UNCERTAINTY DETERMINATION WITH MONTE-CARLO BASED ALGORITHM

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ABSTRACT

The measurement result is complete only if it contains the measurand and its units, uncertainty and coverage factor. The uncertainty estimation for the parameters acquired by the FTI is a known process. To execute this task the Institute of Research and Flight Test (IPEV) developed the SALEV© system which is fully compliant with the applicable standards. But the measurement set also includes Derived Parameters. The uncertainty evaluation of these parameters can be solved by cumbersome partial derivates. The search for a simpler solution leads us to a Monte-Carlo based algorithm. The result of using this approach are presented and discussed.

KEY WORDS

Flight Tests, Uncertainty, Derived Parameters, Monte-Carlo.

INTRODUCTION

The Flight Test Campaign (FTC) is an activity of aeronautical engineering which aims to determine the actual characteristics of an aircraft and/or a system (e.g. Inertial Reference Unit - IRU) to be used for product development, certification or qualification. Flight test and flight safety are closely coupled (Figure 1). At one side the primary concern is the test flight safety (i.e. to bring back the test bed) during the FTC. For the other side there is the aircraft operational safety that relies into the accuracy of gathered information and therefore its uncertainty. Therefore a FTC should be executed with very high scientific rigor. This includes the incorporation of measurement science into Flight Test Instrumentation (FTI) and data analysis [1, 2 and 3], that includes the:

- Development of measurement sciences practices;
- Development of measurement uncertainty tools;
- Indoctrination awareness of measurement science; and
- Implementation of measurement sciences policies.

The statement of the result of a measurement is complete only if it contains both the value attributed to the measurand and its associated uncertainty [4]. The uncertainty of measurement is a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand [5].

For flight test, the lack of information about a parameter uncertainty in a final test report is a common practice. Most of flight test engineers do not understands the physical meaning of the uncertainty information that provides the relationship between the gathered measurement and its true value which is always unknown. As example, for a given FTC if the reported value for the aircraft basic pressure (p_b) is 925.5mb ±0.508mb @1σ, this means that there is a 68.26% probability for the p_b true value to be between 925.5mb ±0.508mb range. Also the most probable value for p_b is 925.5 mb (Figure 2).

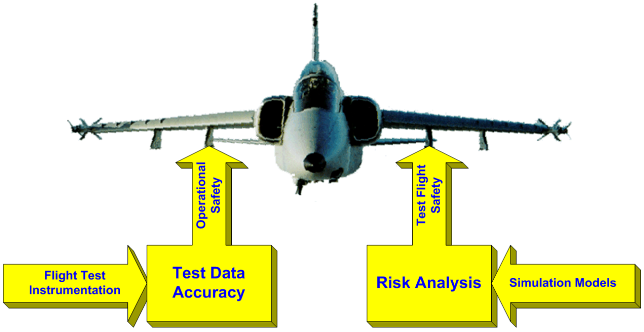

Figure 1. Flight Test and Flight Safety Links

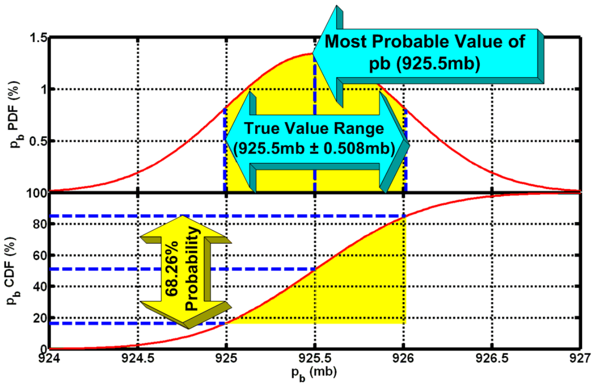

Figure 2. Uncertainty Definition

The certification and qualification processes are related with the compliance degree of the test bed with the applicable regulations (i.e. Part 25 of the Title 14 Federal Administration Regulation - FAR 25) [6]. The measurement uncertainty provides the exact compliance degree for a specific requirement.

As example let's consider the requirement for a single engine failure during takeoff, as specified into FAR-25 §25.121 [6]. In this case the certification FTC should demonstrate that the aircraft climb rate (γ_s) is better than 0% for the 1st segment. If the test report states that γ_s = 0.1%, the certification authority could eventually consider satisfactory such performance.

By the other hand, if the test report includes the full measurement description (i.e. γ_s = 0.1% ±0.28%@1σ), the real interpretation for the test results is: "The probability for γ_s to be acceptable (i.e. $P(\gamma_s) \geq 0$) is only 63.95%" (Figure 3). Therefore this result could be eventually considered unsatisfactory.

To fill this gap at the Flight Test Research Institute (IPEV) it was developed the Automation System for the Flight Test Laboratory (SALEV©) [7] that automatically computes the uncertainty of a direct FTI measurement. The SALEV© architecture entirely complies with International Organization for Standardization (ISO) 17025 standard [8] that defines the requirements of testing and calibration laboratories. One key SALEV© feature is its capability to compute the uncertainty using the FTI full measurement chain (Figure 4). This feature provides more reliable accuracy information because it takes into account all elements involved with the measurement process.

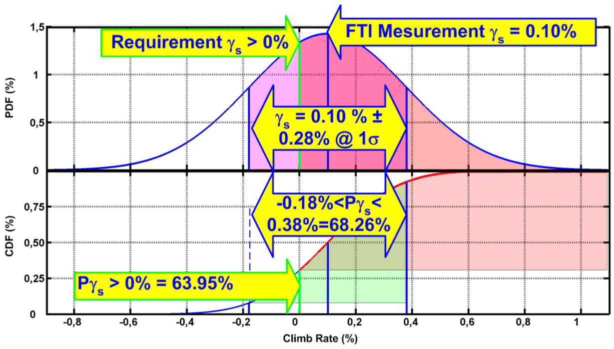

Figure 3. Requirement Compliance

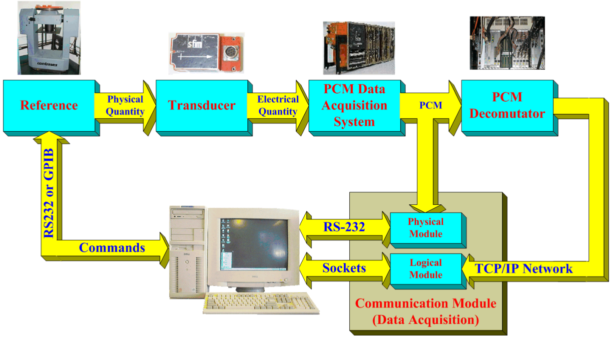

Figure 4. SALEV Architecture

UNCERTAINTY DETERMINATION

For a direct measurement (e.g. Pitot static pressure - p_b) the associated uncertainty could be computed through Type A and B evaluation methods [5]. The A type evaluation is executed by statistical analysis of one series of independent observations of the input quantity, under the same condition of measurement. For each calibration point, will be determined the experimental standard deviation (δ_m) that is computed by:

$$\delta_m = +\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(m_i - \overline{m})^2} \qquad \textbf{Eq. 01}$$

Were:

- n is the number of samples
- m_i is the i^th^ value of quantity m; and
- $\overline{m}$ is the mean value of m.

The associated uncertainty (u_m) is computed by:

$$u_m = \frac{\delta_m}{\sqrt{n}} \qquad \textbf{Eq. 02}$$

The B type evaluation uncertainty is computed through a procedure different than the Type A evaluation. This process is based on scientific judgment of all available information about the variability of the mesurand gathered from:

- Previous measurement;
- Knowledge about the measurement chain behaviour and properties;
- Manufacture's specifications;
- Calibration or certificate data; and
- Literature data.

Unfortunately besides most of FTI parameters that are resulting from a direct measurement (e.g. p_b), the evaluation of a given FTC also requires the acquisition of several Derived Parameters (DP - e.g. Mach Number - M) which are linear or non-linear combinations of direct measurements and conversion coefficients.

In this case the uncertainty estimation for these DP's could result in a very complex mathematical problem that could be solved using stochastic process [9] or partial derivates [10], using:

$$u_D = \left[\sum_{i=1}^{j} (\theta_i u_i)^2 \right]^{\frac{1}{2}} \qquad \textbf{Eq. 03}$$

With:

$$\theta_i = \frac{\partial r}{\partial X_i}, r = f\left(X_1, X_2, ..., X_n\right) \qquad \textbf{Eq. 04}$$

Considering that all FTI parameters are properly calibrated its residuals are white noise (i.e. zero mean, gaussian distribution and uncorrelated). But for some special cases the FTI direct parameters residuals are biased. Therefore the bias (B_D) estimation of a DP is given by:

$$B_D^2 = \left(\sum_{i=1}^{j} (\theta_i B_i)^2 \right) + 2\theta_m \theta_n B_m^{'} B_n^{'} \qquad \textbf{Eq. 05}$$

Where B_i is the bias of each element of the DP.

Let's consider the example of a DP measurement (M_D) computed from FTI measurements M_A and M_B that respectively has the uncertainties u_A and u_B previously computed by SALEV©. Considering that:

$$M_D = +\sqrt{M_A^2 + M_B^2} \qquad \textbf{Eq. 06}$$

And using eqs. 03 and 04 into eq. 06, the resulting uncertainty will be defined by:

$$u_D = \pm \sqrt{\frac{\partial_{M_D}}{\partial_{M_A}} u_A + \frac{\partial_{M_D}}{\partial_{M_B}} u_B} = \pm \sqrt{\frac{M_A}{\sqrt{M_A^2 + M_B^2}} u_A + \frac{M_B}{\sqrt{M_A^2 + M_B^2}} u_B} \qquad \textbf{Eq. 07}$$

Therefore the estimation of u_D (Eq. 07) requires the values of u_A and u_B that can be easily be computed off line by SALEV© and the actual values of M_A and M_B. This process requires at least doubled data processing power and it computes two output variables for each derived measurement (i.e. M_D and u_D).

Obs: If the input variables residuals are not zero mean, the resulting bias should also be computed using Eq. 05. In this case the data processing capability will be tripled and it will be generated tree output variables for each observation.

At this point the uncertainty determination has become a complex process. Initially it is required to compute the partial derivates of the conversion function, then for each observation it should be computed the DP, its associated uncertainty and in special cases its bias.

So the challenge is to develop and validate a novel calibration process that could be used along with SALEV© to automate the determination of the DP uncertainty and bias.

As case study for this development it will be computed the uncertainty for the EMBRAER XAT-26 jet trainer basic Mach number (M_b) Parameter used by the Brazilian Flight Test School (CEV).

BASIC MACH NUMBER UNERTAINTY DETERMINATION

The basic Mach number is a derived parameter computed at the aircraft ADS as:

$$M_b = \sqrt{5\left[\left(\frac{q_b}{p_b}+1\right)^{0.28571}-1\right]} \qquad \textbf{Eq. 08}$$

Were:

- q_b is the measured pitot basic impact pressure (mb);and
- p_b is the measured pitot basic static pressure (mb).

To simplify the work it will be used a Monte-Carlo based simulation algorithm that uses the SALEV© results, to propagate the uncertainty for the overhaul process. The selected simulation point should embed the XAT-26 flight envelope [11]. The validation of such process will be performed by comparison with the partial derivates method.

Using the SALEV© tool it was possible to calibrate and compute the q_b and p_b uncertainties (u_{q_b} and u_{p_b}) that are respectively 0.508mb@1σ (Figure 5) and 0.597mb@1σ (Figure 6).

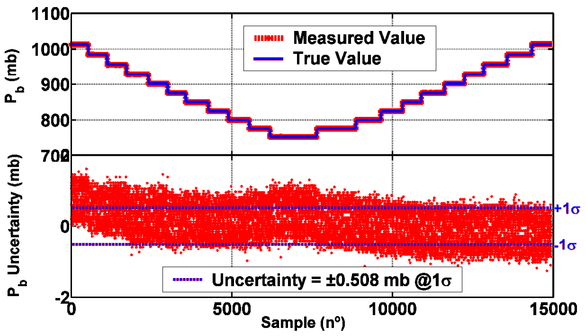

Figure 5. p_b **Calibration & Uncertainty Results**

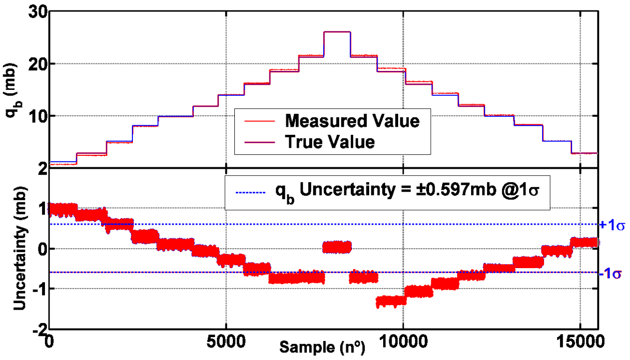

Figure 6. q_b **Calibration & Uncertainty Results**

Then the Monte-Carlo simulation application should execute the following sequence:

1. Initially it is selected the true reference pair values for p_b and q_b measurements.
2. The corresponding reference M_b value for each p_b and q_b measurements is computed and stored.
3. Then p_b and q_b reference values are corrupted by noise with the same statistics of the corresponding uncertainties (i.e. u_{q_b} and u_{p_b}).
4. Now M_b uncertainty (u_{M_b}) can be computed using Type A process.
5. In parallel u_{M_b} is computed using the partial derivates method.
6. Finally both results are compared for validation.

The determination of u_{M_b} requires the knowledge of the partial derivates of the Mach number function, applying eq. 08 into eq. 04 the results are:

$$\frac{\partial M_b}{\partial q_b} = \frac{0.31944}{p_b\left[\frac{q_b}{p_b}+1\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_b}+1\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}} \qquad \textbf{Eq. 09}$$

$$\frac{\partial M_b}{\partial p_b} = \frac{-0.31944 p_b}{p_b^2\left[\frac{q_b}{p_a}\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_a}\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}} \qquad \textbf{Eq. 10}$$

Using eq.s 09 and 10 into eq. 03 the resulting uncertainty is:

$$u_{M_b} = \pm\sqrt{\left(\frac{-0.31944 p_b}{p_b^2\left[\frac{q_b}{p_a}\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_a}\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}}u_{pb}\right)^2+\left(\frac{0.31944}{p_b\left[\frac{q_b}{p_b}+1\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_b}+1\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}}u_{qb}\right)^2} \qquad \textbf{Eq. 11}$$

Also using eq.s 09 and 10 into eq. 05 the resulting bias is:

$$B_{M_b} = \pm\sqrt{\left(\frac{-0.31944 p_b}{p_b^2\left[\frac{q_b}{p_a}\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_a}\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}}B_{pb}\right)^2+\left(\frac{0.31944}{p_b\left[\frac{q_b}{p_b}+1\right]^{\frac{5}{7}}\left[\left(\frac{q_b}{p_b}+1\right)^{\frac{2}{7}}-1\right]^{\frac{1}{2}}}B_{qb}\right)^2+2\frac{-0.102 p_b}{p_b^4\left[\frac{q_b}{p_a}\right]^{\frac{10}{7}}\left[\left(\frac{q_b}{p_a}\right)^{\frac{2}{7}}-1\right]}B'_{pb}B'_{qb}} \qquad \textbf{Eq. 12}$$

ALGORITHM EVALUATION

The Monte-Carlo based algorithm setup used 270 Test Points (TP) uniformly distributed into the flight envelope (Figure 7). For statistical consistence each TP was formed by 100 $\boldsymbol{q_b}$ and $\boldsymbol{p_b}$ pair samples corrupted by synthetic noise with the same statistics as the FTI measurement.

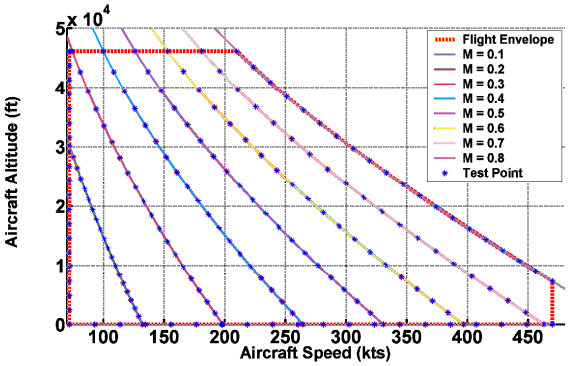

Figure 7. XAT-27 Flight Envelope and Test Points

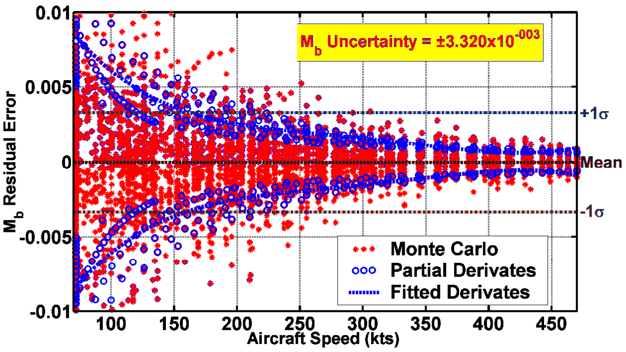

Figure 8. M_b Residual Errors

Running the simulation algorithm it is possible to perceive that the uncertainty computed by this new algorithm is close enough with the uncertainties computed by the partial derivates and its fitted envelope (Figure 8). Applying eqs. 01 and 02 into ***M_b*** residuals (Figure 8), u_{M_b} = $3.32\text{x}10^{-3}$ the resulting bias was considered negligible (i.e. B_{M_b} = $-1\text{x}10^{-6}$).

The execution of a deeper analysis with the ***M_b*** residuals errors show that 86.83% of the residual errors computed by the proposed algorithm are inside the partial derivates limits (Figure 9). In addition the difference mean between both processes is positive (i.e. $3.47\text{x}10^{-4}$), so the computed uncertainty bounds is conservative and therefore the proposed method can be validated.

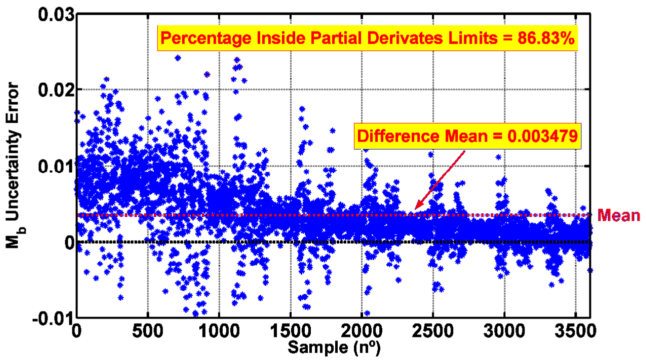

Figure 9. M_b Uncertainty Difference

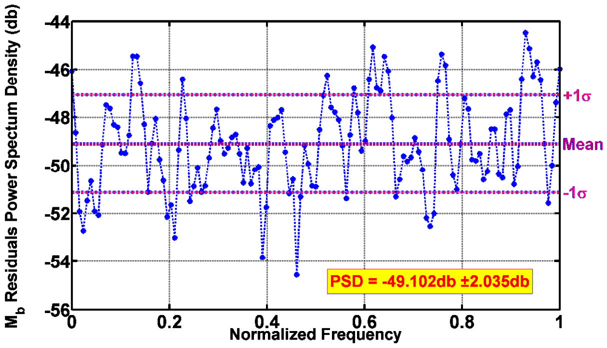

Figure 10. M_b Residuals PSD

Further analysis shows that the residuals are very close to white noise because:

- The mean values is equal to zero (i.e. B_{M_b} = $-1\text{x}10^{-6}$);
- Its Normalized Power Spectrum Density (PSD) function (Figure 10) is almost flat over the entire spectrum range (i.e. PSD = -49.10 db ±2db @1σ); and
- Its normalized autocorrelation function shows uncorrelated residuals (Figure 11).

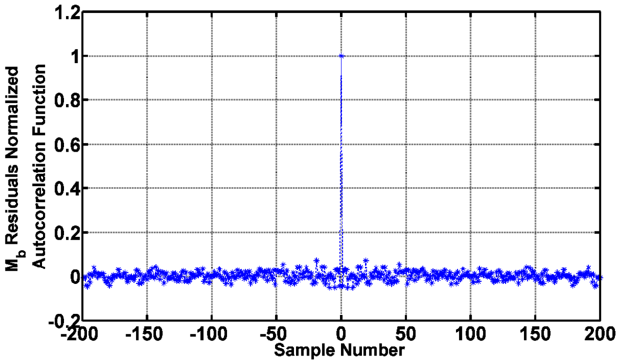

Figure 11. M_b Normalized Autocorrelation Function

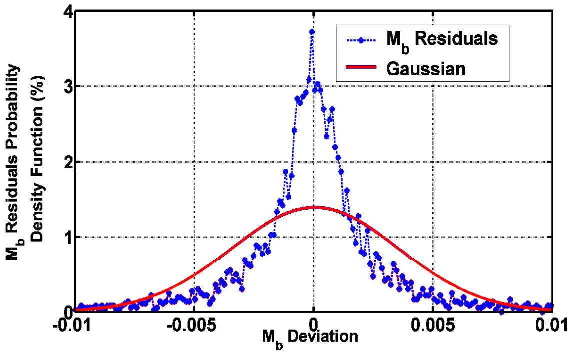

Figure 12. M_b Residuals PDF

The only discrepancy for white noise characterization is regarding ***M_b*** Probability Distribution Function (PDF) which does not fits the corresponding reference Gaussian curve (Figure 12). Also the execution of statistical Lilliefors test [12] shows that ***M_b*** PDF cannot be considered Gaussian.

CONCLUSIONS AND FUTURE WORK

A new tool that computes the M_b uncertainty, that could be used along with SALEV for process automation was developed, tested and validated. The major achievements are the minimization of:

- Real-time mathematical computation workload at the telemetry processor; and
- Storage space required for DP data achieving.

This method embeds the same methodology used for Type A calibration, with the exception that now the calibration reference should simulate multiple physical quantities (e.g. p_b and q_b) and not just one.

Now the next challenge is to validate this process for other DP's and to insert, evaluate and validate this process into SALEV© operational functions.

Future works should include:

- The expansion of this process to compute Mach (M) uncertainty which is a function of M_b and the resulting ADC Calibration model [13];
- The expansion of this process for the evaluation of the uncertainty of the aircraft true airspeed (V_t) that also requires the measurement of the impact temperature (t_i) and the temperature probe recovery factor (K) [14]; and
- The validation of all these models using modified SALEV© functions in a fully automated process.

ACKNOWLEDGEMENT

We wish to thank the partial support given by the IPEV, specially the CEV 2011 class students, for supporting the measurement.

Also we like to thank the Financiadora de Estudos e Projetos (FINEP) agency, under agreement 01.07.0663.00 that funded the development of this tool and the presentation trip.

At last I would specially recognize the help provided by the flight test engineer Leandro Roberto and Msc. Fábio Henrique Lameiras Pinto who helped me with the process review.

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GLOSSARY

ADS:	Air Data System
CEV:	Brazilian Flight Test School
DP	Derived Parameter
FAR:	Federal Administration Regulation
FINEP:	Financiadora de Estudos e Projetos.

FTC: Flight Test Campaign
FTI: Flight Tests Instrumentation
IPEV: Flight Test Research Institute
ISO: International Organization for Standardization
PDF: Probability Distribution Function
PSD: Power Spectrum Density
SALEV: Automation System for the Flight Test Laboratory
TP: Test Point