# ENVELOPE DELAY IN A TAPE RECORDER SYSTEM

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**Introduction** Envelope delay is a term first used by Nyquist and Brand of Bell Telephone Laboratories in 1930. It indicates the degree of phase linearity in a transmission network, and is the first derivative of the phase shift with respect to frequency.

The phase response of a transmission network such as a magnetic tape recorder/ reproducer system has become more significant with the increased use of more complex modulation schemes and wider bandwidth predetection recording.

This paper develops the definition of envelope delay and related terms and describes several methods of measuring it. A method of envelope correction is also described which reduces total variation in envelope delay to less than 200 nanoseconds over the frequency range of 100 Kc to 1.5 Mc.

**I. Envelope Delay** The electrical performance characteristics most generally described in a tape recorder/reproducer system are frequency response, harmonic and intermodulation distortion, signal to noise ratio and dynamic range. The definition and measurement techniques of these parameters are well known and no elaboration is necessary.

Phase linearity, delay distortion, phase distortion, transmission delay, phase delay, envelope delay and group delay are all related terms which are derived from the phase shift versus frequency transfer function of a network. These parameters have an effect on the ability of a recorder to accurately store and reproduce complex data, but are generally not specified. Amplitude distortion can be corrected and controlled quite readily but phase distortion involves more complex measurement devices and compensation networks.

The transfer characteristic of any complex LRC four terminal network may be specified by two basic plots, amplitude vs frequency and phase shift vs frequency.

Amplitude non-linearities with frequency create the well known harmonic and intermodulation distortion products. The phase shift vs frequency plot reveals equally important but less emphasized characteristics. Figure 1 shows a linear phase shift vs frequency plot for all frequencies from 0 to $\omega_1$. Phase delay or transmission delay is the transmission time for a single frequency component of a waveform through a network and is defined as $T_d = \frac{\phi}{\omega}$, where $\phi$ is phase shift in radians, $\omega$ is frequency in Radians per unit time. Figure 1 shows that $\frac{\phi}{\omega}$ is a constant for all values of $\omega$ from 0 to $\omega_1$, and in this network all frequency components up to $\omega_1$ will have an equal transmission time.

In practice, the phase shift vs frequency curve or a complex network is not generally linear nor will it intercept the origin. Let us consider a periodic complex signal which can be represented as a summation of Fourier frequency components. This signal if passed through such a non-linear network will have unequal transmission times for each discrete frequency. The distortion caused by these unequal delay times is called phase distortion. A simple example of this distortion is shown in Figure 1, where it can be seen that the phase delays of $\omega_1$ $\omega_2$ $\omega_3$ and $\omega_4$ are not equal.

The reconstructed output signal will contain distortion introduced as a function of transmission time since the components of the complex waveform ($\omega_1$ - $\omega_4$) will be displaced in time relative to each other. Phase distortion may be represented numerically as 1/2 the difference in transmission time between the minimum and maximum delays existing between any two frequencies in the passband of interest (1/2 the relative phase delay or .17 unit time in the example of Figure 1).

The second form of phase description, and the form most significant for a magnetic tape recorder, is envelope or group delay. It is defined as the first derivative of phase shift, $\phi$, with respect to frequency, w, and is therefore the slope of the phase curve at the frequency of interest.

$$T_e = \frac{d\phi}{d\omega} \approx \frac{\Delta\phi}{\Delta\omega} = K \frac{\phi_1 - \phi_2}{\omega_1 - \omega_2}$$

The term envelope delay is used since the transmission time of the modulation equency (the envelope of an amplitude modulated signal) is equal to the slope of e phase shift curve at the carrier frequency. [1,2,3]

Group delay is used synonymously with envelope delay but is a more significant rm. when related to frequency modulation since no modulation envelope is produced in the case of FM.

The difference between phase delay and envelope or group delay is shown in Figure 2. At $\omega c_1$, the envelope delay and the phase delay both equal $(\tan^{-1} a_1) = K_1$, while at $\omega c_2$, the envelope delay $= K_2 = \tan^{-1} a_2 = \frac{d\phi}{d\omega}$ which is not equal to the phase delay, $\frac{\phi}{\omega} \neq (\tan^{-1} a_2)_1$.

An amplitude modulated waveform will consist of the major components $\omega c_2 \pm \omega m$. At $\omega c_2$, $\omega c_2 + \omega m$ and $\omega c_2 - \omega m$, the time delay of each component will be equal to the phase delay $\frac{\phi}{\omega}$ at each discrete frequency. The time delay suffered by the modulating envelope will not necessarily be equal to any of the discrete phase delays but will be equal to the slope of a line tangent to the curve at $\omega c_2$ or $\tan^{-1} a_2$ in Figure 2, which is the envelope delay. Note that if $\phi_1 = 0$, both the phase delay and the envelope delay are equal $(K_2)$. (See appendix.)

Referring again to Figure 1, the region between $\omega_2$ and $\omega_3$ will have constant envelope delay but not constant phase delay while $\omega_1$ and $\omega_4$ will have equal phase delays. Constant phase or time delay is therefore a special case of constant envelope delay where the phase plot intersects the origin.

The absolute value of phase delay is generally of little importance other than in the evaluation of the insertion delay of a network. A more significant parameter <u>relative phase delay</u> where the delay at any frequency is compared to the delay at selected reference frequency. The same can be said of envelope delay in a tape recorder/reproducer system. Commercial equipment is available which will onl measure relative envelope delay not absolute delay. (4)

<u>Envelope delay distortion</u> is defined as that form of distortion which occurs when the rate of change of phase shift with frequency of a network is not constant over the passband under consideration.

There are three major reasons for specifying and using envelope delay parameters as opposed to phase delay in a tape system.

1. The direct measurement of phase shift is highly inaccurate since real time measurements cannot easily be made. The effects of flutter, jitter and other nor mal tape system dynamics as well as the path length between record and reproduce head prevent a simple phase shift measurement.

Envelope delay measurements are possible where the input and output terminals of a network are not available at one location. Phase shift measurements cannot be made under these conditions.

2. Relative or differential envelope delay is a conveniently obtained parameter with commercially available test equipment. Relative phase delay measurements are not possible.

3. Envelope delay in a predetection recording system is a more meaningful parameter since the information recorded is a modulated signal. Phase delay information would apply to the various components of the modulated signal but it ma not necessarily be equal to the envelope delay at any given frequency.

**II Measurement of Envelope Delay** The measurement of envelope delay consists of determining the slope of the phase shift curve at the frequency of interest. Two basic types of measurement techniques may be used, an open loop and a closed loop system. A closed loop system requires that the input and output terminals of the network be available at one location and that no time translation occurs.

The dynamics of the tape transport prevent the direct measurement of phase shift through a tape recorder/reproducer system. The open loop technique is used where the reference signal is combined with the data and both are transmitted together. This multiplexing of reference and data reduces the effects of minute tape speed changes and variations in tape tension or dimension. The open loop system developed for the evaluation of long transmission lines is ideally suited for a tape recorder system. Two types of open loop systems will be described; one where the reference signal is frequency multiplexed with the data and one where the reference signal is time multiplexed with the data.

## A. Closed Loop Measurement

1. Phase shift reading, Figure 3. Since envelope delay is the first derivative of the phase shift curve, we can measure the phase angle $\varnothing_1$ and $\varnothing_2$ at two different but closely spaced frequencies, $\omega_1$ and $\omega_2$. The envelope delay can be expressed as the slope of a straight line between two frequencies.

$$T_e = \frac{d\varnothing}{d\omega} \approx \frac{\Delta\varnothing}{\Delta\omega} = K \frac{\varnothing_1 - \varnothing_2}{\omega_1 - \omega_2}$$

This graphical method is accurate between any two closely spaced points but is laborious and slow.

2. Nyquist Method 1, Figure 4. It is possible to measure envelope delay directly by using the method of Nyquist where an amplitude modulated signal is passed through a network and a phase meter is used to measure the phase difference between the modulating frequency directly and the demodulated modulating frequency transmitted through the network. As the carrier frequency is varied through the passband of the network, the envelope delay at each frequency can be

$$T_e = \frac{\phi}{2\pi f_m} = \frac{\phi}{\omega_m}$$

$\phi$ = phase reading in radians

$\omega_m$ = modulating frequency, radians/unit time

The phase meter can be calibrated directly in time (Envelope delay). For good resolution $\omega_c$ must be high compared to $\omega_m$. This measurement gives the absolute value of envelope delay at any frequency. If the reference could be placed in the passband, relative envelope delay could be obtained, which is more meaningful. The obvious inaccuracies which evolve from the subtraction of two large numbers lead us to find a measurement system which will measure relative envelope delay directly.

## B. Open Loop Measurements of Envelope Delay

1. Frequency multiplexed reference, Figure 5. In order to obtain phase shift data through a tape recorder, it is necessary to transmit the reference information concurrently with the data information. This multiplexing of data tends to nullify the frequency translation effects of flutter, jitter and other tape dynamics.

Where the bandwidth permits, the reference carrier can be mixed with the modulated data and both transmitted together through the network. Figure 5 represents a typical example of an open loop, frequency multiplexed delay measuring system as applied to a tape recorder. In the recorder under test, the data passband is 100 kc to 1.5 mc, the reference modulation frequency is 10 kc and the phase meter operates at 20 kc.

A complete signal consisting of a low frequency reference, $\omega_m$, plus a carrier, $\omega_c$, modulated by $\omega_m$, is passed through one channel of the recorder. At the output, the signal is separated, the modulated carrier demodulated and the modulation signal compared in phase with the original low frequency signal carried through as a reference. As the carrier w, is raised from 100 kc to 1. 5 mc, the relative phase shift is observed.

The balanced modulator is used for best S/N ratio in preference to straight AM with a carrier. The output of the balanced modulator is a double sideband suppressed carrier signal.

The upper branch in Figure 5 rejects 10 kc from the composite signal, detects the composite signal, passes it through a 20 kc bandpass filter and becomes one data input to the phase meter. The lower branch filters and passes only the 10 kc reference frequency, full wave rectifies and doubles to 20 kc, passes it through an identical 20 kc bandpass filter and then to the phase meter.

Both 20 kc filters are to be identical in phase vs frequency for approximately ±200 cps about 20 kc, so that the effects of flutter affect both channels equally. Flutter therefore introduces no relative phase shift.

A serious problem exists with this method in that the passband of the system must extend approximately one decade below the lower data frequency. In this example, to measure from 100 kc to 1.5 me, the passband must extend to 10 kc to handle the reference modulation frequency.

2. Time Multiplex Reference. If the measuring system could time multiplex the reference signal with the data, the full passband of the system could be used for measurement.

The system used by Wandel & Goltermann in their LD-1 test set[4] uses the open loop, time multiplexed reference, covers a range of 100 kc to 14 mc, and has an envelope delay resolution approaching 2 nanoseconds as a function of system noise. They achieve this resolution by measuring through narrow bandwidths and sweeping across the desired frequency band. The system operates as follows: Figures 6 and 7.

Two RIP carriers $f_1$, $f_2$ are generated and become the reference and data frequency respectively. The reference frequency is held constant and the data frequency is varied throughout the frequency band. The system measures the <u>relative</u> attenuation and envelope delay of the data frequency compared to the reference frequency rather than the absolute value of attenuation or envelope delay.

The two carriers are alternately switched at a rate of approximately 600 cps, ($f_4$) and then coherently amplitude modulated with a 20 kc “split” frequency ($f_3$) so that the envelope of the modulated signal shows no surge or discontinuity at the switching points. (The switch frequency of 600 cps is above the dominant flutter frequency of the recorder.)

The multiplexed signal is then fed into the recorder input. Since the channel presents different delay and attenuation characteristics to the two carrier frequencies $f_1$ and $f_2$, a phase surge is produced in the modulation envelope of the signal received for each switching point. This surge or discontinuity is reflected after demodulation in corresponding phase shifts of the received split frequency input (20 kc). The demodulated 20 kc signal is thus a square wave modulated in phase by the switching frequency $f_4$, the phase shift being proportional to the delay difference in the measurements of output between the reference frequency $f_1$ and the test frequency $f_2$. After demodulation in the receiver, the split frequency voltage is filtered out, limited and applied to a circuit that translates phase variations into amplitude variations. The output signal of this phase shift detector circuit is a square wave voltage of the switching frequency ($f_4$) having an amplitude corresponding to the delay difference. The reference frequency is additionally "tinted" by an identification frequency $f_5$ so that the receiver can identify the carrier representing the reference frequency.

The output signal is further processed to have information with the correct sign i.e., positive or negative, depending on whether the test or reference frequency is more delayed. The information is presented on a meter or oscilloscope for observation.

**III. Envelope Delay Correction** With accurate measurement of relative envelope delay available it is now possible to correct for it in a tape recorder system. The particular tape recorder/reproducer system under discussion is a high performance wideband unit, especially designed for use in a predetection recording system as specified by the Eastern Test Range. The unit has capability for 4-speed operation with either 7 or 14 channels of data. At the highest speed, 120 ips, it has a specified frequency response of 100 kc to 1.5 mc and a maximum permissible envelope delay variation within this passband of 200 nanoseconds. The theory and technique of envelope delay correction as applied to this recorder are described in this section. The unit is in standard production and being delivered to the customer at this time.

The front elevation view of the unit is shown at the beginning of the article. A system block diagram of the signal electronics is shown in Figure 8. The record amplifier, head driver and reproduce pre-amplifier are linear with frequency, containing no amplitude equalization. The frequency response of these elements extends sufficiently beyond the desired passband so that phase distortion within the passband is minimized (3 db amplitude points are 120 cps and 2. 7 mc).

The reproduce output signal. of a playback head is proportional to the rate of change of flux through the coils intercepting the flux changes:

$$E \approx N \frac{d\phi}{dt}$$

Theoretically, for a given value of recorded flux density, doubling the rate of change of flux by either doubling the frequency or tape speed will double the output voltage. This characteristic causes a voltage increase at a rate of 6 db/octave. In practice, however, this characteristic is modified in the high frequency region as the recorded wavelength becomes short compared to the head gap width, by high frequency core losses, head to tape spacing, and effects of bias erasure and record demagnetization.

Figure 9 shows the area of interest for correction, the region between 100 kc and 1.5 mc. The voltage rises at the 6 db/octave slope and peaks at approximately 250 kc. Theoretically the maximum occurs where the reproduce head magnetic gap length equals one half the recorded wavelength.

$$E = K \log \frac{\pi L}{\lambda}$$

$L$ = gap length

$$\lambda = \text{recorded wavelength} = \frac{\text{tape speed, inches per second}}{\text{frequency, cycles per second}}$$

Note that this equation shows that a null would occur when the gap length equals the recorded wavelength.

The various high frequency losses that occur during the recording and reproducing mode and modify the 6 db/octave slope are tabulated below. The losses will be mentioned only briefly since they are well described in the literature by Stewart.[5]

Record Process

Self magnetization

Penetration loss

Bias demagnetization loss

Eddy current and other core and coil losses

Spacing loss

Reproduce Process

Gap loss (aperture effect)

$$20 \log \frac{\sin \frac{\pi L}{\lambda}}{\pi L/a} \text{ db}$$

Contour effects

Azimuth misalignment loss

$$30 \log \frac{\sin \frac{\pi w \tan d}{x}}{\frac{\pi w \tan \gamma}{\lambda}} \text{ db}$$

w = width, a = angle of misalignment

Core losses

Spacing losses - (54. 6 $(\frac{d}{\lambda})$ db)

**A. Amplitude Equalizations** The unprocessed signal from the reproduce head contains large amplitude variations (15-20 db) with frequency and would not be usable in a data recording system. In order to obtain data with a uniform frequency reponse, the reproduce amplifier must have its frequency response modified by a curve which is the complement of the unequalized head output curve.

In the region from 100 kc to approximately 500 kc and the region from 500 kc to 1.5 mc, corrections for amplitude and phase characteristics must be treated differently.

In general, an LRC network which introduces a non-linear amplitude response contains accompanying frequency dependent phase shifts. Another LRC network which is used as an equalization network to restore a linear frequency response will in general contain complementary frequency dependent phase shifts which will tend to make the overall amplitude and phase response linear with frequency.

The tape system in the frequency area below the region of gap effect (500 kc) exhibits a reasonably uniform 901 phase shift and a 6 db/octave amplitude slope such as would be expected from a pure differentiating network. In this region an excellent equivalent circuit can be used. As the frequency increases to the region where the various losses alter the amplitude response curve directly, the amplitude/phase response radically departs from a passive LRC network and the equivalent circuit becomes quite complex. Here a non-uniform frequency response is not accompanied by the normally expected frequency dependent phase shifts since the dominant high frequency losses result from an aperture or gap effect as opposed to that of an LRC network.

In this region conventional amplitude equalization networks can easily restore a uniform frequency response but the phase distortion or envelope delay introduced by this network must be negated by another passive network, a phase correction filter. Here an all-pass network with a uniform frequency response has an envelope delay characteristic which is the complement of the resulting delay introduced by the high frequency equalization network.

Figure 10 shows the group delay and amplitude response of the recorder system through a completely linear amplifier, i.e., no amplitude or phase correction networks.

The region from 100 kc to approximately 500 kc reflects the transition region of the 6 db/octave rise and the introduction of losses due to spacing, gap, and other effects. The envelope delay excursion below 500 kc reflect frequency dependent phase shifts which can be compensated for by using an integration network with an appropriate corner frequency to restore uniform frequency response. An amplifier with low end equalization would have uniform phase shift and minimum differential envelope delay with frequency over the passband as long as no other equalization was used.

The region from 500 kc to 1.5 me exhibits a large (≈15 db) drop in amplitude but a minimum of delay excursion. The coil and core losses of the reproduce head cause the small delay variations at the upper end of the passband.

Figure 11 shows a block diagram of a reproduce amplifier.

It is necessary to amplitude equalize the high frequency region to restore flat frequency response. This is done with a tuned circuit in the feedback loop of the reproduce amplifier in Figure 12. The LCR circuit is tuned to approximately 1.6 mc and its Q adjusted to give the proper amplitude response.

Figure 13 shows the amplitude and phase response of an amplitude equalized amplifier with no phase correction filter. The amplitude response is acceptable out to 1.5 me but the severe phase distortion is not tolerable.

Certain filter networks having uniform frequency response and specific phase response (all-pass networks) can be used as phase correction networks. These filters can be of many forms, active or passive, balanced or unbalanced, and of various orders. The particular type used in this recorder is fundamentally a constant resistance unbalanced all-pass network of second order. The basic second order filter has a form shown in Figure 14. The envelope delay characteristics of this network are designed to complement the delay curve of the high end equalizer and maintain a linear amplitude response. The required number of all-pass sections are cascaded together to develop the desired delay. By inserting this network in series with the reproduce amplifier, see Figure 11, it is possible to obtain a linear phase response without altering the already equalized amplitude response.

Figure 13 (dashed lines) shows the resultant amplitude and envelope delay curves of a fully compensated channel. The specification of 6 db amplitude excursion and 200 nanosecond delay excursion is easily accomplished. Figure 15 shows a module containing all the components of the delay correction network for insertion in a printed wiring board.

Figure 16 is a photograph of the reproduce amplifier containing two boards, one the active and switching component and the second showing the 4 sets of delay and amplitude equalizers. The unit is a four speed amplifier where the proper amplitude and phase correction filters are automatically relay activated correlating the electronics with the transport speed.

**IV. Production** The development of a single filter to precisely cancel undesired phase distortion is not a difficult problem, but when the machine becomes a high production item, additional complications occur.

Normal production tolerances of the record head, reproduce head, reproduce amplifier and delay filter must be accounted for in the design and adjustment range of the equalizers in order to maintain a satisfactory production yield. The solution to this design problem lies in a statistical evaluation of all production tolerances and determination of usable variations.

Figure 17 shows the envelope delay characteristic of various components in the reproduce system. Figure 17a is the envelope delay vs frequency plot of the reproduce head output with no equalization; Figure 17b shows the envelope delay plot of the equalized amplifier. Note that the region below 500 kc is self-compensating (both in amplitude and phase). An adequate range of adjustment is provided in the mid-frequency region to accommodate production tolerances. The largest remaining <u>delay</u> variation exists in the area of high frequency amplitude equalization (1.0 to 1.5 mc).

The largest <u>amplitude</u> variation from <u>unit</u> to <u>unit</u> lies in the high frequency roll-off characteristics of the reproduce head. Normal acceptable head manufacturing tolerances of its peak output at 200 kc: to 1.5 mc response can range from 14 to 20 db. The high frequency amplitude equalizer must encompass this range for proper equalization and is actually designed to cover 12 to 24 db.

Unfortunately the extent of high frequency equalization determines the over-all envelope delay, i.e., as the circuit Q is increased, the phase changes more rapidly through resonance and the envelope delay increases. This increase consequently requires more correction in the all-pass network.

Figure 17c shows the three ranges of delay introduced by the all-pass network to use as required by the particular channel. The desired over-all delay is selected by strapping of the proper terminals at the top of the filter (Figure 15).

Examination of Figures 17a, 17b and 17c shows a sufficient number of adjustments to enable the recorder system to meet the imposed frequency response and envelope delay specification with reasonable production costs.

**V. Conclusion** A system with an envelope delay specification of 200 nanosecond total variation indicates that the phase shift vs frequency plot will exhibit a certain departure from a straight line. Since this line in general will not intersect the zero or $2\pi N$ radians phase shift axis it does not necessarily imply 200 nanoseconds variation in relative phase or transmission delay.

This envelope delay specification is meaningful only in the case of a modulated waveform. For example, two carriers are placed anywhere in the passband and are either amplitude or frequency modulated with a single frequency, the two demodulated frequencies will not have a relative time delay displacement of greater than 200 nanoseconds. (The carriers must be sufficiently displaced from the band edges to allow the AM sidebands and all significant FM sidebands to be contained within the band edges.) Further, if an FM or AM carrier and its sidebands are transmitted through a network where the rate of change of $\frac{d\phi}{dt}$ is zero over the passband occupied by the carrier and its sidebands, no distortion of the demodulated signal will occur as a function of the phase characteristics of the network. This specification is related to differential envelope delay which is not necessarily equal to differential phase delay.

To insure that the Fourier components of a complex unmodulated waveform contain no differential time delay greater than some value $T_d$ in transmission through a network, it is necessary to specify the network in terms of phase delay $(\frac{\phi}{\omega})$.

Techniques are available using complex networks which will control the maximum envelope delay variation in a tape recorder/reproducer system. The system of phase compensation described in the paper represents one possible method, other classes of networks which combine amplitude equalization with linear phase response could be used.

The parameter of envelope delay adds another index of performance to a tape recorder system. This paper has shown that envelope delay is a convenient method of describing the phase linearity of a tape recorder where the dominant recorded data consists of modulated signals. It is important that this term be well understood to accurately correlate over all system performance with recorder performance. Additional analytical work is required to more closely correlate envelope delay variations with FM distortion, for example, so that meaningful specifications can be established. Perhaps the second derivative of the phase shift $\frac{d^2\phi}{dt^2}$ is the parameter to investigate?

## REFERENCES

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5. Stewart, W. E., Magnetic Recording Techniques, McGraw Hill, 1958, Chapters 1 and 3. Also Appendices I, V, VI.

Appendix. -Using amplitude modulation theory the difference between envelope and phase delay can be illustrated.

An unmodulated carrier e of peak amplitude, $E_c$ and frequency $\frac{\omega_c}{2\pi}$ can be represented as

$$e = E_c \ (\omega_c t + \phi), \quad \phi = \text{an arbitrary phase angle} \tag{1}$$

if the carrier is amplitude modulated by a pure sine wave signal of the form.

$$e_m = E_m \cos \omega_m t, \qquad E_m = \text{peak modulation signal amplitude} \tag{2}$$

$$\frac{\omega_m}{2\pi} = \text{the modulating frequency}$$

where $\omega_c$ is much larger than $\omega_m$, the resulting modulated signal has the form

$$e = (E_c + K_a E_m \cos \omega_m t) \cos \omega_c t \tag{3}$$

which can be written as

$$e = E_c \cos \omega_c t \ (1 + m_a \cos \omega_m t) \tag{4}$$

where

$$m_a = \frac{K_a E_m}{E_c} = \text{the modulation index}$$

This equation (4) can be considered as the modulation form of the signal since the $\cos \omega_m$ and $\cos \omega_c$ terms can be examined separately. Further expansion using trigonometric identities yields;

$$e = E_c \cos \omega_c t + \frac{m_a E_c}{2} \cos (\omega_c + \omega_m) t + \frac{m_a E_c}{2} \cos (\omega_c - \omega_m) t \tag{5}$$

The terms $\omega_c t$, $(\omega_c + \omega_m) t$ and $(\omega_c - \omega_m) t$ represent the carrier, upper sideband and lower sideband respectively. If these frequency components are passed through a transmission network having a constant amplitude characteristic with frequency and the phase characteristics of Figure 2, the output signal where $\omega_c = \omega_{c1}$ can be represented as:

$$e = E_c \cos \omega_{c1} \ (t + K_1) + \frac{m_a E_c}{2} \cos \ (\omega_{c1} + \omega_m) \ (t + K_1) + \frac{m_a E_c}{2} \cos \ (\omega_{c1} - \omega_m) \ (t + K_1) \tag{6}$$

rewriting (6) into the modulation form similar to (4) by using the identity "cos A + cos B = 2 cos 1/2 (A + B) cos 1/2 (A - B)"

$$e = E_c \cos \omega_{c1} \ (t + K_1) + \frac{m_a E_c}{2} \ 2 \left\{ \cos 1/2 \left[ (\omega_{c1} + \omega_m) \ (t + K_1) + (\omega_c - \omega_m) \ (t + K_1) \right] \cos 1/2 \left[ (\omega_{c1} + \omega_m) \ (t + K_1) - (\omega_{c1} + \omega_m) \ (t + K_1) \right] \right\} \tag{7}$$

$$e = E_c \cos \omega_{c1} \ (t + K_1) + m_a E_c \cos \ \omega_{c1} \ (t + K_1) \cos \omega_m \ (t + K_1) \tag{8}$$

$$e = E_c \cos \ \omega_{c1} \ (t + K_1) \left[ 1 + m_a \cos \omega_m \ (t + K_1) \right] \tag{9}$$

where both the phase delay $\frac{\phi}{\omega}$ and the envelope delay $\frac{d\phi}{d\omega}$ are equal to $K_1$, $(\phi_1 = 0)$ both the carrier and the modulation envelope will be delayed the same amount.

Now consider the case of $\omega_{c2}$ where the phase shift curve does not intersect the origin or $2\pi n$ radians. In this case the phase delay $\frac{\phi}{\omega}$ and the envelope delay $\frac{d\phi}{d\omega}$ are not equal.

$$e = E_c \cos \left[\omega_{c2} \ (t + k_2) + \phi_1\right] + \frac{m_a E_c}{2} \cos \left[(\omega_{c2} + \omega_m) \ (t + k_2) + \phi_1\right]$$

$$+ \frac{m_a E_c}{2} \cos \left[(\omega_{c2} - \omega_m) \ (t + k_2) + \phi_1\right] \tag{10}$$

using the identity cos A + cos B = 2 cos 1/2 (A + B) cos 1/2 (A - B)

$$\begin{aligned} e = E_c \cos \left[\omega_{c2} \ (t + k_2) + \phi_1\right] + m_a E_c \ \Big\{ \cos 1/2 \ \Big[ (\omega_{c2} + \omega_m) \ (t + k_2) \\ + \phi_1 + \ (\omega_{c2} - \omega_m) \ (t + k_2) + \phi_1 \Big] \ \cos 1/2 \ \Big[ \omega_{c2} + \omega_m) \ (t + k_2) + \phi_1 \\ - (\omega_{c2} - \omega_m) \ (t + k_2) - \phi_1 \Big] \Big\} \end{aligned} \tag{11}$$

$$\begin{aligned} e = E_c \cos \left[\omega_{c2} \ (t + k_2) + \phi_1\right] + m_a E_c \cos \left[\omega_{c2} \ (t + k_2) + \phi_1\right] \\ x \cos \omega_m \ (t + k_2) \end{aligned} \tag{12}$$

$$e = E_c \cos \left[\omega_{c2} \ (t + k_2) + \phi_1\right] \left[1 + m_a E_c \cos \omega_m \ (t + k_2)\right] \tag{13}$$

here it can be seen that the modulation envelope has been delayed by a factor $\frac{d\phi}{d\omega} = k_2$ which is the envelope delay and the carrier delayed by the phase delay $\frac{\phi}{\omega} = k_2 \omega_{c2} + \phi_1$. If the extension of the line with slope $k_2$ intersected a multiple of $2\pi$ radians or if $\phi_1 = 0$, it can be seen that only under these special conditions would the group delay and phase delay be equal. Since in a tape recorder, the phase delay is indeterminate, the emphasis is on envelope delay.

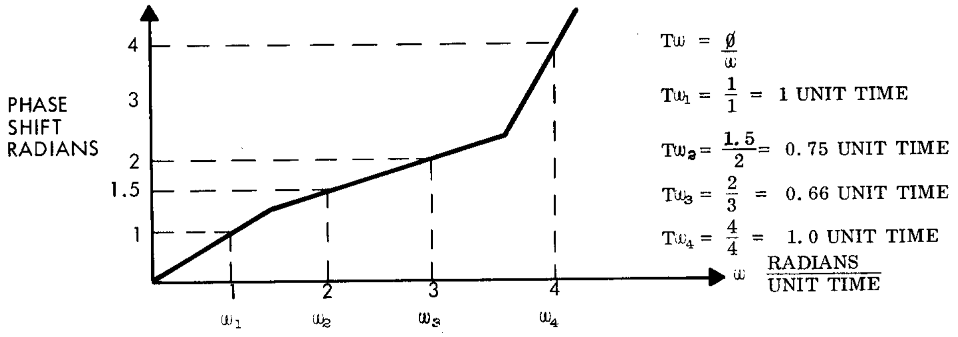


Fig. 1 Phase shift 0 vs ω

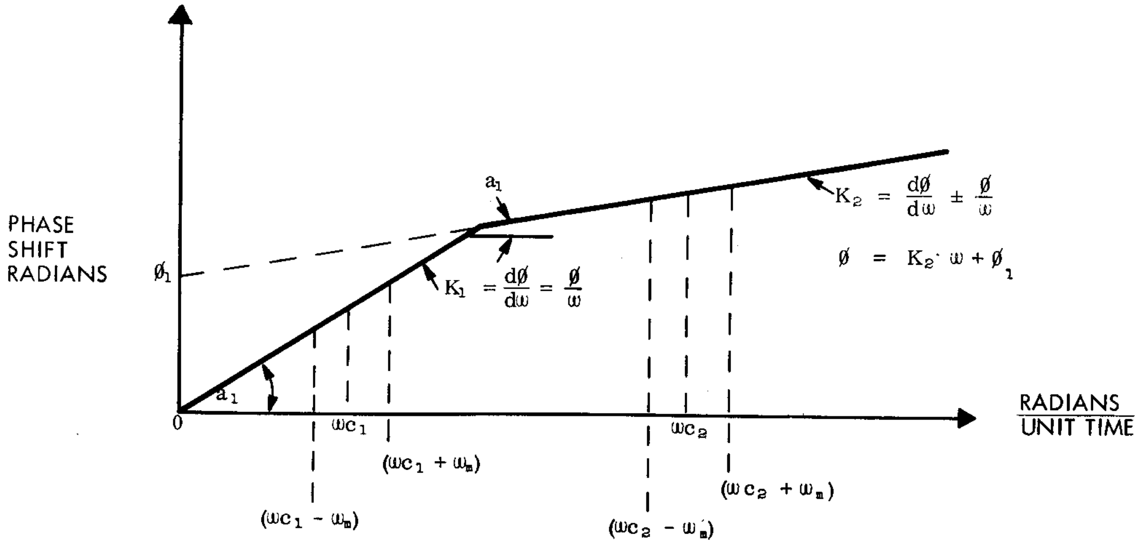


Fig. 2 Phase shift $\phi$ vs $\omega$

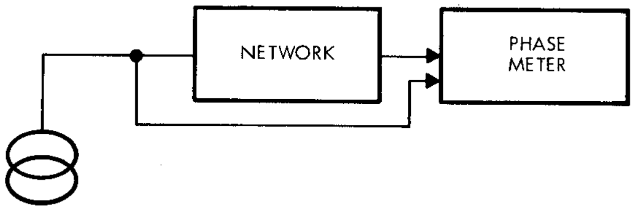


Fig. 3 Phase shift method of envelope delay measurement

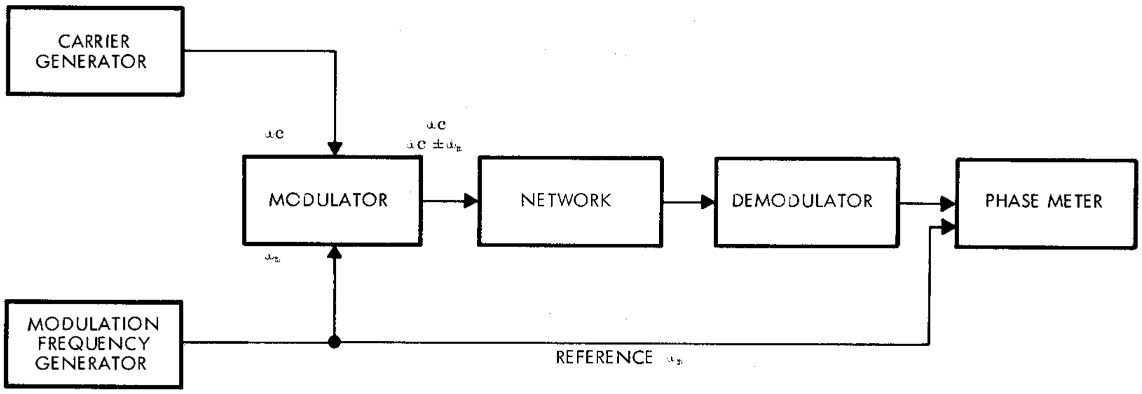


Fig. 4 Nyquist method of envelope delay measurement

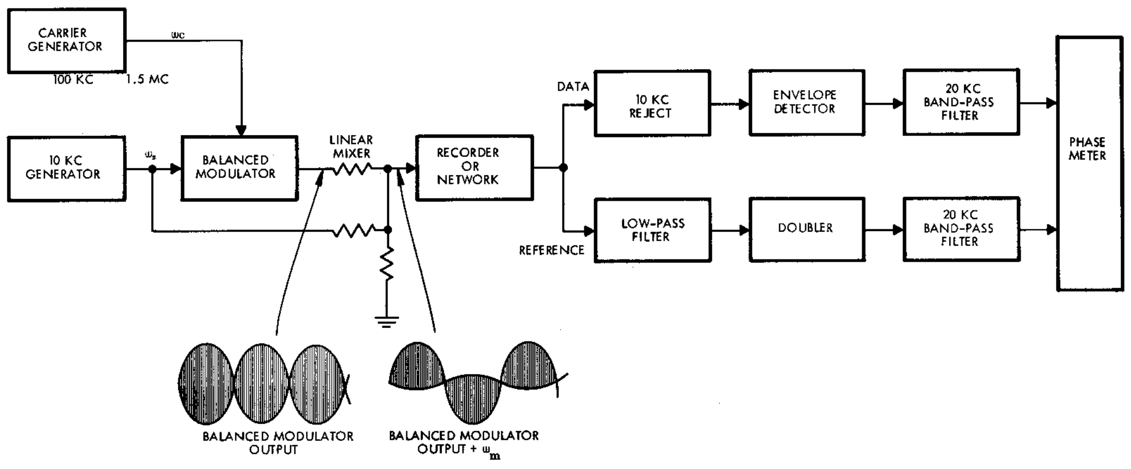


Fig. 5 Open loop, frequency multiplexed method of envelope measurement

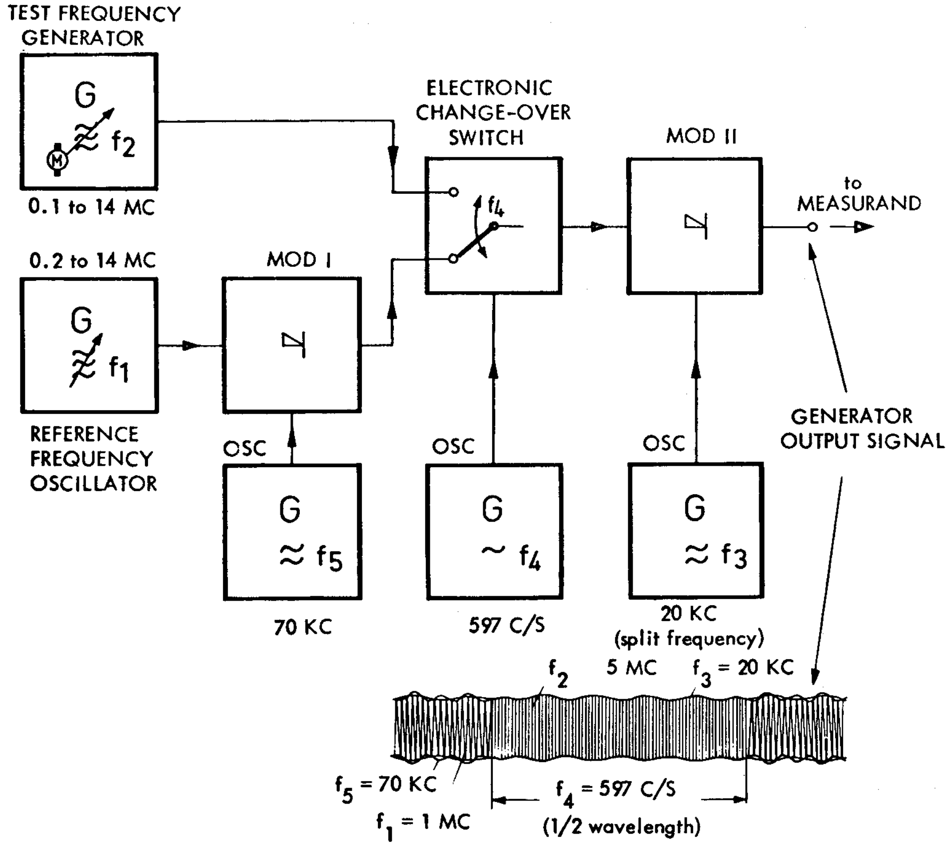


Fig. 6 W and G, LD-1 generator block diagram

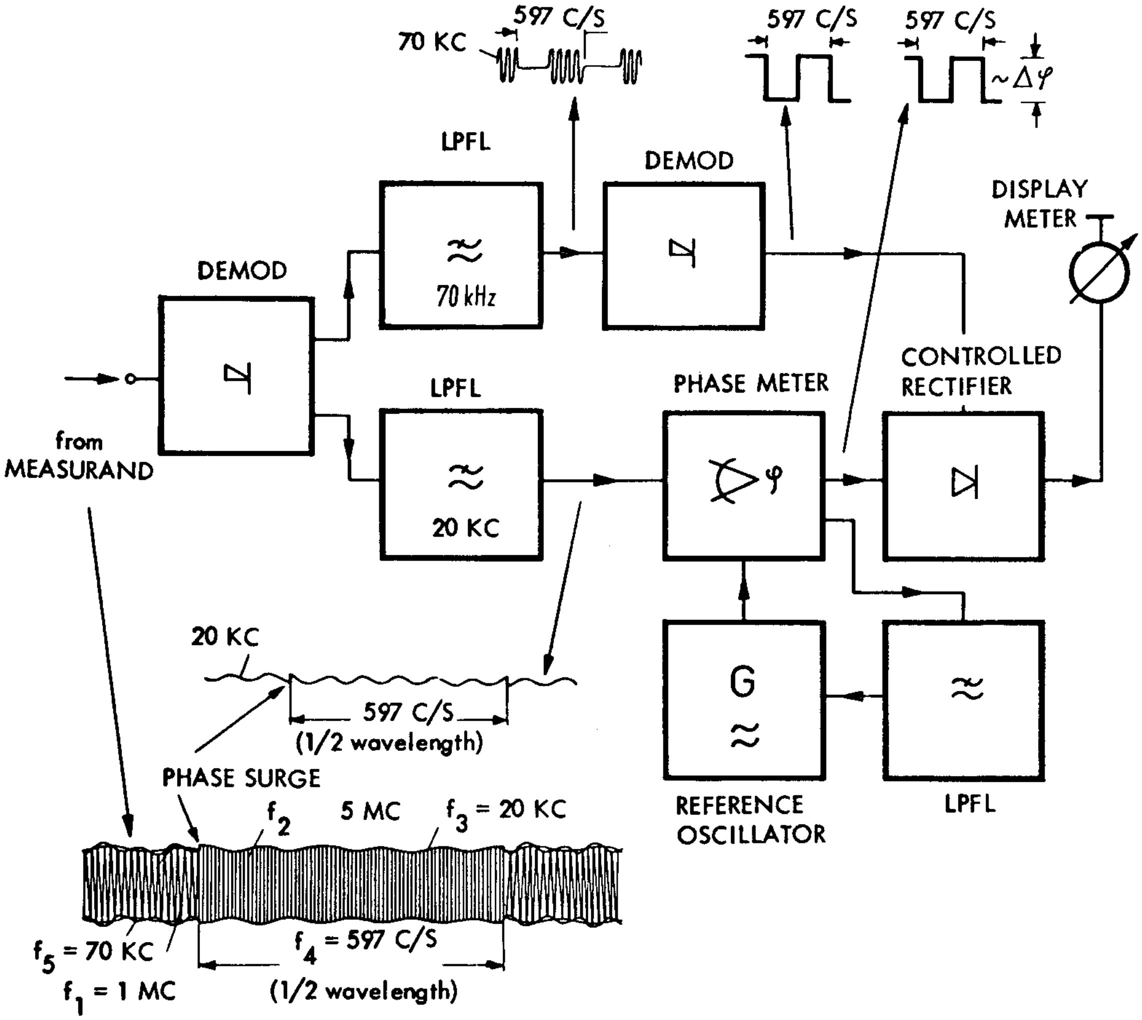


Fig. 7 W and G, LD-1 receiver block diagram

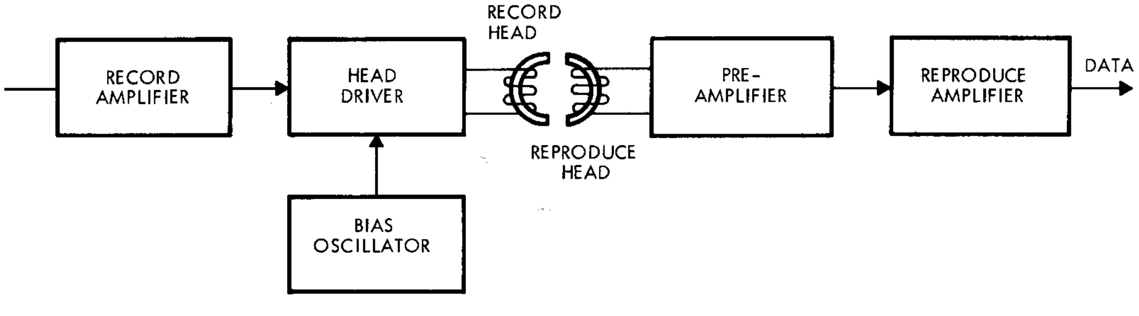


Fig. 8 Magnetic tape recorder system block diagram

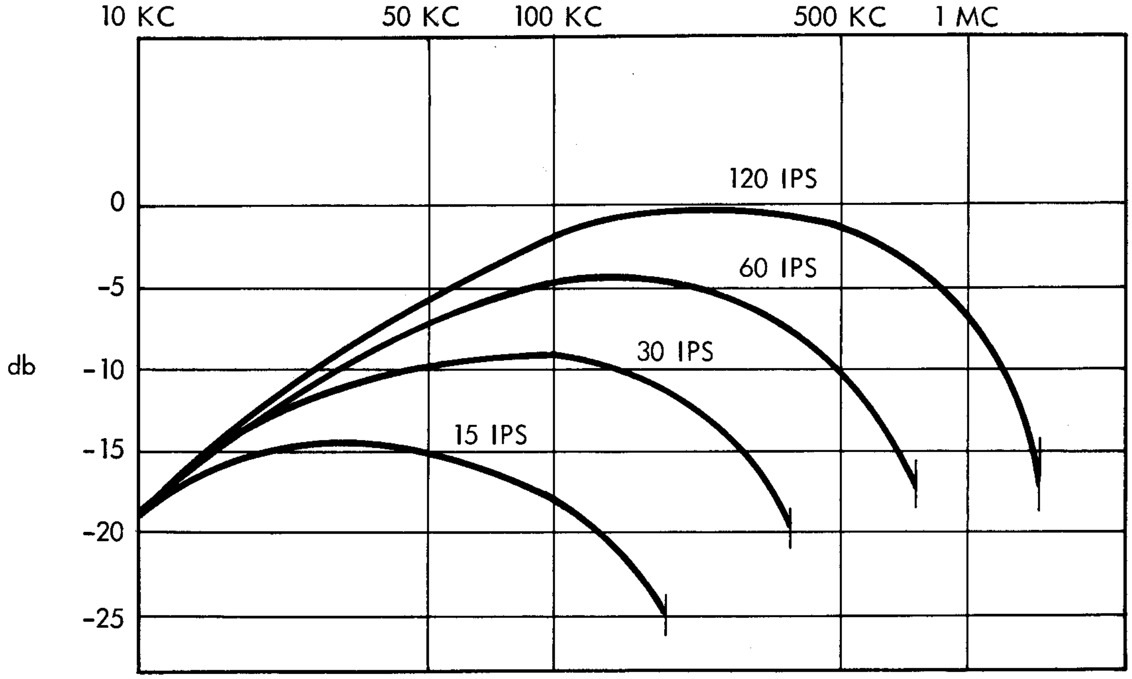


Fig. 9 Reproduce head output at different tape speeds

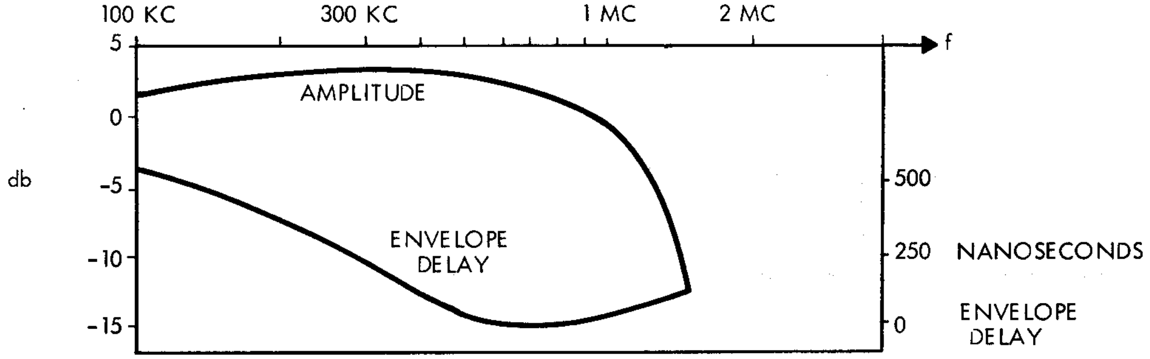


Fig. 10 Reproduce signal at 120 ips without amplitude or phase correction

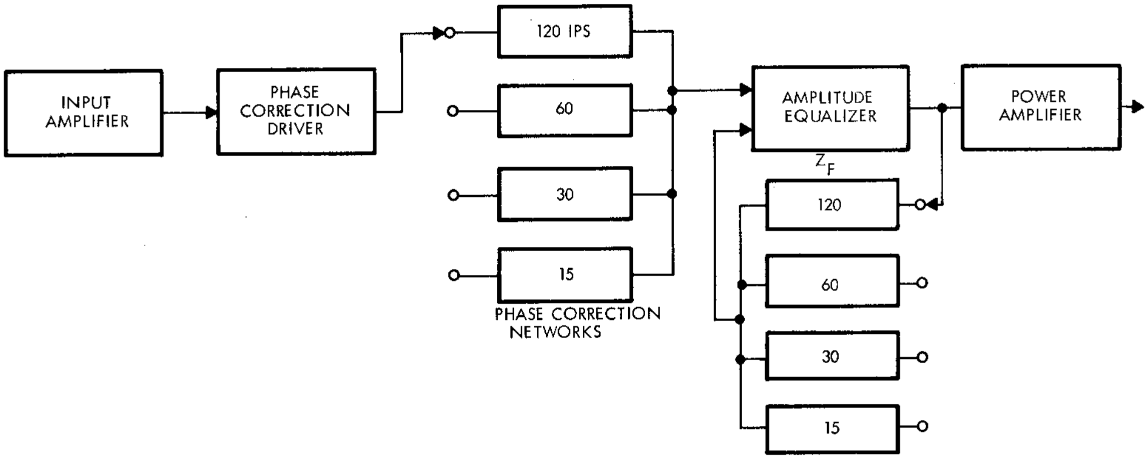


Fig. 11 Block diagram reproduce amplifier

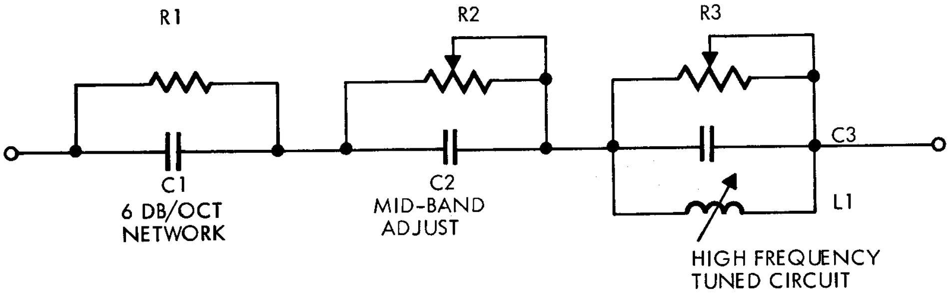


Fig. 12 Typical $Z_F$ , amplitude equalization network

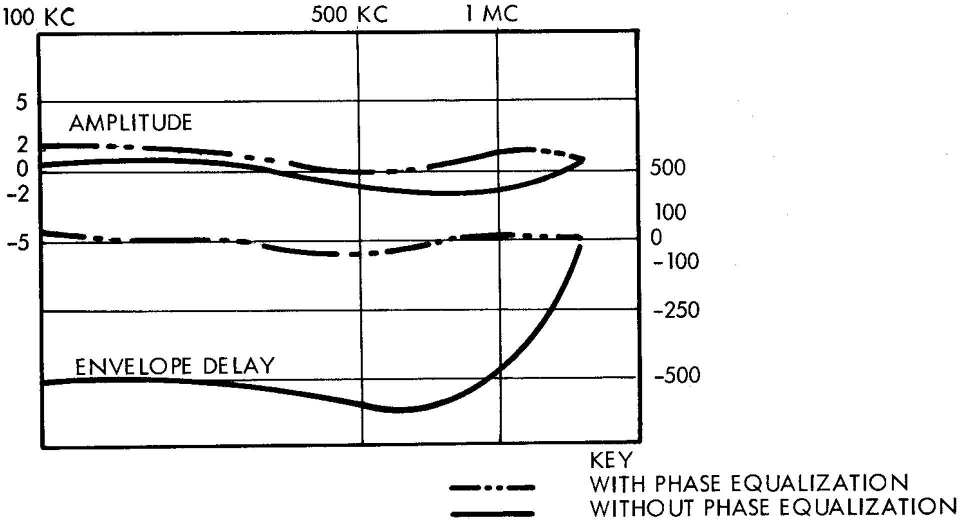


Fig. 13 Amplitude equalized reproduce signal, with and without phase correction

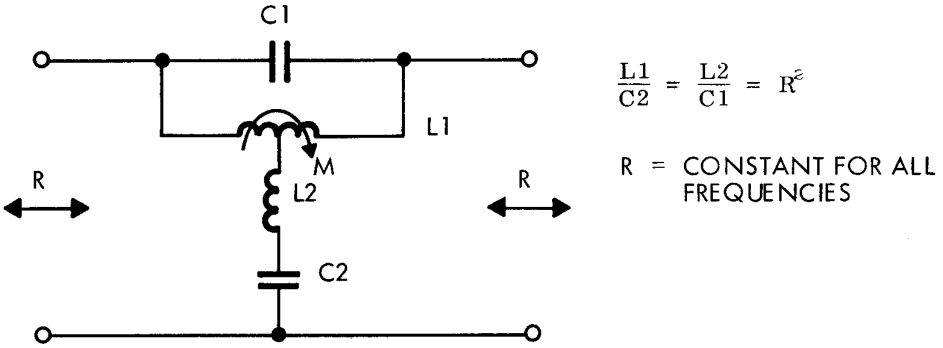


Fig. 14 Second order all-pass network

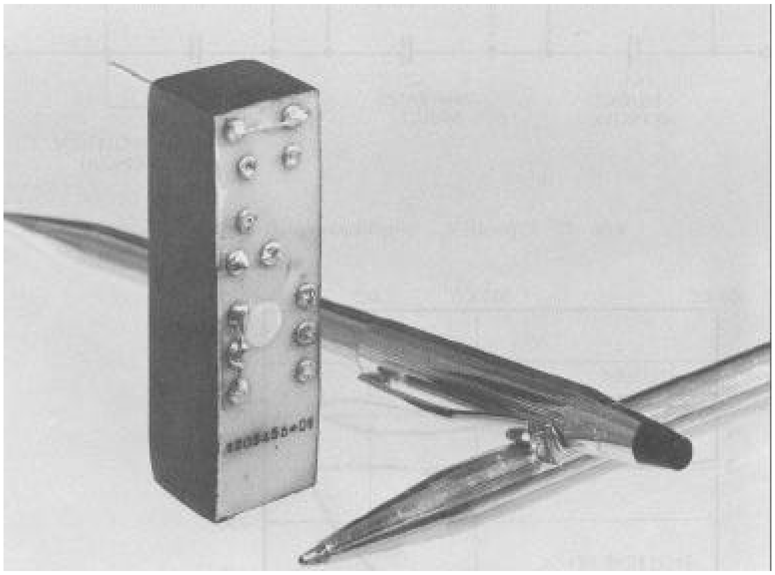


Fig. 15 Phase correction module

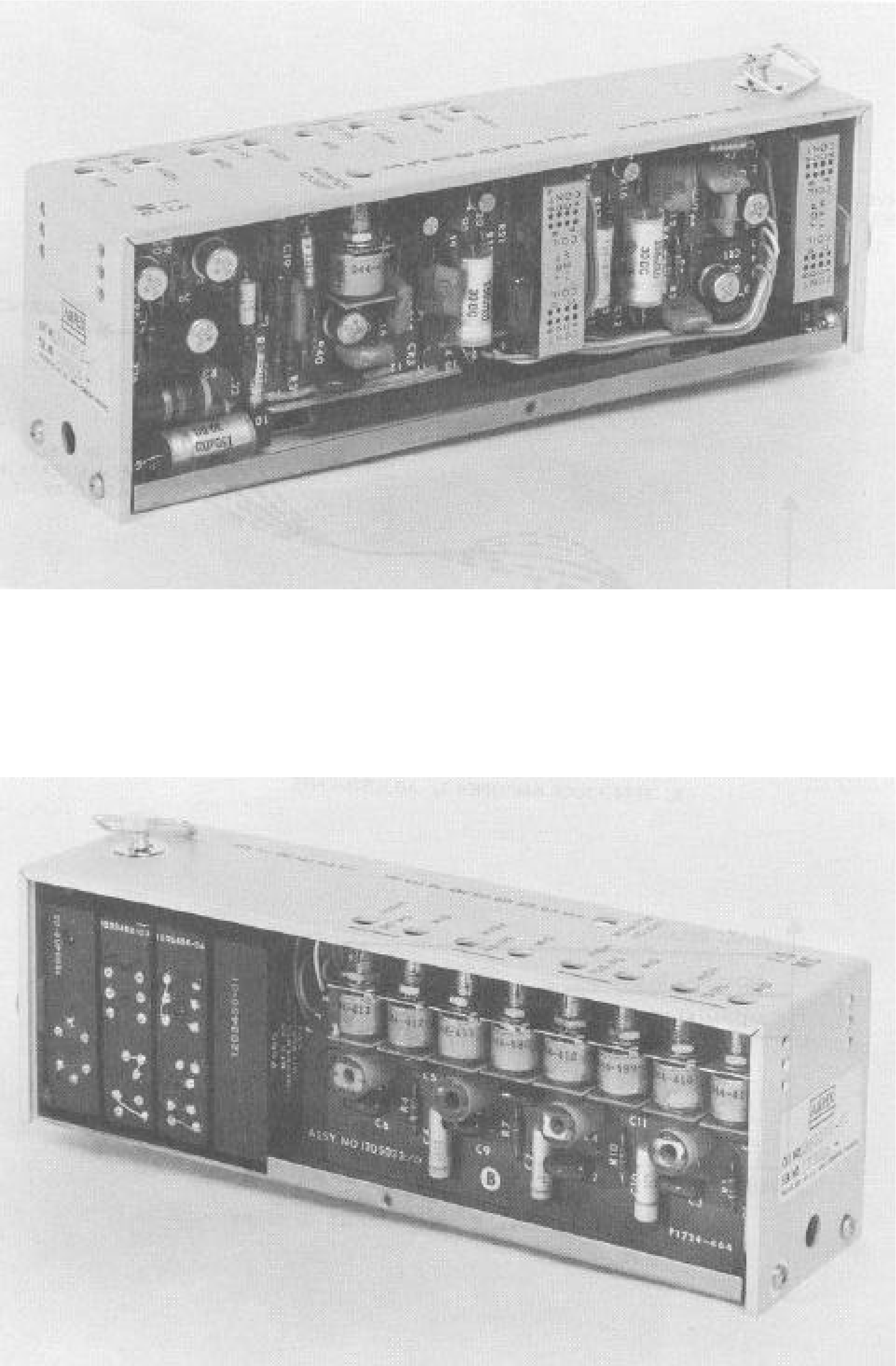
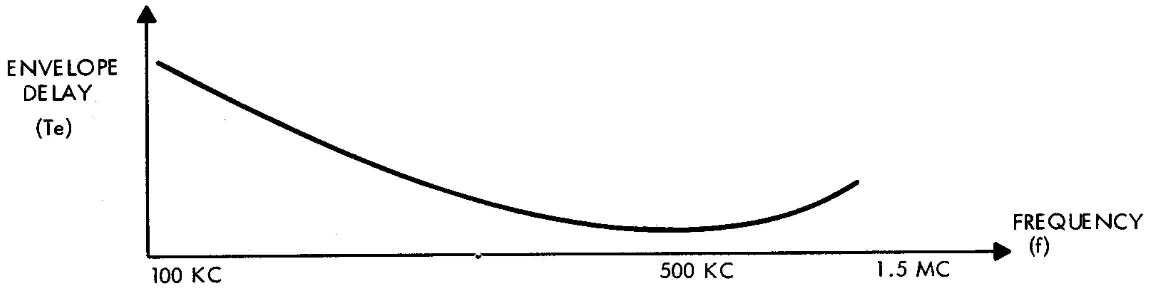
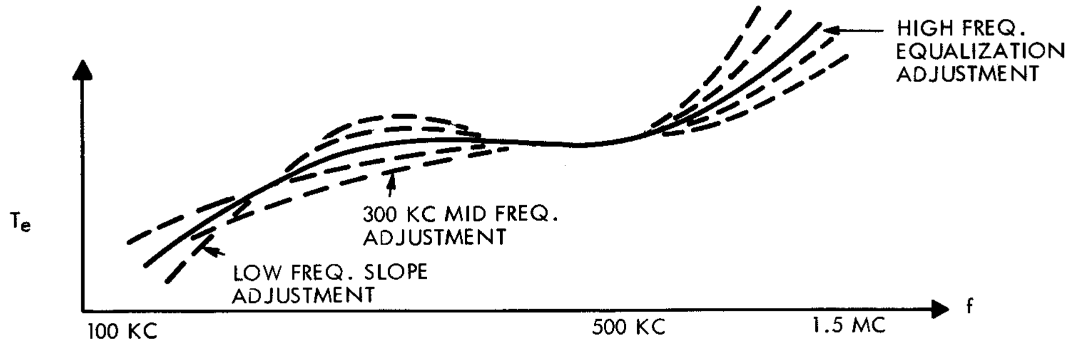


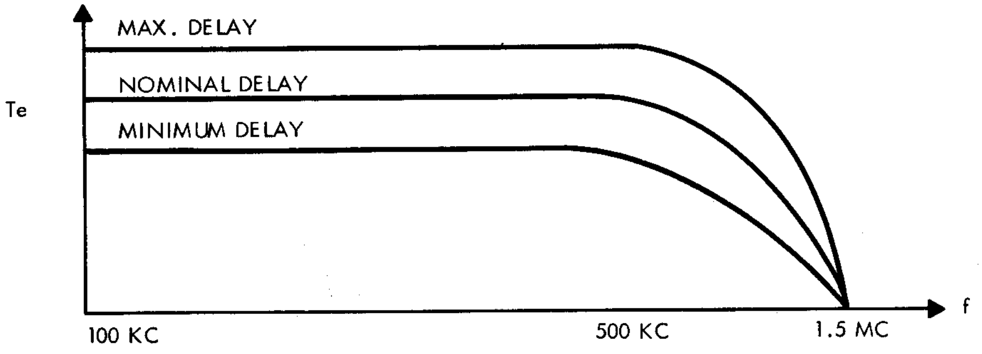
Fig. 16 Reproduce amplifier



A. REPRODUCE HEAD OUTPUT (120 IPS)



B. REPRODUCE AMPLIFIER $T_e$ ADJUSTMENTS



C. ALL PASS NETWORK RANGES OF DELAY

Fig. 17 Reproduce system envelope delay