by doing this the radiologist has put this thermogram into the positive classification.

This decision is interpreted as a point on the pattern asymmetry axis $P_c$ of Fig. A2. Since this discussion point is subjective, it is constantly changing for any radiologist over a small region on the pattern asymmetry axis.

The area under the positive curve to the left of $P_c$ represents the false negatives. Correspondingly, the area under the negative curve to the right of $P_c$ represents the false positives. A very large number of false negatives will exist if the decision point, $P_c$, is chosen to the far right. Therefore, the radiologist must decide whether to be pessimistic, optimistic, or in between in actual practice of deciding.
Fig. A2. Population Distribution.
Therefore, a reciprocal relationship exists between the false negatives and false positive for a given $P_c$. If one assumes the two distribution functions to be Gaussian with equal variances, an explicit relationship between all the various outcomes can be interrelated. For example, the false negatives can be expressed as

$$r = \frac{1}{2} - \text{erf}(P_c)$$

Similarly the false positive can be called

$$s = \frac{1}{2} - \text{erf}(P_1 - P_c)$$

Plotting $r$ versus $s$ produces a hyperbola indicating an interrelationship between false positives and false negatives as shown in Fig. A3. Similar mathematical relationships can be established between any of the other four outcomes.
Fig. A3. Relationship between Incorrect Diagnoses.
APPENDIX B

NOMENCLATURE

[subscript e indicates power (W), subscript q indicates number (photons/sec)]

A: operational amplifier open loop gain

\( A_{BB} \): area of blackbody

\( A_c \): area of collector

\( A_d \): detector area

\( C_1 \): 2C

\( C_2 \): 1.43879 cm °K

\( C_f \): feedback capacitance

\( C_s \): stray capacitance

\( D_\lambda \): dielectric constant

D: spectral detectivity

\( E_b \): error of reciprocal of integral temperature

\( E_c \): error of reciprocal of ratio temperature

\( E_g \): preamp input voltage

\( E_o \): preamp output voltage

\( E_q \): irradiance of background (photon/sec cm²)

F: reflectance

\( F_f \): form factor for converting peak-to-peak square wave to rms

G: amplifier gain

I: current through detector
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_e$</td>
<td>integrated radiance (W/cm²sr)</td>
</tr>
<tr>
<td>$L_e(\lambda,t)$</td>
<td>spectral radiance</td>
</tr>
<tr>
<td>$L_q$</td>
<td>integrated photons radiance (photon/sec cm²sr)</td>
</tr>
<tr>
<td>$L_q(b)$</td>
<td>integral temperature radiance (photons/sec cm²sr)</td>
</tr>
<tr>
<td>$L_q(\lambda,T)$</td>
<td>spectral radiance</td>
</tr>
<tr>
<td>$M$</td>
<td>optical magnification</td>
</tr>
<tr>
<td>$M_q$</td>
<td>radiant emittance (exitance) (photon/sec cm²)</td>
</tr>
<tr>
<td>$M_q(\lambda,T)$</td>
<td>spectral radiant emittance (exitance) (photon/sec cm² μm)</td>
</tr>
<tr>
<td>$N$</td>
<td>number of photons (photon/sec)</td>
</tr>
<tr>
<td>NEP</td>
<td>noise equivalent power</td>
</tr>
<tr>
<td>NEL</td>
<td>noise equivalent radiance</td>
</tr>
<tr>
<td>$N_L$</td>
<td>number of lines per frame</td>
</tr>
<tr>
<td>$N_R$</td>
<td>number of resolution elements per line</td>
</tr>
<tr>
<td>$Q$</td>
<td>photoconductive gain</td>
</tr>
<tr>
<td>$R$</td>
<td>range between source and collector</td>
</tr>
<tr>
<td>$R_D$</td>
<td>detector resistance</td>
</tr>
<tr>
<td>$R_f$</td>
<td>load/feedback resistor</td>
</tr>
<tr>
<td>$S_i$</td>
<td>detector responsivity</td>
</tr>
<tr>
<td>S/N</td>
<td>signal-to-noise ratio</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$T_a$</td>
<td>temperature of surface a</td>
</tr>
<tr>
<td>$T_b$</td>
<td>background temperature</td>
</tr>
<tr>
<td>$T_B$</td>
<td>integral (brightness) temperature</td>
</tr>
<tr>
<td>$T_c$</td>
<td>ratio temperature</td>
</tr>
<tr>
<td>$T_d$</td>
<td>detector temperature</td>
</tr>
<tr>
<td>$T_R$</td>
<td>radiation temperature</td>
</tr>
</tbody>
</table>
\( V_B \) detector biasing voltage
\( V_D \) voltage across detector
\( Z_D \) detector impedance
\( Z_f \) feedback resistor impedance
\( c \) velocity of light \((2.998 \times 10^{10}) \text{ cm/sec}\)
\( d \) detector electrode separation
\( e \) electron charge \(1.6 \times 10^{-19}\) coulomb
\( e_{b} \) integral temperature deviation
\( e_{c} \) ratio-temperature deviation
\( e_i \) output noise voltage due to current
\( e_T \) total noise voltage
\( e_V \) output noise voltage due to voltage
\( f_0 \) electrical frequency
\( h \) \(6.6 \times 10^{-34}\) Joule-sec
\( i_f \) 1/f noise current
\( i_J \) Johnson-noise current
\( i_n \) noise current
\( i_p \) photon noise current
\( k \) Boltzmann constant \((1.38 \times 10^{-23} \text{ J}^\circ\text{K})\)
\( s \) Laplace transform variable
\( t_d \) dwell time
\( t_F \) frame time
\( \alpha \) constant
\( \beta \) constant
\( \varepsilon \) emissivity
\( \varepsilon_a \) emissivity of surface \( a \)

\( \varepsilon_1 \) emissivity at wavelength one

\( \varepsilon_2 \) emissivity at wavelength two

\( \varepsilon_\lambda \) spectral emissivity

\( \zeta \) scan efficiency

\( \eta \) quantum efficiency

\( \lambda \) wavelength (cm)

\( \rho \) reflectivity

\( \sigma \) Stefan-Boltzmann constant \([5.67 \times 10^7 \text{ W/cm}^2 \text{°K}]\)

\( \sigma_b^2 \) integral-temperature variance

\( \sigma_c^2 \) ratio-temperature variance

\( \sigma_d^2 \) ratio-temperature radiance variance

\( \sigma_e \) 5.67 \( \times \) 10^{12} \text{ W/cm °K}

\( \sigma_{LB}^2 \) integral-temperature radiance variance

\( \sigma_q \) 1.52 \( \times \) 10^{11} \text{ photon/sec cm °K} = \frac{\sigma}{2.75K}

\( \sigma_{eB}^2 \) integral-temperature emissivity variance

\( \sigma_{e_1/e_2}^2 \) ratio-temperature emissivity variance

\( \tau_a(\lambda) \) atmospheric transmission

\( \tau_f(\lambda) \) filter transmission

\( \tau_0 \) optical transmission
APPENDIX C

PHYSIOLOGY

Body heat is produced by a continuous metabolic process consisting of the oxidation of carbohydrates and fats, which are the chief sources of energy. Internal body temperatures represent the balance or equilibrium between the heat produced in the tissue and heat lost to the environment. A detailed account of this process is described by Benzinger and Kitzinger (1963), Burton (1941), and Winslow and Pierce (1941). The role of the skin is an unquestioned and significant organ of the body in the thermoregulatory process. Its outer surface is about 20 square feet or 1.9 square meter, and it comprises 6% of the total body weight. The temperature of the skin and its spatial and temporal variations are determined by a number of different factors. First, it is subject to environmental processes such as radiation, conduction, convection, and evaporation. For a 22°C room temperature, this accounts for about 200 watts for an average sized person. Radiation accounts for 66% of the heat loss (Haberman, 1971; Hardy, Milhort and Dubois, 1941). Second, there are physiological mechanisms of heat regulation such as change of the skin circulation and sweat secretion. Third, there is conduction of heat from the warmth inside, through the fat and the dermal layers to the skin surface, and there is convection of heat from the warmth inside by means of blood flow. These conditions are described in detail in the literature (Hardy and Dubois, 1941; Sheard, Williams,
and Horton, 1941). A typical average temperature of the forehead is 34°C while that of a thumb is around 28°C, and that of a big toe is around 25°C. These values hold for a room temperature of 20°C and uncovered skin, exposed to the room temperature for at least 20 minutes (Wissler, 1963; Mali, 1969). Here the temperature of the breast skin of women is of greatest concern. Here the average temperature is about 37°C, again for a room temperature of 20°C. Local variations in the conditions, determining the skin temperature as described above, will result in deviations from this average temperature.
Thermography, as commonly performed, is based on the "blackbody" radiation of the skin. The radiance \( L_E \) (the radiant power per unit area per solid angle) of the skin, as that of any black or graybody, is a function of its temperature \( T \) and its emissivity

\[
L_E = \frac{\varepsilon \sigma T^4}{\pi}.
\]

This is known as the Stefan-Boltzmann law, where \( \varepsilon \) is a measure of the "blackness" of the body and \( \varepsilon = 1 \), denotes a perfect blackbody. Bodies with a constant emissivity less than 1 are usually called graybodies. The Stefan-Boltzmann constant is usually designated \( \sigma \), but here \( \sigma_{\varepsilon} \) is used to indicate that it is for power. The description of radiance as a function of the wavelength was first correctly given by M. Planck

\[
L_{\varepsilon}(\lambda, t) = \frac{\varepsilon_\lambda c_1 \lambda^{-4}}{\lambda T} (\exp \frac{c_2}{\lambda T} - 1)^{-1}.
\]

Figure D.1 shows the spectral distribution of the intensity of three different radiant objects: the sun, a glowing metal, and the human skin (Herstel, 1969). It is important to note that the maximum emission for the human skin of the breast, which is at a temperature of around 310°K, occurs at about 10 μm, and has virtually dropped to zero below 3 μm and above 40 μm.
Fig. D-1. Spectral Distribution of the Intensity of Three Radiant Objects: the Sun, Glowing Metal, and the Human Body (Herstel, 1969).
In order to determine the temperature of the skin from the spectral radiance, it is necessary to know the emissivity as a function of the wavelength. This has been measured by Hardy (1934c, 1939), Elam et al. (1963), and Steketee (1973b). Steketee found that in the wavelength region 2 \(\mu m\) to 20 \(\mu m\), the emissivity is almost constant, \(\varepsilon = 0.98\), with an accuracy of about 1\% (Fig. 2.5b). Elam et al. and Hardy and Muschenheim (1936) however, showed that there may be departures from the blackbody emission in the region around 3 to 6 \(\mu m\) (Figs. 2.5a, D-2). These departures from the blackbody emission have been shown to be significant in the case of imaging in narrow bands by Lloyd-Williams et al. (1963). However, for broadband imaging, it did not seem to be of concern. The authors suggest that the emissivity in certain parts of the 3 to 6-\(\mu m\) region is related to the water content of the epidermis.

Differential scanning (Dodd et al., 1973) is the most recent technique introduced in medical thermography. It involves active photography to obtain a temperature-independent image of the venous structure directly under the skin. Such active photography, of course, depends on the transmission and the reflection of the skin, in particular the epidermis, the subcutaneous layer, the blood vessels, and the blood.

Data on the spectral transmission of the epidermis is reported by Cartwright (1930), Hardy and Muschenheim (1934) and by Elam et al. (1963) (see Fig. 2.5a). They show that the transmission of the epidermis increases with increasing wavelength from 0.9 \(\mu m\) up to 1.8 \(\mu m\) except for an absorption band at 1.5 \(\mu m\). A preliminary study at the Optical Sciences
Fig. D-2. Transmission Spectrum of Human Epidermis, Wet and Dry (after Hardy and Muschenheim, 1936).
Center, using epidermis and some of the subcutaneous layer confirmed this trend. It seems clear that the transmission at around 1.8 μm is much higher than that at 0.9 μm.

Elam et al. (1963) also give data on the reflectivity of the epidermis. They show that the reflectivity peaks at around 0.8 μm with about 60% and then decreases by almost a factor of 6 with increasing wavelength out to around 1.5 μm. From there on, it stays virtually constant up to about 2.8 μm.
APPENDIX E

RADIATION CONTRAST ANALYSIS

The determination of maximum contrast for the photon flux starts with the expression for the photon radiance

\[ L_q(\lambda, T) = \frac{\varepsilon}{\pi} \frac{c_1}{\hbar c} \lambda^{-4} \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right]^{-1}. \]

The maximum is obtained by taking the derivative of the temperature contrast and setting it equal to zero. The contrast is given by

\[ \frac{\partial m}{\partial T} = \frac{\varepsilon c_1 \lambda^{-4}}{\pi \hbar c} \cdot \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right]^{-2} \exp\left(\frac{c_2}{\lambda T}\right) \cdot \frac{c_2}{\lambda T^2}. \]

To maximize this contrast with respect to wavelength, the equation is

\[ \frac{\partial^2 m}{\partial \lambda \partial T} = \frac{\varepsilon c_1 c_2}{\pi \hbar c T^2} \lambda^{-5} \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right]^{-2} \exp\left(\frac{c_2}{\lambda T}\right) \cdot \frac{c_2}{\lambda^2 T} \]

\[ - \exp\left(\frac{c_2}{\lambda T}\right) \left\{ \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right]^{-2} 5 \lambda^{-6} + \lambda^{-5} \cdot 2 \right\} \]

\[ \cdot \left[ \exp\left(\frac{c_2}{\lambda T}\right)^{-1} \exp\left(\frac{c_2}{\lambda T}\right) \cdot \frac{-c_2}{\lambda^2 T} \right] = 0. \]

Then for \( x = \frac{c_2}{\lambda T} \) the result is

\[ \frac{x e^x}{e^x - 1} = \frac{x + 5}{2}. \]

or

\[ x = 5.068 \text{ or } \lambda T = 2880 (\mu m^0 K). \]
APPENDIX F

BAND-LIMITED RELATIVE FLUX DENSITY

\[ M_{\Delta \lambda} = \int_{\lambda_1}^{\lambda_2} \epsilon c_1 \lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T} \right) - 1 \right] d\lambda \]

\[ dM_{\Delta \lambda} = \epsilon \int_{\lambda_1}^{\lambda_2} c_1 \lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T} \right) - 1 \right]^{-2} \exp \left( \frac{c_2}{\lambda t} \right) \cdot \frac{c_2}{\lambda T^2} d\lambda d\epsilon \]

\[ + \int_{\lambda_1}^{\lambda_2} c_1 \lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T} \right) - 1 \right]^{-1} d\lambda d\epsilon. \]

The relative exitance difference is

\[ \frac{dM_{\Delta \lambda}}{M_{\Delta \lambda}} = \frac{\epsilon \int_{\lambda_1}^{\lambda_2} \frac{c_1 c_2}{\lambda^5} \left[ \exp \left( \frac{c_2}{\lambda T} \right) - 1 \right]^{-2} \exp \left( \frac{c_2}{\lambda T} \right) d\lambda d\epsilon}{\epsilon \int_{\lambda_1}^{\lambda_2} c_1 \lambda^{-4} \left[ \exp \left( \frac{c_2}{\lambda T} \right) - 1 \right]^{-1} d\lambda} + \frac{d\epsilon}{\epsilon}. \]

Letting

\[ x = \frac{c_2}{\lambda T}, \]

\[ dx = -\frac{c_2}{\lambda^2 T} d\lambda, \]

then

\[ \frac{dM_{\Delta \lambda}}{M_{\Delta \lambda}} = \frac{\int_{x_1}^{x_2} x^3 e^x (e^x - 1)^{-2} dx}{\int_{x_1}^{x_2} x^2 (e^x - 1)^{-1} dx} \cdot \frac{dT}{T} + \frac{d\epsilon}{\epsilon}. \]
APPENDIX G

DERIVATION OF ERROR ANALYSIS

\[
\frac{1}{T} - \frac{1}{T_B} = \frac{\lambda_2}{c_2} \ln \varepsilon_B
\]  \hspace{1cm} (G1)

\[
\frac{1}{T} - \frac{1}{T_C} = -\frac{\lambda_1 \lambda_2}{c_2(\lambda_1 - \lambda_2)} \ln \frac{\varepsilon_1}{\varepsilon_2}
\]  \hspace{1cm} (G2)

\[
z = \frac{\rho_C}{\rho_B} = \frac{T_C - T}{T_B - T}
\]  \hspace{1cm} (G3)

\[
T_B = \frac{T_C - T}{z} + T
\]  \hspace{1cm} (G4)

\[
\frac{1}{T_B} = \frac{1}{T} - \frac{\lambda_2}{c_2} \ln \varepsilon_B = \frac{1}{T} \left( 1 - \frac{\lambda_2 T}{c_2} \ln \varepsilon_2 \right)
\]  \hspace{1cm} (G5)

\[
\frac{1}{T_C} = \frac{1}{T} + \frac{\lambda_1 \lambda_2}{c_2(\lambda_1 - \lambda_2)} \ln \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{T} \left( 1 + \frac{\lambda_1 \lambda_2 T}{c_2(\lambda_1 - \lambda_2)} \ln \frac{\varepsilon_1}{\varepsilon_2} \right)
\]  \hspace{1cm} (G6)

\[
x_2 = \frac{c_2}{\lambda_2 T}
\]  \hspace{1cm} (G7)

\[
\frac{1}{T_B} = \frac{1}{T} \left( \frac{x_2 - \ln \varepsilon_2}{x_2} \right)
\]  \hspace{1cm} (G8)

\[
\frac{1}{T_C} = \frac{1}{T} \left( 1 + \frac{\lambda_1}{x_2(\lambda_1 - \lambda_2)} \ln \frac{\varepsilon_1}{\varepsilon_2} \right)
\]  \hspace{1cm} (G9)

\[
T \left( \frac{x_2}{x_2 - \ln \varepsilon_2} \right) = \frac{T \{x_2(\lambda_1 - \lambda_2) / [x_2(\lambda_1 - \lambda_2) + \lambda_1 \ln \varepsilon_1/\varepsilon_2] \} - T}{z} + T
\]  \hspace{1cm} (G10)
\[ \frac{x_2}{x_2 - \ln \varepsilon_2} - 1 = \frac{x_2(\lambda_1 - \lambda_2) - x_2(\lambda_1 - \lambda_2) - \lambda \ln \varepsilon_1 / \varepsilon_2}{[x_2(\lambda_1 - \lambda_2) + \lambda_1 \ln \varepsilon_1 / \varepsilon_2]z} \]  
\[ \text{G11} \]

\[ \frac{\ln \varepsilon_2}{x_2 - \ln \varepsilon_2} = \frac{-\lambda_1 \ln \varepsilon_1 / \varepsilon_2}{z[x(\lambda_1 - \lambda_2) + \lambda_1 \ln \varepsilon_1 / \varepsilon_2]} \]  
\[ \text{G12} \]

\[ \frac{x_2 - \ln \varepsilon_2}{z \ln \varepsilon_2} = -\frac{x_2(\lambda_1 - \lambda_2) + \lambda_1 \ln \varepsilon_1 / \varepsilon_2}{\lambda_1 \ln \varepsilon_1 / \varepsilon_2} \]  
\[ \text{G13} \]

\[ \frac{x_2 - \ln \varepsilon_2}{z \ln \varepsilon_2} = -\frac{x_2(\lambda_1 - \lambda_2) + \lambda_1 \ln \varepsilon_1 / \varepsilon_2}{\lambda_1 \ln \varepsilon_1 / \varepsilon_2} = -\frac{x_2(\lambda_1 - \lambda_2)}{\ln \varepsilon_1 / \varepsilon_2} - 1 \]  
\[ \text{G14} \]

\[ \frac{x_2 - \ln \varepsilon_2 + z \ln \varepsilon_2}{z \ln \varepsilon_2} = -\frac{x_2(\lambda_1 - \lambda_2)}{\lambda_1 \ln \varepsilon_1 / \varepsilon_2} \]  
\[ \text{G15} \]

\[ \frac{z \ln \varepsilon_2}{x_2 - \ln \varepsilon_2 + z \ln \varepsilon_2} = -\lambda_1 \left( \ln \varepsilon_1 - \ln \varepsilon_2 \right) \]  
\[ \frac{x_2(\lambda_1 - \lambda_2)}{x_2(\lambda_1 - \lambda_2)} \]  
\[ \text{G16} \]

\[ \ln \varepsilon_1 - \ln \varepsilon_2 = \frac{x_2(\lambda_1 - \lambda_2)}{\lambda_1} \left( \frac{z \ln \varepsilon_2}{x_2 - \ln \varepsilon_2 + z \ln \varepsilon_2} \right) \]  
\[ = \frac{\lambda_1 - \lambda_2}{\lambda_1} \ln \varepsilon_2 \]  
\[ \frac{(1/z) - [(\ln \varepsilon_2)/z x_2] + [(\ln \varepsilon_2)/x_2]}{\} \]  
\[ \text{G18} \]

\[ \ln \varepsilon_1 = \ln \varepsilon_2 \left[ 1 + \frac{\lambda_1 - \lambda_2}{\lambda_1} \right] \]  
\[ \frac{1}{(\lambda_2 T / c_2) \ln \varepsilon_2 + (\sigma_B / \sigma_C) \left[ 1 - (\lambda_2 T / c_2) \ln \varepsilon_2 \right]} \]  
\[ \text{G19} \]
APPENDIX H

CALCULATIONS FOR FINITE SPECTRAL BANDS FOR RATIO-TEMPERATURE

The ratio of radiances for any two finite spectral bandwidths can be calculated by a computer program.

The expression for the ratio is (as discussed in the text)

\[ R = \frac{\int \lambda^{-4} \left[ \exp\left(\frac{hc}{kT}\lambda\right) - 1\right] d\lambda}{\int \lambda^{-4} \left[ \exp\left(\frac{hc}{kT}\lambda\right) - 1\right] d\lambda} \]

This ratio was calculated using the computer program given on the following page. This program can be extended to any temperature range and for any spectral regions.

The ratios for the spectral regions used in the actual system are plotted in Fig. H1.
PROGRAM STAFFET(INPUT,OUTPUT,TAPF=INPUT,TAPF=OUTPUT)
C THIS PROGRAM CALCULATES THE RATIO R OF TOTAL PHOTON
C RADIANCE,\lambda, IN A BANDWIDTH DEFINED BY (WAVELENGTHS) \lambda P-\lambda 1, TO
C TOTAL PHOTON RADIANCE,\phi, IN A BANDWIDTH DEFINED BY \lambda P-\lambda P
C FOR TEMPERATURE T
C TCI IS THE INITIAL TEMPERATURE IN THE RANGE OF INTEREST
C TCF IS THE FINAL TEMPERATURE IN THE RANGE OF INTEREST
C DELTC IS THE INCOREMENT OF TEMPERATURE IN THE RANGE OF INTEREST
C DELW IS THE WAVELENGTH UNIT OF INTEGRATION
INTEGER TCI,TCF,DELTC,T,R
DIMENSION T(10),G(100),H(10),W(100),WH(100),R(100)
DATA CP/1.38153X+109/
C CP IS CH/X IN UNITS OF MICRON KELVIN
DO 97 T=1,100
READ (5,11) W1,W2,WP,TCI,TCF,DELTC,DELW
11 FORMAT(RF10.2)
IF(EOF(N)) GO TO 1000
14 PRINT 100 VT,V:W?,WIP,W2P,TCI,TCF,DELTC,DELW
100 FORMAT(1H1,FI10.2)
C NUMBER OF TEMPERATURE POINTS IS CALCULATED
P=INT((TCF-TCI)/DELTC)+1
C SET UP ARRAY T FOR TEMPERATURE VALUES
DO 98 J=1,P
98 T(J)=TCI+J*DELTC
C MAIN DO LOOP TO CALCULATE RATIO R FOR EACH TEMPERATURE T
DO 99 M=1,P
99 R(M)=G(M)/H(M)*G1=0.*EXP(CP/(W1)*T(M)))-1.0
V=M-2
A=0.0
DO 100 K=1,V
100 A=A+G1=K+1)*DELW/V
C CALCULATION FOR DENOMINATOR B OF RATIO
N=INT((WP-\lambda P)/(DELW)+0.0001)+1
101 FORMAT(1H1,FI10.2)
DO 102 I=1,N
102 W(I)=H(I)*DELW
DO 103 I=1,N
103 H(I)=H(I-1)+DELW
DO 104 I=1,N
104 H(I)=H(I-1)+DELW
C WRITE(6,12)
WRITE(6,12)
12 FORMAT(13X,*RATIO*,AX,*TEMPERATURE*,PSX,*KELVIN*)
DO 105 M=1,P
105 FORMAT(1H1,FI10.2)
106 STOP
END
Fig. H1. Ratios for Spectral Regions Used in Actual System.
APPENDIX I

BACKGROUND FLUX CALCULATIONS

5-µm Channel

\[ Q_B = \int_{4.5}^{5.5} Q_B(300, \lambda) d\lambda = 2.66 \times 10^{16} - 5.5 \times 10^{15} \]

\[ = 2.05 \times 10^{16} \text{ photon/sec-cm}^2 \]

\[ E = Q_B \frac{1}{4F/No.2} \tau_{KCl} \tau_{sapp} \tau_F \]

\[ E = 2.05 \times 10^{16} \cdot \frac{1}{4(13)} \cdot (0.93) \cdot (0.95)(0.6) \]

\[ E = 1.6 \times 10^{13} \text{ photon/sec-cm}^2 \]

13-µm Channel

\[ Q_B = \int_{12.9}^{14} Q_B(300, \lambda) d\lambda = 1.16 \times 10^{18} - 9.8 \times 10^{17} \]

\[ = 1.8 \times 10^{17} \text{ photon/sec-cm}^2 \]

\[ E = Q_B \frac{1}{4F/No.2} \tau_{KCl} \tau_{sapp} \tau_F \]

\[ E = 1.8 \times 10^{17} \cdot \frac{1}{4(13)^2} \cdot (0.93)(0.9)(0.5) \]

\[ E = 1.1 \times 10^{14} \text{ photon/sec-cm}^2 \]

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APPENDIX J

RADIANT SIGNAL LEVEL CALCULATIONS

\[ \phi_d = L_s \frac{A_d A_{bb}}{R^2} \cdot \tau \]

5-\mu m Channel

\[ L_s = \int_{4.5}^{5.5} W(500, \lambda) d\lambda - \int_{4.5}^{5.5} W(300, \lambda) d\lambda \]

\[ = 0.113(0.22 - 0.22) - 0.015(0.025 - 0.006) \]

\[ L_s = 0.0121/\pi = 3.87 \times 10^{-3} \text{ watt/cm}^2\text{-sr} \]

\[ \phi_d = 3.87 \times 10^3 \frac{\pi(0.075)^2}{4} \frac{\pi(0.5)^2}{4} \frac{0.586}{(215)^2} \]

\[ \phi_d = 4.25 \times 10^{-11} \text{ watt} \]

13-\mu m Channel

\[ L_s = \int_{12.9}^{14} W(500, \lambda) d\lambda - \int_{4.5}^{5.5} W(300, \lambda) d\lambda \]

\[ = 0.113(0.81 - 0.77) - 0.015(0.52 - 0.46) \]

\[ L_s = 1.15 \times 10^{-3} \text{ watt/cm}^2\text{-sr} \]

\[ \phi_d = 1.15 \times 10^{-3} \frac{\pi(0.075)^2}{4} \frac{\pi(0.5)^2}{4} \frac{0.586}{(215)^2} \]

\[ \phi_d = 1.26 \times 10^{-11} \text{ watt} \]

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