NOVEL BICM HARQ ALGORITHM BASED ON ADAPTIVE MODULATIONS

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ABSTRACT

A novel type-II hybrid automatic repeat request (HARQ) algorithm using adaptive modulations and bit-interleaved coded modulation (BICM) is presented. The algorithm uses different optimized puncturing patterns for different transmissions of the same data packet. The proposed approach exploits mapping diversity through BICM with iterative decoding. The modulation order is changed in each transmission to keep the number of symbols transmitted constant. We present new bit error rate and frame error rate analytical results for the proposed technique showing good agreement with simulation results. We compare the throughput performance of our proposed HARQ technique with a reference HARQ technique that uses different mapping arrangements but keeps the modulation order fixed. By using optimized puncturing patterns and adaptive modulations, our method provides significantly better throughput performance over the reference HARQ method in the whole signal-to-noise ratio (SNR) range, and achieves a gain of 12 dB in the medium SNR region.

KEY WORDS

Automatic Repeat Request, Bit-Interleaved Coded Modulation, Iterative Decoding, Packet combining.

INTRODUCTION

Hybrid automatic repeat request (HARQ) techniques are used in wireless communication systems to achieve reliable communication links by combining automatic repeat request (ARQ) with forward error correcting (FEC) codes [1]. In a conventional ARQ technique, when an uncoded data packet is received in error, the receiver simply discards the packet and sends a negative-acknowledgment (NACK) signal to the transmitter requesting a retransmission. This process is repeated until the frame is detected free of errors. In contrast, in HARQ algorithms, the transmitter encodes data using an FEC code so that the receiver can correct many of the possible errors, and the receiver requests retransmission only if the FEC fails to correct all the errors in the data packet. HARQ technique has been used in latest-generation wireless systems such as IEEE 802.16e and third generation partnership project - long term evaluation (3GPP-LTE) [2]. There are two main HARQ schemes, classified as type-I and type-II [1], [3]. In a type-I HARQ scheme, the receiver sends an acknowledgment (ACK) signal to the transmitter if the data packet is decoded correctly. Otherwise, the receiver discards the received data packet and asks for retransmission by sending a NACK signal. In the type-II HARQ technique, if the decoded data packet is in error, the receiver asks for retransmission, but keeps the previously received erroneous data packets. During subsequent retransmissions, the receiver jointly decodes the received data packet with the erroneously received data packets from previous transmissions. This process is repeated until the data packet is received correctly or the retransmission reaches a maximum number, in which case the data packet is considered lost [4]. In addition to type-I and type-II HARQ techniques, there is also a less known third category of HARQ technique, called the type-III HARQ. In fact, type-III HARQ can be considered as a special case of type-II HARQ with the additional property that each transmission is self-decodable [5].

Both type-I and type-II HARQ techniques have been studied extensively. The error performance of type-I HARQ scheme with adaptive code rate is studied in [6]. A type-I HARQ technique has also been considered in [7], where the importance of combining ARQ with FEC is discussed. A significant improvement in throughput in the low signal-to-noise ratio (SNR) region of type-I HARQ scheme over the traditional FEC and ARQ schemes is shown. The works in [1] and [8] consider a type-II HARQ technique using convolutional codes and different puncturing patterns for each transmission. Similarly, a HARQ technique using turbo codes has been investigated in [9], where parallel concatenated convolutional code with two recursive systematic convolutional encoders is used. In the first transmission, only systematic and the first parity bits are sent and the received bits are decoded using a convolutional decoder at the receiver. In subsequent retransmissions, the parity bits from the second encoder, the systematic bits and parity bits from first encoder are sent consecutively, performing turbo decoding at the receiver. Type-II HARQ technique using turbo codes is also studied in [10], where decoded data packets are split into segments, and the segments with low averaged log-likelihood ratio (LLR) are identified and asked for retransmission. Consequently, in this approach, an erroneous decoded data packet not only generates a NACK signal, but also a signal identifying the segment to be retransmitted.

Both conventional ARQ and HARQ techniques produce delay due to data packet retransmission. One way to reduce the delay and hence to improve the throughput is to use higher order modulations. However, higher order modulations using the classical trellis coded modulation (TCM) with symbol level interleaving do not provide good performance in the presence of fading. Instead, bit-interleaved coded modulation (BICM) is found to outperform TCM that uses symbol interleavers [11], [12]. In BICM, data after an FEC encoder are interleaved using a bit interleaver to extract bit level diversity of the fading channel. The interleaved data is modulated and transmitted. At the receiver, the demodulator calculates soft information about the coded bits from the received symbols. The soft information is propagated between the demodulator and the decoder to decode the data. The information rate and error probability analysis of BICM was illustrated in [13]. Later, Li and Ritcey [14] proposed the BICM with iterative decoding (BICM-ID) which gives significant error performance improvement over non-iterative BICM decoding. BICM is found in many of the wireless standards, such as high-speed packet access (HSPA), IEEE 802.11 a/g, IEEE 802.16 and high-speed downlink packet access (HSDPA).

Several authors have studied HARQ algorithms with higher order modulations. A type-II HARQ technique with higher order modulation and mapping rearrangement without using BICM has been studied in [15]. The work in [16] considers a type-III HARQ algorithm using turbo code and BICM with mapping diversity. It uses different symbol constellations for subsequent retransmissions, but keeps the same modulation order. Effect of changing the modulation order in retransmission of data packets in HARQ technique without using BICM is studied in [17].

The goal of this paper is to develop a novel BICM HARQ algorithm. In our algorithm, we propose to employ optimized puncturing patterns for each data packet transmission and then use changing modulation order to improve the overall throughput of a type-II HARQ algorithm. The changes in the puncturing patterns and the modulation order are carried out in such a way that the number of transmitted symbols is maintained constant in each transmission. We also develop new bit error rate (BER) and frame error rate (FER) performance analysis results for our technique showing good agreement with simulation results. Finally, we compare the throughput of our proposed technique with a fixed modulation BICM HARQ reference technique, which changes the bit-to-symbol mapping during retransmissions. Our method provides significant throughput improvement over the reference technique and achieves a gain of about 12 dB in the medium SNR region.

SYSTEM DESCRIPTION

The block diagram of our proposed HARQ BICM system is shown in Fig.1. Each group of K information bits is encoded using a cyclic redundancy check (CRC) code. The CRC adds q additional bits so that each group of $(K+q)$ bits is next encoded using a convolutional code of rate R and constraint length L, and is stored in the transmitter's buffer. The encoded bits are next punctured, interleaved, and mapped into complex symbols. In our method, each transmission uses a separate optimized puncturing pattern and a separate pseudo-random bit interleaver [14]. During the i-th transmission of a data packet, each group of m_i consecutive bits of interleaved data are mapped into a complex transmitted signal $x_i \in \chi_i$ via a memoryless mapper, where χ_i is the energy normalized 2^{m_i}- ary constellation, and I is the maximum number of allowed transmissions for a given packet. We consider 16-QAM, 8-PSK and QPSK modulations for the first, second and third transmissions of the data packet respectively. The constellation diagrams for these modulations are shown in Fig. 2.

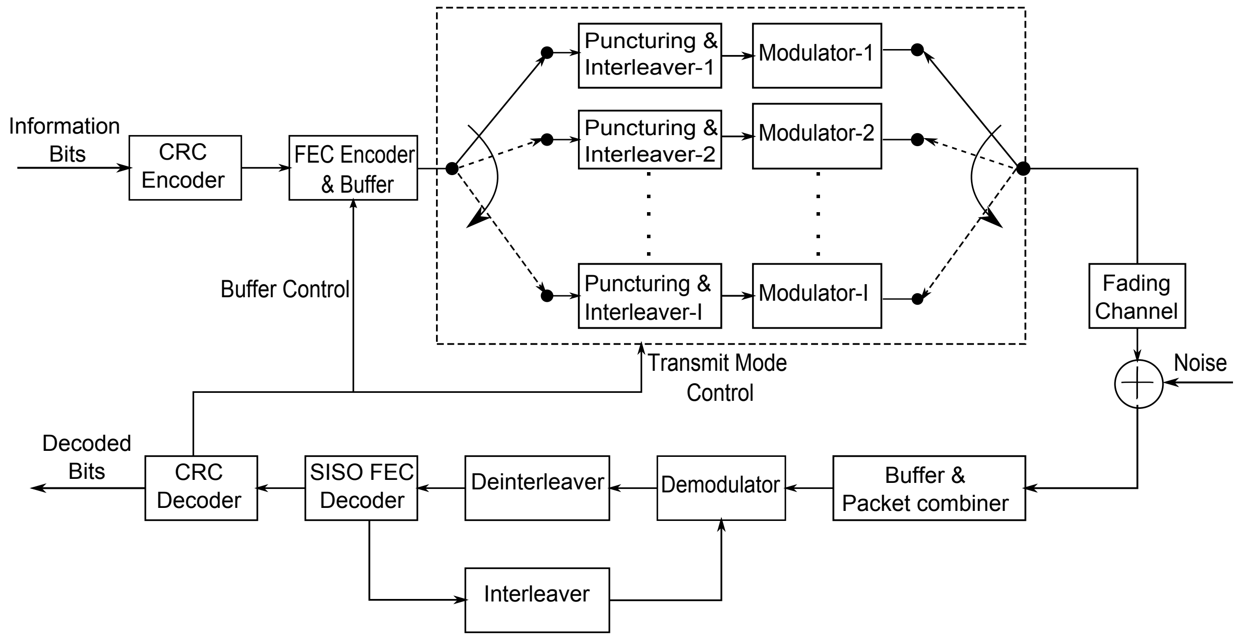

Fig. 1. Block diagram of the proposed HARQ system.

The received signal due to transmitted signal x is modeled as

$$y = hx + \eta \tag{1}$$

where h is complex channel gain with zero mean and normalized power $E\{|h|^2\} = 1$, η is additive white Gaussian noise (AWGN) with variance σ^2, and $\sigma^2 = N_o/2$ is also the two sided noise power spectral density. The received signal is then demodulated and decoded using BICM-ID [11], [14]. At the demodulator, the most likely information about the bits corresponding to the received complex symbols is calculated [13]. The demodulator uses posterior probability given by

$$P(b_k = b|y) = \sum_{x \in \chi_b^k} p(y|x)P(x) \tag{2}$$

where b_k is the k-th labeled bit of the symbol, $b \in \{0, 1\}$, χ_b^k is the set of complex symbols with the k-th labeled bit being b, and $P(x)$ is the a *priori* symbol probability. The logarithm of the conditional probability density function $p(y|x)$ is given by

$$\log p(y|x) = \log \left(\frac{1}{\pi N_o} \right) - \frac{1}{N_o} ||y - hx||^2 \tag{3}$$

After deinterleaving, this information is passed through the soft-input-soft-output (SISO) decoder which uses an *additive log-map* algorithm [14]. Soft information values about the most likely bits at the input of the encoder and the corresponding coded bits are generated. This information is exchanged between the demodulator and the decoder until the incremental growth of the extrinsic information is negligible and then the final decision is made for the decoded bits.

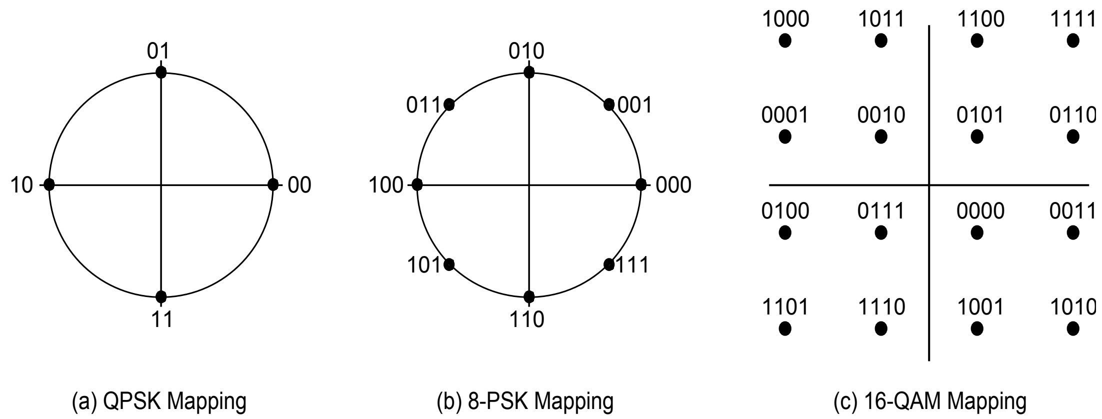

Fig. 2. Constellation diagram of anti-Gray mapping for QPSK, semi-set partitioning (SSP) mapping for 8-PSK [14] and modified-set partitioning (MSP) mapping [11] for 16-QAM modulation

PROPOSED HARQ ALGORITHM

Our proposed HARQ algorithm differs from existing algorithms. In our method, we change the modulation order for different retransmissions of a data packet. Although [16] uses BICM-ID, it does not change the modulation order in packet retransmissions. In [17], different modulation orders are used for different transmissions of the same data packet, but BICM-ID is not considered. So our novel method integrates BICM-ID with changing modulation order to give better matching to the channel conditions. Besides, we use different optimized puncturing patterns for each transmission to maintain the number of symbols constant during each transmission of a given data packet.

For every U input bits to the channel encoder, the output bits are punctured using the optimized puncturing pattern shown in Table I. An example of puncturing pattern from Table I is shown in Fig. 3, where 'X' denotes a punctured output bit, 'O' denotes an unpunctured output bit, and U denotes the puncturing period or window. Let H_i be the number of coded bits transmitted per puncturing window in the i-th transmission and $\rho_i = U/H_i$ be the code rate during that transmission. The effective code rate is given by $R_i = U/(\sum_{n=1}^{i}(H_n)$, and the spectral efficiency is defined as $C_i = \rho_i m_i$ [18]. To keep the number of symbols constant for each transmission, the spectral efficiency also has to be kept constant. We select the values of ρ_i and m_i such that $C_1 = C_2 = ... = C_I$. In the first transmission of a data packet, we choose a high code rate with high order modulation. The idea is to get the highest data transfer possible right in the first transmission. If retransmissions are required (which indicates that the channel is weaker), both the number of retransmitted bits and the modulation order are reduced to take care of the weaker channel conditions. In this paper, for the first transmission, we always consider $H_1 = U + 1$. The proposed algorithm is summarized as follows:

1) For all the transmissions i, $1 \leq i \leq I$, choose a modulation index set $Z = \{m_1, m_2, ..., m_I\}$, with $m_1 \geq m_2 \geq ... \geq m_I$.

2) Find ρ_i for each transmission of a given data packet, keeping the number of symbols in each transmission constant. Equivalently, keep the spectral efficiency constant i.e., $\rho_1 m_1 = \rho_2 m_2 = ... = \rho_I m_I$.

3) In the first transmission, find the coefficient $A(w, d_1)$ of the convolutional encoder corresponding to a code of rate ρ_1 as described in [19], [20], where $A(w, d_1)$ is the number of codewords with input weight w and output weight d_1 corresponding to the first transmission. Compute the BER using (9), given in the next section, for all possible puncturing patterns with code rate ρ_1. Select the combination with the lowest BER, which corresponds to the optimized puncturing pattern for the first transmission.

4) For the i-th transmission, with $i > 1$, find the coefficients $A(w, d_1, d_2, ..., d_i)$ of the convolutional encoder corresponding to a code rate R_i keeping $A(w, d_1, d_2, ..., d_{i-1})$ fixed from previous transmissions. Compute the BER (9) using the coefficients $A(w, d_1, d_2, ..., d_i)$ for all possible puncturing pattern combinations with code rate R_i. Select the one with the lowest BER, which is the optimized puncturing pattern for the i-th transmission.

The obtained puncturing patterns are similar to those found in [21], and are given in Table I for two different convolutional codes with polynomial generators $[5, 7]$ and $[15, 17]$, given in octal notation, using $U = 7$, $m_1 = 4$, $m_2 = 3$ and $m_3 = 2$.

ERROR ANALYSIS OF THE PROPOSED ALGORITHM

We now present an asymptotic error analysis of our scheme based on the concept of error-free feedback [11], [13]. Let $\underline{\mathrm{X}}$ be the true and $\hat{\underline{\mathrm{X}}}$ be the erroneous symbol sequences with a Hamming

TABLE I

OPTIMIZED PUNCTURING PATTERNS IN OCTAL NOTATION.

i	R_i	Encoder Polynomials	Puncturing Pattern
1	7/8	[5, 7] [15, 17]	[137 140] [102 175]
2	1/2	[5, 7] [15, 17]	[040 037] [075 002]
3	7/18	[5, 7] [15, 17]	[000 053] [074 000]

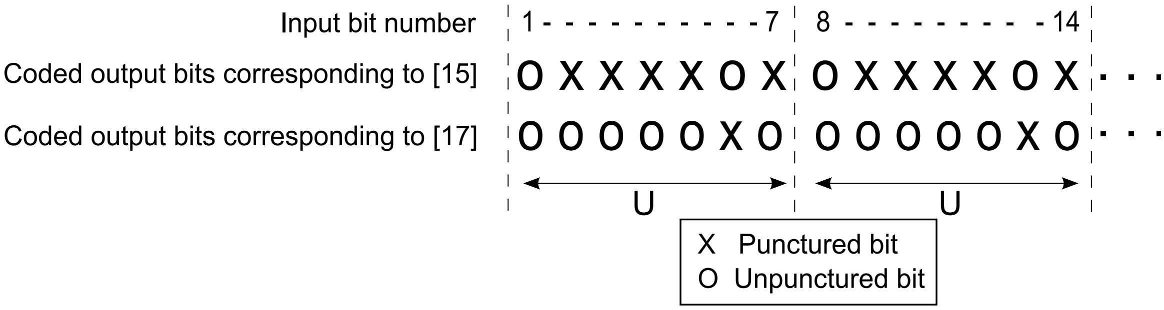

Fig. 3. Puncturing pattern in the first transmission for [15,17] code with $U = 7$.

distance d between the true and the erroneous codeword sequences. Let $\underline{\mathrm{h}}$=$[h_1, h_2, ..., h_d]$ denote the complex channel values corresponding to the d symbols. Then the conditional pairwise error probability (PEP) between the true and the erroneous symbol sequences given $\underline{\mathrm{h}}$ is [22]

$$P(\underline{\mathrm{X}} \to \hat{\underline{\mathrm{X}}}|\underline{\mathrm{h}}) = Q\left(\sqrt{\frac{1}{2N_o}\sum_{l=1}^{d}|h_l|^2||\mathbf{X}_l - \hat{\mathbf{X}}_l||^2}\right) \tag{4}$$

where $\mathbf{X}_l$ and $\hat{\mathbf{X}}_l$ are the true and erroneous symbols at the l-th position of the true and erroneous symbol sequences respectively, and $Q(\gamma) = 1/\sqrt{2}\pi\int_{\gamma}^{\infty}\exp(-y^2/2)dy$ is the Gaussian tail probability. For an ideal interleaver, h_l with $1 \leqslant l \leqslant d$, can be assumed to be independent and identically distributed (i.i.d) random variables [13]. To further simplify the PEP expression for Rayleigh fading channels, we use the Gaussian probability integral given as

$$Q(\tau) = \frac{1}{\pi}\int_{0}^{\pi/2}\exp(-\tau^2/2\sin^2\theta)d\theta \tag{5}$$

As discussed in [22], we can average (4) over the i.i.d. Rayleigh random variable sequence $\underline{\mathrm{h}}$ by using (5) as follows

$$P(\underline{\mathrm{X}} \to \hat{\underline{\mathrm{X}}}) = E_{\underline{\mathrm{h}}}[P(\underline{\mathrm{X}} \to \hat{\underline{\mathrm{X}}}|\underline{\mathrm{h}})] = \frac{1}{\pi}\int_{0}^{\pi/2}\left[\prod_{l=1}^{d}\left(1 + \frac{1}{4N_o}\frac{||\mathbf{X}_l - \hat{\mathbf{X}}_l||^2}{\sin^2\theta}\right)^{-1}\right]d\theta \tag{6}$$

where $E_{\underline{\mathrm{h}}}[.]$ denotes averaging over the vector $\underline{\mathrm{h}}$. Averaging over only one channel coefficient h of the vector $\underline{\mathrm{h}}$, we get $E_h[\exp(-\tau h^2)]$=$1/(1 + \tau)$. To find the average PEP, we assume that the symbols in $\underline{X}$ and $\hat{\underline{X}}$ differ in only one bit. Also, the average pairwise probability depends upon the Hamming distance, mapping and constellation of the symbols. For the i-th transmission, let $\mathbf{X}^{(i)}$ denote the symbols in 2^{m_i}-ary constellation χ_i and let $\hat{\mathbf{X}}^{(i)}$ denote the corresponding symbols which differ in one bit position. Taking the average over all the symbols, the average PEP is

$$\bar{P}_i \leqslant \frac{1}{\pi} \int_0^{\pi/2} \left[E\left\{ \left(1 + \frac{1}{4N_{o,i}} \frac{||\mathbf{X}^{(i)} - \hat{\mathbf{X}}^{(i)}||^2}{\sin^2\theta} \right)^{-1} \right\} \right]^{d_i} d\theta \tag{7}$$

where

$$E\left\{ \left(1 + \frac{1}{4N_{o,i}} \frac{||\mathbf{X}^{(i)} - \hat{\mathbf{X}}^{(i)}||^2}{\sin^2\theta} \right)^{-1} \right\} = \frac{1}{m_i 2^{m_i}} \sum_{\mathbf{X}^{(i)} \in \chi_i} \left[\left(1 + \frac{1}{4N_{o,i}} \frac{||\mathbf{X}^{(i)} - \hat{\mathbf{X}}^{(i)}||^2}{\sin^2\theta} \right)^{-1} \right] \tag{8}$$

Please note that in our proposed HARQ BICM method, the modulation order changes in the subsequent transmissions of a data packet, and hence the SNR and the output weight d_i of the convolutional code also change. The bound on BER for HARQ BICM-ID with a maximum transmissions of I for any data packet is obtained as

$$P_b \leqslant \sum_w \sum_{d_1} \sum_{d_2} .... \sum_{d_I} \left(\frac{w}{K} \right) \times A(w, d_1, d_2, ..., d_I) \times \bar{P}_1 \times \bar{P}_2 ..... \times \bar{P}_I \tag{9}$$

where $A(w, d_1, d_2, ..., d_I)$ is the number of codewords with input weight w and output weights $d_1, d_2, ..., d_I$ corresponding to $1, 2, ..., I$-th transmissions respectively. With no retransmission, i.e., for $I = 1$, the BER expression for HARQ BICM-ID reduces to the simple BER expression for BICM-ID. Similarly, the bound on FER is given by

$$FER \leqslant \sum_w \sum_{d_1} \sum_{d_2} .... \sum_{d_I} A(w, d_1, d_2, ..., d_I) \times \bar{P}_1 \times \bar{P}_2 ..... \times \bar{P}_I \tag{10}$$

NUMERICAL RESULTS AND DISCUSSIONS

A rate $1/2$ convolutional code with encoder polynomial $[15, 17]$ in octal notation is used. We consider uncoded data packets of 2002 bits, which include information, flush-out and CRC bits. We use optimized puncturing patterns given in Table I.

The performance of our proposed HARQ algorithm can be affected by several factors such as the number of iterations used in BICM-ID, the maximum number of transmissions allowed for a data packet, and the method of packet combining at the receiver. In order to understand the effect of iterations used in BICM-ID, we present the analysis and simulation results for 8-iterations in Fig. 4. After 5 iterations of BICM-ID, BER is improved by a factor of nearly 10^6 at 4 dB SNR. Equivalently, more than 3 dB gain in SNR is obtained. Although in the low SNR range, a considerable improvement in BER is observed by increasing the iterations beyond 5, the BER floors after 4 dB SNR. In other words, there is no improvement in BER if we increase the iterations beyond 5 for SNR higher than 4 dB. Our asymptotic error analysis agrees well with the simulation results.

In Fig. 5, we show the effect on FER when the number of retransmissions allowed for a data packet is increased. The case $I = 1$ allows no retransmission, whereas $I = 2$ and 3 allow a maximum

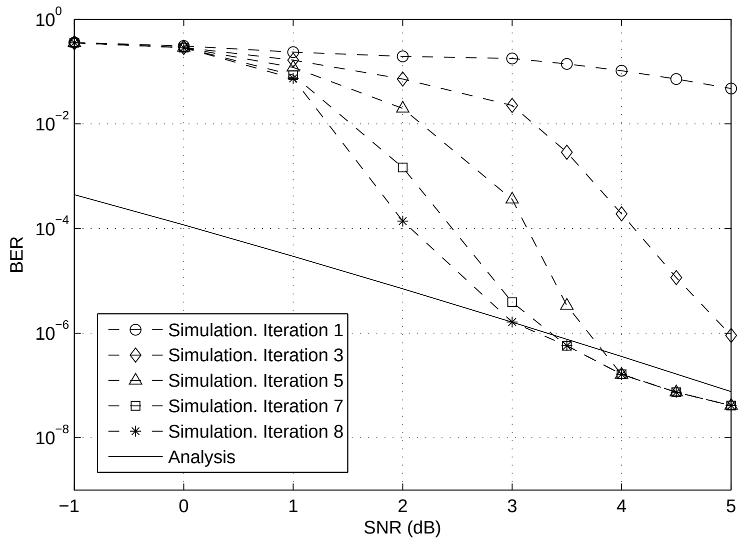

Fig. 4. BER performance of the proposed type-II HARQ technique with I=3

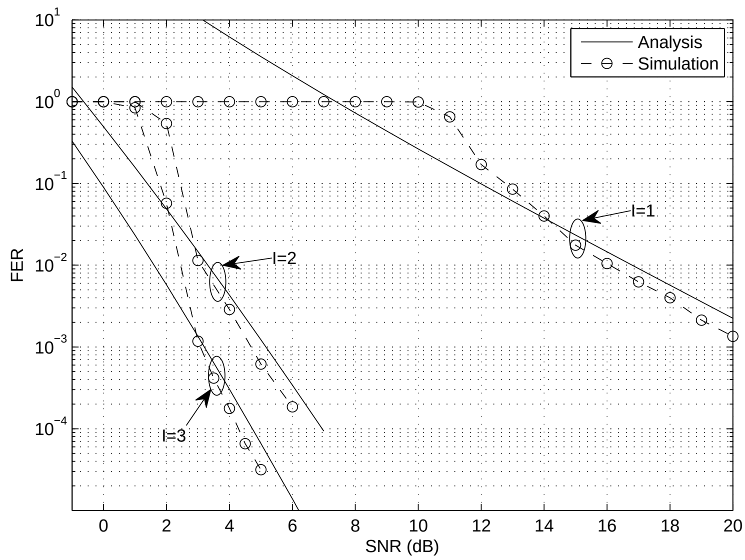

Fig. 5. FER performance of the proposed type-II HARQ technique with I=1, 2 and 3.

of 2 and 3 transmissions respectively. More than 14 dB gain in SNR is achieved by increasing the maximum allowed transmissions of data packet from $I = 1$ to $I = 2$. The effective code rate for $I = 2$ decreases from $R_1 = 7/8$ to $R_2 = 1/2$, and since none of the coded bits is repeated in the second transmission in the optimized puncturing pattern, the FER improves significantly. A further gain of about 2.5 dB in SNR is observed by increasing I from $I = 2$ to $I = 3$. Please note that in every transmission of the same data packet, we decrease the modulation order. Consequently, the number of transmitted coded bits is reduced to keep the number of transmitted symbols constant. Hence, the gain in FER performance is reduced as we keep increasing the maximum number of transmissions I.

Finally, Fig. 6 shows the throughput performance of the proposed method. We compare the throughput of our method with a reference HARQ scheme that keeps the modulation order and puncturing patterns unchanged during retransmissions, but changes the symbol mapping arrangement. This reference technique is similar to the type-III HARQ technique discussed in [16]. For each SNR value, 5000 data packets are transmitted. The throughput is obtained as $N(K-q-L)/(K(\sum_{i=1}^{I} N_i/\rho_i))$, where N is the total number of successfully transmitted frames, and N_i is the total number of i-th transmissions of the same data packet over all packets. In the reference HARQ technique with fixed modulation using minimizing bound on BER (MBER) symbol mapping given in [16], we use the same optimized puncturing pattern given in Table I for $i=1$, equal to $[102, 175]$ in octal notation, and $m_i = 4$ for all the transmissions. Different mapping arrangements for each transmission as given in [16] are considered and $I=3$ is used. We send the same number of symbols in every transmission of the same data packet in both our method as well as in the reference scheme. The results show that our scheme outperforms the reference HARQ technique with fixed modulation order in the whole SNR region, offering a maximum throughput gain of about 12 dB in the medium SNR and a throughput gain of about 5 dB in the high SNR regions.

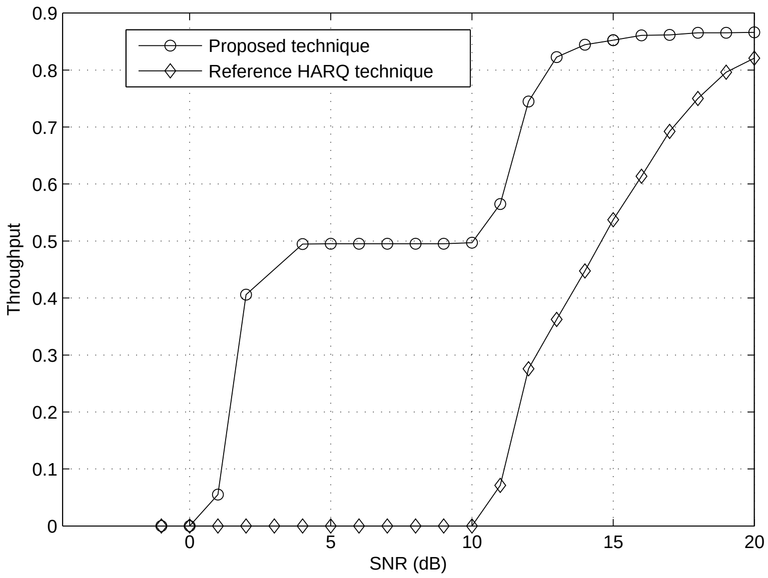

Fig. 6. Throughput performance comparison of the proposed HARQ technique with a HARQ scheme that uses a fixed modulation order in retransmissions.

CONCLUSIONS

In this paper, we propose a novel type-II HARQ technique using convolutional codes and BICM-ID. To obtain high spectral efficiency and throughput, we use different optimized puncturing patterns in each transmission and also change the modulation order. We begin with a high modulation order in the first transmission. If retransmissions are required, modulation order is reduced along with changing puncturing patterns to more effectively accomodate the channel conditions. The optimized puncturing patterns are obtained by minimizing the BER for each transmission. We derive new analytical expressions for the BER and FER. In both BER and FER results, good agreement between analysis and simulation in the medium to high SNR region is obtained. Throughput performance of our technique is compared to a reference HARQ technique that keeps the modulation order unchanged in retransmissions. In this reference technique, although the modulation order is unchanged over all the transmissions, the symbol mappings are changed as reported in [16]. The results show that our proposed method achieves a throughput gain of about 12 dB in the medium SNR region and about 5 dB in the high SNR region over the reference HARQ technique.

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