ROCKET TRAJECTORY ANALYSIS FROM TELEMETERED ACCELERATION AND ATTITUDE DATA[1]

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Summary Double integration of the longitudinal acceleration of a sounding rocket is useful as a simple means of determining its trajectory. Reasonably accurate altitude calculations can be made by this method except when surface winds alter the launch angle of the rocket. Surface wind velocity corrections can be introduced to correct velocity and position information in the horizontal direction, but accurate wind correction data is difficult to obtain for all rockets. A special solar aspect sensor was designed to be used with a commercially available magnetic aspect sensor for rocket attitude determination. This attitude data allows the longitudinal acceleration to be broken more accurately into three vector components. A feasibility study of the aspect system was made using three Aerobee-150 rockets. A digital computer trajectory program was written to utilize aspect and acceleration data for trajectory analysis. It is evident that rocket attitude data is a useful supplement to the longitudinal acceleration data for trajectory determination. More accurate magnetic aspect data is necessary, however, to refine the longitudinal acceleration technique.

Introduction During the past several years an unusually large amount of upper atmospheric research has been done by means of sounding rockets, long range probes and earth satellites. A wide variety of experiments are being performed including measurements of the earth's magnetic field, measurements of electron and ion density, electron temperature, various types of spectrum analysis of radiation from the sun, atmospheric ionization experiments and many others. Most of the upper atmosphere experiments are borne by relatively low altitude sounding rockets such as the Aerobee, Nike-Cajun, Astrobee-200, and Nike-Apache.

When a scientist is performing a research experiment in the upper atmosphere by means of a sounding rocket, it is of interest to him, when reducing data, to know the altitude of the rocket at all times during the period of the rocket flight. In some instances, especially

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when there are external measuring probes on the rocket, it is also important to the experimenter to know the attitude of the rocket in its trajectory. There are many methods for obtaining trajectory information for practically all rockets that are launched today. Most rocket launching ranges are well equipped with various types of auxiliary equipment, such as radar, for trajectory determination. Most of this equipment is rather complex.

In recent years, the Research Foundation of Oklahoma State University has made calculations of rocket trajectories that involve the time variation of rocket acceleration, velocity, and position. These calculations have been based primarily on data obtained from longitudinal accelerometers mounted within the rocket payload. The acceleration information is transmitted to ground-based recording stations by means of radio telemetry. Past efforts in rocket trajectory analysis have been limited in accuracy by the design and calibration of the longitudinal accelerometers and the quality of the telemetry records available for use in extracting data. Recently, a digital computer program has been developed for calculating trajectories of multistage rockets. This computer program, which is applicable primarily to the IBM 1410 computer, has greater versatility than the desk calculator methods used in the past for trajectory calculations.

Historical Background Several methods have been utilized in determining the trajectory of a sounding rocket. The most common of these is radar which is sometimes supplemented with a beacon transponder for reinforcing the echo signal. Modern radar equipment can skin track a rocket to ranges of 150 miles or more. The trajectory information from radar is available immediately after the rocket flight from the plotting board and computing equipment operated in conjunction with the radar.

Radar triangulation has been used to some extent in the past for trajectory work. This system includes an airborne transponder which, after being interrogated from a main interrogation transmitter, responds with a pulsed signal which is received by several widely spaced ground receiving stations. These stations, in turn, retransmit the delayed transponder signal back to the main interrogation station where a computer and associated plotting boards are ready to present the rocket position da ta.

The interferometer principle is used with the airborne telemetry radio frequency signal for trajectory analysis in some instances. Systems using this principle have two or more ground receiving stations which are accurately surveyed with respect to each other. The range of the rockets from the groups of receiving stations may be obtained by measuring the phase difference of the signal at the receivers.

The Doppler principle with radio signals is used with some tracking equipment for obtaining rocket velocity. The rocket velocity can be integrated to obtain rocket position at any instant of time. One such system is a Doppler Velocity and Position System, DOVAP, which uses a ground based transmitter and an airborne transponder. The

Doppler frequency shift is a function of the rocket velocity. The ground transmitting equipment is usually located almost directly below the rocket as it progresses on the upward leg of its trajectory.

A somewhat simple means for obtaining a rocket trajectory is by simple “free-fall” or “vacuum” calculations during the upper portion of the trajectory where atmospheric retardation is not appreciable. If two points, one on the up-leg and one on the down4eg of the trajectory at the same altitude, can be determined, the remainder of the altitude-time trajectory can be calculated (11). Locating these two points is usually a difficult problem and no method has been devised which will define accurately these points. Some of the methods which have been used in the past are the use of ionization type pressure measuring equipment which is capable of making measurements of minute pressures and the use of Lyman-Alpha spectrum indications which are prominent at discrete altitudes. Cosmic ray intensity measuring devices are also useful for defining two equi-altitude trajectory points called the Pfotzer maxima (5).

Another simple means of obtaining trajectory information is by integrating the acceleration data obtained from a longitudinal accelerometer mounted on a sounding rocket. Work with accelerometers of this type for calculating rocket trajectories has been done by Oklahoma State University for many rockets which have been launched in recent years. A digital computer program has been written and utilized quite extensively for trajectory calculations using acceleration data. This work has been done in connection with a phase of a project for the United States Air Force whereby the trajectory of a rocket can be obtained by means of the telemetry system and without the aid of auxiliary trajectory equipment such as some of those mentioned earlier (2, 3, 4).

Most of the systems mentioned are elaborate and require the use of many personnel for field operations and final data analysis of the rocket trajectory. They have the advantage, however, of being quite accurate and in most cases are quite rapid in providing trajectory information following the launching of the rocket. At an established launch site the more elaborate equipment is always available. However, at remote launch sites an expedient such as the “vacuum” or parabolic trajectory or the integrated accelerometer trajectory is advantageous even though extremely accurate results are not provided.

Some of the reasons for exploiting the use of the longitudinal accelerometer method for trajectory determination are that the equipment is light in weight, can be utilized with the existing telemetry system, requires no auxiliary trajectory apparatus during flight, and is reasonably accurate.

Methods For Calculating Accelerometer Trajectories From the laws of mechanics, it is known that the time-rate of change of distance is velocity, and the time-rate of change of velocity is acceleration. Conversely, if a variation of acceleration versus time is integrated between time limits, velocity is obtained, and by integrating the velocity information between the same time limits, position is obtained. This technique is utilized by numerical methods to obtain the velocity and position of the rocket when acceleration-time data are available. The area enclosed by an acceleration curve bounded by two time limits gives a value of g-seconds which is equivalent to velocity at the upper time limit. To obtain the total velocity of the rocket one must add the new velocity to the velocity at the beginning of the period which is being integrated. A second integration will provide a change of distance. This distance added to the value at the beginning of the integration period gives cumulative distance for the vehicle. Because the acceleration curves may not be expressed as continuous mathematical functions, it is not practical to do mathematical integration using indefinite integrals. Therefore it is necessary to integrate the area under acceleration curves by numerical techniques. Some of the earlier methods used for integration, when making trajectory calculations with desk calculators, have utilized the Trapezoidal rule and Simpson’s One-Third rule. When using these integration methods with a desk calculator, it is necessary to divide the time scale over the period being integrated into equal increments.

One of the simplest methods of numerical integration for this particular problem is the Trapezoidal rule. Its use is based upon the fact that the area under the acceleration curve being integrated is divided into equal increments of time. The area of a series of small trapezoids is accumulated over a long period of time to obtain the total g-second area. Figure 1 illustrates the development of the equations used for acceleration integration by means of the Trapezoidal rule. It may be seen in the figure that the areas of each of the increments include the area of the rectangle as well as the area of the small triangle above the rectangle, both of which form the incremental trapezoid. The segment of area is the area between to and tn which involves discrete acceleration figures between a_o and a_n.

A total area for the segment may be expressed as the sum of the areas of successive trapezoids and is expressed as:

$$\text{Area} = \Delta t \left[\frac{a_o}{2} + a_1 + a_2 + \ldots + a_{n-1} + \frac{a_n}{2}\right] \text{ g-seconds} \qquad (1)$$

Where “a” is given in terms of g, acceleration due to gravity, and Δt is the time increment in seconds.

Simpson’s one-third rule is a more accurate process for obtaining the g-second area under an acceleration curve. Simpson’s rule is based upon the assumption that successive values of acceleration data lie on segments of parabolic curves while the trapezoidal rule is based on the assumption that each data point is connected to adjacent ones by straight lines. Simpson’s One-Third rule, as in the Trapezoidal rule, requires an equal division of time for the segments of area being calculated. This means that the time increments, Δt, must be the same and also that an even number of acceleration values must be available for each segment of area. The expression for Simpson's One-Third rule is:

$$\text{Area} = \frac{\Delta t}{3}\left(a_o + 4a_1 + 2a_2 + 4a_3 + 2a_4 + \ldots + 4a_{n-1} + a_n\right) \quad (2)$$

When acceleration information is integrated by means of these methods, the acceleration curve is usually broken into segments which are readily identifiable by prominent features of the curve as when using the trapezoidal rule.

Desk calculator techniques are not considered to be inaccurate as far as acceleration integration and the determination of rocket trajectories are concerned except that human error may be eminent in making calculations. The technique is laborious however, and for this reason the Digital Computer trajectory routine was written as a replacement for the earlier desk calculator methods. The computer technique allows calculation refinements such as g re-evaluation with altitude, and the use of spherical angular coordinates at launch rather than a vertical launch.

The trajectory program is written for the IBM 1410 Computer in FORTRAN so that it is compatible with other types of computers. The computer program utilizes acceleration data obtained from telemetry records. It will make trajectory calculations of acceleration, velocity, and position in three orthogonal directions utilizing a preselected increment of time for the atmospheric portion of the rocket flight. This portion includes all of the thrust and the drag periods of the up-leg flight. A different increment of time, usually larger than the first, is used for velocity and position calculations following the atmospheric portion.

The integration method for the acceleration data does not use the Trapezoidal rule nor Simpson's rule as illustrated earlier but considers the accelerometer curve as a series of small rectangles as shown in Figure 2. The acceleration is assumed to be constant from one value of elapsed time to the next. As can be seen in the figure, the area under a rising accelerometer curve is somewhat greater than the true area depending upon the time increments being used while the area under the falling portion of the curve is somewhat smaller than the true area. Thus the errors partially cancel. The program will accommodate any number of thrust or drag periods making it useful for multi-stage rockets.

The computer program will introduce the effects of winds as forces operating at any given altitude and considers the winds as occupying de-finite altitude strata. The winds have the effect of changing the elevation and azimuth angles of a rocket as measured at the instantaneous position of the rocket in its trajectory. These wind corrections of the elevation and azimuth angles are included when calculating the vector components of velocity such as when rocket attitude are not available.

A least-squares-curve-fit data smoothing feature was originally included in the trajectory program. This curve fit routine utilizes five adjacent acceleration points for calculating a smooth curve. The mid point of this curve is the value of acceleration used in making trajectory calculations. It was found, however, that in most cases the curve fit routine was unnecessary because the data was smoothed at the time it was read from the telemetry records and no improvement could be made by smoothing the data for the computer calculations.

The computer program is also designed to re-evaluate the gravitational attraction constant of the earth, g, as an inverse square function of the altitude of the rocket. This re-evaluation is done with each incremental calculation of the altitude of the rocket and each new g value introduced into the subsequent calculations (12).

There are some features to be desired in the longitudinal accelerometer technique of determining a rocket trajectory. Even though fairly good accuracy may be obtained by integrating the acceleration when the effective launch angle of a rocket is known, there are no accurate means available for determining the corrections that should be made as a result of the ground winds that change the course of the rocket immediately following launch. When a rocket is being launched, the launcher is given an azimuth and elevation attitude which will cause the rocket to impact at a previously determined point, considering the effects of winds. The accelerometer trajectory technique will indicate that the impact point is directly in alignment with the azimuth of the launcher unless wind correction or attitude data have been introduced. Until the present time there has been no definite scheme for providing good attitude correction that will conform to the ground wind data. The latest method utilizes a direct alteration of the transverse velocity components of the rocket, considering that the rocket will weather-vane into the winds with a velocity which is a function of the winds, but the wind corrections have not been accurate enough to indicate that the rocket trajectory will conform to the radar as far as altitude is concerned. If one can measure the attitude of the rocket at all times during the thrust portion of the rocket trajectory, the trajectory can be corrected so that the zenith altitude will be more nearly correct, and the impact point will also be corrected to the proper azimuth, which in most cases is not the same as the azimuth setting of the launcher.

Trajectory Calculations With Digital Computer The accelerometers used for obtaining acceleration data for the upper atmosphere sounding rockets are of the swept resistance type in which a spring-loaded mass actuates an output pickoff. These accelerometers are constructed in such a manner that five linear resistance elements are wound end to end and are traversed in one sweep by the mass and pickoff wiper. The resistance elements are electrically connected in parallel so that the voltage-acceleration variation is presented as five linear sawtooth sweeps between zero and five volts. The accelerometers used for Aerobee rockets, for example, are designed to give acceleration data up to 20 g in the positive direction and 10 g in the negative direction. This is a total of 30 g and is covered by five segments, therefore each segment covers six g acceleration. Based on 50 turns per segment, this is 0.12 g per turn.

One of the features of this segmented type accelerometer over the conventional accelerometer where only one segment is available is that it is self calibrating. As the rocket acceleration transverses through the range of a segment, the output voltage automatically shifts from five to zero volts, thereby setting the maximum and minimum limits of the telemetry channel as far as voltage excursion is concerned.

The factory calibration of an accelerometer is not extensive enough for purposes of accurate trajectory determination, and it is necessary to recalibrate each unit. This is done on a centrifuge where, by means of equations of rotary motion, the acceleration as a function of revolutions-per-second of the centrifuge is computed.

The usual type of telemetry used for data transmission from the sounding rockets with which Oklahoma State University is associated is the FM/FM type. This system, which operates in the vicinity of 225 megacycles, has low power transmitters which are capable of being frequency modulated by subcarrier channel oscillators. For an accelerometer, a subcarrier frequency in the neighborhood of three kc is quite adequate to show the variations in the acceleration and the segmentation of the accelerometer.

It is not the purpose of this paper to discuss mechanical features of rocket flight or the exterior ballistics of rockets. It might be mentioned, however, that rockets which are fired with an appreciable angle of attack are subject to large stresses and bending moments and, in some cases, are subject to breaking apart. In most cases, the rockets are launched with a zero angle of attack where the thrust is parallel to the velocity of the rocket. This zero angle of attack is also referred to as “zero lift” or a “gravity turn” trajectory. Referring to Figure 3, the equations for a rocket trajectory in two dimensions are as follows:

$$\ddot{x}(t) = \frac{F(z)}{m(t)} \sin \gamma(t) - \frac{D(z,v)}{m(t)} \sin \gamma(t) \quad (3)$$

$$\ddot{z}(t) = \frac{F(z)}{m(t)} \cos \gamma(t) - \frac{D(z,v)}{m(t)} \cos \gamma(t) - g(z) \quad (4)$$

where F is the thrust forces and D is the atmospheric drag forces. The rocket velocity is given by

$$v = \sqrt{\dot{x}^2 + \dot{z}^2} \qquad (5)$$

and the elevation angle of the tangent to the flight path is

$$\gamma = \arc \tan (\dot{x}/\dot{z}) \qquad (6)$$

A flat non-rotating earth is assumed here. The horizontal and vertical components are denoted by x and z respectively. Normally the equations of motion are integrated numerically, assuming zero initial velocity and initial altitude. The equations above may be expanded to include three dimensions, x, y, and z. It can be seen that the value, g(z), is the gravity factor that creates the turning moment that directs the trajectory back toward the earth (6, 10).

The numerical integration technique for the computer program is illustrated in Figure 2. The area under the acceleration curve is considered to be the sum of small rectangles. The acceleration is assumed to be constant from one value of time to the next, the values being separated by the increment of time, Δt t, mentioned previously. As can be seen in Figure 2, the area of a rising acceleration curve is somewhat greater than the true area depending upon the increments being used while the area under the falling portion of the curve is smaller than the true area. This partially compensates for the error in total area.

Figure 4 illustrates acceleration vectors on an X, Y, Z, set of orthogonal axes. The measured value of acceleration, a_m, obtained from the telemetry records is the magnitude of the accelerometer readings in terms of feet per second per second and a_x, a_y, and a_z are the vector components of acceleration. It can be seen that the Z component is corrected by g_h, the value of g at the concerned altitude. In analyzing the diagram of Figure 4 and the mathematical treatment used in obtaining velocity and position results, it is assumed that at liftoff when the launcher ceases to support the rocket and before the thrust actually begins, the accelerometer theoretically reads zero.

During atmospheric flight the accelerometer reads the longitudinal thrust or drag from external forces not including the action of gravity. During the "free fall" portion of the trajectory the accelerometer reads "zero" even though the rocket is under the influence of the force of gravity.

The magnitudes of the vector components of acceleration are:

$$a_{x_i} = a_{m_i} \sin E_i \sin A_i \qquad (7) \qquad\qquad a_{z_i} = a_{m_i} \cos E_i - g_h \qquad (9)$$

$$a_{y_i} = a_{m_i} \sin E_i \cos A_i \qquad (8) \qquad\qquad a_{t_i} = \sqrt{a_{x_i}^2 + a_{y_i}^2 + a_{z_i}^2} \qquad (10)$$

where a_{t_i} is the tangential acceleration. R_o is the radius of the earth and

$$g_h = g_o \left[\frac{R_o}{R_o + Z_i}\right]^2 \tag{11}$$

The magnitudes of the vector components of velocity are:

$$v_{x_i} = a_{x_i} \Delta t + v_{x_{i-1}} \tag{12}$$

$$v_{y_i} = a_{y_i} \Delta t + v_{y_{i-1}} \tag{13}$$

The computer trajectory program is written to include the effects of winds on rocket attitude. This is done by making a direct addition of the corrected wind velocities at altitude to the X and Y component velocities of the rocket during thrust. Since the rocket will vane into the wind, the horizontal velocity of the rocket will be increased in the direction of the wind source. To be more accurate in making transverse velocity corrections for the effects of winds, certain wind ballistic factors which vary with wind height and type of rocket should be used. These factors are not always available, however, the computer provides for a fixed wind modif in factor. The wind-corrected velocities are:

$$v_{x_i} = v_{x_i} + W_{x_i} \tag{14}$$

$$v_{y_i} = v_{y_i} + W_{y_i} \tag{15}$$

The vertical component of velocity is:

$$v_{z_i} = a_{z_i} \Delta t + v_{z_{i-1}} \tag{16}$$

$$v_{xy_i} = \sqrt{v_{x_i}^2 + v_{y_i}^2} \tag{17}$$

and

$$v_{t_i} = \sqrt{v_{x_i}^2 + v_{y_i}^2 + v_{z_i}^2} \tag{18}$$

The position components are:

$$X_i = v_{x_i} \Delta t + X_{i-1} \tag{19}$$

$$Y_i = V_{y_i} \Delta t + Y_{i-1} \tag{20}$$

$$Z_i = v_{z_i} \Delta t + Z_{i-1} \tag{21}$$

$$\text{Range} = \sqrt{X_i^2 + Y_i^2 + Z_i^2} \tag{22}$$

From the above expressions, obtain attitude angles, E and A.

$$\sin A_i = \frac{v_{x_i}}{v_{xy_i}} \quad (23) \qquad \cos A_i = \frac{v_{y_i}}{v_{xy_i}} \quad (24)$$

$$\sin E_i = \frac{v_{xy_i}}{v_{t_i}} \quad (25) \qquad \cos E_i = \frac{v_{z_i}}{v_{t_i}} \quad (26)$$

These functions are used in the next iteration as functions of the rocket attitude angles, E and A, in spherical coordinates. The angles, E and A, can readily be obtained from these expressions for print-out purposes.

From the equations it can be seen that when the value of measured acceleration, amp becomes zero, as in the case of “vacuum” trajectory following rocket burnout, the rocket is influenced only by the gravitational attraction, g_h . This g-value is re-evaluated with altitude, and this prevents the trajectory from appearing as a “true” parabola, which would occur, depending upon the smallness of time increments being used in making the calculations.

A simplified flow-chart for the IBM 1410 computer trajectory program is shown in Figure 5. This flow-chart gives a general description of the computer program. The first block of the flow-chart indicates that all input constants are to be read into the computer. These include the spherical coordinates of the sun, the magnetic field, and the rocket launcher as well as the time increments to be used in making calculations, the value of g at the launch site, the radius of the earth and various factors which may be used for correcting, or modifying the dimensional units of the data. The sun and magnetic coordinates are necessary for attitude determination in some cases as described later. The initializing steps of the program are used to set variables such as the rocket velocities and positions and values of winds to zero at the outset of the program. The third block is for reading in all of the wind data that might be used in the program. If no wind data are available, the value of zero with an unattainable rocket altitude is read for the winds.

It has been found that attitude calculations from velocity component vectors are not accurate during the first few seconds of rocket flight because of the method by which the incremental velocities are calculated. For this reason, the attitude of the rocket is established to be the same as that of the rocket launcher for the first 500 feet of altitude. After the rocket has exceeded 500 feet of altitude, the attitude is calculated from the velocity vectors, or attitude input data are used if these are available. It can be seen in the flow-chart that the three blocks that are used to establish the rocket attitude are functionally in parallel immediately before the block which is used to calculate all of the vector components of rocket acceleration, velocity, and position. These calculations are based on the series of equations given previously.

After a round of calculations has been made, a print-out listing is made for the time, acceleration, velocity, and position components as well as the angular position of the rocket from the launch site. Following the print-out the value of g, the acceleration due to gravity, is re-evaluated in accordance with the altitude of the rocket based on the inverse square law. An inverse-cube-law magnetic field intensity factor is also re-evaluated with the rocket altitude for the next round of aspect calculations. In case the trajectory data includes wind information, the wind strata altitudes are examined to see if new wind strata data should be introduced to conform to the rocket altitude so that new X and Y wind component values may be made available.

If the rocket has reached impact, which is indicated by a negative value of altitude, Z, the machine is made to stop and the trajectory calculations are complete. If the trajectory is not complete the acceleration data are examined to see if all significant values have been used. If not, the program returns to "free fall" portion of the calculations where the acceleration is considered to be zero except for the value of g_h and the rocket attitude, which is here assumed tangent to the trajectory, is determined from "gravity turn" velocity ratio calculations. If more acceleration data are available the storage locations for the seven values which were read in at the beginning are examined to see if all have been used. If not, the computer program returns to a point subsequent to the read-in block for acceleration data. If all seven values have been used, the program returns to the beginning of the block which indicates that a new card with a set of seven acceleration and aspect data values should be read in.

It has been found that the inclusion of wind data for adjusting the attitude of the rocket, which is computed from vector velocity components, does not correct the trajectory so that it will conform to the information which may be supplied by radar. It is difficult to obtain wind data that exactly fit the actual conditions, even though a great attempt is made to do this in most instances by the personnel who are available for impact prediction. Most wind data are obtained by a series of balloon launchings where the balloons are tracked either optically or by radio signals to obtain wind velocities and directions from the surface to the desired altitude. The surface winds are most effective immediately after rocket launch because at this time the rocket has not accumulated a large vertical velocity as it does after several seconds of flight time. This is the reason for having different wind ballistic factors at different altitudes. The launch attitude of the rocket is adjusted to compensate for these low altitude winds.

Prior to launch, the attitude of the launcher is adjusted to take into account the anticipated effects of the winds. The effective launch attitude of the rocket launcher is the same as the setting would be if there were no winds. If these effective attitude settings are used in the accelerometer trajectory computer program the rocket zenith and impact point should closely approximate the actual values experienced by the rocket in flight. These effective launch angles are not always available, however.

In equations (23) through (26), E and A are the attitude angles of the rocket in its trajectory. They are determined from velocity ratios and are used in the subsequent iteration.

If the true attitude of the rocket were known at the time longitudinal acceleration is used for calculating a trajectory a more accurate trajectory analysis can be effected. This is based on the assumption that the acceleration is along the rocket axis and that there is a zero angle of attack. This will be more nearly true immediately after launch where the effects of surface winds are greatest. After the rocket has achieved a large tangential velocity in its trajectory, it is obvious that a change of attitude will not immediately cause a corresponding change of velocity or position in the direction of the new attitude. For example, the rocket would not perform as an automobile turning a corner but more like a boat side-slipping in water while turning. The velocity in a certain direction which has been accumulated prior to the time of attitude change still must be accounted for.

By having attitude data available, the trajectory of a rocket can be improved in accuracy by using this information along with the integrated accelerometer data. A more accurate zenith altitude can be calculated as well as a more accurate location of the impact point for the rocket. The attitude of the rocket is also useful for many scientific investigations, usually after burnout, when the rocket is in the higher regions of the atmosphere where data are being taken. The utility of a useful aspect sensing system would then be twofold.

Sun-Magnetic Field Rocket Aspect System There are various types of aspect sensing systems being used currently in work with rockets, satellites, and space probes. Some of these systems are simple, using only the earth’s magnetic field and others are more complicated, especially when they are used for guidance and attitude control. Some systems use magnetic and solar sensors, magnetic and horizon sensors, earth sensors, and various combinations of these devices (1, 8). One of the most well known types of aspect determining devices is the gyroscope. If a gyroscope, or stable platform were used in a rocket, attitude information would be quite accurate and would make the accelerometer trajectory problem more straight forward. Gyroscopes such as those that are designed for rocket installation are very expensive, and the cost would be prohibitive to use with a system such as is proposed for this usage, where simplicity and low cost are the primary objectives.

A very large number of sounding rockets that are used are equipped with magnetic aspect sensors. One or more of these sensors are used for obtaining performance data such as roll rate, angle of attack, aspect during flight, and other information. Because these magnetic sensors are generally available in a rocket payload, it was decided to devise a rather simple and straight forward type of solar aspect sensor which can be used in conjunction with the magnetic aspect sensor for rocket attitude determination for accelerometer trajectory analysis. One of the design, objectives was to devise a solar sensor which would have aspect angular resolution greater than that presently available with the magnetic aspect sensors and one that would occupy a small amount of space, be light weight, and consume a minimum amount of electrical power. Solar sensors are presently available which meet some of these requirements, but not all features are combined in one unit.

The spherical coordinates of the sun vector can be obtained from spherical trigonometric calculations using data from an ephemeris. The longitude and latitude of the observing point must be known. The magnetic field vector, expressed in spherical coordinates can be measured at the observing point by various methods.

Within certain limitations the attitude of a rocket can be determined by measuring the angle between the rocket axis and the solar vector and the angle between the rocket axis and the magnetic field vector. It is impossible to get a unique mathematical solution for the rocket attitude when only one of these vector angles is known. If, for example, the angle between the rocket axis and the solar vector is measured, it can be seen that the rocket occupies some position around a cone with the sun vector as its axis as illustrated in Figure 6. With only this source of information, this is all that is known about the rocket attitude. If the angle between the rocket axis and the magnetic vector can be measured, the rocket, as far as the magnetic field is concerned, lies somewhere on another cone with the magnetic field vector as the axis. In order to satisfy both conditions the rocket then must lie along a line of intersection of the two cones as shown in Figure 6. In most cases there are two lines of intersection for the two cones. One will represent the rocket vector, and the other is extraneous. This second intersection of the two conical surfaces can be eliminated by knowing other facts concerning the rocket attitude. It is possible to have only one solution such as when the cones are tangent. No solution is obtained when the sun and magnetic vectors are coincident.

There are various methods for obtaining the solution for cone intersection such as descriptive geometry and simultaneous algebraic solution.

A simple mathematical method for obtaining the direction of the rocket vector is by means of direction cosines of the vectors. This method is well adapted to digital computer programming. If a vector in XYZ space makes angles, λ , with the X-axis, μ , with the Y-axis and ν with the Z-axis, these angles are the direction angles of the vector. The cosine of the angles, α, β, and γ, respectively are the direction cosines of the direction angles. The sum of the squares of the direction cosines equals unity, and from the cosine law it may be shown that the cosine of the an le between two vectors is

$$\cos \varphi = \alpha_1 \alpha_2 + \beta_1 \beta_2 + \gamma_1 \gamma_2 \qquad (27)$$

Consider the sun vector, S, and the magnetic vector, M, both of which have known directions, and the unknown vector, R. The angles 8 and 6 between the rocket axis vector, R, and the sun and magnetic vectors respectively are also known. Refer to Figure 7. The vectors may be considered to be of unit length and are expressed in terms of direction cosines in a right hand coordinate system with the positive Y-axis to the north. The angle between two vectors such as S and R may be expressed in terms of their direction cosines with the X, Y, and Z axes, α , β , and γ respectively

$$\cos \Theta = \alpha_S \alpha_R + \beta_S \beta_R + \gamma_S \gamma_R \tag{28}$$

similarly

$$\cos \Phi = \alpha_M \alpha_R + \beta_M \beta_R + \gamma_M \gamma_R \tag{29}$$

also

$$1 = \alpha_R^2 + \beta_R^2 + \gamma_R^2 \tag{30}$$

From these equations one can solve for the unknown direction cosines of the rocket vector, α_R , β_R , and γ_R . This may be done by simultaneously solving for α_R , and β_R in terms of γ_R and the known constant cosine terms. By rearranging equations (28) and (29) we obtain

$$\cos \Theta - \gamma_S \gamma_R = \alpha_S \alpha_R + \beta_S \beta_R \tag{31}$$

$$\cos \Phi - \gamma_M \gamma_R = \alpha_M \alpha_R + \beta_M \beta_R \tag{32}$$

from which by determinants

$$\alpha_R = \frac{\beta_M \cos \Theta + \gamma_R(\beta_S \gamma_M - \beta_M \gamma_S) - \beta_S \cos \Phi}{(\alpha_S \beta_M - \alpha_M \beta_S)} \tag{33}$$

is obtained.

$$\text{Let } B = (\beta_S \gamma_M - \beta_M \gamma_S) \text{ and } D = (\alpha_S \beta_M - \alpha_M \beta_S) \tag{34}$$

$$\alpha_R = \frac{\beta_M \cos \Theta + B\gamma_R - \beta_S \cos \Phi}{D} \tag{35}$$

also from equations (31) and (32) we obtain

$$\beta_R = \frac{\alpha_S \cos \Phi + (\alpha_M\gamma_S - \alpha_S\gamma_M)\,\gamma_R - \alpha_M \cos \Theta}{(\alpha_S\beta_M - \alpha_M\beta_S)} \tag{36}$$

Let $F = (\alpha_M\gamma_S - \alpha_S\gamma_M)$ (37)

$$\beta_R = \frac{\alpha_S \cos \Phi + F\gamma_R - \alpha_M \cos \Theta}{D} \tag{38}$$

By substituting these expressions for α_R and β_R into equation (30) a quadratic equation in γ_R is obtained.

$$1 = \left[\frac{\beta_M \cos \Theta + B\gamma_R - \beta_S \cos \Phi}{D}\right]^2 + \left[\frac{\alpha_S \cos \Phi + F\gamma_R - \alpha_M \cos \Theta}{D}\right] + \gamma_R^2 \tag{39}$$

By rearranging this equation, expanding and collecting coefficients of $\cos \Theta$ and $\cos \Phi$, since they are not fixed terms with the sun and magnetic references, the following equation is obtained.

$$\begin{aligned}&(B^2 + F^2 + D^2)\,\gamma_R^2 + 2\left[(B\beta_M - F\gamma_M)\cos\Theta + (F\alpha_S - B\beta_S)\cos\Phi\right]\gamma_R \\ &+ (\beta_M^2 + \alpha_M^2)\cos^2\Theta + (\beta_S^2 + \alpha_S^2)\cos^2\Phi \\ &- 2(\beta_M\beta_S + \alpha_M\alpha_S)\cos\Theta\cos\Phi - D^2 = 0\end{aligned} \tag{40}$$

By substituting K's for the constant terms the following equation is obtained.

$$\begin{aligned}&\hat{K}_1\,\gamma_R^2 + 2(K_2 \cos\Theta + K_3 \cos\Phi)\,\gamma_R + K_4 \cos^2\Theta \\ &+ K_5 \cos^2\Phi - 2K_6 \cos\Theta\cos\Phi - D^2 = 0\end{aligned} \tag{41}$$

which is a quadratic in γ_R. Using the quadratic formula, solve for γ_R.

$$\gamma_R = \frac{-2(K_2 \cos\Theta + K_3 \cos\Phi) \pm \sqrt{\begin{aligned}&4(K_2 \cos\Theta + K_3 \cos\Phi)^2 \\ &-4K_1(K_4 \cos^2\Theta + K_5 \cos^2\Phi \\ &-2K_6 \cos\Theta\cos\Phi - D^2)\end{aligned}}}{2K_1} \tag{42}$$

This, being a quadratic equation, yields two solutions, γ_{R1} and γ_{R2}. These can be substituted into equations (35) and (37) to give two solutions each for α and **â**, hence the two solutions for the rocket attitude are the directions cosines α_{R1}, β_{R1}, γ_{R1} and α_{R2}, β_{R2}, γ_{R2} These direction cosines may readily be used to express the vectors in terms of spherical coordinates where the rocket attitude may be expressed in terms of elevation and azimuth angles for compatibility with the remainder of the trajectory program.

$$\theta_R = \arc\cos\left[\gamma_R \Big/ \sqrt{\alpha_R^2 + \beta_R^2 + \gamma_R^2}\right] \tag{43}$$

$$\varphi_R = \arc\tan\left[\alpha_R \big/ \beta_R\right] \tag{44}$$

It has been found that the root resulting from the positive radical, when solving for γ, provides the proper set of direction cosines for this work. Since the rockets are launched very nearly vertical and remain in that direction throughout the thrust period, a value of γ which approaches unity is required. The negative radical produces a direction cosine, γ, which deviates much too far from vertical to be the proper root. If conditions were such that this criterion were not sufficient to choose the proper root, such as in the case of a rocket not having an almost vertical attitude, then other known facts about the attitude must be utilized.

If the magnetic field vector and the sun vector should both lie in the same vertical plane, the determinate, D, becomes zero and there is no solution for the direction cosines for the rocket vector using this method. If the solar vector and the magnetic vector are coincident only one cone is defined, and the attitude is undetermined.

Solar Aspect Sensor Solar and magnetic aspect sensing devices are commercially available. However, most solar sensors are structurally complicated and their cost prohibits their use in many cases. In order to obtain rocket attitude data to supplement the accelerometer data for trajectory analysis a simple solar aspect sensor was devised. This unit together with the Schonstedt RAM-3 magnetic sensor were used with the calculations for attitude developed earlier.

The solar unit described here uses two slits, a vertical slit called the reference slit and a diagonal slit, used for measuring the solar vector angle. Located in back of these slits are three silicon junction photosensors. Each of the three sensors is in view of the sun through the diagonal slit over a specified range of solar vector angles, and only the upper and lower sensors can view the sun through the reference slit. The point along the diagonal slit at which the sensor and the sun are in alignment represents an angle of rotation which is a function of the solar vector angle. If the roll rate of the rocket is assumed to be constant then the ratio of the time between impulses from the sun through the reference slit and the diagonal slit to the time required for a complete roll of the rocket can be used as solar aspect data (4).

Figure 8 shows sketches of an aspect sensor unit and how each photosensor is used to receive impulses from the sun. The total range of angles is shared by the three sensors as shown. As the rocket rotates and the solar impulses are detected by the photosensing devices, a transistor amplifier circuit is used to condition these signals for the telemetry system. These signals appear on the telemetry records in the form of pulses, one pulse being the reference pulse and the following pulse being the delayed pulse representing the solar vector angle. The electronic circuit was devised so that the signal from the lower and the upper sensors, which are the same ones used for the reference pulse, have a lower voltage amplitude than the signal from the middle sensor. This facilitates identification of the signals in order to know what range of solar angles is being presented.

The photosensors used are Philco L4412 silicon junction devices which are encapsulated in small glass envelopes. They are designed primarily for use in punch card reading machines and other optical sensing devices. The response curve favors the infrared region of the optical spectrum. The Philco L4412 sensor is a photo-voltaic device, which generates a voltage when receiving electromagnetic radiation in the form of light. The devices will supply 0.050 milliamperes of current into a 1000 ohm load when illuminated from a 600 foot-candle source. Since the sun provides an intensity of illumination of some 10,000 foot candles, there is adequate intensity for the sensors even when the light beam is not normal to the sensitive area of the photosensor.

The electronic circuit used in conjunction with the solar aspect sensors is illustrated schematically in Figure 9. Three silicon type 2N338 high-beta transistors are used. One is an input amplifier for the upper and lower photosensors. Another is an amplifier for the middle sensor, while a third transistor is a mixer-buffer which provides a signal to the input of the FM/FM telemetry channel. The total amount of current normally required to operate the electronics circuit is approximately 1.5 milliamperes.

Since the output data from the solar aspect sensors are in the form of pulses, it is desirable that the center frequency of the subcarrier oscillator for the FM/FM telemetry be as high as possible. By using a high center frequency with a wide bandwidth, one can more faithfully preserve the rise-time characteristics of the signal pulses so that data can be read from the telemetry records more accurately. If a signal pulse from the solar aspect sensor unit represents one degree of rocket roll and the roll rate is two revolutions per second, then the pulse would be approximately 1.4 milliseconds in duration. In order to reproduce accurately the leading edge of this pulse, the subcarrier frequency should be at least as high as 14.5 kc. There are other limitations in reading out data from the telemetry, such as the response of the galvanometers being used in the recorder and the recorder paper speed.

Since the first models of the solar aspect sensors were to be flown on Aerobee rockets they were tailored for these rockets. Also since the first units were going through a feasibility phase of their development they were designed to occupy space which was readily available and which would require no structural modifications of the rocket. The solar aspect sensors were designed to occupy the space behind doors which are used for access to fuel valves and other rocket motor adjustments. The slits were cut in’ the surface of the doors as illustrated in Figure 10. The locations of the photosensors within the box-type enclosure behind the curved access door were arranged so that the viewing angle to the sun would not be obscured by the tail fin which is adjacent to one side of the door. This causes the useful rotation angle of the aspect sensor to be nonsymmetrical with respect to the aspect sensor center line. This does not harm the design nor performance of the device, however.

To calibrate the solar aspect sensors in terms of solar vector angles, a calibration setup was constructed which could be used to duplicate the roll of a rocket. The calibration device can be tilted in the presence of the sun to vary the solar vector angles. The sun may be used as an accurately positioned source of light for use in calibrating the aspect sensor unit. The physical location of the sun can be calculated from spherical trigonometry by knowing the latitude and the longitude at the observing point and the Greenwich hour angle and declination of the sun as obtained from a nautical almanac.

Magnetic Aspect Sensors The magnetic aspect sensors used for data obtained for this project are the Schonstedt RAM-3 magnetic aspect sensors. These devices are being used quite extensively in rocket payloads for determining roll rate, magnetic aspect, re-entry of payload, and other information. The unit consists of an electronic compartment and a sensing probe connected with a five-foot cable. The circuit consists basically of an input voltage regulator and a five kilocycle oscillator which supplies a signal frequency to the sensing probe. When the probe is under the influence of an external magnetic field, its core becomes saturated and a second-harmonic frequency is generated. This frequency, when phase compared with the fundamental frequency in the electronics compartment, is used to develop a dc output voltage proportional to the strength of the magnetic field being sensed. The polarity of the output voltage from the special phase detector depends upon the direction of the magnetic field in the probe (9).

The largest signal voltage is developed when the direction of the magnetic field is in alignment with the axis of the sensing probe. A reversal of the magnetic field will produce a maximum output voltage in the opposite direction. The output connections from the magnetic aspect sensor are arranged so that the output voltage may range between ± 2.5 volts, or they may be arranged to give an output signal between zero and five volts dc. The latter is the usual arrangement and makes the signal compatible with the input circuits of most FM/FM subcarrier oscillators. The output voltage, when the magnetic field is perpendicular to the axis of the probe, is midway between these two extreme voltages.

The output voltage from the magnetic aspect sensor follows a cosine variation as the probe is rotated 180 degrees in the presence of the magnetic field, moving from perfect alignment in one direction to perfect alignment in the other direction. Factory calibration data generally denote that the output voltage for no magnetic field, that is, when the magnetic field is perpendicular to the probe axis, is approximately 2.4 volts.

It is known that the strength of the earth's magnetic field approximately follows an inverse cube law with altitude. This must be taken into account in the aspect sensor calibration since it represents a reduction of approximately two per cent at an altitude of 140 thousand feet (7).

When a magnetic aspect sensor probe is mounted in a sounding rocket payload with its axis parallel to the rocket axis, the output from the aspect sensor will be essentially a constant dc level of voltage varying only by the amount that the attitude of the rocket changes with respect to the magnetic field. As mentioned before, this voltage will, in general, be greater or less than the midrange voltage depending upon the orientation or the sensor, whether forward or reversed. If the magnetic aspect sensor is mounted so that its axis is perpendicular to the axis of the rocket and the rocket is rolling, then a sine wave voltage output will be produced as the sensor cuts the earth's magnetic field. In general, the transverse installation is more accurately read from the telemetry records because a peak-to-peak voltage is available rather than a single voltage level whose calibration might be in question.

In reading data from the magnetic aspect sensor one is not concerned with the actual strength of the earth's magnetic field but is concerned only with aspect. From Figure lla it can be seen that the maximum and minimum points of the cosine curve for the longitudinal sensor represent, magnetic aspect angles with respect to the rocket axis which are zero and 180 degrees. The equation for the output voltage may be stated as follows:

$$v = P \cos \Phi + Q \tag{45}$$

where P is the peak amplitude of the cosine curve and Q is the fixed dc level of the midpoint of the cosine curve. This level would represent a magnetic vector angle of 90 degrees with respect to the rocket axis. With reference to Figure 11b, where a peak-to-peak sinusoidal voltage is available to represent the spin of the rocket, the equation for the peak-to-peak output voltage for the transverse sensor is

$$v = R \sin \Phi \tag{46}$$

where Φ is the magnetic aspect angle with respect to the rocket axis. R is the peak-to-peak amplitude of the sine wave which would occur if the rocket axis were perpendicular to the magnetic field vector. Since the peak-to-peak amplitude follows a sinusoidal variation as the aspect angle varies between zero and 90 degrees, it can be seen that the peak amplitude would be exactly zero when the rocket is moving in the direction of the magnetic vector. The amplitude constants, P and R, can be obtained by using the boundary conditions obtained at launch. The launcher coordinates are always known, and from these the magnetic vector angle, Φ, can be obtained.

As mentioned before, the values of P and R in the above equation, are modified by the inverse cube law relative to the magnetic field, which is a function of a rocket altitude. The magnitude of P and R are reduced by the following factor where R_o is the radius of the earth, and h is the rocket altitude (7).

$$K = \left[\frac{R_o}{R_o + h}\right]^3 \tag{47}$$

There are several factors which might alter the calibration of the magnetic aspect sensor. The presence of metals, especially magnetic materials within the payload may sometimes distort the magnetic field which is being sensed and cause erroneous readings. There is also the possibility that electric currents flowing through the metal members of the payload structure will setup fields which will disturb the calibration of the aspect sensor. The motion of the rocket in flight can create currents flowing in the metal structure which might cause distortion of the calibration. Calibration errors such as these are difficult to analyze and would make a study within themselves.

It is desirable to calibrate a magnetic aspect sensor after it has been installed in the nose cone, if possible. If the nose cone payload is not too unwieldy, this can be done by physically positioning it so that the output of the magnetic sensor will indicate the peak voltage values of the earth's magnetic field. If there is any error in the calibration it will, at least, be corrected at the peak values. The mid-point of the sine wave calibration curve is taken to be midway between these two peak values.

The magnetic sensors installed on the rockets used in testing the aspect system for trajectory analysis were used, in all cases, because they were already available in the rocket payloads. Since the system was flown on the three rockets primarily as a feasibility study for aspect devices of the type used here, there was no alternative but to use these magnetic aspect sensing devices. Without reflection on the general utility of the devices, it is indicated that they are the weakest link of the aspect-trajectory system.

A more desirable aspect sensor would be one which would provide more resolution in the output voltage reading, in that only a small spread of aspect angles would be re-ad at the output terminals. Such sensing devices have been devised and tailored especially for rockets which maintain a predetermined magnetic aspect. In these cases, only the portion of the cosine curve which is desired for extracting aspect data is available, and this portion is used throughout the zero-to-five volt range of the telemetry capabilities.

Conclusion The accelerometer method for determining sounding rocket trajectories has been used by Oklahoma State University on several rockets recently. These have included the Aerobee, Nike-Cajun, and Astrobee-200. The greatest effort has been made with the unmodified accelerometer data where results have been considered to be unusually good. In many cases the altitude-time trajectory will agree with radar within an accuracy of one or two per cent. This accuracy is largely attributed to extreme care in calibrating the accelerometers and in extracting data from the telemetry records.

The unmodified accelerometer data produces a gravity-turn trajectory with an azimuth direction throughout the flight the same as the direction of launch. The only thing that can alter the trajectory is the introduction of rocket attitude data other than that caused by gravity. Wind velocities will affect the attitude of a rocket and will, as a result, affect the entire trajectory. Wind data has not successfully been used to improve the accuracy of the calculations. Use of the proper ballistic factors, however, might be used to improve the wind corrections in the computer calculations. The effective launch coordinates, which are usually different from the true launch coordinates, will create an improvement in trajectory accuracy. These effective launch coordinates are determined by impact predictors from wind data and are usually available to use in the computer program.

Attitude corrections from the solar-magnetic aspect system have been used on three Aerobee rockets to date. These three systems were used to study the feasibility of attitude supplemented acceleration data. Various degrees of success were experienced in improving the accuracy of the trajectory. The most significant improvement is in the azimuth angle calculations. In one instance the impact point was improved from 26° error to 2° error as compared with an uncorrected trajectory using the radar trajectory as the reference. Another rocket showed an improvement from 16° error to 1° error. In the former case the impact range was not improved while in the latter case it was improved almost 100 per cent. No significant altitude improvement was indicated for these two rockets. Results for the third rocket were impaired by clouds and undesirable relative positions of the solar and magnetic vectors. Accuracy in the aspect system should be improved considerably for further work. The magnetic system is, at present, the most inaccurate factor. Relative directions of the solar and magnetic vectors also have a great bearing on calculated attitude accuracy.

The accelerometer method for trajectory determination is most useful for altitude-time calculations at present. Further work with attitude methods should improve the accuracy to such an extent that azimuthal and range corrections in the trajectory can be made. This refinement would also require corrections due to the effect of the rotation of the earth.

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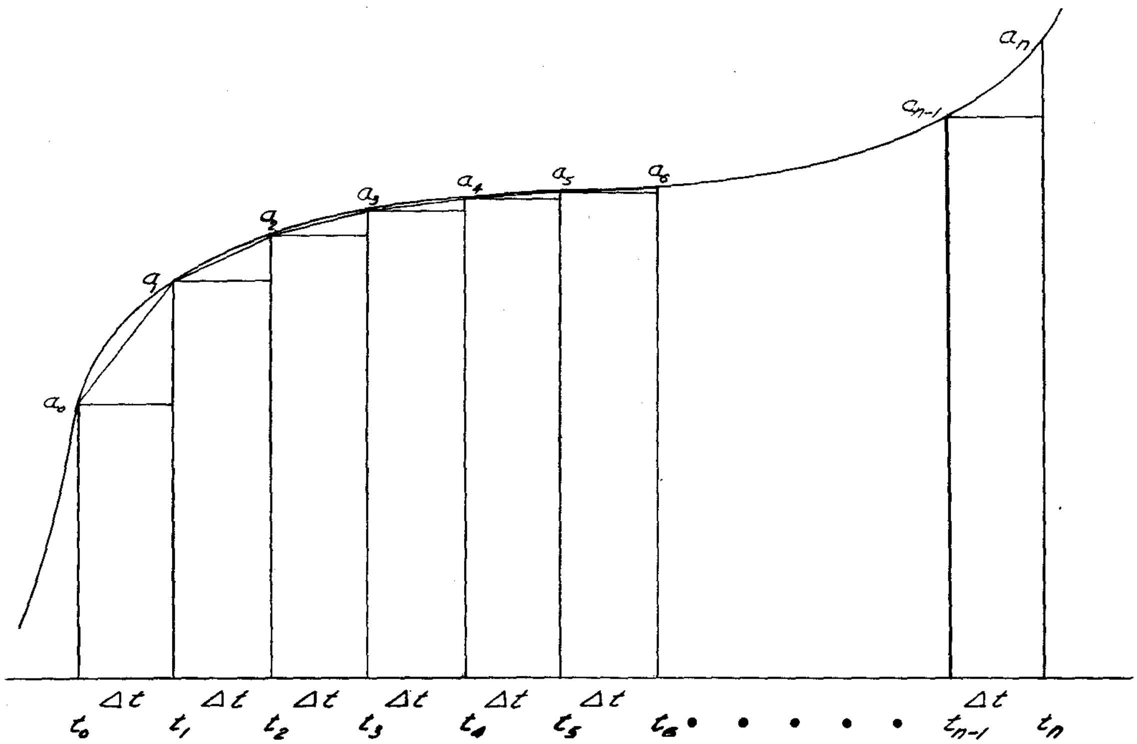

Figure 1. Trapezoidal and Simpson's Rule Method for Numerical Integration

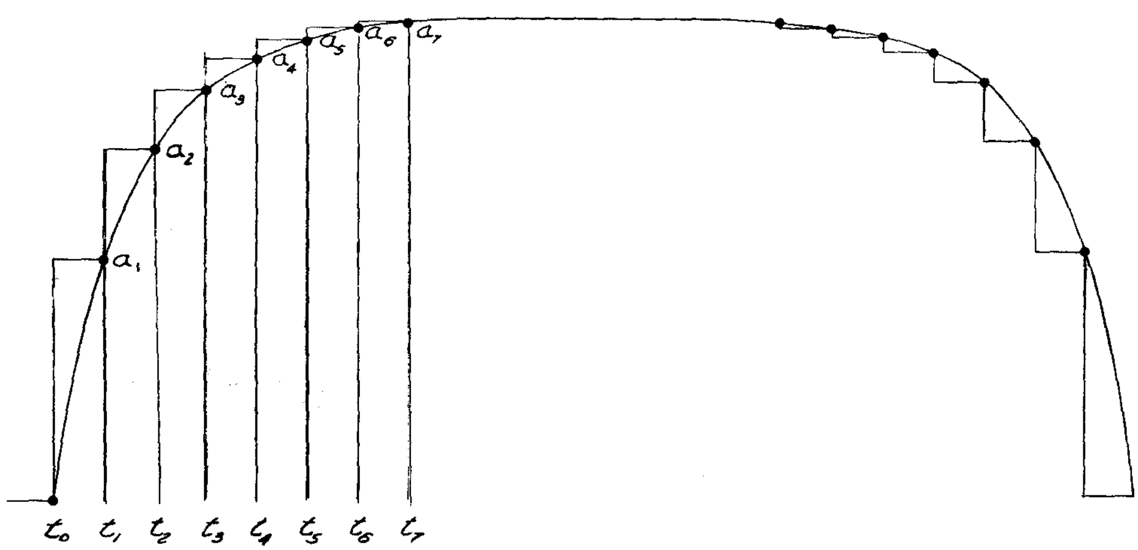

Figure 2. Computer Method for Numerical Integration

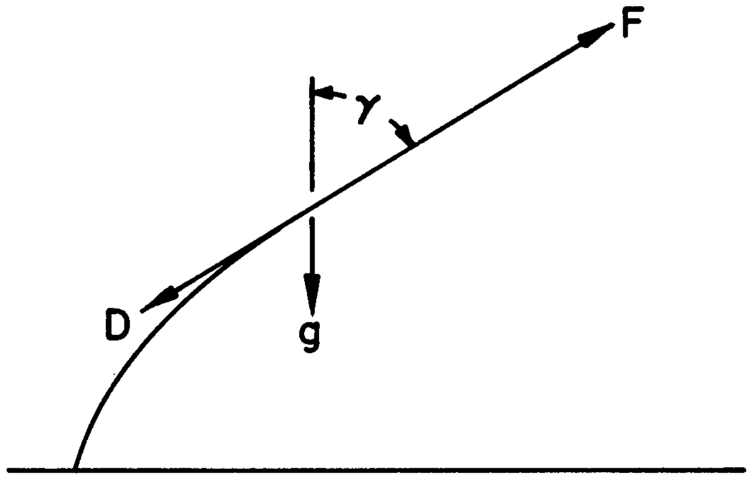

Figure 3. Forces Acting on Rocket

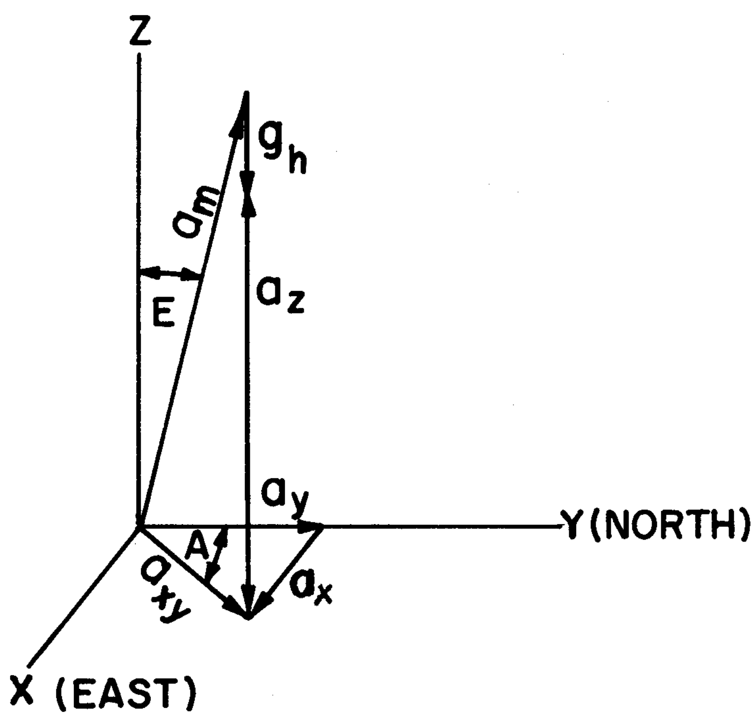

Figure 4. Acceleration Vector Diagram

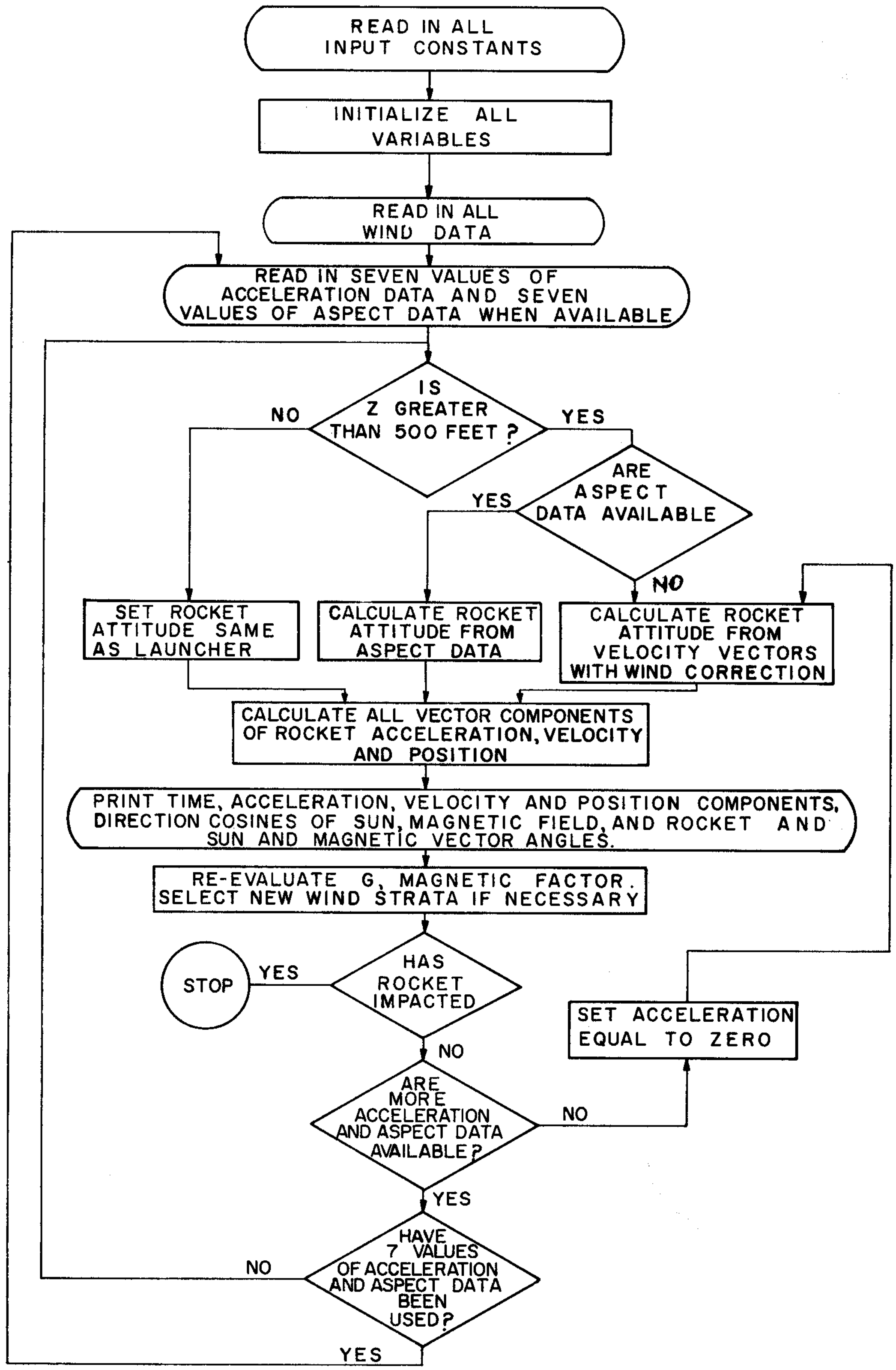

Figure 5. Simplified Flow Chart for Trajectory Program

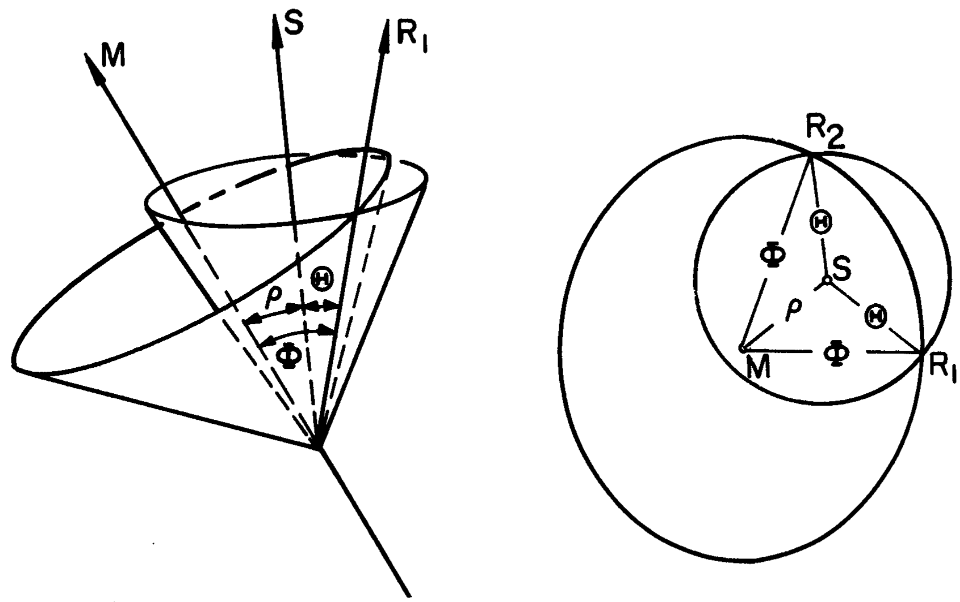

Figure 6. Intersecting Cone Representation of Rocket Vector

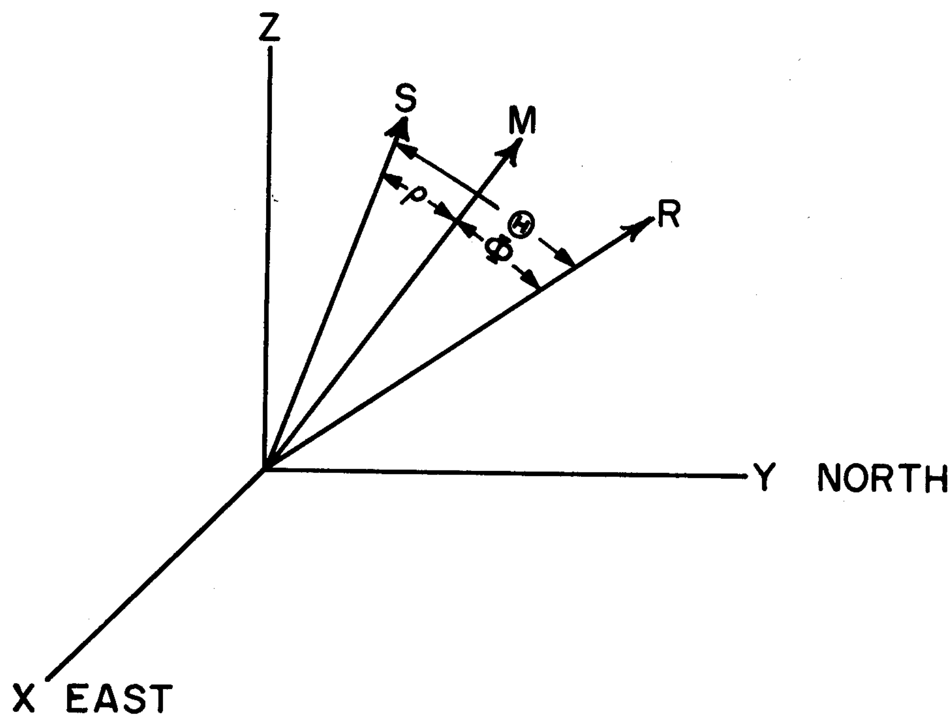

Figure 7. Sun, Magnetic Field, and Rocket Vector Diagram

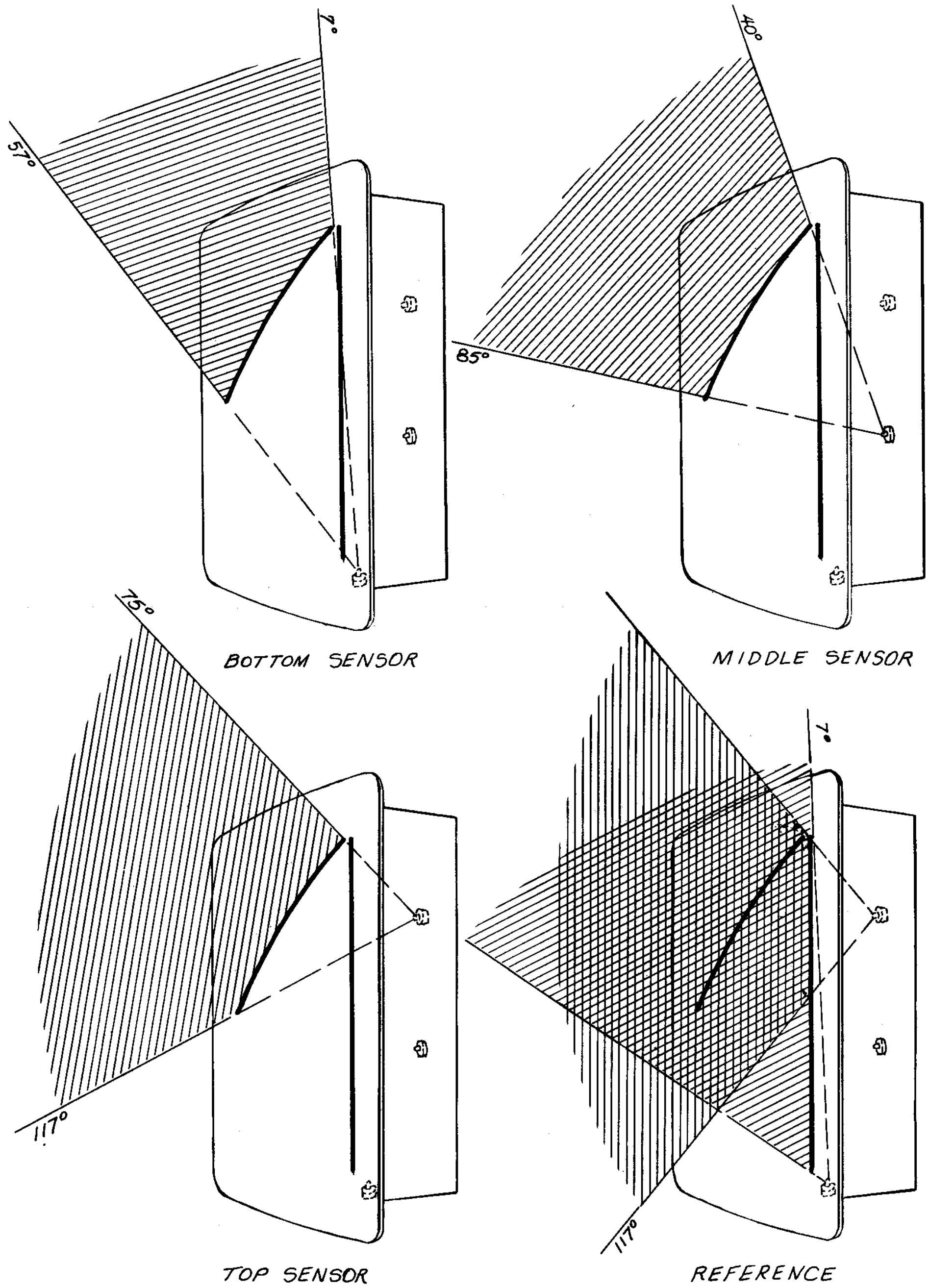

Figure 8. Functional Sketches of Solar Aspect Sensor

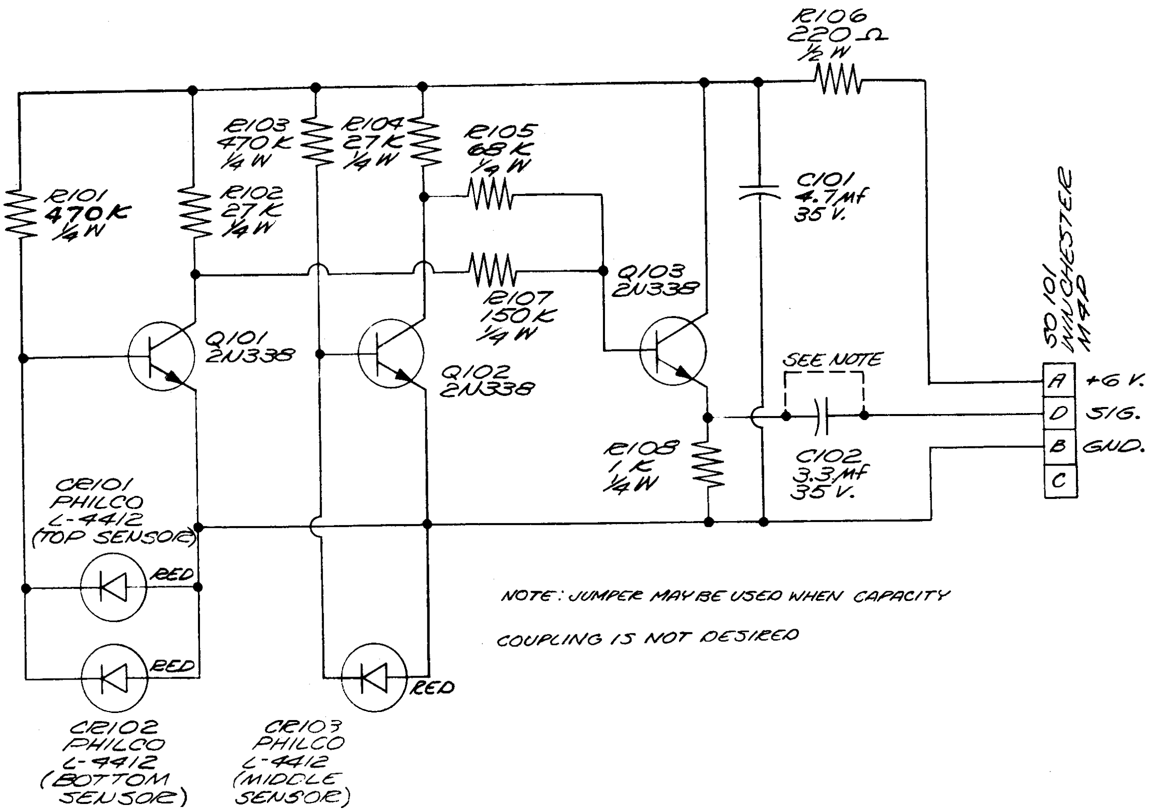

Figure 9. Schematic, Electronic Circuit

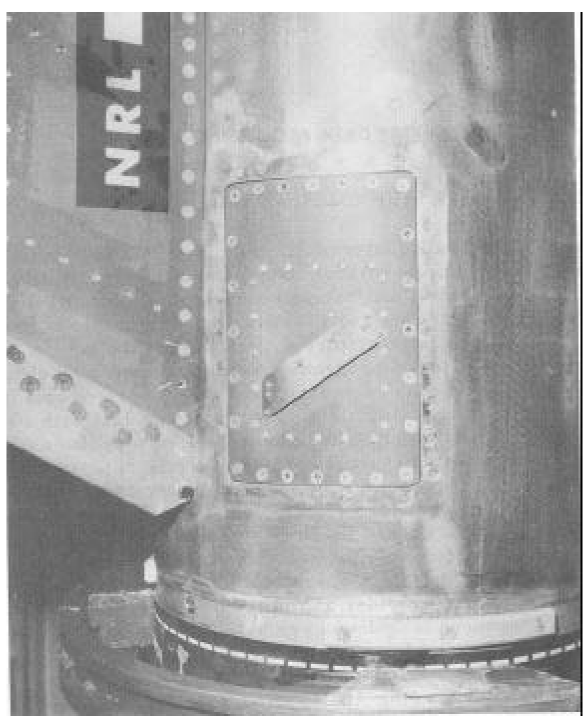

Figure 10. Solar Aspect Sensor Installed

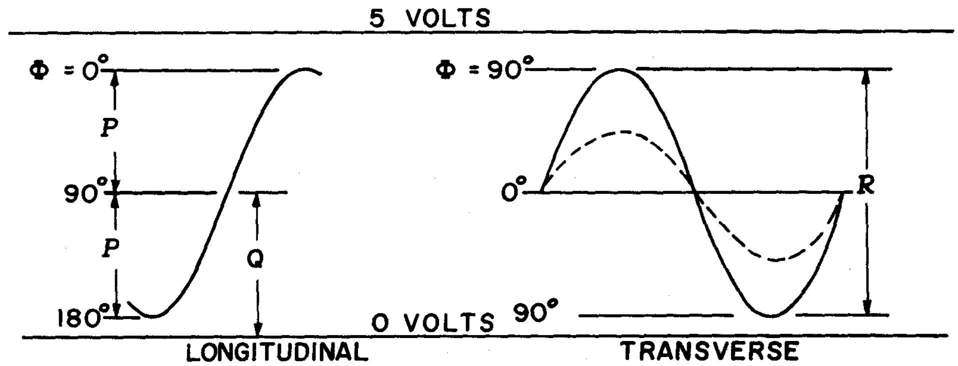

Figure 11. Output Voltage Variation, Magnetic Aspect Sensor