DOES A SPINNING MISSILE CAUSE TRACKING ERROR AT C-BAND?

Darren Kartchner and Michael Rice (Faculty Advisor)
Brigham Young University, Provo, Utah, USA

ABSTRACT

The amplitude fluctuation induced by the spinning missile acts as a disturbance on conscan. In addition, if a tracking system converts from S-band to C-band, the beamwidth is narrower and the wrap-around antenna on the missile requires more patches, and so the margin of error for tracking decreases. Tracking performance is simulated with a spinning missile with ballistic and fly-by trajectories while running at C-band. The spinning missile causes a periodic component in the pointing error, and when the conscan frequency is an integer multiple of the roll rate, conscan may lose track of the target. Remedial techniques are discussed, including increasing the conscan frequency and using monopulse tracking rather than conscan.

INTRODUCTION

As frequency bands are auctioned, allocated, and reallocated, equipment designed to operate at specific frequencies experience changes in performance. In particular, the transition from lower S-band (2000-2300 MHz) to lower C-band (4000-5500 MHz) is of interest. If the carrier frequency of a transmitter-receiver system changes, the gain patterns of the antennas change as well. A missile is usually equipped with a wrap-around antenna comprising patches spaced approximately half a wavelength apart. At higher frequencies, more patches are needed to maintain proper spacing (assuming the radius of the missile remains constant). Generally speaking, the gain pattern of a wrap-around transmit antenna exhibits more lobing at a higher carrier frequency. Figure 1 compares two actual roll patterns of conformal wrap-around antennas designed for a missile with a 5-inch diameter.

The radiation pattern of the transmit antenna is non-isotropic. Therefore, if the missile spins, the amplitude of the received signal will fluctuate. For tracking systems that estimate the target's position based on signal amplitude, these fluctuations induced by the spinning of the missile act as a disturbance on the tracker.

In addition to the changes on the transmitting end, the receiver's performance differs as the carrier

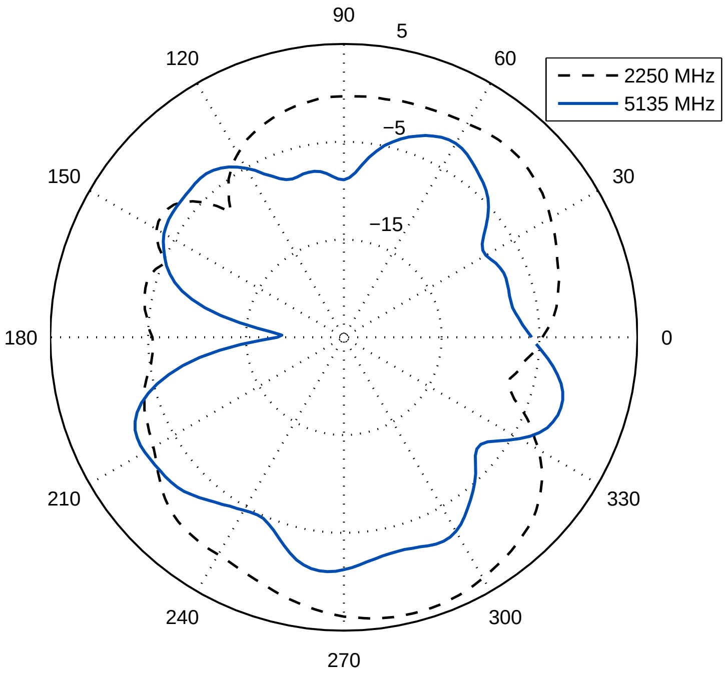


Figure 1: Roll patterns for two conformal wrap-around antennas, operating at S-band (2250 MHz) and at C-band (5135 MHz).

frequency changes. For an ideal, uniformly illuminated parabolic reflector, the gain pattern is described as

$$G(\phi) = G_0 \times 2\frac{J_1\left(0.5kD\sin\left(\phi\right)\right)}{0.5kD\sin\left(\phi\right)} \tag{1}$$

where $J_1(\cdot)$ is the modified Bessel function of the first order, k is the wavenumber of the carrier frequency, D is the diameter of the reflector, and G_0 is the boresight gain, given by

$$G_0 = \left(\frac{\pi D}{\lambda}\right)^2 \eta \tag{2}$$

where λ is the wavelength of the carrier frequency and η is the antenna efficiency. Figure 2 illustrates the gain patterns for an 8-foot parabolic reflector operating at 2250 MHz and 5135 MHz. The well-known tradeoff between boresight gain and beamwidth is apparent upon first glance. For the purposes of this paper, beamwidth is of greater concern. The beamwidth narrows upon transitioning from S-band to C-band; this imposes a smaller margin of error in terms of tracking.

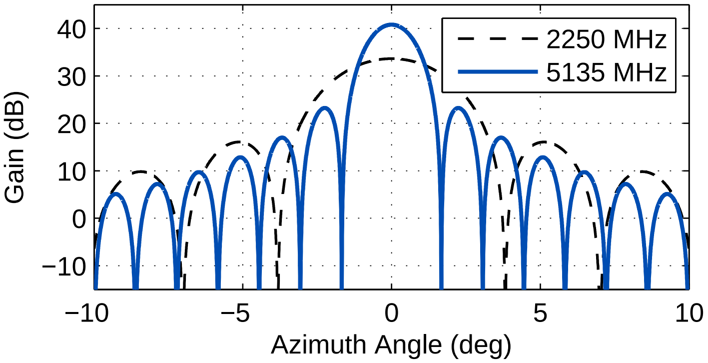


Figure 2: Gain patterns for an 8-foot parabolic reflector antenna with η = 0.7, operating at S-band (2250 MHz) and at C-band (5135 MHz).

For a parabolic reflector adjusted to operate at C-band, the disturbance from a spinning target, combined with the narrower beamwidth, may be enough to cause the tracker to lose sight of the target. To eliminate the uncertainty of how tracking performance is affected in this scenario, we will first examine some common tracking methods.

CONICAL SCAN (CONSCAN)

Conical scan (or conscan) is a tracking technique where the receiver feed deviates slightly off boresight, at an angle called the squint angle. The squint angle is selected so that the loss from pointing away from boresight is about 0.1 dB [1]. A motor rotates the feed about the boresight axis, and the rotating gain pattern follows a conical trajectory (hence the name). The received signal amplitude over one conscan cycle, $a(t)$, is then used to estimate azimuth and elevation pointing error. The component of $a(t)$ at the conscan frequency is used to estimate the azimuth and elevation pointing errors. In continuous time, this is usually done by multiplying $a(t)$ by either one period of a sinuosoidal or a square wave at the conscan frequency–which mixes the frequency component to baseband–then passing the result through a low-pass filter to isolate the conscan frequency component. The in-phase and quadrature elements of the conscan frequency component are used to determine azimuth and elevation pointing error, respectively. In discrete time, the signal amplitude $a(t)$ is sampled N times, with a spacing of $T = 1/(f_0 N)$ seconds:

$$a(t) \rightarrow a(nT),\ n = 0, 1, \cdots, N-1 \tag{3}$$

where f_0 is the conscan frequency in rotations per second. Two methods can be used to find these estimates given $a(nT)$: the least squares method and the DFT method.

Least Squares Method The angular displacement between boresight and the target as a function of time is given by [2]:

$$\epsilon(t)^2 = r^2 + \epsilon_{\text{az}}^2 + \epsilon_{\text{el}}^2 - 2r\epsilon_{\text{az}}\cos(f_0 t) - 2r\epsilon_{\text{el}}\sin(f_0 t) \tag{4}$$

where r is the squint angle, and ϵ_{az} and ϵ_{el} are the azimuth and elevation components of the pointing error, respectively. The angular displacement $\epsilon(t)$ determines the amplitude of the received signal according to the gain pattern, as seen in (1). Under the assumption that the target is relatively stationary during the conscan cycle, the only time-varying elements of (4) are the sine and cosine. Using the least squares method, the azimuth error is estimated by least squares fitting $a(nT)$ to one period of a cosine wave with frequency equal to the conscan frequency, dividing by the average signal amplitude, then dividing by the slope of the resulting S-curve. The elevation error is estimated the same way, substituting the cosine wave with a sine wave.

DFT Method In this method, pointing error is estimated from the frequency component of $a(nT)$ at the conscan frequency. This can be found by using the inverse FFT [3]:

$$A(k) = \text{IFFT}\left\{a(nT)\right\} = \frac{1}{N}\sum_{n=0}^{N-1} a(nT)e^{j\frac{2\pi kn}{N}}, \text{ for } k = 0, 1, \cdots, N-1 \tag{5}$$

The azimuth and elevation error estimates, then, are simply the real and imaginary parts of $A(1)$, divided by the slope of the resultant S-curves [4]. Figure 3 compares examples of S-curves using both least squares and DFT methods for a carrier frequency of 5135 MHz and a conscan frequency of 25 Hz (note that the slopes of the curves are normalized to 1).

IMPACT OF SPINNING MISSILE ON CONSCAN

Roll Rate and Conscan The main feature of conscan is the use of variation in received signal amplitude over one conscan period to estimate pointing error. Therefore, when the amplitude changes due to the missile spinning, the result is a disturbance on conscan. The missile's roll rate has a large role in dictating how much disturbance occurs. To illustrate this point, Figure 4 plots the pointing error estimates produced by conscan when a rotating target is at boresight (note that when using either method of conscan, a target at boresight should output zero error). The gain pattern of the missile is the C-band gain pattern from Figure 1. The abscissa is the roll rate of the missile, and the ordinate is the average output of conscan after the missile spins several times. In studying this figure, there are large peaks whenever the conscan frequency is an integer multiple of the roll rate. This is because conscan picks up the harmonic generated by the spinning missile, resulting in an especially bad disturbance.

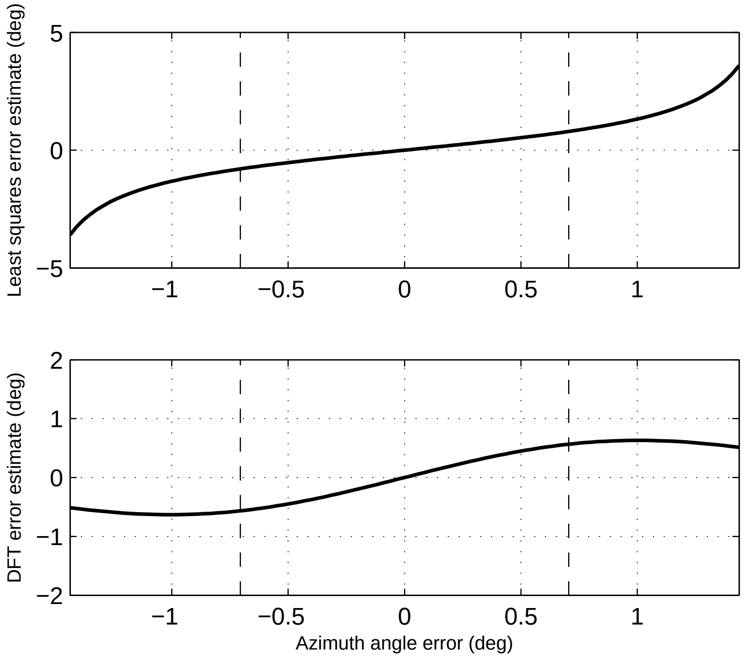


Figure 3: Normalized S-curves for least squares conscan and DFT conscan. The single dotted line denotes the half–power beamwidth.

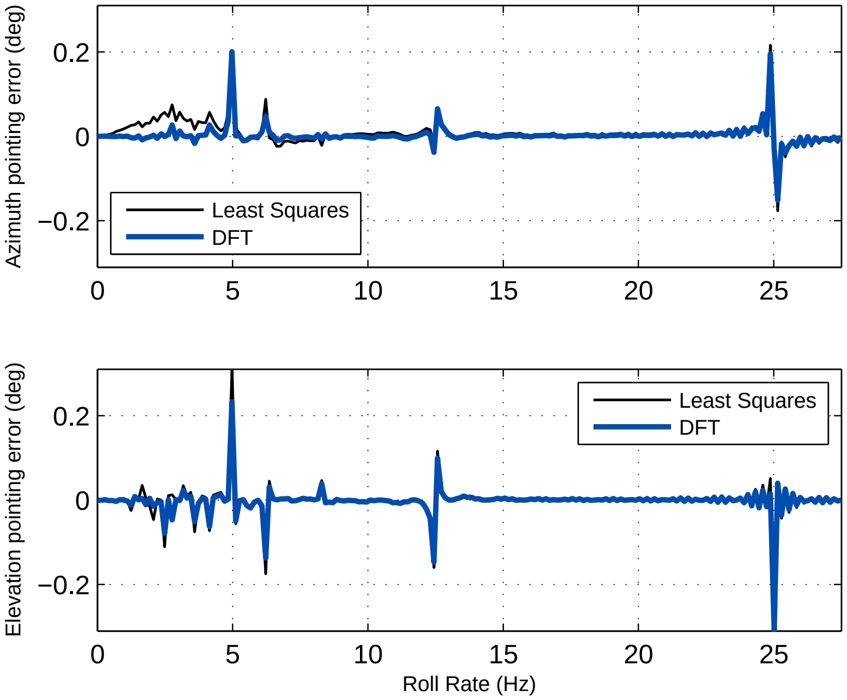


Figure 4: Average azimuth and elevation error estimates of a stationary, rotating target at boresight, using both methods of conscan.

CONSCAN SIMULATION

Ballistic Trajectory To simulate conscan's performance while tracking a spinning ballistic missile, we used the scenario illustrated in Figure 5. The azimuth and elevation pointing angles of the tracking antenna are controlled by PI controllers tuned to a loop bandwidth of 3 Hz using successive loop closure [5]. Figure 6 is a block diagram of the controller. Figures 7-9 plot the azimuth and elevation pointing errors of both implementations of conscan for three cases: when the missile is not spinning, when the missile spins at a rate of 2 Hz (a rate coprime with the conscan frequency), and when the missile spins at a rate of 5 Hz (a harmonic of the conscan frequency). The gain pattern of the target is the C-band pattern from Figure 1. Based on the results, when the target is not spinning, conscan is able to track with relatively small error, with the largest error occuring at the time of largest angular velocity. When the missile spins at 2 Hz, conscan is able to continue tracking, but a periodic component is introduced to the error. When the missile spins at 5 Hz, the RMS pointing error increases.

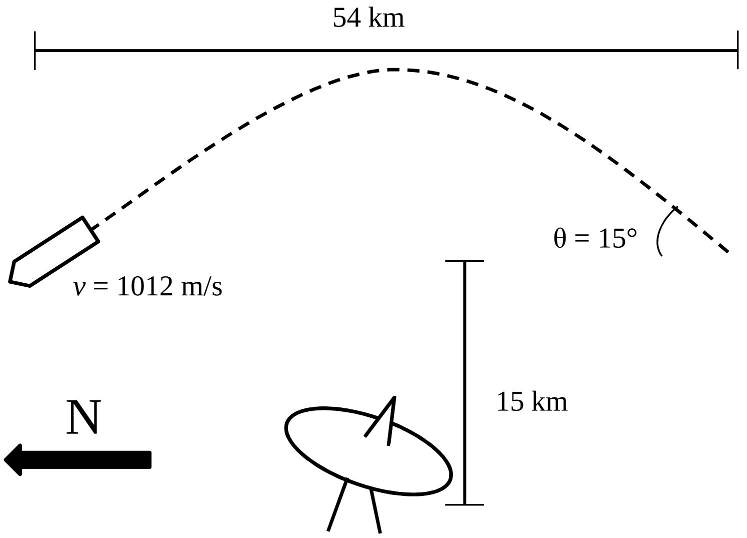


Figure 5: Diagram of the ballistic missile simulation

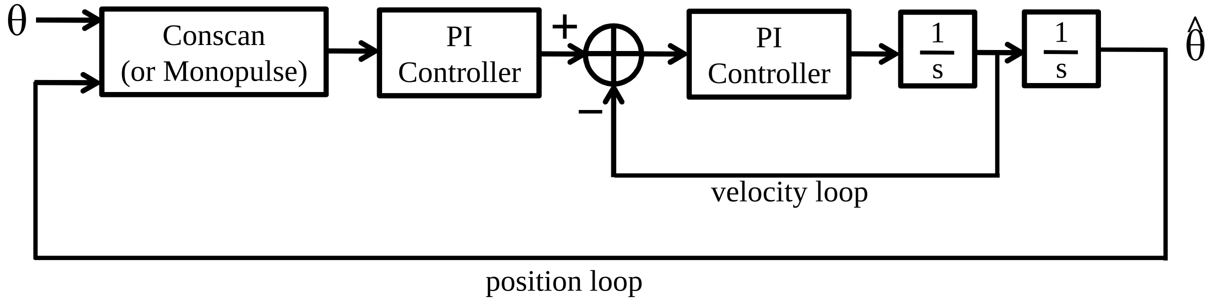
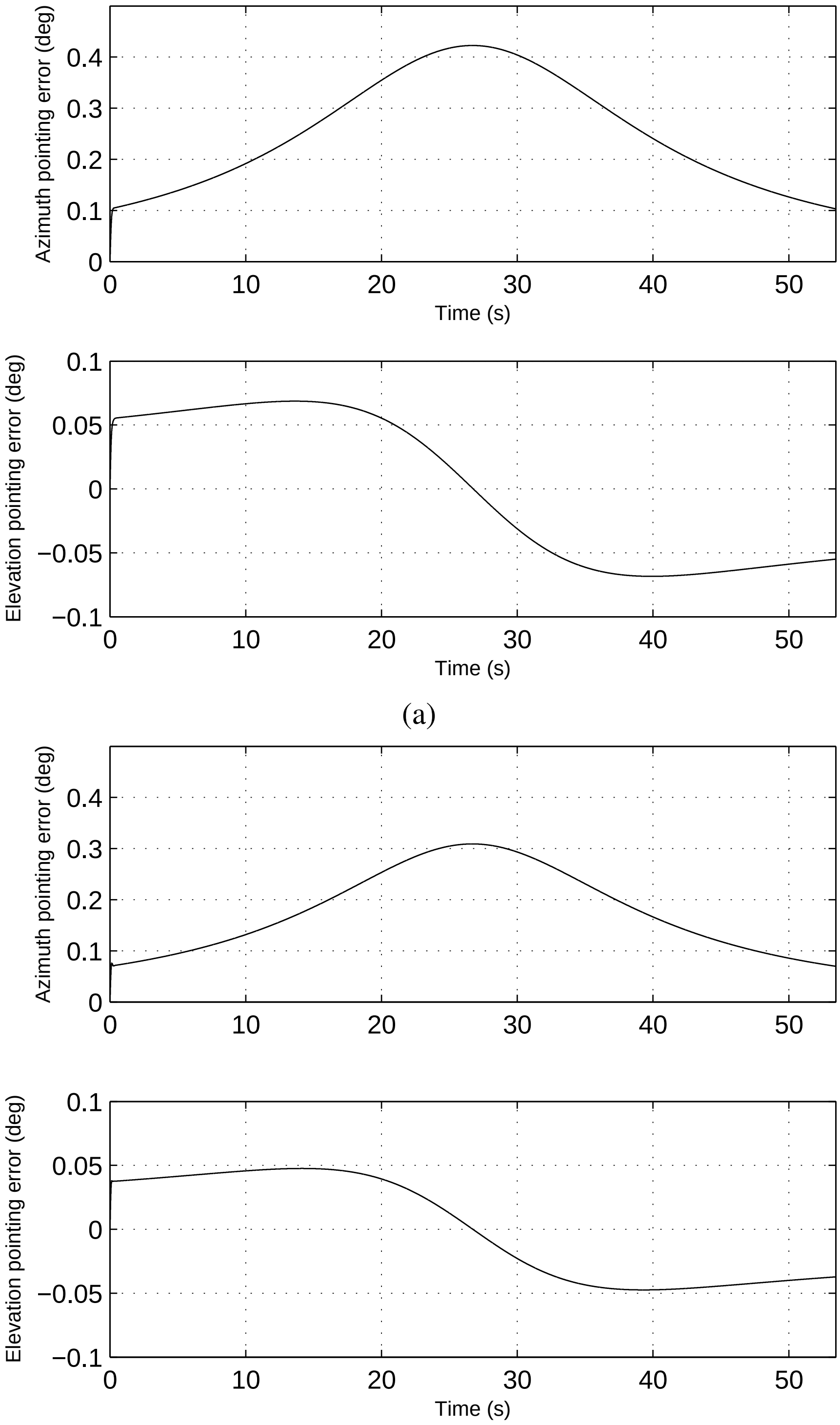


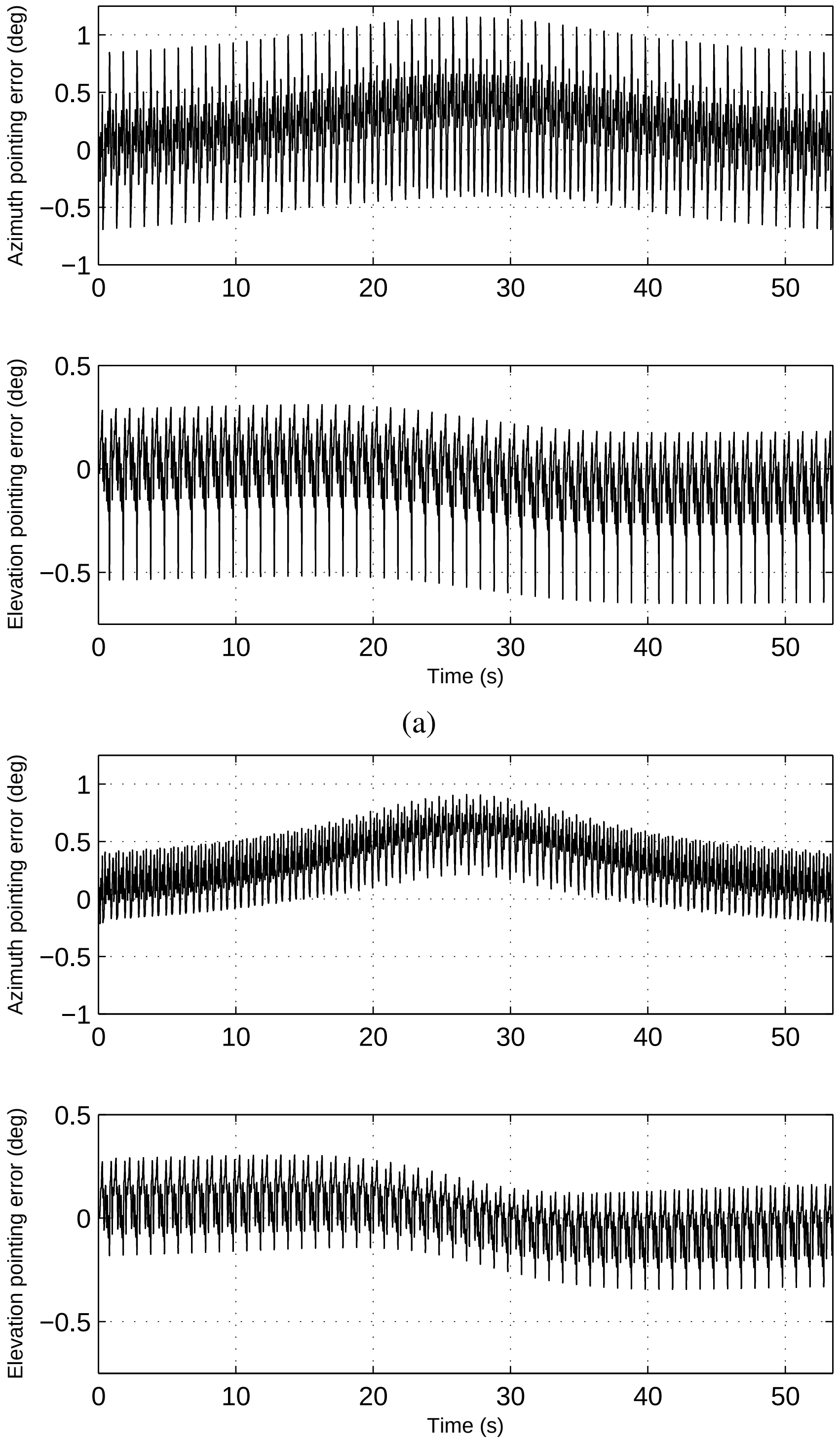
Figure 6: Block diagram of the antenna controller



(a)

(b)

Figure 7: Azimuth and elevation pointing errors for a non-spinning ballistic missile, using (a) least squares conscan and (b) DFT conscan.



(a)

(b)

Figure 8: Azimuth and elevation pointing errors for a ballistic missile spinning at 2 Hz, using (a) least squares conscan and (b) DFT conscan.

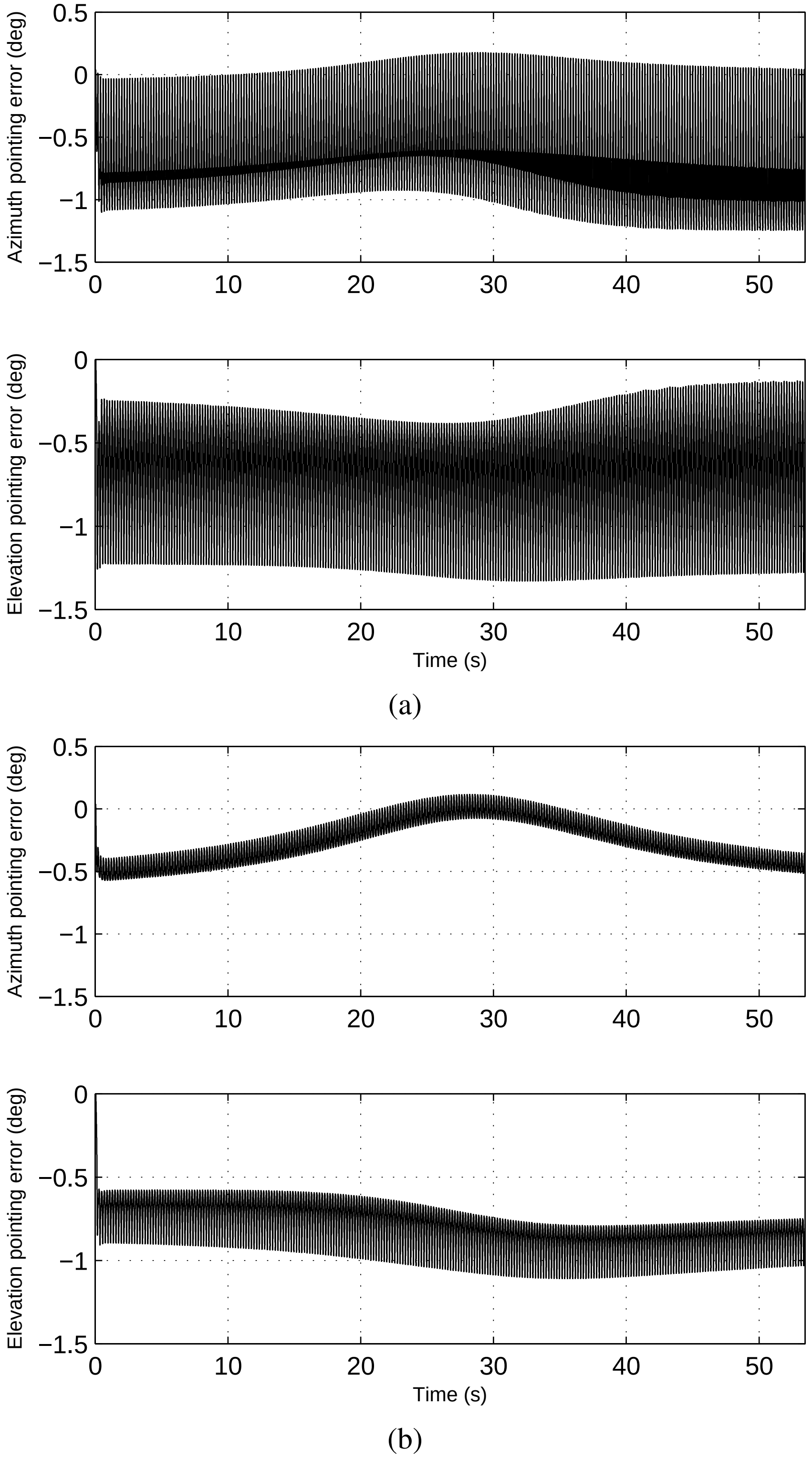


Figure 9: Azimuth and elevation pointing errors for a ballistic missile spinning at 5 Hz, using (a) least squares conscan and (b) DFT conscan.

Fly-by Trajectory To simulate a fly-by, assume the missile flies from north to south at a constant altitude of 11000 feet. The velocity and gain pattern remain the same, and the flight path is 15 km east of the tracking antenna. The total flight time is 20 seconds. Figure 10 illustrates the simulation. Figures 11-13 plot the pointing error of both implementations of conscan when the missile spins at 0, 2, and 5 Hz. Since the elevation angle is nearly constant for this scenario, only the azimuth pointing error will be considered.

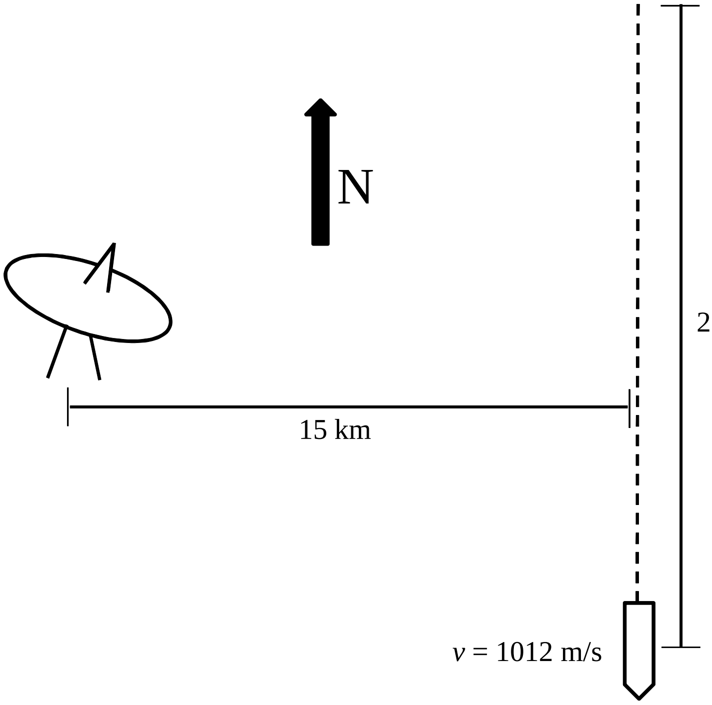


Figure 10: Diagram of the fly-by missile simulation

Changing Conscan Frequency As seen in previous figures, the tracking error is highest when the angular velocity is highest. In such moments, the angular displacement of the target per conscan cycle is at its peak. By increasing the conscan frequency, the target does not move as far per cycle; thus, the displacement per cycle is reduced, and so the assumption that the target is stationary during a conscan cycle is closer to the truth. In addition to a lower angular displacement per cycle, the target experiences less roll per cycle, so there is less disturbance on the amplitude of the received signal. Figure 14 illustrates the difference in tracking in the fly-by scenario when the conscan frequency is 45 Hz instead of 25 Hz, while the missile roll rate is 5 Hz (note that the conscan frequency is still an integer multiple of the roll rate). There are a few positive changes when the conscan frequency increases to 45 Hz. First, DFT conscan does not lose track of the target; second, the RMS pointing error of least squares conscan is lower; and third, the amplitude of the periodic component of the error is lower for least squares conscan.

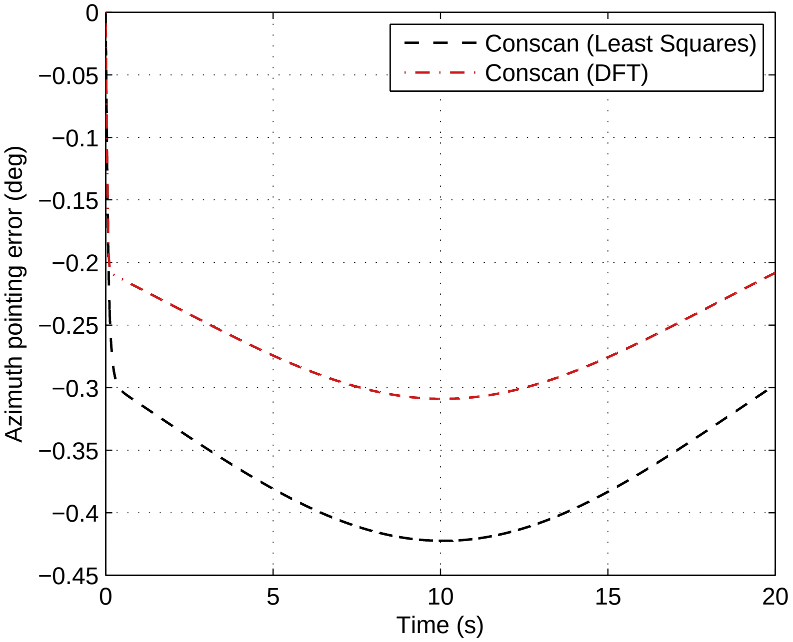


Figure 11: Pointing error for least squares conscan and DFT conscan for a non-spinning fly-by missile.

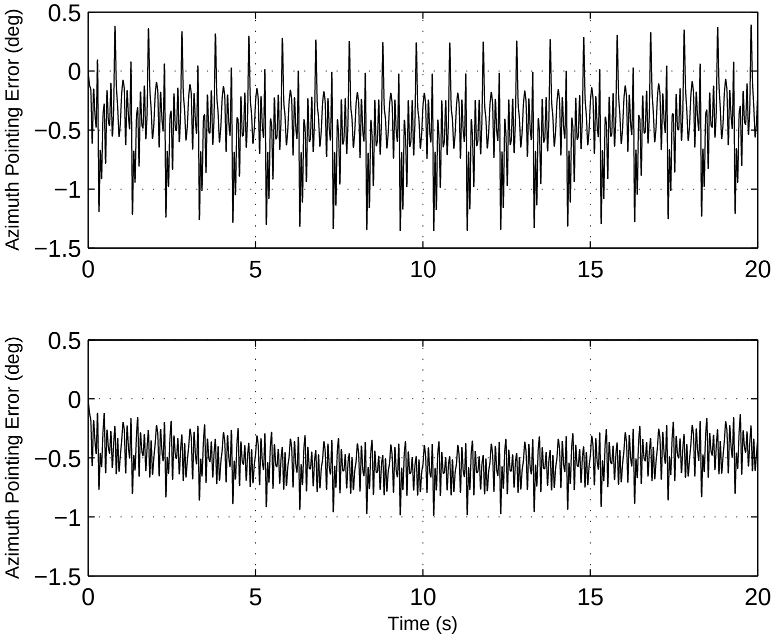


Figure 12: Pointing error for least squares conscan and DFT conscan for a fly-by missile spinning at 2 Hz.

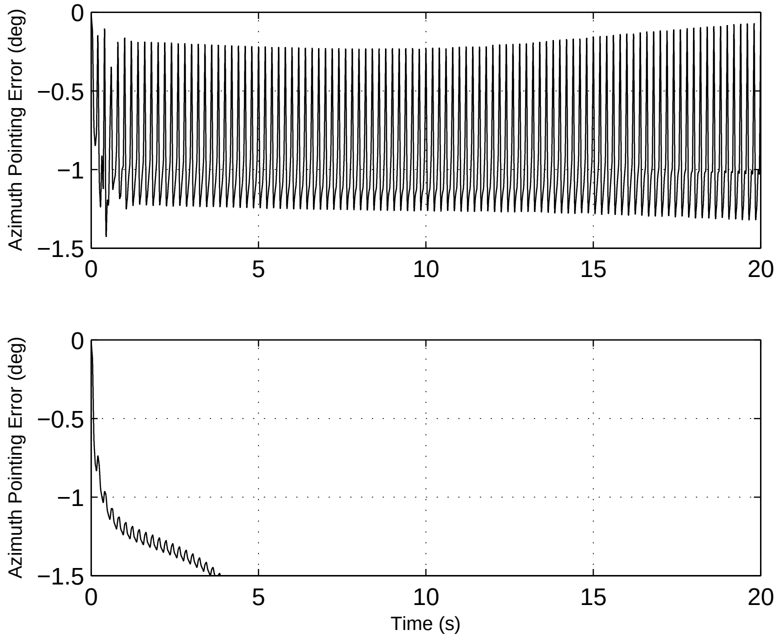


Figure 13: Pointing error for least squares conscan and DFT conscan for a fly-by missile spinning at 5 Hz. Note that DFT conscan loses track of the target.

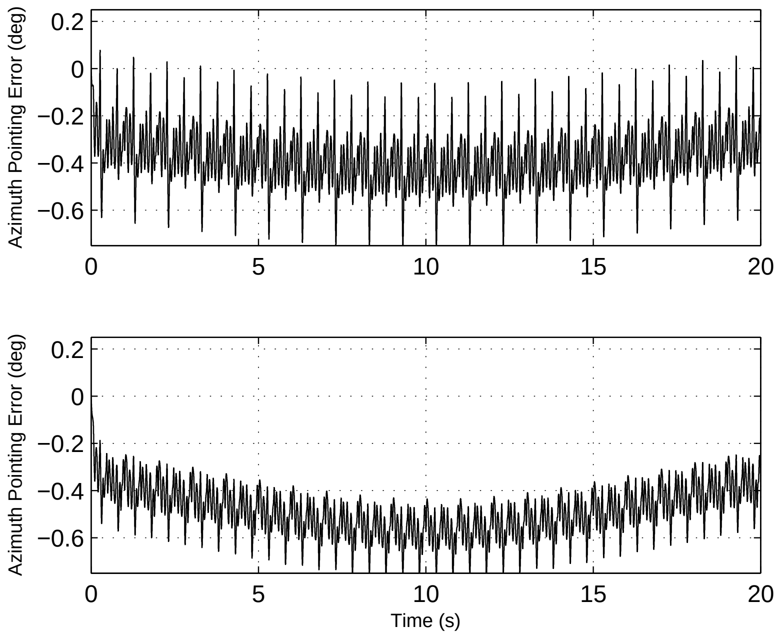


Figure 14: Pointing error for least squares conscan and DFT conscan for a fly-by missile spinning at 5 Hz, for a conscan frequency of 45 Hz. The number of samples per cycle remains constant at 40. Compare with Figure 13.

ANOTHER TRACKING TECHNIQUE: MONOPULSE

Monopulse is a tracking technique similar to conscan, but it uses four stationary feeds pointed away from boresight at the same squint angle used in conscan. The feeds are positioned like four corners of a square. Azimuth and elevation differences are produced using sums and differences of the feed output amplitudes, as summarized in Figure 15. The azimuth and elevation pointing error estimates are the azimuth and elevation differences divided by the sum signal [6], then divided by the slope of the S-curve. The normalized S-curves are shown in Figure 16. Sometimes a "scan frequency" is reported with the monopulse method, but this is not a "scan" in the conscan sense, but rather the rate at which the feed output amplitudes are sampled.

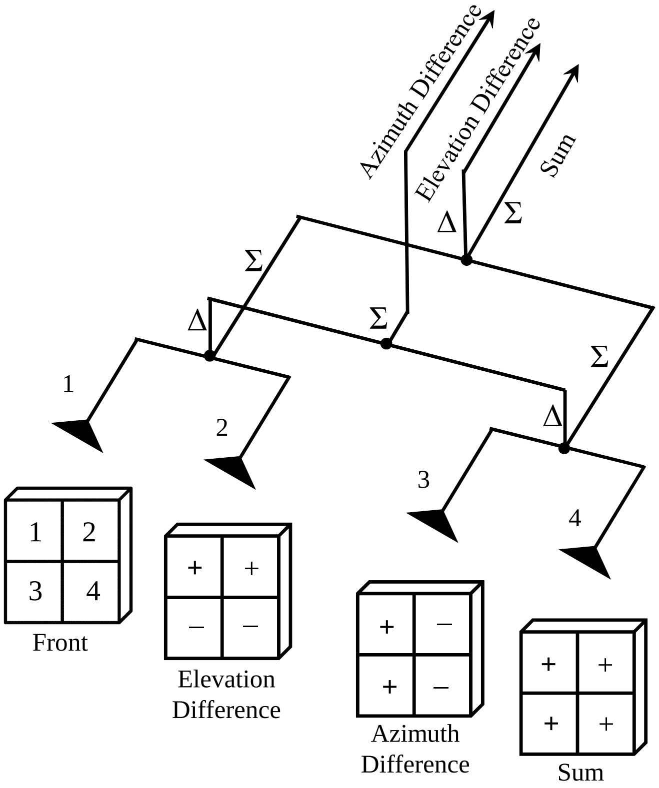


Figure 15: A block diagram of the monopulse tracking method, reproduced from [7].

With respect to the issue of a spinning target, monopulse has a couple advantages over conscan. For one, the temporal delay between signals goes away, since the four feeds are receiving simultaneously. In addition, monopulse divides out the fluctuating amplitude of the received signal. Therefore, monopulse is unaffected by the disturbance induced by a spinning target. If there were a figure simular to Figure 4, but for the monopulse method, the output would be zero for all roll rates.

Simulating a Monopulse Tracker Under the same scenario of a spinning fly-by missile, monopulse tracking can be compared to conscan tracking. Figure 17 is a plot of the azimuth pointing error for monopulse and both implementations of conscan–with a conscan frequency of 45 Hz–for a non-spinning missile, while Figure 18 is a plot of the error while the missile spins at 5 Hz. These two figures reveal that monopulse is slightly more effective than conscan when the target does not spin, and much more effective than conscan when the target does spin.

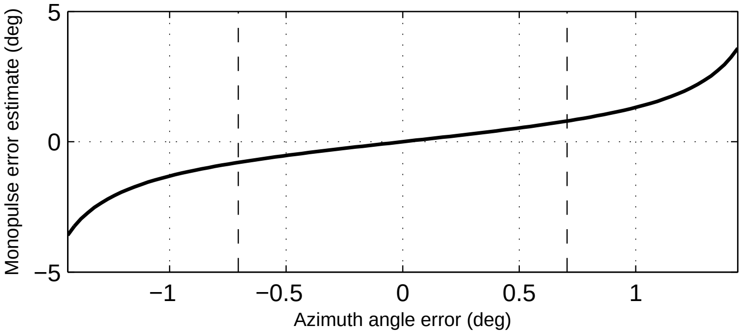


Figure 16: Normalized s-curves for a monopulse tracker with a carrier frequency of 5135 MHz. The single dotted line is at the half-power beamwidth.

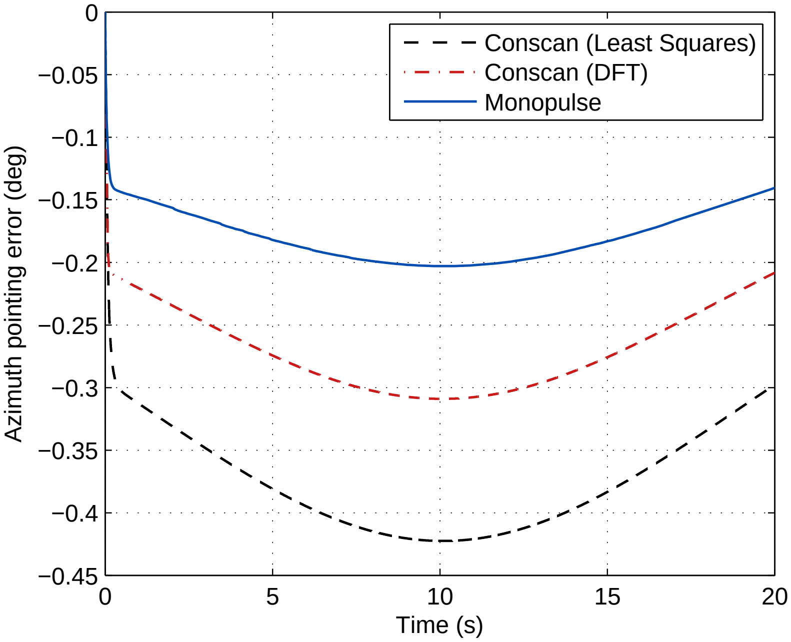


Figure 17: Monopulse tracking error vs. conscan tracking error for a non-spinning fly-by missile. The conscan frequency is 45 Hz.

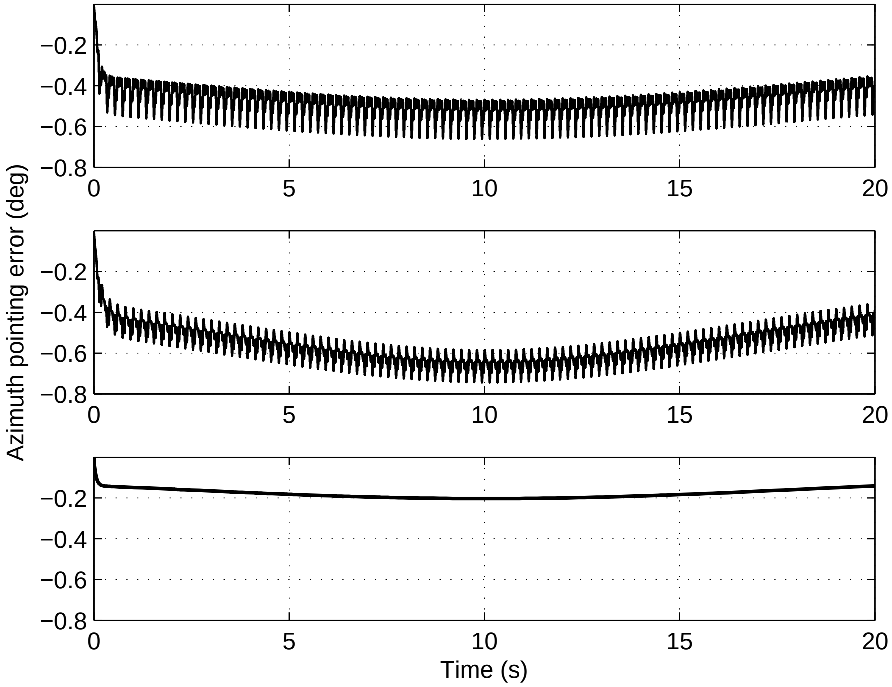


Figure 18: Tracking error for a fly-by missile spinning at 5 Hz, using least squares conscan, DFT conscan, and monopulse, respectively. The conscan frequency is 45 Hz.

CONCLUSION

The simulations demonstrate that monopulse exhibits superior tracking performance over the two implementations of conscan described in this paper. However, the physical implementation of monopulse is significantly more complex than conscan. The four feeds needed to execute monopulse tracking must be rigidly attached to the antenna at a specific squint angle (which changes as the carrier frequency is adjusted), while the conscan feed, already designed to rotate, can have its squint angle modified to be adequate for the carrier frequency. In the event that monopulse is not a viable option for a tracking antenna, the performance of an antenna implementing conscan can be improved by increasing the conscan frequency.

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