DETERMINATION OF POSITION AROUND NEAR-EARTH ASTEROIDS USING COMMUNICATION RELAYS

Evan Nelson, Charles D. Creusere, Eric Butcher *

Klipsch School of Electrical and Computer Engineering
New Mexico State University, Las Cruces, NM
* University of Arizona, Tucson, AZ

ABSTRACT

In this paper we consider the possibility of using a communications system that is operating between probes on the surface of an asteroid and an orbiting satellite to more accurately determine spatial positions. This is done by measuring the round trip communication delay between the orbiter and various surface probes to estimate distance. From these distance measurements, the position can be determined using trilateration—the same basic technique behind the earth-based GPS system. Within the framework of this scenario, the location of the probes or the orbiter can be determined depending on the scenario.

Keywords: Space communications, space communication networks, NEA missions, Position Determination, Trilateration

1 INTRODUCTION

Recently, there has been a great deal of interest in sending robotic precursor missions to near-earth asteroids (NEAs) for scientific study and to prepare for a possible manned mission in the mid-2020s. In particular, increased understanding of these small irregularly-shaped bodies is essential to the development of an NEA deflection strategy which might someday be implemented for planetary protection. In the far future, NEAs might also serve as fueling stations, mining sites, or remote observatories, and recent interest in asteroid exploration has been demonstrated by past, present, and future missions such as NEAR Shoemaker, Hayabusa, DAWN, and OSIRIS REx.

The research presented here considers the need for determining the positions of objects either in orbit or on the surface of the asteroid. This paper considers a low cost way of determining the relative positions of objects in the vicinity of an NEA. Specifically, we consider two different but closely related application scenarios, both of which use the round-trip communication delay between different objects to determine the distance between them. The first is a way of determining position in space above a set of probes on the surface. Using

*Research supported by NASA/EPSCORE Grant 1065603

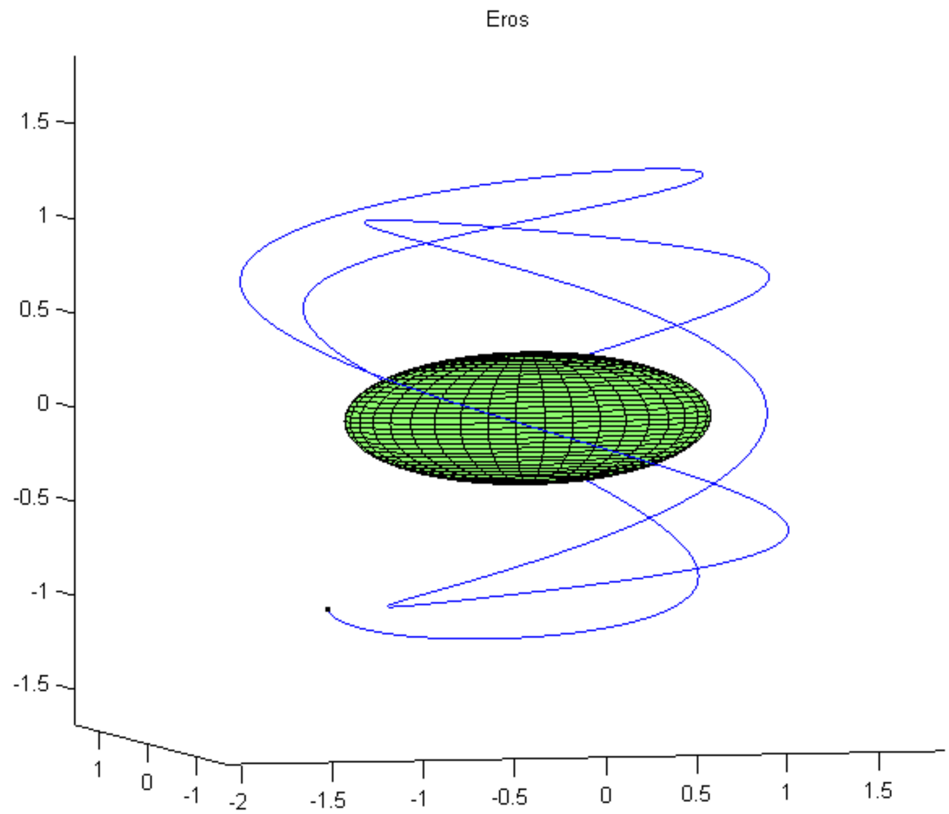

Figure 1: Simulated spacecraft orbit around asteroid Eros shown in the asteroid body-fixed frame.

the distances between the probes on the surface with known positions the location of the orbiter in space can be determined, and we consider how this can be applied to tracking the path of an object in orbit while reducing the number of calculations required. The application scenario considers using an orbiting satellite to determine the location of a fixed point on the surface given that the satellite's path is known. The orbiter takes multiple measurements of distance along its path, and it uses these to trilaterate the location of the object on the surface. We note that the orbit around an asteroid can be irregular compared to that of a standard orbit as seen in figure 1. This can be helpful here because it results in orbital range sample points that are less co-linear than those of standard orbit which can improve the results of the trilateration. On the flip side, however, this also makes it harder to model how the orbiter is moving in order to accurately estimate its position. We modeled the asteroids in our simulations as scalene ellipsoids with constant densities and the orbiter as a point mass which is based on the work of Scheeres [**?**]. For more on the orbital modeling we used here see our previous paper [**?**].

2 TRILATERATION

Trilateration is a method to determine the location of a point by using measurements of distances. In the three-dimensional case, this is done by measuring the distance to at least three known points. Trilateration is then accomplished by taking each of these measurements and forming a set of spheres were each point is the center with the radii being the different distance measurements. The desired location is then determined to be the point were the spheres intersect. The equations for these spheres is

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = r_i \tag{1}$$

where (x, y, z) is the point in space being evaluated, (x_i, y_i, z_i) is the center point of the sphere, and r_i is the radius. Solving these set of equations directly is not very practical, however.

A common way to solve this problem in a more computationally tractable manner is to linearize it. This can be accomplished by using a fourth point which changes the problem from that of finding the point of intersection between spheres into one of finding the point of intersection between several planes [**?**]. This results in a linear system that can be solved using least squares. This linear system written in matrix form is

$$\mathbf{A}\vec{x} = \vec{b} \tag{2}$$

where

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ \vdots & \vdots & \vdots \\ x_i - x_1 & y_i - y_1 & z_i - z_1 \end{pmatrix}, \ \vec{x} = \begin{pmatrix} x - x_1 \\ y - y_1 \\ z - z_1 \end{pmatrix}, \ \vec{b} = \begin{pmatrix} b_{21} \\ b_{31} \\ \vdots \\ b_{n1} \end{pmatrix} \tag{3}$$

with

$$b_{ij} = \frac{1}{2}\left[r_j^2 - r_i^2 + \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}\right]. \tag{4}$$

This linear method works very well in the noiseless case, but when there is measurement noise its performance degrades significantly.

A potentially better way to solve the trilateration problem is to use an iterative non-linear least squares method [**?**]. For this application, we chose to use the Gauss-Newton algorithm. We begin by defining a cost function which is the sum of the squared errors of the distances:

$$F(x) = \sum_{i=1}^{n} f_i(x, y, z)^2 \tag{5}$$

with

$$f_i(x, y, z) = \hat{r_i} - r_i = \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2} - r_i. \tag{6}$$

The next step is defining a vector $\vec{f}$ such that $F(x) = \vec{f}^t \vec{f}$ and finding the Jacobian $\mathbf{J}$ of $\vec{f}$, i.e.,

$$\vec{f} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}, \mathbf{J} = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x} & \frac{\partial f_n}{\partial y} & \frac{\partial f_n}{\partial x} \end{pmatrix}. \tag{7}$$

We then compute the first derivative and approximate the second derivative as follows:

$$\mathbf{F}' = 2\mathbf{J}^T \vec{f}, \tag{8}$$

$$\mathbf{F}'' \approx 2\mathbf{J}^T \mathbf{J}. \tag{9}$$

From this, we can iteratively solve for the solution $\vec{R}$

$$\vec{R} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{10}$$

using the equation

$$\vec{R}_{k+1} = \vec{R}_k - (\mathbf{J}_k^T \mathbf{J}_k)^{-1} \mathbf{J}_k^T \vec{f}_k \tag{11}$$

where $\mathbf{J}^T\mathbf{J}$ and $\mathbf{J}^T\vec{f}$ are defined as

$$\mathbf{J}^T\mathbf{J} = \begin{pmatrix} \sum_{i=1}^{n} \frac{(x-x_i)^2}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(x-x_i)(y-y_i)}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(x-x_i)(z-z_i)}{(f_i+r_i)^2} \\ \sum_{i=1}^{n} \frac{(x-x_i)(y-y_i)}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(y-y_i)^2}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(y-y_i)(z-z_i)}{(f_i+r_i)^2} \\ \sum_{i=1}^{n} \frac{(x-x_i)(z-z_i)}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(y-y_i)(z-z_i)}{(f_i+r_i)^2} & \sum_{i=1}^{n} \frac{(z-z_i)^2}{(f_i+r_i)^2} \end{pmatrix}, \mathbf{J}^T\vec{f} = \begin{pmatrix} \sum_{i=1}^{n} \frac{(x-x_i)f_i}{(f_i+r_i)^2} \\ \sum_{i=1}^{n} \frac{(y-y_i)f_i}{(f_i+r_i)^2} \\ \sum_{i=1}^{n} \frac{(z-z_i)f_i}{(f_i+r_i)^2} \end{pmatrix}. \tag{12}$$

This method requires that an initial guess to be made about the position; the linear method described earlier provides a good initial guess for this algorithm. Equation (11) is iterated until its rate of change drops below a certain threshold or until a set amount of iterations have occurred.

3 ORBITER POSITION DETERMINATION

The first scenario we consider is one in which the location of the probes on the surface are known and the location of the orbiter is determined by trilateration. In this simulation, four probes are placed on the surface of the asteroid in the formation of a triangle with one inside the triangle. For the simulations using only three probes, the probe inside of the triangle is not used. We then numerically evaluate the relative performance of the linear and nonlinear position-finding algorithms described above. For the nonlinear algorithm, sets of both 3 and 4 probes are used find the orbiter's location. In all of the plots that follow, the linear algorithm is referred to as 'LLS 4 probes' and the performance over one orbital pass is evaluated. Furthermore, the previously calculated spacecraft location is always used as the initial guess for the new location calculation, and a fixed number of iterations is used for the nonlinear algorithm. By fixing the number of iterations, we also fix the computational complexity of the algorithm and thus can more fairly compare the different cases. Finally, zero mean Gaussian noise with a variance of 0.1m is added to the distance measurements and the results are averaged over 100 simulations.

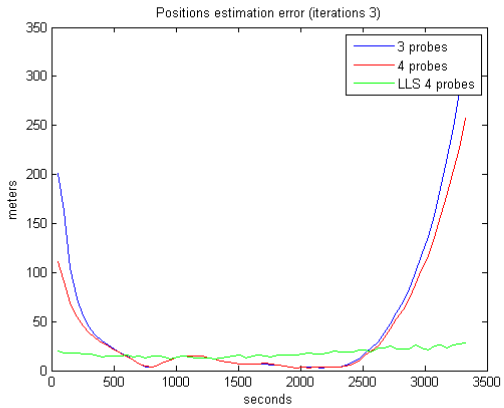

Figure 2: Position error after 3 iterations

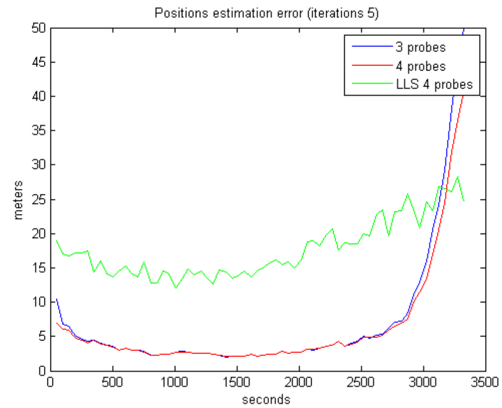

Figure 3: Position error after 5 iterations

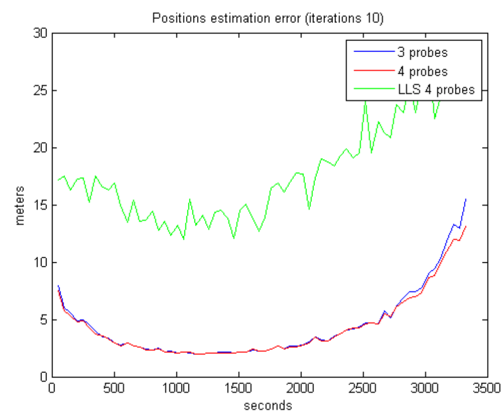

Figure 4: Position error after 10 iterations

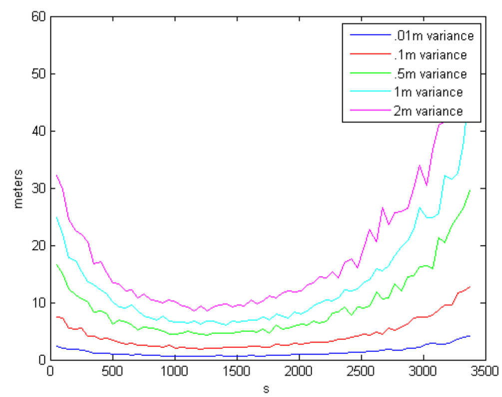

Figure 5: Position error with 4 probes at different noise levels

Figures 2 through 4 show the position estimation error of the orbiter's location for varying numbers of (nonlinear algorithm) iterations at each spatial location in the orbital path. When looking at the graphs, we note that the error is much higher at the beginning and at the end of the plotted time interval. This corresponds with the beginning and end of the orbital pass over over the set of surface probes. At these times, the spacecraft is at a low angle relative to the probes on the surface, and thus, from the perspective of the spacecraft, some the probes appear to be nearly on top of others. At these low angles, the surface positions are close together, making accurate trilateration much more difficult. The linear method does not appear to have as much trouble with these poor angles, but it does have a serious problem with the measurement noise in general. The results from using 4 probes show slight improvement over 3 probes which is to be expected since having additional information from the added surface location should lead to better results.

We have performed another simulation that compares the positional accuracy with varying amounts of Gaussian noise added to the distance measurements. In this case only the nonlinear algorithm with 4 probes was used. Instead of using a fixed number of iterations, however, we allowed the algorithm to iterate to convergence. The resulting position estimates for each different noise level were averaged over 100 simulations. Figure 5 shows the position estimation error for the spacecraft as it passes over the surface probes for the differing amounts of noise, and we see that the error cause by measurement noise has a much greater effect at the beginning and end of the orbital pass. These are the times when the orbiter as a bad angle relative to the probes, so it comes as no surprise that the impact of the additive noise is greatly increased.

4 SURFACE POSITION DETERMINATION

The other scenario we consider in this paper is the situation in which the location of the orbiter is known and the location of a probe on the surface must be estimated. In this scenario, the distance to the probe is measured at different times as the orbiter passes overhead. The location of the probe is then estimated using the distances measured from different locations in orbit. Gaussian noise with variance $0.1m$ is again added to each distance measurements and the results are averaged over 100 simulations.

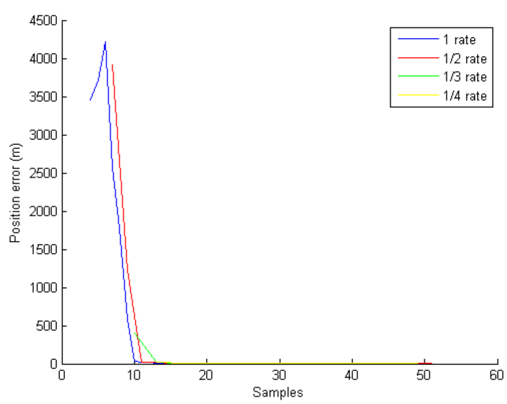

Figure 6: Surface position error.

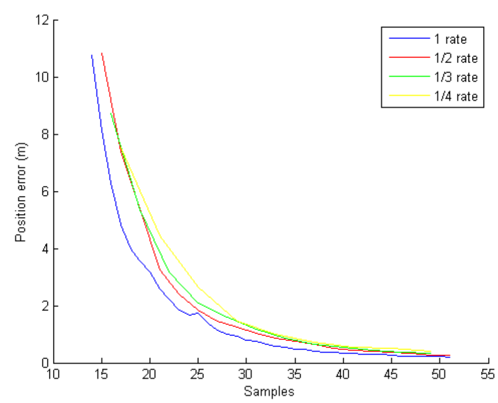

Figure 7: Surface position error starting from 14 samples.

Figures 6 and 7 show the resulting position estimation error for the surface probe with 51 different measurements taken as the spacecraft makes one pass across the surface. Figure 7 shows results from figure 6, but zoomed on the results from 14 samples on. The different rates shown on the plots are of the trilateration being performed using only every other spacecraft position point (1/2), every 3rd position point (1/3), and every 4th position point (1/4). We do this downsampling in order to compare the effects of spatial separation in our distance samples to the effects of using more data points in general. The results from figure 7 show that, while using more distance measurements results in less errors, it is more important to have measurements taken sufficiently far apart in order to get good results.

5 DELAY ANALYSIS

One issue in using the time it takes for a signal to be received and then retransmitted is that there are delays involved, during which time the spacecraft will have moved. These delays will caused by more than just the time it takes for a radio signal to travel through space to the asteroid surface and back— they will also be caused by the processing of the signal in the surface probe prior to retransmittion. In this section, we consider the effects of such delays by measuring how much the spacecraft moves with varying levels of delay around two asteroids, Vesta and Gaspra. The two asteroids will provide very different results, because they are very different in size: Vesta is 265 km wide while Gaspra is 9.5 km wide.

Table 1: Orbit details

Asteroid	Vesta	Gaspra
Average Velocity	329.99 m/s	7.605 m/s
Average distance to surface	496 km	13.3 km
Minimum distance to surface	223 km	4.79 km
Maximum distance to surface	754 km	22.4 km
Average round trip time delay	1.655 ms	0.0447 ms
Minimum round trip time delay	0.747 ms	0.01599ms
Maximum round trip time delay	2.518 ms	0.07498ms

Table 2: Distance moved in orbit during communications with delays

Asteroid	Vesta	Gaspra
Average distance moved in orbit with 0 ms delay	0.5464 m	0.00034 m
Minimum distance moved in orbit with 0 ms delay	0.24635 m	0.00012 m
Maximum distance moved in orbit with 0 ms delay	0.830928 m	0.00057 m
Average distance moved in orbit with 0.1 ms delay	0.579399 m	0.00110 m
Minimum distance moved in orbit with 0.1 ms delay	0.279349 m	0.00088 m
Maximum distance moved in orbit with 0.1 ms delay	0.863927 m	0.00133 m
Average distance moved in orbit with 1 ms delay	0.87639 m	0.00795 m
Minimum distance moved in orbit with 1 ms delay	0.57634 m	0.00773 m
Maximum distance moved in orbit with 1 ms delay	1.16092 m	0.00818 m
Average distance moved in orbit with 10 ms delay	3.8463 m	0.07639 m
Minimum distance moved in orbit with 10 ms delay	3.54625 m	0.07617 m
Maximum distance moved in orbit with 10 ms delay	4.13083 m	0.07662 m
Average distance moved in orbit with 100 ms delay	33.5454 m	0.7609 m
Minimum distance moved in orbit with 100 ms delay	33.2454 m	0.7606 m
Maximum distance moved in orbit with 100 ms delay	33.8299 m	0.7611 m

Table 1 shows the details of the orbital simulations such as the average velocity of the spacecraft in orbit, and also the distances and time it take for communications to reach the surface. Table 2 shows the distance the spacecraft moves for when there are 0, 0.1ms, 1ms, 10ms, and 100 ms delays for each of the two asteroids. For a small asteroid like Gaspra, the processing delay does not have much of an effect as the velocities and distances involved are small. For a large asteroid like Vesta, on the other hand, the delays can result in a significant amount of movement which would result in larger errors in position determination.

6 CONCLUSION

In this paper, we have looked at possibility of using a communications system to determine relative position in the vicinity of an asteroid. First, we considered a scenario in which a set of probes on the surface are used to determine the relative position of a spacecraft in orbit. Second, we looked at a scenario where an orbiting spacecraft is used to determine the relative position of a probe on the surface. Furthermore, we also considered the effect of probe processing delays on the position estimate of the spacecraft. In future work, we hope to study the effects of combining the two approaches and using the spacecraft to improve the estimation of the position of the probes and visa versa by alternating between spacecraft position estimation and probe position estimation. This could result in low cost system for determining position around an asteroid.

References

[1] D. Scheeres, "Dynamics about uniformly rotating triaxial ellipsoids: application to asteroids," *Icarus*, vol. 110, no. 2, pp. 225 –238, august 1994.

[2] E. Nelson, C. Creusere, T. Critz, and E. Butcher, "Analysis of communication rates in the proximity of near-earth asteroids," in *Proc. International Telemetering Conference*, October 2013.

[3] W. S. Murphy, "Determination of a position using approximate distance and trilateration," Master's thesis, Colorado School of Mines, 2007.

[4] Y. Xiong, "Nonlinear least squares," 2014.