

PERFORMANCE STUDY OF PRECODING TECHNIQUES ON GENERALIZED SPATIAL MODULATION WITH CORRELATED ANTENNAS

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ABSTRACT

Spatial modulation (SM) reduces transceiver complexity and inter-channel interference over traditional multiple input multiple output (MIMO) systems. It has been shown recently in the literature that the use of a precoder in an SM or a generalized spatial modulation (GSM) system can significantly improve error performance. This paper investigates two issues related to precoders: 1) the use of a precoder in Alamouti-GSM systems, and 2) the effects of power constraints on the precoder design. The results in this paper show that Alamouti-GSM can improve system performance by several dB. On power constraint issues, the paper shows that there is a trade-off between limiting antenna power fluctuations and the potential gain due to precoders.

KEY WORDS

MIMO, spatial modulation, Precoder, correlated antennas, Alamouti technique.

INTRODUCTION

It is well known that the use of multiple antennas at the transmitter and the receiver can increase the system capacity as well as the system's diversity gain, providing improved bit error rate (BER) performance [1]. Vertical Bell Labs Layered Space-Time (V-BLAST) [2] architecture is one of the most well-known techniques for a multiple-input multiple-output (MIMO) system. It allows data to be transmitted through different transmit antennas at the same frequency simultaneously in order to achieve a very high spectral efficiency. However, when multiple transmit antennas are employed, they require multiple RF chains. Further, since all antennas transmit information at the same frequency and time, there is also interchannel interference (ICI) which degrades the system performance at the receiver [3]. Therefore, a modulation technique called the spatial modulation (SM) has been investigated extensively in recent times [4]. Although SM avoids multiple RF chains and ICI, its spectral efficiency is low. As a result, generalized spatial modulation (GSM) has been studied to obtain trade-offs among complexity, ICI and spectral efficiency [5]. In GSM, more than one transmit antenna can be active to transmit data simultaneously [4], [6].

The performance of SM/GSM systems can be improved by using precoders. Examples of precoders include the use of codebook in [7], the power allocation scheme of [8] and quantized amplitude and phase precoders [9]. Recently, a precoder for SM/GSM systems is proposed in [10], where the minimum distance between received sequences is maximized. The precoder design problem is approximated as a convex program and solved in an iterative fashion.

In this paper, we consider two issues related to the precoder design in GSM systems: 1) the precoders proposed in [10] use a power constraint that the total power over all the available symbols is below a maximum allowable limit. This can lead to significant power fluctuations from one symbol transmission to the next. Let's call this constraint as the constraint on power over multiple use (PMU) of the channel. One problem with PMU is that it can allow wide power fluctuations at the antennas from symbol to symbol, which is not helpful when efficient non-linear power amplifiers are required. Therefore, in contrast to PMU, we consider in this paper a power constraint where the total power transmission in each channel use is constant. Let's call it the constraint on power over single use (PSU) of the channel. Finally, we can limit the power fluctuations in each individual antenna. We call this the power per antenna use (PAU) constraint. We investigate the effects of PAU on precoder performance. We observe that while severe PAU restrictions lead to less power fluctuations, the benefit of precoding decreases. 2) We investigate the precoder effect on an Alamouti-GSM system that we studied previously in [11]. We derive the precoder design requirement for Alamouti-GSM and provide numerical results with convex iterative optimization procedures following [10]. Our results show that the Alamouti-GSM can provide several dBs of gain in BER performance.

SYSTEM MODEL

In GSM, the data bits are first split into two groups before being modulated. During each use of the transmission channel, the first group of bits represents the selected active antennas. To see this, let N_t denote the total number of available transmit antennas and N_a denote the number of active transmit antennas during each channel use. There are $\binom{N_t}{N_a}$ possible antenna combinations, and hence the corresponding number of bits, k_a , is given by $k_a = \lfloor \log_2 \binom{N_t}{N_a} \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function. Thus, there are k_a bits in the first group, and the number of corresponding symbols is $M_a = 2^{k_a}$. This can be achieved by using an antenna selection set G . Let g_m be the m -th element in set G , and it represents the m -th group of active antenna selection. As an example, consider a 4×4 MIMO system with $N_a = 2$ active antennas for each transmission. Then $k_a = \lfloor \log_2 \binom{4}{2} \rfloor = 2$, and an example set G would be $G = \{(1, 2), (1, 3), (1, 4), (2, 4)\}$ containing all the valid active antenna selections. Thus, $k_a = 2$ bits are represented by each active group of antennas.

The second group of the transmit data is mapped to symbols of a given modulation alphabet, such as MPSK and MQAM. For an M -ary modulated system, the number of bits per symbol is $k = \log_2 M$. Thus, the total bits in each transmission is $k_t = k_a + N_a k$ and there will be $M_t = 2^{k_t}$ total different transmit signals.

Let $\mathbf{y} = [y_1, y_2, \dots, y_{N_r}]^T$ be the received signal, where N_r is the number of receive antennas, y_i is the received sample at the i -th receive antenna and $[\cdot]^T$ denotes vector transpose. If this received signal vector is caused due to the transmission of the m -th signal vector, $\mathbf{x}^{(m)}$, where $m = 1, 2, \dots, M_t$, then \mathbf{y} can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x}^{(m)} + \mathbf{n} \quad (1)$$

where \mathbf{n} is the additive white Gaussian noise (AWGN) vector containing noise samples of zero mean and variance σ^2 , and \mathbf{H} is the channel matrix. The transmit signal vector is given by $\mathbf{x}^{(m)} = [x_1^{(m)}, x_2^{(m)}, \dots, x_{N_t}^{(m)}]^T$, where $x_i^{(m)}$ is the signal transmitted by the i -th transmit antenna while transmitting the m -th signal. Each signal $x_i^{(m)}$ is obtained by multiplying an M -ary complex symbol with a coefficient $p_n^{(m)}$ during the precoding stage, and each transmit signal vector $\mathbf{x}^{(m)}$ can

be considered as the sum of N_a SM vector signals. Thus, we can write

$$\mathbf{x}^{(m)} = \sum_{n=1}^{N_a} p_n^{(m)} \mathbf{s}_n^{(m)} \quad (2)$$

where $\mathbf{s}_n^{(m)}$ is the $N_t \times 1$ vector representing one SM transmit signal before precoding. Take the example of the 4×4 MIMO system considered above. If for the m -th signal, the active antenna pair (1, 3) transmits symbols 1 and $1 + j$ before precoding, and if $p_1^{(m)} = 0.7$ and $p_2^{(m)} = 0.2$ are the precoding weights, we can express the transmit signal vector as $\mathbf{x}^{(m)} = [0.7, 0, 0.2 + 0.2j, 0]^T = 0.7[1, 0, 0, 0]^T + 0.2[0, 0, 1 + j, 0]^T$, where $\mathbf{s}_1^{(m)} = [1, 0, 0, 0]^T$, and $\mathbf{s}_2^{(m)} = [0, 0, 1 + j, 0]^T$ that represent the SM vectors activating the first and the third antennas respectively.

The channel matrix \mathbf{H} used in (1) is defined as

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r,1} & h_{N_r,2} & \cdots & h_{N_r,N_t} \end{bmatrix} \quad (3)$$

where h_{ij} is the channel coefficient between the i -th receive antenna and the j -th transmit antenna. According to [7], we can write the channel matrix \mathbf{H} as $\mathbf{H} = \mathbf{H}_r \mathbf{B}_t$, where \mathbf{H}_r is the $N_r \times N_t$ frequency-flat channel matrix with all elements in \mathbf{H}_r distributed as i.i.d. Rayleigh. Matrix \mathbf{B}_t identifies the transmit antenna correlation. The $N_t \times N_t$ transmit antennas correlation matrix is $\mathbf{R}_t = \mathbf{B}_t^H \mathbf{B}_t$, where $[\cdot]^H$ denotes the Hermitian transpose. Therefore $\mathbf{B}_t = \mathbf{R}_t^{\frac{1}{2}}$, and \mathbf{R}_t is determined by the correlation coefficient, ρ , of the transmit antennas, so that the (i, j) -th element of \mathbf{R}_t is $\rho^{|i-j|}$. For a $N_t = 4$ system,

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix} \quad (4)$$

Using (2) we can rewrite (1) as

$$\mathbf{y} = \mathbf{H} \sum_{n=1}^{N_a} p_n^{(m)} \mathbf{s}_n^{(m)} + \mathbf{n} = \sum_{n=1}^{N_a} p_n^{(m)} \mathbf{H} \mathbf{s}_n^{(m)} + \mathbf{n} = \sum_{n=1}^{N_a} p_n^{(m)} \mathbf{c}_n^{(m)} + \mathbf{n} \quad (5)$$

where $\mathbf{c}_n^{(m)} = \mathbf{H} \mathbf{s}_n^{(m)}$. Since all elements but one in vector $\mathbf{s}_n^{(m)}$, are zero, the vector $\mathbf{c}_n^{(m)}$ essentially selects the column of the matrix \mathbf{H} that corresponds to the active antenna and scales it with the symbol transmitted by the antenna (excluding the precoding weight).

PRECODER DESIGN FOR ALAMOUTI-GSM SYSTEMS

In this section, we describe the precoder design for a GSM system that combines the Alamouti space-time modulation technique. Our derivation is an extension of the design given in [10]. In [11], we have studied the Alamouti technique combined with GSM. In Alamouti-GSM, $N_a = 2$ antennas are active for each transmission. Define T as the transmission duration. At each even time slot (time 0, time $2T$, time $4T$, etc.), the symbols transmitted by two antennas are determined from

the transmit data. At the odd time slots (time T , time $3T$, time $5T$, etc.), the same active antennas are used, but the symbols that will be transmitted by them are determined according to the Alamouti scheme [12]. As an example, suppose at time 0, Antenna 1 and 3 are activated, and they transmit signals s_1 and s_2 respectively. Next, at time T , the active antennas remain the same. However, Antenna 1 transmits $-s_2^*$, and Antenna 3 transmits s_1^* , where $*$ denotes complex conjugation. The process then repeats with different data for the next time instants. When a precoder is used in the Alamouti-GSM scheme, then at an even time slot, the pair of symbols $s_1^{(m)}$ and $s_2^{(m)}$ are multiplied by $p_1^{(m)}$ and $p_2^{(m)}$ respectively to transmit $s_1^{(m)} p_1^{(m)}$ and $s_2^{(m)} p_2^{(m)}$. At the next immediate odd slot, the same antennas transmit symbols $-s_2^{(m)*} p_2^{(m)}$ and $s_1^{(m)*} p_1^{(m)}$. As an example, Fig. 1 shows an Alamouti-GSM system where the precoding weights are $p_1^{(1)} = 0.7$ and $p_2^{(1)} = 0.2$ with the symbols $s_1^{(1)} = 1$ and $s_2^{(1)} = 1 + j$.

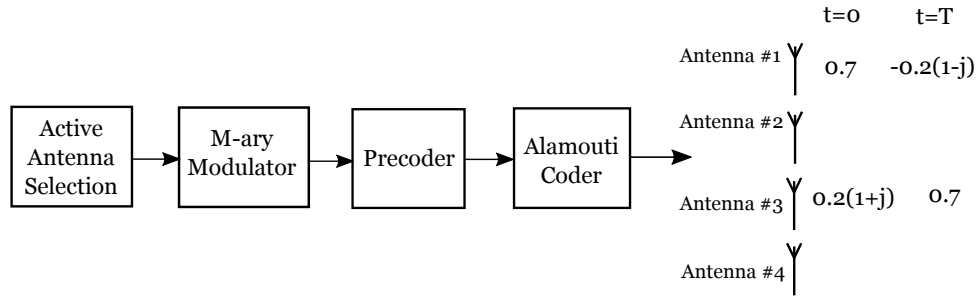


Fig. 1. Block diagram showing precoding with Alamouti-GSM.

We next describe the mathematical expressions for the precoder design using $N_a = 2$ active antennas for the Alamouti-GSM. One complete Alamouti transmission can be described as

$$\begin{aligned} \text{odd time slot: } \quad \mathbf{y} &= \mathbf{H}\mathbf{x}^{(m)} + \mathbf{n} = (p_1^{(m)} \mathbf{c}_1^{(m)} + p_2^{(m)} \mathbf{c}_2^{(m)}) + \mathbf{n} \\ \text{even time slot: } \quad \mathbf{y}' &= \mathbf{H}\mathbf{x}'^{(m)} + \mathbf{n}' = (p_2^{(m)} \mathbf{c}_1'^{(m)} + p_1^{(m)} \mathbf{c}_2'^{(m)}) + \mathbf{n}' \end{aligned} \quad (6)$$

where $\mathbf{x}'^{(m)}$ is the transmit signal at the even time slot determined by the Alamouti scheme. For instance, if $\mathbf{x}^{(m)} = [s_1^{(m)} p_1^{(m)}, 0, s_2^{(m)} p_2^{(m)}, 0]$, then $\mathbf{x}'^{(m)} = [-s_2^{(m)*} p_2^{(m)}, 0, s_1^{(m)*} p_1^{(m)}, 0]$, $\mathbf{c}_1^{(m)} = \mathbf{H}(-s_2^{(m)*})$, and $\mathbf{c}_2'^{(m)} = \mathbf{H}(s_1^{(m)*})$.

The maximum likelihood (ML) detection of the symbol m for the Alamouti-GSM is given by

$$\hat{m} = \arg \min_m \left\| \begin{bmatrix} \mathbf{y} \\ \mathbf{y}' \end{bmatrix} - \begin{bmatrix} \mathbf{H}\mathbf{x}^{(m)} \\ \mathbf{H}\mathbf{x}'^{(m)} \end{bmatrix} \right\|^2 \quad (7)$$

where $\|\cdot\|$ denotes the Euclidean norm. It is known that the upper bound on the error probability in ML detection can be minimized by maximizing the minimum Euclidean distances between received sequences. Therefore, we need to maximize the following minimization

$$\min_{i,j,i \neq j} \left(\left\| \sum_{n=1}^{N_a} (\mathbf{c}_n^{(i)} p_n^{(i)} - \mathbf{c}_n^{(j)} p_n^{(j)}) \right\|^2 + \left\| \sum_{n=1}^{N_a} (\mathbf{c}_n'^{(i)} p_n'^{(i)} - \mathbf{c}_n'^{(j)} p_n'^{(j)}) \right\|^2 \right) \quad (8)$$

For the Alamouti-scheme, $N_a = 2$, $p_1^{(i)} = p_2^{(i)}$, and $p_2'^{(i)} = p_1^{(i)}$. However, to keep the notation more general, we will still use N_a in stead of 2. Define the precoder weight vector, $\mathbf{p} =$

$[p_1^{(1)}, \dots, p_{N_a}^{(1)}, \dots, p_1^{(i)}, \dots, p_{N_a}^{(i)}, \dots, p_{N_a}^{(M_t)}]$. Also define a matrix $\mathbf{C}_i = [\mathbf{c}_1^{(i)}, \mathbf{c}_2^{(i)}, \dots, \mathbf{c}_{N_a}^{(i)}]$ for each $i = 1, 2, \dots, M_t$. Next, formulate a $M_t N_a \times M_t N_a$ positive semidefinite matrix \mathbf{D} . Consider this matrix to be made up of $M_t \times M_t$ blocks, each block containing N_a rows and N_a columns. The (k, l) -th block of this matrix is denoted by $[\mathbf{D}]_{k,l}$ and it refers to the elements in this matrix lying between the $((k-1)N_a + 1)$ to (kN_a) -th row and $((l-1)N_a + 1)$ to (lN_a) -th column. Define this matrix \mathbf{D} in terms of block by block as

$$[\mathbf{D}^{(i,j)}]_{k,l} = \begin{cases} \mathbf{C}_i^H \mathbf{C}_i & \text{if } k = i \text{ and } l = i \\ -\mathbf{C}_i^H \mathbf{C}_j & \text{if } k = i \text{ and } l = j \\ \mathbf{C}_j^H \mathbf{C}_j & \text{if } k = j \text{ and } l = j \\ -\mathbf{C}_j^H \mathbf{C}_i & \text{if } k = j \text{ and } l = i \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Considering that for the Alamouti scheme, $N_a = 2$, we also define $\mathbf{C}'_i = [\mathbf{c}'_2^{(i)}, \mathbf{c}'_1^{(i)}]$. Then we can define another matrix $\mathbf{D}'^{(i,j)}$ as in (9) with \mathbf{C}_i and \mathbf{C}_j being replaced by \mathbf{C}'_i and \mathbf{C}'_j respectively. The optimization problem (8) can now be re-written along with the constraint as

$$\max_{\mathbf{p}} \quad d \quad (10)$$

subject to

$$\mathbf{p}^H (\mathbf{D}^{(i,j)} + \mathbf{D}'^{(i,j)}) \mathbf{p} \geq d \quad \forall i, j, \quad i \neq j \quad (11)$$

and necessary power constraints, to be described in the next section. Note that if $\mathbf{D}'^{(i,j)} = \mathbf{0}$, then the above problem becomes the standard GSM precoder design problem as described in [10].

POWER CONSTRAINTS IN PRECODER DESIGN

The optimization problem (10), coupled with the condition (11), needs to be solved under a power constraint. The power constraint used in [10] is the PMU constraint

$$\|\mathbf{p}\|^2 \leq P_t, \quad (12)$$

where P_t is the maximum power allowed over all the possible symbols. Note that the power constraint (12) is an overall power constraint over many uses of the channel. The resulting precoding coefficients can vary widely in magnitudes. To keep the power transmission in each channel use constant, we propose a new PSU condition,

$$\sum_{n=1}^{N_a} |p_n^{(i)}|^2 \leq \frac{P_t}{M_t}, \quad (13)$$

for all i . Finally, to avoid wide fluctuations in each individual antenna's gain, we introduce another constraint, called PAU, as

$$a \frac{P_t}{M_t N_a} \leq |p_n^{(i)}|^2 \leq b \frac{P_t}{M_t N_a} \quad (14)$$

for all i , where a and b ($0 \leq a \leq b \leq 1$) are constants that control antenna gain fluctuations. If $a = b = 1$, then there will be no precoding.

We follow the procedure given in [10] to express the precoding optimization as an approximated convex optimization problem, and iteratively solve the problem using the convex optimization procedures [13], [14]. The iterations are stopped when no further improvement in the solution is observed.

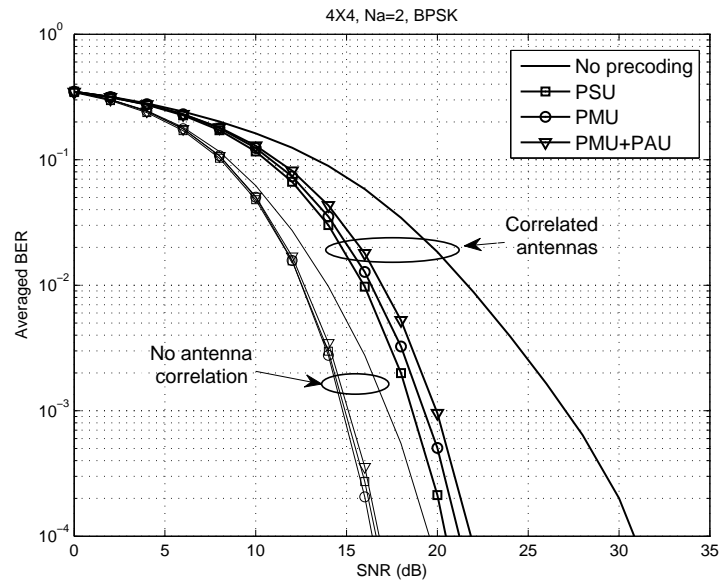


Fig. 2. Performance of BPSK GSM systems with precoders. In the case of correlated antennas, $\rho = 0.9$, while $\rho = 0$ for antennas with no correlation.

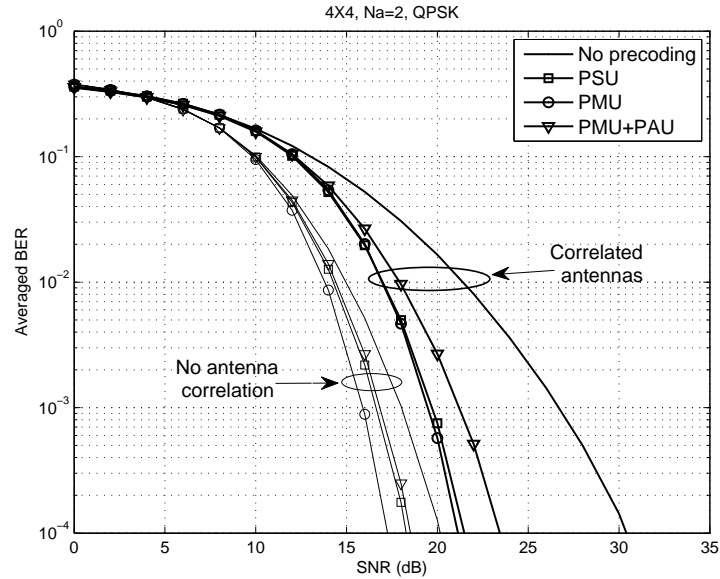


Fig. 3. Performance of QPSK GSM systems with precoders. In the case of correlated antennas, $\rho = 0.9$, while $\rho = 0$ for antennas with no correlation.

NUMERICAL RESULTS AND DISCUSSIONS

We use $N_t = 4$ and $N_r = 4$. The number of active antennas is $N_a = 2$ in all our results. The signal-to-noise ratio (SNR) is defined as E_s/N_o , where E_s is the symbol power and N_o is noise power spectral density.

1) *Effects of Precoder Constraints*: Figures 2 and 3 show the performance of BPSK and QPSK GSM schemes for different precoder power constraints. Due to high numerical complexities involved, our results are obtained by averaging over only 10 realizations of the random matrix \mathbf{H} . Both figures show that precoding provides significant improvement in BER for correlated antennas. The proposed PSU constraint performs slightly better than PMU for BPSK in case of correlated antennas. For PAU, we use $a = 0.25$ and $b = 0.75$. Observe that when PAU is used in conjunction with PMU, the SNR gain reduces. However, the antenna gain fluctuations reduce too. This is shown in Fig. 4 for one of the antennas. For PMU only constraint, the precoder coefficients range from 0.64 to 1.65, while for PMU+PAU, fluctuations range from about 0.68 to 0.87. Still there is significant SNR gain over a system with no precoding. For QPSK, the gain of PMU+PAU is still more than 6 dB.

2) *Precoder performance study for Alamouti-GSM*: Fig.5 presents the BER results for Alamouti-

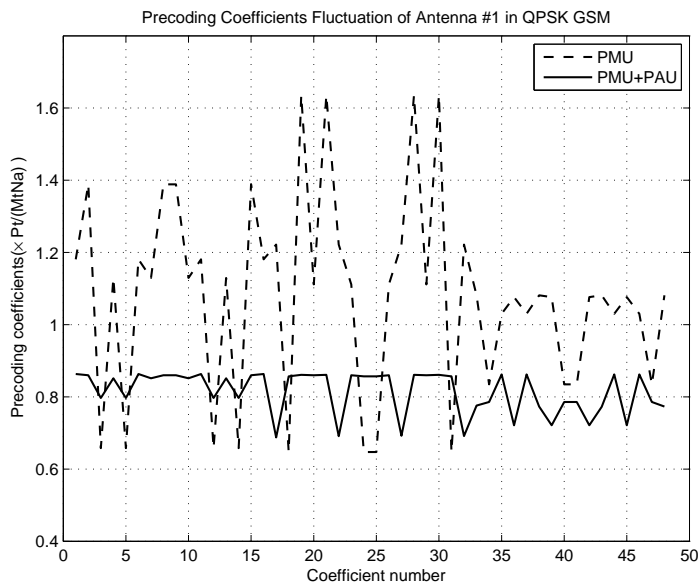


Fig. 4. Variations of the precoding coefficients under PMU and PAU constraints for QPSK GSM.

GSM using 8 PSK symbols. We have compared its performance with BPSK GSM so that the spectral efficiency of the systems being compared is same. Our results are shown for only one specific realization of the channel matrix \mathbf{H} for complexity reasons. The rows of the matrix \mathbf{H} for this specific realization are $[0.3309 + 0.2599j, -0.2951 + 0.0457j, -0.0659 - 0.4223j, 0.1509 - 0.0905j]$, $[-0.3272 - 1.1734j, 0.5747 - 0.3238j, 0.0149 + 0.1168j, 0.1294 + 0.6845j]$, $[-0.9439 + 1.3103j, -0.8254 - 0.2419j, 0.0944 - 0.2592j, 0.1155 + 1.2289j]$, $[-1.2820 - 0.8414j, 0.2230 - 0.7150j, 1.1956 - 1.3072j, 0.7751 - 0.1836j]$. The proposed Alamouti-GSM for correlated antennas (for this specific randomly chosen \mathbf{H}) provides an SNR gain of about 3 dB over BPSK GSM without Alamouti. Also note that the precoder coefficients obtained through optimizing the 8 PSK GSM without taking Alamouti repetitions into consideration show degradation at high SNR.

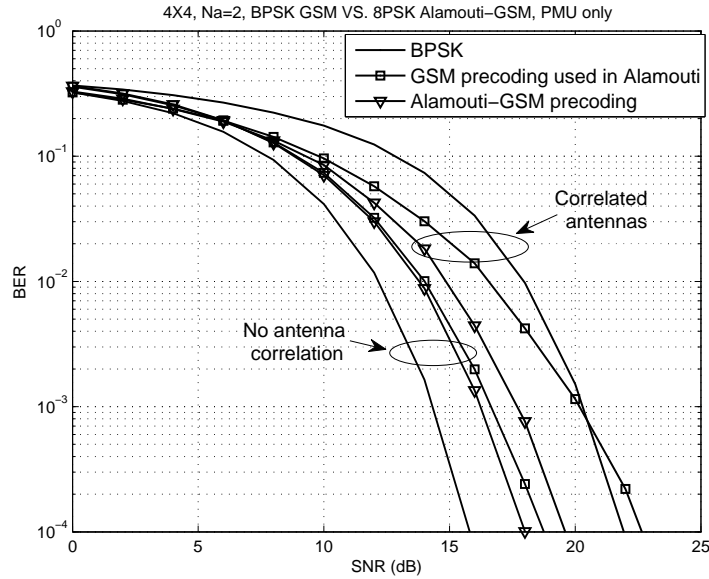


Fig. 5. Performance of precoded Alamouti-GSM systems.

CONCLUSION

We have investigated the effects of using precoders in Alamouti-GSM systems as well as the effects of different power constraints on the precoders for GSM. Our approach is based on a precoding technique used in the literature for studying SM/GSM systems. The precoder maximizes the minimum Euclidean distances between received sequences, and computes the precoder weights by solving an approximate convex program using iterations. The results show that precoders using Alamouti-GSM system can achieve significant gain in BER performance. We also presented a study showing trade-offs that can be achieved by limiting the antenna gain fluctuations.

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