THERMAL CONDUCTIVITY AND HEAT TRANSFER CHARACTERISTICS
OF INVESTMENT CASTING MOLDS

by
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1977
STATEMENT BY AUTHOR

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SIGNED: [Signature]

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

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Professor of Metallurgical Engineering

[Date]
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This work was made possible with the help of many people. My advisor, Dr. G. H. Geiger, whose patience and guidance were invaluable; and the people at Desert Crucible, Tucson, who are trying to make a living at investment casting. Of course my family, but most particularly my wife and her calming spirit. And finally all the guys down at the Department who assisted me in my various casting exploits.
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ABSTRACT

The first part of this work offers values for the density and thermal conductivity of silica investment materials as well as silica materials with zircon added. Typically the values range from 0.0010-0.0022 cal/sec cm °C for the silica, and 0.0010 cal/sec cm °C for the silica materials containing zircon. The densities for all materials were about 1.7-1.9 g/cc.

The second part develops and analyzes a computer program designed to solve for a temperature profile in an investment wall considering mold preheat and metal superheat. It was found that increasing the superheat and preheat extends solidification times and that mold wall thickness determines the means by which heat is removed (i.e., radiation, convection). Also it was found that the silica plus zircon mold material apparently extends the solidification time of a casting due to the attenuation of radiation through the wall.
CHAPTER I

INTRODUCTION

Casting of metals into exotic shapes has become an integral part of industry in the world. One such procedure for obtaining these shapes is called the shell investment casting process, a process which is versatile enough to produce parts with close tolerances utilizing various casting materials.

A wax replica of the final product is produced by injection of wax into a latex or metal mold, and then this pattern is mounted on a sprue by means of a runner (Fig. 1). The entire wax assembly is then dipped into a slurry of a liquid silicate and finely divided ceramic particles. This layer is allowed to dry. Then the assembly is again dipped into the slurry, but now and in each subsequent dipping operation, granular ceramic stucco is sprinkled on the assembly. The granular stucco is larger than the particles used in the slurry, so these larger particles act as strengthening agents. The initial layer of binder, besides binding the finely divided ceramic particles together, reproduces the surface of the wax figure with great precision. This high degree of surface replication is often as good as the surface that is obtained in machining.
1. Procedure of process

(1) The wax holding tree
(2) Wax patterns are mounted on tree
(3) Pattern is dipped in slurry
(4) Wet slurry is coated with stucco; steps 3 and 4 are repeated to obtain the desired thickness
(5) Wax is burned out to obtain pouring cavity
(6) Hot metal is cast into mold
(7) The mold is broken away from the casting
(8) The workpiece is removed from sprue

Fig. 1. Procedure of process.
The ceramic stucco coating is built up on the wax until it is almost one centimeter thick, whereupon it is considered to be strong enough to support the metal casting. The entire shell is then fired to remove the wax, and also to strengthen the shell by diffusion bonding of the ceramic.

The shell is either cooled before casting or else the hot metal is poured while the mold is still hot. After the casting is solid, the shell is broken away and the part is cut off the runner and sprue, and cleaned.

The purpose of this thesis was to study the rate of heat extraction from a shell investment casting. For example, if one can predict the heat transfer through a given shell investment mold wall, then the time to complete solidification of the metal can be determined. These predictions are of particular value when designing the gating and risering system for a given casting.

To complete this project the thermal conductivity measurements had to be made on the ceramic shell. These measurements, along with the porosity and density of the shell are crucial to the prediction of the heat flow in the wall. With these properties, a mathematical model was made to predict freezing times of simple cylindrical castings.

The thesis is divided into two parts: (a) the property evaluation section which included the theory and experimental results pertaining to the thermal conductivity
measurements and (b) the heat transfer results including the computer simulation of the heat flow in the wall and the results of solidification experiments used to verify the computer simulation.
CHAPTER II

THE THERMAL CONDUCTIVITY OF SHELL INVESTMENT MATERIALS

Available information on the thermal properties of investment mold materials is very scarce. One possible reason is the use of such a wide variety of processes and materials that employ the general investment casting concept. Another reason is the experimental difficulties in finding these data.

Literature Review

Various authors [1, 2] have noted the importance of the thermal conductivity ($k$), the density ($\rho$), and the heat capacity ($C_p$). In most heat transfer evaluations for investment walls, the thermal conductivity is assumed to be constant and similar in magnitude to that of ordinary sand molds. Swaminathan et al. [3] offers some thermal conductivity values for silica, zircon, and magnesite base investment materials. Unfortunately, their work is only carried out to 800°C, which is just at the point where radiation becomes an important constituent to thermal conduction. Also, very little documentation as to their technique is given. Other work on the thermal conductivity of investment casting materials was not found; however, various authors [4, 5, 6] have measured $k$ of packed beds of similar
materials and while packings of these materials have continuous pores in them, they have similar attributes to investment walls which makes them a good starting point for an investigation of the thermal conductivity of investment walls. The results of these studies are found in Figs. 2 and 3.

One important attribute found in both the packed beds and investment walls is the degree of porosity. However, porosity in a packed bed is considered to be continuous, while the porosity in an investment wall appears to be semi-continuous at best (Fig. 4). This distinction is an important one when looking at the various theories describing the thermal conductivities of porous materials.

Extensive theoretical analysis of the thermal conductivity of porous materials has been done. However, only a modest amount has been done toward predicting the thermal conductivity at elevated temperatures. The general approach to preparing a mathematical model is to relate the thermal properties of the major constituents (solid phase, gas phase) and the porosity of the material with the effective thermal conductivity of the porous material. One major distinction between the various models depends on whether the porosity is assumed to be continuous or discontinuous. Since the investment material appears to be semi-continuous, we will investigate models that are used for both isolated porosity and continuous porosity.
Fig. 2. Thermal conductivity of a packed bed.
Fig. 3. Thermal conductivity of investment materials.
Fig. 4. Comparison of a packed bed and an investment sample.
A model for thermal conductivity of materials with isolated porosity was developed by Maxwell and later enhanced by Eucken [7]. The result was the Maxwell-Eucken equation:

\[
k_{\text{min}} = k_c \left( \frac{1}{1+2V_d \frac{k_c/k_d}{2k_c/k_d+1}} \right) \frac{1-k_c/k_d}{1-V_d \frac{k_c/k_d}{1+k_c/k_d+1}}
\] (1)

or if \(k_c \gg k_d\)

\[
k_{\text{min}} \approx k_c \left( \frac{1-V_d}{1+V_d/2} \right)
\] (2)

where: \(V_d = \) void fraction.

\(k_c\) = thermal conductivity of the continuous phase.

\(k_d\) = thermal conductivity of the discontinuous phase (all units in cgs, unless otherwise noted).

This work does not consider radiation and its effects at high temperature. Saegusa et al. [8] modified this equation to account for radiation, using the altered form of the Maxwell-Eucken equation:

\[
k_{\text{eff}} = k_s \left( \frac{1+2\omega(1-K)/(2K+1)}{1-\omega(1-K)/(2K+1)} \right)
\] (3)

\[
K = k_s/k_g e^{-0.2\text{Nu}}
\] (4)

\[
\text{Nu} = \frac{2rhr}{k_g}
\] (5)
where: \( r \) = radius of the void.

\( h \) = radiative heat transfer coefficient.

\( T^3 \) = cube of the mean temperature in the void, °K.

This evaluation assumes that there is no convection in the void because the void is so small. Loeb [9] offered another analysis in which he looked at pores oriented in specific directions to the flow of heat. By relating the porosity, direction of orientation and the contribution of radiation at elevated temperatures, Loeb offers this equation:

\[
keff = k_c \left( 1 - \frac{4\gamma e\sigma T^3}{k_c} \right) \left( 1 + \frac{4\gamma e\sigma T^3}{k_c} \right)^{-1} \frac{1-P_L}{P_L} \]

where: \( P_c \) = fraction of pores of a cross-sectional area cut perpendicular to the direction of heat flow.

\( P_L \) = fraction occupied by pores of length of a line of heat flow that passes through pores.

\( \gamma \) = geometrical pore factor (which is a function of orientation and size of the pores).

\( k_c \) = thermal conductivity of the continuous phase.

\( d \) = dimension of pore in direction of heat flow.

\( T \) = average temperature, °K.

Unfortunately, Loeb's work is impractical for randomly oriented grains in an investment wall. In Fig. 5, predicted...
Fig. 5. Thermal conductivity of PIW flour using various theoretical studies.
values are presented for the same material using all three models. Even taking the porosity of Pl and Pc as the same, Loeb's model results in a curve that is shifted above those based on the Maxwell-Eucken or Saegusa equation.

The other area of interest to workers in this field is the solution to the thermal conductivity of a packed bed. The packed bed is at the other extreme; the pores here are considered continuous. Schotte [10] offers an evaluation that is as complete as any that have been published. He begins with the gas phase in the bed. The thermal conductivity of a gas is known to vary with the mean free path of the gas, which varies with the temperature. As the temperature and mean free path increase, so does the thermal conductivity. However, as the effective pore size decreases to the same order of magnitude of the mean free path, the true thermal conductivity of the gas decreases. This condition occurs when the particle size, $D_p$, is about 1000 times the mean free path. Deissler and Eian [11] have developed a criterion for determining if this condition exists by introducing the break away pressure of the gas in the pores:

$$P_B = 1.77 \times 10^{-21} \frac{T}{D_p d^2}$$ (8)
where: \( D_p \) = particle diameter, ft.

\( d \) = gas molecule size, ft.

\( T \) = absolute temperature, R°.

When the break-away pressure is less than the actual pressure, the thermal conductivity of the gas is given by:

\[
\frac{k_g^0}{d^2} = \frac{1}{2} \sqrt{\frac{K_B T}{3 M}} \text{ BTU/hr-°F-ft} \tag{9}
\]

Otherwise a correction factor is needed, resulting in:

\[
k_g = \frac{k_g^0}{1 + 2.03 \times 10^{-22} \left( \frac{C_p}{C_v} \right)^2 \left( \frac{1 - \omega}{\omega} \right) \left( \frac{T_{kg}}{P_B d^2 C_p \eta} \right)} \text{ BTU/hr-°F-ft} \tag{10}
\]

where: \( k_g^0 \) = the thermal conductivity of the gas in normal conditions, BTU/hr-°F-ft.

\( P_B \) = break-away pressure, lbs/ft².

\( \eta \) = viscosity of the gas, lb/ft-hr.

After finding the thermal conductivity of the gas by Equation (9) or (10), the next step is to solve for the conductivity of the bed using Fig. 6 and the ratio of the intrinsic solid phase thermal conductivity and to the gas phase thermal conductivity. The bed thermal conductivity is then found directly.

For higher temperatures, the radiative contribution must be considered. Schotte [10] shows that the radiation heat transfer across the void is given by
Fig. 6. Deissler and Eian's [11] correlation of $\frac{k_s}{k_g}$ vs. $\frac{K_B}{k_g}$. 
where:

\[ k_r^0 = 0.692 \varepsilon \frac{D}{P} \frac{T^3}{10^8} \text{ BTU/hr-°F-ft} \]

and \( \omega \) is the void fraction. Finally, adding \( k_r \) and \( k_b \) the effective thermal conductivity of the bed is obtained.

Using the information in Table 1, the effective thermal conductivity of beds made up of these materials was calculated using Schotte's [10] work. When making the calculations for the P-3 material (Sillimanite) the thermal conductivity of the solid was assumed higher than the listed values for silica, but lower than those for alumina (since it is an alumina silicate). Other pertinent data were the void fraction = 0.4, and the pore size that was assumed to be one-half the particle size. Results are shown in Fig. 7.

**Basis for the Experimental Technique**

Measurement of the thermal conductivity of the investment wall was carried out by a method developed by Haupin [12], using a variation of the hot wire technique. The hot wire technique employs the use of a heating wire placed in a specimen, and then at some distance from the hot wire, a measuring thermocouple is placed. Since the heat generated by the hot wire is extracted from the area at a rate related to the material's thermal conductivity,
Table 1. Materials used for thermal conductivity study.

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Product No.</th>
<th>Size Distribution</th>
<th>Particle Dia.</th>
<th>Chemical Analysis</th>
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<tbody>
<tr>
<td>Nalcast</td>
<td>PIW</td>
<td>100% + 200</td>
<td>0.0017&quot;</td>
<td>100% SiO₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>75% + 325 mesh</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30% 10 μ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nalcast</td>
<td>S-1</td>
<td>Less than 8% 50 mesh</td>
<td>0.00467&quot;</td>
<td>100% SiO₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>89% 140 mesh</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3% + 200 mesh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nalcast</td>
<td>S-2</td>
<td>6% 20 mesh</td>
<td>0.00149&quot;</td>
<td>100% SiO₂</td>
</tr>
<tr>
<td></td>
<td></td>
<td>86% 50 mesh</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>8% 70 mesh</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ione Grain</td>
<td>S-3</td>
<td>4% 14 mesh</td>
<td>0.0323&quot;</td>
<td>SiO₂ 53.5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60% 20 mesh</td>
<td></td>
<td>Al₂O₃ 43.8%</td>
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<tr>
<td></td>
<td></td>
<td>36% 28 mesh</td>
<td></td>
<td>TiO₂ 2.3%</td>
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<td></td>
<td></td>
<td>1% + 28 mesh</td>
<td></td>
<td>Fe₂O₃ 0.5%</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CaO 0.1%</td>
</tr>
<tr>
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<td>MgO 0.1%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>K₂O-Na₂O 0.2%</td>
</tr>
<tr>
<td>N.L. Industries</td>
<td>G</td>
<td>85% r/μ</td>
<td>0.0013&quot;</td>
<td>97% ZrSiO₄</td>
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<tr>
<td>Zircon</td>
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<td>13% + 325 mesh</td>
<td></td>
<td>0.29 TiO₂</td>
</tr>
<tr>
<td>Milled coarse</td>
<td></td>
<td>2% + 200 mesh</td>
<td></td>
<td>0.05% Fe₂O₃</td>
</tr>
<tr>
<td>&amp; Fine Grind</td>
<td></td>
<td></td>
<td></td>
<td>0.35% Al₂O₃</td>
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Fig. 7. Thermal conductivity of silica investment material according to Schotte's [10] theory.
one can arrive at a thermal conductivity by measuring the rate of heat loss from the area. This is done with a thermocouple placed some distance away from the hot wire.

In Haupin's [12] variation of this technique, he uses the thermocouple itself as the hot wire. The hot wire method is based on the transient temperature rise for a line heat source at a constant rate of energy input. By impressing an AC current on the thermocouple/hot wire, one wire is heated. The heat is removed from the wire by conduction through the material surrounding the wire. A DC voltage is generated at the thermocouple junction. This results in a shift of the AC waveform which is related to the DC voltage. The DC voltage is obtained after correct electronic filtering of the signal. Since the DC voltage is related to the temperature, the heat flux away from the hot wire can be calculated from the rate of change of the wire temperature, and from this the thermal conductivity. The equation resulting from analysis of the situation is:

\[
k = \frac{\theta_2 - \theta_0}{\ln(\frac{\theta_2}{\theta_1})} \cdot \frac{Q \ln(\frac{\theta_2}{\theta_1})}{2(T_1 - T_2)}
\]

where: \(k\) = thermal conductivity, cal/sec-cm-\(^\circ\)C
\(T_1, T_2\) = temperatures at times \(\theta_1\) and \(\theta_2\), \(^\circ\)C
\(Q\) = power input, cal/sec-cm
\(\theta_1\) = initial time, sec
\(\theta_2\) = later time, sec
\( \theta_2 \) = later time, sec
\( \theta_0 \) = correction factor for wire diameter.

The correction, \( \theta_0 \), can be ignored for very thin sources and is taken to be zero for wires less than 1 mm in diameter.

Since the heat input must be constant, it is important that both wires used for the thermocouple be of the same resistance. Varying the diameter of the wires used solves this problem. A 0.5 mm Alumel wire and a 0.8 mm Chromel wire will have nearly equal resistances per unit length. Since the actual temperature change at the point of detection is only a few degrees, the technique will not change the temperature of the furnace. However, thermal fluctuations caused by furnace cycling can be a problem and it helps to pack the sample in a high heat capacity material.

The calculations are simple. The wire must first be calibrated for its specific resistance and its thermocouple calibration. The calibration constant is:

\[
C = 0.002595 \frac{R}{f}
\]  \hspace{1cm} (13)

where \( f \) = thermocouple calibration, \( ^\circ C/mV \); and \( R \) = resistivity of wire, ohms/cm\(^2\). The thermal conductivity then is found from:

\[
k = \frac{I^2C}{\Delta E}
\]  \hspace{1cm} (14)

where: \( I \) = current used, amps

\( k \) = thermal conductivity, cal/sec-cm-\( ^\circ C \)
C = calibration constant, and

\[ E_{1-10} = \text{mV change over a ten-minute interval, mV.} \]

Testing entails measuring the change in mV at the thermocouple junction by taking readings over a ten-minute period. A typical plot generated by the emf of the thermocouple is shown in Fig. 8. Over the plateau region, the rise in mV should be linear when fit to a logarithmic plot. By taking readings at intervals when the plateau is reached, a logarithmic graph can be made and accurate readings of millivoltages at the first and tenth minutes can be made. A least square logarithmic fit is made to obtain an even more accurate mV difference. For the apparatus used, an error analysis of Equation (14) is found in Appendix A.

**Experimental Procedure**

To apply Haupin's work to investment casting mold materials, it was felt that the orientation of the layers of investment material would have to be kept perpendicular to the flow of heat. Therefore by repeatedly dipping a wire in a slurry of investment material, as a candle is repeatedly dipped in wax, and appropriately applying the stucco material on the outside, the specimen was built up to the correct thickness required for the hot wire technique. For this situation, the sample had to have a radius of 2.5 cm (1 in) and a half-length of 7.6 cm axially (3 in) from the thermocouple junction. The end result of the sample
Fig. 8. Typical mV response of the thermocouple over 10 minute period.
preparation was a cylindrical shape 5,1 cm (2 in) in diameter, and 17.8-20.3 cm (8-10 in) long. The excess liquid in the sample was then driven off in a drying oven at about 80-100°C.

The stucco materials are described in Table 1, and the formulations used in the slurries are given in Table 2. Samples with the dimensions described above were prepared for thermal conductivity testing from the following material combinations:

- PIW
- PIW + S1
- PIW + S2
- PIW + S3

Table 2. Slurry formulations for thermal conductivity studies.

<table>
<thead>
<tr>
<th>Slurry</th>
<th>Formula</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PIW</td>
<td>Nalcoag 1030 29 w/o</td>
<td>29 lbs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Nalcast PIW 71 w/o</td>
<td>71 lbs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ultrawet --</td>
<td>5 ml</td>
<td></td>
</tr>
<tr>
<td>H-2</td>
<td>H-2 Binder 20 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>&quot;G&quot; Fine Grind 64 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Bircon)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fused silica 16 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(PIW)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H-4</td>
<td>H-4 Binder 28.6 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>&quot;G&quot; Fine Grind 14.3 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(zircon)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fused Silica 57.1 w/o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(PIW)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The specimens were then cut in half using a high speed rock saw. The wire was removed so that the specimens had a small groove in the face (see Fig. 9a). The halved samples were then fired to 1000°C for four hours to drive off any volatiles and to bond the stucco.

The test thermocouple was made from 0.8 mm (0.032 in) Chromel wire and 0.5 mm (0.020 in) Alumel wire. The thermocouple was to be placed in between the sample halves, so the wires were cut to the required lengths as shown in Fig. 9b. Leaving 20-30 cm of wire exposed, the remainder of the wire was placed in ceramic protection tubes. After bending the thermocouple wires as shown in Fig. 9b, they were butt welded using a capacitive discharge welder.

To insure that the surroundings about the sample were of high heat capacity material, two techniques were used. The first employed a castable material placed about the samples. After the sample was cut in half, each half was mounted in the castable alumina. The castable was allowed to dry, and then the two halves were put back together with the thermocouple wire in the center. Later it was found that the sample could simply be placed in a pan and covered with silica sand. The sample halves with the thermocouple wire in the center were held together by stainless steel hose clamps (Fig. 9c). Both techniques yielded the same results and it was felt that the latter would be satisfactory.
Fig. 9. Schematic of testing procedure.
Fig. 9.—Continued Schematic of testing procedure.

(b) Thermocouple configuration
(c) Sample preparation

Fig. 9.—Continued Schematic of testing procedure.
The samples were tested at room temperature, 200, 500, 800, 1000, and 1100°C. The furnace temperature was allowed to stabilize until the specimen's temperature was varying less than ± 0.5°C. This was accomplished by letting the sample stay at the elevated temperature for 1-2 hours after the furnace began cycling at the test temperature.

Constant AC was delivered to the wire by way of the thermocouple leads. The DC voltage was also obtained from these same leads. The electronic circuit used to obtain this voltage is shown in Fig. 10. The AC was filtered out so that the DC signal could be read on a digital voltmeter. Readings were taken at one minute intervals, starting as soon as the AC was impressed on the thermocouple wire. The AC source initially had to be adjusted slightly to insure that the current was constant and did not change with time.

After the readings had been taken, a least squares analysis was made to obtain the coefficients of the equation

\[ \log t = a(mV) + b \]

This resulted in the most accurate reading of the change in mV for a given test. The thermal conductivity was then obtained using Equation (14), and the error associated with that specific test was also computed using the analysis in Appendix A. Calibrations were also run at various
Fig. 10. Circuit diagram of the equipment used for the thermal conductivity apparatus.
temperatures on materials with known thermal conductivities and the results are given in Table 3.

**Density of the Materials**

It is known that the porosity is related to the bulk density of a porous material by the equation:

\[
\omega = 1 - \frac{\rho_B}{\rho}
\]

where: 
\(\rho_B\) = the density of the bulk porous material, g/cm\(^3\)
\(\rho\) = the density of the non-porous sample, g/cm\(^3\)
\(\omega\) = the porosity of the porous material.

This porosity (void fraction) is important to the solution of the various models that have been proposed for describing the thermal conductivity of a porous sample. The density of the porous sample is important to the solution of the heat flow in the wall. The actual measurement of the density was done using small samples which were weighed and then waterproofed so that their displacement in water could be measured. The bulk density of the materials then was the weight divided by the displacement. Ten samples of each type of mold material were tested in this manner after the sample had been fired and thoroughly dried out. The results are in Table 4.

**Thermal Conductivity and Density Results**

The results of the thermal conductivity measurements are presented in Figs. 11 and 12, the former for the silica
Table 3. Results of thermal conductivities of known materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Temperature</th>
<th>( k(\text{cal/sec-cm-°C}) )^a</th>
<th>( k(\text{manufacturer}) )^b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiberfrax</td>
<td>200°F</td>
<td>0.0023</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>400°F</td>
<td>0.0024</td>
<td>0.0027</td>
</tr>
<tr>
<td></td>
<td>600°F</td>
<td>0.0020</td>
<td>0.0030</td>
</tr>
<tr>
<td>HW-30</td>
<td>200°F</td>
<td>0.0084</td>
<td>0.0091</td>
</tr>
<tr>
<td></td>
<td>400°F</td>
<td>0.0094</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>800°F</td>
<td>0.0106</td>
<td>0.0109</td>
</tr>
<tr>
<td></td>
<td>1200°F</td>
<td>0.0119</td>
<td>0.0122</td>
</tr>
<tr>
<td></td>
<td>1600°F</td>
<td>0.0130</td>
<td>0.0134</td>
</tr>
</tbody>
</table>

^a**Averaging of 5-10 test runs.**

^b**Modern Refractory Practice, Harbison-Walker Co. [13], William Feather Pub. Co., Cleveland, Ohio, 1961.**
Table 4. Density results.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cc)</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIW</td>
<td>1.73 ±0.08</td>
<td>0.40</td>
</tr>
<tr>
<td>PIW + S1</td>
<td>1.72 ±0.09</td>
<td>0.40</td>
</tr>
<tr>
<td>PIW + S2</td>
<td>1.65 ±0.12</td>
<td>0.42</td>
</tr>
<tr>
<td>PIW + S3</td>
<td>1.92 ±0.19</td>
<td>0.33</td>
</tr>
<tr>
<td>H2 + S2</td>
<td>1.93 ±0.10</td>
<td>--a</td>
</tr>
<tr>
<td>H4 + S2</td>
<td>1.84 ±0.13</td>
<td>--a</td>
</tr>
</tbody>
</table>

*Due to this material's varied makeup a porosity factor is unable to be determined.*
Fig. 11. Experimental thermal conductivity results on silica materials.
Fig. 12. Experimental thermal conductivity results of silica/zircon materials.
materials, and the latter for the materials containing zircon in the slurry. The mean values (those graphed) and their corresponding standard deviations are listed in Tables 5 and 6. It was found that the standard deviation about the mean of many tests was greater than the error that was associated with any one test, so this standard deviation was the dominant error. The results of the density and porosity determinations were given in Table 4.

Silica Materials (PlW, P1W+S1, S2, S3)

Comparing Figs. 7 and 11, one sees that the experimental results are higher than the predicted values of Schotte [10]. This is caused by the increase in the continuity of the solid phase by the bridging between particles by the silica binder. The results are as would be expected of a silica material, with a noticeable increase in thermal conductivity above 800°C due to the increase in the photon contribution to the thermal conductivity.

Figure 13 compares the thermal conductivity of pure silica, the experimental results, and theoretical results. Only the PlW results are shown on this figure, and for the theoretical results only those for the PlW flour. The experimental values are lower than those of the pure silica because of the porosity in the sample. However, this porosity is not great enough to make the sample the same as a true packed bed, with only
Table 5. Thermal conductivity of silica materials.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Temp (°C)</th>
<th>k cal/sec cm °C</th>
<th>± k (st. dev.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIW</td>
<td>25</td>
<td>0.0023</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0016</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0016</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0015</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0018</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>0.0020</td>
<td>0.0002</td>
</tr>
<tr>
<td>PIW + S1</td>
<td>25</td>
<td>0.0018</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0013</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0015</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0011</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0012</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>0.0013</td>
<td>0.0001</td>
</tr>
<tr>
<td>PIW + S2</td>
<td>25</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0017</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0019</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0018</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td>PIW + S3</td>
<td>25</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0011</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0014</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0017</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
Table 6. Thermal conductivity of samples containing zircon flour.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Temp (°C)</th>
<th>k cal/sec cm °C</th>
<th>+k</th>
</tr>
</thead>
<tbody>
<tr>
<td>H2-S2</td>
<td>25</td>
<td>0.0013</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0009</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0011</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0011</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0011</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>0.0013</td>
<td>0.0003</td>
</tr>
<tr>
<td>H4-S2</td>
<td>25</td>
<td>0.0012</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.0015</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.0017</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>0.0017</td>
<td>0.0004</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>0.0012</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>1100</td>
<td>0.0011</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
Fig. 13. Comparison of solid silica, theoretical packed bed results, and present experimental work.
point contact between particles. Thus, the results lie where they would be expected.

The porosity of the samples is found in two forms: macroporosity and microporosity. The microporosity is associated with the PlW slurry material, whereas the macroporosity is related to the packing of the stucco material. Figure 14 shows micrographs of these materials, with these two forms of porosity present. This bi-modal pore size distribution is not taken into account in any theoretical models for prediction of thermal conductivity.

Figure 15 shows the data for the predicted values as well as the experimental data of the thermal conductivity for the PlW materials. The predicted values were based on the data for the PlW flour only. The experimental results are significantly higher due to the particle-to-particle bridging. Figure 16 shows the same type of results for the PlW+S2 material. The predicted values were based on data for the S2 grit alone.

H2+S2

The thermal conductivity of this material is shown on Fig. 12. The weight ratio of zircon to silica in the slurry is 4:1, and the slurry was made up according to the formula in Table 1. After 1000°C, the effect of the lower inherent thermal conductivity of the zircon negates any increase in the bulk thermal conductivity due to photon
Fig. 14. Micrographs of investment materials.
Fig. 14.—Continued Micrographs of investment materials.
Fig. 14.—Continued  Micrographs of investment materials.
Theoretical results

Fig. 15. Comparison of experimental results and theoretical results for thermal conductivity of PIW material.
Fig. 16. Comparison of experimental results and theoretical results for the thermal conductivity of PIW + S2 material.
compaction. The projection above 1100°C was made by considering that since the thermal conductivity of the zircon does not increase with temperature as does that of silica, the rate of increase of thermal conductivity with temperature will not be as fast as that of pure silical material. The density is given as $1.93 \pm 0.10 \text{ g/cm}^3$, however the porosity could not be evaluated since this was a composite material. The sample is pictured in Fig. 14, and appears to have a pore size on the microscopic scale similar in structure to that of the PlW+S1 material.

H4+S2

This material's thermal conductivity is shown on Fig. 12. The effect of the lower concentration of zircon in the slurry (a ratio of 1:3, zircon flour to silica flour) is seen in the higher temperature region of the graph as the thermal conductivity of this material increases at temperatures above 800°C. The structure of the material is shown in Fig. 14, and it can be seen to be similar to the H2+S2 material.
CHAPTER III

HEAT TRANSFER IN INVESTMENT SHELL MOLDS

The analysis of heat transfer in a thin walled mold has received very little attention in the literature. Roshan et al. [14] offered an analysis of a thin wall mold; however, they made no provisions for metal superheat, preheated mold, or the change of thermal conductivity of the mold with temperature. Ball [15] has also generated a program to describe the casting situation for a backed up shell mold. While he published some results in his paper, details of the program are not given, except that it was a finite difference approach. Consequently, a solution to the heat transfer problem in thin wall shells including the conditions of the mold preheat, metal superheat, and variation of mold thermal conductivity with temperature is required.

**Heat Transfer Model**

The model developed is for a cylindrical casting long enough to be considered infinite in length, with a mold wall made up of layered materials, finite in thickness. The various radius designations are given in Fig. 17.

To begin the analysis, the temperature profile in the wall has to be determined so that the rate at which heat
Fig. 17. Wall dimensions for computer program.
leaves the casting can be determined. To obtain the temperature profile in the wall, the solution of the general energy equation must be found. The equation, in cylindrical coordinates, is used with the assumption that the heat flow will only be in the radial direction. The equation considered is:

\[
\frac{\partial T}{\partial t} = \alpha \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial T}{\partial r})
\]  

(15)

within the limits \( r_i < r < r_o \) where

- \( r \) = the radius in the wall from the center of the casting
- \( T \) = the temperature at \( r \) and \( t \)
- \( t \) = time
- \( \alpha \) = thermal diffusivity of the mold. The initial conditions and the boundary conditions must also be defined.

Initial Conditions

The initial conditions of the wall are first considered. The wall is at some preheated temperature defined by:

\[
T(r^*t) = \text{preheat temperature}
\]  

(16)

for the range: \( r_i > r^* > r_o \). The other initial condition is the temperature first seen by the inside surface of the wall, which is the temperature of the superheated metal.
As the solidification begins, the wall's inside temperature and outside temperature vary according to various boundary conditions. The inside boundary condition will be considered first.

Inside Boundary Conditions

Within the melt itself, it is assumed that there will be no temperature gradient. This approximation is reasonable if a small radius of cast metal is considered and if the ratio of the thermal conductivities of the metal to that of the mold are high. (A more complete analysis of this assumption is given in Appendix B.)

The inside surface of the wall (i.e., r* = r_i) absorbs heat from the metal provided by one of the following sources of heat: (a) superheat, (b) latent heat of solidification, (c) sensible heat from the solid across the shrinkage gap.

Superheat Removal. The superheat is available when the hot metal is above its fusion temperature. The amount of energy that is given off over a specific amount of time is:

\[ Q = C_p' V' \rho' \frac{dT}{dt} \]  

(17)

where \( Q \) = the rate of heat given up by the metal, cal/sec
\( C_p' = \) specific heat of the metal, cal/g-°C
\( V' = \) volume of metal, cm³
\[ \rho' = \text{density of metal, gm/cm}^3 \]
\[ \frac{dT}{dt} = \text{rate of change of metal temperature, °C/sec.} \]

This heat given up by the metal must equal the heat that is absorbed by the wall;

\[
\frac{C PV}{A} \frac{dT}{dt} = -k \left( \frac{dT}{dr} \right) \bigg|_{r=r_i}
\]

where \( A = \text{area of the metal-mold interface, cm}^2 \)

\( k = \text{thermal conductivity of the wall at the inside surface, cal/cm-sec-°C} \)

\( \left( \frac{dT}{dr} \right) \bigg|_{r=r_i} = \text{the change of temperature with respect to the wall radius in the mold, °C/cm.} \)

This assumes little or no resistance to heat flow across the metal-mold interface \((h = \infty)\). The method of handling this boundary condition and computing the temperature that is on the inside surface of the mold is given in Appendix B.

This condition exists until the temperature of the metal equals the fusion temperature of the metal; at that point the second boundary condition comes into play.

**Latent Heat Removal.** The latent heat is evolved during the solidification period. During this time the temperature of the inside mold wall is equal to the solidification temperature. It is required to have an expression to determine when the solidification is complete, however.
To accomplish this, the analysis looks at the change in the solid metal/liquid metal interface.

The heat evolved over the solidification period is:

$$Q = \frac{L \rho' \Delta V'}{\Delta A' \Delta t}$$  \hspace{1cm} (19)

where:  
- $L$ = latent heat of fusion, cal/g
- $\rho'$ = density of the metal, g/cm$^3$
- $\Delta t$ = increment of solidification time, sec
- $\Delta V'$ = change in liquid metal volume over $\Delta t$ period, cm$^3$
- $\Delta A'$ = change in liquid metal/solid metal interfacial area over $\Delta t$ period, cm$^2$

By equating this heat to the heat that is absorbed by the wall, we obtain

$$\frac{L \rho' \Delta V'}{\Delta t \Delta A'} = -k \left( \frac{dT}{dr} \right) \bigg|_{r=r_1}$$  \hspace{1cm} (20)

where:  
- $k$ = thermal conductivity of inside surface of mold wall, cal/cm·sec·°C

$$\left( \frac{dT}{dr} \right) \bigg|_{r=r_1} = \text{change in temperature with respect to the wall radius in the mold } °C/cm$$

The solid metal/liquid metal interface is moving during this solidification process toward the center of the casting. Therefore the $\frac{\Delta V'}{\Delta A'}$ ratio can be expressed as:

$$\frac{\Delta V'}{\Delta A'} = \frac{\pi \Delta r' h}{2 \pi \Delta r' h} = \frac{\Delta r'}{2}$$  \hspace{1cm} (21)
where: \( \Delta r' \) = radius of resulting solid metal/liquid metal interface, cm

\( h \) = unit height of respective casting, cm.

Substituting this into Equation (21), we obtain:

\[
Lp' \frac{\Delta r'}{\Delta t^2} = k \left( \frac{dT}{dr} \right) \bigg|_{r>r_i}
\]

Total solidification occurs when the \( \Delta r'' \) is equal to zero. Solving (22) for \( \Delta r' \) we arrive at:

\[
\Delta r' = \frac{-2}{Lp'} k \Delta t \left( \frac{dT}{dr} \right) \bigg|_{r>r_i}
\]

Equation (23) is used in the program and solved for until \( r' \) equals zero. This analysis assumes that there is no resistance to heat flow across the metal and mold interface occurring.

**Sensible Heat Removal.** As the solid metal cools, the metal contracts and leaves a gap through which heat must escape to the wall. Sensible heat must be removed across this gap. The heat crossing this gap will satisfy the equation:

\[
h_{gap}(T_M - T_W) = -k \frac{T_W - T_{r=1}}{\Delta r}
\]

where: \( h_{gap} \) = heat transfer coefficient across the gap, cal/cm\(^2\)-°C-sec

\( T_M \) = surface metal temperature, °C
\[ T_W = \text{surface wall temperature, °C} \]
\[ T_{r=1} = \text{temperature at one radial increment} \]

This equation is used to solve for the inside surface of the mold wall. Again it is assumed that there is no temperature gradient in the metal itself.

Outside Wall Boundary Conditions

Heat is removed from the outside wall of the mold (i.e., \( r = r_0 \)) by both radiation and convection. The heat arriving to this exterior surface must equal the heat removed. Therefore:

\[
-k \left( \frac{dT}{dr} \right) \bigg|_{r=r_0} = T_W - T_A \left[ h_c + \sigma \varepsilon \left( \frac{T_W^4 - T_A^4}{T_W - T_A} \right) \right] 
\]

where: 
\( T_C = \text{the ambient temperature, °C} \)
\( T_W = \text{temperature of the wall, °C} \)
\( k = \text{thermal conductivity of the wall at the outside surface, cal/sec-cm-°C} \)
\( h_c = \text{convective heat transfer coefficient, cal/cm}^2-\text{sec-°C} \)
\( \sigma = \text{Stefan-Boltzman constant, cal/cm}^2-\text{sec-°C}^4 \)
\( \varepsilon = \text{emissivity of the mold outside surface} \)

The convective heat transfer coefficient can be calculated from the equation (15):

\[
h_c = 0.00112 \left( \frac{T_W - T_A}{2r_0} \right)^{1/8} \]

\( \text{cal/sec-cm} \times \text{°C} \)
for

\[ 10^{-2} < 8r_o^3(T_W - T_A) < 10^3 \]

where \( r = r_o \).

Solution Method

The general procedure in numerical simulations is to solve for a temperature at a given location and time. This temperature is then used to solve for the temperature in the next time interval at this same location. The end result is a grid of temperatures at given times and radii. The forward difference method solves a differential equation using small time intervals and small radii intervals.

To solve Equation (15), dimensionless numbers were used to facilitate the ease of solution:

\[
\theta = \frac{T^* - T_p}{T_A - T_p}
\]

\[
\tau = \frac{\alpha t^*}{r_i^2}
\]

\[
x = \frac{r^*}{r_i}
\]

where \( t^* = \) time (sec)

\( r^* = \) radius (cm)

\( \theta = \) dimensionless temperature

\( \tau = \) dimensionless time
After plugging these dimensionless numbers into Equation (15) we arrive at:

\[ \frac{\partial \theta}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) \]  

(28)

This derivation is shown in Appendix B. The mold wall is broken up into 10 nodal points, from n=1 at the inside surface of the wall, to n=10 at the outside surface (see Fig. 18). Applying the forward difference technique to Equation (28), we obtain:

\[ \theta_{n+1}^i = \theta_n^i + \Delta \tau \left[ \frac{1}{r} \frac{\theta_{n+1}^i - \theta_{n+1}^i}{\Delta r} + \frac{\theta_{n+1}^i + 2\theta_{n+1}^i - \theta_{n-1}^i}{\Delta r^2} \right] \]  

(29)

where: \( \theta_n^i = \) dimensionless temperature of radial increment n, and time increment i

\( \Delta \tau = \) dimensionless time interval

\( \Delta r^2 = \) dimensionless radius interval

This equation is derived in Appendix B.

Interfaces in the Mold Wall

In investment mold walls, there are interfaces between the various mold materials. The walls in the
Fig. 18. Nodes used for computer problem.
program were listed as being made up of either the four silica materials, or the two zircon additive materials. The thicknesses for the various types of materials were held constant, however could be changed when the need arose. The thicknesses were held to the following percentages:

- P1W, 7% of the mold wall thickness
- P1W+S1, 13% of the mold wall thickness
- P1W+S2, 23% of the mold wall thickness
- P1W+S3, 57% of the mold wall thickness

For the zircon additive materials:

- H2+S2, 3% of the mold wall thickness
- H2+S2, 27% of the mold wall thickness
- H4+S2, 70% of the mold wall thickness

These various materials have different thermal properties and therefore the solution to Equation (29) will depend on the type of material present in each node of the mold wall. When an interface between the different materials occurred between nodes, the effect of the change of materials on the thermal diffusivity within that radius interval had to be accounted for (see Fig. 19). This was done by calculating a weighted average of the property based on the amount of material present in the radius interval.

Program Stability

When dealing with a numerical solution to a partial differential equation, there is a related limit
Fig. 19. Interface dividing radial interval.
which must be met in order for the computation to converge on the solution. In the finite difference technique, the condition that must be met is that:

\[
\frac{\alpha \Delta t^*}{\Delta r^*} < 0.5
\]  \hspace{1cm} (30)

where: \( \Delta t^* \) = time interval (sec)
\( \Delta r^* \) = radii interval (cm)
\( \alpha \) = thermal diffusivity (cm\(^2\)/sec).

When this condition is met, the computations will be stable. In the solution for this mold however, the thermal diffusivity is not constant throughout the entire mold. Therefore the thermal diffusivity that must be used in solving for the above condition is the largest one that is encountered in the solution. In this case that value is 0.0033 cm\(^2\)/sec. Therefore the ratio of time interval to the square of the radii interval must satisfy:

\[
151.52 > \frac{\Delta t^*}{\Delta r^*}^2
\]  \hspace{1cm} (31)

The execution of the program with the above boundary conditions and the initial conditions is described in Appendices D and E. A copy of the program used is in Appendix F. Output from the program was in the form of temperature for a given location in the wall, and time into the solidification process. The output was taken and curves for the temperature-time profile for the inside wall
surface, temperature-time profiles for the outside surfaces of the wall, and also various curves showing the effect of different variables on solidification time. All these results will be discussed in the computer results section.

Experimental Procedure for Verification of the Computer Program

Two techniques were used to prove that the solution offered by the computer program was accurate. The first used thermocouples to determine the time to solidification of aluminum and copper castings. The second experiment involved small diameter copper castings using a pour out technique to determine the amount of metal that solidified after a specified time interval.

The first used a cylindrical risers with a 2.54 cm (1 in) radius and a 25.4 cm (10 in) height were chosen for the first technique. Walls of investment were made on the wax pattern according to Table 7. The binder formulations and the stucco were the same used for the thermal conductivity test. The wax was melted out of the molds and the molds were fired at 1000°C for four hours. This firing was done to strengthen the wall. Copper was used due to the lack of suitable melting apparatus and thermocouple problems at the temperatures of nickel. The copper was melted in a Speedymelt® furnace. Flux was added and the melt was deoxidized using phosphorus pellets. The schedule...
Table 7. Wall compositions for computer runs.

<table>
<thead>
<tr>
<th>Metal Cast.</th>
<th>Wall Ratio</th>
<th>Wall Composition (%)&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Super Heat °C</th>
<th>Pre Heat °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SL02 PIW PIW+S1 PIW+S2 PIW+S3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al 1</td>
<td>1:2.6</td>
<td>7 13 25 55</td>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>Cu 1</td>
<td>1:4</td>
<td>10 15 22 53</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>Cu 2</td>
<td>1:3</td>
<td>8 12 30 50</td>
<td>50</td>
<td>300</td>
</tr>
<tr>
<td>Cu 3</td>
<td>1:5</td>
<td>10 12 19 59</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>Cu 4</td>
<td>1:5</td>
<td>8 10 27 55</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Cu 5</td>
<td>1:3</td>
<td>3 22 70</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Cu 6</td>
<td>1:4</td>
<td>2 30 68</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>Cu 7</td>
<td>1:4</td>
<td>3 27 70</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>Cu 8</td>
<td>1:1</td>
<td>8 15 30 47</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Cu 9</td>
<td>1:1</td>
<td>12 19 25 44</td>
<td>100</td>
<td>25</td>
</tr>
<tr>
<td>Cu 10</td>
<td>2:1</td>
<td>10 10 23 57</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Cu 11</td>
<td>2:1</td>
<td>5 25 20 50</td>
<td>100</td>
<td>25</td>
</tr>
</tbody>
</table>

<sup>a</sup>% of total wall thickness.
for testing is also on Table 7. It shows the wall preheat temperature used as well as the metal superheat used.

The mold was removed from the muffle furnace used for preheating, and immediately placed in two supporting ringstands. The top and bottom of the mold were packed in glass wool, then an exterior Chromel-Alumel thermocouple was placed on the outside surface 12.7 cm (5 in) from the bottom, while a quartz shielded platinum-platinum/rhodium (10%) thermocouple was mounted in the center radially and 12.7 cm (5 in) axially into the mold. The thermocouple outputs were read on a strip recorder for at least ten minutes after the pour was made. Results were then graphed in terms of dimensionless temperature and compared with the results of the numerical solution.

A second testing procedure was used to verify the program and the prediction it made for the larger mold wall thickness to casting radius ratios. Samples of the same type that were made for the thinner investment wall/to casting radius ratios were made, only with small inner diameters and thicker wall ratios (1:1, 2:1). The copper melt was poured into the cavity and then after some time interval the casting was drained out and the amount of metal wall solidified was measured. Since the computer output gave the location of the solidification front at any time, the thickness of the casting and the time for solidification could be compared.
The molds made for this second technique were not preheated as it was felt they would cool too much before pouring. The mold was 20.3 cm (8 in) long with a .8 cm (0.31 in) inner radius. The wall thicknesses were 1.6 cm (0.62 in) for the 2:1 and 0.8 cm (0.31 in) for the 1:1 ratios. The various layers were held to the layer thickness percentages that were used on the computer program, i.e., PlW, 7%; P1W+S1, 13%; etc. The molds were open at both ends, with the bottom set in an exothermic material to hold the metal as well as keeping the end heated and the metal liquified. The top was uncovered. After the cast was made, and a suitable time interval had transpired the mold was picked up. The liquid metal ran out of the bottom, leaving a shell of the metal. This shell thickness was measured for comparison with the computer predicted values.

Results of the Computer Program

Figures 20, 21, and 22 show the results of the computer program evaluating the heat transfer situation in a cylindrical casting. Both silica and zircon investment mold cases were analyzed. The values of casting radius and mold wall thicknesses were varied to cover the range of ratios (wall thickness to casting radius) from 1:5 to 4:1. The computer runs were made for aluminum, copper, and nickel. The conditions of the melts included either 50°C or 150°C superheat, and the wall initial conditions were either room
Fig. 20. Computer results of nickel casting in silica mold wall.
Fig. 21. Computer results of copper casting in silica mold wall.
Preheat/Superheat

Dimensionless time = $\frac{t^*}{\frac{s}{r_i^2}}$

Fig. 22. Computer results of nickel casting in silica/zircon mold wall.
temperature (25°C), 200°C, or 500°C. The results are presented as a function of dimensionless solidification time versus the ratio of wall thickness to casting radius. This dimensionless time value is given by

\[ t_s = \frac{\bar{u}t^*_s}{r_s^2} \]

where: \( \bar{u} \) = average thermal diffusivity, cm\(^2\)/sec
\( t^*_s \) = predicted time to solidification, sec
\( t_s \) = predicted dimensionless time to solidification
\( r_s \) = inside radius of casting, cm.

The average thermal diffusivity was found by taking the mean of all the thermal diffusivities at each location node at the moment of complete solidification. The average thermal diffusivities used are in Appendix C.

Results of the Castings

The results of the test castings using the first technique are listed in Table 8. The results of each test casting are given in Appendix D and on Fig. 23. They are shown with their respective computer run. Measured solidification times were from 4-15% short of the predicted values; however, even with this error margin, the trends shown with respect to superheat, preheat of the mold, and other variables were well supported by the experiment.
Table 8. Results of actual castings made (technique 1).

<table>
<thead>
<tr>
<th>Cast No.</th>
<th>Metal Cast</th>
<th>Wall Ratio</th>
<th>Metal Temp (°C)</th>
<th>Wall Temp</th>
<th>Predicted $t^*$ (sec)</th>
<th>Actual $t^*$ (sec)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Al</td>
<td>1:3</td>
<td>710</td>
<td>R.T.</td>
<td>175</td>
<td>155</td>
<td>11%</td>
</tr>
<tr>
<td>2</td>
<td>Ni</td>
<td>1:4</td>
<td>1502</td>
<td>200</td>
<td>265</td>
<td>&gt;240</td>
<td>9%</td>
</tr>
<tr>
<td>3</td>
<td>Cu</td>
<td>1:3</td>
<td>1435</td>
<td>200</td>
<td>370</td>
<td>300</td>
<td>19%</td>
</tr>
<tr>
<td>4</td>
<td>Cu</td>
<td>1:3</td>
<td>1410</td>
<td>350</td>
<td>375</td>
<td>350</td>
<td>7%</td>
</tr>
<tr>
<td>5</td>
<td>Cu</td>
<td>1:5</td>
<td>1410</td>
<td>200</td>
<td>308</td>
<td>285</td>
<td>7%</td>
</tr>
<tr>
<td>6</td>
<td>Cu</td>
<td>1:5</td>
<td>1460</td>
<td>200</td>
<td>350</td>
<td>325</td>
<td>7%</td>
</tr>
<tr>
<td>7</td>
<td>Cu</td>
<td>1:4</td>
<td>1410</td>
<td>200</td>
<td>403</td>
<td>370</td>
<td>8%</td>
</tr>
<tr>
<td>8</td>
<td>Cu</td>
<td>1:3</td>
<td>1460</td>
<td>200</td>
<td>555</td>
<td>&gt;540</td>
<td>3%</td>
</tr>
<tr>
<td>9</td>
<td>Cu</td>
<td>1:5</td>
<td>1410</td>
<td>200</td>
<td>388</td>
<td>370</td>
<td>5%</td>
</tr>
</tbody>
</table>
Fig. 23. Comparison of experimental results of the castings and their respective computer predictions. Numbers are casting number; ----- computer prediction.

Dimensionless time = $\frac{t_s^* \alpha}{r_i^2}$
The errors in the measurements have multiple sources. The thermocouple measuring the temperature of the melt was placed in a quartz protection tube when it was found that the mullite protection tube could not handle the sudden thermal shock of the casting. This quartz tube therefore would slow down the initial response time of the thermocouple. It also would act as a chill at the center of the casting.

Since the wall was preheated, it had to be taken from a warm furnace and placed in position and then the metal cast. This lag of time from furnace to casting could have been a minute at least. Therefore a cooling profile had already begun in the wall. The initial temperature of the wall was taken to be the measured exterior temperature of the wall at the start of the casting. The wall was generally thin enough (0.51 cm thickness for a 2.54 cm radius casting void) that Newtonian cooling was assumed in the wall.

The total effect of these two errors when viewed together with their subjective contribution to the solidification time are as follows:

1. The chilling factor of the quartz tube will decrease the actual time to solidification.

2. The wall gradient in the wall will increase the actual time to solidification.
It was felt that if the same procedure was followed in each casting as closely as possible, the errors present would effect the solidification time in the same manner on each run.

The conditions and results of the second casting procedure are shown in Table 9. The mold walls were at room temperature, and the metal was cast at 50°C superheat in all cases. Four castings were made with only one being successful, i.e., pouring out partially frozen. The other three apparently were frozen at the ends after the specified times. (In these latter cases it was noted that the exothermic material that was to keep the pour-out end warm during the specified time period did not ignite.) The comparison of this result with the prediction was close.

Discussion of the Results

The discussion will cover the various parameters and their effect on the solidification time of a casting.

Effect of Mold Wall Thickness

The general shape of all the curves indicates that there are three regions of interest. The first segment concerns those results for the ratios of 1:5 through 1:3. These ratios show very little change in dimensionless solidification time for the same given casting conditions, even though the wall thickness casting radius ratio changes drastically.
### Table 9. Results of actual castings made (technique 2)

<table>
<thead>
<tr>
<th>Casting Number</th>
<th>Metal Cast</th>
<th>Wall Ratio</th>
<th>Metal Temp</th>
<th>Per Cent Solidified</th>
<th>Predicted $t_s^*$</th>
<th>Actual $t_s^*$</th>
<th>% Diff.</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Cu</td>
<td>2:1</td>
<td>+50°C</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Frozen Ends &lt; 10 sec</td>
</tr>
<tr>
<td>11</td>
<td>Cu</td>
<td>2:1</td>
<td>+50°C</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Frozen Ends &lt; 10 sec</td>
</tr>
<tr>
<td>12</td>
<td>Cu</td>
<td>1:1</td>
<td>+50°C</td>
<td>20%</td>
<td>6.3 sec</td>
<td>5 sec</td>
<td>21%</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Cu</td>
<td>1:1</td>
<td>+50°C</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Frozen Ends &lt; 10 sec</td>
</tr>
</tbody>
</table>
The second region of interest deals with the thicker walls, or ratios from 1:3 through 3:1. This region shows a tremendous change in the dimensionless solidification time for the same casting conditions.

The final region covers the walls or ratios greater than 3:1. These approach a semi-infinite mold wall and can be described using equations for this semi-infinite condition. A complete analysis is given in Appendix E.

Heat transfer from the mold wall to the surroundings is by two modes of heat transfer: convection and radiation. In the thinner wall, radiation predominates over the convective mode, especially at high temperatures. In the thicker walls the outside surface temperature is less and radiation becomes less important. The main point is that for a given casting radius, the time to solidification is increased when the mold wall thickness increases.

Effect of the Metal Cast

Figure 24 shows the various computed results for different metals under the same casting conditions. Nickel, which has the highest melting point and hence the greatest amount of energy associated with it, solidifies first in the thin wall region, before copper, and then aluminum. This is due to radiation being the dominant heat transfer mechanism in the nickel casting and less important for copper and
Fig. 24. Comparison of various metals cast at 200°C preheat and +50°C superheat.

Wall Ratio

1:5  1:4  1:3  1:2  1:1  2:1  3:1  4:1  5:1

Dimensionless time = $\frac{t* \alpha}{r^2}$

Al  Cu  Ni
negligible in aluminum. This makes the nickel casting very dependent on a change in wall thickness.

In the third region of each curve, the total time to solidification for the semi-infinite case is related to the amount of total heat available in the casting. Obviously, the nickel has the greatest followed by copper, then aluminum; and for a given size casting, the nickel takes the longest to solidify.

Effect of Superheat and Preheat on Solidification Time

On Figs. 20, 21 and 22 an increase in the superheat or preheat will always increase the solidification time for the casting. This effect is much more noticeable in the second and third regions than in the first region. When the ratios reach the semi-infinite case, the preheat of the mold affects the solidification more than the superheat of the metal. When the mold material containing zircon is used for a given casting the effects of the superheat are much more pronounced at ratios less than 1:2.

Effect of Mold Material on Solidification Time

Comparing Figs. 20 and 22 we see the casting situations for nickel casting using two types of mold material. One immediately sees a vast difference between the two.
The same three regions are present in the material containing a portion of zircon, however the curves for the silica/zircon material are spread out much more than with the pure silica material. The effect of the superheat and preheat on the curves is also more pronounced in this material. Thus, the small addition of zircon to the slurry of the investment material greatly extends the solidification time of the casting.
CHAPTER IV

CONCLUSIONS

The thermal conductivity of the investment materials was found to vary with temperatures as shown in Figs. 7 and 8. The major controlling factors in all materials were the porosity of the wall and the particle size of the stucco. A model presented by Schotte [10] offers analysis of the thermal conductivity of packed beds, and this analysis can be applied to investment materials with limited success. This limited success is due to the increase of conduction in the sample from the presences of the binder. Radiation at high temperatures increases thermal conductivity of most samples, except the samples with the higher percentage of zircon flour in its slurry. This high zircon containing sample had a lower thermal conductivity than the other samples in this same high temperature range.

The computer simulation developed will generate a temperature profile and solve for the solidification time for a given cylindrical casting situation. The solution indicates there are three regions on a curve relating the mold thickness/casting radius ratio to the dimensionless time to solidification. The heat transfer mechanisms which
dominates in these three areas are conduction plus radiation in the first, conduction plus convection in the second, and pure conduction in the third. The mold wall thickness greatly affects this control.

The metal cast affects the range of dimensionless solidification times over which the curve acts. The greater the amount of heat removed will cause a casting in the third region to take longer to solidify. Higher temperature metals when cast in thinner walled molds will have more radiation and hence shorter solidification times.

The greater the superheat or preheat for the same given set of casting conditions, the longer the solidification time. At mold wall/casting radius ratios greater than 3:1 the preheat of the wall affects the solidification time more than the superheat of the metal.

The mold material was found to exhibit control over the solidification time of the casting. Zircon added to the slurry extended the solidification time in all regions when compared to the pure silica material.
CHAPTER V

SUGGESTIONS FOR FUTURE WORK

One area of study would be the solidification of alloys. Since alloys are usually used in most investment castings, information dealing with the solidification rates of these alloys would be very useful.

The study of the various investment materials and their thermal properties is another important area of study which should be continued. The addition of various other mold materials and their effects on solidification time would be an extremely important study.

A technique to selectively apply these various materials to different areas of investment wall to obtain the desired effects would also be a great addition to the control of solidification in castings.

One last area would be a study of the relationship of the positions of the runners to the risers. Since the riser, which has a larger volume, requires longer times for solidification, the mold surrounding the runner is being irradiated from the riser mold wall, thus slowing down the freezing of the runner. We have seen that an increase in mold preheat is going to affect the solidification rate of a casting. If the runner's solidification time was increased
by this irradiation, this would give a cast product more time to solidify and potentially decrease the rejects due to microporosity caused by a runner solidifying prior to the product solidifying.
APPENDIX A

ERROR ANALYSIS OF THERMAL CONDUCTIVITY MEASUREMENTS

The equation for the thermal conductivity includes measured values of current and changes in the EMF. This equation has an error that is associated with each measurement. The equation of interest is:

$$k = \frac{I^2 C}{\Delta E}$$

where:  
- \( k \) = thermal conductivity,  
- \( I \) = current, amps  
- \( \Delta E \) = millivolt change  
- \( C \) = constant.

The limits for the current and the millivoltage are:
- \( \theta(I) = \pm 0.05 \) amps  
- \( \theta(\Delta E) = \pm 0.0005 \) mV

The constant \( C \) also has an associated error with it because it is defined by measured values. The constant is

$$C = 7.53 \frac{R}{f}$$

where:  
- \( R \) = resistivity of the thermocouple, \( 0.00048 \pm 0.000005 \) ohm/cm\(^2\)  
- \( f \) = change in degrees per millivolt, °C/mV

The change in temperature for a change in millivoltage is
dependent on temperature, and its associated errors are as follows:

<table>
<thead>
<tr>
<th>T(°C)</th>
<th>f °C/mV</th>
<th>± f</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>44</td>
<td>0.39</td>
</tr>
<tr>
<td>100</td>
<td>43</td>
<td>0.38</td>
</tr>
<tr>
<td>200</td>
<td>45</td>
<td>0.41</td>
</tr>
<tr>
<td>500</td>
<td>42</td>
<td>0.42</td>
</tr>
<tr>
<td>800</td>
<td>43</td>
<td>0.38</td>
</tr>
<tr>
<td>1000</td>
<td>45</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The constant C therefore has a related error as follows:

<table>
<thead>
<tr>
<th>T(°C)</th>
<th>± C</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.076 ± 0.0002</td>
</tr>
<tr>
<td>100</td>
<td>0.077 ± 0.0002</td>
</tr>
<tr>
<td>200</td>
<td>0.075 ± 0.0002</td>
</tr>
<tr>
<td>500</td>
<td>0.079 ± 0.0002</td>
</tr>
<tr>
<td>800</td>
<td>0.077 ± 0.0002</td>
</tr>
<tr>
<td>1000</td>
<td>0.074 ± 0.0002</td>
</tr>
</tbody>
</table>

The two measured values, the current and millivoltage, have an associated error as follows:

\[ \delta(mV) = \delta(\Delta E) = 0.0005 \]

\[ \delta(I^2) = 0.1(I) \]

The total error is given then by the following equation:

\[ \delta k = \frac{\Delta E \cdot \delta(I^2C) + (I^2C) \cdot (\delta \Delta E)}{(\Delta E)^2} \]

For any temperature, the error function associated with that measurement can now be computed from the following table:
<table>
<thead>
<tr>
<th>°C</th>
<th>°F</th>
<th>$\alpha (I^2C)$</th>
<th>$\Delta k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>77</td>
<td>$0.0002I^2+0.00765I$</td>
<td>$(\Delta E(0.0002I^2+0.00765I)+0.000035I^2)241.75/\Delta E^2$</td>
</tr>
<tr>
<td>100</td>
<td>212</td>
<td>$0.0002I^2+0.00779I$</td>
<td>$(\Delta E(0.0002I^2+0.00779I)+0.000039I^2)241.75/\Delta E^2$</td>
</tr>
<tr>
<td>200</td>
<td>392</td>
<td>$0.0002I^2+0.00752I$</td>
<td>$(\Delta E(0.0002I^2+0.00752I)+0.000036I^2)241.75/\Delta E^2$</td>
</tr>
<tr>
<td>500</td>
<td>932</td>
<td>$0.0002I^2+0.00799I$</td>
<td>$(\Delta E(0.0002I^2+0.00799I)+0.000039I^2)241.75/\Delta E^2$</td>
</tr>
<tr>
<td>800</td>
<td>1472</td>
<td>$0.0002I^2+0.00773I$</td>
<td>$(\Delta E(0.0002I^2+0.00773I)+0.000038I^2)241.75/\Delta E^2$</td>
</tr>
<tr>
<td>1000</td>
<td>1832</td>
<td>$0.0002I^2+0.00739I$</td>
<td>$(\Delta E(0.0002I^2+0.00739I)+0.000036I^2)241.75/\Delta E^2$</td>
</tr>
</tbody>
</table>
For example: \( I = 3 \) amps

\[ \Delta E = 0.150 \text{ mV} \]

<table>
<thead>
<tr>
<th>( ^\circ C )</th>
<th>T/( ^\circ F )</th>
<th>( k \text{ cal/cm}^{-2} \text{C-sec} )</th>
<th>( \pm k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>77</td>
<td>0.00150</td>
<td>( \pm 0.00006 )</td>
</tr>
<tr>
<td>100</td>
<td>212</td>
<td>0.00159</td>
<td>( \pm 0.00006 )</td>
</tr>
<tr>
<td>200</td>
<td>392</td>
<td>0.00155</td>
<td>( \pm 0.00006 )</td>
</tr>
<tr>
<td>500</td>
<td>932</td>
<td>0.00163</td>
<td>( \pm 0.00006 )</td>
</tr>
<tr>
<td>800</td>
<td>1472</td>
<td>0.00159</td>
<td>( \pm 0.00006 )</td>
</tr>
<tr>
<td>1000</td>
<td>1832</td>
<td>0.00153</td>
<td>( \pm 0.00006 )</td>
</tr>
</tbody>
</table>
APPENDIX B

ADDITIONAL INFORMATION ON COMPUTER PROGRAM

Justification that the Casting
Cools Newtonianly

The assumption that there is no gradient in the metal during the solidification process can be proven to be true by the following analysis.

First of all assuming no interfacial resistance to heat flow, the temperature profile from the casting to the wall appears as shown in Fig. A. We must show that

\[ T_s \approx T_m \]

for our given condition.

Using an analysis using the equation for cooling in a semi-infinite mold wall, we can arrive at the solution which is described in Figure B, given by Reynolds [16]. Using this graph, we see that if

\[ \frac{T_s - T_o}{T_m - T_o} \approx 1.0 \] (1)

then \( T_m \approx T_s \). Below are the cases for nickel, copper, and aluminum worked out to show how close they meet the criteria given in (1).
The Energy Equation

The original energy equation in cylindrical coordinates for the case where heat flow is directed radially from the center can be written:

\[
\frac{\partial T}{\partial t} = \frac{\alpha}{r^*} \left( \frac{\partial T}{\partial r^*} + r^* \frac{\partial^2 T}{\partial r^* \partial T} \right)
\]

(2)

Use of dimensionless numbers can expedite the majority of the calculations and therefore substitution into (1) has been done. The dimensionless numbers used are:

\[
\theta = \frac{T-T_0}{T_s-T_p} ; \quad \tau = \frac{at}{r_i^2} ; \quad r = \frac{Ar^*}{r_i}
\]

(2a) \quad (2b) \quad (2c)

We know that \( T^* \) is a function of \( t \) and \( r^* \) or

\[
T = f(f,r^*)
\]

therefore:

\[
\frac{\partial T}{\partial r^*} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial r^*} + \frac{\partial T}{\partial \tau} \frac{\partial \tau}{\partial r^*}
\]

and

\[
\frac{\partial T}{\partial t} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial t} + \frac{\partial T}{\partial \tau} \frac{\partial \tau}{\partial t}
\]

But there is no relationship between \( r \) and \( t \) or \( R^* \) and \( \tau \); therefore \( \frac{\partial r}{\partial t} \) is zero, and \( \frac{\partial r^*}{\partial \tau} \) is zero and this leaves:

\[
\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \tau} \frac{\partial \tau}{\partial t}
\]

(3a)

\[
\frac{\partial T}{\partial r^*} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial r^*} \quad \text{and} \quad \frac{\partial^2 T}{\partial r^* \partial r} = \frac{\partial T}{\partial r} \frac{\partial^2 r}{\partial r^* \partial r^*} + \frac{\partial^2 T}{\partial r^* \partial r} \frac{\partial r}{\partial r^*}
\]

(3b)
Substituting (3a) and (3b) into (1), we arrive at:

$$\frac{\partial T}{\partial t} \frac{\partial t}{\partial t} = \alpha \left( \frac{\partial T}{\partial r} \frac{\partial r}{\partial t} + r^*(\frac{\partial T}{\partial r} \frac{\partial^2 r}{\partial r^2} + \frac{\partial r}{\partial t} \frac{\partial^2 T}{\partial r^2} \right)$$

(4)

From (2c) and (2b) we see:

$$\frac{\partial r}{\partial r^*} = \frac{1}{r} \frac{\partial^2 r}{\partial r^2} = \phi$$

and also:

$$\frac{\partial t}{\partial t} = \frac{\alpha}{r_i^2}$$

Substituting these in (4) we obtain:

$$\frac{\partial T}{\partial t} \frac{\partial r}{\partial r^*} = \frac{\alpha}{r} \left[ \frac{\partial T}{\partial r} \frac{1}{r} + r^* \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \right) \right].$$

(5)

Rewriting and canceling terms:

$$\frac{\partial T}{\partial r} = \frac{r_i}{r^*} \left( \frac{\partial T}{\partial r} + r \frac{\partial^2 T}{\partial r^2} \right)$$

(6)

Substituting (2c) into (6):

$$\frac{\partial T}{\partial r} = \frac{r_i}{r_i^*} \left( \frac{\partial T}{\partial r} + r_i r \frac{\partial^2 T}{\partial r^2} \right)$$

(7)

We know that

$$\theta = f(T)$$

$$T = g(r^*, t)$$

and also that T can be a function of r and t. Therefore:

$$\frac{\partial \theta}{\partial \theta} = \frac{\partial \theta}{\partial T} \frac{\partial T}{\partial r}$$

(8a)
Differentiating (2a) we obtain

\[
\frac{\partial \theta}{\partial T} = \frac{\partial \theta}{\partial T} \frac{\partial T}{\partial T} = \frac{1}{\Delta T}
\]  

(8b)

and substituting (9) into (8a) and (8b) we obtain:

\[
\frac{\partial \theta}{\partial r} = \frac{1}{\Delta T} \frac{\partial T}{\partial r}
\]  

(10a)

\[
\frac{\partial \theta}{\partial \tau} = \frac{1}{\Delta T} \frac{\partial T}{\partial \tau}
\]  

(10b)

Knowing that T is a function of r by r* we see

\[
\frac{\partial T}{\partial r} = \frac{\partial T}{\partial r^*} \frac{\partial r^*}{\partial r}
\]

\[
\frac{\partial T}{\partial r} = \frac{\partial T}{\partial r^*} r_i
\]

(11)

Substituting (11) into (10a) we get:

\[
\frac{\partial \theta}{\partial r} = \frac{r_i}{\Delta T} \frac{\partial T}{\partial r}
\]

(12)

Differentiating (12):

\[
\frac{\partial^2 \theta}{\partial r^2} = \frac{r_i}{\Delta T} \frac{\partial^2 T}{\partial r^2}
\]

(13)

Substituting (13), (10a), (10b), into (7) and simplifying we finally obtain:

\[
\frac{\partial \theta}{\partial T} = \frac{1}{r} \left( \frac{\partial \theta}{\partial r} + r \frac{\partial^2 \theta}{\partial r^2} \right);
\]

\[
\frac{\partial \theta}{\partial \tau} = \frac{1}{r} \left( \frac{\partial \theta}{\partial r} + r \frac{\partial^2 \theta}{\partial r^2} \right);
\]

and

\[
\frac{\partial \theta}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right).
\]
Application of the Finite Difference Technique

For the purpose of solving the equation derived above, we will apply the finite difference technique. The technique we will use will be the forward finite difference technique [17]. The equation of interest is:

\[
\frac{\partial \theta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right)
\]

It can be rewritten as

\[
\frac{\partial \theta}{\partial t} = \left( \frac{\partial \theta}{\partial r} \frac{1}{r} + \frac{\partial^2 \theta}{\partial r^2} \right)
\]

In the forward different technique, the derivatives are solved for by using difference equations [18]. Their forms are:

\[
\frac{\partial \theta}{\partial t} = \frac{\theta^{i+1}_n - \theta^i_n}{\Delta t}
\]

\[
\frac{\partial \theta}{\partial r} = \frac{\theta^i_n - \theta^{i+1}_n}{\Delta r}
\]

\[
\frac{\partial^2 \theta}{\partial r^2} = \frac{\theta^{i+1}_n + 2\theta^i_n - \theta^{i-1}_n}{(\Delta r)^2}
\]

where: \( \theta^i_n \) = dimensionless temperature at the ith time interval, and the nth location

\( \Delta r \) = dimensionless radial increment

\( \Delta t \) = dimensionless time increment

\( r = (n)(\Delta r) \)
\[ \tau = (n)(\Delta t) \]

\[ n = 1, 2, 3 \ldots \text{ number of radial divisions}; \ i = 1, 2, 3 \ldots \text{ number of time intervals}. \]

These are substituted into (1) and then the expression is simplified to get the temperature at the next time interval for that location, or:

\[ \frac{\theta_{i+1}^n - \theta_i^n}{\Delta t} = \left[ \frac{\theta_i^n - \theta_{n+1}^i}{\Delta r} \right] \left( \frac{1}{r} \right) + \frac{\theta_i^{n+1} + 2\theta_i^n - \theta_i^{n-1}}{(\Delta r)^2} \]

\[ \theta_{i+1}^n = \theta_i^n + \Delta t \left[ \frac{\theta_i^n - \theta_{n+1}^i}{\Delta r} \right] \left( \frac{1}{r} \right) + \frac{\theta_i^{n+1} + 2\theta_i^n - \theta_i^{n-1}}{(\Delta r)^2} \]

**Inside Boundary Conditions**

After the appropriate condition of the inside wall has been established, the equations are handled similarly to the method used for the outside wall.

The information needed is the inside surface wall temperature. At any given time the surface inside temperature can not be found until the temperature at the first radial increment has been found. From this temperature the amount of heat entering the wall over the specified time interval can be determined by:

\[ q = \frac{k(\Delta T)}{\Delta r} \]  

(1)

This heat entering the wall must equal the heat leaving the metal. There are three conditions under which heat
leaves the metal: (a) superheat, (b) latent heat of fusion, (c) sensible heat from the solid metal.

Superheat Removal

If we use the previous temperature of the inside surface as a first approximation to solve for the actual temperature at the inside surface, we obtain:

\[ q = -k \frac{(T_w - T_1)}{\Delta r} \]  

This is the heat provided by the melt. It will be equal to heat given up by the melt, or:

\[ q_m = \frac{C_p \rho V}{A} \frac{dT}{dt} = \frac{C_p \rho V}{A} \frac{T_w - T_w'}{\Delta t} \]  

Therefore,

\[ -k \frac{(T_w - T_1)}{\Delta r} = \frac{C_p \rho V}{A} \frac{T_w - T_w'}{\Delta t} \]

Solving for \( T_w' \)

\[ T_w' = T_w - \frac{k \Delta t}{C_p \rho V} \frac{A}{\Delta r} (T_w - T_1) \]  

This temperature is compared to \( T_w \) and if the difference is less than a specified limit (in this case 1.0°C), then the remainder of the program is executed, otherwise, Equation (4) is solved again using the new found \( T_w' \) as the old \( T_w \) and the subroutine is repeated. When \( T_w' \) equals the solidification temperature of the melt, the program assumes that solidification begins, and latent heat is evolved.
Latent Heat Removal

When latent heat is given up, it is assumed that the inside surface of the wall is constant and equal to the solidification temperature. The amount of heat that is absorbed by the wall is now related to the change in the solidification front's radius. The heat into the wall is still

\[ q = -k \frac{(T_w - T_l)}{\Delta r_{wall}} \]  

(5)

The latent heat provided is

\[ q_L = \frac{L \rho \Delta V}{\Delta t \Delta A} \]  

(6)

Equating the two:

\[ \frac{L \rho \Delta V}{\Delta t \Delta A} = -k \frac{(T_w - T_l)}{\Delta r} \]  

(7)

The ratio of \( \Delta V/\Delta A \) is equal to:

\[ \frac{\Delta V}{\Delta A} = \frac{\pi \Delta r^2 h}{2\pi \Delta rh} = \frac{\Delta r_{liquid}}{2} \]

Plugging this into Equation (7) and solving for \( r \), we obtain

\[ \Delta r_{liquid} = \frac{2\Delta th}{L_T} \frac{T_w - T_l}{\Delta r_{wall}} \]  

(8)

where: \( \Delta r_{liquid} = r_{old \ inside \ radius} - r_{new \ inside \ radius} \)

The equation is rewritten and used in the program in the form
The new radius of the liquid zone is determined until it is found to be zero, at which time total solidification has occurred. The final source of heat is now to be removed through the wall.

Sensible Heat Removal

This solution is similar to the first section. The heat absorbed by the wall is given up by the metal, but now heat must transfer across the shrinkage gap. The heat absorbed is equal to:

\[ q_w = h_{\text{gap}} (T_{MS} - T_w) \]  

where:  
- \( T_{MS} \) = temperature of the metal surface  
- \( T_w \) = temperature of the mold wall  
- \( h_{\text{gap}} \) = heat transfer coefficient of the gap  
- \( q_w \) = heat absorbed by the wall.

The value for the heat transfer coefficient for the gap was assumed to be on the order of 0.010-0.001 cal/sec-cm\(^2\)-°K. The value for the program was a constant and assumed to be 0.005 cal/sec-cm\(^2\)-°K.

Equating Equation (10) to Equation (2) we obtain

\[ h_{\text{gap}} (T_{MS} - T_w) = -k \frac{(T_w - T_1)}{\Delta r} \]
T_w is then solved for to arrive at:

$$T_w = h \Delta r \frac{T_{ms} + k T_1}{h \Delta r + k}$$  \hspace{2cm} (11)

All the variables are known except for T_{ms}. T_{ms} is related to the change in the temperature of the casting, which is determined by the amount of heat lost from the casting. Assuming that T_{ms} = T_m (Newtonian cooling), the change in heat for the casting is given by Equation (3). Equating this to (10), we obtain

$$\frac{C \rho V \rho}{A} \frac{T_{ms} - T_m}{\Delta t} = h_{gap} (T_m - T_w)$$  \hspace{2cm} (12)

If we assume that initially T_w is equal to the old surface temperature we can then solve for T_m and obtain:

$$T_m' = \frac{C V}{\Delta A \Delta t} T_m h_{gap} T_w \left[ \frac{C V}{A \Delta t} + h_{gap} \right]$$  \hspace{2cm} (13)

Using this value we solve Equation (11) for T_w. This new T_w is then compared to the old T_w. If the difference is less than the predetermined allowable difference (in this case 1°C) then the program continues; if this limit is not met then the subroutine is repeated and a new T_w is determined.

**Outside Wall Boundary Conditions**

The outside wall temperature and the heat removed from its surface has to be inserted into the program after
the program has generated a temperature profile through the mold wall. The equation that controls the outside wall is:

\[-k \frac{dT}{dr} \bigg|_{r=r_o} = h_t (T_w - T_A)\]  \hspace{1cm} (1)

where: \(k\) = thermal conductivity of the wall, cal/sec-cm\(^2\)-°C

\(\frac{dT}{dr} \bigg|_{r=r_o}\) = the change in temperature with respect to radius evaluated at \(r=r_o\), °C/cm

\(h_t\) = overall heat transfer coefficient, cal/sec-cm\(^2\)-°C

\(T_w\) = wall temperature, °C

\(T_A\) = ambient temperature, °C

The overall heat transfer coefficient is related to the convective [18] and the radiative heat transfer coefficients as follows:

\[h_c \approx 2.85 \times 10^{-5} (T_w - T_A)^{1/3} \text{ cal/sec-cm}^2\text{-°C} \]  \hspace{1cm} (2)

\[h_r = \sigma \varepsilon \left(\frac{T_w^4 - T_A^4}{T_w - T_A}\right) \text{ cal/sec-cm}^2\text{-°C} \]  \hspace{1cm} (3)

where: \(\sigma\) = Stephan-Boltzman constant--1.37 × 10\(^{-12}\) cal/cm\(^2\)-sec-°C\(^4\)

\(\varepsilon\) = emissivity for the mold wall

Equation (1) then becomes, upon substituting these into it:
\[-k \frac{dT}{dt} \bigg|_{r=r_0} = (T_w - T_A) \left[ (2.85 \times 10^{-5}) (T_w - T_A)^{1/3} \right. \\
\quad \left. + \sigma \varepsilon \left( \frac{T_w^4 - T_A^4}{T_w - T_A} \right) \right] \tag{4} \]

This equation is a very complex one to solve and therefore it will be solved by successive iterations. First the heat arriving at the wall can be approximated by:

\[ q = -k \frac{r=r_{10}}{r=r_9} \left( T_w' - T_w \right) \tag{5} \]

where: 
- \( q' \) = heat arriving to the wall, cal/sec-cm\(^2\)
- \( k \) = thermal conductivity of that section of wall, cal/sec-cm-°C
- \( T_w' \) = temperature at wall surface when heat arrives there
- \( T_w \) = temperature at 9th interval in the wall, °C
- \( \Delta r = r_{10} - r_9 \), cm

By rewriting (4) as:

\[ \left( \frac{r=r_{10}}{\Delta r} \right) = (T_w' - T_A) \bigg|_{r=r_{10}} \tag{6} \]

and then solving for \( T_w' \), we arrive at:
Solving for $h_t$ is done by using the previously solved $T_w'$. Comparing the old $T_w'$ with the new $T_w'$ calculated from (7), one can arrive at the $T_w'$ for the solution.

This comparison takes place in the program as a difference limit. The difference of the old $T_w'$ and the new $T_w'$ is found. If this difference is greater than a predetermined limit (in this case 1°C) then the new $T_w'$ is plugged into Equations (2) and (3) and $h_t$ is solved. This $h_t$ is plugged into (7) and a new $T_w'$ is solved. The following flow chart describes this section.

This flow chart and the program following are used for the solution to the temperature profile in the wall and the solidification time of the melt.
MAIN PROGRAM ROSH3.F4

START

INPUT: metal prop.,
mold prop., dimensions,
no. of time intervals,
no. of wall intervals

Calculate time intervals
and the radius intervals

Print out the input
information

Generate the radial distances

INSERT:
The initial conditions

Counters initialized

Begin iteration

$T(x,t) = T(x,t-1)$
use previous wall
temp. for thermal
prop.

Call TC2 for thermal
properties

Use thermal properties
to solve for new temp.

is

yes

Evaluate superheat

Call SPRHT

no

if temp FT

yes

radius = 0

no

Call SPRHT

for sensible
heat

Temp = FT

Compute new radius

Continue

END

Get T for last
iteration for
outside wall

Call HTC

Obtain outside
temp.

Evaluate no. of
iterations before
printout

Printout dimension-
less temp.
PROGRAM ROSH3(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

COMMON/BLK1/TH(5,20), T(5,20), TH(75), TH1(5), R(75)
COMMON/BLK2/TE(10), CK(4,10), DEN(6), IHMAT(4)
COMMON/BLK3/TD1(5,20)

C THIS PROGRAM IS USED TO DETERMINE THE HEAT TRANSFER FROM AN
C INVESTMENT CASTING. IT IS BASED IN PART ON THE WORK BY ROSHEN.
C ADDED TO IT IS THE CAPABILITY TO CAST AT SUPERHEAT AND TO ACCOUNT
C FOR SHRINKAGE OF THE METAL FROM THE MOLD WALL.
C
C 1) THE METAL PROPERTIES ARE INTRODUCED, INCLUDING THE
C    THE DIMENSIONS OF THE METAL CASTING;
C 2) THE WALL PROPERTIES ARE READ INTO THE PROGRAM WHICH IN-
C    CLudes THE DIMENSIONS OF THE INDIVIDUAL
C    PHASES OF MATERIALS;
C 3) THE NUMBER OF SEGMENTS THE WALL IS TO BROKEN INTO WILL
C    BE ENTERED AS WELL AS THE NUMBER OF TIME ITERATIONS.
C
C THIS PROGRAM ALSO WILL GENERATE THE SOLIDIFICATION FRONT, FROM THIS
C THE FRONT'S VELOCITY CAN BE COMPUTED. THE END RESULT OF THIS WORK
C IS TO RELATE THE THICKNESS OF THE WALL TO THE TIME FOR COMPLETE
C SOLIDIFICATION FOR A GIVEN AMOUNT OF SOLIDIFYING MATERIAL.
C THE INPUT IS DONE IN THREE PARTS:
C
C THE METAL PROPERTIES ARE READ IN
C
READ 23, METAL, ST, DENM, CP, RF, RI, HT, FT

C THE MOLD PROPERTIES ARE READ INTO THE PROGRAM
C
READ 27, EP, RO, WI, HC
READ 24, (THMAT(IK), IK=1,4)
READ 26, NW, NIT

C THE INITIAL TEMPERATURE OF THE MOLD IS CALLED AT; AND IS INSERTED AT
C THIS POINT; ALSO THE STEPHAN-BOLTZAN CONSTANT.
C

BC = 1.36821E-12
AT = 298.0

C THE INTERVALS ARE GENERATED AT THIS POINT. DX IS THE CHANGE IN
C THE RADIUS, AND DT IS THE CHANGE IN TIME. DELX IS THE DIMENSIONLESS
C CHANGE IN RADIUS. THE DT VALUE IS ENTERED DIRECTLY INTO
C THE PROGRAM.

RI = RO = WT
DX = WT/(NIW - 1)
DELX = DX/RI

C THE METAL PROPERTIES AND MOLD PROPERTIES ARE READ OUT

PRINT 51

51 FORMAT (1H1)
PRINT 10, METAL, DENM, CP, ST, HF, RI, FT
PRINT 18, (THMAT(IQ), IQ = 1, 4)
PRINT 16, EP, HC, RO, DX, DT

C THE READINGS FOR THE RADIUS INCREMENT AND TIME INCREMENT ARE
C GENERATED.

DO 210 I = 1, NIW
RH(I) = DX*(I - 1)

210 CONTINUE
PRINT 11, (RH(IY), IY = 1, NIW), RI

C THE INITIAL CONDITIONS ARE INSERTED IN THE PROGRAM.

99 T(1, 1) = FT
TH(1, 1) = 0.0
TH1(1) = 0.0
DO 201 KF = 2, 20
TH(1, KF) = 0.745
T(1,KF) = (TH(1,KF)*(AT-FT)) + FT

CONTINUE
PRINT 12, TH1(1)*(TH(1,N)*N=1 NIW)

C CONSTANT AND COUNTERS THAT ARE NEEDED IN THE PROGRAM ARE INTIALIZED
C
IT=0
KI=0
NC=1
R(1)=RI
TW=AT

C THE ITERATION BEGINS. THIS PROGRAM USES A FORWARD DIFFERENCE
C SOLUTION TO THE SECOND ORDER DIFFERENTIAL EQUATION THAT DESCRIBES
C THIS SYSTEM.

DO 400 K=1,NIT
KI=1
TH1(KI)=DT*K
501 MW=NIW+1
37 WT=0.0

C THE ITERATION PROCEEDS ROW BY ROW DETERMINING
C EACH RADIUS LOCATION AS DETERMINED BY DELX DELT, WHICH IS THE
C DIMENSIONLESS TIME STEP IS DETERMINED BY THE THERMAL DIFFUSIVITY
C AND THE DT VALUE. A SUBROUTINE IS USED TO DETERMINE
C THE THERMAL DIFFUSIVITY IN THE WALL FOR A GIVEN LOCATION AND
C TEMPERATURE. THE PROPERTIES DETERMINED IN THIS SUBROUTINE
C ARE THE THERMAL CONDUCTIVITY DENSITY AND HEAT CAPACITY.

DO 300 KJ=2,MW
TEMP=T(KI,KJ)
ID=0
67 CALL TC2(ID,THMAT,KJ,TEMP,DENS,TC,HCAP,ID)
IF(ID.EQ.0) GO TO29
\[ \text{TH}(\text{KI} \cdot \text{KJ}) = (\text{T}(\text{KI} \cdot 1 \cdot \text{KJ}) - \text{FT}) / (\text{AT} - \text{FT}) \]

GO TO 300

\[ \text{TD} = \text{TC} / (\text{DENS} \cdot \text{HCAP}) \]

\[ \text{RC} = (\text{RI} \cdot (\text{KJ} \cdot \text{DELX})) / \text{RI} \]

\[ \text{TD1} (\text{KI} \cdot \text{KJ}) = \text{TD} \]

\[ \text{DELT} = \text{DT} \cdot \text{TD} / (\text{RI} \cdot \text{RI}^2) \]

\[ \text{A} = (\text{TH}(\text{KI} \cdot \text{KJ} \cdot 1) - \text{TH}(\text{KI} \cdot \text{KJ})) / \text{DELX} \]

\[ \text{B} = (\text{TH}(\text{KI} \cdot \text{KJ} \cdot 1) - 2 \cdot \text{TH}(\text{KI} \cdot \text{KJ}) \cdot \text{TH}(\text{KI} \cdot \text{KJ} = 1)) / \text{DELX} \cdot \text{RI}^2 \]

\[ \text{TH}(\text{KI} \cdot 1 \cdot \text{KJ}) = \text{TH}(\text{KI} \cdot \text{KJ}) \cdot \text{DELX} \cdot ((\text{A} / \text{RC}) - \text{B}) \]

\[ \text{T}(\text{KI} \cdot 1 \cdot \text{KJ}) = (\text{TH}(\text{KI} \cdot 1 \cdot \text{KJ}) \cdot (\text{AT} - \text{FT})) \cdot \text{FT} \]

A CHECK IS MADE NOW TO INSURE THAT THE NEWLY FOUND VALUE IS WITHIN 100 DEGREES OF THE TEMPERATURE THAT IS USED TO FIND THE PROPERTIES OF THE WALL. IF IT IS NOT WITHIN THE HUNDRED DEGREE LIMIT, THE PROGRAM RETURNS TO THE BEGINNING OF THE ITERATION TO GET THE PROPERTIES FROM THE SUBROUTINE THAT ARE MORE COMPATIBLE WITH THIS TEMPERATURE.

\[ \text{IF} (\text{TH}(\text{KI} \cdot 1 \cdot \text{KJ}) = \text{TEMP-\text{LT}} \cdot 100) \text{ GO TO 81} \]

\[ \text{TEMP} = \text{T}(\text{KI} \cdot 1 \cdot \text{KJ}) \]

GO TO 67

THE TEMPERATURE OF THE INSIDE OF THE WALL IS DETERMINED NOW. THIS TEMPERATURE IS DETERMINED HEAT THAT IS TRANSFERRED TO THE MOLD WALL FROM THE CASTING. THIS HEAT IS FUNCTION OF ONE OF THREE THINGS: 1) SUPERHEAT IN THE MELT, 2) HEAT OF FUSION DURING SOLIDIFICATION AND 3) THE ABILITY OF HEAT TO BE TRANSFERRED ACROSS THE GAP FORMED BY METAL SHRINKAGE. ONE OF THESE IS CONSIDERED DEPENDING ON THE CONDITIONS OF THE MELT, i.e., WHETHER OR NOT SOLIDIFICATION HAS OCCURRED OR NOT.

\[ \text{IF} (\text{KJ} \cdot \text{NE} \cdot 2) \text{ GO TO 300} \]

\[ \text{DT1} = \text{T}(\text{KI} \cdot 1) - \text{T}(\text{KI} \cdot 1 \cdot 2) \]

\[ \text{TEMP1} = \text{T}(\text{KI} \cdot 1 \cdot 2) \]

\[ \text{TEMPA} = \text{T}(\text{KI} \cdot 1) \]
CALL SPRHT(DX, DT1, DENM, RI, HGT, TC, DT, TEMPI, ST, TEMPA, IT, TP)

T(KI*1, 1)=TEMPA
TH(KI*1, 1)={T(KI*1, 1)-FT}/(AT-FT)

C THE TEMPERATURE FROM THE SPRHT(SUPERHEAT) SUBROUTINE MUST BE
C EVALUATED TO SEE IF IT IS GREATER THAN THE SOLIDIFICATION
C TEMPERATURE. IF IT IS, THEN THE PROGRAM SKIPS THE SOLIDIFICATION
C RADIUS CHANGE SECTION, BUT IF THE TEMPERATURE IS EQUAL TO OR
C LESS THAN THE FUSION TEMPERATURE, THE RADIUS IS CHECKED
C TO SEE IF SOLIDIFICATION IS COMPLETE. IF THE INTERNAL RADIUS IS ZERO
C A FLAG IS GENERATED AND THE WALL TEMPERATURE WILL BE COMPUTED
C AS A FUNCTION OF THE HEAT CROSSING THE GAP BETWEEN THE
CMOLD AND CASTING.

C IF (T(KI*1, 1).GT. ST) GO TO 39
IF (R(KI).EQ.0.0) GO TO 300
T(KI*1, 1)=ST
GNU={(-TC*2*DT)/(HF*DENM*DX))
R(KI*1)=R(KI)*(GNU*T(KI*1, 1)-T(KI*1, 2))
IF (R(KI*1).GT.0.00) GO TO 300
R(KI*1)=0.000
IT=IT+1
GO TO 300

39 R(KI*1)=RI
300 CONTINUE

C THE OUTSIDE WALL TEMPERATURE IS GENERATED NOW. THE OUTSIDE TEMPERATURE
C IS A FUNCTION OF THE HEAT TRANSFERRED BY CONVECTIVE AND RADIATIVE
C PROCESSES. BOTH OF THESE COEFFICIENTS ARE A FUNCTION OF THE TEMPERA-
C TURE OF THE WALL AS WELL AS THE AMBIENT. THE SUBROUTINE
C HTC COMPUTES THESE VALUES AND IT GENERATES THE TEMPERATURE OF THE
C WALL.

870 T1=T(KI*1, MW-1)
T2=T(KI*1, MW)
DT2=ABS(T1-T2)
GO TO 46
PRINT 57, T1, T2, DT2
57 FORMAT (/9X, "T1 VALUE", F10.5, "T2 VALUE", F10.5, "DT2 = ", F10.5)
46 IF (T1 NE T2) GO TO 58
TH = AT
GO TO 59
58 CALL HTC (AT, T1, T2, TH, DX, TC, BC, EP, HGT, DT2)
59 T(KI, 1, NIW) = TH
TH(KI, 1, NIW) = (TH - FT) / (AT - FT)
C A COUNTER IS INSERTED TO PRINT THE RESULTS AFTER THE SPECIFIED
C NUMBER OF ITERATIONS.
C**********************************************************************
40 IF (NC NE 10) GO TO 42
NC = 0
C**********************************************************************
C THE RESULTS OF ITERATION ARE PRINTED OUT IN DIMENSIONLESS TEMPERATURE
C BUT THE SOLIDIFICATION FRONT IS PRINTED OUT IN CM.
C**********************************************************************
.56 PRINT 12, TH1(KI), (TH(KI, 1, NIW), N = 1, NIW), R(KI), TDI(KI, KJ = 1)
42 R(KI) = R(KI + 1)
DO 400 LI = 1, NIW
   TH(LI) = TH(KI, LI)
   T(LI) = T(KI, LI)
400 CONTINUE
1000 CONTINUE
C FINALLY THE FORMAT STATEMENTS
C**********************************************************************
COINSIDE DIAMETER OF THE CASTING$F10.5/\times9$
COPOURING TEMP.$F10.5$

18 FORMAT(/\times9*THE PROPERTIES OF THE INVESTMENT MATERIAL*9
C$1X9*WALL DIMENSIONS*9/1X9*BINDER*F10.5/91X9*AFS=140*F10.59
C$1X9*AFS=80*F10.5/91X9*AFS=40*F10.5$

16 FORMAT(/91X9*EMISSIVITY*F10.59
C$1X9*HEAT TRANSFER COEFFICIENT*F10.5/91X9*OUTSIDE RADIUS*9
CE10.5/91X9*DELTA VALUES*9/92X9*DELTA RADIUS*F6.49
C$2X9*DELTA TIME*F10.5/9$

11 FORMAT(1X9TIME*6X13(F4.2*CM*),5X9INITIAL RADIUS=*F7.3)
14 FORMAT(1H19*CHANGE IN THE TEMPERATURE*)
12 FORMAT(1X9F5.0*SEC*1X13F6.37X9F10.5F10.5)
15 FORMAT(1X9F5.0*SEC*4X20F5.3)
23 FORMAT(A5,F7.2,F7.2,F7.4,F7.4,F5.2,F7.2,F7.2)
24 FORMAT(4F7.5)
27 FORMAT(4F10.5)
26 FORMAT(215)
STOP
SUBROUTINE TC2(DX, THMAT, KJ, TEMP, DENS, TC, HCAP, ID)
COMMON/BLK2/T(10), CK(4,10), DEN(6), THMAT(1)

C THIS SUBROUTINE FIND THE THERMAL CONDUCTIVITY, DENSITY, AND HEAT
C CAPACITY FOR THE RESPECTIVE TEMPERATURE THAT IS BEING USED IN THE
C MAIN PROGRAM. THE THERMAL CONDUCTIVITY DATA IS SELECTED BY
C TEMPERATURE RANGES. ENTRY INTO THIS PROGRAM PRESUPPOSES THE FACT THAT
C THE TEMPERATURE RANGE AT WHICH THESE VALUES ARE TO BE FOUND IS
C KNOWN.

C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

C THERE ARE NINE TEMPERATURE RANGES THAT ARE OF INTEREST. THEY ARE
T(1)=298.0
T(2)=473.0
T(3)=773.0
T(4)=1073.0
T(5)=1273.0
T(6)=1373.0
T(7)=1573.0
T(8)=1773.0
T(9)=2073.0

C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

C THE VALUES FOR THE THERMAL CONDUCTIVITY FOR THE WALL ARE FED INTO
C THE PROGRAM
DATA(CK(1,IK),IK=1,9)/0.503,0.33,0.38,
C0.35,0.55,0.5878/ 
DATA(CK(2,IK),IK=1,9)/0.25,0.21,0.28,0.32,0.33,
C0.35,0.42,0.52,0.65/ 
DATA(CK(3,IK),IK=1,9)/0.32,0.28,0.31,0.32,
C0.33,0.35,0.34,0.5,0.55/ 
DATA(CK(4,IK),IK=1,9)/0.31,0.2,0.25,0.3,0.31,
C0.38,0.48,0.55,0.62/ 

C~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
C THE VARIOUS DENSITIES VALUES FOR THE WALL ARE FED INTO THE PROGRAM
  DATA (DEN(IF), IF = 1:4) / 1.73, 1.72, 1.60, 1.9 /
C

C THE HEAT CAPACITY OF THE WALL IS NOW FED INTO THE PROGRAM
C THE HEAT CAPACITY IS A FUNCTION OF THE APPROPRIATE TEMPERATURE.
  IF (TEMP.GT.848) GO TO 71
    HCAP = (11.22) * (8.20E-3 * TEMP) - (2.70E5 / TEMP)**2
    GO TO 45
  71
    HCAP = (14.41) * (1.94E-3 * TEMP)
  45
    CONTINUE
    HCAP = HCAP / 60.00
C
C THE POSITION IN THE WALL IS NOW DETERMINED. IF THE NODE OF
C INTEREST STRADLES A MATERIAL INTERFACE, A WEIGHTED AVERAGE OF THE
C MATERIALS PRESENT MUST BE CALCULATED. THIS WEIGHTED PERCENTAGE
C IS USED LATER IN THE DETERMINATION OF THE THERMAL CONDUCTIVITY.
  72
    TH1 = 0
    J1 = 0
    ALOC = DX*(KJ-1)
    BLOC = DX*(KJ-2)
    DO 100 I = 1, 4
      TH1 = TH1 + THMAT(I)
      DIFF1 = ALOC = TH1
      DIFF2 = BLOC = TH1
      IF (DIFF1.GE.0.0 .AND. DIFF2.LE.0.0) GO TO 30
      IF (DIFF1.LT.0.0 .AND. DIFF2.GT.0.0) GO TO 20
      100 CONTINUE
  20
    PT1 = 1.0
    PT2 = 0.0
    DENS = DEN(I)
    GO TO 24
PT2 = DIFF/1/DX
PT1 = 1.0 - PT2

C THE DENSITY OF INTEREST IS A FUNCTION OF LOCATION IN THE WALL
DENS = (PT2 * DEN(1:1)) * (PT1 * DEN(I))

C THE THERMAL CONDUCTIVITY OF THE WALL AT THE KNOWN TEMPERATURE
C AND THE KNOWN POSITION IS NOW CALCULATED. THE NINE TEMPERATURE RANGES
C ARE SCANNED TO SEE WHICH ONE SHOULD BE USED FOR INTERPOLATION.

DO 200 IW = 2, 9
   IF (T(IW) - TEMP) 200, 35, 35
   CONTINUE
200
   P = (TEMP - T(IW-1)) / (T(IW) - T(IW-1))
   TCB = ((P * (CK(I, IW) - CK(I, IW-1))) * CK(I, IW-1))
   TC1 = ((P * (CK(I, 1, IW) - CK(I, 1, IW-1))) * CK(I, 1, IW-1))
   TC = TCB * PT1 * TC1 * PT2
   TC = 4.1365E-3 * TC
   CONTINUE

C THE PROGRAM RETURNS TO THE MAIN PROGRAM TO COMPUTE THE THERMAL
C DIFFUSIVITY FROM THE VALUES ARRIVED AT IN THIS PROGRAM.

RETURN
END
SUBROUTINE SPRHT(DX,DTI,DENMI,RI,HGT,TC,DT,TEMP1,ST,TEMPA,IT,TP)
C*******************************************************************************
C THIS SUBROUTINE GENERATES THE INSIDE WALL TEMPERATURE FOR ONE OF
C THE SITUATIONS THAT EXISTS ON THAT INSIDE SURFACE.
C THE PROGRAM IS CONCERNED WITH TWO SITUATIONS:
C 1) THE CONDITION OF SUPERHEAT
C 2) THE CONDITION OF INTERIOR METAL SHRINKAGE.
C*******************************************************************************

C 1) THIS SECTION FINDS THE INSIDE WALL TEMPERATURE FOR A CASTING MADE
C WITH A GIVEN AMOUNT OF SUPERHEAT.
SLOPE=DT1/DX
EP=7.0/26.0
VOL=3.14159*(RI#2)*HGT
AREA=3.14159#2*RI#HGT
IF (IT.GT.0) GO TO 45
A=(TC#TEMP1)/DX
B=(CP#RI#DENM#TEMPA)/(2#DT)
C=(DX#2#DT)/(TC#2#DT)*(CP#DENM#RI#DX))
TEMPA=C*(A*B)
GO TO 35

C*******************************************************************************

C 2) THIS SECTION OF THE PROGRAM FINDS THE TEMPERATURE OF THE INSIDE
C WALL ASSUMING THAT A SHRINKAGE GAP IS PRESENT.
IF (IT.GT.1) GO TO 46
TP=TEMPA
A1=TC/(CP#DENM#VOL)
B1=(DT1#AREA#DT)/(DX)
TB=TP-(A1*B1)
HI=0.005
A2=(DX)/(TC*(HI#DX))
B2=(TEMP1#TC)/DX
TWN=A2*(B2*(HI#TB))
30    CK=ABS(TWN-TEMPA)
      IF (CK.LT.5.0) GO TO 55
      TEMPA=TWN
      TP=TB
      GO TO 45
55    TP=TB
      TEMPA=TWN
      IT=IT+1

CTHE CONTROL IS RETURNED TO THE MAIN PROGRAM NOW KNOWING THE INSIDE
CSURFACE TEMPERATURE OF THE WALL.
C
35    RETURN
END
SUBROUTINE HTC(ATT, T1, T2, TW, DX, TC, BC, EP, HGT, DT2)

C THIS SUBROUTINE COMPUTES THE OUTSIDE SURFACE MOLD WALL TEMPERATURE
C USING KNOWN EQUATIONS FOR THE CONVECTIVE HEAT TRANSFER COEFFICIENTS,
C AND RADIATIVE HEAT TRANSFER COEFFICIENTS. THIS SURFACE TEMPERATURE IS
C THEN RETURNED TO THE MAIN PROGRAM.

IF(TW EQ AT) GO TO 35

30 A1 = (TW**4 - AT**4) / (TW - AT)
HR = BC*EP*A1
B1 = (TW - AT)**0.33333333
HC = 0.21*H1
HC = (HC**4 - 1365E-3) / (12*2.54)
H = HC + HR
A2 = DX / (TC - (H*DX))
A3 = (TC*T2) / DX
A4 = H*A

T3 = A2*(A3*A4)
CHK = ABS(T3 - TW)
IF(CHK LE 1) GO TO 40
TW = T3
GO TO 30

35 TW = AT
GO TO 40

END
APPENDIX C

THE AVERAGE THERMAL DIFFUSIVITY

The average thermal diffusivity was found by taking the average of all the thermal diffusivities that were used on the last iteration of the program. This value therefore is a function of the conditions of the casting. In Table C.1 are listed the thermal diffusivities of the pure silica material given as a function of temperature and material. On Table C.2 are listed the thermal diffusivities used for the various results given by the computer program. The value varies around 0.002 cm²/sec.
Table C.1. Thermal diffusivities as a function of temperature and material.

<table>
<thead>
<tr>
<th>°K</th>
<th>PIW</th>
<th>PIW+S1</th>
<th>PIW+S2</th>
<th>PIW+S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.0019</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0020</td>
</tr>
<tr>
<td>400</td>
<td>0.0019</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.0018</td>
</tr>
<tr>
<td>500</td>
<td>0.0019</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.0017</td>
</tr>
<tr>
<td>600</td>
<td>0.0020</td>
<td>0.0018</td>
<td>0.0018</td>
<td>0.0020</td>
</tr>
<tr>
<td>700</td>
<td>0.0020</td>
<td>0.0019</td>
<td>0.0018</td>
<td>0.0022</td>
</tr>
<tr>
<td>800</td>
<td>0.0020</td>
<td>0.0019</td>
<td>0.0018</td>
<td>0.0022</td>
</tr>
<tr>
<td>900</td>
<td>0.0020</td>
<td>0.0019</td>
<td>0.0018</td>
<td>0.0022</td>
</tr>
<tr>
<td>1000</td>
<td>0.0021</td>
<td>0.0020</td>
<td>0.0019</td>
<td>0.0022</td>
</tr>
<tr>
<td>1100</td>
<td>0.0021</td>
<td>0.0020</td>
<td>0.0019</td>
<td>0.0023</td>
</tr>
<tr>
<td>1200</td>
<td>0.0022</td>
<td>0.0021</td>
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<td>0.0023</td>
</tr>
<tr>
<td>1300</td>
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<td>0.0021</td>
<td>0.0020</td>
<td>0.0023</td>
</tr>
<tr>
<td>1400</td>
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<td>0.0021</td>
<td>0.0020</td>
<td>0.0023</td>
</tr>
<tr>
<td>1500</td>
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<td>0.0022</td>
<td>0.0020</td>
<td>0.0024</td>
</tr>
<tr>
<td>1600</td>
<td>0.0025</td>
<td>0.0022</td>
<td>0.0021</td>
<td>0.0024</td>
</tr>
</tbody>
</table>
Table C.2. Average thermal diffusivities used for Figures 20, 21, 22.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Superheat °C</th>
<th>Preheat °C</th>
<th>( \bar{\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Nickel cast into silica mold</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:5</td>
<td>+50</td>
<td>200</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>200</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0024</td>
</tr>
<tr>
<td>1:4</td>
<td>+50</td>
<td>200</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>200</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+50</td>
<td>500</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0024</td>
</tr>
<tr>
<td>1:2</td>
<td>+50</td>
<td>200</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>200</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>+50</td>
<td>500</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0024</td>
</tr>
<tr>
<td>1:1</td>
<td>+50</td>
<td>200</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>200</td>
<td>0.0021</td>
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<td></td>
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<td>200</td>
<td>0.0022</td>
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<td>500</td>
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<tr>
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<td>+150</td>
<td>200</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td>+50</td>
<td>500</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0022</td>
</tr>
<tr>
<td>4:1</td>
<td>+50</td>
<td>200</td>
<td>0.0019</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>200</td>
<td>0.0019</td>
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<tr>
<td></td>
<td>+50</td>
<td>500</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>+150</td>
<td>500</td>
<td>0.0021</td>
</tr>
<tr>
<td></td>
<td><strong>Copper cast into silica mold</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1:5</td>
<td>50</td>
<td>--</td>
<td>0.0022</td>
</tr>
<tr>
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<td>50</td>
<td>200</td>
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<tr>
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<tr>
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Table C.2.--Continued  Average thermal diffusivities used for Figures 20, 21, 22.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Superheat °C&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Preheat °C</th>
<th>$\bar{\alpha}$</th>
</tr>
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Nickel cast in silica/zircon mold

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<th>Ratio</th>
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<th>Preheat °C</th>
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Table C.2.—Continued  Average thermal diffusivities used for Figures 20, 21, 22.

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<sup>a</sup>Amount of superheat above fusion temperature.
APPENDIX D

RESULTS OF THE TEST CASTINGS

The following graphs are the plots of the test castings and the respective computer simulations. The results are shown in time (sec) versus dimensionless temperature. The formula used for dimensionless temperature was:

\[ \theta = \frac{T^* - T_S}{T_A - T_S} \]

The top curve is the plot of the temperature in the metal and the bottom is the temperature of the outside surface of the mold wall.
Fig. D.1. Casting #1, Al, metal—710°C, mold—25°C, wall ratio—1:3.
Fig. D.2. Casting #2, Ni, metal—1502, preheat—200, wall ratio—1:4.
Fig. D.3. Casting #3, Cu, metal—1435, mold—200, wall ratio—1:3.
Fig. D.4. Casting #4, Cu, metal—1410, preheat—350, wall ratio—1:3.
Fig. D.5. Casting #5, Cu, metal—1410, preheat—200, wall ratio—1:5.
Fig. D.6. Casting #6, Cu, metal—1460, preheat—200, wall ratio—1:5.
Fig. D.7. Casting #7, Cu, metal--1410, preheat--200, wall ratio--1:4.
Fig. D.8. Casting #8, Cu, metal—1460, preheat—200, wall ratio—1:3.
Fig. D.9. Casting #9, Cu, metal—1410, preheat—200, wall ratio—1.5.
APPENDIX E

SOLUTIONS TO THE SEMI-INFINITE CASE

The computer simulation is limited by the case of the semi-infinite wall as the mold wall becomes thicker. This tendency is discussed in the discussion section of the text.

The solution for the semi-infinite wall is as follows.

The heat flux of a semi-infinite wall is given by:

\[ q = \frac{\sqrt{k \rho C}}{\sqrt{\pi t}} (T_m - T_0) \]

Remembering that

\[ \alpha = \frac{k}{\rho C_p} \]

and

\[ Q = q(At) \]

Equation (1) becomes:

\[ Q = \frac{k}{\sqrt{\alpha \pi}} (T_m - T_0)A(t)^{1/2} \]

The heat given up by the melt to the wall which has a certain amount of superheat and heat of fusion is:

\[ Q_m = \rho' \gamma H_f + C_p \rho' \Delta T \]
Equating $Q$ and $Q_m$ we obtain:

$$\rho' V H_f + C_p V \rho \Delta T = \frac{k}{\sqrt{\pi \alpha T}} (T_m - T_o) A(t)^{1/2}$$

Solving for $(t)^{1/2}$:

$$t^{1/2} = \left[ \frac{\pi \alpha}{k A (T_m - T_o)} \right]^{1/2} \rho' V [H_f + C_p \Delta T]$$

where: $\Delta T = T_m - T_s$.

- $T_m = \text{temp of original melt, °C}$.  
- $T_s = \text{solidification temp, °C}$.  
- $T_o = \text{initial temp of wall, °C}$.  
- $\rho' = \text{density of metal = g/cm}^3$.  
- $k = \text{thermal conductivity of mold material cal/sec cm °C}$.  
- $\alpha = \text{thermal diffusivity of mold material sec/cm}^2$.  
- $A = \text{area of mold-metal interface, cm}^2$.  
- $V = \text{volume of metal, cm}^3$.

Rewriting (5) we obtain:

$$t^{1/2} = \left[ \frac{\pi \alpha}{2k (T_m - T_o)} \right]^{1/2} \rho' r [H_f + C_p \Delta \tau]$$

where: $r = \text{radius of the casting, cm}$.

We can solve (6) and find the limits of the computer program.
Table E.1. Application of semi-infinite solution to computer cases.

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$^a$Amount of superheat above fusion temperature.
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