SYNTHESIS AND SENSITIVITY ANALYSIS OF ELLIPTIC NETWORKS

by

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A Dissertation Submitted to the Faculty of the

DEPARTMENT OF ELECTRICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

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THE UNIVERSITY OF ARIZONA
GRADUATE COLLEGE

I hereby recommend that this dissertation prepared under my direction

by David Jose Miguel Baez Lopez.

entitled Synthesis and Sensitivity Analysis of Elliptic Networks.

be accepted as fulfilling the dissertation requirement for the Degree

of Doctor of Philosophy.

L P Huelsman 1/25/79

Dissertation Director Date

As members of the Final Examination Committee, we certify that we have read this dissertation and agree that it may be presented for final defense.

Jan 25 '79
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To

Luz del Carmen

and

David Alfredo

ACKNOWLEDGMENTS

The author wishes to thank Professor Lawrence P. Huelsman for his encouragement, suggestion, and expert advice during all phases of this work. Thanks are also due to Profesors W.J. Kerwin and J.V. Wait, who carefully read the entire manuscript and offered valuable suggestions.

A note of thanks is due to the Computer Center of The University of Arizona for its cooperation and assistance.

The author would also like to express his gratitude to the Consejo Nacional de Ciencia y Tecnologia, CONACYT, Mexico, and to the Department of Electrical Engineering of The University of Arizona, for the scholarship aid during the development of this work.

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ABSTRACT

Elliptic filters are very useful in electrical engineering because of their steeper fall off at the band edge, compared to other filters such as Butterworth or Chebychev. However, the design of elliptic filters involves a great deal of computational effort. Tables for the design of elliptic passive filters were published in Germany and the United States between 1963 and 1967 for different values of passband ripple and stopband frequency; however, these two parameters are specified in terms of reflection coefficient and modular angle. Although these parameters can be converted to quantities having a direct physical meaning, such as dB and radians per second, the values obtained are not easy to work with. In addition, the resistance terminations are normalized to unity for most cases, or they are fixed by the value of passband ripple.

The purpose of this dissertation is to describe the theory of elliptic passive filters, and from this theoretical treatment develop an algorithm to obtain the desired elliptic passive filter element values in terms of specified resistance termination, passband ripple, and stopband frequency.

CHAPTER 1

THE ELLIPTIC NETWORK FUNCTION

The first step in the design of an elliptic network is to obtain a function that approximates the desired characteristic. Once the approximating function is obtained, we can proceed to synthesize a network with the desired response. In this work we will be interested in the lowpass characteristic since the passive lowpass network obtained can be used as a prototype to obtain highpass, bandpass, and band elimination networks. The passive lowpass network will be described as a two port network. In the next sections, a brief treatment of the elliptic transfer function will be given.

The Elliptic Approximation Problem

The ideal lowpass magnitude characteristic we want to approximate is illustrated in Fig. 1.1a. $N_0(s)$ represents the analytical expression for this function, where s is the complex frequency variable. The frequency w_c separating the passband and the stopband is called the cutoff frequency. Unfortunately, this characteristic cannot be obtained by any real network. Instead, we have to find a function that approximates the ideal characteristic in

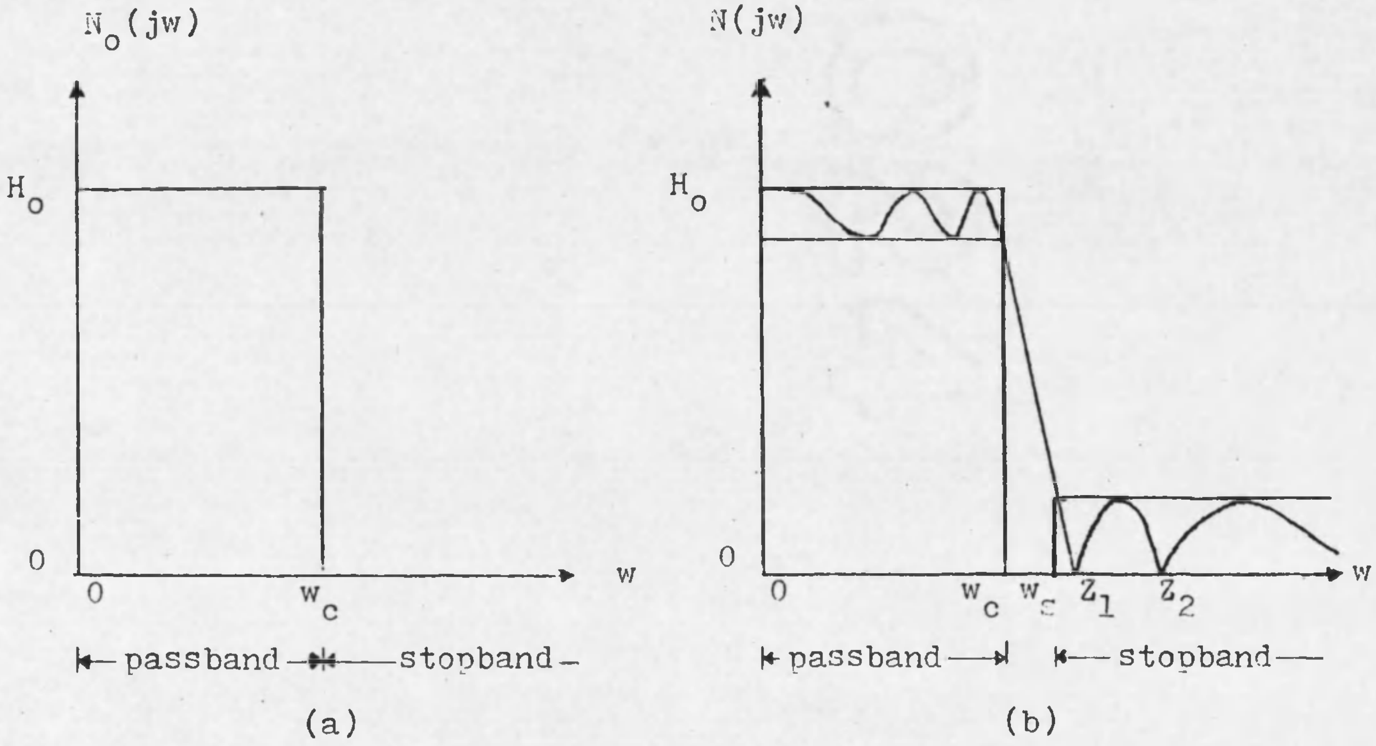


Fig. 1.1 Lowpass Characteristics.

a) Ideal, b) Elliptic.

such a way that a network can be synthesized from the approximating function. One of the most useful approximating functions is the elliptic one, illustrated in Fig. 1.1b. In this function, the transition from passband to stopband has a finite slope. The frequency where the stopband starts is called the stopband frequency and is denoted by w_s. The region between w_c and w_s is called the transition region. As shown, a ripple is allowed in both the passband and the stopband.

The parameters describing the elliptic characteristic are: a) the passband ripple, usually measured in dB and denoted by A_{max}; b) the minimum attenuation at the stopband with respect to the maximum value at the passband, measured in dB and denoted by A_{min}; c) the stopband frequency w_s. Another important parameter is the degree of the denominator polynomial of the transfer function N(s), as will be discussed in the next section. The frequencies z_i, shown in Fig. 1.1b, are called transmission zeros.

Since the ideal lowpass characteristic rejects a band of frequencies while leaving another band unaltered, this characteristic is also called a lowpass filter. Thus, we will refer to the elliptic characteristic as an elliptic filter.

Another quantity used to specify the stopband fre- is the modular angle θ defined by $w_s = 1/\sin\theta$.

Transfer and Characteristic Functions

When we are treating two port networks, we are usually interested in the voltage transfer function

$$N(s) = \frac{V_2(s)}{V_1(s)} \qquad (1\text{-}1)$$

where $V_2(s)$ is the voltage at port 2 and $V_1(s)$ is the voltage at port 1 as shown in Fig. 1.2. We now define the characteristic function by the following relation, called the Feldtkeller equation,

$$|N(jw)|^{-2} = 1 + |K(jw)|^2 \qquad (1\text{-}2)$$

For elliptic ladder networks, the characteristic function K(s) is a rational function of the complex variable s and it has the form:

n odd:
$$K(s) = H\, s \prod_{j=1}^{\frac{n-1}{2}} \frac{s^2 + P_j^2}{s^2 P_j^2 + 1} \qquad (1\text{-}3)$$

n even:
$$K(s) = H \prod_{j=1}^{n/2} \frac{s^2 + P_j^2}{s^2 P_j^2 + 1} \qquad (1\text{-}4).$$

where n is the prescribed degree of K(s), H is a positive constant, and the quantities P_j are called the Cauer parameters. The value of the constant H and the parameters P_j are chosen such that the value of $|K(jw)|$ has the smallest maximum deviation A_{max} in the passband and the largest minimum deviation A_{min} in the stopband. The solution to this extremal problem was given by Cauer (1958) and involves

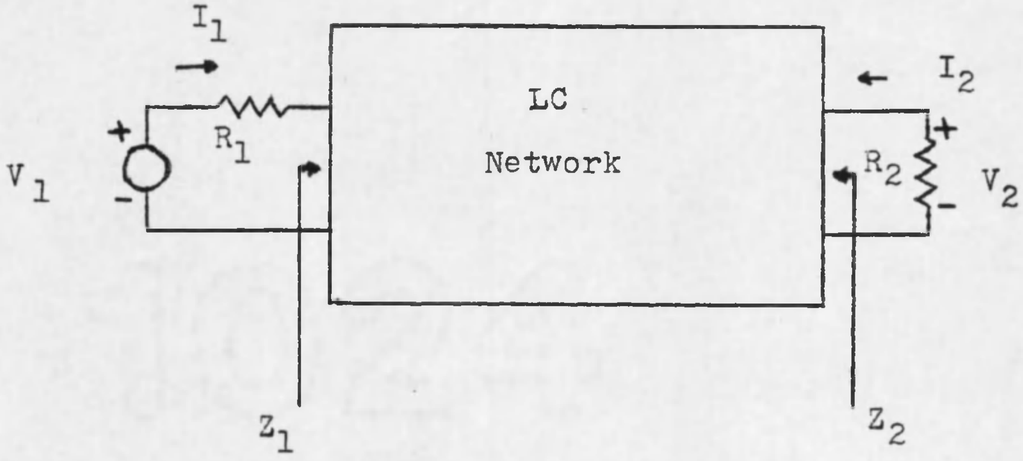


Fig. 1.2 Representation of a Doubly Terminated LC Network.

the evaluation of elliptic integrals and Jacobian elliptic functions. The values of the Cauer parameters are given by the following expressions:

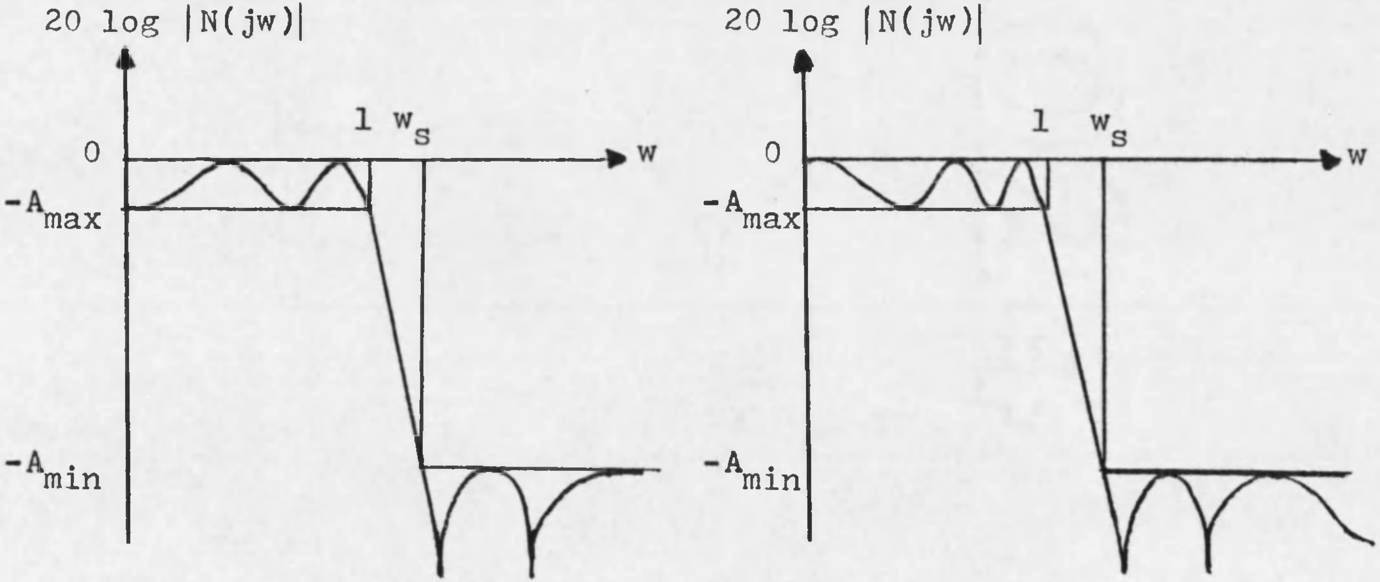
$$P_j = \text{sn}\left[\frac{n-2j-1}{n}K,\ 1/w_s\right] \qquad (1\text{-}5)$$

where K is the elliptic integral of the first kind with the modulus $1/w_s^2$, and sn(a,b) is the Jacobian elliptic function with modulus b.

In Eqs.(1-2) to (1-5) we are assuming that the frequency w_c is normalized to unity. In Fig. 1.3, representative plots of 20 log|N(jw)| for n=4 and n=5 are shown. In these plots, the maximum value of 20 log|N(jw)| is normalized to 0 dB. From Fig. 1.3a, we note that for n=4 the value of |N(jw)| for w approaching infinity is the finite value $-A_{min}$. This gives place to a ladder network realization containing mutual inductances. We will refer to this case of an even order function as case 1.

To avoid the mutual inductances in the ladder realization we need to make a frequency transformation to shift the rightmost transmission zero to infinity. This will give a realization without mutual inductances but with unequal terminating resistances. We refer to this case of even order functions as case 2.

Unfortunately, the transformation shifts the stop-band frequency to a new value given by



(a) (b)

Fig. 1.3 Plots of 20 log $|N(jw)|$.

a) $n=4$, b) $n=5$.

$$w_{s2} = \frac{w_s}{sn\left[\frac{n-1}{n}K, 1/w_s\right]} \qquad (1\text{-}6)$$

Since the elliptic function is less than one for b less than one, the new stopband frequency is greater than the value given for case 1. The new characteristic function and Cauer parameters are given by

$$K(s) = H \frac{\prod_{j=1}^{n/2} (s^2 + PO_j^2)}{\prod_{j=1}^{\frac{n-1}{2}} Z_j^{-2}(s^2 + Z_j^2)} \qquad (1\text{-}7)$$

$$PO_j^2 = \frac{P_j^2 \, cn(K/n, 1/w_s)dn(K/n, 1/w_s)}{1 - P_j^2 P_{n/2}^2} \qquad (1\text{-}8)$$

$$Z_i^2 = \frac{cn(K/n, 1/w_s)dn(K/n, 1/w_s)}{P_i^2 - P_{n/2}^2} \qquad (1\text{-}9)$$

where $cn(u,v) = \sqrt{1-sn^2(u,v)}$ and $dn(u,v) = \sqrt{1-v^2 sn^2(u,v)}$.

In order to eliminate the restriction of unequal terminating resistances, we need to make a second frequency transformation, this time to shift the first zero of Eq.(1-4) to the origin. The resulting characteristic function is given by

$$K(s) = H\, s^2 \prod_{j=1}^{\frac{n-1}{2}} \frac{s^2 + PO_j^2}{s^2 PO_j^2 + 1} \qquad (1\text{-}10)$$

where the parameters PO_j are given by

$$PO_j^2 = \frac{P_j^2 - P_{n/2}^2}{1 - P_j^2 P_{n/2}^2} \qquad (1\text{-}11)$$

As in case 2, the stopband frequency shifts to a greater value given by

$$w_{s3} = \frac{w_s}{sn^2\left[\frac{n-1}{n}K, 1/w_s\right]} \qquad (1\text{-}12)$$

From this equation, we see that the new stopband frequency increases to a value greater than the stopband frequency for case 2. We will refer to this case of even order functions as case 3. Two examples showing the magnitude of the transfer function are plotted in Fig. 1.4 for cases 2 and 3 of sixth order.

If we write K(s) and N(s) as

$$K(s) = \frac{F(s)}{C_1 P(s)} \qquad (1\text{-}13)$$

$$N(s) = \frac{C_2 P(s)}{E(s)} \qquad (1\text{-}14)$$

where C_1 and C_2 are positive constants and F(s), P(s), and E(s) have unity leading coefficient, then, substituting Eq.(1-13) in Eq.(1-2) and replacing s by jw we obtain

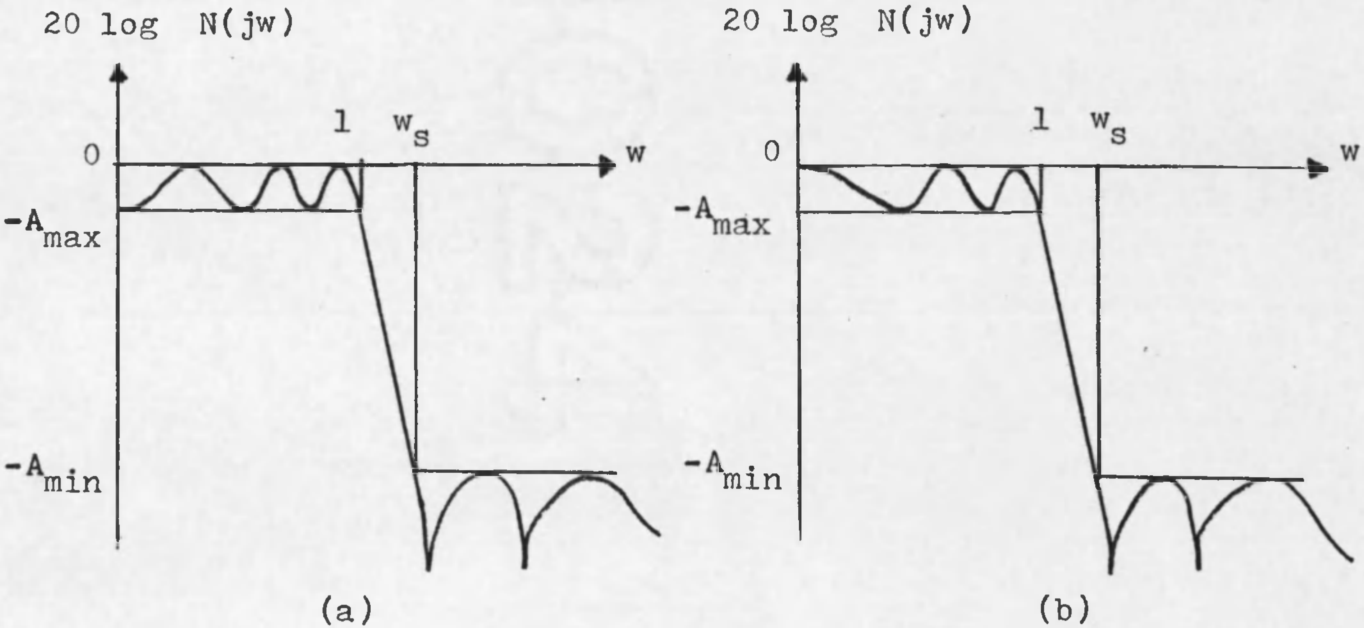


Fig. 1.4 Plots of $|N(jw)|$ for Even Order Cases 2 and 3.

a) Case 2, b) Case 3.

$$E(s)E(-s) = F(s)F(-s) + C^2P(s)P(-s) \qquad (1\text{-}15)$$

where $C = C_1C_2$.

For N(s) to be realizable as a network, it is required that its poles be located on the left half plane. We designate by E(s) the polynomial whose roots are all located in the left half plane and E(-s) the polynomial with roots on the right half plane. The zeros of N(s), called the transmission zeros, are the poles of K(s). Note that the degree of F(s) is greater than or equal to the degree of P(s). Furthermore, the degree of F(s) is the prescribed degree n of K(s). Therefore, the degree of E(s) is equal to the degree F(s).

Even Order Transfer Functions with Prescribed Value of w_s

As discussed in the preceding section, when we want to obtain the parameters of the transfer function for even order cases 2 and 3 with unity cutoff frequency and stopband frequency w_s, we end up with a value of stopband frequency greater than the prescribed one. That is, the resulting values of stopband frequencies will be w_{s2} and w_{s3} for cases 2 and 3, respectively, and they satisfy

$$w_{s3} > w_{s2} > w_s \qquad (1\text{-}16)$$

If we want to find the transfer function for cases 2 and 3 with a prescribed value of w_s, we will have to find a new value of w_s, denoted by w_s', such that when substituted in the corresponding equation, either Eq.(1-6) or (1-12), gives the desired value of w_s. Since the form of Eqs.(1-6) and (1-12) is similar, we will describe the solution for case 2, that is, we will work with Eq.(1-6). The results are then extended to case 3.

Let us write Eq.(1-6) in the form

$$w_s - \frac{w_s'}{\operatorname{sn}\left[\frac{n-1}{n}K', 1/w_s'\right]} = 0 \qquad (1\text{-}17)$$

where K' indicates that the elliptic integral K depends on w_s'. The problem can now be stated as follows: given w_s, find a value of w_s' such that when substituted in Eq.(1-17) the equality is valid. Thus, the problem reduces to solving Eq.(1-17) for w_s'. Since K' is a function of w_s', Eq.(1-17) cannot be solved in closed form. Instead, we have to use numerical methods to find the solution of Eq.(1-17).

The root solving algorithm used to solve Eq.(1-17) is the chord method (Henrici 1964, p.87). Although this is a first order method, it is found that it converges very fast if the initial estimate is chosen as the prescribed value w_s. The iteration equation is then given by

$$w'_{s,k+1} = w'_{s,k} - m\ f(w'_{s,k}) \qquad (1\text{-}18)$$

where m is a constant to be determined, and $f(w'_{s,k})$ is given by

$$f(w'_{s,k}) = w_s - \frac{w'_{s,k}}{sn\left[\frac{n-1}{n}K(w'_{s,k}), 1/w'_{s,k}\right]} \qquad (1-19)$$

In practice, the evaluation of the elliptic function requires the reciprocal of $1/w'_s$ in each iteration. To avoid this and reduce computational effort, we rewrite Eq.(1-17) as

$$x - x' \; sn\left[\frac{n-1}{n} K' \; , \; x'\right] = 0 \qquad (1-20)$$

where $x=1/w_s$ and $x'=1/w'_s$. The new iteration equation is

$$x_{k+1} = x'_k - m\; g(x'_k) \qquad (1-21)$$

$$g(x'_k) = x - x'_k \; sn\left[\frac{n-1}{n}K(x'_k), x'_k\right] \qquad (1-22)$$

When the solution to Eq.(1-20) is obtained, we just take the reciprocal of x' and that is the value of w'_s needed to obtain the transfer function with a prescribed value of w_s. The value of m that gives the best results is $-\frac{1}{2}$. When the value of w_s is very close to one, we also use the value -1/10. The algorithm usually converges in less than 20 iterations, and the accuracy is 10^{-10} . For case 3, the final value of w'_s is given by

$$w'_s = 1/x'^2 \qquad (1-23)$$

CHAPTER 2

SYNTHESIS OF LC DOUBLY TERMINATED NETWORKS

The synthesis of LC networks which are resistively terminated is an important topic in network design. This is due to the fact that in practice most two ports are doubly loaded. Furthermore, by choosing adequate values of terminating resistances, we can match simultaneously generator and load. For simplicity, we will assume that the generator impedance is normalized to unity. The general form of an LC doubly terminated network is shown in Fig. 2.1.

Case of Equal Terminating Resistances

As discussed in Chapter 1, only odd order transfer functions and case 3 of even order ones can have a passive ladder realization with equal terminating resistances. Case 1 requires mutual inductances and will not be discussed here. Case 2 has unequal terminating resistances and will be treated in the next section.

We start the synthesis procedure by considering Eqs.(1-13), (1-14), and (1-15), repeated below for easy reference,

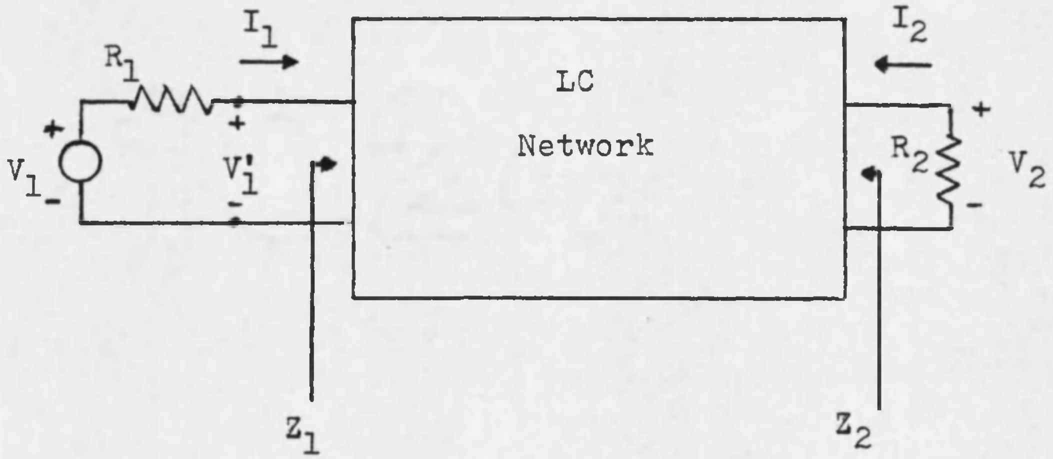


Fig. 2.1 Elliptic LC Network Representation.

$$K(s) = \frac{F(s)}{C_1 P(s)} \tag{2-1a}$$

$$N(s) = \frac{C_2 P(s)}{E(s)} \tag{2-1b}$$

$$E(s)E(-s) = F(s)F(-s) + C^2 P(s)P(-s) \tag{2-2}$$

Denoting the even and odd part of the polynomials P(s), E(s), and F(s) by the subscripts e and o, respectively, we have that

$$E(s) = E_e(s) + E_o(s) \tag{2-3a}$$

$$F(s) = F_e(s) + F_o(s) \tag{2-3b}$$

$$P(s) = P_e(s) + P_o(s) \tag{2-3c}$$

To synthesize the LC network of Fig. 2.1 we need to find the driving point z or y-parameters of this network. These can be readily evaluated from Table 2.1 (Temes and LaPatra 1977, p.209) where in this case $R_2=1$. The only requirement the driving point function must satisfy is that the degree of either numerator or denominator must be equal to n. Once these parameters are evaluated we can start the actual synthesis procedure by removing poles from the driving point immittance corresponding to transmission zeros of the transfer function.

Table 2.1 Immittances of the LC Network.

$$z_{11} = \frac{E_e - F_e}{E_o + F_o}$$

$$z_{22} = \frac{E_e + F_e}{E_o + F_o} R_2$$

$$y_{11} = \frac{E_e + F_e}{E_o - F_o}$$

$$y_{22} = \frac{E_e - F_e}{E_o - F_o} \frac{1}{R_2}$$

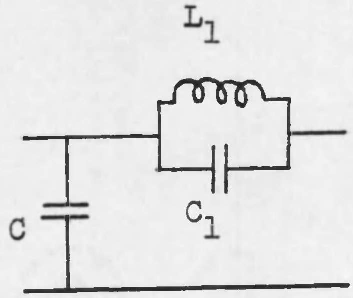
$$Z_1 = \frac{E - F}{E + F}$$

Since N(s) has jw-axis zeros, we will have to use the zero shifting technique to remove the zeros. Thus, for each zero we will have a section of the form shown in Fig.2.2a. In the case of odd order networks there is only a zero at infinity whereas for even order networks there are two zeros at infinity. The corresponding sections of the network are shown in Figs. 2.2b and 2.2c, respectively.

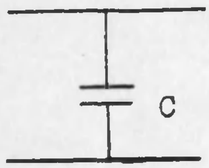
When the synthesis procedure is implemented in a digital computer, we will have round off error because only a finite number of digits is retained. To avoid a large accumulating round off error, that could lead to incorrect results, we start the synthesis from both ports and stop when we reach the central branch of the ladder. Configurations for odd and even order networks are shown in Fig.2.3. The synthesis method is best described by the following procedure(Saal and Ulbrich 1958). Obtain first the corresponding driving point function. If we work with z_{11}, then define $B(s)=1/z_{11}$. If we work with y_{11}, then $B(s)=y_{11}$. Second, make a partial removal of the zero and obtain the new function $B_1(s)$ as follows

Partial Removal $$C=\left.\frac{B(s)}{s}\right|_{s^2=-\mathbb{Z}_1^2} \qquad (2\text{-}4)$$

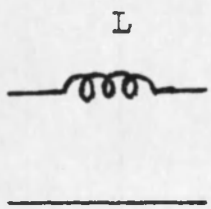
$$B_1(s)=B(s)-s\,C \qquad (2\text{-}5)$$



(a)



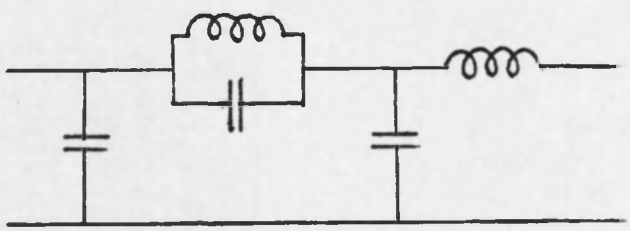
(b)



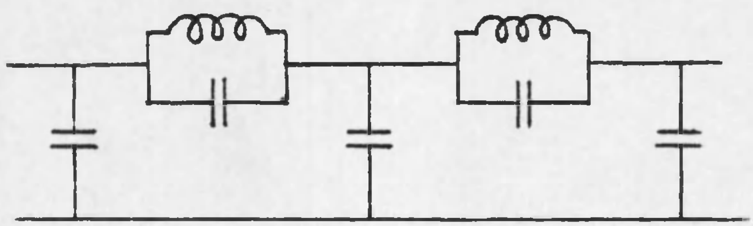
(c)

Fig. 2.2 Branches of the Elliptic Ladder Network.

a) Section for a Transmission Zero, b) and c) Zero at Infinity.



(a)



(b)

Fig. 2.3 Even and Odd Order Configurations.

a) Even Order,
b) Odd Order.

The partial fraction expansion of $B_1(s)$ will have a term of the form $k_i s/s^2+Z_i^2$. The removal of this term will produce a corresponding zero in the transfer function. The shunt branch is given by

$$C_1 = \left. \frac{s\ B_1(s)}{s^2+Z_i^2} \right|_{s^2=-Z_i^2} \qquad (2\text{-}6)$$

$$L_1 = 1/C_1 Z_i^2 \qquad (2\text{-}7)$$

and the new function B_2 is given by

$$\frac{1}{B_2(s)} = \frac{1}{B_1(s)} - \frac{s/C_1}{s^2+Z_i^2} \qquad (2\text{-}8)$$

Once we obtain the function $B_2(s)$ the process is repeated. For the synthesis from port 2 we use z_{22} and y_{22} instead of z_{11} and y_{11}. The section obtained from Eqs.(2-5), (2-6), and (2-7) is shown in Fig.2.2a.

There are certain guidelines that must be followed to avoid elements with negative value wherever possible. These gives place to a removal of zeros in a certain order (Saal and Ulbrich 1958). In general, care should be taken to insure that zeros lying closer to the stopband frequency are attributed to the circuit elements situated in the middle part of the network.

Case of Unequal Terminating Resistances

For the case when R_2 is different from one, we have two possibilities, namely, case 2 of even order transfer functions, where the value of R_2 is fixed by the value of $|N(j0)|$, and the case when the value of R_2 is prescribed.

In case 2 of even order functions, the synthesis procedure is the same as in the preceding section. The only difference is in the determination of the resistance value at port 2.

The reason that R_2 is different from one is due to the fact that $|N(jw)|$ does not obtain a maximum at $w=0$, but rather it obtains a minimum. Furthermore, the value of $|N(jw)|$ is related to the passband ripple. Therefore, a relation between the passband ripple A_{max} and the value of R_2 exists. This relation will have a simpler form if we introduce another measure of the passband ripple, the reflection coefficient p given by

$$p = (1 - 10^{A_{max}/10})^{\frac{1}{2}} \qquad (2\text{-}9)$$

and the relation between R_2 and p is

$$R_2 = \frac{1-p}{1+p} \qquad (2\text{-}10)$$

When the value of R_2 is prescribed, we consider two cases. The value of R_2 is finite and different from zero, or it is infinite.

When R_2 is infinite the procedure is the same as in the preceding section, but in this case the polynomial F(s) is not included in the evaluation of the driving point functions. Thus, the resulting network can be synthesized entirely from a knowledge of the transfer function N(s). This kind of network is called singly terminated. In this case, only current or voltage can be transmitted, whereas when R_2 is finite, power can be transmitted too.

For the case of finite R_2, let us consider Eq.(2-2)

$$E(s)E(-s) = F(s)F(-s) + C^2P(s)P(-s) \qquad (2\text{-}11)$$

If $R_2 \neq 1$, we will have to reduce the value of $|N(jw)|$ by a constant decrement given by (Cauer 1958, p.493)

$$g = \tfrac{1}{2}(t+t^{-1}) \quad ; \; t = \sqrt{R_2} \qquad (2\text{-}12)$$

The decremented magnitude of the transfer function is shown in Fig. 2.4 for n=3. The new transfer function is

$$|N_1(s)|_{s=jw} = g|N(s)|_{s=jw} \qquad (2\text{-}13a)$$

$$= gC\left|\frac{P(s)}{E(s)}\right|_{s=jw} \qquad (2\text{-}13b)$$

$$= C_1\left|\frac{P(s)}{E(s)}\right|_{s=jw} \qquad (2\text{-}13c)$$

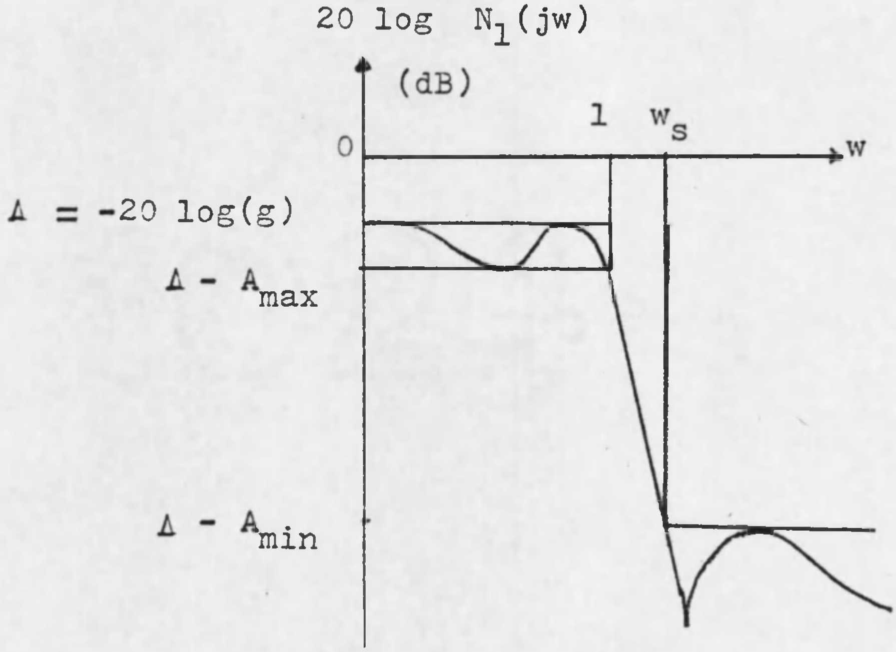


Fig. 2.4 Magnitude Function Decremented Due to Unequal Resistance Termination.

The new function N(s) must also satisfy Eq.(1-2). This means we need to find a compatible polynomial $F_1(s)$ so that together with E(s), P(s), and C_1 satisfy Eq.(2-11),

$$E(s)E(-s) = F_1(s)F_1(-s) + C_1^2P(s)P(-s) \qquad (2\text{-}14)$$

Note from Eq.(2-12) that the same procedure is followed if the reciprocal value of R_2 is specified. At this point we should mention a limitation for the case when R_2 is greater than one in the even order case 3. From the expression for Z_1 from Table 2.1, we see that(Saal 1965)

$$R_2 = \lim_{s\to 0} Z_1(s) = \frac{e_o - f_o}{e_o - f_o} < 1 \qquad (2\text{-}15)$$

where e_o and f_o represent the constant terms of E(s) and F(s), respectively. This limitation does not apply to odd order networks since the networks obtained are structurally symmetrical, and thus can be reversed, that is, the two ports can be interchanged.

Then, in case we want an even order network with R_2 greater than one, we will have to synthesize a network with a terminating resistance of $1/R_2$ and use the dual network that will have the desired termination.

Once the new polynomial $F_1(s)$ is found, the driving point immittances are evaluated from Table 2.1 and the synthesis procedure of the preceding section is applied.

CHAPTER 3

THE PROGRAM ELLIP

Based on the theory developed in the two preceding chapters, a digital computer program has been written to synthesize the passive ladder structure of elliptic filters. This program is written in FORTRAN IV and is named ELLIP.

Description

The program ELLIP will be able to synthesize the elliptic ladder network provided the stopband frequency and a set of the parameters described below is specified.

Parameter	Symbol	Mnemonic
1. Ripple in the passband.		
a) In decibels.	A_{max}	AMAX
b) As a numeral.	R_{max}	RMAX
c) Reflection coefficient.	ρ	RO
2. Minimum attenuation in the stopband.		
a) In decibels.	A_{min}	AMIN
b) As a numeral.	R_{min}	RMIN
3. Order.	n	N
4. Stopband frequency.		
a) Radians per second.	w_s	WS
b) Modular angle.	θ	TETA
5. Case number.	case	CASE
6. Terminating resistance.	R_2	R2

In addition to these parameters, there are four extra parameters. 1) FCN indicates that only the transfer function is desired. 2) ORD is included only when the transmission zeros are removed according to their magnitude. If Z_1, Z_2, ..., Z_r are the zeros and Z_1 Z_2 ... Z_r, then Z_1 is removed first, then Z_2, and so on. If ORD is omitted, the removal of zeros is done as explained in chapter 2. 3) PNC indicates that punched output is desired. The information included in the punched output is discussed in the examples. 4) COMP indicates that the coefficients of numerator and denominator polynomials of the transfer functions are desired. Otherwise, only pole and zero locations will be given in addition to the network elements.

The necessary parameters to synthesize the ladder network are 4 and 6. The default value for R_2 is unity. The remaining parameters can be arranged in three possible combinations to give the desired network.

	Possible combination.		
	A	B	C
Stopband frequency and R_2.	x	x	x
Passband ripple.	x	x	
Stopband attenuation.	x		x
Order.		x	x

FCN, ORD, PNC, and COMP are included according to the information desired.

User's Guide

The program will accept data cards in the following order:

Card number	Columns	Variable	Field	Explanation
1	1-80	LETT	8A10	Title card.
2	1-4	Specification mnemonic	A4	
	5-10			Blank spaces.
	11-20	X	E10.0	Specification value.
3	1-80			Separator.

In the case of parameters FCN, ORD, PNC, and COMP no value is given. Since the number of parameters is greater than one, there will be as many No. 2 cards as needed. Repeat as many decks of cards 1 to 3 as desired.

Examples

Elliptic filters are sometimes called Cauer filters because Cauer (1958) was the first one to describe the approximation described in chapter 1.

The notation used to describe these filters is the following:

$$C\ n\text{-case}/A_{max}\ \text{or}\ A_{min}/w_s\ \text{or}\ \theta/R_2$$

Examples showing the use of ELLIP are shown in Figs. 3.1 and 3.2. These examples have different resistance termination values.

```
C 05/.1/1.5/R2=2.0

     INPUT  DATA
AMAX            .10000
N              5.00000
WS             1.50000
R2             2.00000
PNC

                                                     2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                                 C(1)                C(2)                   ZEROS

-.649753176666E+00
-.417037455064E+00 + J  .775766106185E+00   .834074910128E+00   .775733290431E+00   .155740639076E+01
-.114129434187E+00 + J  .106615071581E+01   .228258868374E+00   .114970287656E+01   .233187577188E+01
   C( 1)= .59635244562996E+00      L( 2)= .11713736258119E+01      C( 2)= .15699786203662E+00
   C( 3)= .13732958199057E+01      L( 4)= .11355393084175E+01      C( 4)= .36307297387865E+00
   C( 5)= .10960826311221E+01

WS=1.50000000   AMIN= .434152E+02   AMAX= .10000000   N= 5   CONST= .439371E-01   R2= .200000E+01

----------------------------------------------------------------------------------------------------

C 05/.1/2.0/R2=INF.

     INPUT  DATA
AMAX            .10000
N              5.00000
WS             2.00000
R2          1000.00000

                                                     2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                                 C(1)                C(2)                   ZEROS

-.590933447131E+00
-.429091677736E+00 + J  .721329244665E+00   .858183355471E+00   .704435547110E+00   .208924650220E+01
-.138912551519E+00 + J  .107356744377E+01   .277825103037E+00   .117184375330E+01   .325080487443E+01
   C( 1)= .50028486422972E+00      L( 2)= .11398596027824E+01      C( 2)= .83016961381393E-01
   C( 3)= .13904889547689E+01      L( 4)= .13393293616646E+01      C( 4)= .17105400327130E+00
   C( 5)= .12568046083690E+01

WS=2.00000000   AMIN= .599009E+02   AMAX= .10000000   N= 5   CONST= .105752E-01   R2= .100000E+04
```

Fig. 3.1 Examples of Use of ELLIP.

```
C 04-3/.1/1.5/R2=2.0

     INPUT  DATA
AMAX            .10000
N              4.00000
WS             1.50000
CASE           3.00000
R2             2.00000
PNC

                                                               2
          THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                    POLES                                  C(1)                 C(2)                 ZEROS

-.206295931924E+00 + J  .113643087521E+01   .412591863848E+00   .133403314565E+01   .161606884189E+01
-.863021989336E+00 + J  .597833382666E+00   .172604397867E+01   .110221170751E+01  0.
   L( 1)= .20334142973691E+01      C( 2)= .47980903069704E+00      L( 2)= .79801649462560E+00
   L( 3)= .20630639657037E+01      C( 4)= .28484757787600E+00

WS=1.50000000   AMIN= .237362E+02   AMAX= .10000000   N= 4   CONST= .563005E+00   R2= .200000E+01
-------------------------------------------------------------------------------------------------
C 04-3/.1/1.5/R2=.5

     INPUT  DATA
AMAX            .10000
N              4.00000
WS             1.50000
CASE           3.00000
R2              .50000


                                                               2
          THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                    POLES                                  C(1)                 C(2)                 ZEROS

-.206295931924E+00 + J  .113643087521E+01   .412591863848E+00   .133403314565E+01   .161606884189E+01
-.863021989336E+00 + J  .597833382666E+00   .172604397867E+01   .110221170751E+01  0.
   C( 1)= .20334142973691E+01      L( 2)= .47980903069704E+00      C( 2)= .79801649462560E+00
   C( 3)= .20630639657037E+01      L( 4)= .28484757787600E+00

WS=1.50000000   AMIN= .237362E+02   AMAX= .10000000   N= 4   CONST= .563005E+00   R2= .500000E+00
```

Fig. 3.1 Examples of Use of ELLIP.-Continued

```
C 04-3/.01/1.5

     INPUT  DATA
AMAX          .01000
N            4.00000
CASE         3.00000
WS           1.50000


                                                       2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                                C(1)                 C(2)                   ZEROS

-.231209162785E+00 + J  .133257070774E+01   .462418325570E+00   .182920236810E+01   .161606884189E+01
-.130689910375E+01 + J  .920441836644E+00   .261379820751E+01   .255519844204E+01  0.
   C( 1)= .26959066022438E+00     L( 2)= .62558204985428E+00     C( 2)= .61206283149548E+00
   C( 3)= .10061406814328E+01     L( 4)= .65014929166873E+00

WS=1.50000000   AMIN= .138552E+02   AMAX= .01000000   N= 4   CONST= .178964E+01   R2= .100000E+01
-------------------------------------------------------------------------------------------------

C 03/.1/1.5

     INPUT  DATA
WS           1.50000
AMAX          .10000
N            3.00000
FCN


                                                       2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                                C(1)                 C(2)                   ZEROS

-.129818203824E+01
-.289646240908E+00 + J  .121242785156E+01   .579292481816E+00   .155387624012E+01   .167511614228E+01

WS=1.50000000        AMIN= .148478E+02   AMAX= .10000000     N= 3               CONST= .719890E+00
```

Fig. 3.1 Examples of Use of ELLIP.-Continued

```
C 05/.01/1.5

     INPUT  DATA
AMAX            .01000
N              5.00000
WS             1.50000


                                  2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                          C(1)                C(2)                ZEROS

-.104879025213E+01
-.603366415246E+00 + J  .946045842589E+00   .120673283049E+01   .125905376733E+01   .155740639076E+01
-.148803611832E+00 + J  .117168189959E+01   .297607223664E+00   .139498098873E+01   .233187577188E+01
   C( 1)= .636807104757781E+00      L( 2)= .11343501940434E+01      C( 2)= .16212202886220E+00
   C( 3)= .12983101074346E+01       L( 4)= .78253828492108E+00      C( 4)= .52685426592877E+00
   C( 5)= .39852461595274E+00

WS=1.50000000   AMIN= .333719E+02   AMAX= .01000000   N= 5   CONST= .139665E+00   R2= .100000E+01
----------------------------------------------------------------------------------------------------
C 04-2/.01/1.5

     INPUT  DATA
AMAX            .01000
N              4.00000
CASE           2.00000
WS             1.50000


                                  2
      THE QUADRATIC FACTORS ARE OF THE FORM S +C(1)S+C(2).

                 POLES                          C(1)                C(2)                ZEROS

-.221824706205E+00 + J  .128925907876E+01   .443649412411E+00   .171139517245E+01   .161106092790E+01
-.111051845594E+01 + J  .825102230917E+00   .222103691188E+01   .191404493245E+01  0.
   C( 1)= .37170671808354E+00       L( 2)= .67279369264807E+00      C( 2)= .57265645235872E+00
   C( 3)= .11194262275070E+01       L( 4)= .68186192168858E+00

WS=1.50000000   AMIN= .163667E+02   AMAX= .01000000   N= 4   CONST= .126060E+01   R2= .908474E+00
```

Fig. 3.1 Examples of Use of ELLIP.-Continued

```
C 03/.1/1.5

    INPUT  DATA
AMAX           .10000
N             3.00000
WS            1.50000
COMP

C 03/.1/1.5

THE ORDER OF THE NETWORK FUNCTION DENOMINATOR IS   3

THE ZEROS ARE


ZERO (  1)=   .167511614228E+01


THE POLES ARE


POLE( 1)=  -.129818203824E+01

POLE( 2)=  -.289646240908E+00 + J    .121242785156E+01

POLE( 3)=  -.289646240908E+00 - J    .121242785156E+01

THE COEFFICIENTS OF THE QUADRATIC FACTORS
 2
S +C(1,J)S+C(2,J), WHERE J IS THE INDEX OF THE CORRESPONDING POLE

C(1, 2)=   .579292481816E+00  C(2, 2)=   .155387624012E+01
```

(a)

```
C 03/.1/1.5

COEFFICIENTS OF THE DENOMINATOR OF THE NETWORK TRANSFER FUNCTION.
                                                   N
THE DENOMINATOR IS OF THE FORM A(1)+A(2)S+...+A(N+1)S .

A( 1)=      .201721422457E+01


A( 2)=      .230590333490E+01


A( 3)=      .187747452005E+01


A( 4)=      .100000000000E+01
```

(b)

Fig. 3.2 Example when COMP is included.

a) First page, b) Second page

```
 C 03/.1/1.5
COEFFICIENTS OF THE NUMERATOR OF THE NETWORK TRANSFER FUNCTION.

                                                      2
THE NUMERATOR IS OF THE FORM  B(1)+B(3)S  +...

B( 1)=      .280601409014E+01

B( 3)=      .100000000000E+01
```

(c)

```
C 03/.1/1.5

THE RIPPLE IN THE PASSBAND IS EQUAL TO   .100000000000E+00  DB   ( RO=15.09 O/O  )

THE MINIMUM ATTENUATION IN THE STOPBAND IS EQUAL TO   .148477587928E+02  DB.

THE VALUE OF WS IS   1.50000000

THE VALUE OF THE CONSTANT THAT MAKES THE MINIMUM VALUE

OF THE NETWORK TRANSFER FUNCTION EQUAL TO 0 DB IS   .718890E+00

THE VALUES OF THE NETWORK ELEMENTS ARE :

C( 1)= .77030799267464E+00     L( 2)= .74560954966492E+00     C( 2)= .47796795608562E+00
C( 3)= .77030799267464E+00

THE VALUE OF THE SOURCE RESISTANCE IS EQUAL TO 1 OHM

THE VALUE OF THE LOAD RESISTANCE IS EQUAL TO   1.0000000
```

(d)

Fig. 3.2 Example when COMP is included.
Continued

c) Third page, d) Fourth page.

The examples of Fig. 3.1 show the standard format output of this program. When COMP is included, we obtain the output shown in Fig. 3.2, in this case the coefficients of numerator and denominator polynomials of the transfer function are included. The complete output for each example when COMP is included is four pages.

The listing of the input cards for the examples of Figs. 3.1 and 3.2 are shown in Fig. 3.3. The first and third sets of data cards include a PNC card. The punched output for these two examples is shown in Fig. 3.4. The first card of the punched output is the title card given in the input data. The following card includes the order, case number if any, the passband ripple, the minimum attenuation at the stopband, and a constant that makes the maximum value of the transfer function equal to one. This card has a field I3,I2,4E15.7. The following cards include the zeros of transmission, the poles of the transfer function, and the value of the network elements. These cards have a field 4E20.14.

When it is desired to synthesize a network with open termination, that is, with $R_2 = \infty$, the value of R_2 can be any value greater than or equal to 1000. One example of this case is given in Fig. 3.1.

```
C 05/.1/1.5/R2=2.0
AMAX        .1
N           5.
PNC
WS          1.5
R2          2.0

C 05/.1/2.0/R2=INF.
AMAX        .1
N           5.
WS          2.
R2          1000.

C 04-3/.1/1.5/R2=2.0
PNC
AMAX        .1
N           4.
WS          1.5
CASE        3.
R2          2.0

C 04-3/.1/1.5/R2=.5
AMAX        .1
N           4.
WS          1.5
CASE        3.
R2          0.5

C 05/.01/1.5
AMAX        .01
N           5.
WS          1.5

C 04-2/.01/1.5
AMAX        .01
N           4.
CASE        2.
WS          1.5

C 04-3/.01/1.5
AMAX        .01
N           4.
CASE        3.
WS          1.5

C 03/.1/1.5
FCN
WS          1.5
AMAX        .1
N           3.

C 03/.1/1.5
AMAX        .1
N           3.
WS          1.5
COMP
```

Fig. 3.3 Listing of Input Data for Examples of Figs. 3.1 and 3.2.

```
C 05/.1/1.5/R2=2.0
  5     .1000000E+00   .4341521E+02   .1500000E+01   .4393713F-01
 .596352445629965E+00 .117137362581119E+01 .156997862036662E+00 .137329581990057E+01
 .113553930641795E+01 .363072973878655E+00 .109609263112221E+01
 .155740639076480E+01 .233197577189753F+01
-.649753176666480E+000.              -.417037455063966E+00 .775766106180470E+00
-.114129434187222E+00 .105615071580067E+01
C 04-3/.1/1.5/R2=2.0
  4 3   .1000000E+00   .2373619E+02   .1500000E+01   .5630046E+00
 .203341429736910E+01 .479809030697040E+00 .798016494625600E+00 .206306396570370E+01
 .284847577875000E+00
 .101605884189500E+01
-.206295931923980E+00 .113643087520790E+01-.963021989335500E+00 .597833382665520E+00
```

Fig. 3.4 Punched Output for Input of Fig. 3.3.

CHAPTER 4

SENSITIVITY ANALYSIS

One of the most important applications of the elliptic ladder network is in the synthesis of active filters. This method of synthesis is called the direct method. Here, the filter function is realized by a single circuit configuration.

Among the methods of realization we have the inductance simulation method (Huelsman and Allen in press, Chapter 5). Here, each inductor of the passive network is simulated by an active network. This method has the advantage of having basically the same sensitivities that characterize the prototype passive network. Thus, it is important to know the sensitivities of the overall network response with respect to the different elements of the network.

Function Sensitivity

Function sensitivity is defined as the relative sensitivity of N(s) with respect to x, where x represents a parameter of N(s), that is,

$$S_x^{N(s)} = \frac{\partial N(s)}{\partial x} \frac{x}{N(s)} \qquad (4\text{-}1)$$

When s=jw, the function N(jw) can be separated into its real and imaginary parts and Eq.(4-1) becomes

$$S_x^{N(jw)} = S_x^{|N(jw)|} + j\,\frac{\partial \operatorname{Arg} N(jw)}{\partial x/x} \qquad (4\text{-}2)$$

Since the elliptic approximation is a magnitude approximation, we will be interested only in the real part of Eq.(4-2).

The basic components of an elliptic network are those shown in Fig. 2.2. Thus, a sensitivity analysis will include these elements. A network containing these elements is the third order one. The realizations selected are, from Appendix B,

C 3/0.1/1.05/1.0
C 3/0.1/1.20/1.0
C 3/0.1/2.00/1.0

The configuration for these networks is shown in Fig. 4.1, and their magnitude response $|N(jw)|$ is plotted in Fig. 4.2

The complexity of the equation defining the functional dependence of $|N(jw)|$ on each element of the ladder makes the use of numerical techniques attractive to evalate function sensitivity. In this case, the digital computer program SENSITIVITY (Huelsman 1976) is used to obtain the sensitivities of $|N(jw)|$ with respect to R_1, R_2, C_1, C_2, C_3, and L_2. The results are shown in Figs. 4.3 to 4.5.

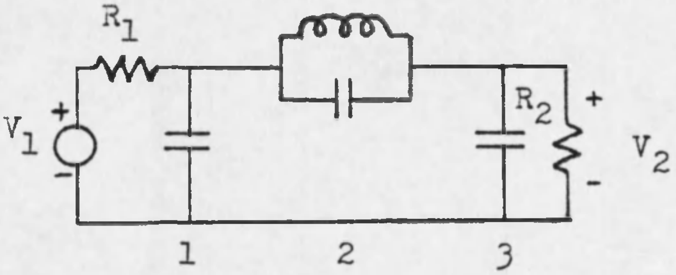
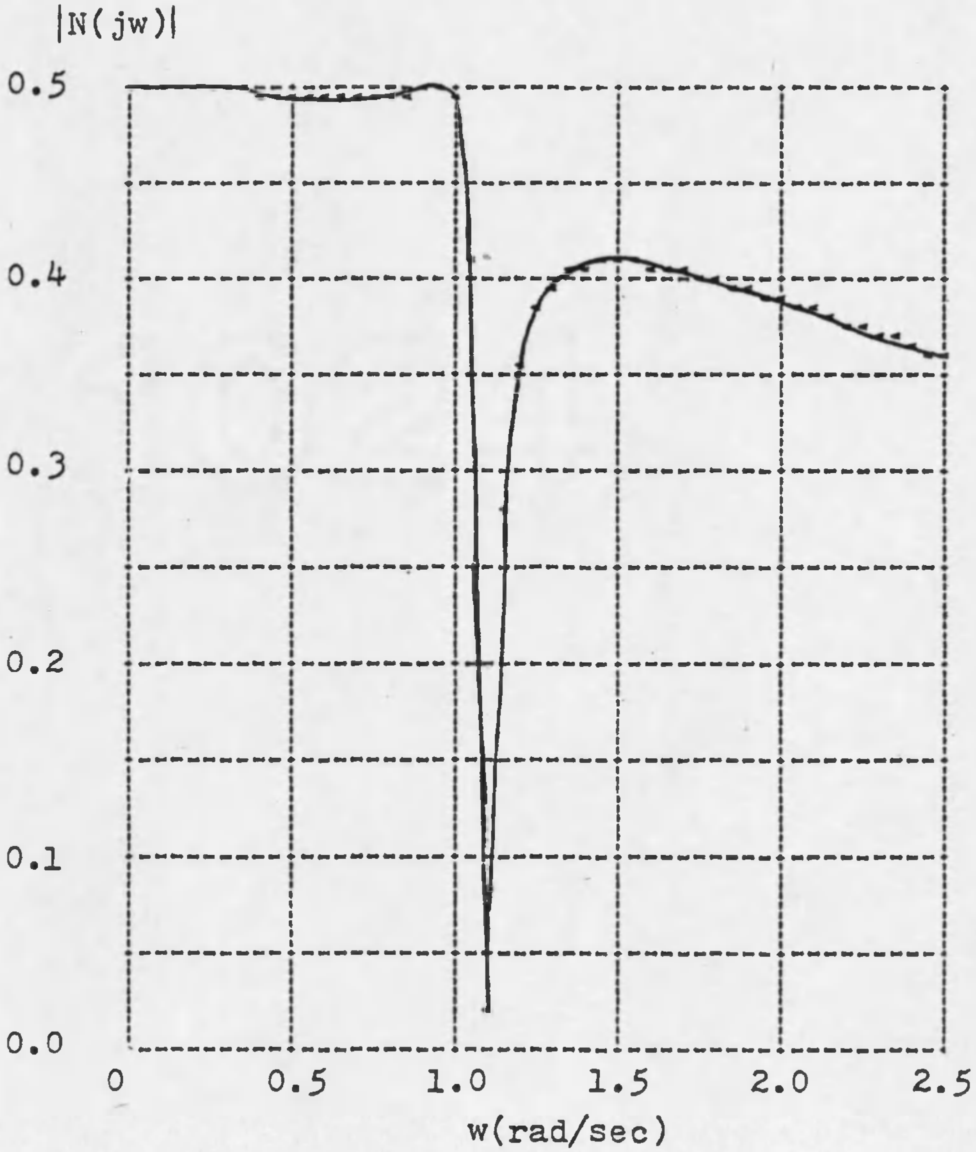


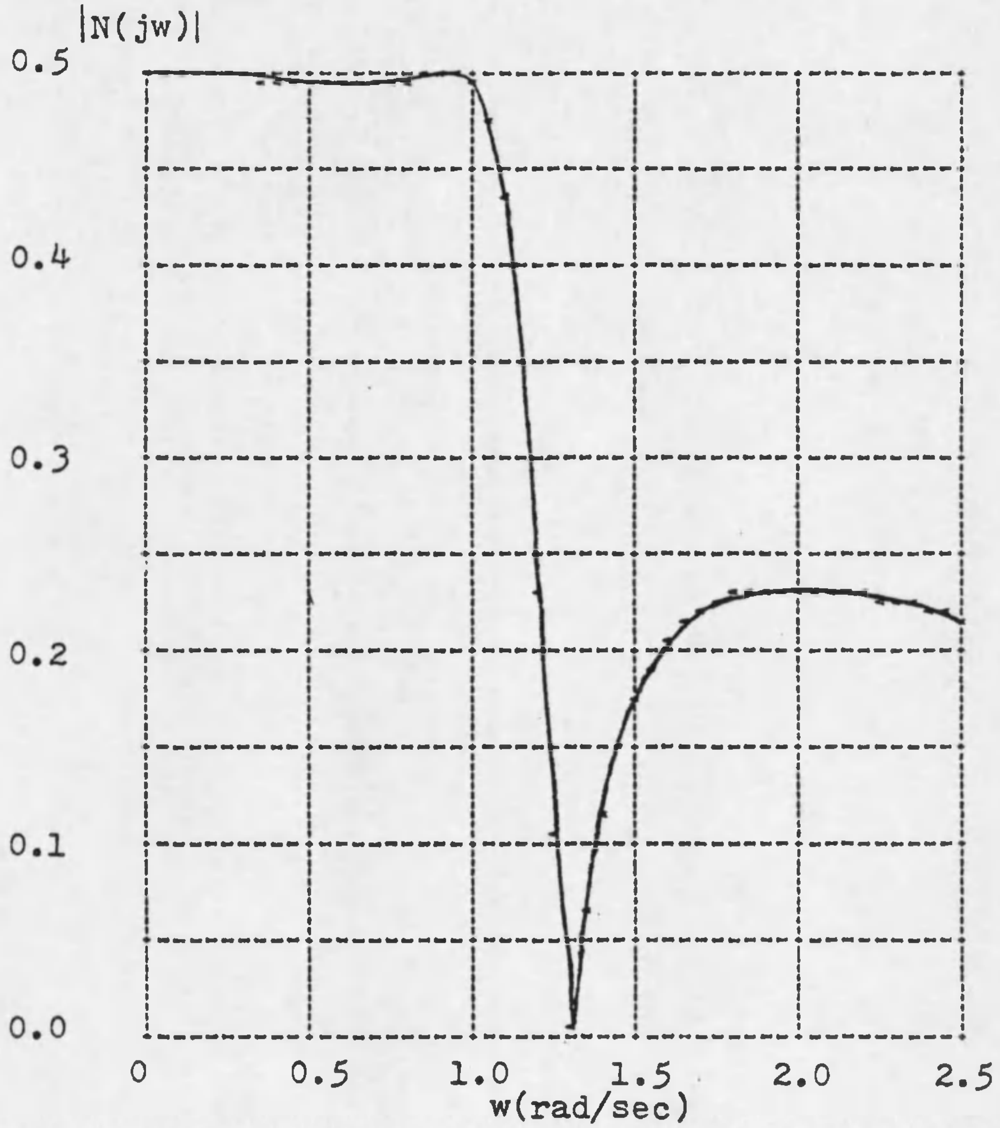
Fig. 4.1 Third Order Network for Sensitivity Analysis.



(a)

Fig. 4.2 Plots of |N(jw)| for Third Order Networks.

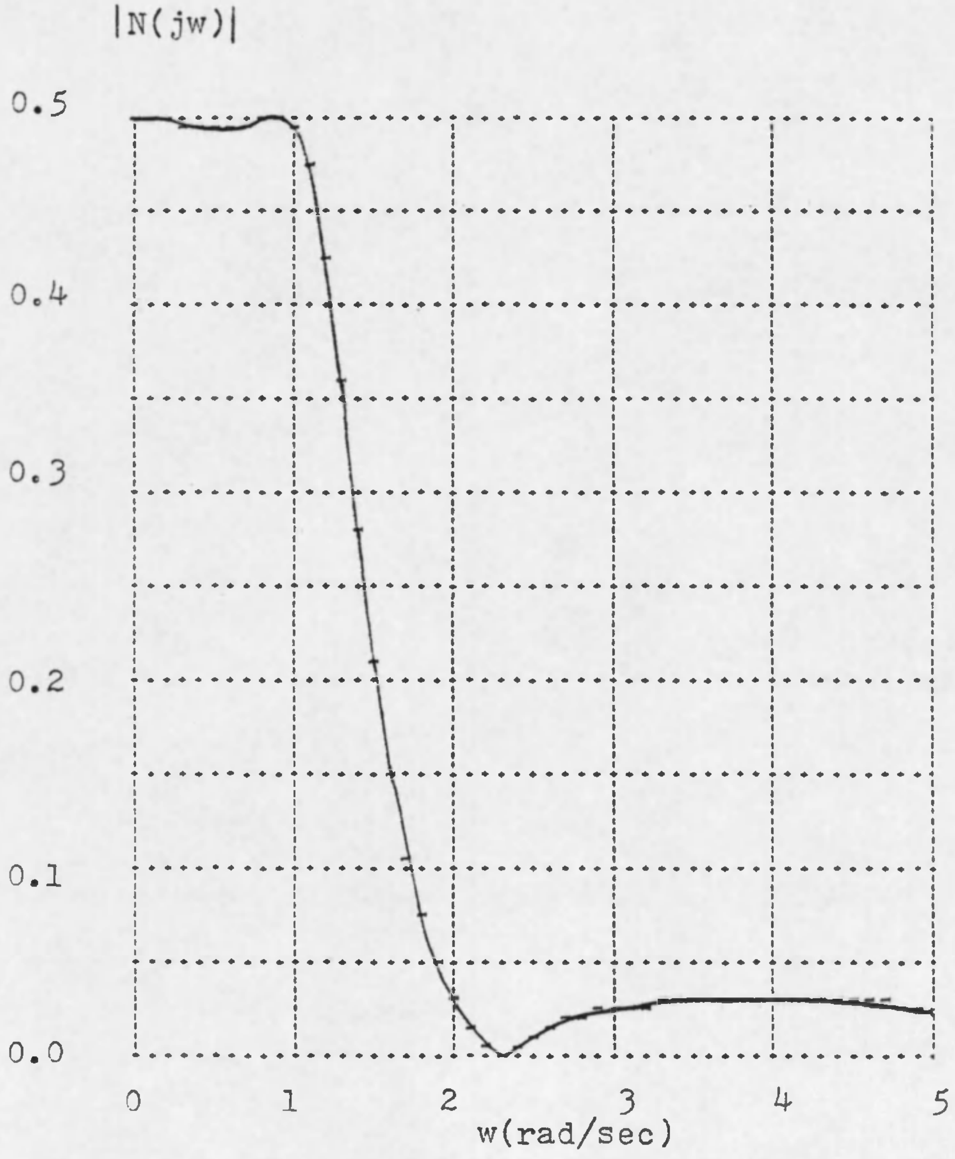
a) C 3/0.1/1.05/1.0.



(b)

Fig. 4.2 Plots of |N(jw)| for Third Order Networks.-Continued

b) C 3/0.1/1.20/1.0.



(c)

Fig. 4.2 Plots of |N(jw)| for Third Order Networks.-Continued

c) C 3/0.1/2.00/1.0.

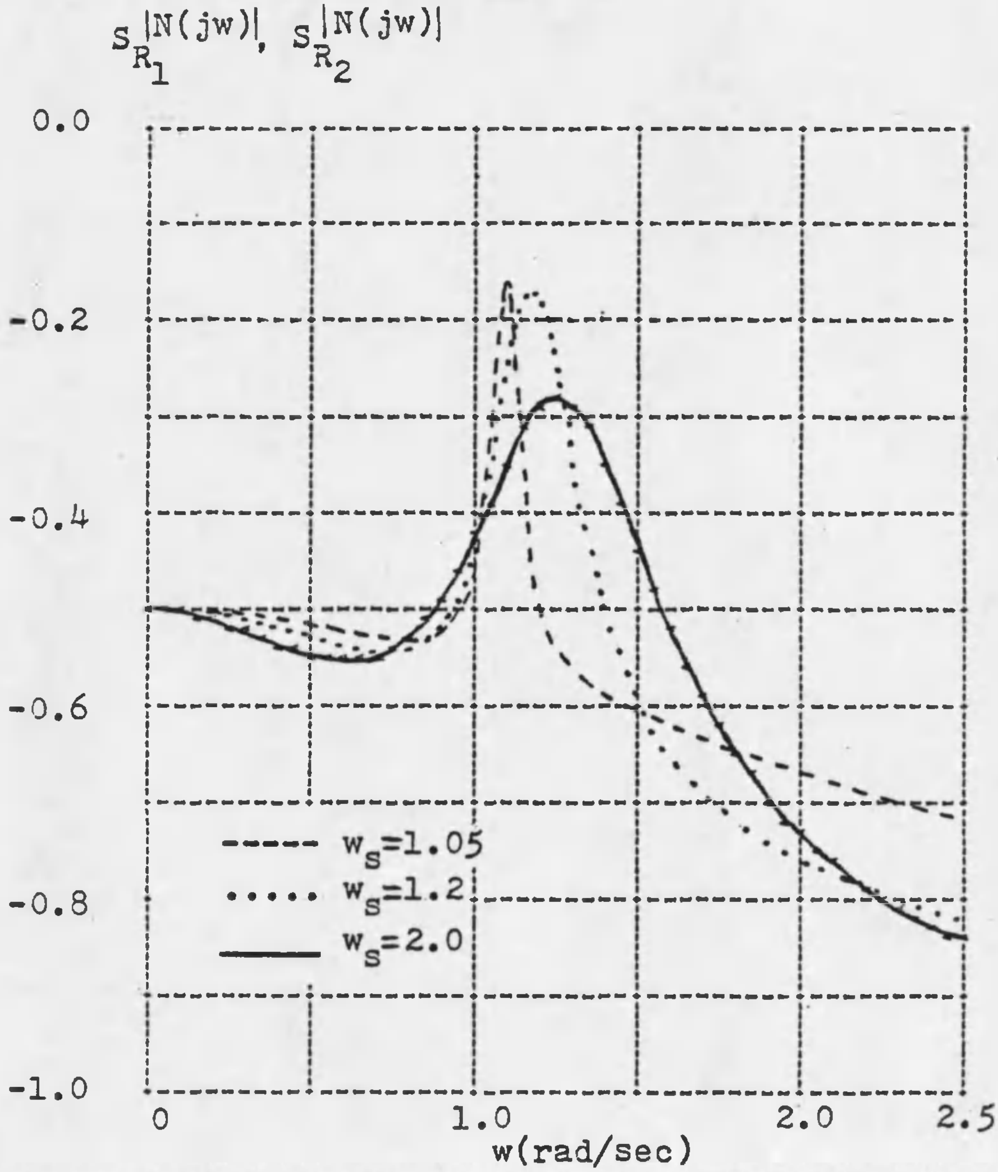


Fig. 4.3 Sensitivity of $|N(jw)|$ with Respect to R_1 and R_2.

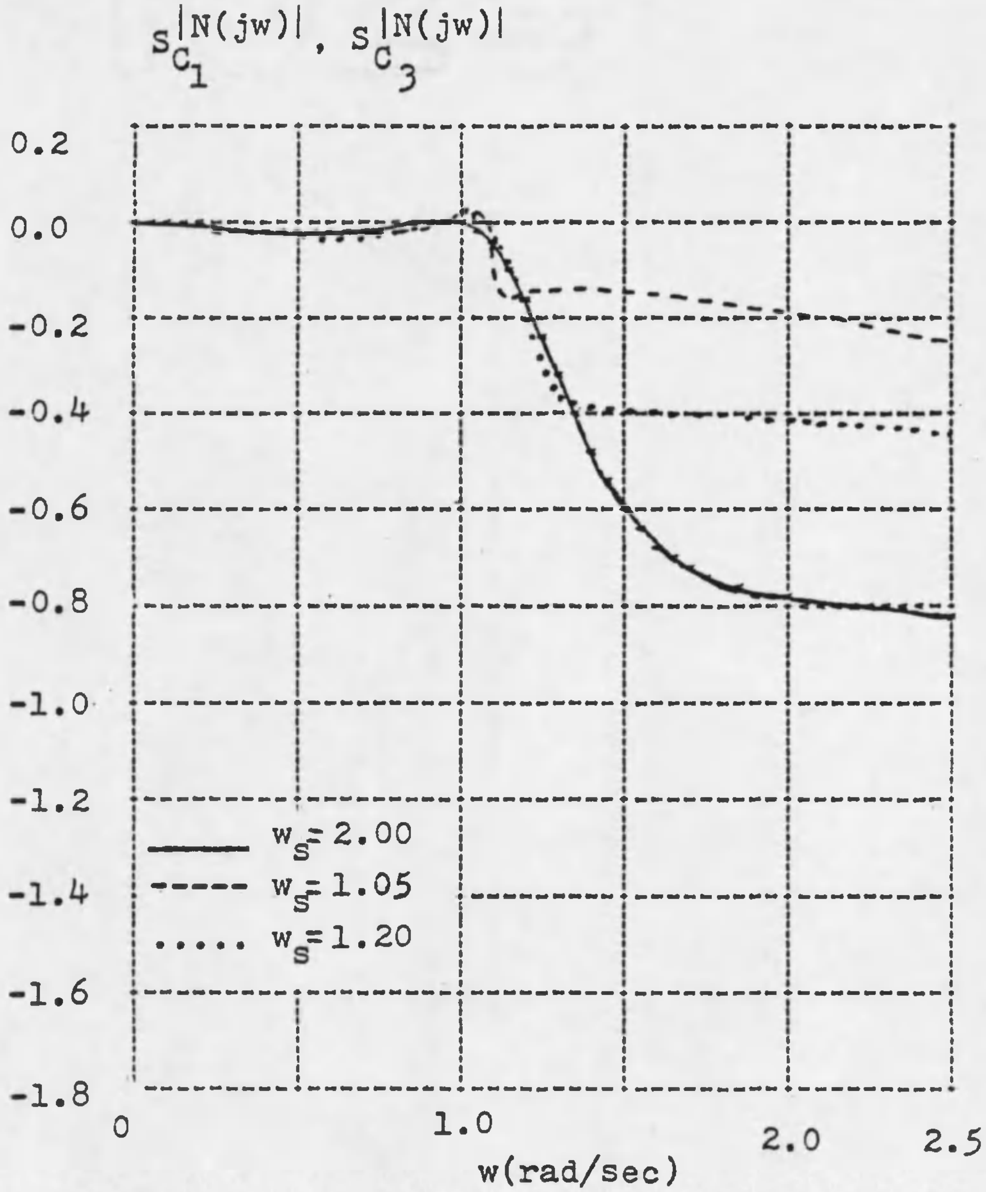


Fig. 4.4 Sensitivity of |N(jw)| with Respect to C_1 and C_3.

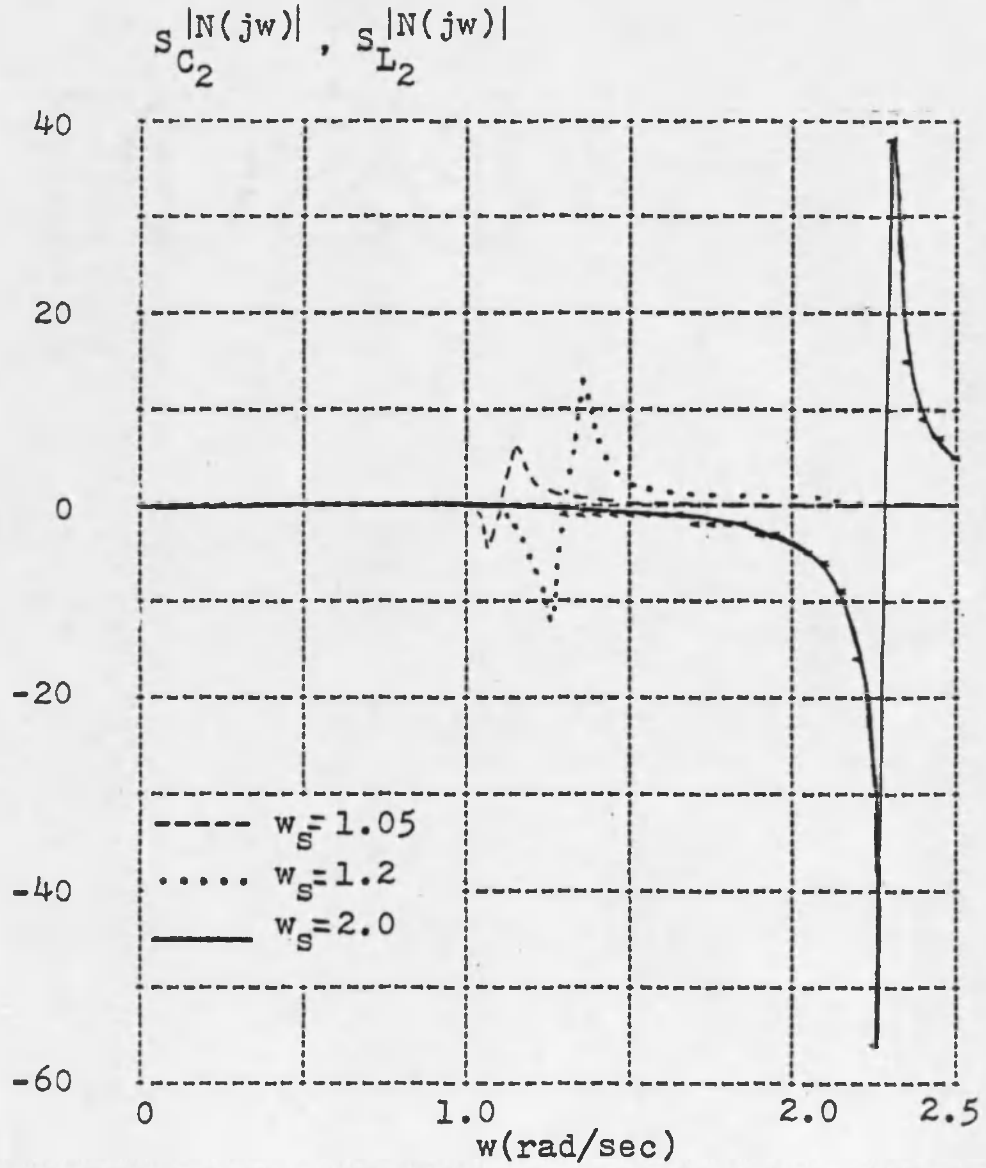


Fig. 4.5 Sensitivity of $|N(jw)|$ with Respect to L_2 and C_2.

Here, we observe that for the LC shunt branch, the sensitivity $S_{L,C}^{|N|}$ has two sharp peaks around the transmission zero frequency. We also note that except for $S_R^{|N|}$, where R is any termination, the sensitivities increase as w_s increases. Except for $S_{L,C}^{|N|}$ near the transmission zero frequency, the sensitivities are less than unity, and in the passband the sensitivities are close to zero for the reactive elements.

Finally, we should mention that as A_{max} decreases, the sensitivities also decrease (Temes and LaPatra 1978, Chapter 10). However, for any given value of stopband frequency and order, as A_{max} decreases, A_{min} decreases too. Then, to keep A_{min} from decreasing we must change w_s to a greater value that in turn increases the sensitivity. Thus, instead of increasing w_s we can increase the order of the network keeping the sensitivities as low as possible.

CHAPTER 5

CONCLUSIONS

In this work, we have treated and solved the synthesis problem for elliptic lowpass passive ladder networks. One important point is that for even order cases 2 and 3, we are now able to synthesize the network with a specified value of stopband frequency.

A function sensitivity analysis for the branches due to a finite transmission zero has been carried out, and we have found out that the sensitivities to the reactive elements, in the passband, are all less than unity.

Finally, the ability to synthesize networks with unequal resistance terminations makes this method very attractive since published tables (Saal 1963; Zverev 1967) are usually given for unity valued terminations, except for even order case 2 networks.

APPENDIX A

PROGRAM LISTINGS

In this appendix are shown the main program and the subroutines used in the synthesis of the elliptic network. The subroutines used in the evaluation of the network transfer function are shown in Baezlopez (1976.) The subroutine FPOLY finds the coefficients of the polynomial F(s) needed in the synthesis of the ladder network as explained in Chapter 2. Subroutines EVNET and ODNET synthesize the even and odd order network, respectively. In these two subroutines, all the calculations are carried in double precision arithmetic. Subroutine FIX rearranges the transmission zeros to ensure that positive elements are obtained. Subroutine DIV is used to evaluate the quotient of Eq.(2-6).

Program ELLIP.

```
      PROGRAM ELLIP(INPUT,OUTPUT,PUNCH,TAPE1=INPUT,TAPE2=OUTPUT)
C       THIS PROGRAM FINDS THE TRANSFER FNCTION
C       OF ELLIPTIC FILTERS AND SYNTHESIZES THE
C       PASSIVE LADDER FILTER.
      DIMENSION ZEVEN(25),XNU(25),ZA(25),PP(50),AA(101),Q(50),XK(100),P(
     125),A(101),B(101),C(101),BB(24),ZA2(24),LETT(8),PO(25),ARPR(25),
     2ARPIM(25),U(50),V(50),NARR(50),F(51),PO1(25),PO2(25)
      COMMON/DAVE/EP(25),EQ(25)
      COMMON/GARY/R2,PUNCA,TABLE,NETWRK,ORD
      INTEGER PUNCA,TABLE,ORD
      EXTERNAL SNF,DIV,COEF
      REAL NMIN,NMAXX
      FA(Y,SN,XN)=XN**2-Y**2*SN
      G(Y,SN,XN)=XN-Y*SN
      NCNT=0
      WRITE(2,77)
   77 FORMAT(1H1,//,31X,*ELLIPTIC NETWORK PROGRAM*,//,38X,*DAVID BAEZ*,
     1//,38X,*FALL    1978*,//,35X,*VERSION 12/18/78*,//)
    3 READ(1,78)(LETT(LET),LET=1,8)
      IF(EOF(1))199,612
   78 FORMAT(8A10)
  612 DO 500 J=1,100
      A(J)=0.0
      B(J)=0.0
      C(J)=0.0
      IF(J.GT.24)GO TO 501
      ZA2(J)=0.0
      EP(J)=0.$EQ(J)=0.
      BB(J)=0.0
  501 IF(J.GT.25)GO TO 502
      ARPR(J)=0.0$ARPIM(J)=0.0
      PO(J)=0.0
      P(J)=0.0
      XNU(J)=0.0
      PO1(J)=0.
      PO2(J)=0.

      ZEVEN(J)=0.0
      ZA(J)=0.0
  502 IF(J.GT.50)GO TO 500
      U(J)=0.$V(J)=0.
      F(J)=0.
      NARR(J)=0
      PP(J)=0.0
      AA(J)=0.0
      Q(J)=0.0
      XK(J)=0.0
  500 CONTINUE
      IC=0$IC1=0
      H=0.0
      R2=0.
      AL=0.0
      CLEAN=4.
      CALL FH(A,N,P,H,ZEVEN,PO,CLEAN)
      AMAX=0.0$AMIN=0.0
      CALL DATOS(WS,AMIN,AMAX,NMIN,Y,NMAXX,CASE,LETT,NCNT)
      IF(WS)610,610,611
  610 WRITE(2,609)
  609 FORMAT(/,5X,*THE VALUE OF WS IS LESS THAN OR EQUAL TO 1.*,/)
```

```
      GO TO 3
611   IF(PUNCA.EQ.1)PUNCH 78,(LETT(LET),LET=1,8)
      IF(NETWRK.EQ.1.AND.R2.LT.1..AND.CASE.EQ.3.)GO TO 13
      GO TO 12
   13 RO=SORT(1.-10.**(-AMAX/10.))
      IF(ABS((1.-RO)/(1+RO)-R2).LT.1.E-03)CASE=2.
   12 X=1./SQRT(WS)
      CALL ELLIN( XK,ELIN,J,X,FI)
      IF(WS.NE.0.0.AND.AMIN.NE.0.0.AND.AMAX.NE.0.0.AND.NMIN.NE.0.0)GO TO
     1 196
      N=Y
      RN=N
      RND=RN/2.
      ND=N/2
      IX=N+1
      IF(N.NE.0.AND.AMIN.NE.0..AND.WS.NE.0..AND.(CASE.EQ.1..OR.CASE.EQ.0
     1.))GO TO 197
      IF(N.NE.0.AND.AMIN.NE.0..AND.WS.NE.0.)ITEST=1
C
C      COMBINATION B.
C
      IF(ITEST.EQ.1 )GO TO 103
  195 AAMAX=AMAX/10.
   88 IF((RND-ND).GT.0.0)GO TO 31
      IF(CASE.EQ.1.)GO TO 200
C
C      CASES 2 AND 3
C
  103 XN=X
      M=1
      XO=XN
      OR=(RN-1.)/RN
      DO 101 KL=1,150
      CALL ELLIN(XK,ELIN,J,XO,FI)
      AR=OR*ELIN
      CALL SNF(FI,XK,N,AR,SN,J,M)
      IF(CASE.EQ.2.)AAA=FA(XO,SN,XN)
      IF(CASE.EQ.3.)AAA=G(XO,SN,XN)
      X1=XO+AAA/2.
      IF(X1.GT.1.)X1=XO+AAA/10.
      IF(ABS(X1-XO).LT.1.E-10)GO TO 102
  101  XO=X1
  102 X=X1
      IF(ITEST.EQ.1 )GO TO 197
      CALL ODPOLE(XK,ELIN,N,FI,X,XNU,J,SNF,R)
      AL=1.0
      DO 1 L=1,ND
    1 AL=AL*XNU(L)**2
      IF(ITEST.EQ.2)GO TO 86
      H=SQRT(10.**AAMAX-1.)/AL
      HH=H*AL
   52 CALL EVENPO(XK,X,ELIN,SNF,XNU,N,FI,J,WSN,ZEVEN,PO,CASE)
      WS=WSN
      IF(TABLE.EQ.0)WRITE(2,79)(LETT(LT),LT=1,8),N
      SQWSN=SQRT(WSN)
      NM2=ND-1
      IF(CASE.EQ.3.)GO TO 300
      DO 303 NEV=1,NM2
      ZA(NEV)=ZEVEN(NEV)
  303 CONTINUE
```

```
      CALL FH(C,N,P,HH,ZEVEN,PO,CASE)
      IF(NETWRK-1)301,126,126
  126 DO 127 I=1,ND
      PO1(I)=0.
  127 PO2(I)=PO(I)**2
      GO TO 301
  300 CALL FH(C,N,P,H,ZEVEN,PO,CASE)
      DO 302 J=1,NM2
      ZA(J)=ZEVEN(J)*SQWSN
  302 CONTINUE
      IF(NETWRK-1)301,128,128
  128 NX=N-1
      DO 124 I=1,NM2
      PO1(I)=0.
  124 PO2(I)=(PO(I)*SQWSN)**2
   27 FORMAT(5X,4E20.12)
  301 N2=2*N+1
      IF(TABLE.EQ.0)WRITE(2,41)(J,ZA(J),J=1,NM2)
   75 FORMAT(4E20.14)
      IF(TABLE.EQ.0)WRITE(2,38)
      DO 2 J=1,N2,2
      K=(J+1)/2
    2 A(K)=C(J)
      CALL PROOT4(N,A,U,V)
  109 DO 108 I=1,ND
      J=2*I-1
      PO(I)=-2.*U(J)
  108 Q(I)=U(J)**2+V(J)**2
      AA(1)=Q(1)
      AA(2)=PO(1)
      AA(3)=1.
      IF(N.EQ.3.OR.N.EQ.2)GO TO 122
      CALL COEF(AA,PO,Q,ND,IX)
  122 DO 104 LUZ=1,IX
      IF(ABS(A(LUZ)-AA(LUZ)).GT.1.E-05)GO TO 105
  104 CONTINUE
      GO TO 107
  105 WRITE(2,106)
  106 FORMAT(3X,*ROOT SOLVING ALGORITHM FAILS TO CONVERGE*)
  107 IF(CASE.EQ.1.)GO TO 110
      IF(CASE.EQ.5.)GO TO 114
      IF(TABLE.EQ.1)WRITE(2,26)
      DO 8 J=1,ND
      IC1=2*J
      IC=IC1-1
      PR=U(2*J-1)
      PIM1=ABS(V(2*J-1))
      OO1=ATAN2(PIM1,PR)
      AS1=(PIM1**2+PR**2)**.25
      PRS1=-AS1*COS(OO1/2.)
      PIMS1=AS1*SIN(OO1/2.)
      IF(CASE.EQ.3.)PRS1=PRS1*SQWSN
      IF(CASE.EQ.3.)PIMS1=PIMS1*SQWSN
      PP(J)=2.*ABS(PRS1)
      Q(J)=PRS1**2+PIMS1**2
      EQ(J)=Q(J)$EP(J)=PP(J)
      IF(TABLE.EQ.1)WRITE(2,25)PRS1,PIMS1,PP(J),Q(J),ZA(J)
   25 FORMAT(2X,E18.12,* + J *,E18.12,3E20.12)
      IF(TABLE.EQ.0)WRITE(2,23)IC,PRS1,PIMS1
      IF(TABLE.EQ.0)WRITE(2,24)IC1,PRS1,PIMS1
```

```
      ARPR(J)=PRS1$ARPIM(J)=PIMS1
    8 CONTINUE
      DO 3000 I=1,ND
 3000 NARR(I)=I*2-1
      IF(TABLE.EQ.0)WRITE(2,2001)
      IF(TABLE.EQ.0)WRITE(2,2000)(NARR(J),PP(J),NARR(J),Q(J),J=1,ND)
      AA(1)=Q(1)
      AA(2)=PP(1)
      AA(3)=1.
      CALL COEF(AA,PP,Q,ND,IX)
      IF(TABLE.EQ.1)GO TO 2112
      WRITE(2,46)(LETT(LT),LT=1,8)
      WRITE(2,62)(I,AA(I),I=1,IX)
 2112 N21=ND-1
      DO 310 I=1,N21
      PP(I)=0.
  310 ZA2(I)=ZA(I)**2
      BB(1)=ZA2(1)
      BB(3)=1.
      IF(N.EQ.4)GO TO 54
      CALL COEF(BB,PP,ZA2,NM2,N)
      GO TO 54
C
C   CASE 1
C
  200 CALL ODPOLE(XK,ELIN,N,FI,X,PO,J,SNF,RR)
      AL=1.
      DO 201 L=1,ND
  201 AL=AL*PO(L)**2
      H=SQRT(10.**AAMAX-1.)/AL
  520 IF(TABLE.EQ.0)WRITE(2,79)(LETT(LT),LT=1,8),N
      DO 202 J=1,ND
      ZA(J)=1./(PO(J)*X)
      IF(TABLE.EQ.0)WRITE(2,41)J,ZA(J)
  202 ZEVEN(J)=1./PO(J)
      CALL FH(C,N,P,H,ZEVEN,PO,CASE)
      N2=2*N+1
      DO 212 J=1,N2,2
      K=(J+1)/2
  212 A(K)=C(J)
      CALL PROOT4(N,A,U,V)
      GO TO 109
  110 IF(TABLE.EQ.0)WRITE(2,38)
      IF(TABLE.EQ.1)WRITE(2,26)
      DO 208 J=1,ND
      IC1=2*J
      IC=IC1-1
      PR=U(2*J-1)
      PIM=ABS(V(2*J-1))
      OO1=ATAN2(PIM,PR)
      AS1=(PIM**2+PR**2)**.25
      PRS1=AS1*COS(OO1/2.)
      PIMS1=AS1*SIN(OO1/2.)
      PRA=-PRS1/X
      PIMA=PIMS1/X
      ARPR(J)=PRA
      ARPIM(J)=PIMA
      PP(J)=2.*ABS(PRA)
      Q(J)=PRA**2+PIMA**2
      IF(TABLE.EQ.0)WRITE(2,23)IC,PRA,PIMA
```

```
      IF(TABLE.EQ.0)WRITE(2,24)IC1,PRA,PIMA
      IF(TABLE.EQ.1)WRITE(2,25)PRA,PIMA,PP(J),Q(J),ZA(J)
  208 CONTINUE
 2001 FORMAT(/,5X,*THE COEFFICIENTS OF THE QUADRATIC FACTORS*,/,6X,*2*,/
     1,5X,*S +C(1,J)S+C(2,J), WHERE J IS THE INDEX OF THE CORRESPONDING POL
     2POLE*,/)
      DO 2003 JMR=1,N
 2003 NARR(JMR)=JMR*2-1
      IF(TABLE.EQ.0)WRITE(2,2001)
      IF(TABLE.EQ.0)WRITE(2,2000)(NARR(J),PP(J),NARR(J),Q(J),J=1,ND)
      AA(1)=Q(1)
      AA(2)=PP(1)
      AA(3)=1.
      IF(N.EQ.2)GO TO 209
      CALL COEF(AA,PP,Q,ND,IX)
  209 IF(TABLE.EQ.1)GO TO 2113
      WRITE(2,46)(LETT(LT),LT=1,8)
      WRITE(2,62)(I,AA(I),I=1,IX)
 2113 DO 210 I=1,IX
      PP(I)=0.
  210 ZA2(I)=ZA(I)**2
      GO TO 203
C     ODD CASE
C
   31 CALL ODPOLE(XK,ELIN,N,FI,X,P,J,SNF,AL)
      H=SQRT(10.**AAMAX-1.)/AL
   43 NNEW=N-1
      IF(NETWRK-1)131,132,132
  132 DO 121 JOY=1,ND
      PO1(JOY)=0.
  121 PO2(JOY)=(P(JOY)/X)**2
  131 IF(TABLE.EQ.0)WRITE(2,79)(LETT(LT),LT=1,8),N
   79 FORMAT(/,5X,8A10,//5X,*THE ORDER OF THE NETWORK FUNCTION DENOMINAT
     1OR IS*,I4,//,5X,*THE ZEROS ARE*,/)
      DO 40 NK=1,ND
      ZA(NK)=1./(P(NK)*X)
   41 FORMAT(//,5X,*ZERO ( *,I2,*)=*,E20.12)
   40 CONTINUE
      IF(TABLE.EQ.0)WRITE(2,41)(NK,ZA(NK),NK=1,ND)
      CASE=5.
      CALL FM(A,N,P,H,ZEVEN,PO,CASE)
      CALL PREAL(A,B,Z,N)
      IF(TABLE.EQ.0)WRITE(2,38)
   38 FORMAT(//,5X,*THE POLES ARE*,/)
      ZX=Z/X
      EQ(1)=ZX
      IF(TABLE.EQ.1)WRITE(2,26)
      IF(TABLE.EQ.0)WRITE(2,42)ZX
      IF(TABLE.EQ.1)WRITE(2,20)ZX
   20 FORMAT(2X,E18.12)
      IF(PUNCA.EQ.1)ARPR(1)=ZX
   42 FORMAT(//,5X,*POLE( 1)=*,E20.12)
      DO 9 LV=1,100
      LVS=LV+1
    9 A(LV)=B(LVS)
      II=(N+1)/2
      CALL PROOT4(NNEW,A,U,V)
      IX=N
      GO TO 109
```

```
 114 DO 60 JIM=2,II
     IC1=2*JIM-1
     IC=IC1-1
     R=2.*ABS(U(2*JIM-3))
     S=U(2*JIM-3)**2+V(2*JIM-3)**2
     PR=(-R/2.)/X
     PIM=SQRT(ABS(S-(R/2.)**2))/X
     PP(JIM)=2.*ABS(PR)
     Q(JIM)=PR**2.+PIM**2.
     EP(JIM)=PP(JIM)$EQ(JIM)=Q(JIM)
     IF(TABLE.EQ.1)WRITE(2,25)PR,PIM,PP(JIM),Q(JIM),ZA(JIM-1)
     IF(TABLE.EQ.0)WRITE(2,23)IC,PR,PIM
     IF(TABLE.EQ.0)WRITE(2,24)IC1,PR,PIM
     ARPR(JIM)=PR$ARPIM(JIM)=PIM
  60 CONTINUE
     DO 3001 J=2,II
3001 NARR(J)=2*J-2
     IF(TABLE.EQ.0)WRITE(2,2001)
     IF(TABLE.EQ.0)WRITE(2,2000)(NARR(JIM),PP(JIM),NARR(JIM),Q(JIM),JIM
    1=2,II)
2010 AA(2)=1.
     AA(1)=-ZX
     IX=N+1
     DO 123 I=3,IX
 123 AA(I)=0.
     CALL COEF(AA,PP,Q,II,IX)
     IF(TABLE.EQ.0)WRITE(2,46)(LETT(LT),LT=1,8)
  46 FORMAT(1H1/5X,8A10,//,5X,*COEFFICIENTS OF THE DENOMINATOR OF THE N
    1ETWORK TRANSFER FUNCTION.*,//,58X,*N*,/,5X,*THE DENOMINATOR IS OF
    2THE FORM A(1)+A(2)S+...+A(N+1)S .*)
     IF(TABLE.EQ.0)WRITE(2,62)(I,AA(I),I=1,IX)
  62 FORMAT(/,5X,*A(*,I2,*)=*,3X,E20.12,/)
     DO 53 I=1,ND
     PP(I)=0.0
  53 ZA2(I)=ZA(I)**2
 203 BB(1)=ZA2(1)
     BB(3)=1.0
     IF(N.EQ.2.OR.N.EQ.3)GO TO 54
     IF(CASE.EQ.5.)IX=N
     CALL COEF(BB,PP,ZA2,ND,IX)
      IX=N+1
  54 IF(TABLE.EQ.1)GO TO 191
     IF(TABLE.EQ.0)WRITE(2,59)(LETT(I),I=1,8)
  59 FORMAT(1H1,5X,8A10,//,5X,*COEFFICIENTS OF THE NUMERATOR OF THE NET
    1WORK TRANSFER FUNCTION.*,//,45X,*2*,/,5X,*THE NUMERATOR IS OF THE
    2FORM  B(1)+B(3)S +...*)
     IF(CASE.EQ.2..OR.CASE.EQ.3.)WRITE(2,50)(IH,BB(IH),IH=1,N,2)
     IF(CASE.EQ.5..OR.CASE.EQ.1.)WRITE(2,50)(IH,BB(IH),IH=1,IX,2)
  50 FORMAT(/,5X,*B(*,I2,*)=*,3X,E20.12,/)
 191 AMAX=10.*ALOG10(1.+(H**2)*(AL**2))
     AMIN=10.*ALOG10(1.+(H**2)/(AL**2))
     RO=SQRT(1.-10.**(-AMAX/10.))
     ROP=RO*100.
  49 FORMAT(1H1)
 190 FORMAT(5X,8A10,/)
  63 FORMAT(5X,*THE RIPPLE IN THE PASSBAND IS EQUAL TO*,E20.12,*  DB*,
    13X,*( RO=*,F5.2,* O/O  )*,//
    25X,*THE MINIMUM ATTENUATION IN THE STOPBAND IS EQUAL TO*,E20.12,
    3*  DB.*)
     CON35=AA(1)/BB(1)
```

```
      CON23=AA(1)/(BB(1)*(10.**(AMAX/20.)))
      IF(TABLE.EQ.1)GO TO 15
      WRITE(2,49)
      WRITE(2,190)(LETT(I),I=1,8)
      WRITE(2,63)AMAX,ROP,AMIN
      WRITE(2,600)WS
  600 FORMAT(/,5X,*THE VALUE OF WS IS    *,F10.8)
      IF(CASE.EQ.2..OR.CASE.EQ.1.)WRITE(2,602)CON23
      IF(CASE.EQ.3..OR.CASE.EQ.5.)WRITE(2,602)CON35
  602 FORMAT(/,5X,*THE VALUE OF THE CONSTANT THAT MAKES THE MINIMUM VALU
     1E*,//,5X,*OF THE NETWORK TRANSFER FUNCTION EQUAL TO 0 DB IS   *
     2,E12.6)
   15 NCAS=CASE
      IF(PUNCA.EQ.0)GO TO 14
      IF(NCAS.EQ.1.OR.NCAS.EQ.2)PUNCH 2004,N,NCAS,AMAX,AMIN,WS,CON23
      IF(NCAS.EQ.3)PUNCH 2004,N,NCAS,AMAX,AMIN,WSN,CON35
      IF(CASE.EQ.5.)PUNCH 2007,N,AMAX,AMIN,WS,CON35
14      IF(CASE.EQ.1..AND.TABLE.EQ.1)GO TO 192
      IF(CASE.EQ.1..AND.TABLE.EQ.0)GO TO 2111
      IF(NETWRK.EQ.0.AND.TABLE.EQ.1)GO TO 192
      IF(NETWRK-1)2111,2123,2123
 2123  IF(TABLE.EQ.0)WRITE(2,2118)
 2114 CALL FPOLY(R2,N,F,PO1,PO2,CASE,COEF,P,C)
      IF (CASE.EQ.5.)CALL ODNET(N,ZA,F,AA,DIV)
      IF(CASE.EQ.2..OR.CASE.EQ.3.)CALL EVNET(N,ZA,F,AA,RO,CASE)
      IF(TABLE.EQ.1)GO TO 192
 2117 FORMAT(//,5X,*THE VALUE OF THE SOURCE RESISTANCE IS EQUAL TO 1 OHM
     1*,//,5X,*THE VALUE OF THE LOAD RESISTANCE IS EQUAL TO  *,F10.7,/)
 2118 FORMAT(//,5X,*THE VALUES OF THE NETWORK ELEMENTS ARE :*,//)
 2119 FORMAT(//,5X,*THE VALUE OF THE SOURCE RESISTANCE IS EQUAL TO 1 OHM
     1*,//,5X,*THE VALUE OF THE LOAD RESISTANCE IS INFINITE*,/)
      IF(R2-1000.)2121,2120,2120
 2120 WRITE(2,2119)
      GO TO 2111
 2121 IF(CASE-2.)2116,2115,2116
 2115     WRITE(2,2117)R2
 3100 FORMAT(*R*,7X,*2.0*,5X,*0.0*,5X,F8.5)
      GO TO 2111
 2116 WRITE(2,2117)R2
 2111 IF(PUNCA.NE.1)GO TO 3
 2004 FORMAT(I3,I2,4E15.7)
 2007 FORMAT(I3,2X,4E15.7)
      IF(CASE.EQ.1.)PUNCH 75,(ZA(J),J=1,ND)
      IF(CASE.EQ.2..OR.CASE.EQ.3.)PUNCH 75,(ZA(J),J=1,NM2)
      IF(CASE.EQ.5.)PUNCH 75,(ZA(J),J=1,ND)
      IF(CASE.NE.5.)PUNCH 75,(ARPR(J),ARPIM(J),J=1,ND)
      IF(CASE.EQ.5.)PUNCH 75,(ARPR(J),ARPIM(J),J=1,II)
      GO TO 3
   26 FORMAT(//,49X,*2*,/,10X,*THE QUADRATIC FACTORS ARE OF THE FORM S +
     1C(1)S+C(2).*,//,19X,*POLES*,27X,*C(1)*,16X,*C(2)*,16X,*ZEROS*,/)
C
C      COMBINATION C.
C
  197 AAMIN=AMIN/10.
      IF((RND-ND).GT.0.0)GO TO 81
      CALL ODPOLE(XK,ELIN,N,FI,X,XNU,J,SNF,PEZ)
      AL=1.0
      DO 51 L=1,ND
   51 AL=AL*XNU(L)**2
      H=AL*SQRT(10.**AAMIN-1.)
```

```
      IF(CASE.EQ.1.)GO TO 521
      HH=H*AL
      ITEST=0
      GO TO 52
   81 CALL ODPOLE(XK,ELIN,N,FI,X,P,J,SNF,AL)
      H=AL*SQRT(10.**AAMIN-1.)
      GO TO 43
  521 DO 522 LN=1,ND
  522 PO(LN)=XNU(LN)
      GO TO 520
C
C       COMBINATION A
C
  196 AAMAX=AMAX/10.
      N=NMIN
      NMAX=NMAXX
      XLOC=X
   84 IF(N.GT.NMAX)GO TO 691
      RN=N
      RND=RN/2.
      ND=N/2
      IF((RND-ND).GT.0.0)GO TO 82
      IF(CASE-2.)194,111,111
  111 X=XLOC
      ITEST=2
      GO TO 103
  194 CALL ODPOLE(XK,ELIN,N,FI,X,XNU,J,SNF,R)
      AL=1.0
      DO 85 L=1,ND
   85 AL=AL*XNU(L)**2
      GO TO 86
   82 CALL ODPOLE(XK,ELIN,N,FI,X,P,J,SNF,AL)
   86 H=SQRT(10.**AAMAX-1.)/AL
      AMINN=10.*ALOG10(1.+(H**2)/(AL**2))
      IX=N+1
      IF(AMINN-AMIN)89,87,87
   87 ITEST=0
      X=XLOC
      GO TO 88
   89 N=IX
      GO TO 84
C
C
C
 2000 FORMAT(/,5X,*C(1,*,I2,*)=*,E20.12,2X,*C(2,*,I2,*)=*,E20.12,/)
   23 FORMAT(/,5X,*POLE(*,I2,*)=*,E20.12,* + J *,E20.12)
   24 FORMAT( /,5X,*POLE(*,I2,*)=*,E20.12,* - J *,E20.12)
  192 IF(CASE.EQ.3..OR.CASE.EQ.5.)CONST=CON35
      IF(CASE.EQ.1..OR.CASE.EQ.2.)CONST=CON23
      IF(NETWRK.EQ.1)WRITE(2,1193)WS,AMIN,AMAX,N,CONST,R2
 1193 FORMAT(/,2X,*WS=*,F10.8,3X,*AMIN=*,E12.6,3X,*AMAX=*,F10.8,3X,*N=*,
     1I2,3X,*CONST=*,E12.6,3X,*R2=*,E12.6)
      IF(NETWRK.EQ.0)WRITE(2, 193)WS,AMIN,AMAX,N,CONST
      WRITE(2,1002)
 1002 FORMAT(/,2X,100(1H-),/)
  193 FORMAT(/,2X,*WS=*,F10.8,9X,*AMIN=*,E12.6,3X,*AMAX=*,F10.8,5X,*N=*
     1,I2,15X,*CONST=*,E12.6)
      GO TO 2111
  691 WRITE(2,692)
  692 FORMAT(/,5X,*THE VALUE OF N EXCEEDS THE VALUE OF NMAX*,//)
      GO TO 3
  199 STOP
      END
```

Subroutine FPOLY.

```
      SUBROUTINE FPOLY(RK2,N,F,PO1,PO2,CASE, COEF,H,AA)
      DIMENSION PO1(50),PO2(50),F(101),H(101),AA(101),A(101),U(50),V(50)
     1,B(101),PP(50),Q(50),POA(50),EEP(50),EEQ(50),E(101)
      COMMON/DAVE/EP(25),EQ(25)
      IF(RK2.GE.1000.)RETURN
      IX=N+1
      ND=N/2
      IF(CASE-3.)2,3,1
CASE ODD
    1 IF(RK2-1.)9  ,5,9
    5 F(1)=PO2(1)
      F(2)=0.
      F(3)=1.
      IF(N.NE.3 )CALL COEF(F,PO1,PO2,ND,N)
      DO 133 I=1,IX,2
      F(I+1)=F(I)
  133 F(I)=0.
    4 RETURN
C CASE 2
   2  F(1)=PO2(1)
      F(2)=0.
      F(3)=1.
      CALL COEF(F,PO1,PO2,ND,IX)
    6 RETURN
CASE 3
    3 IF (RK2-1.)10,11,10
   11 F(1)=PO2(1)
      F(2)=0.
      F(3)=1.
      NM2=ND-1
      NX=N-1
        IF(NX.EQ.3)GO TO 126
      CALL COEF(F,PO1,PO2,NM2,NX)
126       DO 125 I=1,NX,2
      IXA=IX+1-I
  125 F(IXA)=F(IXA-2)
      F(1)=0.
    7 RETURN
9       T=SQRT(RK2)
        GAM=(2/(T+1./T))**2
      NDD=2*N+1
      II=IX/2
      DO 200 J=2,II
      I=2*J-2
      EEP(I)=EP(J)
      EEP(I+1)=-EP(J)
      EEQ(I)=EQ(J)
      EEQ(I+1)=EQ(J)
  200 CONTINUE
      E(1)=EQ(1)**2
      E(2)=0.
      E(3)=-1.
      CALL COEF(E,EEP,EEQ,N,NDD)
      NL=NDD-2
      NX=N-1
      DO 201 J=1,ND
      I=2*J-1
      POA(I)=PO2(J)
  201 POA(I+1)=PO2(J)
      F(1)=POA(1)
      F(2)=0.
      F(3)=1.
      CALL COEF(F,PO1,POA,NX,NL)
```

```
      DO 202 J=1,NDO,2
      K=NDO+1-J
  202 F(K)=F(K-2)
      F(1)=0.
      DO 203 J=1,NDO,2
  203 H(J)=(E(J)+F(J))*GAM
      DO 204 J=1,NDO,2
  204 F(J)=H(J)-E(J)
      N2=NDO+1
      DO 13 J=1,N2,2
      K=(J+1)/2
   13 A(K)=F(J)
      CALL PREAL(A,B,Z,N)
      DO 19 LV=1,N2
      LVS=LV+1
   19 A(LV)=B(LVS)
      CALL PROOT4(NX,A,U,V)
      DO 60 J=2,II
      PR=U(2*J-3)
      PIM=ABS(V(2*J-3))
      DO1=ATAN2(PIM,PR)
      AS1=(V(2*J-3)**2+U(2*J-3)**2)**.25
      PRS1=-AS1*COS(DO1/2.)
      PP(J)=2.*ABS(PRS1)
   60 Q(J)=PRS1**2+(AS1*SIN(DO1/2.))**2
      F(1)=SQRT(ABS(Z))
      F(2)=1.
       F(3)=0.
      CALL COEF(F,PP,Q,II,IX)
      IF(RK2-1.)16,16,17
   16 DO 15 J=2,IX,2
   15 F(J)=-F(J)
      RETURN
   17 DO 18 J=1,IX,2
   18 F(J)=-F(J)
      RETURN
   10 T=SQRT(RK2)
      GAM=(2/(T+1./T))**2
      DO 20 I=1,ND
      J=2*I-1
      EEP(J)=EP(I)
      EEP(J+1)=-EP(I)
      EEQ(J)=EQ(I)
   20 EEQ(J+1)=EQ(I)
      NDO=2*N+1
      E(1)=EEQ(1)
      E(2)=EEP(1)
      E(3)=1.
      CALL COEF(E,EEP,EEQ,N,NDO)
  100 FORMAT(5X,5E20.12,/)
      NM2=ND-1
      DO 21 I=1,NM2
      J=2*I-1
      POA(J)=PO2(I)
   21 POA(J+1)=PO2(I)
      N2=N-2
      F(1)=POA(1)
      F(2)=0.
      F(3)=1.
      N3=2*N-3
      CALL COEF(F,PO1,POA,N2,N3)
      DO 22 I=1,N3,2
      K=NDO+1-I
   22 F(K)=F(K-4)
```

Subroutines DIV and FIX.

```
      F(1)=0.
      F(3)=0.
      DO 23 I=1,NDO,2
   23 H(I)=(E(I)-F(I))*GAM
      DO 24 I=1,NDO,2
   24 F(I)=E(I)-H(I)
      DO 25 J=1,NDO,2
      K=(J+1)/2
   25 A(K)=F(J)
      CALL PROOT4(N,A,U,V)
      DO 26 J=1,ND
      PR=U(2*J-1)
      PIM=ABS(V(2*J-1))
      OO1=ATAN2(PIM,PR)
      AS=(PIM**2+PR**2)**.25
      PRS=-AS*COS(OO1/2.)
      PIMS=AS*SIN(OO1/2.)
      PP(J)=2.*ABS(PRS)
   26 Q(J)=PRS**2+PIMS**2
      F(1)=Q(1)
      F(2)=PP(1)
      F(3)=1.
      CALL COEF(F,PP,Q,ND,IX)
      RETURN
      END
      SUBROUTINE DIV(N,A,B,X)
C  N+1 IS THE ORDER OF THE NUMERATOR POLYNOMIAL
      DOUBLE PRECISION A(51),B(51),X
      NM=N-1
      DO 700 I=1,NM,2
      NN=N+3-I
      L=NN-2
      B(L)=A(NN)
  700 A(L)=A(L)-B(L)*X
      IF((L-2).EQ.0)RETURN
      B(L-2)=A(L)
      RETURN$END
      SUBROUTINE FIX(ZA,ND,ZN)
      DIMENSION ZA(25),ZN(25)
      L=ND
      I=1
      ZN(1)=ZA(ND)
   70 L=L-2
      IF(L)51,51,53
   53 I=I+1
      ZN(I)=ZA(L)
      GO TO 70
   51 LA=I+1
      IDJ=2
      IF(L.EQ.0)J=-1
      IF(L.LT.0)J=0
      DO 80 I=LA,ND
      J=J+IDJ
   80 ZN(I)=ZA(J)
      RETURN
      END
```

Subroutine EVNET.

```
      SUBROUTINE EVNET(NH,ZA,F,E,RO,CASE)
      DIMENSION EL(74),NEL(74,2),E(51),F(51),ZA(25),ZN(25)
      DOUBLE PRECISION ANUM(51),DEN(51),ZOM(25),ANUM1(51),DEN1(51)
     1,SUM,SUMA,CX,AB,CI,CX1,YL,USQ
      INTEGER PUNSA,TABLE,ORD,CNT
      COMMON/GARY/R2,PUNSA,TABLE,NETWRK,ORD
      NELE=1.
        NU=0
      NDEL=NH+(NH-1)/2
      N=NH
        ND=N/2
        NDA=ND-1
      IF(ORD.EQ.0)CALL FIX(ZA,NDA,ZN)
        DO 29 I=1,NDA
   29 ZOM(I)=-ZN(I)**2
      NP=N+1
      DO 1 I=1,NP,2
    1 ANUM(I)=E(I)+F(I)
      DO 2 I=2,N,2
    2 DEN(I)=E(I)-F(I)
        DO 41 JA=1,NDA
      SUM=0.
      SUMA=0.
      DO 3 I=1,NP,2
    3 SUM=SUM+ANUM(I)*ZOM(JA)**(I/2)
      DO 9 I=2,N,2
    9 SUMA=SUMA+DEN(I)*ZOM(JA)**(I/2)
        NU=NU+1
        CX1=SUM/SUMA
      EL(NELE)=CX1
      NEL(NELE,1)=1HC
      IF(R2.GT.1..AND.R2.LT.1000.)NEL(NELE,1)=1HL
      NEL(NELE,2)=NU
      NELE=NELE+1
      IF(R2.GE.1000.) GO TO 12
   13 IF(NU.GE.(NH/2))GO TO 500
   12 DO 4 I=3,NP,2
    4 ANUM(I)=ANUM(I)-CX1*DEN(I-1)
        AB=-ZOM(JA)
      NA=N-2
      NM1=N-1
      CALL DIV(NM1,ANUM,ANUM1,AB)
        SUM=0.
      SUMA=0
      DO 5 I=1,NM1,2
      SUMA=SUMA+DEN(I+1)*ZOM(JA)**(I/2)
    5 SUM=SUM+ANUM1(I)*ZOM(JA)**(I/2)
        NU=NU+1
        CX=SUM/SUMA
        YL=1./(CX*AB)
      EL(NELE)=YL
      NEL(NELE,1)=1HL
      IF(R2.GT.1..AND.R2.LT.1000.)NEL(NELE,1)=1HC
      NEL(NELE,2)=NU
      NELE=NELE+1
      EL(NELE)=CX
      NEL(NELE,1)=1HC
      IF(R2.GT.1..AND.R2.LT.1000.)NEL(NELE,1)=1HL
      NEL(NELE,2)=NU
      IF(NH.EQ.4)GO TO 500
      NELE=NELE+1
        CI=1./CX
      DO 6 I=2,N,2
    6 DEN(I)=DEN(I)-CI*ANUM1(I-1)
```

```
      CALL DIV(NA,DEN,DEN1,AB)
        N=NA
      NP=N+1
      DO 7 I=1,NP,2
    7 ANUM(I)=ANUM1(I)
      DO 8 I=2,N,2
    8 DEN(I)=DEN1(I)
41        CONTINUE
      EL(NELE)=ANUM(3)/DEN(2)
      EL(NELE+1)=DEN(2)/ANUM(1)
      NEL(NELE,1)=1HC
      NEL(NELE+1,1)=1HL
      NEL(NELE,2)=NH-1
      NEL(NELE+1,2)=NH
      GO TO 900
  500 NDAB=NDA-1
      NELE=NOEL
      NU=NH
      N=NH
      NP=NH+1
      DO 128 I=1,NP
      ANUM1(I)=0.
      DEN(I)=0.
      ANUM(I)=0.
  128 DEN1(I)=0.
      USQ=(1-RO)/(1+RO)
      IF(CASE.EQ.3.)USQ=R2
      IF(USQ.GT.1.)USQ=1./USQ
      DO 10 I=2,N,2
   10 ANUM(I)=E(I)+F(I)
      DO 11 I=1,NP,2
   11 DEN(I)=E(I)+F(I)
      YL=DEN(NP)*USQ/ANUM(N)
      CNT=1.
      EL(NELE)=YL
      NEL(NELE,1)=1HL
      IF(R2.GT.1.)NEL(NELE,1)=1HC
      NEL(NELE,2)=NU
      NELE=NELE-1
      DO 129 I=3,NP,2
  129 DEN(I)=USQ*DEN(I)-YL*ANUM(I-1)
      DEN(1)=USQ*DEN(1)
      DO 141 JB=1,NDA
      NU=NU-1
      JA=ND-JB
      SUM=0
      SUMA=0
      DO 131 I=1,N,2
  131 SUM=SUM+ANUM(I+1)*ZOM(JA)**(I/2)
      DO 139 I=1,N,2
  139 SUMA=SUMA+DEN(I)*ZOM(JA)**(I/2)
      CX=SUM/SUMA
      EL(NELE)=CX
      NEL(NELE,1)=1HC
      IF(R2.GT.1.)NEL(NELE,1)=1HL
      NEL(NELE,2)=NU
      CNT=CNT+1.
      IF(CNT.GE.(NOEL/2))GO TO 900
```

Subroutine ODNET.

```
      NELE=NELE-1
      DO 134 I=2,N,2
  134 ANUM(I)=ANUM(I)-CX*DEN(I-1)
      AB=-ZOM(JA)
      NM2=N-2
      NX=N-3
      CALL DIV(NM2,ANUM,ANUM1,AB)
      SUM=0.
      DO 135 I=2,NM2,2
  135 SUM=SUM+ANUM1(I)*ZOM(JA)**(I/2)
      CX=SUM/SUMA
      YL=1./(CX*AB)
      NU=NU-1
      EL(NELE)=CX
      NEL(NELE,1)=1HC
      IF(R2.GT.1.)NEL(NELE,1)=1HL
      NEL(NELE,2)=NU
      NELE=NELE-1
      EL(NELE)=YL
      NEL(NELE,1)=1HL
      IF(R2.GT.1.)NEL(NELE,1)=1HC
      NEL(NELE,2)=NU
      CNT=CNT+2.
      IF(CNT.GE.(NOEL/2))GO TO 900
      NELE=NELE-1
      CI=1./CX
      DO 136 I=3,N,2
  136 DEN(I)=DEN(I)-CI*ANUM1(I-1)
      CALL DIV(NX,DEN,DEN1,AB)
      N=NM2
      DO 142 I=2,N,2
  142 ANUM(I)=ANUM1(I)
      DO 143 I=1,N,2
  143 DEN(I)=DEN1(I)
  141 CONTINUE
  900 WRITE(2,800)((NEL(I,J),J=1,2),EL(I),I=1,NOEL)
  800 FORMAT(3(5X,A1,*(*,I2,*)=*,E20.14    ))
      IF(CASE.EQ.2.)R2=USQ
      IF(PUNSA.EQ.0)RETURN
      PUNCH 910,(EL(I),I=1,NOEL)
  910 FORMAT(4E20.14)
      RETURN
        END
      SUBROUTINE ODNET(NH,ZA,F,E,DIV)
      DOUBLE PRECISION DEN(51),ZOM(25),ANUM(51),ANUM1(51),ADEN(51)
      DIMENSION ZN(25),EL(74),NEL(74,2),E (51),F(51),ZA(25)
      COMMON/GARY/R2,PUNSA,TABLE,NETWRK,ORD
      INTEGER PUNSA,TABLE,ORD
      NOEL=NH+NH/2
      NELE=1
      N=NH
      ND=N/2
      IF(ORD.EQ.0)CALL FIX(ZA,ND,ZN)
      DO 4 I=1,ND
    4 ZOM(I)=-ZN(I)**2
      NU=0
      DO 1 I=1,N,2
      ANUM(I+1)=E(I+1)+F(I+1)
    1 DEN(I)=E(I)-F(I)
```

```
   61 DO 14 JA =1,ND
      SUM=0.
      SUMA=0.
      DO 3 I=1,N,2
    3 SUM=SUM+ANUM(I+1)*ZOM(JA)**(I/2)
      DO 5 I=1,N,2
    5 SUMA=SUMA+DEN(I)*ZOM(JA)**(I/2)
      NU=NU+1
      CX1=SUM/SUMA
      EL(NELE)=CXI
      NEL(NELE,1)=1HC
      NEL(NELE,2)=NU
      IF(R2.GE.1000.)GO TO 12
      IF(NU.GE.((NOEL/2)-1))GO TO 60
12    NELE=NELE+1
      DO 6 I=1,N,2
    6 ANUM(I+1)=ANUM(I+1)-CXI*DEN(I)
      AB=-ZOM(JA)
      NM2=N-2
      NM1=N-1
      CALL DIV(NM1,ANUM,ANUM1,AB)
      SUM=0.
      DO 7 I=2,N,2
    7 SUM=SUM+ANUM1(I)*ZOM(JA)**(I/2)
      DO 8 I=2,N,2
    8 SUMA=SUMA+DEN(I)*ZOM(JA)**((I-1)/2)
      NU=NU+1
      CX=SUM/SUMA
      YL=1./(CX*AB)
      EL(NELE)=YL
      NEL(NELE,1)=1HL
      NEL(NELE,2)=NU
      EL(NELE+1)=CX
      NEL(NELE+1,1)=1HC
      NEL(NELE+1,2)=NU
      NELE=NELE+2
      CI=1./CX
      NM3=N-3
      DO 9 I=3,N,2
    9 DEN(I)=DEN(I)-CI*ANUM1(I-1)
      CALL DIV(NM2,DEN,ADEN,AB)
   16 N=N-2
      DO 10 I=1,N,2
   10 ANUM(I+1)=ANUM1(I+1)
      DO 11 I=1,N,2
   11 DEN(I)=ADEN(I)
   14 CONTINUE
      EL(NELE)=ANUM(2)/DEN(1)
      NEL(NELE,1)=1HC
      NEL(NELE,2)=NH
      GO TO 900
   60 DO 42 I=1,NH,2
      ANUM(I+1)=0.
      DEN(I)=0.
      ANUM(I+1)=E(I+1)+F(I+1)
      DEN(I)=R2*(E(I)+F(I))
   42 CONTINUE
      NELE=NOEL
      NU1=NH
      NU=NH
      N=NH-1
```

```
      NP=NH
      DO 114 JAN=1,ND
      JA=-JAN+ND+1
      SUMA=0.
      SUM=0.
      DO 106 I=1,NP,2
  106 SUM=SUM+ANUM(I+1)*ZOM(JA)**(I/2)
      DO 107 I=1,NP,2
  107 SUMA=SUMA+DEN(I)*ZOM(JA)**(I/2)
      CX1=SUM/SUMA
      EL(NELE)=CX1
      NEL(NELE,1)=1HC
      NEL(NELE,2)=NU1
      NELE=NELE-1
      NU1=NU1-1
      DO 108 I=1,NP,2
  108 ANUM(I+1)=ANUM(I+1)-CX1*DEN(I)
      NM1=N-1
      AB=-ZOM(JA)
      CALL DIV(N,ANUM,ANUM1,AB)
      SUM=0.
      DO 110 I=2,N,2
  110 SUM=SUM+ANUM1(I)*ZOM(JA)**(I/2)
      NU=NU-1
      CX=SUM/SUMA
      YL=1./(CX*AB)
      EL(NELE)=CX
      EL(NELE-1)=YL
      NEL(NELE,1)=1HC
      NEL(NELE,2)=NU1
      NEL(NELE-1,1)=1HL
      NEL(NELE-1,2)=NU1
      NELE=NELE-2
      NU1=NU1-1
      IF(NU1.LE.(NDEL/2   ))GO TO 900
      NU=NU-1
      CI=1./CX
      DO 112 I=3,NP,2
  112 DEN(I)=DEN(I)-CI*ANUM1(I-1)
      NM2=N-2
      CALL DIV(NM1,DEN,ADEN,AB)
      N=N-2
      NP=N+1
      DO 117 J=1,NP,2
  117 ANUM(J+1)=ANUM1(J+1)
      DO 118 J=1,NP,2
  118 DEN(J)=ADEN(J)
  114 CONTINUE
  900 WRITE(2,800)((NEL(I,J),J=1,2),EL(I),I=1,NOEL)
  800 FORMAT(3(5X,A1,*(*,I2,*)=*,E20.14    ))
      IF(PUNSA.EQ.0)RETURN
      PUNCH 910,(EL(I),I=1,NOEL)
  910 FORMAT(4E20.14)
      RETURN
      END
```

APPENDIX B

TABLES OF ELLIPTIC REALIZATIONS

Complete calculations for four different values of passband ripple and stopband frequency have been carried out. The results have been arranged in the form of tables.

The tables presented here, include odd order and even order case 3. These tables illustrate the fact that for even order case 3, networks with exact value of w_s can be synthesized.

The elements listed on top of the tables correspond to the network structure shown in Fig. B.1a. The primed elements in the bottom of the tables correspond to the dual network shown in Fig. B.1b.

For low values of passband ripple and order, we obtain low values of stopband attenuation, and these values can be increased as w_s increases. Thus, we only have included realizations with stopband attenuations greater than 10 dB.

Finally, we should mention that the tables presented here are representative of the results that can be obtained with ELLIP. Thus, tables with values of stopband frequency and passband ripple other than those shown can be obtained. These results are also valid for even order case 2 networks.

Table B.1 Element Values for A_{max} =0.001 dB, R_2=1.

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
3	2.50	10.838	.29185724	.49650526	.24686859	.29185724						
	3.00	15.692	.32758766	.56674712	.14917848	.32758766						
	3.50	19.844	.34934082	.60953856	.10150456	.34934082						
	4.00	23.431	.36343080	.63726216	.07414664	.36343080						
	4.50	26.579	.37305116	.65619335	.05679786	.37305116						
4	2.00	16.080	.21685832	.58613730	.36042055	.83817888	.46889989					
	2.50	24.651	.30785049	.72137328	.18599423	.87475484	.46123205					
	3.00	31.436	.35280339	.79226639	.11714730	.89586650	.45640350					
	3.50	37.059	.37855093	.83393606	.08150139	.90879727	.45341214					
	4.00	41.869	.39475434	.86052754	.06044334	.91723407	.45146087					
5	1.50	23.385	.40759607	.94069150	.19549784	1.09174034	.52231275	.78934247	.09538853			
	2.00	38.852	.47577101	1.03418456	.09149980	1.17566926	.80542980	.28444148	.32957737			
	2.50	49.719	.50191432	1.06829846	.05472520	1.22282991	.92529999	.15767238	.41420217			
	3.00	58.235	.51505909	1.08545793	.03673780	1.24925892	.98751396	.10233377	.45603398			
	3.50	65.277	.52267155	1.09543646	.02646721	1.26534763	1.02408749	.07239007	.48007649			
6	1.20	16.168	.12174296	.63826328	.65159007	.89938156	.41225949	1.61528411	.58855620	.55915795		
	1.50	34.154	.32789849	.88824901	.26612082	1.08002324	.84473467	.49715554	.87650484	.55144289		
	2.00	52.712	.43272975	1.02848041	.12089168	1.22987857	1.11598029	.20984965	1.02589023	.54403785		
	2.50	65.746	.47281570	1.08517447	.07139798	1.29657199	1.22771774	.12165247	1.08426765	.54076313		
	3.00	75.960	.49295576	1.11431926	.04763319	1.33194513	1.28546929	.08054023	1.11391688	.53902921		
7	1.20	30.917	.44702658	1.04544764	.19258835	.99910774	.65716637	1.02825412	.91992301	.54794054	.94007994	.04592815
	1.50	52.079	.51931961	1.14525619	.09162224	1.23698438	1.04628120	.40905495	1.15612040	.85879152	.35136705	.32231347
	2.00	73.759	.55507161	1.19428335	.04416002	1.37906872	1.28477821	.18620490	1.32929435	1.04449560	.15437511	.45808707
	2.50	88.975	.56854151	1.21280808	.02662843	1.43717304	1.38382483	.11035698	1.40470736	1.11984872	.09027116	.50954685
	3.00	100.896	.57526583	1.22206844	.01794566	1.46716117	1.43527825	.07379143	1.44446121	1.15853795	.05995889	.53532855
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.1 Element Values for A_{max}=0.001 dB, R_2=1.-Continued

n	w_s	A_{min}	C_1	L_2	C_2	C_4	L_4	C_4	C_5	L_6	C_6	C_7
8	1.10	26.359	.21835763	.78328453	.48935759	.78534500	.45447958	1.78938519	.71796635	.57389806	1.20282864	.67646498
	1.20	39.590	.33265435	.92492988	.30297629	.97131953	.73066504	.92964144	.90929861	.82338489	.66760848	.85071811
	1.50	63.757	.45291834	1.08569855	.13937796	1.22491701	1.12090844	.38467701	1.21399988	1.15069853	.28506316	1.04611869
	2.00	88.525	.51390439	1.17195508	.06608171	1.37938075	1.36417814	.17692805	1.41195303	1.34337264	.13187914	1.15153145
	2.50	105.911	.53714123	1.20562862	.03960587	1.44321649	1.46568625	.10518245	1.49527751	1.42153722	.07846898	1.19297203
9	1.05	27.235	.42528830	1.02672759	.24387776	.86627155	.49056348	1.69110249	.56167296	.31305567	2.87412543	.68218300
	1.10	38.658	.47755574	1.10257515	.17115026	1.01713445	.70551831	1.03436005	.73295303	.52763172	1.54797666	.81231807
	1.20	53.579	.52227143	1.16617701	.11205682	1.17779306	.94826620	.62046692	.96306078	.79258427	.86195734	.99146781
	1.50	80.793	.56727628	1.23014874	.05414222	1.37702891	1.26907328	.28081704	1.29950702	1.16817164	.37192931	1.25652645
	2.00	108.667	.58921853	1.26127756	.02625229	1.49078403	1.46015601	.13289939	1.51099772	1.40363337	.17345024	1.42421670
10	1.05	33.529	.27044544	.85549806	.41674531	.79956983	.48819217	1.72625845	.53685461	.38295745	2.35236545	.63693995
	1.10	46.213	.35723912	.96639671	.28644762	.95545493	.70145724	1.06331196	.72250753	.60141738	1.36055582	.81465455
	1.20	62.780	.43390924	1.06934774	.18415331	1.12596125	.95042401	.63728190	.96098920	.86960434	.78767488	1.02983830
	1.50	93.006	.51396796	1.18149779	.08735682	1.34395188	1.28621020	.28798363	1.30472793	1.25165095	.34842472	1.32707534
	2.00	123.973	.55415465	1.23944589	.04197240	1.47108087	1.48856126	.13626854	1.52031562	1.49180452	.14392523	1.50889604
11	1.10	58.008	.53077711	1.18490096	.11196204	1.17168192	.92774330	.68071791	.85019452	.64703910	1.26730142	.74268777
	1.20	76.246	.56146865	1.22962902	.07381913	1.30654273	1.14020257	.43119630	1.08641049	.90832640	.75624977	.99202042
	1.30	89.490	.57651989	1.25152998	.05517870	1.38177615	1.26303926	.31696018	1.23205871	1.07445601	.54370031	1.13880850
	1.40	100.270	.58576892	1.26495557	.04371516	1.43140571	1.34551043	.24876552	1.33330567	1.19262652	.42178867	1.26258036
	1.50	109.506	.59208717	1.27410686	.03587341	1.46690300	1.40512961	.20291613	1.40818249	1.28092344	.34176016	1.34748406
12	1.10	65.910	.43598219	1.07616261	.19185485	1.09846734	.89515956	.73835517	.82627870	.67595243	1.21437689	.74499002
	1.20	85.799	.49139040	1.15340348	.12514717	1.24676393	1.11810339	.46540244	1.06428126	.94177002	.73048723	.99736693
	1.30	100.244	.51930067	1.19300915	.09304119	1.33122569	1.24910594	.34143651	1.21205943	1.11125344	.52663921	1.15761814
	1.40	112.002	.53671190	1.21793731	.07346001	1.38755216	1.33783005	.26770864	1.31518554	1.23131219	.40935217	1.27072242
	1.50	122.077	.54872317	1.23523018	.06013868	1.42811759	1.40233118	.21824102	1.39161979	1.32115634	.33206399	1.35514091
13	1.05	60.847	.53563772	1.19489701	.11104352	1.17063372	.91896717	.70675098	.79217461	.55896082	1.56236517	.55928760
	1.10	77.358	.56208050	1.23410247	.07922662	1.28501029	1.09701591	.49019943	.98845775	.77103978	1.01516411	.77172431
	1.15	89.246	.57567266	1.25424623	.06290992	1.35145202	1.20402176	.38437555	1.11566609	.91498428	.77390412	.92039594
	1.50	138.220	.60653720	1.29970343	.02554492	1.52652528	1.49623938	.15225825	1.49723105	1.37226691	.29274447	1.39915702
	1.60	147.854	.60984957	1.30454075	.02148505	1.54760307	1.53226884	.12775217	1.54749543	1.43240818	.24531865	1.44004756
14	1.05	67.735	.43691001	1.07897036	.19564762	1.08269976	.86360402	.79618758	.74856213	.55900207	1.56977441	.54504733
	1.10	85.510	.48537354	1.14677072	.13841131	1.21012069	1.05171631	.54826848	.94927187	.77761721	1.01353693	.77440627
	1.15	98.311	.51078605	1.18270324	.10945220	1.28507483	1.16614742	.42885815	1.07779549	.92646501	.77071402	.91257405
	1.20	108.718	.52737264	1.20632408	.09093545	1.33747285	1.24739748	.35415582	1.17325250	1.03732645	.62751153	1.02476355
	1.25	117.655	.53931313	1.22340540	.07777563	1.37699965	1.30935814	.30172646	1.24807217	1.12495931	.53014837	1.12412454
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.1 Element Values for A_{max}=0.001 dB, R_2=1.-Continued

n	w_s	A_{min}	L_8	C_8	C_9	L_{10}	C_{10}	C_{11}	L_{12}	C_{12}	C_{13}	L_{14}
8	1.10	26.359	.58647178									
	1.20	39.590	.58501079									
	1.50	63.757	.58064840									
	2.00	88.525	.57726375									
	2.50	105.911	.57575518									
9	1.05	27.235	.42774460	1.46640317	-.11308440							
	1.10	38.658	.60789833	.84894258	.08845169							
	1.20	53.579	.79916346	.48517172	.25915189							
	1.50	80.793	1.03112319	.20917729	.43527542							
	2.00	108.667	1.15958839	.09665506	.52396411							
10	1.05	33.529	.65534125	1.03788276	.73701597	.59804844						
	1.10	46.213	.85429392	.66714309	.87276057	.59905141						
	1.20	62.780	1.06302796	.41053452	1.00042582	.59871977						
	1.50	93.006	1.31725032	.18849992	1.14365516	.59676901						
	2.00	123.973	1.45947614	.08949538	1.21989285	.59505222						
11	1.10	58.008	.69642916	1.09706221	.93379894	.80414577	.50161565	.25687684				
	1.20	76.246	.94456083	.66112678	1.11172639	.95668924	.30721185	.37520534				
	1.30	89.490	1.09777771	.47653628	1.22033005	1.03930771	.22211840	.43509640				
	1.40	100.270	1.20438515	.37015230	1.29526615	1.09280136	.17255224	.47264044				
	1.50	109.506	1.28324803	.30003081	1.35037197	1.13057429	.13976027	.49863942				
12	1.10	65.910	.75472347	1.02407352	.95737386	1.04966158	.44349361	.99572933	.60716182			
	1.20	85.799	1.01045364	.62774687	1.15799762	1.22106873	.28326826	1.09449175	.60748302			
	1.30	100.244	1.16903942	.45568445	1.27798774	1.31512644	.20876140	1.14661528	.60726679			
	1.40	112.002	1.27965970	.35538115	1.36007580	1.37652026	.16401955	1.17998980	.60698095			
	1.50	122.077	1.36162804	.28881576	1.42015601	1.42010928	.13385363	1.20340821	.60671065			
13	1.05	60.847	.47733311	1.09298969	.62359432	.65033096	1.22692285	.90753785	.80962316	.50479487	.25851399	
	1.10	77.358	.68866891	1.19337296	.83068278	.85375906	.81210188	1.05601137	.93802136	.33947559	.35831990	
	1.15	89.246	.83128623	.90341051	1.18224623	.98544301	.62372733	1.15028586	1.01025082	.26203328	.41128139	
	1.50	138.220	1.30925421	.33581198	1.31621635	1.37874024	.23811732	1.42412011	1.19228897	.10013873	.53734422	
	1.60	147.854	1.19297050	.32374061	.84599268	1.43045627	.19919313	1.45939154	1.21344287	.08369527	.55143493	
14	1.05	67.735	.49269321	1.83478595	.61317638	.68812623	1.18133307	.91664901	1.04146957	.46071666	.99259982	.61120460
	1.10	85.510	.71908185	1.14366054	.82641165	.90166437	.78832539	1.08615937	1.18881565	.32067855	1.07862346	.61285168
	1.15	98.311	.87047736	.86344701	.96731776	1.03900817	.60895684	1.19100681	1.27241456	.25181619	1.12563630	.61332094
	1.20	108.718	.99349148	.69423096	1.07301660	1.14034990	.49927608	1.26648707	1.32950154	.20834579	1.15713530	.61349726
	1.25	117.655	1.06678080	.59545728	1.15917444	1.21937709	.42349012	1.32456613	1.37186747	.17769202	1.18022374	.61355062
			C'_8	L'_8	L'_9	C'_{10}	L'_{10}	L_{11}	C'_{12}	L'_{12}	L'_{13}	C'_{14}

Table B.2 Element Values for A_{max}=0.01 dB, R_2=1.

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
3	2.50	20.508	.53604300	.79616329	.15395278	.53604300						
	3.00	25.590	.56541673	.85105400	.09934326	.56541673						
	3.50	29.808	.58274721	.88345311	.07003308	.58274721						
	4.00	33.417	.59383687	.90418857	.05225774	.59383687						
	4.50	36.575	.60136695	.91826944	.04058763	.60136695						
4	2.00	25.987	.44796998	.89497974	.23604549	1.08897495	.64196519					
	2.00	25.987	.44796998	.89497974	.23604549	1.08897495	.64196519					
	3.00	41.438	.55262812	1.06726250	.08696255	1.14810190	.63346752					
	3.50	47.063	.57300703	1.10176349	.06174979	1.16039332	.63163686					
	4.00	51.874	.58593465	1.12379262	.04628359	1.16831253	.63045456					
5	1.50	33.372	.63680710	1.13435019	.16212203	1.29831011	.78253828	.52605427	.39852462			
	2.00	48.856	.69698005	1.22020715	.07755050	1.42448779	1.03140951	.22212094	.57753701			
	2.50	59.724	.72014337	1.25310888	.04665424	1.48094601	1.13503380	.12853736	.64692211			
	3.00	68.239	.73180803	1.26973659	.03140599	1.51094399	1.18875583	.08500991	.68203876			
	3.50	75.282	.73856805	1.27940079	.02266150	1.52879638	1.22034698	.06074811	.70245190			
6	1.20	26.077	.36909325	.87420670	.47572761	1.01691102	.66049179	1.00821270	.87816572	.72947150		
	1.50	44.157	.53879523	1.10527194	.21386733	1.25667075	1.07079653	.39219824	1.10727912	.72667664		
	2.00	62.716	.62882539	1.23604065	.10059113	1.41788929	1.32402434	.17687596	1.23671948	.72336917		
	2.50	75.750	.66385307	1.28851177	.06013082	1.48645646	1.42880812	.10453111	1.28879393	.72178358		
	3.00	85.965	.68156768	1.31537959	.04035229	1.52232317	1.48310723	.06980749	1.31551631	.72092037		
7	1.10	29.317	.60482287	1.11004026	.25630935	1.01569414	.53405017	1.51725237	.87045593	.53722925	1.22143818	.14508144
	1.20	40.918	.66757000	1.20256033	.16742697	1.19920983	.79596350	.84895102	1.05462033	.75731764	.68017419	.34519534
	1.50	62.084	.73288770	1.29802145	.08083914	1.44395444	1.15979128	.36902028	1.34563691	1.04596356	.28849097	.56475177
	2.00	83.764	.76554251	1.34603145	.03918153	1.59029923	1.38489872	.17274332	1.53532597	1.21467112	.13274714	.68077703
	2.50	98.979	.77789278	1.36425173	.02367245	1.65028715	1.47862940	.10328127	1.61538361	1.28277584	.07880570	.72590977
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.2 Element Values for $A_{max} = 0.01$ dB, $R_2 = 1.$ -Continued

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
8	1.05	26.233	.33868373	.84582536	.56701386	.77306341	.37943906	2.36399029	.62797061	.53975232	1.47097095	.77846811
	1.10	36.354	.44463248	.98321345	.38985047	.93673278	.60005849	1.35526628	.79557309	.75758828	.91118229	.92156203
	1.20	49.594	.54240909	1.11563434	.25118609	1.12776679	.87015868	.78061222	1.01457063	1.00319256	.54794937	1.06422065
	1.50	73.762	.64891530	1.26625328	.11950409	1.38576541	1.25068053	.34476247	1.33198653	1.32355801	.24783331	1.23425185
	2.00	98.529	.70409897	1.34674538	.05750515	1.54213897	1.48796603	.16220893	1.53096157	1.51196071	.11717423	1.32918412
9	1.05	37.232	.64617058	1.17624619	.21287723	1.04725856	.58738633	1.41234667	.63312905	.41352383	2.17583896	.77310704
	1.10	48.662	.69292650	1.24543798	.15151780	1.19880659	.79212117	.92127314	.83646321	.62131670	1.31456565	.94211928
	1.20	63.584	.73348420	1.30572047	.10008121	1.36107802	1.02495043	.57404514	1.08863201	.87588619	.77998014	1.14026261
	1.50	90.797	.77482070	1.36755058	.04870239	1.56444037	1.33287249	.26737546	1.44396257	1.23638134	.35141041	1.43512451
	2.00	118.671	.79514263	1.39804411	.02368410	1.68139024	1.51613922	.12799210	1.66407791	1.46215946	.16650752	1.61150720
10	1.05	43.532	.48785660	1.04145982	.34233179	.94385762	.59304554	1.42104747	.59429283	.47503815	1.89638636	.71405167
	1.10	56.217	.56370058	1.14418546	.24193808	1.10119407	.80466464	.92693010	.79611379	.69398528	1.17907676	.90036606
	1.20	72.785	.63250081	1.23969268	.15884899	1.27368631	1.04994900	.57687375	1.04427860	.96276823	.71145419	1.11905860
	1.50	103.011	.70588135	1.34376371	.07680807	1.49475347	1.37907307	.26859163	1.39373313	1.34442170	.32438196	1.41557156
	2.00	133.978	.74321250	1.39753936	.03722437	1.62384717	1.57689527	.12863510	1.61068930	1.58391008	.15440261	1.59526409
11	1.10	68.012	.74016961	1.31807746	.10064957	1.34472230	.98923059	.63840675	.95126845	.69295901	1.18332190	.83329857
	1.20	86.250	.76830690	1.36057823	.06671439	1.48122663	1.19310546	.41207684	1.19873293	.94716499	.72523392	1.09792510
	1.30	99.495	.78220815	1.38163030	.04998283	1.55793393	1.31106271	.30535012	1.35039158	1.10857081	.52696865	1.26490983
	1.40	110.274	.79078383	1.39462685	.03965056	1.60871852	1.39028814	.24075340	1.45556550	1.22269359	.41141653	1.38243135
	1.50	119.511	.79665642	1.40352871	.03256546	1.64512122	1.44757568	.19696619	1.53323276	1.30803627	.33467619	1.47005608
12	1.05	60.682	.57755415	1.16203640	.23324845	1.09728045	.78165946	.98249509	.68899215	.51989692	1.73656303	.58879713
	1.10	75.915	.63467870	1.24041916	.16644939	1.24030831	.98064236	.67399260	.89533030	.73704886	1.11371316	.79928824
	1.15	86.884	.66545859	1.28305500	.13215421	1.32803688	1.10697171	.52617658	1.03423232	.88805237	.84445240	.94465607
	1.20	95.804	.68577591	1.31134230	.11007437	1.39053500	1.19863072	.43413541	1.13852775	1.00363082	.68546218	1.05530103
	1.25	103.463	.70051271	1.33193094	.09431086	1.43830467	1.26951451	.36972145	1.22109934	1.09624600	.57783405	1.14363189
13	1.05	70.851	.74406404	1.32492555	.10014568	1.33877627	.97371450	.66701373	.88192157	.59142713	1.47662048	.62157249
	1.10	87.362	.76824075	1.36167585	.07180399	1.45375031	1.14397069	.47007898	1.08655536	.79877266	.97991825	.84158658
	1.15	99.251	.78075551	1.38075159	.05714607	1.52104884	1.24647457	.37128437	1.21869399	.93829584	.75467680	1.00557733
	1.20	108.916	.78882181	1.39305759	.04767409	1.56768464	1.31857774	.30835025	1.31560530	1.04389786	.61770538	1.11969097
	1.25	117.216	.79457749	1.40184097	.04089504	1.60265705	1.37316914	.26369774	1.39107540	1.12701461	.52375567	1.20673726
14	1.30	135.577	.73998241	1.38349408	.06062162	1.55209034	1.42367888	.25043986	1.38180177	1.24383173	.44119398	1.23746742
	1.10	95.515	.68056323	1.30180033	.12192809	1.35043466	1.12567454	.51224655	1.01541948	.82769083	.95222000	.80885102
	1.15	108.315	.70438548	1.33449515	.09700258	1.42656658	1.23676721	.40437022	1.14697997	.97386991	.73319811	.95848322
	1.20	118.723	.72002960	1.35600601	.08089760	1.47991735	1.31568391	.33577448	1.24409648	1.08424490	.60035727	1.07208013
	1.25	127.660	.73133614	1.37157456	.06937365	1.52023088	1.37584137	.28714647	1.32012805	1.17194324	.50889439	1.16286344
			L_1'	C_2'	L_2'	L_3'	C_4'	L_4'	L_5'	C_6'	L_6'	L_7'

Table B.2 Element Values for $A_{max} = 0.01$ dB, $R_2 = 1.$-Continued

n	w_s	A_{min}	L_8	C_8	C_9	L_{10}	C_{10}	C_{11}	L_{12}	C_{12}	C_{13}	L_{14}
8	1.05	26.233	.75316911									
	1.10	36.354	.75764016									
	1.20	49.594	.75998155									
	1.50	73.762	.76042726									
	2.00	98.529	.75971151									
9	1.05	37.232	.62416523	1.00493587	.22129824							
	1.10	48.662	.79702165	.64749907	.37024043							
	1.20	63.584	.97580547	.39734508	.50758942							
	1.50	90.797	1.18954742	.18131901	.65815951							
	2.00	118.671	1.30723181	.08573850	.73675538							
10	1.05	43.532	.82016598	.82930456	.96031551	.77066475						
	1.10	56.217	1.01639452	.56074317	1.07271847	.77486306						
	1.20	72.785	1.22240758	.35700832	1.18300606	.77771287						
	1.50	103.011	1.47400174	.16845406	1.31073771	.77941700						
	2.00	133.978	1.61517131	.08086844	1.38006177	.77965881						
11	1.10	68.012	.76372231	1.00039779	1.07907273	.97315558	.41449909	.50322215				
	1.20	86.250	1.00380859	.62210512	1.27119133	1.11409856	.26380635	.60395777				
	1.30	99.495	1.15164825	.45424539	1.38627645	1.19021327	.19395631	.65631880				
	1.40	110.274	1.25438559	.35539784	1.46505906	1.23945091	.15213618	.68956165				
	1.50	119.511	1.33033012	.28941233	1.52273844	1.27420447	.12400629	.71275845				
12	1.05	60.682	.61271492	1.41210361	.85568489	1.03217611	.55461224	1.08250671	.78232125			
	1.10	75.915	.83071027	.93039938	1.03429035	1.20208251	.38725977	1.17332733	.78633020			
	1.15	86.884	.97794070	.71342022	1.14920767	1.30239280	.30444016	1.22502833	.78819208			
	1.20	95.804	1.08870640	.58262642	1.23331009	1.37217131	.25207495	1.26030664	.78927375			
	1.25	103.463	1.17651106	.49292858	1.29876573	1.42461285	.21510646	1.28649236	.78997455			
13	1.05	70.851	.50982856	1.77233433	.69843194	.71032755	1.12329294	1.04750584	.97470894	.41929811	.50334251	
	1.10	87.362	.75847320	1.08354370	6.22159562	.90735127	.76413592	1.20712838	1.09312165	.29130825	.58822135	
	1.15	99.251	.61442553	1.22226810	.33204095	1.03459643	.59409420	1.30691469	1.15964168	.22827684	.63434962	
	1.20	108.916	.98082902	.70252943	1.23486736	1.12783660	.48893545	1.37868217	1.20458990	.18858430	.66498587	
	1.25	117.216	1.00526924	.63123440	1.02303626	1.20042416	.41567455	1.43386268	1.23770933	.16065231	.68731108	
14	1.30	135.577	1.20595484	.48673761	1.27205286	1.35808987	.34702784	1.43385489	1.55273738	.13992452	1.34913450	.79859743
	1.10	95.515	.77744097	1.05781091	.86914044	.97225015	.73109263	1.15485592	1.33665431	.28521037	1.24276201	.79449602
	1.15	108.315	.91829314	.81848708	1.01210964	1.11089812	.56954919	1.25773259	1.42001585	.22564156	1.28435377	.79629590
	1.20	118.723	1.03509626	.66632696	1.11855969	1.21324643	.46927764	1.33162755	1.47711300	.18752529	1.31242443	.79737147
	1.25	127.660	1.12876318	.56275967	1.20296568	1.29326328	.39929545	1.38841431	1.51957601	.16041968	1.33309956	.79808757
			C'_8	L'_8	L'_9	C'_{10}	L'_{10}	L'_{11}	C'_{12}	L'_{12}	L'_{13}	C'_{14}

Table B.3 Element Values for A_{max}=0.1 dB, R_2=1.

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
3	2.00	24.010	.89544350	.93758925	.20697105	.89544350						
	2.50	30.518	.94719793	1.01730981	.12048596	.94719793						
	3.00	35.624	.97394258	1.05853951	.07987087	.97394258						
	3.50	39.849	.98964108	1.08274670	.05714258	.98964108						
	4.00	43.461	.99966588	1.09820667	.04302546	.99966588						
4	1.50	23.736	.62815020	.94008825	.40729742	1.24711362	.93517557					
	2.00	36.023	.77554263	1.17645683	.17956964	1.33473267	.93381846					
	2.50	44.686	.83469209	1.27439647	.10528220	1.37217636	.93247199					
	3.00	51.483	.86497711	1.32910437	.07004117	1.39178964	.93166238					
	3.50	57.108	.88267971	1.35490626	.05021282	1.40338454	.93115798					
5	1.10	20.050	.81296401	.92418384	.49338356	1.22445100	.37193315	2.13500483	.29124883			
	1.20	28.303	.91441022	1.06515862	.31627733	1.38201107	.60131012	1.09329178	.52973828			
	1.50	43.415	1.02789353	1.21516579	.15133997	1.63178519	.93525091	.44082677	.81548774			
	2.00	58.901	1.08757749	1.29321767	.07317228	1.79386740	1.14329581	.20038353	.97719848			
	2.50	69.769	1.11065984	1.32351178	.04417252	1.86208958	1.22960502	.11865131	1.04216990			
6	1.05	18.727	.44176873	.71650818	.90905469	.83142031	.36273977	2.44679731	.80462892	.99857002		
	1.10	26.230	.57630316	.88798176	.61281750	.97304469	.59060382	1.35665956	.94304953	1.01381179		
	1.20	36.113	.70984321	1.06265990	.39136323	1.15974373	.87407363	.76185367	1.09176401	1.02461741		
	1.50	54.202	.86594786	1.27402527	.18553914	1.43106134	1.27234592	.33007102	1.28252928	1.03316730		
	2.00	72.761	.95130704	1.39296923	.08925877	1.60131931	1.51865533	.15420752	1.39520906	1.03621085		
7	1.05	30.470	.91937311	1.07659444	.34219880	1.09623111	.40517914	2.20849860	.84335490	.50341888	1.51827206	.41097750
	1.10	39.357	.98820856	1.16726101	.24374471	1.27743352	.59720106	1.35681087	1.04029345	.67880620	.96668581	.58281525
	1.20	50.963	1.05028831	1.24871668	.16123837	1.48377237	.82869406	.81542038	1.28723140	.87427785	.58918102	.75394977
	1.50	72.129	1.11593076	1.33554029	.07856815	1.75686531	1.15173655	.37160105	1.63827136	1.12501680	.26821915	.95587538
	2.00	93.809	1.14909986	1.37978663	.03822299	1.92025806	1.35220621	.17691976	1.85664179	1.27022732	.12694115	1.06720227
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.3 Element Values for A_{max} =0.1 dB, R_2=1.-Continued

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
8	1.05	36.268	.68104997	1.01039691	.47465371	.91102588	.48837977	1.83666542	.67612113	.70499885	1.12618622	.98461035
	1.10	46.399	.77921439	1.13273405	.33839031	1.08667876	.70666682	1.15080970	.86562368	.92697923	.74467799	1.09626758
	1.20	59.639	.87289593	1.25078549	.22404467	1.28896564	.97154115	.69915361	1.09600623	1.17712930	.46698246	1.21321463
	1.50	83.807	.97779398	1.38442896	.10930315	1.56051058	1.34330769	.32098953	1.41679256	1.50420117	.21807041	1.35777091
	2.00	108.575	1.03311201	1.45551074	.05320798	1.72464044	1.57463414	.15328093	1.61419226	1.69717838	.10438669	1.44022288
9	1.05	47.276	1.02597314	1.21654302	.20582587	1.29803041	.60674549	1.36728356	.76114146	.44745135	2.01085827	.94133819
	1.10	58.707	1.07226179	1.27741148	.14772533	1.46402782	.79046466	.92320376	1.00154063	.63574533	1.28473078	1.14956421
	1.20	73.629	1.11294516	1.33139084	.09815156	1.64256692	.99964363	.58857756	1.29049582	.86538232	.78944741	1.39229470
	1.50	100.842	1.15493232	1.38760790	.04799842	1.86765384	1.27611168	.27926819	1.69055284	1.18960193	.36522912	1.72036674
	2.00	128.717	1.17576274	1.41567584	.02338913	1.99761361	1.44055056	.13470811	1.93638157	1.39225351	.17486797	1.91900237
10	1.05	53.576	.82096373	1.17318754	.30389413	1.09173993	.65563207	1.26608362	.64758855	.54490669	1.65322962	.76830038
	1.10	66.262	.89544259	1.26213208	.21932890	1.25855816	.87194859	.85540350	.86007620	.76734193	1.06635893	.95502768
	1.20	82.830	.96460575	1.34441233	.14647584	1.44189130	1.10918360	.54606651	1.11621615	1.03950595	.65893369	1.17010710
	1.50	113.056	1.03982328	1.43372115	.07198802	1.67757996	1.42539713	.25986266	1.47271919	1.42505163	.30602831	1.45794306
	2.00	144.023	1.07857344	1.47976864	.03515585	1.81552436	1.61465486	.12562689	1.69286399	1.66661521	.14673117	1.63109855
11	1.04	60.291	1.07386285	1.27915610	.15150432	1.42500134	.73555327	1.04708046	.81108932	.43027540	2.13873498	.66592789
	1.05	64.086	1.08520352	1.29413331	.13793156	1.46957117	.78596984	.94325043	.87844376	.48125599	1.87467828	.73369940
	1.10	78.057	1.11799983	1.33767657	.09917490	1.61496522	.95602000	.66058397	1.11799858	.66993852	1.22398333	.98224524
	1.20	96.295	1.14642044	1.37569826	.06598114	1.76495881	1.13885373	.43170700	1.39533028	.90030438	.76298216	1.28131338
	1.50	129.556	1.17536511	1.41469570	.03230840	1.94614259	1.36724271	.20853903	1.76838442	1.22605380	.35705497	1.69772917
12	1.04	66.590	.89068857	1.24875505	.23367904	1.20754335	.77952072	1.02044974	.68455842	.52102559	1.76729540	.32650205
	1.05	70.727	.91019288	1.27100977	.21325028	1.25376401	.83538928	.91930385	.74610612	.56352079	1.60213036	.62361208
	1.10	85.960	.96846146	1.33676982	.15445218	1.40682624	1.02535095	.64460436	.96105852	.77998167	1.05241064	.83805721
	1.15	96.929	1.00034058	1.37236567	.12355389	1.50115151	1.14533165	.50855365	1.10510657	.93071714	.80574207	.98409483
	1.20	105.849	1.02155610	1.39593133	.10340421	1.56853602	1.23213467	.42233049	1.21308380	1.04577596	.65783781	1.09480322
13	1.02	64.480	1.08464449	1.29318362	.14109702	1.44706199	.75519367	1.00962286	.78507621	.37949526	2.47707678	.47903396
	1.04	76.411	1.11264353	1.33026524	.10861379	1.56451409	.89042881	.76431213	.96644178	.51470716	1.73720344	.68863023
	1.10	97.407	1.14538590	1.37396009	.07116201	1.72949332	1.08927244	.49368418	1.26118125	.75473928	1.03708914	.97930519
	1.15	109.296	1.15805259	1.39096634	.05672641	1.80336575	1.18104758	.39185256	1.40841835	.88151725	.80328559	1.15380423
	1.50	158.269	1.18733874	1.43051632	.02320897	2.00077878	1.43265773	.15901550	1.84755054	1.27852099	.31420959	1.70255515
14	1.10	105.560	1.01744679	1.38250752	.11481025	1.52679182	1.15245587	.50034271	1.08665822	.85495685	.92185209	.84551206
	1.12	111.066	1.02905821	1.39466916	.10424580	1.56406500	1.19893645	.45304715	1.14745256	.91816724	.82444012	.91227484
	1.14	116.036	1.03860759	1.40463425	.09563099	1.59565089	1.23847577	.41474968	1.20021578	.97341205	.74776348	.97006212
	1.16	120.593	1.04662451	1.41296292	.08841905	1.62289505	1.27269954	.38286298	1.24670410	1.02246389	.68523915	1.02162917
	1.18	124.817	1.05352781	1.42012738	.08225548	1.64683726	1.30284980	.35571775	1.28827742	1.06672939	.63280849	1.06885074
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.3 Element Values for A_{max} = 0.1 dB, R_2 = 1. - Continued

n	w_s	A_{min}	L_8	C_8	C_9	L_{10}	C_{10}	C_{11}	L_{12}	C_{12}	C_{13}	L_{14}
8	1.05	36.268	1.04903322									
	1.10	46.399	1.06140430									
	1.20	59.639	1.07162648									
	1.50	83.807	1.08093020									
	2.00	108.575	1.08484434									
9	1.05	47.276	.74311493	.84407674	.63917703							
	1.10	58.707	.89544349	.57632981	.77014678							
	1.20	73.629	1.05142831	.36876647	.89696801							
	1.50	100.842	1.23693580	.17437248	1.04130049							
	2.00	128.717	1.33886989	.08371246	1.11842315							
10	1.05	53.576	.97996852	.69407065	1.11484452	1.07974229						
	1.10	66.262	1.18233230	.48204408	1.20555196	1.09090170						
	1.20	82.830	1.39629460	.31254842	1.29689403	1.10031786						
	1.50	113.056	1.65959323	.14961593	1.40488638	1.10918875						
	2.00	144.023	1.80819496	.07223578	1.46427466	1.11310134						
11	1.04	60.291	.51399536	1.71271255	1.05046637	.87749715	.61160098	.75492549				
	1.05	64.086	.56563850	1.51724517	1.10791934	.91749844	.55155639	.78827225				
	1.10	78.057	.75094904	1.01741406	1.30573817	1.04322239	.38665974	.89033594				
	1.20	96.295	.96783326	.64522939	1.52509284	1.16560212	.25214974	.98647738				
	1.50	129.556	1.26185417	.30511762	1.80851593	1.30459620	.12111745	1.09275964				
12	1.04	66.590	.64872837	1.36681653	.84927835	1.14054613	.52895686	1.18287085	1.09753560			
	1.05	70.727	.67488641	1.28201862	.89905789	1.19585412	.47870178	1.20649707	1.10103848			
	1.10	85.960	.89906631	.85966108	1.07158490	1.37387187	.33883669	1.28013806	1.11108615			
	1.15	96.929	1.05042112	.66419329	1.18165404	1.47979582	.26794282	1.32259040	1.11630653			
	1.20	105.849	1.16430461	.54479653	1.26187057	1.55382149	.22260602	1.35175639	1.11963807			
13	1.02	64.480	.31207277	3.07334246	.60705320	.50521344	1.75486755	1.05737758	.90498917	.57318418	.77788350	
	1.04	76.411	.15957238	5.77471154	.17436119	.64541763	1.27685250	1.21186483	1.00830334	.43306936	.86239308	
	1.10	97.407	.67676179	1.21436947	1.05351494	.87477325	.79259321	1.44796536	1.14430729	.27827784	.97003462	
	1.15	109.296	.81097810	.92603329	1.23090284	.98952278	.62115572	1.56059913	1.20196830	.22023820	1.01467736	
	1.50	158.269	1.23344692	.35645089	1.73180193	1.33091228	.24667436	1.88135627	1.34753849	.08860178	1.12528226	
14	1.10	105.560	.80323187	1.02384575	.89518932	1.04391991	.68089986	1.17666259	1.51694936	.25131206	1.33150914	1.12574885
	1.12	111.066	.87120924	.91011418	.95658535	1.10772615	.61151471	1.21978127	1.55746435	.22731962	1.34686972	1.12793719
	1.14	116.036	.93017888	.82242469	1.01012776	1.16311319	.55623295	1.25664792	1.59143587	.20788582	1.35965984	1.12972153
	1.16	120.593	.98293879	.75139335	1.05759661	1.21190938	.51079492	1.28869682	1.62048313	.19170671	1.37050498	1.13118176
	1.18	124.817	1.03136433	.69181075	1.10038960	1.25553107	.47251623	1.31704622	1.64581291	.17794366	1.37993192	1.13244098
			C'_8	L'_8	L'_9	C'_{10}	L'_{10}	L'_{11}	C'_{12}	L'_{12}	L'_{13}	C'_{14}

Table B.4 Element Values for $A_{max}=1.0$ dB, $R_2=1.$

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
3	1.50	25.176	1.69200364	.73340029	.48592481	1.69200364						
	2.00	34.454	1.85199437	.85903447	.22589761	1.85199437						
	2.50	40.974	1.91737073	.91047529	.13462370	1.91737073						
	3.00	46.083	1.95107249	.93700445	.09023060	1.95107249						
	3.50	50.308	1.97083914	.95256642	.06495184	1.97083914						
4	1.20	22.293	1.00329315	.77733225	.79633979	1.26620926	1.49217016					
	1.50	34.179	1.25675103	1.11431069	.34361648	1.38981229	1.53225264					
	2.00	46.481	1.40676837	1.32367159	.15959844	1.46761651	1.55071328					
	2.50	55.145	1.46798354	1.41068059	.09511102	1.50012089	1.55742384					
	3.00	61.942	1.49954328	1.45584789	.06375107	1.51702795	1.56072334					
5	1.05	24.135	1.56190840	.67559969	.83448965	1.55459558	.26584305	3.31881494	.88528110			
	1.10	30.471	1.69690703	.77511481	.58827042	1.79892331	.39922112	1.98907077	1.12108891			
	1.20	38.757	1.82812040	.87004759	.38720356	2.09094738	.56346706	1.16671845	1.38093714			
	1.50	53.875	1.97686687	.97693776	.18824450	2.49160619	.79361787	.51949893	1.71889081			
	2.00	69.360	2.05594437	1.03391839	.09152336	2.73567025	.93560978	.24486453	1.91939393			
6	1.05	29.133	1.07457826	.80115928	.81300327	.92735455	.51752931	1.71497668	.92186101	1.60510522		
	1.10	36.680	1.22058610	.94234629	.57746369	1.10900311	.75718495	1.05819367	1.01676074	1.64681871		
	1.20	46.571	1.37146245	1.08633019	.38283573	1.32609930	1.05109597	.63354463	1.12483980	1.68497540		
	1.50	64.661	1.55425476	1.25875817	.18778949	1.62528520	1.46557475	.28655278	1.26960844	1.72481548		
	2.00	83.221	1.65660534	1.35449802	.09179395	1.80860254	1.72376444	.13585851	1.35729054	1.74423596		
7	1.05	40.926	1.82156355	.86343431	.42667905	1.67631767	.34380966	2.60271209	1.23495588	.46778651	1.63392232	1.22362014
	1.10	49.816	1.91040306	.92661670	.30704572	1.93579273	.48016374	1.68752621	1.55276068	.59277280	1.10698790	1.41993642
	1.20	61.422	1.99167568	.98474158	.20446079	2.22803849	.64444166	1.04855733	1.92724069	.73011731	.70551390	1.62538565
	1.50	82.588	2.07881714	1.04761036	.10016218	2.61371526	.87393050	.48972598	2.44020752	.90483490	.33348741	1.87716581
	2.00	104.268	2.12329184	1.07992854	.04883617	2.84446074	1.01638024	.23537647	2.75306016	1.00566798	.16033534	2.01923650
			L'_1	C'_2	L'_2	L'_3	C'_4	L'_4	L'_5	C'_6	L'_6	L'_7

Table B.4 Element Values for A_{max}=1.0 dB, R_2=1.-Continued

n	w_s	A_{min}	C_1	L_2	C_2	C_3	L_4	C_4	C_5	L_6	C_6	C_7
8	1.05	46.727	1.34672931	1.00921631	.47521497	1.08540032	.54692322	1.64006611	.70772605	.86532840	.91752448	1.00315272
	1.10	56.858	1.46597393	1.10092015	.34816897	1.28841831	.75882994	1.07170130	.90030359	1.11345110	.61996524	1.07811215
	1.20	70.098	1.58346426	1.18748347	.23598798	1.52224159	1.01385542	.66997373	1.12752665	1.39664172	.39358608	1.15904175
	1.50	94.266	1.71868703	1.28336608	.11791059	1.83710258	1.36931701	.31489254	1.43764321	1.77189199	.18512514	1.26147193
	2.00	119.034	1.79131333	1.33357708	.05807298	2.02799974	1.58945553	.15185161	1.62645405	1.99560956	.08877630	1.32074601
9	1.05	57.736	1.95471188	.95672496	.26172206	1.94086985	.46950816	1.76694082	1.12604526	.35392435	2.54224174	1.40978663
	1.10	69.167	2.01502980	.99976001	.18875133	2.18047903	.60062001	1.21501105	1.47338873	.48929224	1.66927148	1.71691122
	1.20	84.089	2.06866938	1.03833333	.12585369	2.43021883	.74985343	.78464376	1.88530565	.65376350	1.04498619	2.06810345
	1.50	111.302	2.12469584	1.07895392	.06172922	2.74601368	.94696286	.37633725	2.45079158	.88531248	.49076149	2.53625129
	2.00	139.176	2.15275074	1.09942169	.03011713	2.92871341	1.06414964	.18235570	2.79679638	1.02978932	.23641782	2.81748358
10	1.05	64.036	1.52460886	1.11269973	.32041421	1.31260306	.68238916	1.23499303	.69671194	.59633719	1.51064850	.77126468
	1.10	76.722	1.62236431	1.17191255	.23621390	1.51411394	.87139058	.85595127	.92029145	.82678905	.98968644	.94303286
	1.20	93.289	1.71564193	1.22500046	.16075416	1.73786985	1.08550730	.55797692	1.18808172	1.10906181	.61760804	1.13757407
	1.50	123.515	1.81947148	1.28114072	.08056249	2.02843018	1.36690079	.27098344	1.55972171	1.50975889	.28885814	1.39429420
	2.00	154.482	1.87378205	1.30964026	.03972276	2.19972103	1.53360466	.13226621	1.78907581	1.76136472	.13883803	1.54730497
11	1.04	70.750	2.01485910	.99883458	.19402380	2.11816056	.55627842	1.38452873	1.18896541	.32684894	2.81550564	.97834110
	1.05	74.546	2.02961623	1.00936309	.17684600	2.17997838	.59214627	1.25199876	1.28470583	.36362117	2.48115410	1.07553684
	1.06	77.880	2.04135770	1.01775655	.16327205	2.23179294	.62264717	1.14915603	1.36786730	.39614713	2.23334213	1.16104269
	1.07	80.878	2.05104862	1.02469568	.15212633	2.27643789	.64922604	1.06586644	1.44161592	.42541727	2.03989169	1.23764495
	1.08	83.618	2.05925819	1.03058255	.14272238	2.31566353	.67279323	.99635320	1.50798847	.45208345	1.88325798	1.30716686
12	1.06	84.822	1.66871343	1.17665481	.21588307	1.57196404	.85886598	.86520822	.87158596	.63393907	1.39670727	.69312446
	1.07	88.090	1.68682602	1.18530385	.20216004	1.61449233	.89498715	.80478941	.92242325	.68034428	1.27663778	.73881731
	1.08	91.078	1.70248775	1.19268556	.19046834	1.65220902	.92701201	.75418872	.96829267	.72244007	1.17959044	.78012453
	1.09	93.840	1.71614636	1.19900828	.18033207	1.68602155	.95573090	.71090212	1.01024962	.76118092	1.09867876	.81802494
	1.10	96.419	1.72823995	1.20452785	.17140909	1.71667234	.98176601	.67322120	1.04892187	.79707114	1.02984661	.85304435
13	1.02	74.939	2.02789506	1.00776237	.18105891	2.14528156	.56819489	1.34190013	1.14683413	.28563663	3.29103062	.70027682
	1.03	81.579	2.04954107	1.02318336	.15639853	2.23888561	.62301185	1.15113089	1.29067071	.33969287	2.69734678	.84010680
	1.04	86.870	2.06426822	1.03370332	.13977430	2.30770892	.66414743	1.02472056	1.40301606	.38349153	2.33160569	.95456107
	1.05	91.356	2.07530591	1.04160412	.12738580	2.36232899	.69725138	.93148751	1.49618889	.42077347	2.07549545	1.05265997
	1.06	95.296	2.08405958	1.04788060	.11759849	2.40764654	.72500171	.85834349	1.57617858	.45341722	1.88246204	1.13901711
			L_1'	C_2'	L_2'	L_3'	C_4'	L_4'	L_5'	C_6'	L_6'	L_7'

Table B.4 Element Values for A_{max}=1.0 dB, R_2=1.-Continued

n	w_s	A_{min}	L_3	C_8	C_9	L_{10}	C_{10}	C_{11}	L_{12}	C_{12}	C_{13}
8	1.05	46.727	1.72154045								
	1.10	56.858	1.75960671								
	1.20	70.098	1.79429364								
	1.50	94.266	1.83032968								
	2.00	119.034	1.84787095								
9	1.05	57.736	.63098644	.99407213	1.47896505						
	1.10	69.167	.73764129	.69962295	1.63779449						
	1.20	84.089	.84652436	.45802758	1.79579980						
	1.50	111.302	.97584494	.22102646	1.97957451						
	2.00	139.176	1.04688310	.10706075	2.07916363						
10	1.05	64.036	1.18326686	.57482163	1.06945036	1.79994596					
	1.10	76.722	1.42283761	.40056313	1.12735433	1.83422699					
	1.20	93.289	1.68092206	.25962517	1.18654826	1.86522421					
	1.50	123.515	2.00434516	.12388165	1.25749372	1.89726572					
	2.00	154.482	2.18933318	.05966035	1.29689606	1.91283710					
11	1.04	70.750	.39548388	2.22594737	1.56876937	.72219921	.74311645	1.61591863			
	1.05	74.546	.43266324	1.98355718	1.65217187	.75006424	.67467839	1.65691262			
	1.06	77.880	.46511003	1.80045631	1.72342076	.77301908	.62104556	1.69048475			
	1.07	80.878	.49399047	1.65533134	1.78576792	.79250804	.57728249	1.71885680			
	1.08	83.618	.52005983	1.53634506	1.84126258	.80941343	.54054103	1.74337550			
12	1.06	84.822	.79763531	1.05930128	.90738870	1.50749591	.36213374	1.12650039	1.86468593		
	1.07	88.090	.85013467	.97122788	.94004407	1.55651277	.33572315	1.13625854	1.87157739		
	1.08	91.078	.89765480	.89935092	.96906868	1.59965538	.31361307	1.14480882	1.87754357		
	1.09	93.840	.94122213	.83905553	.99516263	1.63807789	.29472097	1.15231061	1.88269557		
	1.10	96.419	.98146055	.78749199	1.01887413	1.67270090	.27830331	1.15899750	1.88722339		
13	1.02	74.939	.23484688	4.08396516	.84075306	.38671363	2.29260778	1.57708781	.74009505	.70089034	1.64271162
	1.03	81.579	.28654321	3.28038960	.98327565	.44312148	1.92550208	1.70347120	.78193306	.60320570	1.70398396
	1.04	86.870	.32921400	2.79904386	1.09804731	.48737855	1.69088918	1.79926775	.81183383	.53787520	1.74744056
	1.05	91.356	.36599845	2.46882646	1.19527648	.52417755	1.52220544	1.87701123	.83505301	.48942236	1.78101050
	1.06	95.296	.39853546	2.22374660	1.28014366	.55582532	1.39241052	1.94265707	.85398245	.45126748	1.80827180
			C'_8	L'_8	L'_9	C'_{10}	L'_{10}	L'_{11}	C'_{12}	L'_{12}	L'_{13}

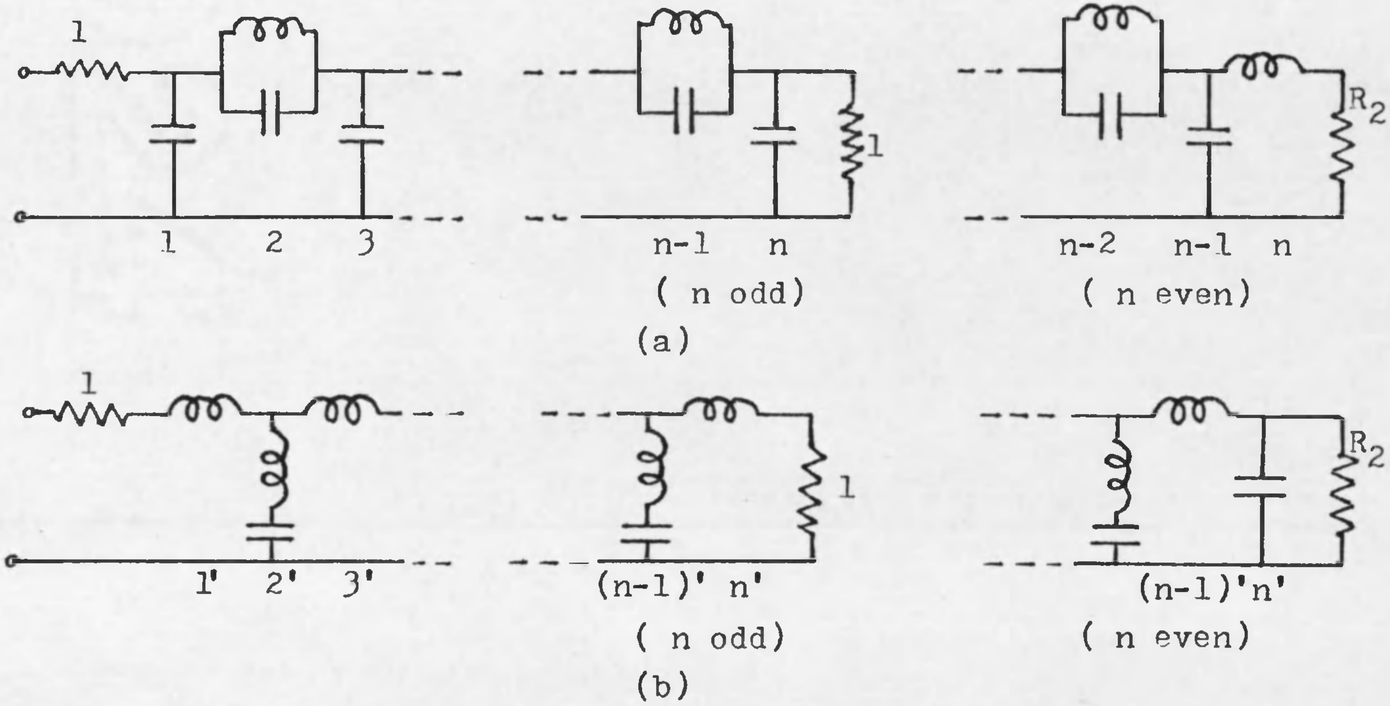


Fig. B.1 Network Configurations for Tables of Appendix B.

a) Direct Realization, b) Dual Network.

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