# Optimized Constellation Mappings for Adaptive Decode-and-Forward Relay Networks using BICM-ID

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## ABSTRACT

In this paper, we investigate an adaptive decode-and-forward (DF) cooperative diversity scheme based on bit interleaved coded modulation with iterative decoding (BICM-ID). Data bits are first encoded by using a convolutional code and the coded bits after an interleaver are modulated before transmission. Iterative decoding is used at the receiver. Optimized constellation mapping is designed jointly for the source and the relay using a genetic algorithm. A novel error performance analysis for the adaptive DF scheme using BICM-ID is proposed. The simulation results agree well with the analytical results at high signal-to-noise ratio (SNR). More than $5.8$ dB gain in terms of SNR over the existing mappings is achieved with proposed mappings.

## KEY WORDS

Coperative diversity, decode and forward, optimized mapping, iterative decoding.

## INTRODUCTION

Diversity techniques can combat the detrimental effects of multiplicative time selective fading in wireless communication systems [1]. In particular, spatial diversity is of special interest where multiple antennas are used at the transmitter and/or at the receiver sides, as in multiple-input multiple-output (MIMO) systems. It is known that MIMO systems can be utilized for enhancing the capacity and error performance of a communication system [2]. On the other hand, the application of MIMO technology to cellular and ad hoc networks often faces the practical problem of installing multiple antennas at the terminals with severe space limitations. This limitation can be overcome through cooperative diversity schemes [3]- [5]. The basic idea of this strategy is that when a transmitter wants to transmit its information, it takes help from nearby single-antenna terminals to cooperatively transmit its information to the destination, thus forming a virtual multi-antenna system. In this process, the cooperating terminals act as relays for the source.

Various relay strategies have been proposed for the cooperative diversity networks, with the two most popular schemes being decode-and-forward (regenerative) and amplify-and-forward (non-regenerative) schemes. In the decode-and-forward (DF) scheme, each relay decodes the received signal transmitted by the source, re-encodes it and then forwards to the destination again [6],[7]. The destination combines the signals received from the relays and the source, and decodes the information. Amplify-and-forward (AF) is a simpler scheme [1], where the relay amplifies the received signal and forwards to the destination.

Both schemes of relaying are suitable for using higher order modulation at the source and the relay. Higher order modulation can increase the efficiency of transmission. Bit-interleaved coded modulation (BICM) is a well established technique used for higher order modulations [8], [9]. In BICM, the coded bits after a forward error correction (FEC) encoder are interleaved using a bit interleaver. The interleaved data is then transmitted after proper modulation. At the receiver, the demodulator calculates soft information about the coded bits from the received symbols, and the decoder decodes the information bits through iterations with the demodulator. The information rate and error probability analysis of BICM was illustrated in [10]. The authors in [11] proposed BICM with iterative decoding (BICM-ID) which gives significant error performance improvement over non-iterative BICM decoding.

In this paper, we focus on DF cooperative diversity scheme using BICM-ID over Rayleigh fading channels. The authors in [6] and [7] study the outage probability and error analysis of DF schemes. The performance of cooperative networks using BICM-ID is analyzed in [12], where M-ary phase shift keying (M-PSK) modulation is used. The authors in [13] investigate the error performance of BICM system over a non-orthogonal amplify-and-forward (NAF) half-duplex single-relay channel. In related works, constellation rearrangement (CoRe) has been considered to improve the received signal quality in hybrid automatic repeat request schemes [14], [15]. Our work is different from the rest of the work in the sense that we present new constellation mappings for BICM-ID when used in DF relay networks. To find the combined optimized symbol mappings at source and relay, we propose to use genetic algorithm. A new cost function is derived for this purpose. We also present novel error performance analysis for the adaptive DF scheme using BICM-ID.

## SYSTEM DESCRIPTION

We consider a dual-hop cooperative communication network where information is transmitted from a source $S$ to a destination $D$ over a fading channel. A relay terminal R will assist the destination to receive a second copy of the original signal through the fading channel links $S-R$ and $R-D$ as shown in Fig.1. All the terminals are equipped with a single antenna and no terminal can transmit and receive at the same time. We assume that the perfect channel-state information is available at each receiver, but not at the transmitter. At the transmitter, the coded bits $\{v_i\}$ after the interleaver are divided into groups of size $m$ bits, for example, $\mathbf{v} = (v_1, ...., v_m)$ denotes such a group. Each group of bits is then mapped to a complex channel symbol $x = \mu(\mathbf{v})$ chosen from an $M$-ary constellation $\chi$, where $\mu$ denotes bit pattern to the constellation point mapping.

In the first time slot, the source transmits signal $x$ and the signals received by the destination and the relay are respectively given by

$$\begin{aligned} y_{S-D} &= h\sqrt{E_s}x + n_{D_1} \\ y_{S-R} &= f\sqrt{E_s}x + n_{R_1} \end{aligned} \tag{1}$$

where $E_s$ is the average energy of the signal transmitted from the source, $h$ and $f$ are the channel gains with $E\{|h|^2\} = E\{|f|^2\} = 1$, and $n_{D_1}$, $n_{R_1}$ are zero mean additive white Gaussian noise

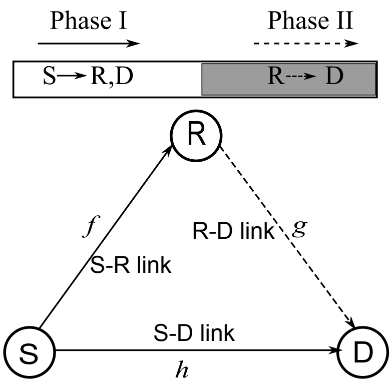


Fig. 1. Cooperative Communication model with a source, destination and a single relay.

(AWGN) terms with two-sided power spectral density (PSD) $N_{o,h}/2$ and $N_{o,f}/2$ respectively for the links $S$-$D$ and $S$-$R$ respectively.

There are two types of DF relaying strategies: (i) A fixed DF Scheme, where the relay detects the received signal and always forwards the data to the destination after encoding. Although this scheme is simple, it gives adverse effect of cooperation if the data is not detected correctly at the relay. (ii) An adaptive DF Scheme, where the relay detects and decodes the received signal only if the signal to noise ratio (SNR) is above a reference SNR level, and then it forwards the decoded data to the destination after encoding it again. Otherwise, it keeps silent in the second phase of transmission. Therefore possible erroneous transmission induced by the relay in fixed DF scheme can be avoided.

In this paper, we consider the adaptive DF scheme. We assume the ideal case in which the relay knows whether the received signal is decoded correctly or not. The forwarding decision at the relay is made on the basis of this knowledge. When the relay assists the source by transmitting the information, the signal received at the destination from the relay is given by

$$y_{R-D} = g\sqrt{E_r}x + n_{D_2} \tag{2}$$

where $g$ is the channel gain with $E\{|g|^2\} = 1$ and $n_{D_2}$ is AWGN with two-sided PSD $N_{o,g}/2$. For simplicity, we consider the case where both source and relay are allocated equal energy, i.e., $E_s = E_r = E_T/2$. The received signal is demodulated and decoded iteratively at each receiving terminal. The receiver consists of an interleaver ($\pi$), a deinterleaver ($\pi^{-1}$), a soft demodulator and a soft-input soft-output (SISO) decoder as shown in Fig. 2. We use pseudorandom interleavers which satisfy the conditions given in [11].

As given in [16], the SISO decoder uses maximum $a\ posteriori$ probability (MAP) algorithm. Let the input and the output of a SISO module be represented by $I$ and $O$ respectively. Extrinsic information of the coded bits $P(v;O)$ and $P(c;O)$ is exchanged between the demodulator and the SISO decoder iteratively through a feedback loop, where the letters $c$ and $v$ denote the coded bits before and after the interleaver respectively. After deinterleaving, $P(v;O)$ becomes $P(c;I)$, $a\ priori$ information at the input of the SISO decoder. Similarly, $P(c;O)$ after interleaving becomes $P(v;I)$, $a\ priori$ information at the input of the soft demodulator. After a number of iterations, the total $a\ posteriori$ probability of the information bits $P(u;O)$ is calculated to make the hard decision

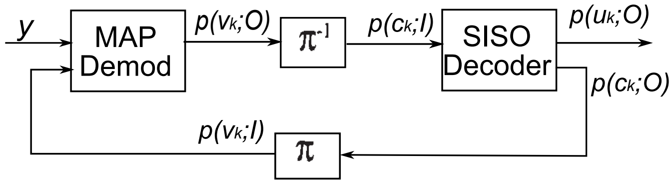


Fig. 2. Block diagram of receiver.

at the output of the decoder. For a received signal $y$, the extrinsic information [9] of the coded bits is calculated as

$$P(v_k = b; O) = \sum_{\mathbf{x} \in \chi_b^i} \left( p(y|x) \prod_{j \neq i} P(v_j = v_j(x); I) \right) \tag{3}$$

where $v_j(x) \in \{0, 1\}$, $1 \leq j \leq m$, is the value of $j$th bit in the label of transmitted symbol $x$ and $\chi_b^i$ is the subset of $\chi$ whose label has the binary value $b$ at the $i$th bit position.

## MAPPING OPTIMIZATION

Symbol mapping is crucial to the performance of a BICM-ID system. An optimized mapping could be found through an exhaustive search, but it is too complex for higher order modulation since we have to check $(2^m)!$ different possibilities. To address this issue, several techniques have been investigated in the literature. A binary switching algorithm is proposed in [17], where a list of symbols is generated and the index of a symbol with the highest cost is switched with the index of another symbol which gives the maximum decrease in the cost. This process of switching is repeated until there is no further reduction in the total cost. The authors in [18] use a tabu search algorithm to find the optimized mapping. In our work, we use a genetic algorithm [19] to find the optimal mapping at the source and the relay.

### *Cost Function*

To find the cost function, we use the analytical error bound expressions. The idea here is to minimize the total cost for the joint mappings at source and relay by switching the symbols of relay mapping and keeping source mapping fixed. First, we find an optimized mapping for the source by using the genetic algorithm. As given in [19], the cost function $\mathcal{F}$ used in the genetic algorithm is based on the Chernoff bound [10] and is given below for a Rayleigh fading channel.

$$\mathcal{F} = \frac{1}{m2^m} \sum_{i=1}^{m} \sum_{b=0}^{1} \sum_{S_k \in \chi_b^i} \sum_{\hat{S}_k \in \chi_{\bar{b}}^i} \frac{1}{\|S_k - \hat{S}_k\|^2} \tag{4}$$

where $\hat{S}_k$ denotes the transmitted symbol whose label differs only at position $k$ compared to the label of $S_k$, $\chi_b^i$ represents the set of constellation points, and the set $\chi_{\bar{b}}^i$ contains the corresponding points that differ from the points in $\chi_b^i$ at the $i$th bit position.

To find the optimized mapping for the relay, we keep the source mapping fixed as obtained from the cost expression (4). We then optimize the relay mapping based on the following cost function.

$$\mathcal{F}^c = \frac{1}{m2^m} \sum_{i=1}^{m} \sum_{b=0}^{1} \sum_{\substack{S_k \in \chi_b^i \\ R_k \in \chi_b^i}} \sum_{\substack{\hat{S}_k \in \chi_{\bar{b}}^i \\ \hat{R}_k \in \chi_{\bar{b}}^i}} \left( \frac{1}{\|S_k - \hat{S}_k\|^2} \times \frac{1}{\|R_k - \hat{R}_k\|^2} \right) \tag{5}$$

where $S_k$ and $R_k$ are the symbols for source and relay mappings respectively.

*Genetic Algorithm*

In this section, we describe the genetic algorithm (GA) used for mapping optimization. The GA is a popular heuristic algorithm to solve optimization problems. After formulation of the problem in genetic representation with a fitness function, the algorithm is initialized with a population size of $N_{pop}$ random mappings (solutions). The GA then improves the population through repetitive use of selection, crossover and mutation operations.

The selection operator is used to select the fittest solution from the population. There are many selection schemes available to select the superior solution out of available solutions. One of the popular selection schemes is the tournament scheme which is used here. It selects the superior solution from randomly selected two or more solutions. Selection operator selects the $N_{best} = p_s.N_{pop}$ mappings for further operations, where $p_s$ denotes the fraction for selection.

Crossover operation is used for breeding. The crossover operator randomly selects two mappings (known as parents) from the $N_{best}$ mappings for breeding. The two parents are selected randomly from the $N_{best}$ mappings other than the best mapping. The crossover operator creates two children by combining subparts of the selected two parents. The cost of the generated children are evaluated. If a child has a lower cost than the cost of its direct parent, then the parent is replaced by the child. If the child's cost is higher than its parent cost, then the mutation process is started.

The mutation operator exchanges the two randomly selected positions of the selected solution with a given probability $p_{mut}$. During the mutation process, a total of $L$ solutions are generated and the cost of each is evaluated. If any of the mutant has a lower cost than a parent, then that parent is replaced by this mutant otherwise it will replace the worst solution present in the population.

In each generation, many new solutions are generated and there are chances of having duplicate solutions present in the population. After creating a certain number of generations, a culling process is performed. During the culling process, the duplicate entries are deleted from the population and new solutions are generated randomly and added to the population to keep its size constant.

The GA algorithm is run for a specified number of generations $(nGen)$ and the mapping with minimum cost value is selected as the optimum mapping. The parameters used in our implementation are: $N_{pop} = 100$, $p_s = 0.4$, $p_{mut} = 0.02$ and $nGen = 10,000$. Fig. 3 shows the flow chart of GA to solve the problem of finding optimal symbol mappings. The optimized mappings thus obtained for the source and the relay are shown in Fig.4.

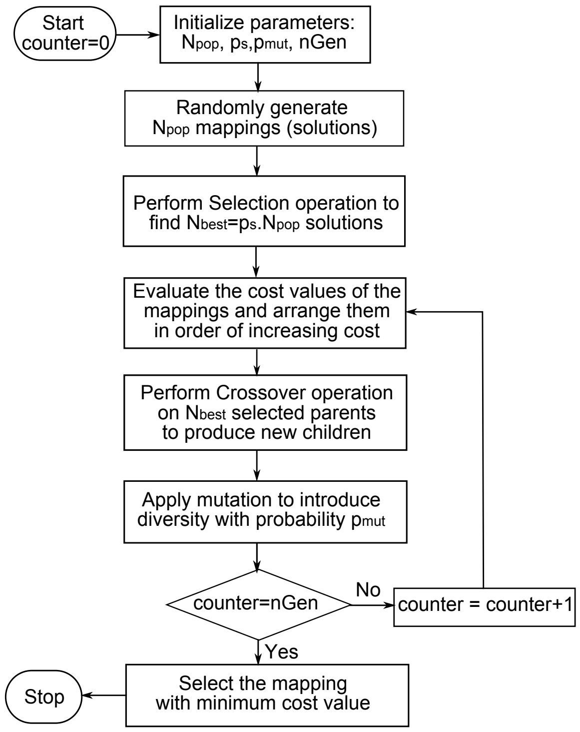


Fig. 3. Flow chart for the GA to find the optimized symbol mapping.

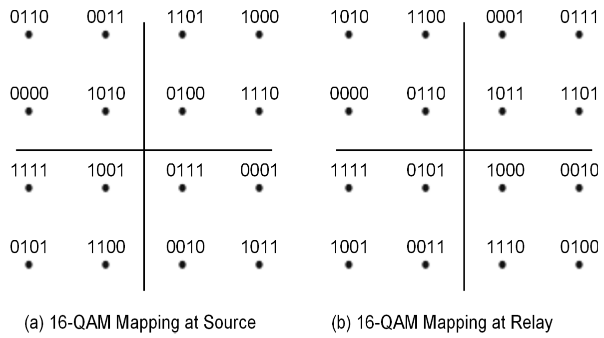


Fig. 4. Optimal symbol mappings obtained at high SNR for 16-QAM modulation at (a) source and (b) relay.

## ERROR ANALYSIS OF THE PROPOSED SCHEME

In this section, we derive expressions for the end-to-end bit error rate for the adaptive DF scheme. As discussed in [7], the error probability for a general DF scheme can be written as

$$P_{e2e}(e) = P(dec) \times P_{div}(e) + (1 - P(dec)) \times P_{nodiv}(e) \tag{6}$$

where $P(dec)$ is the probability that the relay decides to decode and forward, $P_{div}(e)$ is the probability of error in case of combined diversity reception at the destination due to the source and the relay, and $P_{nodiv}(e)$ is the probability of error at the destination when no message is forwarded from the relay. For adaptive DF case, $P(dec)$ can be written as

$$P(dec) = 1 - FER_{(S-R)} \tag{7}$$

where $FER_{(S-R)}$ is the frame error rate (FER) at relay. The FER expression is given later in (14).

To find $P_{nodiv}(e)$, we will calculate bit error probability for the $S$-$D$ link. Consider a single channel BICM-ID case and develop an asymptotic error analysis based on error-free feedback [9], [10]. Let $X$ be the true and $\hat{X}$ be the erroneous symbol sequences with a Hamming distance $d$ between the true and the erroneous codeword sequences. Let $\mathbf{h} = [h_1, h_2, ...., h_d]$ represent the complex channel values corresponding to the $d$ symbols. The conditional pairwise error probability (PEP) between the true and the erroneous symbol sequences given $\mathbf{h}$ for a single channel [20] is given by

$$P(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{h}) = Q\left(\sqrt{\frac{1}{2N_o}\sum_{l=1}^{d}|h_l|^2||\mathbf{X}_l - \hat{\mathbf{X}}_l||^2}\right) \tag{8}$$

where $\mathbf{X}_l$ and $\hat{\mathbf{X}}_l$ are the true and erroneous symbols at the $l$-th position of the true and erroneous symbol sequences respectively, and $Q(\gamma) = 1/\sqrt{2}\pi \int_{\gamma}^{\infty} \exp(-y^2/2)dy$ is the Gaussian tail probability. We further simplify the PEP expression for Rayleigh fading channels by using the following Gaussian probability integral.

$$Q(\tau) = \frac{1}{\pi}\int_{0}^{\pi/2} \exp(-\tau^2/2\sin^2\theta)d\theta \tag{9}$$

Now averaging (8) over Rayleigh random variable sequence $\mathbf{h}$ by using (9) we get

$$P(\mathbf{X} \to \hat{\mathbf{X}}) = E_h[P(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{h})] \tag{10}$$

where $E_h[\cdot]$ denotes averaging over the vector $\mathbf{h}$. Averaging over only one channel coefficient $h$ of the vector $\mathbf{h}$, we get $E_h[\exp(-\tau h^2)]$=$1/(1+\tau)$. Then (10) can be evaluated as

$$P(\mathbf{X} \to \hat{\mathbf{X}}) = \frac{1}{\pi}\int_{0}^{\pi/2}\left[\prod_{l=1}^{d}\left(1 + \frac{1}{4N_o}\frac{||\mathbf{X}_l - \hat{\mathbf{X}}_l||^2}{\sin^2\theta}\right)^{-1}\right]d\theta \tag{11}$$

The average PEP depends upon the Hamming distance, mapping and constellation of the symbols and here we assume that the symbols in $\mathbf{X}$ and $\hat{\mathbf{X}}$ differ in only one bit.

The bound for bit error rate (BER) for simple BICM-ID system employing a rate-$k_c/n_c$ convolutional code is given by

$$P_{nodiv}(e) \leq \sum_{i}\sum_{\mathbf{d}} \frac{i}{k_c} A(i, \mathbf{d}) \bar{P}_1^{d_1} \bar{P}_2^{d_2} \ldots \bar{P}_m^{d_m} \tag{12}$$

where $A(i, \mathbf{d})$ is the information weight covering the error events with input weight $i$ and output weight $\mathbf{d} = [d_1, d_2, \cdots d_m]$, and $d_k$, $1 \leq k \leq m$, is the output Hamming weight of the error pattern corresponding to the $k$-th bit position of the symbol bit label. The average probability $\bar{P}_k$ as explained in [21], is obtained from (11) considering only the $k$-th bit position as

$$\bar{P}_k = \left(\frac{1}{2^m}\right)\sum_{\mathbf{x}\to\hat{\mathbf{x}}^{(k)}} \frac{1}{\pi}\int_{0}^{\pi/2}\left(1 + \frac{1}{4N_o}\frac{||\mathbf{x} - \hat{\mathbf{x}}^{(k)}||^2}{\sin^2\theta}\right)^{-1} d\theta \tag{13}$$

Similarly, the bound on FER can be found from (12) as

$$FER \leq \sum_{i}\sum_{\mathbf{d}} A(i, \mathbf{d}) \bar{P}_1^{d_1} \bar{P}_2^{d_2} \ldots \bar{P}_m^{d_m} \tag{14}$$

We can also define $FER_{(S-R)} = \min(1, FER)$, since FER can be considered as the probability that relay does not decode and forward. In the case of diversity, we keep the total energy constant, i.e., the source and the relay are allocated half of the total energy for one relay scenario. To find $P_{div}(e)$, the probability of error for cooperative diversity transmission when relay decodes successfully, we extend (13) for two channels as

$$\bar{P}_{c,k} = \left(\frac{1}{2^m}\right) \sum_{\mathbf{x}\to\hat{\mathbf{x}}^{(k)}} \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{1}{4N_{o,h}} \frac{||\mathbf{x} - \hat{\mathbf{x}}^{(k)}||^2}{\sin^2\theta}\right)^{-1} \times \left(1 + \frac{1}{4N_{o,g}} \frac{||\mathbf{x} - \hat{\mathbf{x}}^{(k)}||^2}{\sin^2\theta}\right)^{-1} d\theta \quad (15)$$

Similarly, the bound on BER for cooperative diversity case is given by

$$P_{div}(e) \le \sum_i \sum_{\mathbf{d}} \frac{i}{k_c} A(i, \mathbf{d}) \bar{P}_{c,1}^{d_1} \bar{P}_{c,2}^{d_2} \ldots \bar{P}_{c,m}^{d_m} \quad (16)$$

Now by substituting (16), (12) and (7) in (6) we can find a closed form expression for the end-to-end error probability of the adaptive DF scheme for Rayleigh fading channels.

## NUMERICAL RESULTS AND DISCUSSIONS

A rate $1/2$ convolutional code with encoder polynomial $[5, 7]$ in octal notation is used. Each input uncoded data block length is 2002, and the interleaver length is 4008. Independent Rayleigh fading channels are assumed between $S$-$R$, $R$-$D$ and $S$-$D$ links. Total power transmitted is kept constant for all scenarios, i.e., for the scenario with $N$ relays, the source and each relay gets a fraction of $1/(N+1)$ of the total power . All the BICM-ID simulation results use $8$ iterations.

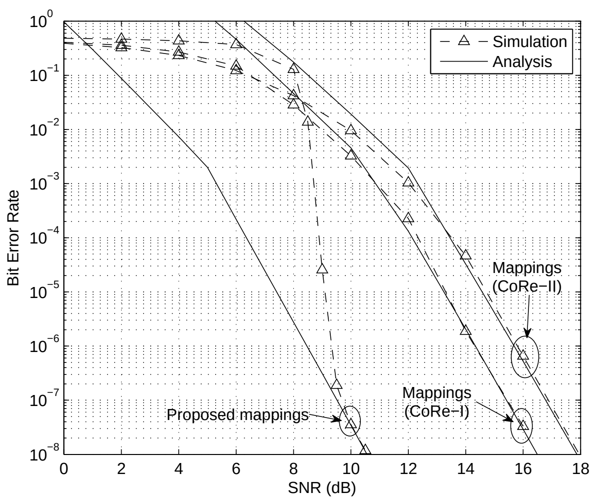


Fig. 5. BER comparison of the existing mappings CoRe-I [14], CoRe-II [15] and the proposed mappings at the source and relay.

Fig. 5 shows the BER performance comparison of adaptive DF scheme using our proposed 16-QAM mappings with existing mappings. More than $5.8$ dB gain in SNR is observed at a BER of about $10^{-8}$ by using proposed mappings over the existing mappings. Note that the analysis results were derived with approximations that are valid only in the high SNR region. Therefore, the analysis and simulation results agree only at high SNR.

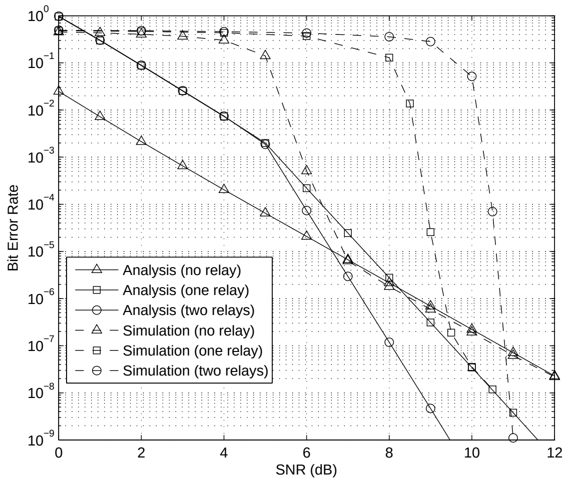


Fig. 6. BER performance with no relay, single relay and two relays.

To see the effect of using relay on the error performance, in Fig. 6, we show the BER performance of adaptive DF scheme without relaying, with one relay and with two relays using optimized mappings. An improvement of more than $1.8$ dB in SNR is observed by using one relay at a BER of about $10^{-8}$, and a further gain in SNR is expected by using a second relay for lower BER values. We found the optimized symbol mapping for second relay by keeping the source and the first relay mappings fixed as obtained from the cost expression (5).

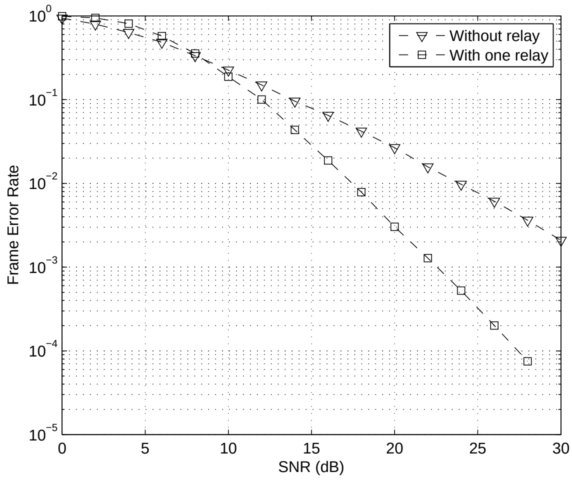


Fig. 7. Error performance of the proposed scheme with and without relay in a block fading channel.

Finally, in Fig. 7 we show the simulation results of our proposed scheme with and without relay in the presence of block fading channel. At an FER value of $10^{-3}$, more than $9$ dB improvement in SNR is observed for single relay network over non-relay network.

## CONCLUSIONS

An adaptive decode and forward cooperative diversity technique using convolutional codes and BICM-ID is investigated. Optimized constellation mappings are designed for the source and the relay. Genetic algorithm is used to find the optimized mappings and a new cost function is derived for the algorithm. A novel error performance analysis for adaptive DF scheme using BICM-ID is proposed. More than $5.8$ dB gain in SNR is observed by using proposed optimized mappings over the existing mappings.

## REFERENCES


[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, 1998.

[3] A. Sendonaris, E. Erkrip, and B. Aazhang, "User cooperation diversity - Part I : System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[4] A. Sendonaris, E. Erkrip, and B. Aazhang, "User cooperation diversity - Part II : Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.

[5] M. O. Hasna and M. S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 1126–1131, Nov. 2003.

[6] N. C. Beaulieu and J. Hu, "A closed-form expression for the outage probability of decode-and-forward relaying in dissimilar Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 10, pp. 813–815, Dec. 2006.

[7] S. S. Ikki, "Performance analysis of adaptive decode-and-forward cooperative diversity networks with best-relay selection," *IEEE Trans. Commun.*, vol. 58, pp. 68–72, Jan. 2010.

[8] E. Zehavi, "8-PSK trellis codes for a Rayleigh channel," *IEEE Trans. Commun.*, vol. 40, pp. 873–884, May 1992.

[9] A. Chindapol and J. A. Ritcey, "Design, analysis and performance evaluation for BICM-ID with square QAM constellations in Rayleigh fading channels," *IEEE J. Select. Areas Commun.*, vol. 19, pp. 944–957, May 2001.

[10] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Trans. Inf. Theory*, vol. 44, pp. 927–946, May 1998.

[11] X. Li, A. Chindapol, and J. A. Ritcey, "Bit-interleaved coded modulation with iterative decoding and 8-PSK signaling," *IEEE Trans. Commun.*, vol. 50, pp. 1250–1257, August 2002.

[12] S. A. K. Tanoli, I. Khan, and N. Rajatheva, "Cooperative networks: Bit-interleaved coded modulation with iterative decoding," *Journal of Commun.*, vol. 5, pp. 562–570, July 2010.

[13] L. J. Rodrguez, N. H. Tran, and T. Le-Ngoc, "Bandwidth-efficient bit-interleaved coded modulation over NAF relay channels: Error performance and precoder design," *IEEE Trans. Veh. Tech.*, vol. 60, pp. 2086–2101, June 2011.

[14] C. Wengerter, A. G. E. Elbwart, E. Seidel, G. Velev, and M. P. Schmitt, "Advanced hybrid ARQ technique employing a signal constellation rearrangement," in *Proc. of IEEE Vehicular Tech. Conf. (VTC)*, vol. 4, pp. 2002–2006, Sept. 2002.

[15] H.-S. Ryu, J.-S. Lee, and C. G. Kang, "BER analysis of constellation rearrangement for cooperative relaying networks over Nakagami-m fading channel," in *Proc. of ICC '11*, pp. 1–5, June 2011.

[16] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "A soft-input-soft-output APP module for iterative decoding of concatenated codes," *IEEE Commun. Lett.*, vol. 1, pp. 22–24, Jan. 1997.

[17] F. Schreckenbach, N. Gortz, J. Hagenauer, and G. Bauch, "Optimization of symbol mappings for bit-interleaved coded modulation with iterative decoding," *IEEE Commun. Lett.*, vol. 7, pp. 593–595, Dec. 2003.

[18] Y. Huang and J. A. Ritcey, "Optimal constellation labeling for iteratively decoded bit-interleaved space-time coded modulation," *IEEE Trans. Inform. Theory*, vol. 51, pp. 1865–1871, May 2005.

[19] M. C. Valenti, R. Doppalapudi, and D. Torrieri, "A genetic algorithm for designing constellations with low error floors," in *Proc. of CISS'08*, (Princeton, NJ), pp. 1155 – 1160, Mar. 2008.

[20] N. Tran and H. Nguyen, "Design and performance of BICM-ID systems with hypercube constellations," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1169–1179, May 2006.

[21] K. Kumar and D. K. Borah, "Hybrid symbols for parallel optical/RF channels using BICM-ID," *Electron. lett.*, vol. 47, pp. 1189–1190, Oct. 2011.