System-Level Algorithm Design for Radionavigation using UWB Waveforms *

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ABSTRACT

A radiolocation/navigation system is considered in which mobile nodes use ultra-wideband (UWB) radios to obtain inter-node ranges via round-trip travel time (RTT). Each node is also assumed to contain an inertial measurement unit (IMU) which generates 2D position estimates subject to Gaussian drift and additive noise errors. The key problem in such a system is obtaining 2 or 3-D position estimates from the nonlinear UWB range measurements and fusing the resulting UWB and IMU estimates. The system presented uses a Steepest Descent Random Start (SDRS) algorithm to solve the nonlinear positioning problem. It is shown that SDRS is a stable algorithim under a realistic communications reciprocity assumption. The SDRS estimates are then treated as measurements by the navigation Kalman filter. The navigation filter also processes separate IMU-derived position estimates to update node position/velocity. Simulation results for an urban corridor are given showing < 6 m. rms position errors.

KEYWORDS

Radiolocation, nonlinear programming, Kalman filtering, navigation.

INTRODUCTION

We develop radionavigation algorithms that fuse UWB range and IMU-derived position estimates. The core of our algorithms is based on maximum-likelihood estimation of quasi-static

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position based on inter-node range measurements. Previous such algorithms include steepest-descent [1], least-squares [2, 3], Semidefinite Programming (SDP) (SDP) and smoothed least-squares [4]. The fundamental problem in range-based localization is the non-convexity of the likelihood function [4]. Conventional steepest descent mimimization fails especially in certain unfavorable node topologies. We thus develop the Steepest Descent Random Start (SDRS) algorithm to handle the non-convex likelihood maximization. The SDRS approach appears easier to implement than SDP and smoothing methods, and avoids the noise enhancement in squared-range approaches.

For dynamic scenarios, it is assumed that the topology is quasi-static during each SDRS position update. The SDRS estimates are then treated as measurements by each node's local Kalman filter. Additionally, each node is assumed to have an Inertial Measurement Unit (IMU) which performs pedestrian dead reckoning [5, 6]. The IMU position estimate is also treated as a measurement with drift modeled as a Gaussian variable. The overall navigation Kalman filter is developed in the sequel. The complete navigation algorithm incorporating the SDRS position estimates, IMU and Kalman filter is then applied to an urban corridor radiolocation scenario and simulation results for position/velocity RMS error are provided.

ALL ANCHORS SCENARIO

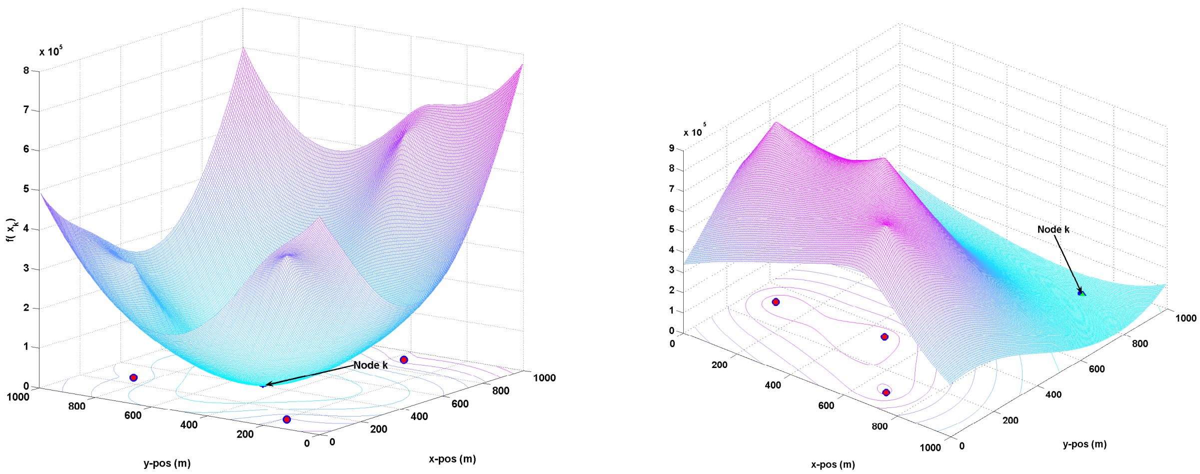
A scenario in which a single static node computes its position using anchor nodes is first considered to motivate the SDRS algorithm. In this "all anchors" scenario, node k obtains round-trip travel time measurements of range [1] given by

$$z_{k,j} = ||\boldsymbol{x}_k - \boldsymbol{x}_j^a|| + v_{k,j}, \forall j \in N_a(k), \tag{1}$$

where $\boldsymbol{x}_k$ is the x-y position of node k. The anchor positions are given by $\boldsymbol{x}_j^a$, and $N_a(k)$ is the set of anchors which are neighbors of k, i.e. from which a round-trip travel time measurement can be obtained. The measurement errors $v_{k,j}$ are approximated as Gaussian here so that the ML position estimate corresponds to the minimum of the objective function

$$f_k(\boldsymbol{x}_k) = \sum_{j \in N_a(k)} \left(z_{k,j} - ||\boldsymbol{x}_k - \boldsymbol{x}_j^a|| \right)^2. \tag{2}$$

The all-anchors objective $f_k(\boldsymbol{x}_k)$ is plotted for two different topologies in Fig. 1. In topology (a), node k is approximately at the center of mass of the triangle formed by the three anchors, and $f_k(\boldsymbol{x}_k)$ has an extensive locally convex region. Hence steepest descent would be expected to converge rapidly to the true position $\boldsymbol{x}_k$ for large initialization errors. However, in topology (b) node k is exterior to the triangle formed by the anchors, and the region of local convexity of $f_k(\boldsymbol{x}_k)$ is much smaller than in (a). Steepest descent would diverge unless initialized close to $\boldsymbol{x}_k$ for this unfavorable topology. Thus we propose the SDRS algorithm for static position estimation using inter-node range measurements.



(a) Favorable (b) Unfavorable

Figure 1: All-anchors objective function for a favorable and unfavorable topology.

STEEPEST DESCENT RANDOM START ALGORITHM

The SDRS algorithm uses multiple initial starting points $\boldsymbol{x}_k^{q,0}$, $q = 1 \ldots N_{start}$ uniformly distributed in the scenario region. N_{start} steepest descent algorithms proceed in parallel, with n_q the iteration number at which convergence is declared from starting point $\boldsymbol{x}_k^{q,0}$. The SDRS estimate corresponds to $\hat{\boldsymbol{x}}_k = \boldsymbol{x}_k^{\hat{q},n_{\hat{q}}}$, where $\hat{q} = \arg\min_q f_k(\boldsymbol{x}_k^{q,n_q})$. SDRS can be interpreted as a lightweight particle filter, in which the random start positions $\boldsymbol{x}_k^{q,0}$ correspond to an initial choice of particles, and with a single particle $\hat{\boldsymbol{x}}_k$ surviving at the end of the algorithm.

The SDRS description using the Armijo rule is given in Algorithm 1 for the all-anchors case. The extension to mobile nodes plus anchors is straightforward.

for $q = 1, \ldots, N_{start}$ **do**
- $\boldsymbol{x}_k^{q,0} = \text{uniform}(S)$;
- **while** $||\boldsymbol{x}_k^{q,n+1} - \boldsymbol{x}_k^{q,n}|| > tol$ **do**
  - $\alpha \in [0,1]$;
  - $\nabla f_k(\boldsymbol{x}_k^{q,n}) = \sum_{j \in N_a(k)} \frac{(||\boldsymbol{x}_k^{q,n} - \boldsymbol{x}_j^a|| - z_{k,j})}{||\boldsymbol{x}_k^{q,n} - \boldsymbol{x}_j^a||} \left(\boldsymbol{x}_k^{q,n} - \boldsymbol{x}_j^a\right)$;
  - **while** $f_k(\boldsymbol{x}_k^{q,n+1}) > f_k(\boldsymbol{x}_k^{q,n}) - \sigma\alpha||\nabla f_k(\boldsymbol{x}_k^{q,n})||^2$ **do**
    - $\boldsymbol{x}_k^{q,n+1} = \boldsymbol{x}_k^{q,n} - \alpha\nabla f_k(\boldsymbol{x}_k^{q,n})$;
    - $\alpha = \alpha/2,$;
  - $n_q = n + 1$;

$\hat{q} = \arg\min_q f_k(\boldsymbol{x}_k^{q,n_q})$;

$\hat{\boldsymbol{x}}_k = \boldsymbol{x}_k^{\hat{q},n_{\hat{q}}}$;

Algorithm 1: SDRS algorithm for all-anchors quasi-static positioning

The SDRS algorithm is next incorporated into a Gauss-Seidel type algorithm for cooperative radiolocation where ranges are measured to both mobile and anchor nodes. Let $j \in N(k)$ be the mobile nodes which are neighbors of k, and for which round-trip travel time measurements $z_{k,j}$ are available at node k. We now introduce a reciprocity assumption which guarantees stability of a nonlinear Gauss-Seidel cooperative radiolocation algorithm.

Assumption 1. *Reciprocity: Assume that $j \in N(k)$, that is node k computes a round-trip travel time measurement $z_{k,j}$. Then node $k \in N(j)$, that is node j also computes a measurement $z_{j,k}$. Futhermore, node k receives measurement $z_{j,k}$ from node j as well as its current position estimate $\boldsymbol{x}_j^n$. Thus node k receives* all *measurements $\{z_{k,j}, z_{j,k} : j \in N(k)\}$ that depend on its position $\boldsymbol{x}_k$.*

Assumption 1 is reasonable when the physical layer communications channel is reciprocal between nodes. Define the concatenated vector of node positions as $\boldsymbol{x} = [\boldsymbol{x}_1^T \dots \boldsymbol{x}_N^T]^T$, with N the total number of mobiles. The global objective function $f_g(\boldsymbol{x}) = f_g(\boldsymbol{x}_1^n \dots \boldsymbol{x}_N^n)$ can then be partitioned as follows under Assumption 1, where $\boldsymbol{x}_{(k)}$ is the vector $\boldsymbol{x}$ with $\boldsymbol{x}_k$ deleted. Also define $\boldsymbol{x}_{N(k)}$ as the vector of concatenated positions of nodes which are neighbors of k.

$$
\begin{aligned}
& f_g(\boldsymbol{x}) = \sum_{k=1}^{N} \left(\sum_{j \in N(k)} (z_{k,j} - ||\boldsymbol{x}_k - \boldsymbol{x}_j||)^2 + \sum_{j=1}^{N_a} (z_{k,j}^a - ||\boldsymbol{x}_k - \boldsymbol{x}_j^a||)^2 \right) \\
& = \sum_{j \in N(k)} (z_{k,j} - ||\boldsymbol{x}_k - \boldsymbol{x}_j||)^2 + (z_{j,k} - ||\boldsymbol{x}_j - \boldsymbol{x}_k||)^2 + \sum_{j \in N_a(k)} \left(z_{k,j}^a - ||\boldsymbol{x}_k - \boldsymbol{x}_j^a|| \right)^2 + f_{(k)}(\boldsymbol{x}_{(k)}) \\
& = f_k(\boldsymbol{x}_k, \boldsymbol{x}_{N(k)}) + f_{(k)}(\boldsymbol{x}_{(k)}).
\end{aligned} \tag{3}
$$

Under reciprocity, observe that the component of the objective function dependent on $\boldsymbol{x}_k$, $f_k(\boldsymbol{x}_k, \boldsymbol{x}_{N(k)})$, as well as its gradient can be computed locally at node k

The nonlinear Gauss-Seidel distributed algorithm under the reciprocity assumption is given in Algorithm 2.

```
for node k = 1, 2, ..., N do
    for all j ∈ N(k) do
        Compute z̄_{k,j} = (z_{k,j} + z_{j,k})/2
for n = 1, ..., N_f do
    for node k = 1, 2, ..., N do
        Approximate using SDRS ;
        x_k^{n+1} ≈
        argmin_{x_k} Σ_{j∈N(k),j<k} (z̄_{k,j} − ||x_k − x_j^{n+1}||)^2 + Σ_{j∈N(k),j>k} (z̄_{k,j} − ||x_k − x_j^n||)^2
        + Σ_{j∈N_a(k)} (z^a_{k,j} − ||x_k − x^a_j||)^2 ;
        ≈ argmin_{x_k} f_k(x_k, x^{n,n+1}_{N(k)}) ;
```

Algorithm 2: Nonlinear Gauss-Seidel algorithm for cooperative radiocation under reciprocity

The following proposition characterizes the stability of the nonlinear Gauss-Seidel algorithm under the reciprocity assumption.

Proposition 1. *Assume that reciprocity 1 holds. Further assume that SDRS is employed with a test on the objective such that at each iteration* $f_k(\boldsymbol{x}_k^{n+1}, \boldsymbol{x}_{N(k)}^{n,n+1}) \leq f_k(\boldsymbol{x}_k^n, \boldsymbol{x}_{N(k)}^{n,n+1})$*. Then the global objective function* $f_g(\boldsymbol{x}^n)$ *is non-increasing and attains a lower limit* f_g^**.*

Proof: Follows directly from the partitioned form of the objective in eq. (3).

NAVIGATION ALGORITHM

The navigation algorithm implemented in each node is shown in Fig. 2. It is assumed that IMU position estimates are obtained on even time samples and radiolocation/SDRS position estimates are computed on odd samples. These estimates are treated by measurements in the local Kalman filter.

The specific blocks in Fig. 2 perform the following functions.

1. Ranging/TOA estimation: This block includes an UWB transceiver that performs round-trip ranging generating measurements $z_{k,j}$.
2. SDRS positioning: This block performs the minimization of the local objective $f_k(\boldsymbol{x}_k, \boldsymbol{x}_{N(k)})$ in Algorithm 1. The local updated estimate on each SDRS iteration $\boldsymbol{x}_k^{n+1}$ is broadcast to neighbor nodes $N(k)$. The SDRS block completes N_f iterations and the final estimate $\boldsymbol{x}_k^{N_f}$ becomes the new radiolocation measurement of position, $\boldsymbol{z}_{k,RL}(2n+1)$.
3. Navigation Kalman filter: As described below, this filter treats the SDRS estimates and IMU position estimates as linear Gaussian measurements to update the final position estimate $\hat{\boldsymbol{x}}_k(n|n)$.
4. IMU and IMU/PDR filter: These blocks use a local IMU and a separate EKF as in [6] to generate a position estimate at even time instants $2n$. These IMU-derived estimates are treated as linear Gaussian measurements of position by the local Navigation Kalman filter.
5. Network Radio: This transmits the SDRS updated estimates $\hat{\boldsymbol{x}}_k^{n+1}$ and Kalman filter position estimates $\hat{\boldsymbol{x}}_k(n|n)$ to neighbor nodes in $N(k)$. This radio also transmits measurements $\{z_{k,j}\}$ to and receives the measurements $\{z_{j,k}\}$ from neighbors to satisfy the reciprocity assumption.

The node state is defined by

$$\boldsymbol{x}_k(n) = [x_k(n), \dot{x}_k(n), y_k(n), \dot{x}_k(n), u_{x,k}, u_{y,k}]^T. \quad (4)$$

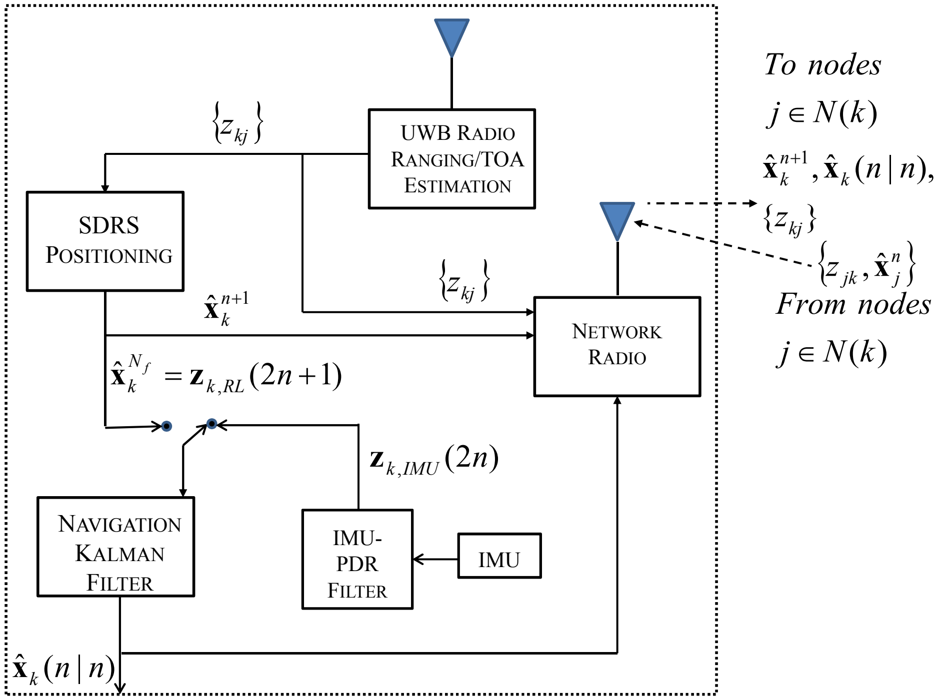


Figure 2: Navigation algorithm.

The quantities $x_k(n), \dot{x}_k(n)$ are position and velocity in a zero-mean Gaussian acceleration model. Node k's IMU x-y drift parameters $u_{x,k}, u_{y,k}$ are modeled as static Gaussian random variables. Following [6], it is assumed that the IMU position error increases linearly with time proportional to $u_{x,k}$ as described in the measurement model eq. (8). It is emphasized that the IMU drift parameters are jointly estimated along with node position/velocity by the Kalman filter.

The process model then has the standard form

$$\boldsymbol{x}_k(n+1) = \boldsymbol{F}\boldsymbol{x}_k(n) + \boldsymbol{G}\mathbf{w}_k(n), \tag{5}$$

where

$$\boldsymbol{F} = \begin{bmatrix} 1 & T & 0 & \dots & & 0 \\ 0 & 1 & 0 & \dots & & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & & & 1 & 0 \\ 0 & 0 & \dots & & 0 & 1 \end{bmatrix}, \quad \boldsymbol{G}^T = \begin{bmatrix} T^2/2 & T & 0 & \dots & & 0 \\ 0 & 0 & T^2/2 & T & 0 & 0 \end{bmatrix} \tag{6}$$

The sample time is denoted by T sec. The acceleration vector is an i.i.d. Gaussian sequence, $\boldsymbol{w}_k(n) \sim \mathcal{N}(\mathbf{0}, (\sigma_w^2/2)\boldsymbol{I}_2)$, where σ_w is acceleration standard deviation in m/s^2.

The measurement models assumed by the Navigation Kalman filter are as follows. The

SDRS-derived position estimes $\hat{\boldsymbol{x}}_k^{N_f}$ are modeled by

$$\begin{aligned} \boldsymbol{z}_{k,RL}(2n+1) &= \hat{\boldsymbol{x}}_k^{N_f} = \boldsymbol{H}_k(2n+1)\boldsymbol{x}_k(2n+1) + \boldsymbol{v}_k(2n+1) \\ \boldsymbol{H}_k(2n+1) &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{7}$$

The SDRS position estimation error is taken to be Gaussian, $\boldsymbol{v}_k(2n+1) \sim \mathcal{N}(\mathbf{0}, \sigma^2_{LOS,(NLOS)}\boldsymbol{I})$. Note that different SDRS position error variances, $\sigma^2_{LOS,(NLOS)}$ can be employed if the node is known to be in a LOS or NLOS environment.

The IMU position measurements are modeled by

$$\begin{aligned} \boldsymbol{z}_{k,IMU}(2n) &= \boldsymbol{H}_k(2n)\boldsymbol{x}_k(2n) + \boldsymbol{v}_k(2n) \\ \boldsymbol{H}_k(2n) &= \begin{bmatrix} 1 & 0 & 0 & 0 & (.05)2nT & 0 \\ 0 & 0 & 1 & 0 & 0 & (.05)2nT \end{bmatrix}. \end{aligned} \tag{8}$$

Eq. (8) corresponds to the linear drift observed for IMU position estimates in [6]. For example, the x position IMU measurement in the absence of additive noise is $x_k(2n) + (.05)2nTu_{x,k}$. With $u_{x,k}$ a Gaussian r.v. with variance $1/\sqrt{2}$, and a nominal velocity of 1 m/s, this corresponds to an IMU position error with standard deviation of 5 % of total distance traveled. An additional additive IMU noise is given by the i.i.d. Gaussian sequence $\boldsymbol{v}_k(2n) \sim \mathcal{N}(\mathbf{0}, \sigma^2_{IMU}\boldsymbol{I})$.

The Kalman filter equations are readily obtained in standard form given the process and measurement definitions in eqs. (5)-(8).

URBAN CORRIDOR SIMULATIONS

The navigation algorithm in Fig. 2 was simulated in an urban corridor. As shown in Fig. 3 the street is 15 m wide with buildings 15 m wide on the north and south sides. The simulation parameters used are given in Table 1. Five anchor nodes are placed on the west end of the street, and 10 mobile nodes move according to the Gaussian acceleration model in (5) at a nominal velocity of 1 m/s heading east. The simulation duration is 10 minutes. Range measurements $z_{k,j}$ are assumed to be LOS when both nodes k, j are in the street, otherwise the measurements are modeled as NLOS. The LOS and NLOS range measurement RMS errors are based on [[7] Fig. 9]. The sample time of $T = 5$ sec. with alternating IMU and SDRS position estimate updates implies that a new SDRS estimate $\hat{\boldsymbol{x}}_k^{N_f}$ is computed every 10 sec., and a new IMU estimate updated also every 10 sec.

A total of 16 simulation runs with independent range errors, IMU drifts and Gaussian accelerations were carried out. Snapshots of the node positions and errors are shown in Fig. 3 for a favorable run. The error circles plotted have radii at sample n given by $||\boldsymbol{x}_k(n) - \hat{\boldsymbol{x}}_k(n|n)||$ where $\hat{\boldsymbol{x}}_k(n|n)$ is the Kalman filter measurement update. As expected, nodes that remain in the street have relatively small error radii of 5 m or less, whereas nodes inside buildings

Mobile-to-mobile radio range	30 m
Anchor-to-mobile radio range	100 m
LOS range measurement RMS error	1.5 m
NLOS range measurement RMS error	9 m
Initial mobile x-velocity (East)	1 m/s
Mobile node RMS acceleration (Gaussian)	.001 m/s^2
IMU drift	5% of total distance travelled
Sample time T	5 sec.

Table 1: Urban corridor simulation parameters

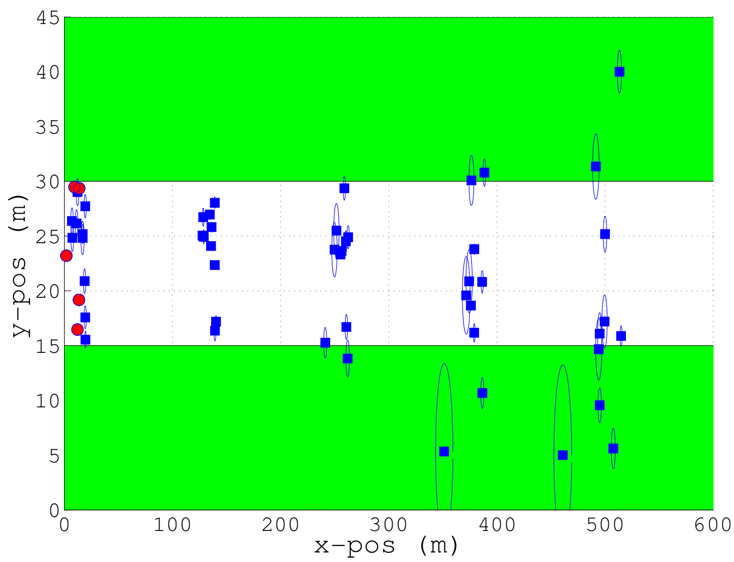


Figure 3: Snapshots of node positions and estimation errors over 10 minutes – favorable run.

develop large errors due to the NLOS range measurement variance. Note that the anchor-to-mobile radio range is assumed to be 100 m. In all simulations, position errors were relatively small up to this 100 m distance from the anchors due to the benefit of known anchor position and additional anchor range measurements. Position errors can rapidly increase after 100 m even with cooperative radiolocation, due to the lack of absolute position information from anchors.

An example of an unfavorable simulation is given in Fig. 4. In this case all nodes experience large position errors when they are outside the anchor radio range. Again, the lack of absolute position estimation when more than 100 m from anchors leads to an overall instability that must be addressed.

The RMS position, velocity, and IMU drift parameter $u_{x,k}, u_{y,k}$ estimation errors were com-

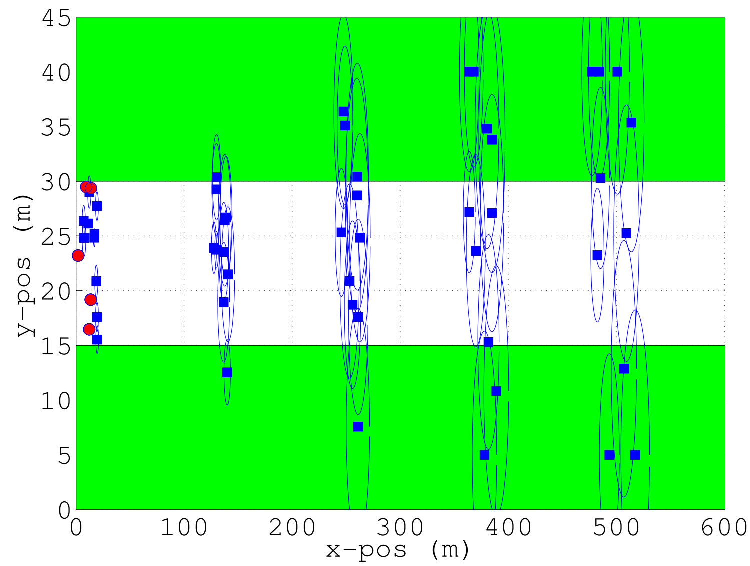


Figure 4: Snapshots of node positions and estimation errors over 10 minutes – unfavorable run.

puted versus time and averaged over the 10 mobile nodes and 16 simulation runs. These RMS average errors are plotted in Fig. 5. Although instability is observed in some individual simulation runs, it is seen that the RMS position error is 6 m after 10 minutes, which is better than commercial GPS accuracy. It is also interesting that the IMU drift estimation error is approximately .2 after 10 minutes, which compares favorably with the drift parameter RMS value of unity.

CONCLUSIONS

A cooperative navigation algorithm using inter-node range measurements and internal IMU position estimation was developed. A key feature is the Steepest Descent Random Start algorithm for quasi-static positioning using round-trip travel time measurements. A stability and convergence result for a nonlinear Gauss-Seidel algorithm employing SDRS was obtained. Initial simulation results given here demonstrate some overall system instability in the urban corridor scenario. However, the ensemble and node-averaged RMS position error was 6 m at the end of a 10 minute simulation, which compares favorably with commercial GPS accuracy.

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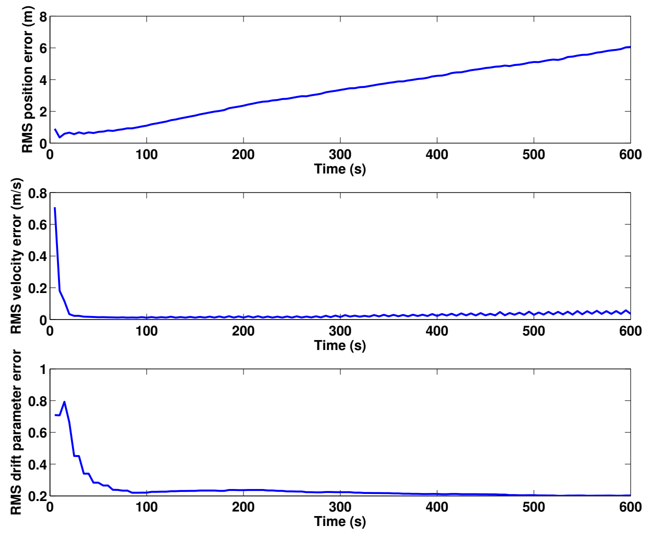

Figure 5: RMS position, velocity and IMU drift parameter estimation errors averaged over mobile nodes and 16 runs.

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