ANALYSIS OF FM/FM TRANSMISSION LINE DISTORTION

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Summary An IBM 1620 computer was programmed to evaluate the effects which the variation of propagation velocity and attenuation in coaxial cables has on a frequency modulated carrier. For a nine mile length of RG-8A/U the distortion of 70kc modulation on a 1 mc carrier was severe, while for a 10 mc carrier the distortion was negligible. A typical system using 4, 6, and 9 mc carriers on a single cable was evaluated.

Introduction Frequently in the discussion of transmitting frequency modulated telemetry signals over moderately long coaxial cables, the question of phase distortion arises. It is generally accepted that the propagation velocity and attenuation of coaxial cables vary with frequency, but the extent of the variation is generally unknown and even less is understood of its effect on frequency modulated waveforms. For this reason the “bigger hammer” approach is generally used and the most expensive cable which can be justified is purchased, or an r.f. relay is chosen.

Equations which quite accurately express the effects of velocity and attenuation have been written, but they are quite difficult to interpret for a signal with a spectrum as complex as frequency modulated telemetry. However, even a small digital computer can quickly evaluate these equations, so that the output can be plotted.

Transfer Functions For A Transmission Line The transfer function of a low-loss, infinitely long transmission is (ref. 1) :

$$H(x,s) = \exp\left[-\frac{x}{u}\left(\alpha + s + \eta\sqrt{s}\right)\right] \qquad (1)$$

where:

x = length of cable

α = damping coefficient = $\frac{1}{2}\left\{\frac{r}{\ell} + \frac{g}{c}\right\}$

η = skin effect coefficient

s = Laplacian variable

ℓ = inductance per unit length

c = capacitance per unit length

r = resistance per unit length

g = conductance per unit length

u = propagation velocity

The derivation of this equation is based on the assumption of low-loss, i.e., that $\frac{r}{\omega \ell} \ll 1$ and $\frac{g}{\omega c} \ll 1$, a condition which is satisfied by most coaxial cables. The last term is empirical.

Equation (1) applies to finite transmission lines only if the terminating impedance matches the line impedance (ref. 1):

$$Z_e(j\omega) = R_s \left[1 + \frac{\sigma}{j\omega} + \frac{\eta}{\sqrt{j\omega}} \right] \qquad \text{where } \sigma = \frac{1}{2}\left(\frac{r}{\ell} - \frac{g}{c} \right) \tag{2}$$

Since there is no lumped component which has a response of √ω, perfect match is impossible. However, for frequencies above 1 mc and lilies over 1 mile long, reflected waves are negligible.

The extent to which equation (1) describes the actual transfer function of a cable can be seen by solving for a step input,

$$V(s) = \frac{E_o}{s}$$

The inverse transform of V(s)H(x,s) is

$$u(x,t) = E_o e^{-\alpha x/u} \operatorname{erfc}\left[\frac{\eta}{u} \frac{x}{2\sqrt{t - x/u}}\right] \tag{3}$$

giving the error function rise time observed for coaxial cables.

This equation can be used to evaluate experimentally the skin effect coefficient η, by measuring the 0 to 50% rise time t of a cable and solving the equation t_r of a cable and solving the equation

$$\operatorname{erfc}\left[\frac{\eta}{u} \frac{\quad}{\sqrt{t_r}}\right] = \frac{1}{2}$$

for η. For several common coaxial cables, t_r can be found on page 181 of Tektronix Catalog 22 (ref. 2).

Application Of Transfer Function To Frequency Modulated Carriers The equation for a frequency modulation carrier is:

$$\begin{aligned} e_s(t) &= E_c \cos(\omega_c t + M_f \sin \omega_m t) \\ &= E_c [J_o(M_f) \cos \omega_c t + J_1(M_f) \cos(\omega_c + \omega_m)t \\ &\quad + J_2(M_f) \cos(\omega_c + 2\omega_m)t + \ldots \\ &\quad - J_1(M_f) \cos(\omega_c - \omega_m)t + J_2(M_f) \cos(\omega_c - 2\omega_m)t + \ldots] \end{aligned} \tag{4}$$

In the frequency domain, the transfer function of a transmission line becomes:

$$H(j\omega) = \exp\{-\frac{x}{u}[\eta\sqrt{\frac{\omega}{2}} + j(\omega+\eta\sqrt{\frac{\omega}{2}}) + \alpha]\} \tag{5}$$

The equation for the modulated carrier in the frequency domain is:

$$V_s(j\omega) = \frac{E_c}{2}\{J_o(M_f)[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]+\sum_{n=1}^{\infty} J_n(M_f)[\delta(\omega-\omega_c-n\omega_m)$$

$$+\delta(\omega+\omega_c+n\omega_m)+(-1)^n\delta(\omega-\omega_c+n\omega_m)+(-1)^n(\omega+\omega_c-n\omega_m)]\} \tag{6}$$

where δ is the Dirac delta.

The real part of the inverse Fourier transform of $H(j\omega)V_s(j\omega)$ is:

$$v(x,\tau) = E_o\{J_o(M_f)\cos\omega_c\tau\exp(-A_o)$$

$$+\sum_{n=1}^{\infty}[J_n(M_f)\cos[\tau(\omega_c+n\omega_m)+A_n]\exp(-A_n)$$

$$+(-1)^n J_n(M_f)\cos[\tau(\omega_c-n\omega_m)B_n]\exp(-B_n)]\} \tag{7}$$

where $A_n = \frac{x}{u}\eta\sqrt{(\omega_c+n\omega_m)/2}$

$B_n = \frac{x}{u}\eta\sqrt{(\omega_c-n\omega_m)/2}$

$E_o = E_c e^{-x\alpha/u}$

$\tau = t-x/u$

The Computer Program Equation (7) is difficult to interpret; however, it is easily solved by a computer for incremental values of τ, so that the plot of the modulated carrier can be made. A Fortran program for an IBM 1620 computer was written to solve Equation (7) for incremental values of τ. Since in most cases of interest, line amplifiers will be required to keep the signal-to-noise ratio at a usable level, provisions were made to include the gain and phase shift characteristics of amplifiers in the solution.

In most cases, the distortion of the modulation rather than the carrier is of primary interest. Therefore, the program also includes a sub-routine to locate the negative going axis crossings of the carrier by successive approximation using smaller and smaller increments of τ. The period T_o is determined by calculating the distance between crossings, so that the average modulation voltage for a cycle can be calculated from the equation:

$$v_m = K \left(\frac{1}{T_s} - \frac{1}{T_c}\right) \tag{8}$$

where T_c is the period of the unmodulated carrier. This gives one point of the modulating waveform for each carrier cycle.

Additional programs were written for estimating the r.m.s. error of a cycle of modulation, and for calculating the gain and phase characteristics of ideal tuned amplifiers (ref. 3).,

A Typical Coaxial Cable Transmission Problem The computer program was used to determine suitable carrier frequencies and cable types for transmitting telemetry data over a nine mile link. RG-8A/U was the lowest priced cable for which rise time and attenuation characteristics were readily available, so its characteristics were used in the calculation. The skin effect coefficient, η, was found to be about 125 for RG-8A/U.

The distortion was measured for various carrier frequencies, modulated by a 70kc cosine wave at a modulation index of $M_f = 1.5$. A ten megacycle carrier was selected first. The modulation recovered at the end of the nine miles of cable was an almost perfect cosine wave, with an estimated r.m.s. error of 0.25%.

Since there was so little distortion with a 10 mc carrier, a 1 mc carrier was selected next, since no intermediate line amplifiers would be required for a 1 mc carrier. The plot of a cycle of modulation is shown in (fig. 1). No measurement of the r.m.s. error was made, but the modulation is obviously badly distorted. The carrier, plotted in (fig. 2), shows a high degree of distortion in the form of amplitude modulation. The corresponding phase shifts, which cause the distortion of the modulation, are not so apparent.

Next an attempt was made to decide on a set of three carrier frequencies which could be used on the cable simultaneously, so that 6 channels could be transmitted on two cables. The selected carrier frequencies were 4, 6, and 9 mc. These are obviously free from interference due to harmonics and second order intermodulation products. They are also free from interference due to third and fifth order intermodulation products (ref. 4).

To make the solution as realistic as possible, the characteristics of line amplifiers were also included. Since experimentally determined phase delay characterisitics of tuned amplifiers were not available, phase and gain characteristics for a five stage, double tuned amplifier were calculated. The characteristics of a 500kc bandwidth amplifier are plotted in (fig. 3). Experimental gain characteristics for a similar amplifier are indicated by the x’s.

In order to maintain a satisfactory signal-to-noise ratio for the 9 mc carrier, the length of cable between line amplifiers was limited to 3 miles. The amplifiers were assumed to have an output power of 10 watts, an equivalent noise input resistance of 950Ω, and a bandwidth of 1 mc. For signals into the line with a signal-to-noise ratio of 40db, the output 9, 6, and 4 mc carriers had signal-to-noise ratios of 32db, 39db, and 40db respectively.

A cycle of modulation for each carrier is shown in (fig. 4). The r.m.s. error of the modulation is 1.5% for the 9 mc carrier, 1.8% for the 6 mc carrier, and 2.1% for the 4 mc carrier. A large amount of error was evidently introduced by the amplifiers, as can be seen by comparing the 0.25% error for the 10 mc carrier with flat amplifier characteristics with the 1.5% error for the 9 mc carrier using three amplifiers with a bandwidth of I mc.

Conclusions A nine mile length of RG-8A/U can be used to relay three channels of FM/FM telemetry data using carrier frequencies of 4, 6, and 9 mc without causing excessive distortion of 70kc subcarrier. The distortion could be reduced to a somewhat lower level by using line amplifiers with a wider bandwidth than the I mc amplifier which was chosen. However, this would be done at the expense of an increase in noise, and cable distortion would then be the limiting factor.

A small digital computer can be used to analyze the distortion of frequency modulated waveforms. The three carrier relay analysis required about 15 hours of 1620 computer time, at a cost well below that of an experimental analysis. This technique can be easily extended to the analysis of distortion of PDM/FM and PAM/FM waveforms by use of the proper Fourier components.

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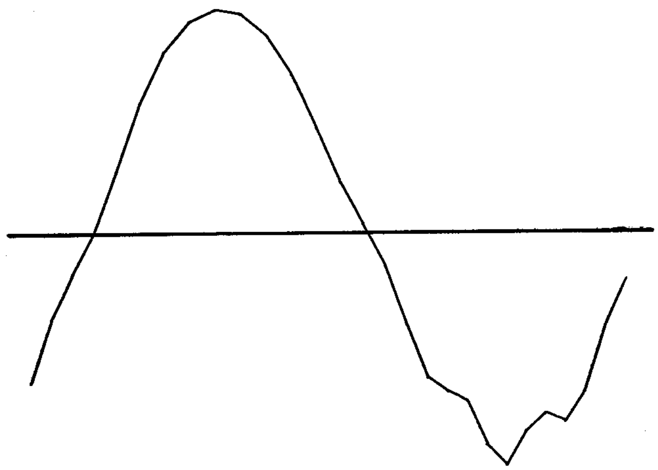

Figure 1: 1 Cycle of Modulation
1 M.C. Carrier

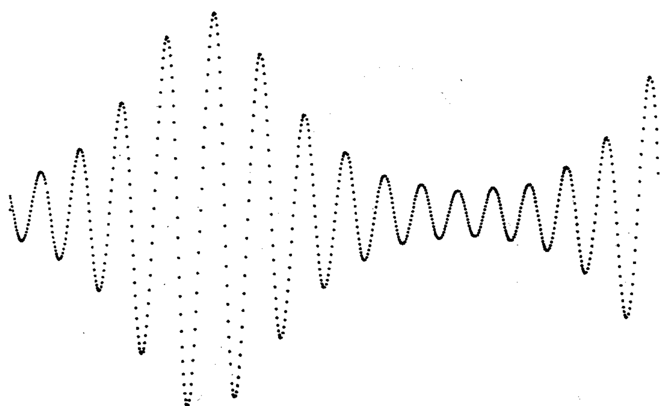

Figure 2: Modulated 1 M. C. Carrier

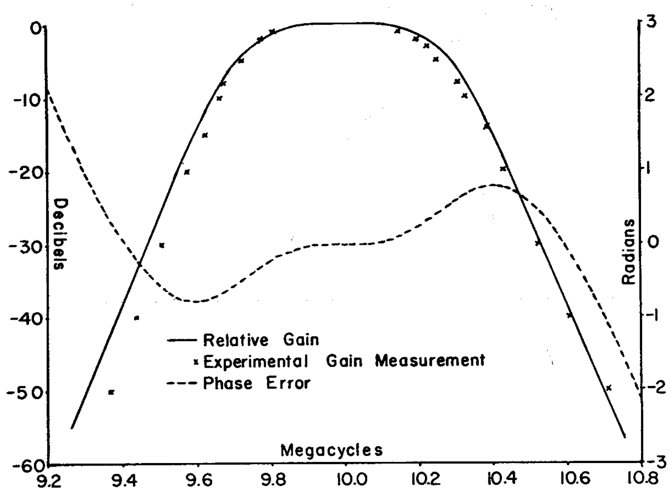

Figure 3: Tuned Amplifier Characteristics

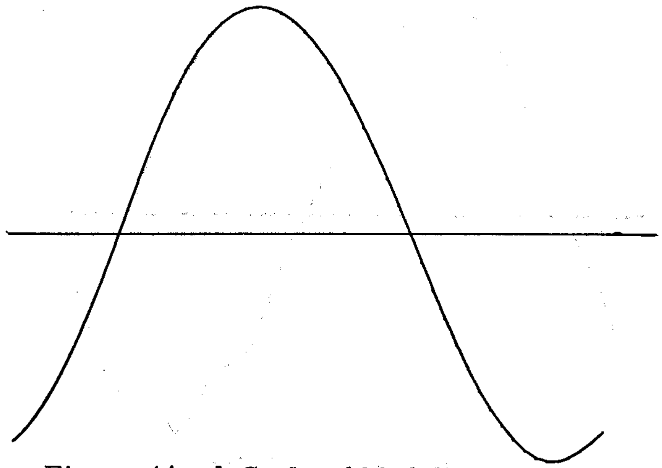

Figure 4A: 1 Cycle of Modulation
4 M.C. Carrier

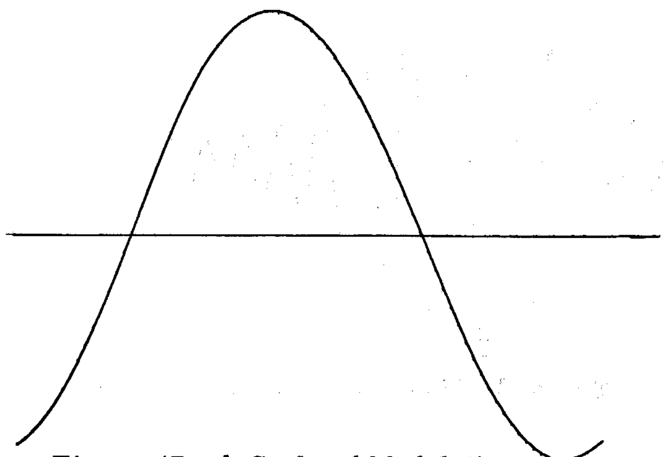

Figure 4B: 1 Cycle of Modulation
6 M.C. Carrier

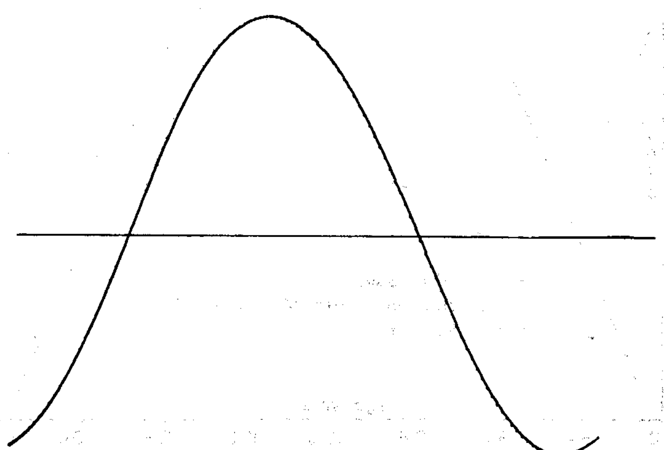

Figure 4C 1 Cycle of Modulation
9 M.C. Carrier