Technology Advancement in Network Markets and Agent Bargaining

by

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Dedication

This work is dedicated to my family; my wife, Karina, without whom this work would never have been completed; my daughter, Amelia, whose heralding inspired the necessary muster of energy, discipline, and ingenuity to traverse the final milestones; and to any future little ones, for whom I hope this provides for you as much as my parents were able to provide for me.

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Abstract

I extend the Katz and Shapiro (1985) oligopoly model with network effects to encompass products with differing technological levels. I focus on a version of the model in which firms can invest in order to improve the probability that they advance their technology from a low level to a high level. I find that better available technology, lower adoption costs, and stronger network effects increase the rate of technological advancement and social welfare. Incompatible networks have lower total surplus but higher adoption rates. The investment competition dissipates to some degree the potential producer rents from successful advancement, particularly in the incompatible network case where increased competition can result in lower total welfare. A policy imposing a technology standard (via a high type technology requirement) yields the highest adoption rates, but negatively affects overall welfare. Analysis of the optimal tax/subsidy policy shows that taxes are optimal in most cases, since the private incentive to advance technology outweighs the social incentive.

Negotiations in the real world can rarely be represented by a simple bargaining session between two parties. Agent bargaining, when one player represents another party in a bargaining situation for some form of compensation, is one such complicating circumstance from the real world. I explore the effects that this third entity has on the outcome of negotiations. I conduct a laboratory experiment emulating a simple example of agent bargaining. I test a hypothesis formulated using sequential-Nash-bargaining and also propose behavioral explanations for the observed behavior. I find that sequential-Nash-bargaining does a poor job of explaining our observations, and that using a weighted minimization of the differences between each of the three parties as a focal point provides a promising alternative.

CHAPTER 1
Introduction

Improving theory to better match reality in meaningful ways is a never ending yet necessary task. As theory better matches reality in important ways, economics can answer more specific questions that are more robust to possible confounding circumstances. Testing proposed theory is equally important, putting proposed ideas through a crucible to strengthen or call into question the theory's underlying assumptions. This dissertation contributes to the economics literature by improving theory in these ways in two disparate situations. The first is that of firms decisions about advancing the technology of the good that they produce in a market when network effects are present. The second involves testing a theory of sequential bargaining situations between 3 or more members.

Chapter 2, based on my work "Technology Advancement in Network Markets", extends the classic network effects model of Katz and Shapiro (1985) to incorporate a technology characteristic into firm goods. I derive analogous results when possible and, leveraging the technology characteristic and techniques found in Reynolds and Isaac (1992), explore a technology advancement game which endogenizes the firm decision about investment into incorporating current technology into firm goods. I use this model to derive results in a single period replacement game. In analyzing the model, I conclude that better available technology, lower adoption costs, and stronger network effects increase the rate of technological advancement and total social welfare. Network structure is important as well, with an incompatible network resulting in a lower total surplus, but higher technology adoption rates than their compatible network counterpart. Further differences in network structure arise when the effects of investment spending are considered, with investments in incompatible network settings being more rent dissipating than in the compatible case. Due to private firm incentives, the incompatible network case can even result in not just lower profits, but lower total welfare. I also consider two potential public policies: an imposed technology standard (meaning all firms must meet a technology threshold to participate) and an optimal tax/subsidy policy to address misaligned incentives. The imposed standard succeeds in increasing technological advancement, but at the expense of lowering total welfare, thus needing some further unaccounted for benefit to be welfare improving. In most cases, firms are individually more incentivized than is socially optimal to advance their technology, and so the optimal tax/subsidy recommends a tax in almost all cases, with monopoly being the largest exception.

Real life negotiations are complicated and often involve more than two parties. Chapter 3 of this dissertation, based on my work “Agent Bargaining” with Alex Roomets, focuses on a particular case of this. Agent bargaining occurs when one player represents another party in a bargaining situation for some form of compensation. The chapter explores the effects that this third entity has on the outcome of the negotiations. We conducted a laboratory experiment emulating a simple example of agent bargaining. We test a hypothesis formulated using sequential-Nash-bargaining and also propose behavioral explanations for the observed behavior. We find that sequential-Nash-bargaining does a poor job of explaining our observations, and that using a weighted minimization of the differences between each of the three parties as a focal point provides a promising alternative.

CHAPTER 2
Technology Advancement in Network Markets

2.1 Introduction

Many modern good and service markets share two important attributes: the use of technology and network effects. The cutting edge of technology rarely stands idle, as new breakthroughs occur with regularity. While the development of available technology marches on, practical technological advancement in the sense of adapting new advances into goods and services in a market is neither smooth nor inevitable. Firms have to spend resources investing into incorporating technologies into their goods in a useful or attractive way. This process can be further complicated by the different network structures that may exist in any given market, such as whether the network effects from the goods are compatible or incompatible between firms. Thus, we are left with questions which are unaddressed by the current literature: How does technology advance in network markets? What affects firm investment decisions about changing the embodied technology in their goods and services? What are the differences in investment decisions between markets with compatible network effects versus incompatible network effects?

I address these questions by extending the stylized model of network effects in oligopoly settings first put forth by Katz and Shapiro (1985) to incorporate a simple additive quality/technology characteristic, and adapt methods from Reynolds and Isaac (1992) to analyze the model with uncertain adoption probabilities for a new quality/technology characteristic. I use this model to derive results in a single period replacement game. In analyzing the model, I conclude that better available technology, lower adoption costs, and stronger network effects increase the rate of technological advancement and total social welfare. Network structure is important as well, with an incompatible network resulting in a lower total surplus, but higher technology adoption rates than their compatible network counterpart. Further differences in network structure arise when the effects of investment spending are considered, with investments in incompatible network settings being more rent dissipating than in the compatible case. Due to private firm incentives, the incompatible network case can even result in not just lower profits, but lower total welfare. I also consider two potential public policies: an imposed technology standard (meaning all firms must meet a technology threshold to participate) and an optimal tax/subsidy policy to address misaligned incentives. The imposed standard succeeds in increasing technological advancement, but at the expense of lowering total welfare, thus needing some further unaccounted for benefit to be welfare improving. In most cases, firms are individually more incentivized than is socially optimal to advance their technology, and so the optimal tax/subsidy recommends a tax in almost all cases, with monopoly being the largest exception.

Every possible technological or quality iteration of a good does not see the market. For example, in the media storage industry, there existed capacities between DVDs (4.7 to 17.8 GB) and the higher capacity Blu-ray (25 to 100 GB) for differing degrees of maximum storage than were created. For instance, one could imagine a larger diameter DVD disk which one could make as large as necessary to store more data, similar to LaserDisc. The video game console industry is another good example. There was a five year gap between the release of the Playstation 2 and the Playstation 3, and a seven year gap between the Playstation 3 and Playstation 4. The difference between each iteration is a large improvement in the specifications of the hardware. If Sony had wanted to release an improved Playstation 2, they could have done so within months of its release given the rapid rate of progress in processing technology. Especially in an industry that innovates as fast and steadily as the computer industry, firms choose the technological level of the good they offer to the market. Constant updates to goods are costly. Applying and adopting a new technology has a cost, usually large, both from designing a good with the new technology and setting up a manufacturing process. Uncertainty in this adoption process can play a role in this process as well. Firms have to balance the value of the improved good to consumers with the costs of offering it, but in many cases this is only a piece of the picture.

When these goods exhibit network effects, the matter can be complicated further. New advances in cellular phones are available for purchase multiple times a year. New video game consoles tend to have new iterations every five years. The most successful massively multiplayer online role playing game (MMORPG), World of Warcraft by Blizzard Entertainment, was released in November of 2004 and continues without a released sequel as of this writing.[1] All of these use or are dependent on computing technology of some form in the production of new goods. While there are differences in the exact types of technology they use, it is not apparent that the differing firm adoption decisions are fully explained just by these characteristics. The structure of network effects in these industries are significantly different from one another and provide further impetus for this paper.

Consider the cellular phone industry. The value of a cell phone is affected by who you can call, which is the source of some network effects. Phones have a fully compatible network; no matter which cell phone you have, you can call any cell phone, old, new, or even older non-cell telephones. This network is something that consumers value, but not something that differentiates one phone company from another, as having any cell phone lets you call anyone else with a phone. In this industry, a firm cares about upgrading to a new technology to the extent that it increases the value of the good to the consumers (allowing the firm to charge a higher price), expands the overall market (which expands the value of the network) and changes the cost that will be incurred. [2]

[1]World of Warcraft does add to their product by creating expansion packs every few years, but the game is limited in some structural ways by the initial build of the game.

[2]In recent years smart phones have expanded their uses beyond simple phone calls to a larger variety of tasks and entertainment. Potential for network fragmentation exists in the use of these

In the case of World of Warcraft, the network looks very different. The consumers who play World of Warcraft care about the number of other players who play World of Warcraft; the network is incompatible with the products of its competitors. There are a great many other MMORPGs that exist, but a large part of the value in an MMORPG is the chance to interact with many people in the game cooperatively and competitively. As such, the network effects are strong and exclusive. Since World of Warcraft has this network advantage over newer games, they do not need to upgrade their product until the upgrade is significant or competition is strong. Activision-Blizzard (the company that makes and runs World of Warcraft) cares about new technology only if the new technology offers a value upgrade to consumers that outweighs their massive network advantage.

With the advent of always connected computers and game systems, updates to games after their initial release is extremely common. Most games, even when bought in store, now force the consumer to download a day one "patch", or update, to their game before it can be played. The current most popular online video game in the world, "League of Legends" by Riot Games, is updated multiple times a month with small changes. Large changes also occur yearly, such as the upcoming graphics overhaul to the game. The timing and cost decisions of technological adoption are becoming very important questions for firms when products, even after first being presented as complete, can be upgraded and added to over time.

Advances in technology for firms that trail current market leaders may present itself as a lucrative option as a way to break the dominance of some firms in network driven markets. Technological advancement is one way the king of a market today can be unseated in the future. This paper explores network effects and endogenous technological advancement and its effects on the market. I first extend the classic

apps. While these additions complicate the consumer demand and network effects of this market, the situation certainly applies to the original telephone market and the telephone call function of the cellular phone and to markets like it.

Katz and Shapiro (1985) model of network effects to include a firm specific technological characteristic and show some general results for this model. This model can encompass equilibria where a strong network could keep a lower technology firm on top; it could also be the case that an advancement in the technology of one firm could eliminate this low-technology high-network firm outcome from the set of possible equilibria. I then add to the game a technological advancement period before production, where firms can choose whether or how likely (depending on the case) it will be that they upgrade their current technology into the top of the line technology. This extension gets at a basic form of endogenous technological advancement in a market with network effects. In other words, the current work makes itself distinct through the combination of endogenous firm technological advancement and by incorporating network effects. Within this model, I investigate the effects of a compatible and incompatible network so that the model can be applied to a wider range of markets. The differing network structures also allows a contrast between the two, showing a possible value of standardization and the differences in rent dissipation. Two possible public policies are also evaluated. The first is a tax/subsidy policy to address over and under technological adoption in the market. The second policy is a forced restriction to the more valuable technology; firms must either present the high technology to the market or exit.

Katz and Shapiro (1985) is one of the seminal works in the network effects literature, providing a model that gives insight into how network effects shape market outcomes. It also introduces the fulfilled expectations equilibrium concept. The model put forth in the next section draws from Katz and Shapiro but adds a firm specific single dimensioned quality characteristic. Katz and Shapiro (1986) themselves looked into the effects of network effects on technological adoption, but with a very different structure. In their model the adoption of technology is exogenous; technology A exists alone in the first period, and then technology B becomes available exogenously in the second alongside A. They also focus heavily on technology sponsorship and compatibility decisions and the different effects these have on the market. Yet again, Katz and Shapiro (1992) make a statement on network markets, this time on the topic of product introduction. In particular, this model uses continuous time and is most interested in the timing of entry for the second technology, though the decision to introduce a new technology remains exogenous. Also at this time, Farrell and Saloner (1985 and 1986) were also making important contributions in the network effects literature. Their work mostly concerns the potential inefficiencies of network effects. Lock in can occur if consumers group on a subpar technology; consumers could be better off if they all switched to the better technology, but inertia keeps them where they are. Another discouraging factor when considering adopting a new technology can be a penalty to the first adopter, as he takes the risk; consumers always prefer to adopt late. Also notable is that it incorporates endogenous timing by allowing consumers to wait indefinitely to adopt. In equilibrium, all firms that would find it valuable to adopt the new technology do so as there is no penalty for waiting.

Reynolds and Isaac (1992) offers a useful tool in the refinement of equilibrium in cases of technological progress. Their work focuses on technological progress in the form of production cost savings. I utilize their approach to analyze a special case of the model with network effects. Further work developing fulfilled expectations equilibrium is that of Amir and Lazzati (2011). With minimal assumptions and leveraging lattice theory in their network analysis, the authors provide detailed insights regarding equilibria under relatively general conditions, providing results on existence in cases of pure network effects and non-additive network effects, as well as some comparative statics. Their model also addresses technological progress in the form of an exogenous cost reduction. The analysis differs in the use of non-symmetric firms and technology represented as an increase in consumer valuation of a product, as well as focusing on the firm decision of technological adoption.

Several studies provide insight into the process and properties of technological adoption by examining relevant real world data. Gatignon and Robertson (1989) find that markets in concentrated markets and limited price intensity encourage a firm's adoption of new technology, as firms in this advantaged position are most able to benefit from the increased surplus the new technology generates. Utterbach and Suarez (1993) depict the search for a dominant design in a market, identified by many firms doing individual research to find a strong enough design. Once one is found, research changes from exploration into improving the dominant design in a market with fewer firms. They also study the structural effects of this innovation on the market. One relevant finding is that more competition leads to more technological change, which this article's model also support. Gandal, Kende, and Rob (2000) address technology diffusion in two sided markets, finding that the software side has very important effects on the decision of consumers to adopt a new product. Besen and Farrell (1994) list and detail the practical concerns and important strategies of firms operating in a market with network effects. Sheremata (2004) lays out a general framework to classify different markets and innovations into how compatible or incompatible the contributions are and the size of innovations. Different real world examples are then fit onto the spectrum. The author finds that incompatible innovations and large innovations are more profitable.

Building off these previous works, I incorporate an additive technology characteristic into the base Katz and Shapiro (1985) network effects model. Within this model, I add an endogenous choice for firms to adopt a new, higher technology good at a cost. To analyze the model, I employ the fulfilled expectations Cournot equilibrium introduced by Katz and Shapiro (1985) when technology levels are given. To analyze the technology adoption portion of the model, I use Nash equilibrium. In particular, in the case of uncertain adoption, I employ the techniques of Reynolds and Isaac (1992) to tackle the problem in a special case of two possible technology levels. I also consider an imposed high technology policy and an optimal taxation/subsidization policy on newly adopting firms in both compatible and incompatible network situations.

We find that better available technology, lower adoption costs, and stronger network effects increase the rate of technological adoption and social welfare. Incompatible networks have lower total surplus but higher adoption rates. Producer surplus can be higher under an incompatible network, as the increased product differentiation can provide high type firms with extra market power; however, total and consumer surplus are lower. While imposing a high type technology yields the highest adoption rates, the loss of competition due to exit negatively affects overall welfare in all cases. In the incompatible imposed requirement policy, monopoly can provide the highest total surplus because the fragmented networks in multi-firm markets lower the value of the good. Consumer surplus remains the lowest under monopoly in this case. The tax/subsidy policy imposes taxes on the adopting firms in most cases as firms overinvest, as the profit incentive does not line up with the smaller value of an additional high type firm. The exception is monopoly, where the monopolist makes strong profits with either technological level. Firms which face incompatible networks overinvest more than their compatible counterpart as individual profits vary more with network size. Thus, optimal taxes are higher in the incompatible case.

2.2 A Model of Network Competition with a Single Quality Characteristic

This model is the same as that of Katz and Shapiro (1985), but with the addition of a product specific additive quality/technology characteristic which homogeneously increases the value to all consumers of owning the firm's product.

2.2.1 Consumers

Consumers act to maximize their surplus. Consumers each buy either one or zero products. Consumers base their valuation of product i on the intrinsic value of owning any product, r, the quality of the specific product, T_i, the price of the product, p_i, and their expectations about network size of the product, y_i^e.

Consumer valuations of quality and network size are assumed to be homogeneous. Consumers are heterogeneous in the intrinsic value r. r is assumed to be distributed uniformly in the domain (-∞,A] where A>0 (as in Katz and Shapiro). A type r consumer has a valuation of $r+T+v(y^e)$ for any product with quality T and expected network size y^e. I normalize $v(0)$ to $v(0) = 0$. The network valuation function v is assumed to be twice differentiable, monotonically increasing ($v' > 0$), strictly concave ($v'' < 0$), and bounded from above ($\exists a$ such that $\lim_{x \longrightarrow \infty} v(x) < a$). A consumer purchases the product for which the net surplus

$$r + T_i + v(y_i^e) - p_i \tag{2.1}$$

is greatest. If the net surplus for every product is negative, the consumer does not make a purchase and receives utility of 0.

2.2.2 Firms

Consider the case where all n firms have positive sales. If the technological level of a firm's product is much lower than the technological levels of the other firms, consumers will not value the low technology enough for purchase even at a price of zero. Thus, a sufficient condition would entail that a firm's technology is not "too far" behind that of other firms. The assumption forgoes network effects, which would expand the possible range of positive production universes but be dependent on equilibrium outcomes. I assume:

$$\sum_{k \neq i} T_k \leq A + nT_i \tag{2.2}$$

Under the positive sales assumption and this technology assumption[3], the consumer valuations between any two products must satisfy

[3]If this assumption is not palatable, an exit process which allows the low technology firms which would not produce in equilibrium to leave the market can achieve the same technology conditions as the assumption.

$$r + T_i + v(y_i^e) - p_i \quad = \quad r + T_j + v(y_j^e) - p_j \tag{2.3}$$

$$p_i - T_i - v(y_i^e) \quad = \quad p_j - T_j - v(y_i^e) \tag{2.4}$$

for all i, j. If this were not the case, there would be some product whose value was higher, which all consumers would buy, or some product whose value was lower, which no consumers would buy. Let $\phi = p_i - T_i - v(y_i^e) = p_j - T_j - v(y_i^e)$. ϕ informs about the product specific value being offered by the individual product; Katz and Shapiro call ϕ the expected hedonic price of a good. Once ϕ is known, it is known which of the consumers will make a purchase, as consumers with $r \geq \phi$ make a purchase. With these assumptions on the distribution of r, there are $A - \phi$ consumers who make a purchase. Let x_i denote firm i's output, and let z denote total output, i.e. $z \equiv \sum_{i=1}^{n} x_i$. Then, $A - \phi = z$, or

$$A + T_i + v(y_i^e) - p_i = z. \tag{2.5}$$

Rearranging, firm i's price is

$$p_i = A + T_i + v(y_i^e) - z. \tag{2.6}$$

Thus, in a Cournot environment, the price a firm receives depends on the total output, the quality of the firm's good, and the expected network size of the firm's good.

As in Katz and Shapiro, for simplicity, I assume a fixed cost of zero. I also assume a constant per unit cost of production equal to zero. Here generality is lost with respect to the structure of costs for different technological levels. In many models, improved technology manifests itself as per unit cost savings for a higher technology firm, or consumer valued technology costs more as its benefits improve. This model only considers technology as improving the benefit offered to consumers, directly through consumption and indirectly through network valuation, without increasing per unit costs. However, beyond the impositions on the interactions of technology in the market, this cost assumption is without loss of further generality. Firm i's profits are equal to

$$\pi_i = x_i(A - z + T_i + v(y_i^e)). \tag{2.7}$$

2.3 Equilibrium

First, I solve for the Cournot equilibrium. From there, I refine the Cournot equilibrium by using the Fulfilled Expectations Equilibrium concept laid out in Katz and Shapiro (1985).

2.3.1 Cournot

Taking consumer expectations (y_i^e), quality levels (T_i), and other firms outputs ($x_k, k \neq i$) as given, firms are faced with the following maximization problem:

$$\max_{x_i} x_i(A - \sum_k x_k + T_i + v(y_i^e)) \tag{2.8}$$

After taking a derivative and solving the first order condition for x_i^*, the result is

$$x_i^* = \frac{A + T_i + v(y_i^e) - \sum_{k \neq i} x_k}{2} \tag{2.9}$$

Simultaneously solving these equations, I get something akin to the standard linear Cournot solution:

$$x_i^* = \frac{A + n(T_i + v(y_i^e)) - \sum_{k \neq i}(T_k + v(y_k^e))}{n+1}. \tag{2.10}$$

Summing this over all n firms, I find that

$$z^* = \frac{nA + \sum_i (T_i + v(y_i^e))}{n+1}. \tag{2.11}$$

Pairing this form with the conditions for all firms producing in equilibrium (namely $p_i - T_i - v(y_i^e) = p_j - T_j - v(y_j^e) = \phi \ \forall i, j$ and $\phi = A - z$), I find:

$$p_i - T_i - v(y_i^e) = A - \frac{nA + \sum_i (T_i + v(y_i^e))}{n+1} \tag{2.12}$$

$$p_i = \frac{(n+1)(A + T_i + v(y_i^e))}{n+1} - \frac{nA + \sum_i (T_i + v(y_i^e))}{n+1} \tag{2.13}$$

$$p_i^* = \frac{A + n(T_i + v(y_i^e)) - \sum_{k \neq i}(T_k + v(y_k^e))}{n+1} = x_i^*. \tag{2.14}$$

Under these conditions, Cournot equilibrium yields $p_i^* = x_i^*$, and $\pi = (x_i^*)^2$ regardless of network expectations. This is due to the assumptions on consumers' reaction to prices and technology.

2.3.2 Fulfilled Expectations Equilibrium

Next I add the additional restriction to equilibrium that consumers have rational expectations about network size. In other words, expected network sizes are realized in equilibrium; $y_i^e = y_i \ \forall i$. I assume firms do not account for this when they calculate their best response function, so the Cournot structure derived above applies. In order to find this equilibrium, I seek to find Cournot structured production outcomes that satisfy the requirements by finding a fixed point for each firm's best response that also satisfies consumer's behavior. In order to proceed, it is useful to split the analysis into different network structures. I consider first a case where networks are fully compatible, meaning the network externality depends on the total number of people who make purchases. Second, I consider a case of fully incompatible networks, where network externalities depend on the total number of people who make purchases of each good.

Fully compatible network ($y_i^e = z$) In a compatible network framework, a consumer's network externality depends only on the total number of people who buy any product, i.e., z. This also means that every purchaser enjoys the same network externality. In the notation to this point, this is represented by setting $v(y_i^e) = v(z)$. Factoring this into the Cournot equilibrium above:

$$x_i^* = \frac{A + v(z) + nT_i - \sum_{k \neq i} T_k}{n+1} \tag{2.15}$$

Summing over all x_i^*:

$$z = \frac{n(A + v(z)) + \sum_i T_i}{n+1} \tag{2.16}$$

It can easily be shown that, given parameters, there exists a unique fulfilled expectations equilibrium z^*. This is demonstrated by a simple manipulation and corresponding Figure 2.1:

$$\frac{n+1}{n} z = (A + v(z)) + \frac{1}{n} \sum_i T_i \tag{2.17}$$

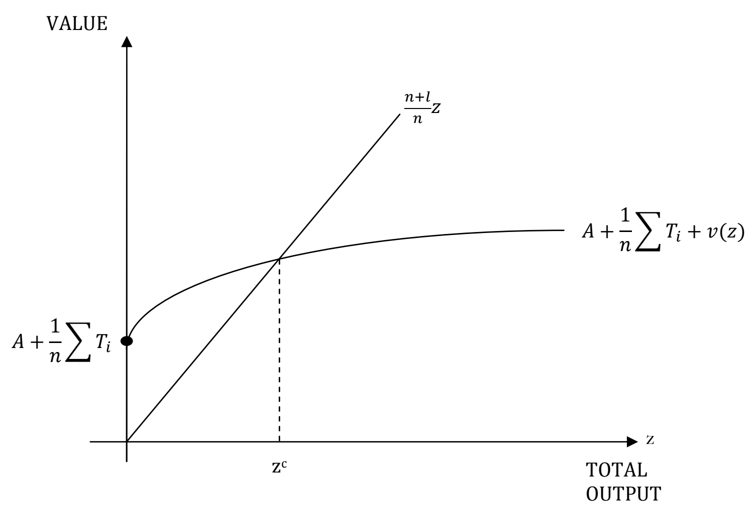


Figure 2.1: Compatible Equilibrium

Set z on the horizontal axis and "value" on the vertical axis. The LHS is an upward sloping line beginning at the origin. The RHS is entirely constant and positive save for the network, so when the network is zero, the rest give the y-axis intercept. Since $v(\cdot)$ is strictly increasing, strictly concave and bounded, the two curves will intersect once and only once. Knowing z^* sets ϕ, which then reveals individual firm prices and quantities. In essence, the prices differ in such a way as to make up the difference in quality levels. Using this characterization of equilibrium, profits can be easily described as well. Going back to the necessary conditions to have all firms produce:

$$p_i - T_i - v(z) = p_j - T_j - v(z) \tag{2.18}$$

$$p_i - p_j = T_i - T_j \tag{2.19}$$

for all i,j.

The shares can be characterized by relating them to prices. It is true that $\phi = p_i - T_i - v(z)$ for all i and $\phi = A - z$. Setting the different expressions for ϕ equal and substituting the solution for z and solving for p_i

$$p_i - T_i - v(z) = A - \frac{n(A + v(z)) + \sum_j T_j}{n+1} \tag{2.20}$$

$$p_i - T_i - v(z) = \frac{A - \sum_j (T_j + v(z))}{n+1} \tag{2.21}$$

$$p_i = \frac{A + nT_i + v(z) - \sum_{j \neq i} T_j}{n+1} \tag{2.22}$$

Looking at the solution for x_i^*, it is true that $x_i = p_i$ in equilibrium. Knowing this, it is also true that $x_i^* - x_j^* = T_i - T_j$ and that maximized profits are equal to $(x_i^*)^2$.

Incompatible network ($y_i^e = x_i$) Now I investigate a case where consumers only realize network externalities from consumers who use products from the same firm as they do. In the notation so far, this translates to $v(y_i^e) = v(x_i^e)$. Factoring this into the Cournot x_i^*:

$$x_i^* = \frac{A + n(T_i + v(x_i)) - \sum_{k \neq i}(T_k + v(x_k))}{n+1} \tag{2.23}$$

Summing these together, I can get an expression for z^*:

$$z^* = \frac{nA + \sum_i (T_i + v(x_i))}{n+1} \tag{2.24}$$

Proposition 1. A fulfilled expectations Nash equilibrium exists with an incompatible network.

Proof. To show existence in the case of incompatible networks, I can apply Brouwer's fixed point theorem to a suitably constructed function. I propose the function $f_i(x_i, x_{-i})$ such that:

$$f_i(x_i, x_{-i}) = \max(\frac{A + n(T_i + v(x_i)) - \sum_{-i}(T_k + v(x_{-i}))}{n+1}, 0) \tag{2.25}$$

Each function f_i is continuous in the x_i's, though not differentiable. It lives in the domain and range of $[0, \ell]$, where 0 is the lower bound, as firms cannot produce a negative number of goods, and ℓ is the assumed upper bound. Consequently, I can construct a function B which consolidates all of the f_i's into one and satisfies the requirements to apply Brouwer's. Let $\vec{x} \equiv (x_1, x_2, ..., x_n)$. Then I define B as:

$$B(\vec{x}) = (f_1(x_1, x_{-1}), f_2(x_2, x_{-2}), ..., f_n(x_n, x_{-n})) \tag{2.26}$$

As each f_i is continuous, B is continuous and maps from $[0, \ell]^n \rightarrow [0, \ell]^n$. Since this set is nonempty, convex, and compact, I can apply Brouwer's fixed point theorem. A fixed point of this function is a joint solution to the maximization problem, i.e. a Nash equilibrium. Thus, there exists at least one equilibrium. QED.

In many incompatible network cases there is a large multiplicity problem. The standard Katz-Shapiro no technology characteristic case demonstrates that by examining the form of the equilibrium reaction correspondences I can break the different equilibria into classes. I start with the special case of quality/technology T being equivalent among all firms ($T_i = T_j, \forall i, j$). The results of Katz and Shapiro (1985) apply, and are summarized here with the addition of the quality/technology parameter where appropriate. First, a unique symmetric equilibrium exists in a similar fashion as the fully compatible case. By assuming every firm produces an equal part of the market total, or $\frac{\hat{z}}{n}$ I find that the equilibrium must satisfy the equation:

$$\frac{n+1}{n}\hat{z} = A + T + v(\frac{\hat{z}}{n}) \tag{2.27}$$

This equation yields a unique solution for $\hat{z}$, which is the symmetric equilibrium's z^*. From there, a division by n determines the equilibrium levels of production for every firm.

A second case in which some of the firms produce zero and the remaining firms act symmetrically can exist if and only if, with k active (non-zero production) firms, the condition $v(\frac{A+T}{k}) \geq \frac{A+T}{k}$ is satisfied. The condition ensures the non-active firms are behaving rationally by producing zero in each proposed equilibrium. The resulting equilibria are the same as the symmetric case but with k firms instead of n firms.

The third class of equilibria with equal quality/technology is asymmetric oligopoly. Simple firm equilibrium reaction correspondence graphs show their existence under some conditions. Notably, these equilibria have different firms producing different positive levels of output in contrast to the symmetric cases. Unfortunately they are hard to characterize in general.

By adding differing quality/technology levels to the model, the picture is complicated further. With a high enough difference in T_i's, a monopoly may be the only equilibrium. The best intuition in this case comes from a thorough examination and understanding of the two firm correspondence graph and its possible appearances. Taking the other firm's output as given and accounting for fulfilled expectations ($y_i^e = x_i$), the equation denoting the equilibrium reaction correspondence for firm 1 is as follows. The equilibrium reaction correspondence for firm 2 is analogous.

$$x_2 = A + T_1 + v(x_1) - 2x_1 \tag{2.28}$$

Interpretation of the equilibrium reaction correspondence is somewhat complicated. The correspondence tells us, for some output of the other firms (shown on the y-axis), which output amounts of the firm could be supported by rational expectations. There can be as many as 3 supportable network expectations for any given other firm outputs. The interior ones are defined by the above equation, but note that in many cases, if consumers expect network size of a firm to be zero, that is a possible fulfilled expectation. An expectation of zero is not defined by the above equation; it is due to a corner condition in the Cournot equilibrium.

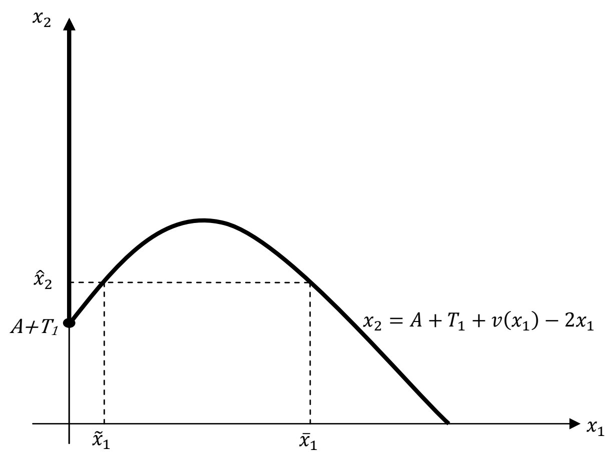


Figure 2.2: Incompatible network equilibrium reaction correspondence for firm 1

Putting x_2 on the vertical axis and x_1 on the horizontal, the vertical intercept is at $A + T_1$.[4] From this intercept, $-2x_1$ is a downward sloping line from which $v(x_1)$ will deviate to create the function or correspondence, depending on the strength of the network effect. Recalling the properties of $v(\cdot)$, that it is strictly increasing, strictly concave, and bounded from above, the shape of the correspondence can be generally determined. The upper bound ensures that eventually the $-2x_1$ portion will come to dominate the direction downward. With a weak network effect, where $v'(0) < 2$, the correspondence will be strictly decreasing in x_{-1}. With a stronger network effect than this, the correspondence will start upward sloping from the vertical intercept

[4]The correspondence extends up along the vertical axis as well to infinity.

and gradually become decreasing as shown in Figure 2.2. Note that the intercepts are determined in part by the quality/technology parameter T_1.

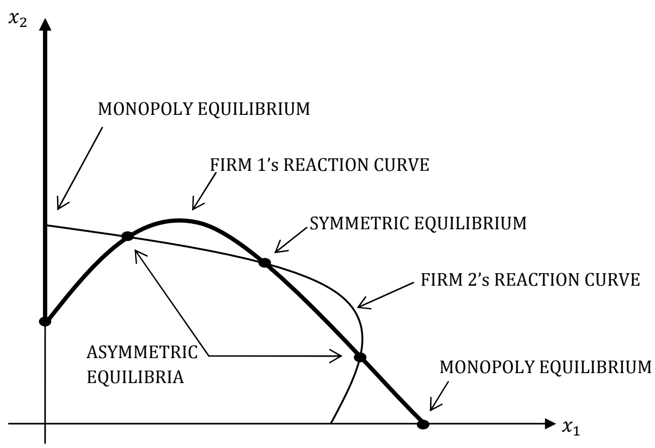


Figure 2.3: Potential duopoly response correspondences under Katz-Shapiro (the special case of $T_1 = T_2$)

When starting from the same axis intercept on both sides as in Katz and Shapiro (1985) and the special case of equivalent quality/technology, the different cases of equilibria can be seen through the different shapes of the curves. This is shown in Figure 2.3. Adding differing quality/technology levels to the firms is represented on the graphs as a shift of the axis intercept. As the quality/technology for the firm increases, the curve shifts outward with it. For instance, suppose $T_1 < T_2$. The correspondence for firm 2 will be shifted more outward than firm 1, even though the shape of firm 2's curve is the same as firm 1's (See Figure 2.4). When this shift occurs, all of the previous equilibria are altered in some direction. There is no longer a symmetric equilibrium, for example, though the outputs may still be similar for a small shift. The set of equilibria in this two firm case can be as high as five, with two monopoly equilibria (one for each firm), two asymmetric equilibria (one where each firm is dominant), and one close to symmetric split equilibrium. The diversity of equilibria is strongly influenced by the role of network expectations. If consumers expect that many people will buy the lower technology good and network effects are strong enough, then the expected network can draw enough consumers to make this dominant low technology firm a fulfilled expectations equilibrium. Consumers would prefer to coordinate than to seek the higher technology good, similar to a case of low technology lock in. Under other circumstances, for instance with weak network effects and similar technology, only three equilibria may remain, which are the close to symmetric equilibrium and the monopoly cases. In other conditions, such as a large technology discrepancy, a solitary monopoly equilibrium may remain for the high technology firm.

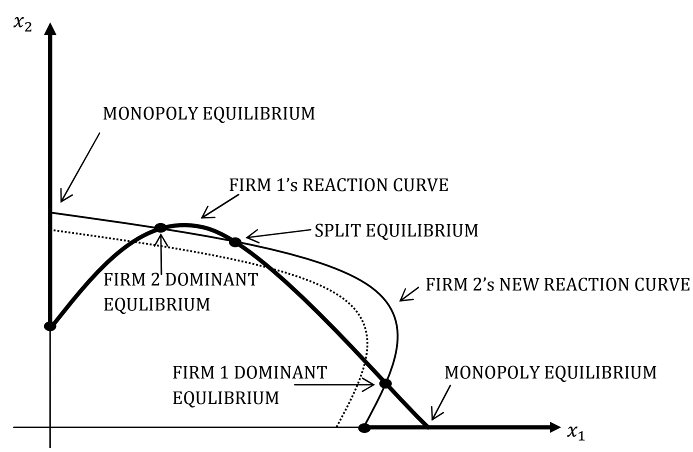


Figure 2.4: Potential duopoly response correspondences under Katz-Shapiro with technologically advantaged firm 2 ($T_1 < T_2$)

2.3.3 Welfare Measurement

Welfare can be measured by adding the producer and consumers surplus. The producer surplus is the total industry profits, or sum of firm profits. In this case, firm

i's profits π_i are equal to $(x_i)^2$. Producer surplus can be expressed as $PS(x_i) = \sum_{i=1}^{n}(x_i)^2$. Consumer surplus is the value to consumers of goods purchased minus their prices. Since goods are identical to each consumer in equilibrium, the value offered by each product is the same. Along with the assumption of uniformly distributed consumers, knowing how many bought any good (z), the highest value consumer A, and the hedonic price, ϕ,[5] is sufficient for finding the surplus. This intuition is formalized by looking at earlier equations. Recall the value of good i to consumer r from equation (1), the hedonic price ϕ from equation (4), and that $A - \phi = z$. The consumer surplus experienced by consumer r is then

$$CS_r = r + z - A \tag{2.29}$$

Integrating over the different values of r for purchasing consumers and with A as a parameter, it is shown that consumers surplus depends entirely on equilibrium z.

$$CS(z) = \int_{A-z}^{A} (\rho + z - A)d\rho = \frac{z^2}{2} \tag{2.30}$$

Total welfare is determined by the summation of producer and consumer surplus.

$$\begin{aligned} \text{Total Welfare} &= \text{Producer Surplus} + \text{Consumer Surplus} \\ W &= \sum_{i=1}^{n}(x_i)^2 + \frac{z^2}{2} \end{aligned} \tag{2.31}$$

Note that T is not named explicitly in this expression of the welfare function. T levels implicitly affect both z and the x_i's, making the quality/technological level of the good a strong driver of overall welfare.

2.4 Quality Choice/Technological Adoption

Since equilibrium has been established in the selling portion of the game, I can back out and take a look at how this shapes firm's choices about replacing the quality/technological level that they currently offer with the top of the line/cutting edge

[5] The hedonic price accounts for attributes which are faced by all consumers, namely quality/technology level and network size in addition to the charged price.

product. I assume that cutting edge technology is driven and provided for sale by an outside technology firm. A real world analogy to this assumption is that of Intel's production of microprocessors. Firms need not develop their own processors for their products; they buy chip sets or contract with Intel to meet their needs for applied technology. If this assumption is extreme to some, a more complicated model can be considered where each individual firm invests in their own research and development to extend the available replacement qualities/technologies. The extra complication is not necessary to gain useful insights into this situation. In other words, I endogenize each firm i's technological level by allowing them to keep their current state, T_i, at no cost or to replace their state with the maximum available state, T_{max}, for a replacement cost, K. I consider two forms of this replacement cost. In the first, a firm may pay a fixed K and produce the upgraded product with certainty. Firms have perfect control over their replacement decision; this generally results in many equilibria. In the second, firms choose some amount of funds to put towards adopting the new technology; the endeavor succeeds with some probability, which increases with the amount spent. This cost and adoption structure leads to a unique equilibrium in some cases.

2.4.1 Deterministic Upgrade Decisions

In this version of the replacement game, firms start with some quality/technological level that can vary among all firms. In the case of a certain fixed cost replacement, firms individually and simultaneously decide whether they want to keep their given technological level or if they would like to "upgrade" their product offering to the most advanced product type, T_{max}. T_{max} is the same for all firms. A firm that "upgrades" incurs a fixed cost, K, which is the same for each firm no matter the firm's current quality. T_{max} and K are assumed to be positive. T_{max} is also assumed to be greater than or equal to the maximum T_i among all firms.

When looking at this decision, the firms compare the profits they would receive between their current level and upgrading. If a firm keeps their current quality, they receive (using a generic network structure and given other players' actions)

$$\pi_i^{Keep}(T_{-i}) = \left(\frac{A + n(T_i + v(y_i^e)) - \sum_{j \neq i}(T_j + v(y_j^e))}{n+1} \right)^2 . \tag{2.32}$$

If a firm upgrades to the higher quality/technology, their net profits would be (again taking other players' actions as given)

$$\pi_i^{Upgrade}(T_{-i}) = \left(\frac{A + n(T_{max} + v(y_{max}^e)) - \sum_{j \neq i}(T_j + v(y_j^e))}{n+1} \right)^2 - K. \tag{2.33}$$

While it is useful to consider the deterministic model in order to improve ones intuition for how the nuts and bolts of the structure work and interact, the following step of finding equilibria brings the usefulness to an end by overprediction. The possible equilibria are numerous in each of the different network types, be it a no network case ($v(\cdot) = 0$), a fully compatible case ($y_i^e = z$), or an incompatible case ($y_i^e = x_i$). Besides obvious statements, such as that an expensive and insignificantly improved technology (in a relative sense) will not be adopted by any firm, equilibria can predict almost any result. There are symmetric all replace or no replace cases, catch up cases where lower type firms replace while high types do not, pre-emption cases where the higher type firm replaces to prevent a low type from replacing, cases where the same type of firms have different strategies, mixtures of some of these, etc.

The flexibility with which the deterministic model can fit different outcomes as equilibria is impressive, but not very useful for public policy recommendations. A model that predicts everything predicts nothing. With this mantra fresh in mind, I turn to a model which better reflects the complexities of reality in accounting for uncertainty in the execution of upgrading the product, and also presents a more manageable set of equilibria.

2.4.2 Probabilistic Upgrades

It is difficult to say much for certain in the deterministic strategy case. In search of a clear result, I turn to a case of uncertain adoption that is affected by firm investment. Specifically, I adapt Reynolds and Isaac's (1992) approach to the Sah and Stiglitz (1987) research program model to the current case. In this formulation, firms choose how much they wish to invest in their adoption process, with increasing investment leading to increasing probability of successful adoption. The model is further simplified by restricting the technology state space to two elements, a low technology T_L and a high technology T_H. Firms with T_H do not face an adoption decision, whereas T_L firms may invest with the hope of becoming a T_H firm in the production period.

Let s_i be the probability of successful adoption chosen by firm i. A firm's adoption cost is determined by cost function $C(s_i)$. I assume that $C(\cdot)$ is increasing ($C'(\cdot) > 0$) and convex ($C''(\cdot) > 0$). The investment stage of the game occurs before the production stage. Let n be the total number of firms in the market, m be the number of high tech (T_H) firms in the market, let i be an arbitrary T_L firm, and let k be the number of non-i T_L firms which successfully adopt the new technology. Firm i faces the following problem during the investment stage of the game with generic network expectations.

$$\max_{s_i} E[\Pi(s_i|s_{-i,m})] = \max_{s_i} \sum_{k=0}^{n-m-1} [s_i \left(\frac{A+n(T_H+v(y_i^e))-((m+k)T_H+(n-m-k-1)T_L+\sum_{j\neq i} v(y_j^e))}{n+1}\right)^2$$
$$+(1-s_i)\left(\frac{A+n(T_L+v(y_i^e))-((m+k)T_H+(n-m-k-1)T_L+\sum_{j\neq i} v(y_j^e))}{n+1}\right)^2]Pr(k|s_{-i,m}) - C(s_i)$$

We can derive the first order conditions from this formulation.

$$\frac{\partial \Pi}{\partial s_i} : \sum_{k}^{n-m-1} [\left(\frac{A+n(T_H+v(y_i^e))-((m+k)T_H+(n-m-k-1)T_L+\sum_{j\neq i} v(y_j^e))}{n+1}\right)^2$$
$$-\left(\frac{A+n(T_L+v(y_i^e))-((m+k)T_H+(n-m-k-1)T_L+\sum_{j\neq i} v(y_j^e))}{n+1}\right)^2]Pr(k|s_{-i,m}) - C'(s_i) = 0$$

The first order condition simply calls for the expected marginal benefit of adoption to equal the marginal cost of replacement probability. I define a_k to be the marginal benefit of replacement conditional on k other T_L firms successfully adopting replacement:

$$\begin{aligned} a_k \equiv & \left(\frac{A + n(T_H + v(y_i^e)) - ((m+k)T_H + (n-m-k-1)T_L + \sum_{j\neq i} v(y_j^e))}{n+1}\right)^2 \\ - & \left(\frac{A + n(T_L + v(y_i^e)) - ((m+k)T_H + (n-m-k-1)T_L + \sum_{j\neq i} v(y_j^e))}{n+1}\right)^2 \end{aligned}$$

With this, I can simplify the first order condition greatly:

$$FOC: \sum_{k}^{n-m-1} a_k Pr(k|s_{-i,-m}) - C'(s_i) = 0 \quad (2.34)$$

This formulation is the same as in Reynolds and Isaac (1992). Existence and uniqueness of the equilibrium can be shown under the same conditions on $C(\cdot)$ and a_k, namely that

1. $\lim_{s_t \to 1} C'(s_t) > a_0 > C'(0)$.

2. $a_0 \geq a_1 \geq a_2 \ldots \geq a_{n-m-1} \geq 0$ with $a_k > a_{k+1}$ for some k.

More formally stated, I make the following proposition.

Proposition R-I. *Let* $\lim_{s_t \to 1} C'(s_t) > a_0 > C'(0)$ *and* $a_0 \geq a_1 \geq a_2 \ldots \geq a_{n-m-1} \geq 0$ *with* $a_k > a_{k+1}$ *for some* k. *Then there exists a symmetric Nash equilibrium in adoption investment decisions that is unique within the class of symmetric equilibria.*

A proof can be found in Reynolds and Isaac (1992). Satisfaction of condition 1 depends on the specification of the cost function. In addition to the initial assumptions of a strictly increasing and a strictly convex cost function, it needs to start flat enough (so firms invest) and end steep enough (so firms do not maximally invest).

Satisfaction of the second condition is not the same in this model as that of Reynolds and Isaac (1992) since the behavior of a_k depends on a different production game and incorporates network effects. The cases are examined individually.

2.4.2.1 ***No Network*** $y_i^e = 0$.

The no network case provides a useful baseline upon which to build. I need to show that the incentive to invest in adoption decreases as more firms are successful in their adoptions. This will show that the game is one of strategic substitutes which, along with restricting attention to equilibria that are symmetric with respect to low technology firms, will grant uniqueness. Recall that a_k is the marginal benefit of replacing T_L with T_H given that k other firms successfully replace. In the no network case, this amounts to:

$$\begin{aligned} a_k \equiv & \left(\frac{A + nT_H - ((m+k)T_H + (n-m-k-1)T_L)}{n+1}\right)^2 \\ - & \left(\frac{A + nT_L - ((m+k)T_H + (n-m-k-1)T_L)}{n+1}\right)^2 \end{aligned} \tag{2.35}$$

Now consider what would occur if one other firm were successful in their replacement efforts. The term inside each of the squared expressions decreases by $\frac{T_H - T_L}{n+1}$. Because of the convexity of the square function, decreasing the inside of both of the squared terms means that the difference between the two has also decreased. In other words, as the number of other firm successes, k, increases, the benefit of replacement, a_k, decreases. Thus, the adoption game is one of strategic substitutes with a unique, symmetric equilibrium in the no network case.

2.4.2.2 ***Compatible Network*** $y_i^e = z$.

Increased adoption by other firms affects a firm in two ways when there are positive network effects. The increased competitiveness in the product market is the first, as firms adjust their prices as more firms have higher technology. This is precisely the downward pressure required on the benefits of adoption as more firms successfully adopt in order to satisfy the conditions for a unique symmetric Nash equilibrium in the adoption game. This effect is captured in the no network case above. The second effect of an increase in k is on the network itself. Higher T's mean more consumers, which increases the value of any good to consumers. This upward pressure has the potential to derail the satisfaction of equilibrium conditions and so must be carefully considered.

Examining a_k, I need to ensure that as more T_L's become T_H's, the benefit reliably decreases. Incorporating the compatible network structure, a_k becomes:

$$\begin{aligned} a_k \equiv & \left(\frac{A + nT_H + v(z_{k+1})) - ((m+k)T_H + (n-m-k-1)T_L)}{n+1}\right)^2 \\ - & \left(\frac{A + nT_L + v(z_k) - ((m+k)T_H + (n-m-k-1)T_L)}{n+1}\right)^2 \end{aligned} \tag{2.36}$$

where z_k is the equilibrium total output when there are k high type firms.

Of particular importance are the parts of the numerators that change as more firms successfully replace. The direct effect of one more k is the same as the no network case. However, network effects must also be considered. z increases as $\sum T$ does. This poses a potential problem, as an increase in the network effect benefits all firms. The concavity of $v(\cdot)$ assumed in the product market helps, as an exploding convex network structure would certainly be a game of strategic complements trying to grow the biggest network possible for firms to share. The threat to the strategic substitutes property stems from the convexity of the square function involved in equilibrium profit. While the marginal benefit directly from the network is decreasing due to its concavity, a very large benefit here increases the value of replacement through the convex profit. With this in mind, ensuring that the adoption game is a game of strategic substitutes requires some further restrictions upon $v(\cdot)$ and the T's. Recall that in the compatible network case, I have previously assumed $v'(\cdot) > 0$, $v''(\cdot) < 0$, and is bounded above. First, I establish a useful property which flows from concavity of $v\cdot$.

Lemma. *Let the model be as specified in, with uncertain replacement and the simplified technology state space of T_H, T_L where $T_H > T_L$, as well as the increasing, concave and bounded $v(\cdot)$. Then $v(z_{i+1}) - v(z_i) > v(z_{i+2}) - v(z_{i+1})$, where i specifies the number of low technology firms who successfully adopt the high technology.*

Proof. Let z be an equilibrium level of total output and let $\hat{z} > z$ be equilibrium total output if one more low technology firm upgrades to high technology. Using the condition for equilibrium total output:

$$\hat{z} - z = \frac{n}{n+1}(v(\hat{z}) - v(z) + T_H - T_L) \tag{2.37}$$

Define $\delta \equiv (\hat{z}) - z$. From 2.37, δ must satisfy:

$$\delta = \frac{n}{n+1}(T_H - T_L) + \frac{n}{n+1}\int_z^{z+\delta} v'(s)ds \tag{2.38}$$

For each z, there is a unique $\delta > 0$ that solves equation (2.38). Moreover, since the RHS of equation (2.38) is strictly decreasing in z, the solution δ of equation (2.38) is strictly decreasing in z. This demonstrates that $z_{i+1} - z_i > z_{i+2} - z_{i+1}$. The result for $v(\cdot)$ follows from concavity of $v(\cdot)$. QED.

We proceed in the fashion of Reynolds and Isaac (1992) as referenced above. Thus, the goal is to show that the replacement game is one of strategic substitutes, as this will then imply a unique symmetric equilibrium. This means I need to show $a_0 > a_1 > \cdots > a_n - m - 1$. I first show that, under some restrictions on the relative strengths of the technological improvement and the network effects, $a_0 > a_1$. The expansion of these terms is messy, but yields useable expressions that allow me to find this result under some circumstances. I then show that, given $a_0 > a_1$ and the above lemma, it is true for $a_0 > a_1 > \cdots > a_{n-m-1}$.

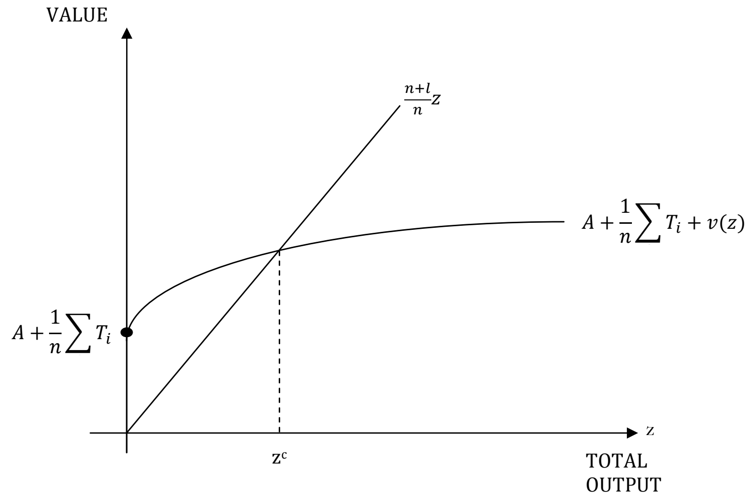


Figure 2.5: Compatible Equilibrium

Proposition 2. *Let $T_H - T_L > \frac{v(z_2)-v(z_0)}{2}$ where z_k denotes the resulting value of z when k firms (including the firm of interest) successfully replace, and $v(\hat{z})v''(\hat{z}) + v'(\hat{z})^2 \leq 0 \ \forall \hat{z} \geq z_0$. Then a unique symmetric equilibrium exists in the adoption game.*

Proof. Consider $a_0 > a_1$. Let $D \equiv A - mT_H - (n-m-1)T_L$ and $F \equiv A-(m+1)T_H-(n-m-2)T_L$, which are useful groupings of messy constants. Note that $D - F = T_H - T_L$. I begin with the full forms of $a_0 > a_1$:

$$\begin{aligned} &\left(\tfrac{A+nT_H+v(z_1)-(mT_H+(n-m-1)T_L)}{n+1}\right)^2 \\ -&\left(\tfrac{A+nT_L+v(z_0)-(mT_H+(n-m-1)T_L)}{n+1}\right)^2 \\ >&\left(\tfrac{A+nT_H+v(z_2)-((m+1)T_H+(n-m-2)T_L)}{n+1}\right)^2 \\ -&\left(\tfrac{A+nT_L+v(z_1)-((m+1)T_H+(n-m-2)T_L)}{n+1}\right)^2 \end{aligned}$$

After expanding both terms and simplifying, I am left with the following.

$$\begin{aligned} &2D(v(z_1)-v(z_0))+2nD(T_H-T_L)+2n(T_Hv(z_1)-T_Lv(z_0))+(v(z_1)^2-v(z_0)^2) \\ >\ &2F(v(z_2)-v(z_1))+2nF(T_H-T_L)+2n(T_Hv(z_2)-T_Lv(z_1))+(v(z_2)^2-v(z_1)^2) \end{aligned}$$

We consider this in three parts, showing each time the inequality to hold true in the individual parts. The three parts are (1) $2D(v(z_1) - v(z_0)) > 2F(v(z_2) - v(z_1))$, (2) $v(z_1)^2 - v(z_0)^2 > v(z_2)^2 - v(z_1)^2$, and (3) $2nD(T_H - T_L) + 2n(T_H v(z_1) - T_L v(z_0)) > 2nF(T_H - T_L) + 2n(T_H v(z_2) - T_L v(z_1))$. This provides a sufficient condition for the overall inequality, though may be stronger than necessary. First, consider (1) $2D(v(z_1)-v(z_0)) > 2F(v(z_2)-v(z_1))$. Since $D > F$ and $v(z_1)-v(z_0) > v(z_2)-v(z_1)$ by the lemma above, the inequality holds and holds even as k increases. Next, consider (2) $v(z_1)^2 - v(z_0)^2 > v(z_2)^2 - v(z_1)^2$. Concavity of $v(\cdot)^2$ would satisfy this equation. The second derivative of $v(\cdot)^2$ set less than zero yields $v(\hat{z})v''(\hat{z}) + v'(\hat{z})^2 < 0$, which was assumed to be true. Finally, consider (3) $2nD(T_H - T_L) + 2n(T_H v(z_1) - T_L v(z_0)) > 2nF(T_H - T_L) + 2n(T_H v(z_2) - T_L v(z_1))$. Recalling that $D - F = T_H - T_L$, this can be reduced to

$$(T_H - T_L)^2 + v(z_1)(T_H + T_L) > T_H v(z_2) + T_L v(z_0).$$

The above lemma implies that $z_1 > \frac{z_2 + z_0}{2}$ and $v(z_1) > \frac{v(z_2)+v(z_0)}{2}$. Thus, replacing for $v(z_1)$,

$$\begin{aligned}
(T_H - T_L)^2 + \frac{v(z_2) + v(z_0)}{2}(T_H + T_L) &> T_H v(z_2) + T_L v(z_0) \\
(T_H - T_L)^2 + \frac{v(z_0)}{2}T_H + \frac{v(z_2)}{2}T_L &> \frac{v(z_2)}{2}T_H + \frac{v(z_0)}{2}T_L \\
2(T_H - T_L)^2 + v(z_0)(T_H - T_L) &> v(z_2)(T_H - T_L) \\
2(T_H - T_L) + v(z_0) &> v(z_2) \\
T_H - T_L &> \frac{v(z_2) + v(z_0)}{2}
\end{aligned}$$

which was assumed to be true. The above lemma, $\frac{v(z_{k+2})+v(z_k)}{2}$ will only decrease as k increases, meaning if it is true for $k = 0$, it is true for all higher k. I have shown that (1), (2), and (3) hold true throughout, for all k. Thus, the entirety of the inequality holds true for all k. QED.

With a game of strategic substitutes established, I conclude that a unique symmetric Nash equilibrium exists in the adoption investment game under a compatible network structure.

2.4.2.3 Incompatible network $y_i^e = x_i$.

The incompatible case is simplified by the high and low type restriction, but remains complicated. The first issue which needs resolution is the post-adoption period competition. If I assume symmetry by technology type offered by the firm, I can simplify the equilibrium reaction correspondences into two dimensions. This case is then similar to the illustrated two firm cases above, with slopes shifting as the number of high and low type firms change. Specifically, assuming all high types produce x_H and all low types produce x_L and letting n_H be the number of high type firms, the equilibrium reaction correspondence for the high type, given x_L, can be characterized by

$$(n - n_H)x_L = A + T_H + v(x_H) - (n_H + 1)x_H. \tag{2.39}$$

Similarly, the equilibrium reaction correspondence for the low type, given x_H, can be characterized by

$$n_H x_H = A + T_L + v(x_L) - (n + 1 - n_H)x_L. \tag{2.40}$$

This means that in any given firm technology state, there could be as many as five potential fulfilled expectations Cournot equilibria.[6] The first possible equilibrium is one in which consumers expect low type technology firms to produce nothing and the low type technology firms do not produce anything. Given the run of the market, the situation is no different to the high type firms than the symmetric firm incompatible network case to which there is a unique symmetric equilibrium. This symmetric equilibrium is the result in this case, which I will call the mono-high-type equilibrium. The second possible equilibrium is the opposite, in which the high type technology firms do not produce anything. A similar situation occurs, resulting in a unique

[6] The situation looks similar to Figure 2.4, but more variable in its intercept and slopes.

symmetric equilibrium among the low type firms. I will call this the mono-low-type equilibrium.

The third and fourth possible equilibria can be thought of as ones where consumers are pessimistic about the success of one type or the other. The lower expectations for one type allows the other type to take a dominant position in the market relative to where they would be if consumers were more optimistic. Within the model, this means that when there are two possible positive production network sizes which could be supported with particular consumer expectations, the consumers expect the lower number to be the outcome. The result is that the type suffering the pessimism performs worse and the other types perform better in equilibrium. These results are akin to a form of lock-in as well, where consumers, for some reason, expect less of one technology type. Specifically, the two possible equilibria are one in which consumers are pessimistic about the high type firms and one in which consumers are pessimistic about the low type firms. I will call these the high-type-pessimistic equilibrium when consumers are pessimistic about the high type firms and the low-type-pessimistic equilibrium when consumers are pessimistic about the low type firms. It is also work noting that, if consumers are generally pessimistic about both firm types, two pessimistic equilibria are attractors; the final outcome of iteration would depend on the starting point. In this sense, there is no need to distinguish an equilibrium where consumers are pessimistic about both firms, as the resulting outcome would be either the high-type-pessimistic or the low-type-pessimistic equilibrium.

The fifth and final possible equilibria can be thought of as one where consumers are optimistic about both firm types' network prospects. The optimism leads to a unique equilibrium where both firms produce more than in their respective pessimistic equilibria; i.e. the high type firms produce more in the optimistic equilibrium than they do in the high-type-pessimistic equilibrium, and vise-versa for the low type firms. Optimism in the context of the model means that when there are two possible positive production network sizes which could be supported with particular consumer expectations, the consumers expect the higher number to be the outcome. I will call this equilibrium the optimistic equilibrium. When there are no pessimistic equilibria, and thus only one non-mono-technology equilibrium, I will also classify this as an optimistic equilibrium.

This situation looks similar to Figure 2.4, which shows the possible compatible network equilibria in a two firm case. The mono-high-type and mono-low-type equilibria correspond to the monopoly equilibria, the pessimistic equilibria correspond to the firm 1 or 2 dominant equilibria, and the optimistic equilibrium corresponds to the split equilibrium. Shifts in the reaction curves due to changes in the number of high and low type firms make the advancement game situation even more complicated, as the slopes and intersects of both curves change.

Narrowing this scope is the first step in the analysis. As the optimistic equilibrium provides a single prediction, I proceed with this when possible. It also does not require consumers to favor one type or another with their expectations. Thus, I will always select the optimistic equilibrium when it exists, and the mono-high-type equilibrium if the optimistic equilibrium does not exist, and the mono-low-type equilibrium if neither of the previous equilibria exist.

These equilibria can be found numerically given a particular network function and model parameters.

Since the equilibrium can be calculated given the technology state, the replacement game can be shown to have a unique symmetric equilibrium in the adoption game in specific cases by calculating the marginal adoption profits from the technology state conditional equilibrium profits and ensuring they are decreasing as more firms successfully adopt. This is sufficient to guarantee a unique optimistic equilibrium in that particular technology adoption game as per the previous results. A more general result is difficult to show without extensive restrictions on parameters. Given this difficulty, I will restrict attention to numerical examples for which a symmetric optimistic equilibrium in investments can be found.

2.5 Simulation of Technological Advancement Models

A purely analytical approach to the single period replacement game equilibrium is not feasible, so I turn to numerical methods to evaluate the properties of the single period replacement game formulation constructed in the previous section. Recall the different parameters of the model: n, the total number of firms; m, the number of the n firms that are initially high type; A, the upper limit of the distribution of the value of owning any single good to the consumer; T_H, the value of the high-type product to the consumer; and T_L, the value of the high-type product to the consumer. I also specify a cost function $C(s_i)$ and a network function $v(\cdot)$. I choose $C(s_i) = \frac{-\ln(1-s_i)}{h}$ (based on an exponentially distributed probability of success with hazard rate h) and $v(y_i^e) = a(y_i^e)^{\frac{1}{2}}$, with the assumption that $v(\cdot)$ is bounded above at some arbitrarily high level. The assumptions to satisfy proposition 2 are checked in each case and found satisfied.

Multiple specifications are evaluated by adjusting these parameters and evaluating the resulting equilibria. I set a base situation of five low type firms with reasonable demand and firm attribute values. From there, I look into marginal changes which reflect different possible situations which may occur in the market. This also provides a basic kind of comparative statics, though the effects can only be trusted locally. The different situations are, in order of their table appearances, the base situation, monopoly ($n = 1$), all but one high type firms ($m = 4$), a lower consumer value on the low-type product ($T_L = 7$), a higher consumer value on the high-type product ($T_H = 12$), a higher cost of technological advancement investment ($h = 0.08$), and a stronger network effect ($a = 1.5$). These varied parameters will allow the observation of some of the interactions which occur in this complicated model. Table 2.1 summarizes the different cases and their parameters. [7]

[7] As a point of reference, the price elasticity of demand in the base case when all firms are low type firms is -0.2.

Table 2.1: Perturbation Cases Considered

Case Name	Parameters						
	n	m	A	T_H	T_L	h	a
Base	5	0	20	10	8	0.1	1
Monopoly	1	0	20	10	8	0.1	1
Single Low Type	5	4	20	10	8	0.1	1
Worse Low Tech	5	0	20	10	7	0.1	1
Better High Tech	5	0	20	12	8	0.1	1
More Costly Investment	5	0	20	10	8	0.08	1
Stronger Network Effects	5	0	20	10	8	0.1	1.5

2.5.1 Methodology

The compatible network case is straightforward in that equilibrium z can be derived through the simple iteration as described in section 3.2.1 and shown in Figure 2.1. I simply iterate between the two expressions from equation 17 until they converge, which solves for equilibrium z. With equilibrium z in hand, I can solve for all desired aspects of the model without adoption. This iteration is done for all possible technological states, so that the payoffs for any possible adoption outcome is known. Using a search algorithm, I then solve for the replacement probability which maximizes expected profit of each firm under the assumption of symmetry. This provides the equilibrium replacement probability, from which the remaining market characteristics, such as welfare, can be calculated.

In the case of incompatible networks, a more complicated process is necessary. I additionally must assume consumers have optimistic expectations in the sense that, in a situation that there are multiple justifiable network sizes, consumers will always expect firms will have the largest network size of the justifiable sizes. I find the equilibrium quantities given the technology state through iteration until convergence between. I then utilize a search algorithm over these quantities to find the adoption game replacement probabilities.

Iteration is accomplished in the incompatible case with the assumption of optimistic consumer expectations. Given the technology state, i.e. the number of high and low type firms, a feasible positive quantity is selected for either firm technology type and used as an input into the other firm type's equilibrium reaction correspondence to get the set of possibly feasible quantities. The assumption of optimistic consumer expectations dictates that I select the highest number from this set. This number is then used as input into the other firm type's equilibrium reaction correspondence and the same process is enacted. Iteration between the equilibrium reaction correspondences continues in this manner until the quantities converge. The quantities which are converged to is the optimistic equilibrium outcome given the technology level. Once the payoffs for each technological state are derived, I can search over the possible symmetric investment strategies to find the equilibrium in the investment game.

Specifically, I use a solver function on equations (39) and (40) with an assumed initial value for the other firm and iterate until convergence for each technology state, taking only the highest value from the solver function to implement optimistic consumer expectations. This provides the equilibria given a technology state, over which I can search for the optimal replacement probability. Alternatively, and as a check on the solver function method, I jointly iterate equation (23) over x_i between the high and low type firms for each technology state until the convergence of equilibrium x's to solve for equilibrium.

All programming was done in Matlab.

2.5.2 Compatible network ($y_i^e = z$)

Reynolds and Isaac (1992) gives a place to start with predicting comparative statics. In a corollary to their second proposition, they show that if the benefits to project success increase, firms will be incentivized to increase their probability of success. Similarly, if costs increase, firm incentives encourage them to decrease the probability of success. With this in hand, I should expect to see a low-type firm's chosen adoption probability increase when the value of the higher technology, T_H, increases, and when the value of the lower technology, T_L, decreases. A higher top technology valuation A would lead to a higher adoption chance as well. Note also that a higher network effect means that every firm gains more benefits for replacing than they would otherwise, and so an increase in network strength a should also have a higher chosen adoption chance. When adoption costs increase, I expect a lower chosen probability of adoption.

All tables for the compatible network policies show the calculated welfare amounts as a percentage difference from their base compatible case counterpart, as the comparison is what is of interest. In other words, Table 2.2 shows the equilibrium values of the standard compatible network case and the subsequent perturbations. Table 2.3 and Table 2.4 contain the percentage differences from the 2.2 perturbation. Thus, the monopoly row will tell you how the monopoly case is different from the base case as a percentage change for easy comparison. This is true for the incompatible network tables and the base incompatible case counterparts as well.[8]

Table 2.2: Base Model (Compatible)

Case	Expected Equilibrium Values				
	s_i	CS	PS	TS	$C(s_i)$
Base	0.4801	408.6695	134.7872	543.4566	6.5412
Monopoly	0.7162	140.5008	268.6383	409.1391	12.5949
Single Low Type	0.4102	429.6950	168.4917	598.1867	5.2797
Worse Low Tech	0.6295	406.8163	121.5430	528.3593	9.9290
Better High Tech	0.7195	459.6518	133.3201	592.9720	12.7118
More Costly Investment	0.3665	402.9213	136.3385	539.1599	7.9188
Stronger Network Effect	0.5189	486.0201	161.8572	647.8772	7.3168

[8]The raw tables are included in the appendix.

Using the top parameters as a default case, I can compare the effects of changes in the model parameters. With regards to the symmetric equilibrium replacement percentage, the changes in the parameters have the predicted and intuitive results. Replacement is more valuable in a monopoly setting as the benefits are exclusive, and so the chosen percentage is higher as in the second row of Table 2.2. Similarly, when there are more firms already presenting the higher technology, the benefits of replacing are less than they would be otherwise, and so the replacement percentage is less as in the third row. Moving away from the number of firms, consumer demand parameters such as a lower technology low type T_L, a higher technology high type T_H, a higher upper limit on consumer's valuation of any good A, and a stronger network effect all make replacement more valuable as reflected by the increased equilibrium replacement probability as predicted by Reynolds and Isaac (1992). When cost is raised or hazard rate (h) lowered (a lower h means a high cost as it interacts through the denominator), I see a lower replacement probability as predicted. Thus, the table is consistent with the predicted results.

There are some important notes concerning surplus differences as well. The expected effect of monopoly is realized, with a decrease in welfare and a large increase in producer surplus. In the other cases the expected effects occur as well compared to the base case. Increases in value and decreases in cost lead to increased welfare, while decreases in value and increases in cost lead to decreased welfare. In the third row, where 4 firms already start with the high technology, welfare is much higher due to the higher technology, and the lack of adoption costs from those 4 firms. The equilibrium replacement rate is decreased as well due to the decreased value of being a high type from the increased competition.

We next consider a couple of potential policies that could be imposed upon the market which are analogous to some real world policies. The first is an imposed high technology requirement, where firms must either offer the high technology good or exit the market. The second is a tax (permit or fee) on or subsidy (investment tax credit) to a firm for moving to a high tech good. This tax or subsidy is set to maximize overall welfare given that firms will act according to their best interest.

2.5.2.1 ***Compatible Policy: Imposed High Technology*** .

At times a government will consider an outright ban of a current technology with eyes on the more widespread adoption of a new one. For instance, in the United States and many other countries around the world, the incandescent light bulb is banned for sale or in the process of being phased out in favor of a new more efficient bulb, or the transition to broadcasting digital signals instead of analogue. Generally, this is done to correct for a negative externality associated with the lower technology, which is not accounted for in this model but could be added. In this case, there are other potential benefits, such as an increased network size or consumer benefits from competition at the higher technology. Here I explore the effects of such a policy on the market currently considered.

The firms face a new payoff function, as failure to adopt yields zero profits instead of the low technology profits. The payoff function can be expressed as

$$\begin{aligned}\max_{s_i} E[\Pi(s_i|s_{-i,-m})] &= \max_{s_i} \sum_{k=0}^{n-m-1} [s_i \left(\frac{A+T_H+v(z_k)}{m+k+1}\right)^2 \\ &+ (1-s_i)*0]Pr(k|s_{-i,-m}) - C(s_i)\end{aligned}$$

This new payoff function pays zero to firms that do not successfully adopt the new technology as they are forced to exit. Adoption costs are still incurred as the firm attempts to successfully adopt. This change in the payoff function does not necessarily retain the same properties under which Proposition 2 held. Fewer firms active in the market increases the profitability as competition slackens, but this condition also eliminates the gap in technology that existed as well. This means that there is no longer a built in profitability gap for the high type firms; consumers are presented with a homogeneous good. Also, fewer firms may also decrease the compatible network size, which would lower the value of the good to consumers and the surplus overall. Thus, there is a tension between the competitive effects of more successfully replacing firms (less profit per firm) and the increased network size with more successfully replacing firms (more profit per firm). If the uniqueness of equilibrium condition is to hold, the possibilities are restricted to cases where the profit shrinking effects of more firms outweigh the network benefits gained, overall resulting in a net decrease in expected profitability as more firms successfully replace to guarantee a game of strategic substitutes and uniqueness of equilibrium. Directly looking at this case, noting that failure to adopt yields zero profits, the condition is simply $x_k > x_{k+1}$, as profits are just output squared. So long as more firms successfully replacing decreases the amount each individual firm produces, there is a unique equilibrium in the technology adoption game. Another way to look at the condition that characterizes this relationship and guarantees uniqueness generally is:

$$x_k = \frac{A + T_H + v(z_k)}{m + k + 1} > v(z_{k+1}) - v(z_k) \quad \forall k. \tag{2.41}$$

Unfortunately, this will depend strongly on the function $v(\cdot)$, as it factors both directly into the equation and indirectly through equilibrium z. For each specific case, I can check to be sure this condition is satisfied. I find that it is easily satisfied for all relevant k's that I investigate.

Consumer, producer, and total surplus account for firm exit, but producer and total surplus incorporate the adoption costs of all firms which attempted replacement whether they were succeeded or failed.

The results for this case are found in Table 2.3. The first takeaway here is that total surplus is lower in all cases in the imposed high technology case than the base case. This holds even in the monopoly case. If technology adoption is an uncertain proposition, forcing the hand of firms to adopt the better technology or shutdown is risky. The loss of competition is an important factor in this risk. While the model may

Table 2.3: Imposed High Technology Policy (Compatible; % Comparison with Analogous Base Model Perturbation)

Case	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base	66.92%	-4.38%	-10.29%	-4.28%	147.12%
Monopoly	34.84%	-3.36%	-7.86%	-6.32%	167.78%
Single Low Type	73.89%	-1.04%	0.36%	-0.64%	136.63%
Worse Low Tech	27.31%	-3.95%	-0.52%	-3.16%	62.80%
Better High Tech	13.66%	-3.18%	4.51%	-1.45%	33.94%
More Costly Investment	109.50%	-0.02%	-15.36%	-7.89%	130.49%
Stronger Network Effect	58.70%	-3.98%	-9.43%	-5.34%	137.05%

not be robust to some circumstances, this still provides a warning to tread carefully when such uncertainty is on the table.

Second, note that the equilibrium replacement probabilities are much higher than their base model counterparts as the consequences of not replacing is no longer low technology profits but exit. The comparative statics here are similar in relative terms to that of the base model with a couple of exceptions. The big exception is found in the third row, where 4 firms already have the high technology. The remaining firm is far more incentivized to successfully replace, and the producer surplus is slightly higher in this case than the base case because of the exit possibility. This policy probably only has a place if there is some unaccounted for outside value to moving to a new technology, such as a negative externality from the low technology good, as it eliminates the low technology good and promotes the highest replacement probabilities.

2.5.2.2 ***Compatible Policy: Subsidy/Tax on High Technology Firms*** .

Another policy to consider is that of a second best policy, similar to Mankiw and Whinston (1986). At first glance, this is not explicitly a game with entry. However, the adoption decision is a choice to exit the market as a low type firm and enter the market as a high type firm. As such, it can be subject to the same problem of too much adoption that manifests itself as too much entry in the entry case. Firms can be individually incentivized to realize the high type profits, while the benefits to the consumer are minimal and the effect on other firm profits is detrimental. One way to regulate this entry type game is through a tax on adoption if entry is too high, or a subsidy on entry if entry is too low, which I pursue.

I assume that a one time tax τ (for instance, a permit or licensing fee) or subsidy ς (such as an investment tax credit) can be imposed on or given to any firm that successfully adopts the high-type technology good in the market. I assume that existing high-types can be identified and are not directly affected by this policy. To find the optimal tax or subsidy the solution to the following maximization problem is required.

$$\max_{\tau} Welfare[s^*(\tau)] \tag{2.42}$$

where $s^*(\cdot)$ is the equilibrium replacement strategy for firms. A tax on the new high-type would mean that firms are overinvesting and that it is welfare improving to incentivize firms to invest less. A tax is represented by a negative number in the table. Tax revenue is presented as a positive amount under the expected equilibrium values. A subsidy for the high-type means firms are underinvesting and that bringing more firms into the high-type market will benefit consumers more than it will cost firms. A subsidy is represented by a positive number in the table. The cost of the subsidy is represented by a negative amount under the expected equilibrium values. I find the optimal tax or subsidy by using a search algorithm over the possible taxes/subsidies given the firms' equilibrium adoption probabilities as a function of the tax or subsidy.

The results for this case are found in Table 2.4. There are improvements in total welfare in all cases, as zero was a possible selection which would have left the situation unchanged from the base model. Of the cases considered, only the monopoly case

Table 2.4: High Technology Second Best Tax/Subsidy Policy (Compatible; % Comparison with Analogous Base Model Perturbation)

Case	Expected Equilibrium Values						
	τ/ς	s_i	CS	PS	TS	τR/ςC	C(s_i)
Base	-4.48	-27.56%	-1.67%	0.33%	1.93%	7.79	-34.72%
Monopoly	17.97	13.38%	1.22%	5.16%	.24%	-14.59	32.67%
Single Low Type	-5.37	-66.58%	-0.67%	1.65%	0.11%	0.74	-72.16%
Worse Low Tech	-6.94	-16.51%	-1.97%	-6.98%	0.33%	18.23	-24.87%
Better High Tech	-10.27	-12.55%	-2.14%	-15.18%	0.38%	32.30	-21.96%
More Costly Investment	-4.33	-39.75%	-1.87%	3.09%	0.29%	4.78	-60.60%
Stronger Network Effect	-4.39	-21.47%	-1.34%	-0.78%	0.18%	8.95	-28.52%

has a subsidy; in all other cases, it is optimal to tax newly adopting firms. The monopolist cares less about the adoption outcome because he has market power in either case, and so under invests. When a subsidy is offered, the monopolist increases his investment to realize the extra revenue. The resulting increased prevalence of high technology boosts the consumers expected surplus and the expected welfare overall.

In all other considered cases, it is optimal to tax the newly adopting firms. This implies the benefits of the high technology are important, but the investments made by firms over provide the high-type good relative to its benefit. The tax makes firms invest less, as the high technology case is less valuable than it was before.

2.5.3 Incompatible Network ($y_i^e = x_i$)

We now look at the same cases under an incompatible network. I assume that, given technological levels, consumers have optimistic expectations about network size. I iterate between firm type equilibrium reaction correspondences until convergence given optimistic expectations to find the optimistic equilibrium as described at the beginning of Section 5. Using the payoffs from the derived equilibria, I search for the equilibrium investment levels.

Table 2.5: Base Model (Incompatible)

Cases	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base	0.5421	341.4571	104.1067	445.5638	7.8110
Monopoly	0.7162	140.5008	268.6383	409.1391	12.5949
Single Low Type	0.4644	357.6007	139.7456	497.3463	6.2437
Worse Low Tech	0.6689	339.3392	94.1529	433.4921	11.0533
Better High Tech	0.7442	384.6648	107.2059	491.8708	13.6336
More Costly Investment	0.4443	337.0629	104.5963	441.6593	7.3441
Stronger Network Effect	0.6084	371.1430	110.1528	481.2957	9.3751

The results for this case are found in Table 2.5. As with the compatible case, the policy changes in later tables are shown as a percentage change from their respective incompatible base case values.[9] The equilibrium replacement probabilities are higher under the incompatible network because the exclusive networks make it more valuable to the firm to have the high technology. The value of the network is less overall as well because of the network incompatibility, leading to the lowered consumer and total surplus. One of the more interesting differences is the increased producer surplus in some cases between the compatible and incompatible networks. When firms offer different values of goods to consumers, the difference has to be made up in price. Because this difference is not just in the technological level of the two goods but also the network sizes in the incompatible case, this increased gap can lead to a higher price for the higher technology good. If this higher price outweighs the effect of the overall loss of network value compared to the compatible case, then high type firms can end up doing better than they would in the compatible case. This is an important result, as it suggests the possibility that firms may not want to create an adapter or otherwise unify the networks under some circumstances. It is also worth noting that the decreased value when networks are fragmented has effect of lowering

[9]Raw tables can be found in the appendix.

overall welfare, despite the increased producer surplus in some circumstances. This is expected as the total amount sold necessarily decreases from the network loss.

2.5.3.1 ***Incompatible Policy: Imposed High Technology*** .

As in the compatible case, I need to confirm that the imposed technology policy can find a relevant equilibrium. The continued assumption of a symmetric equilibrium, coupled with all firms in the market necessarily being high technology firms, means the production game is analogous to the Katz and Shapiro (1985) incompatible symmetric case. To insure a unique symmetric equilibrium in the technology adoption game, the marginal benefit of replacement needs to be decreasing in the number of adopting firms, as demonstrated previously. Recall the firm's first order conditions that $A - \sum_{j=1}^{n} x_j + T_i + v(y_i^e) - x_i = 0$, which in the all high type incompatible case becomes $A + T_H + v(x_H) = (n+1)x_H$. Under the imposed policy, $n = m + k$, where m is the number of existing high type firms and k is the total number of successfully adopting low type firms. Thus, an increase k by 1 is an increase in n by one. According to the FOC, the righthand side becomes steeper as k increases, while the lefthand side does not change with k. The result is that equilibrium x_H must decrease as k increases. I will refer to the equilibrium x_H when k low type firms adopt the new technology as x_H^k. Marginal profits in this case are simply $\pi_H^k - \pi_L^{k-1} = (x_H^k)^2 - 0$, as failure to adopt yields zero profits. Thus, as more firms successfully adopt (as k increases), the profits for each individual firm for adoption decreases. Thus, there is a unique symmetric equilibrium in the adoption game.

The results for this case are found in Table 2.6. In some cases, the incompatible policy would result in negative expected profits for firms. In those cases, I assumed firms would exit until the expected profits were positive. Welfare comparisons between cases are greatly affected negatively by exit. When replacement is more valuable, there is a higher probability of replacement. Compared with the compatible network case, there are fewer firms supported under the incompatible network model. This is

Table 2.6: Imposed High Technology Policy (Incompatible; % Comparison with Analogous Base Model Perturbation)

Cases	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base (n=3)	62.16%	-18.21%	-73.34%	-31.09%	170.49%
Monopoly	34.84%	-3.36%	-7.86%	-6.32%	167.78%
Single Low Type	41.13%	-0.87%	-84.88%	-24.48%	70.63%
Worse Low Tech (n=3)	31.42%	-17.70%	-70.52%	-29.18%	91.15%
Better High Tech (n=3)	19.71%	-17.11%	-68.97%	-28.42%	62.50%
More Costly Investment (n=3)	92.39%	-18.83%	-75.68%	-32.29%	228.43%
Stronger Network Effect (n=3)	46.04%	-16.58%	-71.88%	-29.24%	134.00%

due to the loss of value from the network effects for each firm.

Most interesting in this comparison is the case of monopoly. Since a monopoly is not affected by the change between compatible and incompatible since it has a network made up entirely of its own customers in either case, the numbers are the same. In both cases monopoly provides a lower consumers surplus to the market than any other. However, in the compatible case, monopoly is the worst outcome for total welfare. In the incompatible case, monopoly is the best outcome when considering total welfare. This highlights the conflict in the imposed technology case between the intense homogeneous competition with a fractured network that will result if more than one firm succeeds and the willingness to invest and succeed. Since the monopolist is guaranteed sole monopoly high type profits if he succeeds, he can afford to invest more than the multiple firm counterpart. The monopolist's successful adoption chance is 96.57%,[10] and if he succeeds he will have high type monopoly profits. Compare that with the base case under this policy. Firms in this case pick a lower success rate of 87.91%. The chance of a monopolist from this case is 3.85%, and only 1.28% for a particular firm of being the monopolist. There is only a 4.02%

[10]This precise number and the others mentioned in this paragraph can be found in Table A.5, in the appendix.

chance of having less than 2 of 3 firms in the market. An extra competitor is tough competition wise, bringing profits (and thus investment) down, but even tougher as it fractures the network. The monopolist can offer a more valuable good to consumers as the network is united, whereas more firms have to split the network effects.

The extreme degradation of total surplus brings into question the effectiveness of forced replacement policies in these kind of markets, especially when incompatible networks are in play. Unless there is a strong outside justification, such as extreme pollution by the low technology good or some other negative externality, the policy seems ill considered.

2.5.3.2 ***Incompatible Policy: Subsidy/Tax on High Technology Firms*** .

The results for this case are found in Table 2.7. The optimal tax is higher in the incompatible case compared to the compatible case. This suggests that there is a bigger problem of over adoption, which is not surprising due to the weaker market conditions caused by the fractured networks. Much more is at stake in the incompatible case, as the individual firm network sizes swing greatly depending on if you have the high or low technology. This means firms are more prone to heavily invest in success as it greatly benefits them, even when the resulting benefit to society is outweighed by the cost to adopt. The exception is the monopoly case.[11]

Compared to the compatible case, consumers surplus, producer surplus, and total surplus are all lowered. The optimal tax is also higher in the incompatible case than the compatible one. This suggests that firms are more likely to want to overinvest in the incompatible case than in the compatible case. Since the differences in networks between the high type and low type goods exist, it is much more valuable to be a high type firm than a low type firm in the incompatible case, which drives the additional overinvestment. Higher taxes, which reduce the reward of success, help temper the

[11] The monopoly case in the incompatible case is equivalent to monopoly in the compatible case since under monopoly, $x_M = z$

Table 2.7: High Technology Second Best Tax/Subsidy Policy (Incompatible; % Comparison with Analogous Base Model Perturbation)

Cases	Expected Equilibrium Values						
	τ/ς	s_i	CS	PS	TS	τR/ςC	C(s_i)
Base	-8.99	-43.50%	-3.10%	1.67%	1.11%	13.77	-53.14%
Monopoly	17.97	13.38%	1.22%	5.16%	0.24%	-14.59	32.67%
Single Low Type	-12.42	-99.99%	-1.24%	5.03%	0.52%	$7 * 10^{-4}$	-99.99%
Worse Low Tech	-13.81	-28.06%	-3.75%	-15.35%	1.39%	33.22	-40.65%
Better High Tech	-19.84	-22.08%	-4.15%	-31.70%	1.54%	57.54	-36.41%
More Costly Investment	-8.74	-57.30%	-3.35%	7.90%	1.19%	8.29	-64.19%
Stronger Network Effect	-12.40	-43.08%	-3.35%	-1.40%	1.56%	21.47	-54.67%

profit motive and reduces investment in adoption so that the amount spent is closer to its expected market benefit.

2.6 Rent Dissipation

Rent dissipation through the replacement investment is an important effect, and one that varies by network type in a large way. Entry into an incompatible market can decrease overall welfare as the network effects become diffuse. While technological advancement is beneficial to consumers, the results can be murky for firms. Notably, firms have to invest money to have a chance at keeping up technologically. These investments act as rent dissipators within the market, and the effects differ depending on the type of networks that are present in the market. Competition between firms in short cycle technology markets is not just in prices, but also in these investments, which can quickly eat away at profits. This makes entry in these markets, even without a fixed entry cost or fixed costs, potentially welfare decreasing. With the simulated market outcomes, these rent dissipating effects are shown to be important at the firm level. I compared the two network cases[12] as the number of firms increased

[12] The parameters are those of the base case: m=0, A=20, $T_H = 10$, $T_L = 8$, h=0.1, a=1

Table 2.8: Compatible vs. Incompatible by Number of Firms

	Compatible					Incompatible				
n	s_i	CS	PS	TS	n*C(s_i)	s_i	CS	PS	TS	n*C(s_i)
1	0.72	140.50	268.64	409.14	12.59	0.72	140.50	268.64	4396.54	12.59
2	0.67	258.51	237.36	495.88	22.15	0.69	237.47	238.66	452.57	23.56
3	0.61	330.60	194.35	524.94	28.03	0.64	290.12	196.15	455.43	30.84
4	0.54	377.03	160.25	537.29	31.29	0.59	321.43	165.28	450.99	35.72
5	0.48	408.67	134.79	543.46	32.71	0.54	341.46	143.16	445.56	39.06
6	0.42	431.22	115.81	547.03	32.84	0.50	354.99	127.01	440.62	41.39
7	0.37	447.89	101.52	549.41	32.05	0.46	364.55	114.98	436.44	43.08

to investigate the effects of tighter competition on the market and rent dissipation. The overall result is that increased competition at first increases welfare in both cases, but in the incompatible case welfare gains quickly succumb to the pressure of increased investment costs as more firms are added, while the compatible case continues to improve welfare as the number of firms increases. The results are displayed in Table 2.8.

For both cases total welfare increases for the first four firms in the market. With the fifth firm the two cases begin to part ways. Consumer surplus continues to rise and producer surplus continues to fall in both, as is intuitive with increased entry. However, total welfare in the incompatible case begins to decrease as the number of firms increase. The reasons for this are two-fold. First, the nature of incompatible networks makes it so that consumer surplus from network effects decreases as more consumers are spread among the many firms. So while there is a benefit to consumers through a downward pressure on prices, the benefits are not as pronounced as they are in the compatible case. Second, the replacement probabilities are higher in the incompatible case meaning the amount spent on replacement costs is higher than the compatible one. While the per firm amount spent on replacement is decreasing as the number of firms increase, the total amount continues to increase in the incompatible case even when it decreases in the compatible case. This can be seen by comparing the total cost columns with six and seven firms between the two network cases, where total cost decreases for the compatible network from 32.8390 to 32.0542, while the incompatible total cost goes from 41.3852 to 43.0812.

In other words, the network effects strongly drive the comparative statics. Under the compatible case, more firms always means more network effects, which increases the value of the good, increasing the number of consumers in the market. Since both low and high type firms share the network size in the compatible case, firms only need consider their marginal increase of the network size which would result from a successful switch to a high technology good. Since this benefit decreases as more consumers enter due to the concavity of the network valuation function, over time and in a large enough market it is trivial, and a more standard competition reigns; entry causes welfare to increase, but individual firms get less and less. This contrasts sharply with the incompatible network case. As more firms enter, the market becomes increasingly fractured. While the decrease in market power by firms benefits consumers and overall welfare at first, this effect can eventually become dominated by the decreased benefits from network effects of the fractured market. This drives down the overall valuation to consumers of any one good, and so decreases the number of consumers who decide to enter the market. This is not the only problem in the incompatible case. Since firms get fewer consumers as competition increases, this negatively affects their network size. The concavity of the network valuation function in turn makes it so that as fewer and fewer consumers use any single firm's good, the marginal difference in network valuation between being a high type firm and a low type firm increases as more firms enter, putting an upward pressure on optimal investment. This can then exacerbate the problem of over-investment which was apparent from the optimal taxation policy. In short, the decrease in to value of the good due to decreased network size is what causes the loss in overall welfare from entry in the incompatible market.

2.7 Concluding Thoughts

Firms do not necessarily offer the best possible good to consumers in the market. Firms make important decisions about when to change the goods they offer which can be complicated by a myriad of considerations, such as network effects. This paper explores this area by extending Katz and Shapiro's (1985) network model to incorporate technological differences between goods. Over this model I add a firm decision to adopt a new technology into their good. The current work shows that this situation can be messy and complicated, but in some insights can be found. Using a two technology case, I adapt the methods of Reynolds and Isaac (1992) to analyze the adoption decision when adoption of a new good is uncertain.

I find that better available technology, lower adoption costs, and stronger network effects increase the rate of technological adoption and social welfare. Incompatible networks have lower total surplus but higher adoption rates. Producer surplus can be higher under an incompatible network, as the increased product differentiation can provide high type firms with extra market power; however, total and consumer surplus are lower.

I also considered an imposed high technological adoption policy and a tax/subsidy policy on adoption. While imposing a high type technology yields the highest adoption rates, the loss of competition due to exit negatively affects overall welfare in all cases. In the incompatible imposed requirement policy, monopoly can provide the highest total surplus because the fragmented networks in multi-firm markets lower the value of the good. Consumer surplus remains the lowest under monopoly in this case. The tax/subsidy policy imposes taxes on the adopting firms in most cases as firms overinvest, as the profit incentive does not line up with the smaller value of an additional high type firm. The exception is monopoly, where the monopolist makes strong profits with either technological level and so under invests. Firms which face incompatible networks overinvest more than their compatible counterpart as individual profits vary more with network size. Thus, optimal taxes are higher in the incompatible case.

One of the most distinct differences in outcomes between the compatible and incompatible network types is the effect of increasing competition on rent dissipation. While in both cases more firms increase consumer surplus and lower producer surplus as per a standard market, total welfare starts to decrease in the incompatible case relatively quickly. This is due to the higher adoption costs that incompatible firms pay in equilibrium coupled with the fracturing of the market network effects. Under compatible network effects the value to consumers continues to increase, and so total surplus does not decrease, though the replacement probabilities tend towards zero at a high number of firms.

I add an often ignored step to the process of technological advancement in directly modeling the firm decision to incorporate new technology into their product in a market with network effects. The model suggests that markets with incompatible network effects do better with fewer firms as the fractured market and investment in technology adoption dissipate rents at a high rate. A further extension to this model which incorporates a multi-period structure and an innovation rate would account the dynamic nature of rapidly changing technology adoption decisions, and allow the effects of adoption decisions on innovation to be explored.

CHAPTER 3

Agent Bargaining*

3.1 Introduction

Negotiations take many forms in the real world, and rarely do only two parties have a stake in negotiating outcomes. For example, when a real estate agent is assisting in the search for a home on behalf of interested buyers, or a sports agent talks directly to the team when determining contract details, a third party has been introduced. With only two sides to each negotiation, having three interested parties might muddy the incentives and actions of agents. Does the two party negotiation change when there are three interested parties? Do people care about the equality of the outcomes? To further this line of inquiry, we conduct a laboratory experiment to test some properties and predictions of a simple three player bargaining model. Our experiment is built around the interactions of three agents: an Athlete, a Bargaining Agent, and a Team. The Athlete and Bargaining Agent share in a venture where the Bargaining Agent will negotiate on behalf of the Athlete for a new player contract with the Team. The Athlete and the Bargaining Agent have agreed to a percentage split of however much is negotiated for them to share. The Bargaining Agent is fully in charge of negotiations, whereas the Athlete does not participate in the negotiating process. The Team is interested in the professional services of the Athlete for a profitable business opportunity. The Team and the Bargaining Agent negotiate a split of the profits between the Team and the partnership of the Athlete and the Bargaining Agent.

Purely selfish motivations are not often the sole determinant of realized actions, especially in the laboratory. For many reasons, negotiators will internalize the out-

*This essay is based on joint research with Alex W. Roomets come of other players' payoffs. We propose two potential models to fit this situation. Sequential-Nash-bargaining, or two Nash bargaining periods solved using backwards induction, provides a classic, rational approach to the problem. Alternatively, a weighted minimization of differences between the payoffs of the different agents offers a behavioral approach which tries to capture some form of fairness. We also calculate an alternative Nash specification to best fit our data. Our experiment tests a necessary condition for sequential-Nash-bargaining, which is that the percentage bargaining split in the current period should not be affected by expected or known results in other bargaining periods. We test this condition under the assumption that Nash bargaining power represents a person's general strength at bargaining in any situation, as well as the weaker assumption that the bargaining power represents a person's strength at bargaining in a specific bargaining situation. The experimental results suggest that sequential-Nash-bargaining is a poor fit for the data in both cases, though much worse under the first assumption. The behavioral approach better explains our experimental results.[2] Specifically, the negotiation is set up such that all players cannot receive an equal split. Negotiators tend to focus on a split that makes equal the payoffs between the active negotiators, the Team and the Bargaining Agent. When it is not costly to do so, these two active negotiators have a tendency to match their payoff shares. When it is costly to match the payoffs of the active negotiators, the fairness focal point switches to equating the payoffs of the Team and the Athlete. This kind of split is in line with our behavioral metric, in contrast to the predictions of sequential-Nash-bargaining that the first period bargaining outcomes should be the same across treatments.

Our paper fits into the extensive literature on agent bargaining and middle men, in which an agent serves as an intermediary or as representation for another party. The Bargaining Agent in our model acts in some ways as a vested middle man,

[2]A better fit is expected as the model is more flexible. For this reason, we also include a flexible version of Nash bargaining for comparison.

negotiating the payoff to be split between the Bargaining Agent and the Athlete. Levitt and Syverson (2008a,2008b) show that real estate agents negotiate differently when selling their own properties versus the properties of their clients, keeping their houses on the market longer and realizing a higher price. Bruce and Santore (2006) find that firms in the real estate market do not offer the lowest possible commission to their sellers because it does not sufficiently incentivize agents. Chinloy and Winkler (2010) use real estate data to provide evidence that agents work more and earn more when they collect 100% commission above some flat price versus a more traditional split. This boost is attributed to solving the moral hazard problem which was present; as usual, increasing the risk held by the agents aligns incentives properly. Fisher and Yavas (2010) construct a model in which percentage commissions can get better performance from agents when they compete over who makes the sale. Moving away from real estate, Lefgren, McIntyre, and Miller (2010) discover that lawyers tend to push more profitable options at the expense of their clients. This literature tells us that the incentive structure will be important in the design of our experiment.

Our work also touches the strand of the negotiation literature involving low cost/costless negotiations. From a theoretical standpoint, when negotiations are costless, any percentage stake in the negotiated amount is enough to incite maximum effort on the part of a negotiator to get the highest stake possible. In other words, it makes no difference whether the negotiator is set to receive one percent or one hundred percent of the negotiated amount, they will bargain for the biggest possible pile of cash. We seek to test this theoretical prediction by creating a simple bargaining environment, eliminating the typical costs of bargaining, and varying the share received by a bargaining party in an experimental setting.

We address other gaps in the literature on negotiations as well. The primary area is in the vetting of sequential-Nash-bargaining as a viable equilibrium concept. Proposed by Moresi, Salop, and Sarafidis (2008), the authors develop the concept and prove some of its useful properties. This concept provides a simple structure for analyzing the results of a multi-period bargaining situation involving multiple partners. Kitchens and Roomets (2015) put the concept to use in an eminent domain experiment. They find that sequential Nash bargaining deos not fit the observed behavior of their situation. However, the authors state in their analysis that there were many potentially confounding issues with their experiment that may have made sequential-Nash-bargaining perform poorly. Our experiment directly addresses this issue by providing a simple, best case scenario for the applicability of the sequential-Nash-bargaining solution to multi-period negotiations between more than two agents in order to provide evidence as to the effectiveness of sequential-Nash-bargaining.

Our work also can be connected to the literature on exchange rate bargaining, specifically the results of the experiment conducted by Kagel, Kim, and Moser (1996). They find that changing interest rate and information conditions affect the behavioral focal point that is important. In the experiment, subjects played an ultimatum game. One side offered some number of chips and the other side accepted or rejected the offer. The value of one chip to each subject was varied, as was the knowledge of the other subjects' values. The focal points of this game seem to be an equal split of the number of chips themselves or an equal split of the value to each player depending on the conditions. It is also notable that the Nash prediction of one or zero chips offered to the subject does not do well as a predictor. In our experiment, we observe a similar lackluster performance of the sequential-Nash-bargaining solution and a similar move towards behavioral focal points which switch depending on the given conditions. Kagel, Kim, and Moser (1996) also find that rejection rates become higher when multiple focal points are common knowledge to the players. Our experiment should be more flexible in allowing a back and forth negotiation not present in the ultimatum game, relieving some of the pressure to reach no agreement, but these types of focal point preferences can suggest certain behavior for different negotiators under different circumstances which we can use to make some predictions under this behavioral paradigm. Fairness as a focal point is one that has long been evidenced to influence bilateral bargaining situations, and a preference for equity is one proposed by Fehr and Schmidt (1999) as a unifying explanation for these observations.

3.2 Model

We are interested in the following 3 person bargaining situation. There are 3 players, one each of an Athlete (A), a Bargaining Agent (B), and a Team (T). The players A and B together make up a partnership. A entrusts the partnership's external dealings to B. The partnership and T have an opportunity to participate together in a profitable venture.[3] If either the partnership or T do not participate, there is no profit made. The partnership, represented by B, and T negotiate over a split of the total profit dollar amount, V. We call the partnership's negotiated portion of the profits "compensation". At some other point in time, either before, after, or conditional on the negotiation over the profits between the partnership and T, the negotiation to split the partnership's compensation between the partnership members A and B occurs. The end result is that, if the partnership and T agree to perform the profitable venture, the profits V are divided among the 3 players through two separate negotiations. If there is no agreement, no profits are realized and all parties receive 0.

We represent this situation using a Nash bargaining power structure. Each player i's bargaining power is represented by α_i. The payoff of each player is equal to the amount of V player i gets represented by P_i. To account for the multiple negotiations that occur in the situation, we propose a sequential procedure. If we assume that at least A and B have knowledge of each other's bargaining power, it will not matter when the compensation negotiation occurs; the matter will already be resolved if it is before the profit negotiation, or the result can be deduced if it occurs after the profit negotiation. As we assume for the model that each players bargaining power is

[3] Say, where A will play for T given an agreed upon salary.

common knowledge, the timing of the negotiations is flexible. A simultaneous game does not fit well with the situation we wish to model. A is excluded from the external negotiations of the partnership; if the negotiations were happening simultaneously, A could easily participate. Simultaneity also complicates the Nash bargaining model in ways that confound the situation of interest. A sequential negotiation structure is simpler, better reflects the agent bargaining situation, and does not exclude interesting cases.

Thus, we model the negotiation procedure as follows. In the first stage, B and T bargain over the split of the profit between the partnership and T. In the second stage, the partners A and B negotiate over the compensation $(V - P_T)$. If a deal is not reached in any stage, all players get nothing. All values are common knowledge.

3.2.1 Sequential-Nash-Bargaining

To solve this model, we need an appropriate solution concept which can address the cooperative negotiation portion of the game as well as the multi-period aspects. Standard Nash bargaining does not account for the sequential nature of the model. We can still make use of the structure of Nash bargaining if we apply sequential-Nash-bargaining to this situation. Sequential-Nash-bargaining solves the multi-period Nash bargaining sessions through a backward induction approach. To evaluate the predictions of sequential-Nash-bargaining on the scenario, we start at the second stage (compensation negotiation), solve, and then apply that result in the first stage (profit negotiation). In the second stage, Nash bargaining between the Athlete and the Bargaining Agent can be represented by the following maximization problem:

$$\arg\max_{A}[(P_A)^{\alpha_A} * (V - P_T - P_A)^{\alpha_B}] \tag{3.1}$$

$$\arg\max_{A}[\alpha_A \ln(P_A) + \alpha_B \ln(V - P_T - P_A)] \tag{3.2}$$

$$FOC : \frac{\alpha_A}{P_A} - \frac{\alpha_B}{(V - P_T - P_A)} = 0 \tag{3.3}$$

$$P_A^* = \frac{\alpha_A}{\alpha_A + \alpha_B}(V - P_T) \tag{3.4}$$

Using the solution to the second stage, we can apply backward induction to solve for the first stage negotiation between the Bargaining Agent and the Team:

$$\arg\max_{P_T}[\alpha_T \ln(P_T) + \alpha_B \ln(V - P_T - (\frac{\alpha_A}{\alpha A + \alpha_B})(V - P_T))] \tag{3.5}$$

$$= \arg\max_{P_T}[\alpha_T \ln(P_T) + \alpha_B \ln((1 - \frac{\alpha_A}{\alpha A + \alpha_B})(V - P_T))] \tag{3.6}$$

$$FOC : \frac{\alpha_T}{P_T} - \frac{\alpha_B}{V - P_T} = 0 \tag{3.7}$$

$$P_T^* = \frac{\alpha_T}{\alpha_B + \alpha_T}V \tag{3.8}$$

It is important to note that P_T^* is not a function of α_A. In other words, the bargaining power of the Athlete will have no effect on P_T under sequential-Nash-bargaining. This is because no matter how large or small the amount being negotiated over is, the players still want the largest part of the pie they can get. Whether the past or future negotiation goes well or poorly, the player still wishes to do their best in the current negotiation. This property is one on which we will hang our testable hypothesis. It is difficult to measure any of the bargaining powers because all of the bargaining powers are relative. These bargaining powers are also difficult to make common knowledge, as it is described in relative terms, especially when it is tough to put any kind of measure on one's own bargaining power. To avoid these problems and create a testable hypothesis, we propose using a fixed percentage split for the second period of bargaining (compensation negotiation) which occurs between the Athlete and the Bargaining Agent. This makes clear what the split of compensation in the second period will be, solving the backward induction portion of the problem for the Bargaining Agent. It implies an $\frac{\alpha_A}{\alpha_B}$ that subjects may have some intuitive sense for. It is also easily made common knowledge. While this deviates from the specific situation of interest as the Athlete never personally bargains, this set up allows us to see if a necessary condition for sequential-Nash-bargaining is true, namely, that negotiations of this period are not affected by the prospects of future negotiations. Let ϕ be the percentage of the compensation received by the Bargaining Agent. We thus form our hypotheses.

Null Hypothesis: The percentage split of the compensation between the Bargaining Agent and the Athlete (ϕ) will have no impact on the split of the profit, and, in particular, P_T.

Alternative Hypothesis: The percentage split of the compensation between the Bargaining Agent and the Athlete (ϕ) will have an impact in either direction on the split of the profit, and, in particular, P_T.

We see no reason to think that either the Bargaining Agent or the Team will have a structural advantage over one another, and so with subjects drawn from the same pool, bargaining power should be on average equivalent, resulting in a 50/50 split of the total pot in the first round.

Assumption: There is no structural advantage or disadvantage to bargaining power between the Bargaining Agent and the Team negotiating positions in their bargaining session. In other words, the bargaining power between players is, on average, equal.

Since the Bargaining Agents and the Team will be pulled from the same population, their bargaining powers will on average be equivalent under this assumption, which would predict the aforementioned 50/50 split on average. However, if the assumption fails and some structural advantage does exist, we also perform tests between treatments to see if the Team shares P_T differ which, if the null hypothesis holds, should not be true. Changes between treatments would show that the Athlete's bargaining power is affecting the negotiation between the Bargaining Agent and the Team, an outcome that cannot be if the structure underlying sequential-Nash-bargaining is true.

There are possible reasons to doubt the null hypothesis. If agents have preferences over the payoffs of others, there are many different possible effects. Agents may exhibit inequality aversion. Agents may be generous, or feel guilt. Agents may also harbor resentment, affecting their behavior. In a post-Results section, we propose and analyze a criterion to address some of the possible alternative behavioral explanations for the results.

3.3 Experimental Design

Our design is focused on a simple negotiating situation which involves two negotiators with a percentage stake in the outcome, which can be varied, and a third party that has no direct hand in the negotiating process. Kitchens and Roomets (2015) provides results suggesting that sequential Nash Bargaining equilibrium may have little predictive value in these situations. However, their results are clouded by the complexity of their process. A cleaner design that is aimed specifically at investigating this issue should provide a clearer picture about the uses of the sequential-Nash-bargaining equilibrium concept. This experiment also serves as a robustness check of the results of the exchange rate bargaining experiment performed by Kagel, Kim, and Moser (1996).

Our identification strategy is to vary the fixed percentage split of the compensation between multiple treatments and see if this affects the negotiated split of the profits between the partnership and T. Sequential-Nash-bargaining led to the hypothesis that the bargaining power of A, α_A, should have no effect on the outcome of the profits negotiation. Setting the percentage split of compensation between A and B allows us to create conditions under which α_A is relatively higher or relatively lower. This does deviate from the three player bargaining situation as A no longer directly bargains, but it allows us to test a necessary condition for sequential-Nash-bargaining to be true, namely, that B knows future bargaining results and is not affected by them in the other negotiation. While the absolute bargaining powers of B and T remain unknown, the fact that we are drawing the players randomly from the same population and randomly assigning them to player roles should, on average with a large enough sample size, result in similar bargaining power players in the B and T roles. Since each treatment varies α_A, if the hypothesis generated from sequential-Nash-bargaining that α_A does not affect the profit negotiations between B and T is true, the profit negotiation outcomes should be similar on average between treatments. Put another way, if the hypothesis is true we should not see any change in the payoff of T, P_T, between treatments with enough data.

Importantly, we test this as a one shot game. Each subject plays the game only once. While this does prevent us from trying to get a better measure of each subject's bargaining power, a potentially useful piece of information, it eliminates the much more problematic issues of repeated game and learning effects. To the extent that fairness might be important, an implicit understanding or feeling of fairness should be captured in a one shot game.

The experiment generally proceeds as follows. B and T negotiate over 15 dollars in profit, with the second stage compensation split between B and A commonly known to all parties, as in the model and under sequential-Nash-bargaining. The A does not take part in active negotiations, but receives his payout given the agreed upon split between B and T and the fixed, commonly known split between B and A. We have conducted five treatments, four with the Bargaining Agent receiving different shares of what is negotiated (0, 20, 40, 60 percent). The fifth is a control in which we test a case where we have the same payoff distributions for the negotiators but there is no Athlete. This is a robustness check for whether there is an observable difference between having the extra payoff go to a A or it simply going to waste. The zero percent share is also notable in that sequential-Nash-bargaining makes no explicit prediction. However, the limit at the percentage share approaches zero percent has the same predictions from sequential-Nash-bargaining as the other treatments.

3.3.1 Procedure

This experiment was conducted at the University of Arizona's Economic Science Laboratory (ESL) using the software package ztree. The 253 subjects were drawn from a database of undergraduate students at the University of Arizona in early April of 2012. Average payments for each treatment ranged from 9.47 USD to 9.70 USD per subject for 20 to 30 minutes of time. This resulted in a total of 89 total observations across the 4 treatments and the control. 78 reached an agreement and did not agree in chat to split payoffs after the experiment. By type of treatment, we have 16 observations for the group that faced a share of 60%, 17 observations for the group that faced a share of 40%, 16 observations for 20%, 15 observations for 0%, and 14 observations in the control.

The experiment was conducted as follows. Doors to the waiting room were opened 5 minutes before the requested show up time. Subjects lined up to be checked into the system, and personal items, including bags, computers, cell phones, and food were securely held for the subjects until the experiment was complete. Details of the experiment beyond a simple title were not known to the subjects before arrival, including the session's treatment.

Once subjects were checked in, they were randomly assigned to a role as Bargaining Agent, Athlete, or Team, though these groups and roles would not be apparent to subjects until the beginning of the experiment. If at the beginning of the session the number of participants was not a multiple of 3 (therefore not allowing for the formation of a 3-member group), the extra were given their show up fee and a chance to return to a different session. With the overflow subjects gone, instructions[4] were passed out to subjects. Instructions were the same across treatments save for the percentage split of the compensation and, in the control, the number of players and where the second part of the compensation goes. Once all subjects had a page of instructions, the instructions were read aloud and questions answered by the same investigator throughout the experiment for every session. After the instructions were read, a last chance was given to decline participation. Bargaining Agents and Teams proceeded to their computers in the computer room, while Athletes remained in the waiting room. Athletes were allowed to chat quietly while they waited for their resulting payoffs.

Once all Bargaining Agents and Teams were at their assigned computers, the program was begun. Subjects first answered 5 questions meant to help them understand the negotiation process and software, as well as how payoffs were being determined for each role. Subjects could not proceed until they answered all 5 questions correctly, at which point they made it to a waiting screen. The program then proceeded to the negotiation session.

The negotiation process was as follows. The negotiators have 5 minutes to negotiate a split of the 15 profit dollars. If no agreement is reached, all parties get nothing but the show up fee. If they reach an agreement, the negotiators must wait for the remainder of the five minutes, which was specified in the experiment's instructions.[5] During negotiations, the negotiators can communicate with each other though a chat box. This chat is unfiltered and unlimited. Negotiations are over a

[4] Available in the appendix.

[5] There are potential costs to the bargaining process which could be incorporated into this model. The two most prominent costs are the time cost to bargaining and the effort cost to bargaining. Since time is valuable, using time is costly. Through our experimental design, we combat the effects of time costs by giving a fixed bargaining period which must be endured no matter if or when an agreement is reached. With their phones and other entertainment sources taken away, only the benefits of self amusement were available as an incentive to finish bargaining quickly. In this way we are mostly justified in excluding time costs from the model. Effort costs are more difficult to measure and eliminate. However, since it is practically effortless to say yes to an offer, these costs are mostly accounted for in the bargaining power coefficient.

split in a lump sum of money, and so is one dimensional with discrete choices. In addition to the chat box, subjects have a box to enter proposals, a boxes showing the current proposal under consideration, and three buttons, one labeled “Calculate” and one labeled “Propose”, and one labeled “Accept”. Either negotiator can propose a split to the other at any time by putting a number into the proposal box, calculating the outcome by hitting the “Calculate” button, and clicking the “Propose” button.[6] This allows each negotiator to see the resulting payoffs to all parties of the proposal to help reduce potential confusion. Once a proposal is sent, it appears in the current proposal boxes for both players. Both negotiators are free to suggest new proposals, chat, calculate future potential proposals, or accept the current proposal.

The subject who proposed the current proposal is assumed to have accepted his own proposal. If the negotiator considering the proposal chooses to accept the proposal, he can click on the “Accept” button. If a proposal is accepted, both negotiators go to a final check screen displaying the resulting payoffs from the proposal and both are asked once more if they are sure they accept the proposal.[7] If both click “Accept”, then an agreement has been reached and the screens display a waiting screen for the remainder of the negotiating period. If either negotiator clicks the “Decline” button, both are returned to the negotiating screen, where they can chat, calculate, propose, and accept proposals as before. After the 5 minute negotiating period is over, subjects are paid according to what they negotiated plus their show up fee.

3.4 Results

Our results provide evidence that the sequential-Nash-bargaining equilibrium does not fit the behavior in this situation well. The behavior of subjects seems to be affected by the split they are to receive in the second period in contrast to the predictions of

[6] Proposals resulting in fractions of cents were not allowed.

[7] This is to prevent a last minute quick switch of the numbers being agreed upon, intentional or not.

Table 3.1: Agreement Rates by Treatment

B Share	Obs	Agreements	Agreement Rate
0%	19	15	78.94%
20%	19	16	84.21%
40%	20	17	85%
60%	17	16	94.12%
Control (40%)	14	14	100%

sequential-Nash-bargaining. The proposed minimization of differences criterion seems to perform better, providing justifications for observations in all treatments save the 20% case.

Our data set has a total of 89 observations. 80 of these observations reached an agreement, yielding an overall agreement rate of $\frac{80}{89} \approx 89.89\%$. Additionally, there were two instances which appear to be collusion to split money outside of the experiment to keep money away from the Athlete as determined by the chat logs. As these players were playing a different game than the one we were testing, these observations were also left out of consideration. These two incidents occurred in the 20% and 40% treatments. By treatment, the agreement rates (including collusion outcomes as non-agreements) are shown in Table 3.1.

Theory predicts that an agreement will always be reached, but we see this is not the case in the laboratory. There may be a slight trend toward fewer agreements as the Bargaining Agent gets less of what he negotiates for. This may be because a satisfying outcome is more difficult to find for the negotiators, but is not predicted by theory.

Agreement rates increase as the Bargaining Agent gets a larger share of what they negotiate. When they receive nothing, 0%, the agreement rate is 78.94%. The Bargaining Agent has a very credible threat to not agree, as he receives zero from any agreement or non-agreement. Thus, the 0% treatment yielding the lowest agreement rate is expected. The 20% and 40% treatments have higher agreement rates, at 84.21% and 85% respectively. These treatments are similar in that that amount that the Bargaining Agent keeps of what he negotiates is less than half. This contrasts with the 60% treatment, with an even higher agreement rate of 94.12% when the Bargaining Agent keeps most of what he negotiates. Interestingly, the control case, which differs from the 40% treatment only in that there is Athlete to receive the 60%, has a 100% agreement rate. This could suggest some kind of envy effect in which players have some negative value from negotiating a deal that gives benefits to someone not directly involved in the negotiation, and instead prefer all players receive nothing instead. Also, note that the no agreement case is the only way to get equal payoffs among all players, which could suggest some players may have extreme preferences for equality over positive individual benefits.

Although this is not a focus of the paper, we can still test for whether these agreement rates show evidence of some pattern going on. A chi-squared test on the data, checking whether, by treatment, there is a difference in the agreement versus non-agreement indicator variable distributions results in a p-value of 0.6396. This p-value means there is no evidence that any of the agreement distributions are different.

A test with more power in this situation is the JonckheereTerpstra test. This test has a more specific alternative hypothesis, namely, that the medians of the agreement rate distributions are increasing monotonically between treatments in some order. If we believed that, say, as the active negotiators (the Bargaining Agent and the Team) have more skin in the game, they are more likely to come to an agreement, we could order the samples from lowest to highest as $0\% < 20\% < 40\% < 60\%$, then this test would be appropriate. Using the direct counting method, we get the relevant statistic $S = 6$, with a variance of $8.\overline{6}$. The continuity correction adjusted z-score in this case is $z = \frac{S-1}{\sqrt{8.\overline{6}}} \frac{5}{2.9439} = 1.6984$. Thus, in a one-sided t-test, we would reject the null hypothesis that all medians of the agreement rate distributions are equal, and have some evidence that the medians of the agreement rate distributions are increasing as

Table 3.2: Average P by Treatment (Deal Reached)

Treatment	Obs	P_A	P_B	P_T
0%	15	603.33 †	0.00 †	896.67 †
20%	16	841.25 *	210.31 *	448.44 *
40%	17	605.65 *	403.76 *	490.59 *
60%	16	377.50 *	566.25 *	556.25 *
Control	14	641.14 *	427.43 *	431.43 *

†- No theoretical predictions made.

* - Statistically significant difference from sequential-Nash-bargaining prediction.

the Bargaining Agent receives a larger share.

The following analysis is done using only data in which agreement was reached and no collusion was detected.

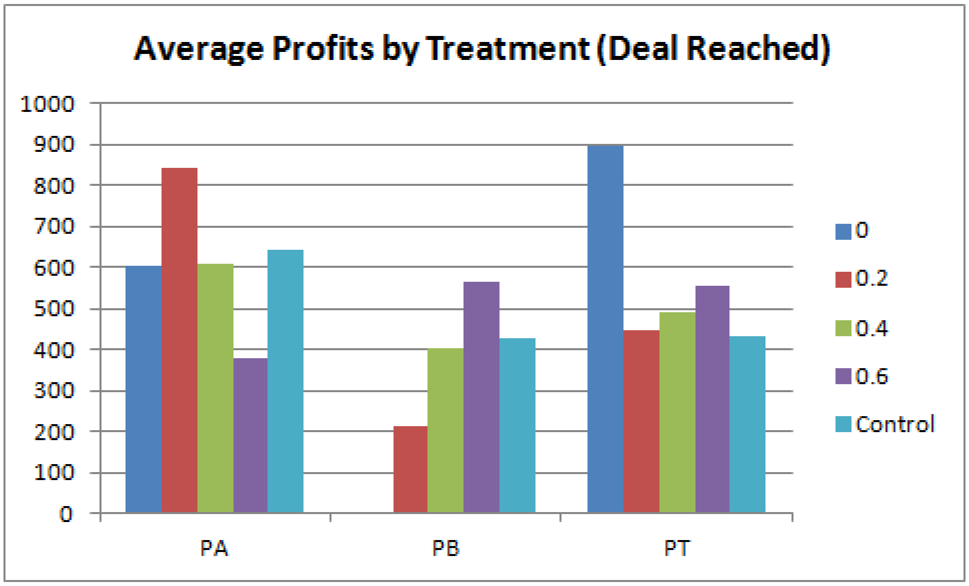
3.4.1 Performance of Sequential-Nash-Bargaining

First, let us evaluate the hypothesis which would hold if sequential-Nash-bargaining were guiding the behavior of subjects. Recall that under sequential-Nash-bargaining, the treatments should not affect the profits realized by the Team, P_T. We perform a t-test to see if the profit split is statistically different than the predicted 50/50 between the Bargaining Agent and the Team, i.e., if the result is statistically different than $P_T = 750$.[8]

Table 3.2 shows the results of the t-tests. Figure 3.1 also displays the shares graphically by treatment. In all situations in which sequential-Nash-bargaining pro-

[8] As noted at the end of section 2, we see no reason to think that either the Bargaining Agent or the Team will have a structural advantage over one another, and so with subjects drawn from the same pool, bargaining power should be on average equivalent, resulting in a 50/50 split of the total pot in the first round. However, if some structural advantage does exist, we also perform tests between treatments to see if the Team shares P_T differ which, if the null hypothesis holds, should not be true. Changes between treatments would show that the Athlete's bargaining power is affecting the negotiation between the Bargaining Agent and the Team, an outcome that cannot be if the structure underlying sequential-Nash-bargaining is true.

Figure 3.1: Overall Share



vides a prediction, we find that the resulting outcomes are statistically inconsistent with the predictions at the 95% level. This suggests that sequential-Nash-bargaining is a poor fit to describe behavior in this situation. More specifically, the 0% treatment makes predictions under sequential-Nash-bargaining, but we see that Athletes are still represented in most cases, with an average of 603.33 cents out of 1500 bargained for them and the remaining 896.67 going to the Team. The 20% treatment has the Team doing the worst of all of their cases, getting on average a share of 448.44. Interestingly, this means that the Bargaining Agent on average negotiates the largest amount for him and the Athlete to split when the Bargaining Agent only gets 20% of what he negotiates. As the Bargaining Agent gets a larger share of the amount negotiated, he negotiates less and less for his side. This could mean that the Bargaining Agent has some important income level to hit in a non-linear utility function, or that something other than self interest is influencing the negotiations. The 40% treatment results in an average share of 490.59 going to the Team, and the 60% treatment with an average share of 556.25 going to the team. As we can see, the Bargaining Agent negotiates less and less of the 1500 pot for his side as he gets a larger share. It is also important to note that the predicted share for the Team of 750 in all cases under sequential-Nash-bargaining is never reached, and there is a statistically significant difference in all cases. The control case, where there is no Athlete to benefit from the negotiation and that share is simply lost, offers another point of interest in that the average Team share, 431.43 is the lowest of all treatments. It seems odd to argue that the Bargaining Agent is even more incentivized to bargain well when only their interest is at stake. A better explanation, when noting that the Bargaining agent average share was only 4 cents different (427.43) than the average Team share, may be one of the equality focal point. When there is no third player to consider, the players can more easily decide they will just split things evenly. We explore this equality focal point in the two and three player case in Section 5.

Another way to test sequential-Nash-bargaining is to jointly test the Team's outcomes to see if they appear from the same distribution between treatments. Recall that under sequential-Nash-bargaining, the outcome of the bargaining between the Bargaining Agent and the Team should not be affected between treatments. We can weaken our assumptions under this test; previously, by assuming there was no structural difference between the bargaining positions, we predicted a split of 50/50, i.e. $P_T = 750$. With a nonparametric test we can see whether there are differences between the treatments even if there are structural benefits to one side of the negotiation or the other. By looking jointly at the distribution of the treatment groups, we can perform a Kruskal-Wallis test to get a broad, nonparametric read on whether the data groups appear to jointly be from the same distribution. Under sequential-Nash-bargaining, there should be no effect from the treatments on the average bargaining split. Table 3.3 shows the results of this test. The resulting P-value of 0.0037 suggests that we have evidence to reject the null hypothesis that the data groups come from the same distribution as sequential-Nash-bargaining would predict.

In addition to the global joint test, we can also test to see whether neighboring

Table 3.3: Kruskal-Wallis Test of P_T between 20%,40%, and 60% Treatments

Source	SS	df	MS	Chi-square	P-Value
Groups	2252.45	2	1126.22	11.19	0.0037
Error	7409.05	46	161.07		
Total	9661.5	48			

Table 3.4: Rank-sum Tests

Variable	Comparison of Bargaining Share Cases				
	0% vs 20%	20% vs 40%	40% vs 60%	20% vs 60%	40% vs Control
P_T	R	NR	R	R	NR

R - Rejection of the null

NR - No Rejection of the null

treatments have similar medians through the use of a rank-sum test. A rejection of the null means that there is some evidence to suggest that the median negotiation outcomes are different between the treatments, which should not be the case if sequential-Nash-bargaining is true. We only look at P_T shares as this determines the negotiating outcome.

Table 3.4 shows the results of the rank-sum tests. We include the 0% and 20% for completion, even though no prediction is made on the 0% treatment. However, the rejection in this case does tell us there is evidence that behavior is different between the two treatments. The rest of the table tells an interesting tale. First note that this test finds no evidence of difference between the 40% and the control case, suggesting that differences between these cases are small. The story of the different treatment comparisons is that there is very little difference between the 20% and 40% treatments, but both of these seem to differ from the 60% treatment, as evidenced by the non-rejection between 20% and 40%, while we see rejection between 60% and both 20% and 40%. Sequential-Nash-bargaining would suggest that we do not reject in any of these cases. Sequential-Nash-bargaining also offers no explanation as to why some would reject and others would not. Overall, this provides some evidence that sequential-Nash-bargaining is a poor fit for this situation, although the evidence is somewhat weak. In section 5, we suggest a behavioral criterion based on a preference for equality of outcomes which could provide some explanation as to why the 20% and 40% treatments could see similar behavior while the 60% differs.

In summary, the prediction generated from a sequential-Nash-bargaining paradigm regarding this experiment is that when the Bargaining Agent has any share of the negotiated outcome, the share the Bargaining Agent negotiates to split with the Athlete will on average be the same. To test this, we varied the size of the share the Bargaining Agent would receive in an open negotiating session between the Bargaining Agent and the Team where both sides could communicate through chat and offer proposals. Our results show that the different share sizes do affect the negotiating outcomes; when the Bargaining Agent gets a smaller share of what is negotiated, the Bargaining Agent on average negotiates a larger portion for the company. This can be seen in Table 3.2. Since the amount any party receives affects the amount the other parties receive, it is not surprising that significance occurs in all cases if it occurs in one. In none of the positive share cases do we see the negotiated split go half and half on average. This leads us to reject the applicability of sequential-Nash-bargaining to this situation. Using a rank-sum test to compare the treatments pairwise, we can see that behavior is, save for the Team's 20% to 40% comparison, different between the respective treatments. It is also useful to note that the rank-sum tests between the 40% treatment and the control (which also let the Bargaining Agent keep 40% but had no one claim the other 60%) did not show any difference, which would be consistent with sequential-Nash-bargaining and as well as many other paradigms. The control shows us that, at least when the Bargaining Agent gets 40% of what he negotiates, the behavior of the subject seems the same whether there is a third party or not. This implies that the important payoffs in this treatment are those of the Bargaining

Agent and the Team.

These results indicate that the second period negotiations have an effect on the first period, which should not be the case if sequential-Nash-bargaining is a good fit. We have attempted to eliminate a time cost of negotiating by making clear nothing can be done but waiting in the time allotted after an agreement is reached. If there is some time dependent effort cost, this could affect the amount of time subjects are willing to negotiate, resulting in worse results for subjects that keep less of what they negotiate. However, the average time to agreement seems to vary randomly by treatment, suggesting that this is not the case when the Bargaining Agent receives a positive share. If we move beyond time and effort costs to the subject, we can open ourselves up to some behavioral explanations as well.

3.5 Alternative Explanations

Sequential-Nash-bargaining seems to come up short in modeling this 3 person sequential bargaining situation. We propose and analyze other potential models and explanations in this section. The first is a behavioral model based on fairness focal points, analyzed first with equal weights among players and later solving for the weighting which best justifies the observations. We also investigate the best case scenario under Nash bargaining, solving for the Nash bargaining powers which would best fit the observations overall as well as the individual treatments. Both of these models are very flexible as applied to the current experiment, but while they do not offer strong testable predictions, they do offer insight into the situation generally.

3.5.1 Minimization of Differences Model

Equality focal points often influence experimental results, as, at least within the environment of a laboratory, some agents seem to prefer an equal split outcome in spite of other incentives. Fehr and Schmidt (1999) note equity as an important consideration in bilateral negotiating scenarios. When we designed our experiment, we did so with the plan to avoid such a focal point. While this was successful in avoiding an equal split between all three agents,[9] we still suspected from the results that some kind of equality focal point was at work. Since sequential-Nash-bargaining proved lackluster in our estimation in predicting the negotiation outcomes, we decided to explore an equality criterion which may be applicable to this situation. Namely, the criterion needs to provide insight in cases where more than two players are involved and equal outcome among players may be preferable but not possible. Thus, we propose a criterion in which players try to minimize the payoff differences between all players. This has the desired property of equal outcome among all players when possible, as well as informing where an equality preferring player might look for a new focal point, i.e. the least costly one that still provides equal outcomes between some agents.

Similar to experiments with exchange rate economics, our bargaining situation can provide multiple possible focal points that the subjects can choose from. The relative value of the resource which is being bargained over by the Bargaining Agent and the Team changes as the bargaining power changes between the Bargaining Agent and the Athlete. If the bargaining power of the Athlete grows, the Bargaining Agent gets less of what he bargains for; in the language of Kagel, Kim, and Moser (1996), the chips being bargained over are worth less to the Bargaining Agent. This introduces a new potential focal point beyond the "even chip" split and the "even payoff" split, as there is a third party. The even payoff splits might now be between the payoffs of the Bargaining Agent and the Team, the Athlete and the Team, or the Bargaining Agent and Athlete. Assumptions can be made about the manner in which this focal point is chosen to generate a behavioral prediction.

We propose a criterion that minimizes the pairwise inequalities between agents, potentially weighted to reflect the importance of different players. One way to char-

[9] Except in cases of no agreement.

acterize this criterion is by summing the absolute value of differences between each of the player's final payoffs.

$$\min_{P_i}[|P_{BA} - P_A| + |P_{BA} - P_T| + |P_A - P_T|] \tag{3.9}$$

This particular formulation applies equal weight to each difference in payoff. Weights could be added to emphasize the importance of different matchups. Inserting our model, we can specify the form of this expression. Let β be the portion of the profit that goes to the Bargaining Agents and Athletes, ϕ be the portion of the compensation that goes to the Bargaining Agent, and 1500 be the total profit from the venture. Our criterion becomes:

$$\min_{P_i}[|\phi\beta - (1-\phi)\beta| + |\phi\beta - (1500-\beta)| + |(1-\phi)\beta - (1500-\beta)|] \tag{3.10}$$

If we assume the agents are trying to minimize this function because the players value making the payoffs more equal, we can generate some alternative predictions about outcomes. Most notable is the prediction that comes from the 0% treatment, or when $\phi = 0$. Sequential-Nash-bargaining does not make a prediction in this case. If we try to minimize the criterion expression above with $\phi = 0$, then we can get a prediction of $\beta = 750$, or a 50/50 split of the profit between the Team and the Athlete.

How this criterion functions has a strong intuitive explanation. Since ϕ is taken as given, there is only one dimension, β, through which to minimize the three different additive portions. Since inequalities are costly, the obvious division if all subjects are weighted equally would be to split the pot completely evenly. However, the structure of the negotiations limits this outcome, as the compensation can never be split evenly between the Bargaining Agent and Athlete.[10] This means that the optimal way

[10]This is by design, to avoid the allure of this focal point and enlighten what occurs when this option is not available.

to minimize the function involves accepting that some of the payoffs will not be even. Due to the linear piecewise nature of the criterion, there are three possible minimization points, one each at the equating of two payoffs.[11] Given some weights over the different pairs, the minimand can be found by checking at these three points and determining which is lowest. This amounts to choosing two players for which payoffs will be equalized and minimizing the damages (inequalities) incurred by this. When testing this criterion, evidence for this would be the equalization of two players at the expense of a third.

One issue with this criterion is that the assumed split of the compensation between the Bargaining Agent and the Athlete does not contain this equality idea. This does not change the potential explanatory power of the criterion, and can be solved with an assumption on the relationship between the Bargaining Agent and the Athlete. If we assume the Athlete is not concerned with this equal split, or that their relationship is as an owner/employer and some inequity in payoffs is expected, then these concerns disappear.

3.5.1.1 ***Predictions of Minimization of Differences*** .

The flexibility of this model makes it so that at most anything can be justified without some restrictions. We choose to restrict ourselves to a case where all players dislike inequalities of outcome between any two players equally and compare the deviations from the possible focal points. An alternative would be to calculate the player weights that would best justify (i.e. minimize the differences) between observations and predictions. Since we can never equate the Athlete's and the Bargaining Agent's payoffs, and because the amount negotiated is fixed, the payoff differences will never be minimized by attempting to minimize the differences between these two players.

[11] The equalization of the Athlete and Bargaining Agent payoffs is unlikely, as it requires the Bargaining Agent to negotiate zero for their profit share, making the differences of the other two pairs rather larger. However, with a weighting scheme that heavily weights the Athlete/Bargaining Agent inequality, it is possible

Table 3.5: Potential Focal Points

Treatment	A=T Minimizer	Difference	BA=T Minimizer	Difference	Prediction
60%	428.57	428.58	562.5	374	BA-T
40%	562.5	374	428.57	428.56	A-T
20%	666.67	500	250	1500	A-T
0%	750	1500	0	3000	A-T

Thus, we focus on the focal points of 1) equating the Athlete and the Team and 2) equating the Bargaining Agent and the Team. For instance, in the 60% case, we know that $BA = 1.5A$. If we check the focal point of $A = T$, this means that

$$A + BA + T = 1500 \tag{3.11}$$

$$A + 1.5A + A = 1500 \tag{3.12}$$

$$A = T = 428.57 \tag{3.13}$$

$$BA = 642.86 \tag{3.14}$$

The differences can be calculated from these numbers: $A - T = 0$, $BA - A = 214.29$, and $BA - T = 214.29$ for a total of 428.58. If the same is done for the $BA - T$ focal point, the difference is 374. This means that, in the 60% case, the $BA-T$ focal point has a less total difference and should be more attractive to the agents.

Doing this for all cases yields Table 3.5. We can see that the Bargaining Agent-Team focal point around the Bargaining Agent and Team payoff of 562.5 each should dominate in the 60% case as the total difference is minimized ($374 < 428.56$). Similarly, as the shares are simply reversed, the 40% case favors the Athlete-Team focal point, this time with the Athlete and the Team at around 562.5. The 20% case makes the Bargaining Agent-Team focal point very costly to achieve, and so Athlete-Team focal point at 666.67 is favored. This metric also gives us a prediction for the 0% case, of 750 each to the Athlete and Team.

Our experiment did not allow for fractional shares, and so our predicted shares

Table 3.6: Feasible predicted shares for the Team

Treatment	A=T Minimizer	BA=T Minimizer	Prediction
60%	430	560	BA-T
40%	560	430	A-T
20%	665	250	A-T
0%	750	0	A-T

Table 3.7: Results under minimization of squared error

Treatment	A=T	BA=T	Average T	Prediction	Observed	SSE A=T	SSE B=T
60%	430	560	556.25	BA-T	BA-T	294750	39950
40%	560	430	490.59	A-T	BA-T	1312250	1292750
20%	665	250	448.44	A-T	BA-T	1068475	948125
0%	750	0	896.67	A-T	A-T	1597550	13335050

will be those which minimize the distance criterion over possible outcomes. Table 3.6 shows the predicted shares given this constraint. We will compare the Table 3.6 values with the actual results to test the predictions.

3.5.1.2 ***Performance of Minimization of Differences*** .

While we have some form of prediction, we can evaluate the performance of the prediction in a few different ways. Our first approach is to use a minimization of squared error approach.

Table 3.7 displays the results compared to the predictions when minimizing the sums of the squared errors. We see that the prediction falls in line with the observed in the treatments of 60% and 0%. The 0% is heavily in favor of equating the Athlete and the Team payoffs. Similarly, in the 60% treatment, the squared error of equating the Bargaining Agent payoffs with the Team payoffs is much less than the Athlete-Team case. The 40% treatment falls against the prediction, favoring instead the Bargaining Agent-Team focal point. However, the squared error in this case is close, meaning

Table 3.8: Closest focal point

Treat.	A=T Share	BA=T Share	Midpoint	Average	Closer to A=T	Closer to BA=T
60%	430	560	495	556.25	1	15
40%	560	430	495	490.59	5	12
20%	665	250	457.5	448.44	10	6
0%	750	0	375	896.67	14	1

small differences could have put it matching the prediction. The 20% treatment is not so close in its defiance of the prediction, landing in favor of the Bargaining Agent-Team focal point.

By this measure, the model is not a perfect fit for explaining the situation. There are other metrics that we can look at to further our understanding of the observations under this paradigm. Another metric would be to simply see which focal point the observation is closest to and compare these numbers. The results of this are found in Table 3.8. These results tell us that the 60% and 40% treatments favor the Bargaining Agent and Team focal point, whereas the 20% and 0% favor the Athlete and Team focal point. This differs from the Table 3.7 minimization of differences results in the 20% treatment, as here it favors the predicted Athlete-Team focal point. The 40% treatment still defies prediction, which suggests that something is unaccounted for in this case.

Finally, we can look generally at how close the shares end up in each treatment. If there are many shares close together, that is evidence that this focal point was important to the players. Table 3.9 displays this comparison when considering the shares to be “close” when they are less than 100 apart from one another. Note that in the 40% and 60% cases it was possible for both the Athlete and the Bargaining Agent to be within 100 of the Team in the same observation. The table tells us once again that the 60% and 40% treatments favor the Bargaining Agent-Team focal point, while 0% favors the Athlete-Team focal point. The 20% in this case appears to be

Table 3.9: Rough Equality of P's

	$\lvert P_A - P_T \rvert < 101$		$\lvert P_{BA} - P_T \rvert < 101$		
	Count	Percent	Count	Percent	Obs
60%	2	12.50%	15	93.75%	16
40%	3	17.65%	14	82.35%	17
20%	1	6.25%	5	31.25%	16
0%	8	53.33%	0	0.00%	15
Control	1	7.14%	14	100.00%	14

out of sorts, as very few observations are close. This is a difficult case to try to make things equal, as the Team has to sacrifice much to find the Bargaining Agent-Team focal point. However, note that over 31% of the cases show that the Team was willing to make that sacrifice.

The proposed minimization of differences criterion is not a perfect fit for this case, but it does help to explain what is going on. The prediction for the 40% treatment fails strongly here, which suggests that something more is needed to account for the observed actions. A possible explanation is that, as the Bargaining Agent and the Team are the active negotiators, their outcomes may matter more. This could make the Bargaining Agent-Team focal point more important in this case; table 3.9 would seem to bear this out, with 82.35% of the observations being around this point. Note also that we have gained a reasonably performing prediction of the 0% treatment, which cannot be approached by the Nash bargaining paradigm.

3.5.2 Best Fit for Nash Bargaining

As we have opened ourselves to exploring flexible explanations with the minimum distance fairness criterion, it is only fair that we also look at the best fit scenario for Nash Bargaining for comparison's sake as it is also incredibly flexible if we move away from our assumption that, on average, bargaining power between the players should

Table 3.10: Best fit α's

Treatment	α_{BA}
All	2.0104
20%	2.345
40%	2.0575
60%	1.6966

be equal. For each treatment, we can calculate the relative bargaining power of the different player positions that would best fit in each scenario. From there, we discuss the patterns and implications of these values.

To conduct the analysis, we first note that the bargaining powers of the Athlete and the Bargaining Agent are fixed relative to each other. If we further normalize the bargaining power of the Team to be 1, we can find the bargaining power of the Bargaining Agent which best fits the observed outcomes under the criterion of the minimization of squared errors between the predicted outcomes given a bargaining power and the observations. The results of this process are presented in Table 3.10. Note that the zero percent treatment is ignored, as Nash has no predictions in this case.

It appears the less the Bargaining Agent gets of what he negotiates, the stronger his bargaining power becomes. In the 60% treatment, his bargaining power is at its lowest, with 1.6966. This increases to 2.0575 in the 40% treatment, and 2.345 in the 60% treatment. This feels like a counter intuitive result; one would expect that as the negotiator is privy to a larger piece of the negotiated amount, he would try harder to secure more. One possible explanation would be that the Bargaining Agent places a very high value on the first amounts received. This would mean he fights hardest when he gets the least, as those small amounts are more valuable, and fights less hard as he gets a larger share since his desired amount is easily achieved. It could also be some form of pity that the Team has for the Bargaining Agent, bargaining less hard as they see the Bargaining Agent as being in a disadvantageous position. However, we have no way to test this, and so are left only with speculation.

It is worth noting that under sequential-Nash-bargaining this is an unexpected result. We would expect that the different treatments would have no effect on the bargaining power, on average, of the participants. We see here again that the treatments do have an effect, and so sequential-Nash-bargaining appears to fall short when modeling this situation.

3.6 Conclusions

Negotiations in real life are very common and can be very complex, but most of the literature focuses on the two party case. We set out to explore the gap by designing and running an experiment involving a simple three person bargaining structure as a simple extension from the two party case. When two agents first bargain over a single pot and one of those agents then splits their portion with a third party, we find that the predictions implied by sequential-Nash-bargaining perform poorly in explaining observed behavior. A fairness paradigm where agents settle on roughly equal payoffs between some agents seems to provide a promising alternative explanation, as well as providing predictions in cases where Nash is silent.

Our findings provide further evidence that fairness is an important consideration when predicting behavior. It also highlights the inadequacy of current equilibrium concepts when addressing multi-agent bargaining situations, at least in this context. It is possible that expert bargainers would be less focused on a fair outcome and more focused on gaining every last ounce of advantage their opponent gave them, but that was not evident in our experiment sample consisting of undergraduates.

Our work opens further opportunities for research. While difficult and less clean, observing behavior in a situation where one agent actually bargains in a first and second period instead of a second period predetermined split would be useful as a robustness check. To further explore how people define and value fairness, a potential experimental design would include a costly production period leading to a negotiation between the two producers over the combined pot of their production. By varying the productivity and production processes of the players, we could determine whether players determine fairness as an equal split of the produced goods or if they feel more productive individuals deserve more of the payoffs.

APPENDIX A

Raw Tables from Chapter 2

Table A.1: Base Model (Compatible) Raw

Case	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base	0.4801	408.6695	134.7872	543.4566	6.5412
Monopoly	0.7162	140.5008	268.6383	409.1391	12.5949
Single Low Type	0.4102	429.6950	168.4917	598.1867	5.2797
Worse Low Tech	0.6295	406.8163	121.5430	528.3593	9.9290
Better High Tech	0.7195	459.6518	133.3201	592.9720	12.7118
More Costly Investment	0.3665	402.9213	136.3385	539.1599	7.9188
Stronger Network Effect	0.5189	486.0201	161.8572	647.8772	7.3168

Table A.2: Imposed High Technology Policy (Compatible) Raw

Case	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base	0.8014	390.7578	120.9142	511.6720	16.1646
Monopoly	0.9657	135.7850	247.5167	383.3017	33.7261
Single Low Type	0.7133	425.2244	169.1061	594.3306	12.4932
Worse Low Tech	0.8014	390.7578	120.9142	511.6720	16.1646
Better High Tech	0.8178	445.0284	139.3281	584.3565	17.0265
More Costly Investment	0.7678	402.8213	115.3972	496.6262	18.2520
Stronger Network Effect	0.8235	466.6537	146.5960	613.2497	17.3443

Table A.3: High Technology Second Best Tax/Subsidy Policy (Compatible) Raw

Case	Expected Equilibrium Values						
	τ/ς	s_i	CS	PS	TS	τR/ςC	C(s_i)
Base	-4.4813	0.3478	401.86	135.23	544.89	7.79	4.27
Monopoly	17.9721	0.8120	142.22	282.49	410.11	-14.59	16.71
Single Low Type	-5.3653	0.1371	426.81	171.28	598.82	0.74	1.47
Worse Low Tech	-6.9369	0.5256	398.82	113.06	530.11	18.23	7.46
Better High Tech	-10.2656	0.6292	449.83	113.08	595.20	32.30	9.92
More Costly Investment	-4.3308	0.2208	395.37	140.55	540.71	4.78	3.12
Stronger Network Effect	-4.3928	0.4075	479.52	160.59	649.06	8.95	5.23

Table A.4: Base Model (Incompatible) Raw

Cases	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base	0.5421	341.4571	104.1067	445.5638	7.8110
Monopoly	0.7162	140.5008	268.6383	409.1391	12.5949
Single Low Type	0.4644	357.6007	139.7456	497.3463	6.2437
Worse Low Tech	0.6689	339.3392	94.1529	433.4921	11.0533
Better High Tech	0.7442	384.6648	107.2059	491.8708	13.6336
More Costly Investment	0.4443	337.0629	104.5963	441.6593	7.3441
Stronger Network Effect	0.6084	371.1430	110.1528	481.2957	9.3751

Table A.5: Imposed High Technology Policy (Incompatible) Raw

Cases	Expected Equilibrium Values				
	s_i	CS	PS	TS	C(s_i)
Base (n=3)	0.8791	279.2625	27.7542	307.0167	21.1279
Monopoly	0.9657	135.7850	247.5167	383.3017	33.7261
Single Low Type	0.6554	354.4834	21.1312	375.6146	10.6537
Worse Low Tech (n=3)	0.8791	279.2625	27.7542	307.0167	21.1279
Better High Tech (n=3)	0.8909	318.8344	33.2693	352.1037	22.1549
More Costly Investment (n=3)	0.8548	273.5950	25.4380	299.0330	24.1205
Stronger Network Effect (n=3)	0.8885	309.6015	30.9741	340.5756	21.9373

Table A.6: High Technology Second Best Tax/Subsidy Policy (Incompatible) Raw

Cases	Expected Equilibrium Values						
	τ/ς	s_i	CS	PS	TS	τR/ςC	C(s_i)
Base	-8.9923	0.3063	330.87	105.85	450.49	13.77	3.66
Monopoly	17.9721	0.8120	142.21	282.49	410.11	-14.59	16.71
Single Low Type	-12.4155	$5 * 10^{-5}$	353.16	146.77	499.93	$7 * 10^{-4}$	$5 * 10^{-5}$
Worse Low Tech	-13.8064	0.4812	326.61	79.70	439.53	33.22	6.56
Better High Tech	-19.8445	0.5799	368.70	73.22	499.47	57.54	8.67
More Costly Investment	-8.7434	0.1897	325.78	112.86	446.93	8.29	2.63
Stronger Network Effect	-12.4005	0.3463	358.70	108.61	488.78	21.47	4.25

APPENDIX B

Experiment Instructions from Chapter 3

Instructions

Be sure to read through these instructions carefully. This is a bargaining experiment. You will have up to 5 minutes to bargain with other players. The experiment will take place through computer terminals. It is important that you do not talk or in any way try to communicate with other subjects during the experiment. The use of portable devices such as cell phones or mp3 players is prohibited.

The Bargaining

Each participant in the experiment will be randomly assigned to the role of either Player 1, Player 2, or Player 3. Players 2 and 3 will decide how to split 1500 cents ($15) between all three players. Player 1 will not be part of the bargaining process, but will receive a share of the agreed upon split.

In order to determine the split, Players 2 and 3 will have 5 minutes to agree on a value called Beta. Beta can be anything between 0 and 1500 cents (in multiples of 5 cents). If an agreement is reached before the time is up, players will be paid as follows:

Player 1 gets 60% of Beta.
Player 2 gets 40% of Beta.
Player 3 gets 1500 – Beta.

If an agreement is not reached in the allotted 5 minutes then all players receive $0.

The Computer Interface

The interface for players 2 and 3 is very similar. First you will be asked to complete a short quiz on the rules of the game and asked your subject number. The subject number is the number on the card you received at the beginning of the experiment. It is important that you fill this number out correctly as it is how you will eventually be paid

Once all players have completed this quiz, the bargaining will begin. On the left half of the screen there will be a chat box that allows you to communicate with your bargaining partner. The right half of the screen will be divided into two boxes: In the upper right hand corner participants can propose values of Beta for consideration, and in the lower right hand corner participants can review and accept the most recent proposal. Only the most recent proposal can be reviewed and accepted. Participants are encouraged to discuss a potential deal in the chat box before (and after) submitting it for review. Note: if you are a player 1, you will simply wait while players 2 and 3 bargain.

Details:

-The proposal window will be in the upper right hand corner of the screen. There you can make a proposal by typing in a number in the field marked "Beta", then clicking "Calculate" to see the resulting value of beta, then clicking on "Submit." You can also investigate possible values of Beta by typing them in the field and clicking "Calculate" and then not clicking submit.

-The acceptance window will be in the lower right hand corner of the screen. There you can review the current proposal and accept it if you choose. The screen shows the value of beta being considered as well as the resulting payoffs for all three players. It also shows which players have accepted the current proposal.

The Bargaining period will last for 5 minutes. If you agree to terms before that time you must wait quietly until the 5 minutes are up. After the 5 minutes are up you will be asked to fill out a short survey. Once everyone completes the survey subjects will be paid by subject number. During payment, please remain seated until your subject number is called. Once you are paid you may leave and have a nice day.

References

Amir, Rabah, and Lazzati, Natalia, (2011) "Network effects, market structure, and industry performance", *Journal of Economic Theory*, Vol. 146, Issue 6, pp. 2389-2419.

Besen, Stanley M., and Farrell, Joseph, (1994) "Choosing How to Compete: Strategies and Tactics in Standardization", *The Journal of Economic Perspectives*, Vol. 8, No. 2, pp. 117-131.

Bruce, Donald, and Santore, Rudy, (2006) "On optimal real estate commissions", *Journal of Housing Economics* Vol. 15, Issue 2, 156-166.

Chinloy, Peter, and Winkler, Daniel, (2010) "Contract Incentives and Effort", *Journal of Real Estate Research* Vol. 32, Number 4, 397-412.

Farrell, Joseph, and Saloner, Garth, (1985) "Standardization, Compatibility, and Innovation", *The RAND Journal of Economics*, Vol. 16, No. 1, pp. 70-83.

Farrell, Joseph, and Saloner, Garth, (1986) "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation", *The American Economic Review*, Vol. 76, No. 5, pp. 940-955.

Fehr, Earnst, and Schmidt, Klaus M. (1999) "A Theory of Fairness, Competition, and Cooperation", *The Quarterly Journal of Economics* Vol. 114, No. 3, 817-868.

Fisher and Yavas, (2010) "A Case for Percentage Commission Contracts: The Impact of a Race Among Agents", *The Journal of Real Estate Finance and Economics* Vol. 40, Issue 1, 1-13.

Gandal, Neil, Kende, Michael, and Rob, Rafael, (2000) "The Dynamics of Technological Adoption in Hardware/Software Systems: The Case of Compact Disc Players", *The RAND Journal of Economics*, Vol. 31, No. 1, pp. 43-61.

Gatignon, Hubert and Robertson, Thomas S., (1989) "Technology Diffusion: An Empirical Test of Competitive Effects", *Journal of Marketing*, Vol. 53, No. 1, pp. 35-49.

Grossman, Sanford J., and Oliver D. Hart, (1983) "An Analysis of the Principal-Agent Problem", *Econometrica* Vol. 51, Issue 1, 745.

Jares, Timothy, James Larsen, and Thomas Zorn, (2000) "An Optimal Incentive System for Real Estate Agents," *Journal of Real Estate Research* Vol. 20, Issue 12, 4959.

Katz, Michael L. and Shapiro, Carl, (1985) "Network Externalities, Competition, and Compatibility", *The American Economic Review* Vol. 75, No. 3, pp. 424-440.

Katz, Michael L. and Shapiro, Carl, (1986) "Technological Adoption in the Presence of Network Externalities", *Journal of Political Economy*, Vol. 94, No. 4, pp. 822-841.

Katz, Michael L. and Shapiro, Carl, (1992) "Product Introduction with Network Externalities", *The Journal of Industrial Economics*, Vol. 40, No. 1, pp. 55-83.

Kitchens, Carl, and Roomets, Alex, (2015) "Dealing with Eminent Domain," *Journal of Behavioral and Experimental Economics* Vol. 54, February 2015, Pages 22-31.

Lefgren, McIntyre, and Miller, (2010) "Chapter 7 or 13: Are Client or Lawyer Interests Paramount?", *The B.E. Journal of Economic Analysis* & *Policy*, Vol. 10, Issue 1.

Levitt and Syverson, (2008) "Market Distortions When Agents are better informed: a theoretical and empirical exploration of the value of information in real-estate transaction" *Review of Economics and Statistics* Vol. , Issue , .

Levitt and Syverson, (2008) "Antitrust Implications of Home Seller Outcomes when Using Flat-Fee Real Estate Agents" *Brookings-Wharton Papers on Urban Affairs 2008*.

Mankiw, N. Gregory and Whinston, Michael D., (1986) "Free entry and social inefficiency", *The RAND Journal of Economics*, Vol. 17, No. 1, pp. 48-58.

McAdams and Schwarz, (2006) "Credible Sales Mechanisms and Intermediaries," MIT Sloan Research Paper No. 4606-06.

Moresi, Serge and Salop, Steven C. and Sarafidis, Yianis, "A Model of Ordered Bargaining with Applications" (October 20, 2008). Available at SSRN: http://ssrn.com/abstract=1287224 or http://dx.doi.org/10.2139/ssrn.1287224.

Munneke, Henry, and Abdullah Yavas, (2001) "Incentives and Performance in Real Estate Brokerage," *Journal of Real Estate Finance and Economics* Vol. 22, Issue 1, 521.

Reynolds, Stanley S. and Isaac, R. Mark, (1992) "Stochastic innovation and product market organization", *Economic Theory*, Vol. 2, No. 4, pp. 525-545.

Sah, Raaj, and Stiglitz, Joseph, (1987) "The invariance of market innovation to the number of firms", *The RAND Jounal of Economics*, Vol. 18, pp. 98-108.

Sappington, David E. M., (1991) "Incentives in Principal-Agent Relationships," *Journal of Economic Perspectives* Vol. 5, Issue 2, 4566.

Sheremata, Willow A., (2004) “Competing through Innovation in Network Markets: Strategies for Challengers”, *The Academy of Management Review*, Vol. 29, No. 3, pp. 359-377.

Utterbach, James M., and Suarez, Fernando F., (1993) “Innovation, Competition, and Industry Structure”, *Research Policy*, Vol. 22, No. 1, pp. 1-21.