ENERGY EFFICIENT WATER-FILLING ALGORITHM FOR MIMO-OFDMA CELLULAR SYSTEM

Hailu Belay Kassa, Dereje H.Mariam
Addis Ababa University, Ethiopia

Farzad Moazzami, Yacob Astatke
Morgan State University
Baltimore, MD 21251

ABSTRACT

In this work we evaluated the performance of different water filling algorithms. We have selected four power allocation algorithms: Conventional water-filling (CWF), Constant power water-filling, Inverse Water-filling (IWF), and Adaptive Iterative Water-Filling (AIWF) algorithms. Capacity is the performance metric we used to compare the above algorithms by taking the optimality of transmission power allocation to each sub-channel into account. The power allocation can be calculated with a reference of the water level value that has different approaches for different algorithms. The water level can either be fixed once it is found, or it may be adaptive or different for different sub-channels. Hence, the results show that the adaptive iterative water filling (AIWF) algorithm has a better effect on the performance of MIMO-OFDM system by allocating power adaptively.
Keywords— water filling algorithms;capacity;power allocation

INTRODUCTION

Wireless cellular communication systems' energy consumption is basically due to the circuit energy consumption and the data transmission energy at the base station (BS). Even though the circuit power consumption in multiple inputs-multiple outputs (MIMO) and orthogonal frequency division multiple access (MIMO-OFDM) systems may increase due to the circuit complexity and the extra overheads that are added on the Single Input Single Output (SISO) system, the spectral efficiency (SE) can be increased without the need for extra transmission energy [1]. Hence, the transmission energy can be managed and minimized with an optimum resource allocation technique. Channel state information at the transmitter is considered to allocate transmission power for sub-channels and multiple users.

The resource allocation technique is fundamentally based on the convex optimization problem on which the water-filling algorithms are working on [2]. In water-filling, more power is allocated to a sub-channel with better signal to noise ratio (SNR), so as to maximize the sum of data rates in all sub-channels, according to the Shannon's Gaussian capacity formula where each sub-channel's data rate is related to the power allocation [3]. Each used sub-channel's SNR information will be fed back to the BS so that the water level is going to be derived from the fading channel coefficient and the noise power.

Since then, different water-filling algorithms have been evolved for efficient resource allocation, of which: Conventional water-filling (CWF), Constant power water-filling, Inverse Water-filling (IWF),

Margin Adaptive Water-Filling (MAWF), Iterative dynamic water-filling and Geometric water-filling (GWF) algorithms [4, 5, 6, 3, 1].
The paper is organized as follows: the system model is discussed in Section 2. The problem formulation of different power allocation algorithms are defined in Section 3. In section 4, performance analysis is presented. Finally, our conclusion and future work are presented in Section 5.

SYSTEM MODEL

Consider a downlink MIMO-OFDMA cellular network with one BS and I users. It is assumed that M transmitter antennas and N receive antennas are configured at the BS and on each mobile user respectively. Overall K subcarriers are shared by different users with no overlap. The index set of subcarriers occupied by user i is denoted as S_i with size k_i. Closed-loop single-user MIMO (SU-MIMO) schemes are considered and capacity-achieving precoder is assumed in this paper. It is assumed that users undergo frequency-selective and spatial non-coherent block fading channels. Denote H_{ij} as the spatial channel matrix from BS to user i on subcarrier j.
Elements in H_{ij} are independent and identically distributed (i.i.d) random variables with zero mean and variance μ_i, which is the large-scale channel gain from the BS to user i. The noise at the receiver of each user is assumed to be additive white Gaussian with zero mean and variance N_0.

PROBLEM FORMULATION

The transmit-receive signal model from the BS to the ith user, for $i = 1, . . , K$, can be expressed as

$$y_i = \sum_{i=1}^{I} L_i S_i H_i x_i + n_i \tag{1}$$

Where$L_i, S_i, and\ H_i$ are path loss, shadow fading and multipath fading gain matrices respectively, H_i is a M×N matrix with its elements $H_i(j,d) \sim CN(0,1)$, and $n_i \sim CN(0,1)$, is additional Gaussian white noise. L_i and S_i are modeled as macro-cell model. Denoting $los_i = L_i * S_i$.

$$L_i = 40(1 - 4 \times 10^{-3} \times h_{bi}) log_{10}(d_i) - 18 \times log_{10}(h_{bi}) + 21 \times log_{10}(f_i) + 80$$
$$S_i = 10^{X_i}, X_i \sim N(0,1) \tag{2}$$

The system tries to evaluate the capacity with an optimum power allocation. To get the optimum power allocation, we needed to go though different water filling algorithms. Thus, we used the transmission power or allocated power to a user i in each water filling algorithms to evaluate the capacity of a user. We assumed that all the users are in the same fading environment and the Eigen value of the sub-channels $\varepsilon_{j,i}(d)$ is the same for different j and d [7].
Considering singular value decomposition (SVD) on every data frame, independent sub-channels of each user have been constituted. Thus h_{ijd} denotes the dth sub-channel's power gain of user i, and $H_{ijd} = \varepsilon_{j,i}(d) = L_i * S_i * h_{ijd}$.
Therefore, the overall power allocated to an active user by a conventional water filling according to their large scale fading $los_i = L_i * S_i$ is:

$$P_i = \left(\frac{1}{\lambda} - \frac{N_0}{los_i(d)}\right)^+$$
$$= \frac{1}{I}\left[P_t + \sum_{i=1}^{I} \frac{N_0}{los_i(d)}\right] - \frac{N_0}{los_i(d)} \tag{3}$$

Where, the water level is,

$$\mu = \frac{1}{\lambda} = \frac{1}{I}\left[P_t + \sum_{i=1}^{I} \frac{N_0}{los_i(d)}\right].$$

The power allocated to a sub-channel or the transmit power in each sub-channel $P_{j,d}$ having power gains h_{ijd} with the water level of user *i*, $\mu_i = \frac{1}{M}\left[P_i + \sum_{d=1}^{M}\frac{N_0}{h_{ijd}}\right]$ is:

$$P_{j,d} = \mu_i - \frac{1}{h_{ijd}}$$

$$P_{j,d} = \frac{1}{MI}\left[P_t + \sum_{i=1}^{I}\frac{N_0}{los_i(d)}\right] - \frac{N_0}{Mlos_i(d)} + \frac{1}{M}\sum_{d=1}^{M}\frac{N_0}{h_{ijd}} - \frac{1}{h_{ijd}} \qquad (4)$$

The sum capacity of user *i* over one transmission frame of duration $\tau_{\text{frame}}$with the optimization problem is:

$$Maximize\ \ C_i = \frac{\omega\tau}{\tau_{\text{frame}}}\sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}})\left(log_2\,(1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0\omega})\right)$$

$$\text{Subjected to} \qquad \sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}} P_{j,d} \ \le \ P_i$$

$$P_{j,d} \ \ge 0 \qquad (5)$$

Hence, the system capacity is the sum of active users' capacity. It can also be denoted as the sum of active users' sub-channel capacity:

$$C = \frac{\omega\tau}{\tau_{\text{frame}}}\sum_{\text{i}=1}^{\text{I}}\sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}}\left(log_2\,(1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0\omega})\right) \qquad (6)$$

We then compared different water filling algorithms based on their effect on the system capacity considering different transmit power allocation approaches for individual sub-channels. The algorithms are: Conventional water-filling (CWF), Constant power water-filling, Inverse Water-filling (IWF), and Dynamic Iterative water-filling.

A. Conventional Water filling Algorithm

The conventional water-filling (CWF) problem has a sum power constraint under non-negative individual powers. The solution for CWF problem is parameterized with a single water level of:

$$\mu = \frac{1}{MI}\left[P_t + \sum_{i=1}^{I}\sum_{d=1}^{M}\frac{N_0}{h_{ijd}}\right]$$

The transmit power that can be allocated to sub-channel *d* of a user *i* is:

$$P_{j,d} = \mu - \frac{N_0}{h_{ijd}}$$

$$= \frac{1}{MI}\left[P_t + \sum_{i=1}^{I}\sum_{d=1}^{M}\frac{N_0}{h_{ijd}}\right] - \frac{N_0}{h_{ijd}} \qquad (7)$$

The system capacity equation (8) is presented on the next page.

B. Constant power allocation without water filling

This approach considers that a constant power is allocated to each sub-channel with no water-filling. In other words, no water level is considered at which the power allocation threshold is based on. The optimization problem to minimize the transmit power required at the BS is with constraints on the upper bound of ergodic capacity for each user. The sum capacity of user *i* over one transmission frame of duration τ_{frame} is:

$$C = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{\text{i}=1}^{\text{I}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \, (1 + \frac{\left(\frac{1}{MI}\left[P_t + \sum_{i=1}^{I}\sum_{d=1}^{M}\frac{N_0}{h_{ijd}}\right] - \frac{N_0}{h_{ijd}}\right)\varepsilon_{j,i}(d)}{N_0\omega}) \right) \quad (8)$$

Where, $\sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}} P_{j,d} \; = \; P_i$

$$R_i = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\text{D}} \left(log_2 \, (1 + \frac{P_i\varepsilon_{j,i}(d)}{KDMN_0\omega}) \right) \quad (9)$$

The constraints over the upper bound of the ergodic capacity are:

$$C_i \le \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\text{D}} \left(log_2 \, (1 + \frac{P_i E[\varepsilon_{j,i}(d)]}{KDMN_0\omega}) \right) \quad (10)$$

The Eigen value of sub-channels $\varepsilon_{j,i}(d)$ is the same for different j and d [8] and is obtained as:

$$E[\varepsilon_{j,i}(d)] = \frac{MN\mu_i}{D} \quad (11)$$

Hence, that the capacity constraint becomes:

$$C_i \le \frac{\omega\tau}{\tau_{\text{frame}}} \text{k}_\text{i}\text{D} \left(log_2 \, (1 + \frac{MN\mu_i P_t}{\text{K}D^2 N_0\omega}) \right) \quad (12)$$

Where the constraint only depends on the large-scale channel gain $\{\mu_i\}_{i=1}^{I}$

The assumption here is that the transmit power is equally allocated over subcarriers and data streams. Therefore, the transmit power on each subcarrier for each data stream is

$$P_{j,d} = \frac{P_t}{KD} \quad (13)$$

Where, $D = min\{M, N\}$ is the number of data streams in an M × N MIMO system.

So that the system capacity becomes:

$$C = \sum_{i=1}^{I} \frac{\omega\tau}{\tau_{\text{frame}}} \text{k}_\text{i}\text{D} \left(log_2 \, (1 + \frac{mN\mu_i P_t}{\text{K}D^2 N_0\omega}) \right) \quad (14)$$

C. Inverse Water filling Algorithm

The water level can be changed iteratively until it reaches to the water-level that can be lower than the next channel metric to be added. It means that the water-level is largest on the first iteration and decreases on each iteration until it can no longer be decreased. Thus, the water level will be set on the channels that contribute a positive power.

The data transmission rate to user *i* with respect to the allocated transmission power to sub-channels is given by:

$$R_i = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \, (1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0\omega}) \right) \quad (15)$$

Considering the power assigned to the each sub-channel is the ratio of the transmission power allocated to a user and the number of data streams that is equivalent to the number of transmit antennas, $P_{j,d} = \frac{P_i}{M}$, we can rewrite:

$$R_i = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \, (1 + \frac{P_i\varepsilon_{j,i}(d)}{MN_0\omega}) \right) \quad (16)$$

The system capacity is:

$$C = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{\text{i}=1}^{\text{I}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \, (1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0\omega}) \right) \quad (17)$$

Where the RF transmission power of user *i* in time slot t is:

$$P_i = \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{a,k}} P_{j,d} \qquad (18)$$

$where\ P_{j,d} \geq 0$

The power that is allocated to each sub-channel $P_{j,d}$ can also be expressed with respect to the water level μ and it is given by:

$$P_{j,d} = \frac{\mu\omega\tau}{\log(2)} - \frac{\omega N_0}{\varepsilon_{j,i}(d)} \qquad (19)$$

Where, the water level μ can be found via an iterative search over the vector or set β_i of channels that contribute a positive power.

$$\log(\mu) = \frac{1}{\beta_i}\left(\frac{B_{target,i}}{\omega\tau}\sum_{d=1}^{\beta_i}\left(log_2\left(\frac{\tau\varepsilon_{j,i}(d)}{N_0\log(2)}\right)\right)\right) \qquad (20)$$

D. Constant Power Water filling Algorithm

The central theme of this algorithm is that a constant power is allocated to all sub-channels that have channel gains $h_{j,i}(d)$ greater than or equal to the cutoff point, $h_{j,i}(0)$.

$$P_{j,d} = \begin{cases} P_{j,0} & if\ h_{j,i}(d) \geq h_{j,i}(0) \\ 0 & if\ h_{j,i}(d) < h_{j,i}(0) \end{cases} \qquad (21)$$

Where, if the same cut-off point $h_{j,i}(0)$ is used as in exact water-filling, we have,

$$P_{j,0} + \min_{h_{j,i}(d)\geq h_{j,i}(0)} \frac{N_0}{h_{j,i}(d)} \leq$$

$$\frac{N_0}{h_{j,i}(0)} \leq \min_{h_{j,i}(d)<h_{j,i}(0)} \frac{N_0}{h_{j,i}(d)} \qquad (22)$$

From the first inequality, i.e. when $h_{j,i}(d) \geq h_{j,i}(0)$ we see that the minimal sum of power and (normalized) noise is less than the water level ($\frac{N_0}{h_{j,i}(d)}$), at which a constant power is allocated for all sub-channels that are under this requirement.

With the analysis of low complexity power allocation algorithm, the constant power allocated in each state $P_{j,0}$ should be such that:

$$P_{j,0} + \min_{h_{j,i}(d)\geq h_{j,i}(0)} \frac{N_0}{h_{j,i}(d)} = \frac{N_0}{h_{j,i}(0)} \qquad (23)$$

Therefore, the constant power allocated to the sub-channels with positive power allocation is:

$P_{j,0} = \frac{|P_i|}{|k_i|D}$, which is the average power.

Hence the system capacity becomes:

$$C = \frac{\omega\tau}{\tau_{\text{frame}}}\sum_{\text{i}=1}^{\text{I}}\sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}}\left(log_2\left(1 + \frac{P_{j,0}\varepsilon_{j,i}(d)}{N_0\omega}\right)\right)$$

$$= \frac{\omega\tau}{\tau_{\text{frame}}}\sum_{\text{i}=1}^{\text{I}}\sum_{j=1}^{|k_i|}\sum_{d=1}^{\varepsilon_{j,i}}\left(log_2\left(1 + \frac{|P_i|\varepsilon_{j,i}(d)}{|k_i|DN_0\omega}\right)\right) \qquad (24)$$

Where, $|k_i|$ is the number of subcarriers per a user i and D is the number of data streams at a give time slot τ.

E. *Adaptive Iterative Water filling Algorithm*

There are different iterative water filling algorithms, but in this paper we concentrated on the single level and multilevel iterative approaches. The water level can be one or more than one per given data frame over each iteration.

The transmit power in each sub-channel with the sub-channels power gain of h_{ijd} is:

$$P_{j,d} = \mu_i - \frac{1}{h_{ijd}} \qquad (25)$$

Where, $\mu_i = \frac{1}{M}\left[P_i + \sum_{d=1}^{M} \frac{N_0}{h_{ijd}}\right]$, hence:

$$P_{j,d} = \frac{1}{MI}\left[P_t + \sum_{i=1}^{I} \frac{N_0}{los_i(d)}\right] - \frac{N_0}{M los_i(d)} + \frac{1}{M}\sum_{d=1}^{M} \frac{N_0}{h_{ijd}} - \frac{1}{h_{ijd}} \qquad (26)$$

The sum capacity of user *i* over one transmission frame of duration τ_{frame} is:

$$C_i = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \left(1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0 \omega}\right)\right) \qquad (27)$$

Therefore the system capacity is:

$$C = \frac{\omega\tau}{\tau_{\text{frame}}} \sum_{i=1}^{I} \sum_{j=1}^{|k_i|} \sum_{d=1}^{\varepsilon_{j,i}} \left(log_2 \left(1 + \frac{P_{j,d}\varepsilon_{j,i}(d)}{N_0 \omega}\right)\right) \qquad (28)$$

PERFORMANCE ANALYSIS

The performance of different water filling algorithms are evaluated based on the numerical values specified in table 1 below. The algorithms we have considered were: constant power allocation, Conventional water-filling (CWF), Constant power water-filling, Inverse Water-filling (IWF), and Iterative dynamic water-filling. Comparing figures 1 and 2, we can see that in Figure 2, because of an optimum power allocation using adaptive iterative water filling algorithm the capacity of the system increased from that of figure 1.

Table 1: List of simulation parameters:

Number of Subcarrier, K	80
Number of users, I	4
Number of BS antennas, M	[1,2,3,4,5]
Number of antennas at each user, N	[1,2,3,4,5]
Minimum distance from S to users	35m
Maximum transmit power, P_{max}	20
User Distribution in a cell	Uniform
Duration of frame/time slot, τ_{frame}/τ	10 ms/1 ms
System/subcarrier bandwidth, W/w	$4x10^{-21}$W/Hz

Taking a similar point on both graphs where the SNR is at 20dB, we can notice that a 4x4 MIMO CWF system of Fig. 2 has almost the same capacity as a 5x5 MIMO system of Fig. 1. Furthermore, we also notice that a 4x4 MIMO AIWF has a higher capacity than a CWF system. Thus, the graphs indicate that the energy efficiency of Fig. 2 is higher than that of Fig. 1, and that a AIWF system has a higher capacity as compared to the CWF system. This is because of the adaptive power allocation to the sub-channels. Since we have considered a spatial multiplexing MIMO mode i.e. the data stream, $D=min(M,N)$, in this paper, we can see from Fig. 3 that the capacities of 2x3 MIMO and 3x2 MIMO modes are aligned. Fig. 4 indicates that the capacity for a 4x4 MIMO mode of constant power water filling algorithm is almost equivalent to that of equal power allocation. However, the performance of both systems shown in Fig. 4 is lower than those shown in Fig. 2 and Fig. 3.

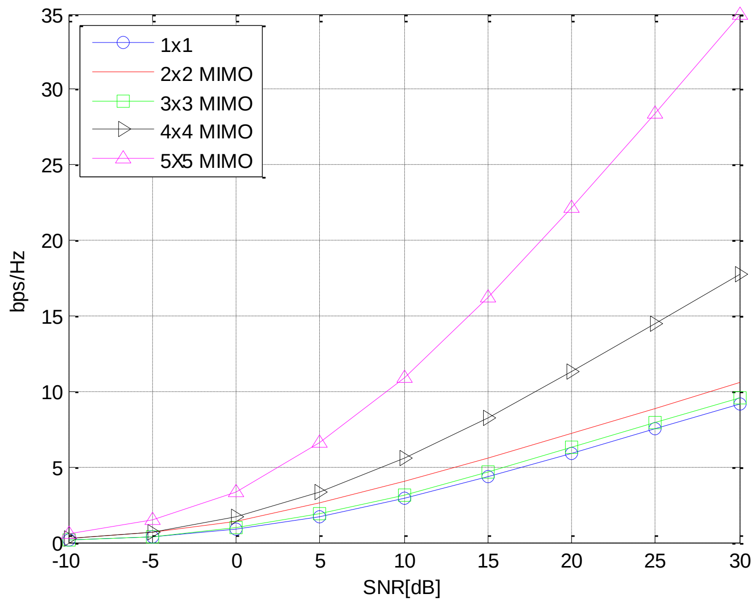

Figure 1: Ergodic capacity of the system with respect to the SNR for different MIMO modes

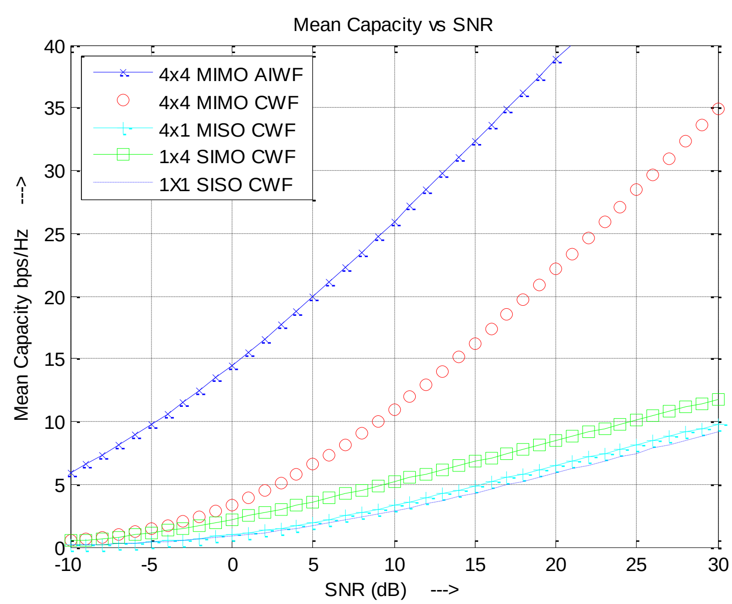

Figure 2: Capacity of the system with respect to the SNR for different MIMO modes using adaptive iterative water filling algorithm.

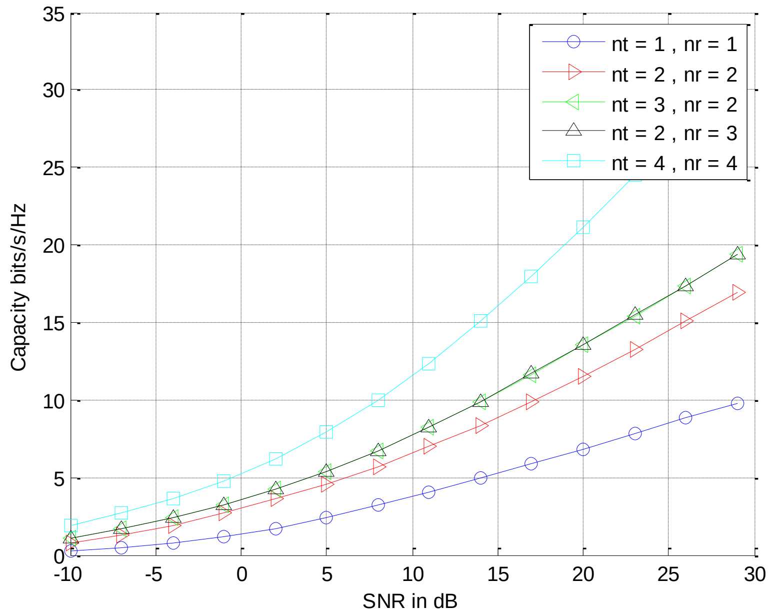

Figure 3: Capacity of the system with respect to the SNR for different MIMO modes and a 4x4 MIMO mode with conventional water filling algorithm.

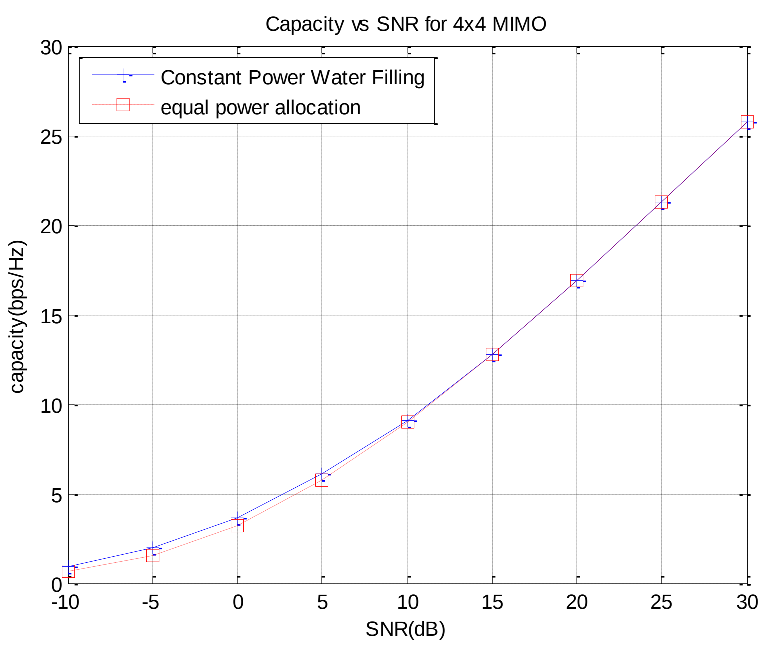

Figure 4: Capacity of the system with respect to the SNR of 4x4 MIMO mode for Constant power water filling algorithm and equal power allocation approach.

Finally, the results presented in figures 1, 2, and 3 all show that as the number of antennas mounted on the transmitter and the receiver increases, the capacity of the system increases accordingly. Even though the transmission energy would not be increased with the increase in spatial diversity, the circuit energy consumption will rise due to the corresponding RF chain increase. This implies that the total energy consumption will increase.

CONCLUSION

In this work, our results show that the adaptive iterative water filling (AIWF) algorithm has a better effect on the performance of MIMO-OFDM system by allocating optimum power to the sub-channels adaptively. It also increases the energy efficiency by providing more data transmission rate with the same number of RF chains. In addition, the following four approaches: AIWF, constant power allocation, constant power water-filling and conventional water filling algorithms provide higher energy efficiency with respect to the corresponding ergodic capacity system by providing more data transmission rate with less number of RF chains. This implies that the circuit energy consumption of the ergodic capacity system is higher than the other four algorithms: AIWF, constant power allocation, constant power water-filling and conventional. Our future work will include the evaluation of other recent water filling algorithms based on different metrics.

REFERENCES

[1] Hauke Holtkamp, Gunther Auer, Samer Bazzi, Harald Haas, "Minimizing Base Station Power Consumption", 2013.

[2] S. Boyd and L. Vandenberghe, "Convex optimization with en-gineering applications," 1998, Course Notes, EE364, Stanford University.

[3] Zhe Wang, Vaneet Aggarwal, and Xiaodong Wang, "Iterative Dynamic Water-filling for Fading Multiple-Access Channels with Energy Harvesting", *IEEEAT&T Labs-Research, 2014.*

[4] Peter He, Lian Zhao, Sheng Zhou, ZhishengNiu, "Water-Filling: A Geometric Approach and its Application to Solve Generalized Radio Resource Allocation Problems*", IEEE Trans. Wireless Commun*, VOL. 12, NO. 7, pp. 3637-3647, 2013.

[5]. Jie Xu, Ling Qiu, Shunqing Zhang, "Energy Efficient Iterative Waterfilling for the MIMO Broadcasting Channels", Huawei Technologies, 2012.

[6] Zhikun Xu, Chenyang Yang, Geoffrey Ye Li, Shunqing Zhang, Yan Chen, and Shugong Xu, "Energy-Efficient MIMO-OFDMA Systems based on Switching Off RF Chains", IMT-Advanced Systems,

[7] Spencer, Q.H. Swindlehurst, A.L. Haardt, M. "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels", *IEEE Trans.Signal Processing*, VOL.52(2): pp. 461-471, 2004,