RELAY SELECTION FOR MULTIPLE SOURCE COMMUNICATIONS AND LOCALIZATION

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ABSTRACT
Relay selection for optimal communication as well as multiple source localization is studied. We consider the use of dual-role nodes that can work both as relays and also as anchors. The dual-role nodes and multiple sources are placed at fixed locations in a two-dimensional space. Each dual-role node estimates its distance to all the sources within its radius of action. Dual-role selection is then obtained considering all the measured distances and the total SNR of all sources-to-destination channels for optimal communication and multiple source localization. Bit error rate performance as well as mean squared error of the proposed optimal dual-role node selection scheme are presented.

KEY WORDS
Fisher information matrix, amplify-and-forward, wireless sensor network, source localization.

INTRODUCTION
A source communicating with a distant destination with the help of relay nodes in a wireless sensor network (WSN) is a topic of great interest in the literature [1], [2], [3]. Recently, in [4], [5] dual-role nodes (DRNs) have been proposed, where each node is not only able to transmit data but also to estimate the distance from the source to the DRN. Selection of DRN for optimal communications and source localization have been studied. However, [4], [5] only consider a single source present in the WSN. When several sources are present in the WSN, the question is: how to select the multiple DRNs?
For communication purposes, it is well known that relay nodes can provide significant performance improvement by exploiting multipath diversity [3], [6]. In a WSN, they can provide the link communication from a source to a remote destination node [1], [3], [6], [7], [8]. In [9], a two-phase repetition-based relaying scheme is proposed. The proposed scheme divides data transmission into two phases. In the first phase, multiple sources transmit data to all the relay nodes. The relay nodes within the transmission range of the multiple sources receive a noisy and attenuated version of the transmitted signal. In the second phase, a subset of relay nodes is selected and they send the data to multiple destination nodes.
In WSNs with known relays positions, source location can be estimated at the destination node by measuring the distance between the selected relay (or sensor) nodes and the source. When

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the sensor nodes are placed at fixed locations, their optimal selection is needed so that the source location estimation is optimal at the destination node. A relaxed convex formulation of this problem is proposed in [10], where the subset of sensor nodes with the highest Fisher information matrix (FIM) determinant is chosen for transmission. An optimal sensor placement analysis for source localization using time-of-arrival (TOA) range measurements is given in [11]. In [12], a source location is estimated by selecting a subset of sensors randomly deployed in a two-dimensional (2D) space. An expression for the source location estimate is derived using received radio-signal-strength (RSS) values, and an algorithm for sensor selection is proposed. The algorithm divides the search space into five equi-angular regions around the source and selects only one from each region. This algorithm offers much better mean squared error (MSE) performance when compared to a scheme that uses five sensors with the highest RSS. Please note that these research activities only focus on source localization, and there is no effort to study the effects when these sensors also act as communication relays.

In this paper, we study the selection of DRN for optimal multiple-source communications and localizations. We use a two-phase amplify and forward scheme, and we consider both sources and DRNs to use orthogonal channels for transmission. A maxmin problem formulation is proposed, where we select DRNs so the minimum $i$-th-source-to-destination signal-to-noise ratio (SNR) is maximized under a multiple-source location estimation target MSE constraint. Numerical results show the effectiveness of the proposed scheme when compared with a scheme where the closest DRNs to the source are selected.

SYSTEM DESCRIPTION

Consider a WSN with $N$ identical DRNs randomly spread over a two-dimensional (2D) geographical area, $M$ sources and a distant destination node. The source locations are given by $x_i = [x_i, y_i]^T$, $1 \leq i \leq M$, and the DRNs are placed at fixed locations $v_j = [x'_j, y'_j]^T$, $1 \leq j \leq N$. The location of the DRNs are known at the destination node. The distance between the $i$-th source and the $j$-th DRN is given by $d_{i,j} = [(x_i - x'_j)^2 + (y_i - y'_j)^2]^{1/2}$.

The sources communicate to the destination node with the help of the DRNs using a two-phase amplify-and-forward protocol. In the first phase, the sources send data to the DRNs using orthogonal transmissions (e.g. using different frequency bands) [9]. The received signal at the $j$-th DRN from the $i$-th source is given by

$$r_{i,j} = \sqrt{P_0} h_{i,j} [d_{i,j}]^{-\alpha/2} a_i + w_{i,j},$$

where $a_i$ is the unit energy transmitted symbol from the $i$-th source, $\alpha$ is the path-loss exponent, $h_{i,j}$ is the $i$-th-source-to-$j$-th-DRN channel gain, and $w_{i,j}$ is AWGN at the $j$-th DRN with normal distribution $\mathcal{CN}(0, \sigma_n^2)$. We assume in this paper that all the sources transmit with the same power $P_0$. In the second phase, $K$ DRNs out of $N$ are selected, and the received data from the different sources are normalized, amplified-and-forwarded to the destination node. Channel coefficients $h_{i,j}$ as well as distance estimates from the $j$-th DRN to the $i$-th source are sent to the destination node as assumed in [5]. The received signal at the destination node sent from the $i$-th source and relayed from the $j$-th DRN is given by

$$d_{j,f}^{(i)} = \frac{\sqrt{P_j} h_{j,f} D^{-\alpha/2}}{\sqrt{P_0} ||h_{i,j}||^2 [d_{i,j}]^{-\alpha} + \sigma_n^2} r_{i,j} + w_f,$$
where $P_j$ is the transmitted power from the $j$-th DRN, $D$ is the distance between any DRN and the destination node (assuming the maximum distance between any source and any DRN $d_{max} << D$), and $h_{j,f}$ is the $j$-th DRN-to-destination node channel gain. The total power is $P_T = \sum_{j=1}^{N} P_j + MP_0$. Each channel gain coefficient has a Ricean distribution and $E[\|h_{i,j}\|^2] = E[\|h_{j,f}\|^2] = 1$, where $E[.]$ is the average operator. We consider Rice factors of source-to-DRN and DRN-to-destination links to be $H_{ij}$ and $H_{jf}$ respectively. We assume the destination knows the channel coefficients $h_{i,j}$, $h_{j,f}$, the distance $D$ and the locations of all DRNs.

We use maximal ratio combining (MRC) to estimate the transmitted symbol $a_i$ at the destination node. Combining the received signals from the selected $j$-th DRN corresponding to the $i$-th source transmission, the soft estimate $\hat{a}_i$ of $a_i$ is given by

$$\hat{a}_i = \sum_{j \in \Gamma} \sqrt{P_0P_j} \frac{\sqrt{P_0} h_{i,j}^* h_{j,f}^* [d_{i,j}]^{-\frac{\alpha}{2}} D^{-\frac{\alpha}{2}}}{\hat{\sigma}_{j,f}^2 \sqrt{P_0}\|h_{i,j}\|^2 [d_{i,j}]^{-\alpha} + \sigma_n^2}}$$

where

$$\hat{\sigma}_{j,f}^2 = \frac{P_j \|h_{j,f}\|^2 D^{-\alpha} \sigma_n^2}{P_0 \|h_{i,j}\|^2 [d_{i,j}]^{-\alpha} + \sigma_n^2} + \sigma_f^2.$$  

and $\Gamma$ is the set containing the indices of the $K$ selected DRNs.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{telemetry_diagram.png}
\caption{Multiple source telemetry data collection using a two phase amplify-and-forward relay network}
\end{figure}

\section*{Optimal Dual-Role Node Selection}

The problem of relay selection for optimal communication have been recently studied in different case scenarios [2], [3], [6]. Similar to this problem is the relay selection for optimal source localization [13], [14], [15]. Recently, in [4], [5], DRNs have been proposed, and different solutions for DRN selection for optimal communication and localization of a single source have been studied. To find the optimal DRN selection, we choose to maximize the minimum SNR corresponding to the $i$-source communication channel under a localization constraint. To illustrate this concept, consider Fig. 1. Suppose we need to select $K = 3$ DRNs, and let $\sigma_{t,i}$, $i = 1, 2$, be the maximum allowed
\( i \)-th source localization MSE. Different DRN selections will occur depending on \( \sigma^2_{t,i} \). For high \( \sigma^2_{t,i} \), DRNs 1, 4 and 5 would be selected. However, for high \( \sigma^2_{t,1} \) and low \( \sigma^2_{t,2} \), DRNs 3, 2, and 5 would be selected. This selection significantly improves the source 2 location estimation MSE while keeping good communications performance for both sources 1 and 2. In the following, we describe the problem formulation for optimal DRN selection.

Consider the SNR at the destination node for the \( i \)-th source communication channel as given by

\[
\gamma_i(c, p) = \sum_{j \in \Gamma} c_i \frac{P_j P_0 \|h_{i,j}\|^2 \|h_{j,f}\|^2 [d_{i,j}]}{1 + P_j \|h_{j,f}\|^2 D^{-\alpha} + P_0 \|h_{i,j}\|^2 [d_{i,j}]^{-\alpha}} \frac{(\sigma^2_n \sigma^2_f)}{\sigma^2_f}
\]

(5)

To introduce a localization performance constraint in our problem formulation, we follow the approach in [5]. Let us consider the distances between the source and the DRNs be obtained through ranging techniques, such as RSS or TOA based measurements. At the \( j \)-th DRN, the distance estimate is

\[
g_{i,j} = b_{i,j} + \eta d_{i,j} + w_{g,i,j}
\]

(6)

where \( w_{g,i,j} \) is the measurement noise with distribution \( \mathcal{N}(0, \sigma^2_g) \), and \( b_{i,j} \) is the distance bias corresponding to the \( i \)-th-source-to-\( j \)-th-DRN. We assume \( \eta = 1 \) for simplicity, and the bias known at the destination node. Now consider \( \hat{x}_i \) to be an unbiased estimation of the \( i \)-th source location \( x_i \).

By the Cramer-Rao lower bound (CRLB), the covariance matrix \( \text{cov} \) is the measurement noise with distribution \( \mathcal{N}(0, \sigma^2_g) \), and \( b_{i,j} \) is the distance bias corresponding to the \( i \)-th-source-to-\( j \)-th-DRN. We assume \( \eta = 1 \) for simplicity, and the bias known at the destination node. Now consider \( \hat{x}_i \) to be an unbiased estimation of the \( i \)-th source location \( x_i \).

By the Cramer-Rao lower bound (CRLB), the covariance matrix \( \text{cov} \) of \( \hat{x}_i \) is lower bounded by [16]

\[
\text{M}_i = E[(\hat{x}_i - x_i)(\hat{x}_i - x_i)^T] \geq [I(x_i)]^{-1}
\]

(7)

where \( \text{M}_i \geq [I(x_i)]^{-1} \) means \( \text{M}_i - [I(x_i)]^{-1} \) is positive semi-definite, and \( I(x_i) \) is the FIM defined as

\[
I(x_i) = E\left[ (\nabla_i \ln p(g_i; d_i, c))^T (\nabla_i \ln p(g_i; d_i, c)) \right]
\]

(8)

where \( g_i = (g_i,1, ..., g_i,N)^T \), \( d_i = (d_{i,1}, ..., d_{i,N})^T \), \( \nabla_i = \left( \frac{\partial}{\partial x_1}, \frac{\partial}{\partial y_1} \right), c = (c_1, ..., c_N)^T \), \( c_i \in \{0, 1\}, i = 1, 2, ..., N \), is a variable denoting the selection of the \( i \)-th DRN for transmission in the second phase (with the node being selected when \( c_i = 1 \)), \( p(g_i; b_i, d_i, c) \) is the probability density function of \( g_i \), and \( E[\cdot] \) denotes the expectation over \( g_i \). Remembering that \( \sigma^2_{t,i} \) is the maximum allowed \( i \)-th source localization MSE, from (7) we know that \( \sigma^2_{t,i} \geq tr([I(x_i)]^{-1}) \), where \( tr(\cdot) \) is the trace of a matrix. From [5] it can be shown that

\[
tr([I(x_i)]^{-1}) = \frac{K}{\sigma^2_g \text{det}(I(x_i))} \leq \sigma^2_{t,i}
\]

(9)

Using (5) and (9), the DRN selection for optimal communication and source localization can be obtained by solving

\[
\max_{p \in \mathbb{R}^+, c \in \{0, 1\}^N} \min_{c \in \{0, 1\}^N} \gamma_i(c, p) \quad \text{subject to} \quad \text{det}(I(x_i)) \geq \frac{K}{\sigma^2_g \sigma^2_{t,i}}; \quad i = 1, 2, ..., M
\]

(10)

\[
\sum_{j=1}^N P_j = P_T - MP_0; \quad \sum_{j=1}^N c_j = K
\]

(11)

\[
0 \leq P_j \leq P_{max} \quad j = 1, 2, ..., N
\]

(12)

\[
0 \leq P_j \leq P_{max} \quad j = 1, 2, ..., N
\]

(13)
To solve (10) we relax the condition $c \in \{0, 1\}$ with $c \in [0, 1]$ and we use the convex approximations proposed in [5] for (5). Note that (11) is not concave. We substitute $\det(I(x_i))$ by $(\det(I(x_i)))^{1/2}$ since the latter expression is concave and share the same maximizers as the former.

NUMERICAL RESULTS

We show the bit error rate (BER) and MSE performance of the proposed DRN selection. In addition, we compare our results with an SNR-based DRN selection technique. In SNR-based selection, the sum of the SNRs between the $j$-th DRN and all sources is found for each $j$, and $K$ DRNs with the highest sum SNR are selected. To find the transmitted DRN power $P_j$, (10) is solved removing constraint (11), with fixed $c$.

We consider two sources deployed in a WSN of size $100 \times 100$ m, and $N = 13$ nodes are randomly deployed as shown in Fig. 2. We use $\alpha = 2.1$, $P_0 = 30$ mW, $P_T = 120$ mW, $\sigma^2_s = 2.25 \times 10^{-4}$, $\sigma^2_g = 0.01$, and $\sigma^2_{t,i} = 0.04$, $i = 1, 2$. We assume $10 \log((P_T - P_0)/(D^\alpha \sigma^2_f)) = 26.5$ dB at the destination node with $D = 1000$ m. LOS between the DRNs and the destination is assumed with $H_{j,f} = 20$ dB, and $H_{i,j} = 5$ dB.

In Figs. 3 and 4, we show the BER and source localization MSE respectively for different values of $K$. Note that the source 1 BER shows worse performance than source 2 for $K < 7$. This is because DRNs surrounding source 1 have poor spatial diversity. As a consequence, DRNs located at longer distances get selected to increase the spatial diversity thus satisfying (11). When $K \geq 7$, the DRNs closest to source 1 are selected. Now, source 1 BER performance is better than source 2 since the latter has less DRNs in its close vicinity. In Fig. 4, we can observe that for $K < 5$, source 1 has a better source localization MSE performance. The selected DRNs have better spatial diversity for source 1 than for source 2. However, when $K \geq 5$ this situation is reversed and now the selected DRNs have better spatial diversity for source 2 than for source 1. Finally, note that
in both the BER and MSE results, the proposed DRN selection always outperforms the combined SNR based selection.

![Graph showing BER comparison]

Fig. 3. BER performance comparison of the proposed DRN selection and combined SNR-based DRN selection for different values of $K$.

![Graph showing MSE comparison]

Fig. 4. MSE performance comparison of the proposed DRN selection and combined SNR-based DRN selection for different values of $K$. 

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CONCLUSIONS

In this paper, we propose a novel formulation for DRN selection for optimal multiple source communications and localizations. We maximize the minimum $i$-th-source-to-destination SNR, limiting the maximum MSE in each source location estimation. We show BER and source localization MSE numerical results of the proposed solution, and compare them with a scheme where the DRNs with the highest combined SNR are selected. Results show that proposed DRN selection always outperforms the combined SNR-based DRN selection in both BER and source localization MSE sense.

REFERENCES