

MAXIMUM LIKELIHOOD DETECTION FROM MULTIPLE BIT SOURCES

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ABSTRACT

This paper deals with the problem of producing the best bit stream from a number of input bit streams with varying degrees of reliability. The best source selector and smart source selector are recast as detectors, and the maximum likelihood bit detector (MLBD) is derived from basic principles under the assumption that each bit value is accompanied by a quality measure proportional to its probability of error. We show that both the majority voter and the best source selector are special cases of the MLBD and define the conditions under which these special cases occur. We give a mathematical proof that the MLBD is the same as or better than the best source selector.

INTRODUCTION

Challenging air-to-ground radio links are a fact of life in aeronautical telemetry. While these challenges exasperate many a program monitor in his quest for pristine flight test data, the challenges make the range engineer's job interesting. Less-than-ideal channel conditions are caused by a number of spatial factors such as RF interference, shadowing (blockage by structures or terrain), multipath propagation, or large transmitter-to-receiver distances. These factors are termed "spatial" because they are location dependent in the sense that when the airborne test article moves to a different location, these factors tend to change. Some factors are not spatial, such as faulty cables and connectors or circuit failure in the transmitter.

For the spatial factors limiting link availability, the obvious approach to use some form of spatial

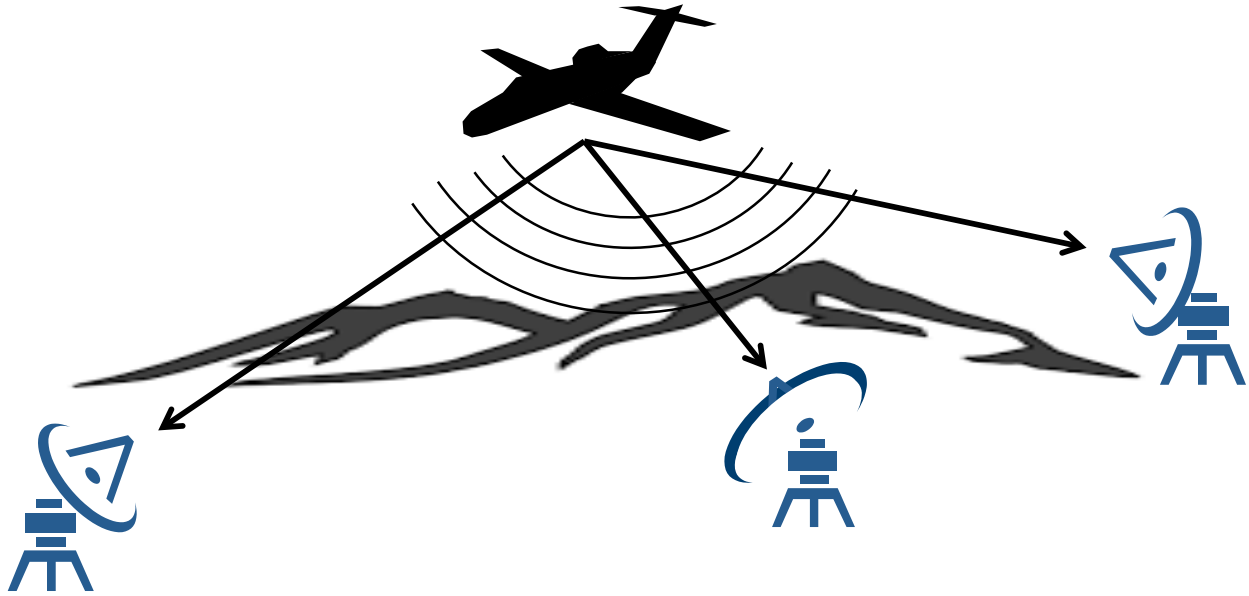


Figure 1: An illustration of the spatial diversity concept.

diversity. The concept is illustrated in Figure 1. The basic idea is that when the propagation path from the airborne test article to one of the receivers is bad, the other propagation paths are good. The greater the spatial separation, the more likely this is to be true. Many ranges deploy receiving stations as shown in Figure 1 for a more simple reason: to provide radio downlink coverage over the entire test range. In these situations, spatial diversity is an added bonus.

The optimum form of spatial diversity is RF/IF combination.¹ Generally speaking, diversity reception operates in one of three modes: *selection diversity* (choose the best signal and ignore the others), *equal gain combiners* (co-phase and add the available signals), or *maximum ratio combiners* (co-phase, weight the available signals based on the signal-to-noise ratio, and add). Essentially all of the work in spatial diversity in aeronautical telemetry has been devoted to selection diversity. Turner and Potter [1] describe a telemetry ground station in Japan using two antennas and selection diversity. Several papers describe methods for identifying the best signal from hypothetical antennas [2] – [7].

Given the large distances separating most ground stations, RF/IF combining has not been considered practical. What is practical is delivering to a central location the bit decisions from de-

¹This is why polarization diversity combiners operate prior to demodulation. But polarization diversity in the aeronautical telemetry environment does not provide the spatial diversity improvement in the sense this paper emphasizes. This is because the “spatial separation” between the two polarizations is insufficient to overcome the type of impairments considered here. Polarization diversity, as performed in aeronautical telemetry, has the benefit of reclaiming the 3 dB loss accompanying the synthesis of the two circular polarizations from the two linear polarizations in the antenna feed.

modulators operating at widely separated receiving sites. This situation has given rise to what is generally called “best source selection” (BSS) [8] – [19]. Because BSS operates on the recovered bit streams, BSS can be thought of as “poor man’s” diversity.

The earliest systems attempting merge different bit streams were summarized by Rigley et al. [10] and Gatton [13], whose descriptions conjure the remarkable mental image of a telemetry engineer switching patch cables in response to what he observed on oscilloscopes. Later versions, described by Peterson [8], Wilson [11], and Endress [19] focused on the post-test phase of creating a “master file” from “source files” recorded at each receiver site. Real-time approaches first took the form of “best source selection” in the sense that the “best” of the input bit streams was passed to the user. Examples of the criteria for “best” include receiver AGC when available [9, 13], signal-to-noise level metrics derived from bit syncs when “analog inputs” were available [9, 14, 18], and frame synchronization statistics (usually extracted from the decommutator) [8, 10, 12, 13, 15]

It quickly became apparent that it was better to produce the “best bit” than select the “best source.” Gatton [13] termed this concept a “best data selector” whereas Fewer, Wilmot [16] and Corry [17] termed the concept a “smart data selector.” The most popular form of best/smart data selector is the majority vote [12, 13, 14, 15, 18, 19]. After aligning the bit stream outputs from the available demodulators, the best/smart data selector examines the available outputs on a bit-by-bit basis and selects for each bit the most common bit value. Generalizations of majority voting, sometimes called “weighted majority vote” [18] are used to break ties when an even number of sources are available or to improve performance. The voting weights described in the open literature are based on the familiar quality measures such as receiver AGC [14], signal-to-noise level metrics derived from bit syncs² [14, 18], frame synchronization statistics [12, 13, 15], and a variety of ad-hoc approaches based on assumptions of oversampling [11], occurrence of fill patterns [19], or assumptions about high error rates adjacent to a data dropout [19]. Fewer, Wilmot [16] and Corry [17] describe a maximum ratio “soft-bit combiner” that is perilously close to the optimum IF combiner.

The best/smart data selector is similar to the best source selector in the sense that it produces one (hopefully better) bit stream from a number of input bit streams. But the best/smart data selector operates on a fundamentally different principle: the goal is to produce a bit estimate based on all

²The signal-to-noise ratio metric is only available when the bit sync input is “analog,” i.e., the (video) output of FM demodulator. This approach applies almost exclusively to PCM/FM. Gerstner and Lillevold [18] point out the problem of transmitting the analog video output over long distances and describe a work-around where this information is quantized at its source and digitally encoded in the form of frame “encapsulation.” An SOQPSK-TG counterpart can be envisioned based on similar principles, but it does not involve a bit sync as in Gerstner and Lillevold because SOQPSK-TG demodulators do not require an external bit sync: symbol timing synchronization is embedded in the detection process internal to the SOQPSK-TG demodulator.

available inputs without attempting to replicate or select any one input. In this sense, the best/smart data selector is not a *selector*, but a *detector*. In this paper, we develop the Maximum Likelihood Bit Detector (MLBD) based on the assumption that each bit value is accompanied by a quality measure proportional to its probability of error. We show that both the majority voter and the best source selector are special cases of the MLBD and give a mathematical proof that the MLBD is the same as or better than the best source selector.

MAXIMUM LIKELIHOOD BIT DETECTOR

The Maximum Likelihood (ML) bit detector (MLBD) is based on the following two assumptions:

Assumption #1 The transmission channel, from the point of view of the input and output bits, is what is called a *binary symmetric channel* or BSC. An example of the BSC is illustrated in Figure 2 (a). The BSC has an input $x \in \{0, 1\}$ and an output $y \in \{0, 1\}$ and a transition probability p . The BSC is completely characterized by the conditional probability $p(y|x)$. Because there are only two possibilities for x and y , the values for the conditional probability $p(y|x)$ can be listed:

$$p(0|0) = 1 - p, \quad p(0|1) = p, \quad p(1|0) = p, \quad p(1|1) = 1 - p. \quad (1)$$

Because all four probabilities depend on the parameter p (this is the probability of bit error), we say the BSC is completely parameterized by the transition probability p .

Assumption #2 The best source selection problem involves N *statistically independent observations* of a bit. The block diagram illustrating this view of the MLBD problem is illustrated in Figure 2 (b). Here, a bit x is simultaneously transmitted over N BSCs, each characterized by their own transition probability p_n , for $n = 1, 2, \dots, N$. The observation is the vector $\mathbf{y} = [y_1, \dots, y_N]$ where y_n is the output of the n -th channel. The conditional probability that defines this system is $p(\mathbf{y}|x)$. Because the N parallel channels are assumed independent, we have

$$p(\mathbf{y}|x) = \prod_{n=1}^N p(y_n|x) \quad (2)$$

where each probability is given by (1) where p is replaced by p_n .

With these assumptions in place, the MLBD can now be defined. The ML estimate of the bit x is given by

$$\hat{x} = 0 \iff p(\mathbf{y}|x = 0) > p(\mathbf{y}|x = 1), \quad (3)$$

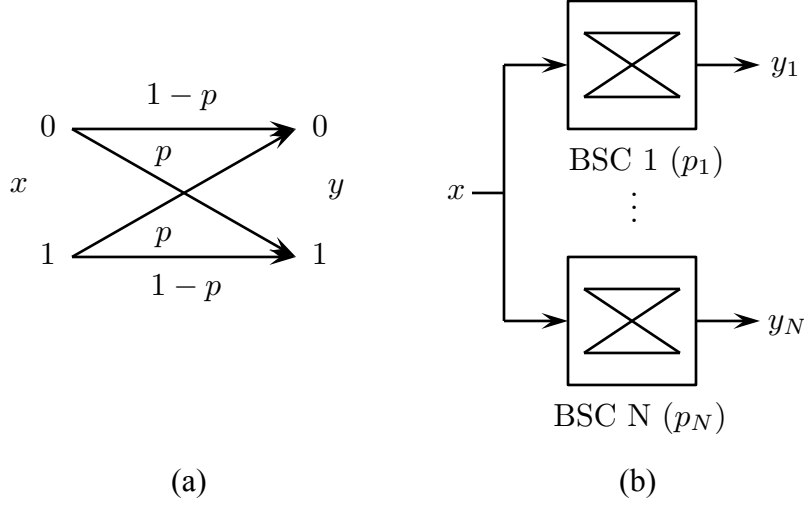


Figure 2: Binary symmetric channels (BSCs): (a) A binary symmetric channel with input x and output y and transition probability p . (b) A conceptualization of the MLBD problem: a data bit x is transmitted over N independent binary symmetric channels.

otherwise $\hat{x} = 1$. Examining condition (3) in more detail, we have

$$\begin{aligned}
 \hat{x} = 0 &\iff p(\mathbf{y}|x = 0) > p(\mathbf{y}|x = 1) \\
 &\iff \prod_{n=1}^N p(y_n|x = 0) > \prod_{n=1}^N p(y_n|x = 1)
 \end{aligned} \tag{4}$$

Let \mathcal{N}_0 be the set of indexes for which $y_n = 0$ and let \mathcal{N}_1 be the set of indexes for which $y_n = 1$. Using these definitions, condition (4) may be expressed as

$$\begin{aligned}
 \hat{x} = 0 &\iff \prod_{n \in \mathcal{N}_0} p(y_n|x = 0) \prod_{n \in \mathcal{N}_1} p(y_n|x = 0) > \prod_{n \in \mathcal{N}_0} p(y_n|x = 1) \prod_{n \in \mathcal{N}_1} p(y_n|x = 1) \\
 &\iff \prod_{n \in \mathcal{N}_0} (1 - p_n) \prod_{n \in \mathcal{N}_1} p_n > \prod_{n \in \mathcal{N}_0} p_n \prod_{n \in \mathcal{N}_1} (1 - p_n) \\
 &\iff \log \left(\prod_{n \in \mathcal{N}_0} (1 - p_n) \prod_{n \in \mathcal{N}_1} p_n \right) > \log \left(\prod_{n \in \mathcal{N}_0} p_n \prod_{n \in \mathcal{N}_1} (1 - p_n) \right) \\
 &\iff \sum_{n \in \mathcal{N}_0} \log(1 - p_n) + \sum_{n \in \mathcal{N}_1} \log(p_n) > \sum_{n \in \mathcal{N}_0} \log(p_n) + \sum_{n \in \mathcal{N}_1} \log(1 - p_n) \\
 &\iff \sum_{n \in \mathcal{N}_0} \log(1 - p_n) - \sum_{n \in \mathcal{N}_0} \log(p_n) > \sum_{n \in \mathcal{N}_1} \log(1 - p_n) - \sum_{n \in \mathcal{N}_1} \log(p_n) \\
 &\iff \sum_{n \in \mathcal{N}_0} \log \left(\frac{1 - p_n}{p_n} \right) > \sum_{n \in \mathcal{N}_1} \log \left(\frac{1 - p_n}{p_n} \right).
 \end{aligned} \tag{5}$$

The last relationship (5) defines the MLBD.

Example Suppose there are $N = 3$ channels with $p_1 = 10^{-4}$, $p_2 = 10^{-3}$, and $p_3 = 10^{-2}$. The log-ratios for the three channels are

$$\log\left(\frac{1-p_1}{p_1}\right) = 9.2 \quad \log\left(\frac{1-p_2}{p_2}\right) = 6.9 \quad \log\left(\frac{1-p_3}{p_3}\right) = 4.6.$$

Now suppose the three channel outputs are $y_1 = 1$, $y_2 = 0$, $y_3 = 1$. This defines the sets

$$\mathcal{N}_0 = \{2\} \quad \mathcal{N}_1 = \{1, 3\}.$$

The summations in (5) are

$$\sum_{n \in \mathcal{N}_0} \log\left(\frac{1-p_n}{p_n}\right) = 6.9 \quad \sum_{n \in \mathcal{N}_1} \log\left(\frac{1-p_n}{p_n}\right) = 13.8.$$

Because the summation over the set \mathcal{N}_1 is greater than the summation over the set \mathcal{N}_0 , the MLBD output is 1. □

Special Case: Equal Channels For the special case where all N channels are the same, $p_n = p$ for $1 \leq n \leq N$ and the ML rule (5) reduces to

$$\hat{x} = 0 \iff |\mathcal{N}_0| > |\mathcal{N}_1|. \tag{6}$$

where $|\mathcal{N}_0|$ is the number of elements in the set \mathcal{N}_0 . In words, the MLBD assigns $\hat{x} = 0$ if there are more zeros than ones, otherwise $\hat{x} = 1$. This is the majority vote rule. In summary, the MLBD reduces to the majority voter when the channels are the same. □

Special Case: Dominant Channel Suppose the channels are ordered so that $p_1 \leq p_2 \leq \dots \leq p_N$. Then we have

$$\log\left(\frac{1-p_1}{p_1}\right) \geq \log\left(\frac{1-p_2}{p_2}\right) \geq \dots \geq \log\left(\frac{1-p_N}{p_N}\right). \tag{7}$$

Channel 1 is a *dominant channel* if

$$\log\left(\frac{1-p_1}{p_1}\right) > \sum_{n=2}^N \log\left(\frac{1-p_n}{p_n}\right). \quad (8)$$

When condition (8) holds, the summands corresponding to channels 2 through N can never sum to a value that changes the decision based on observing the output of channel 1. Consequently, when a dominant channel exists, the MLBD reduces to the output of the dominant channel. That is, the best source selector is optimal. \square

PERFORMANCE ANALYSIS

If the MLBD is to be of practical value, the probability of error at the MLBD output $P(E)$ cannot be greater than any of the bit error probabilities at the MLBD input. In this section we evaluate the probability of error at the MLBD output. The starting point is the conditional probability of error $P(E|0)$ which is the probability of error given $x = 0$. Referring to (5), an error occurs when the inequality is not satisfied. Consequently, the conditional probability of error is

$$P(E|0) = \Pr\left\{\sum_{n \in \mathcal{N}_1} \log\left(\frac{1-p_n}{p_n}\right) > \sum_{n \in \mathcal{N}_0} \log\left(\frac{1-p_n}{p_n}\right)\right\}. \quad (9)$$

Similarly, the conditional probability of error assuming $x = 1$ is

$$P(E|1) = \Pr\left\{\sum_{n \in \mathcal{N}_0} \log\left(\frac{1-p_n}{p_n}\right) > \sum_{n \in \mathcal{N}_1} \log\left(\frac{1-p_n}{p_n}\right)\right\}. \quad (10)$$

The symmetry of the BSCs are such that $P(E|1) = P(E|0)$. The average probability of error is

$$P(E) = \frac{1}{2}P(E|0) + \frac{1}{2}P(E|1) = P(E|0). \quad (11)$$

Because the functional dependence between the channel transition probabilities and the MLBD output errors lies in membership in the sets \mathcal{N}_0 and \mathcal{N}_1 , it is hard to draw any general conclusions from the form (9). An example illustrates how the computations work.

Example Returning to the example of $N = 3$ with $p_1 = 10^{-4}$, $p_2 = 10^{-3}$ and $p_3 = 10^{-2}$, we assume $x = 0$ and construct the following table listing all possible channel outputs, their

probabilities, and the values of the summations:

| y_1 | y_2 | y_3 | probability | $\sum_{\mathcal{N}_0}$ | $\sum_{\mathcal{N}_1}$ |
|-------|-------|-------|---|------------------------|------------------------|
| 0 | 0 | 0 | $(1 - p_1)(1 - p_2)(1 - p_3) = 9.8891 \times 10^{-1}$ | 20.7 | 0 |
| 0 | 0 | 1 | $(1 - p_1)(1 - p_2)p_3 = 9.9890 \times 10^{-3}$ | 16.1 | 4.6 |
| 0 | 1 | 0 | $(1 - p_1)p_2(1 - p_3) = 9.8990 \times 10^{-4}$ | 13.8 | 6.9 |
| 0 | 1 | 1 | $(1 - p_1)p_2p_3 = 9.9990 \times 10^{-6}$ | 9.2 | 11.5 |
| 1 | 0 | 0 | $p_1(1 - p_2)(1 - p_3) = 9.8901 \times 10^{-5}$ | 11.5 | 9.2 |
| 1 | 0 | 1 | $p_1(1 - p_2)p_3 = 9.9900 \times 10^{-7}$ | 6.9 | 13.8 |
| 1 | 1 | 0 | $p_1p_2(1 - p_3) = 9.9000 \times 10^{-8}$ | 4.6 | 16.1 |
| 1 | 1 | 1 | $p_1p_2p_3 = 1.0000 \times 10^{-9}$ | 0.0 | 20.7 |

An error occurs when the $\sum_{\mathcal{N}_1}$ column is larger than the $\sum_{\mathcal{N}_0}$ column. This occurs on the fourth, sixth, seventh, and eighth rows. Adding the probabilities gives

$$P(E|0) = 9.9990 \times 10^{-6} + 9.9900 \times 10^{-7} + 9.9000 \times 10^{-8} + 1.0000 \times 10^{-9} = 1.1098 \times 10^{-5}. \quad (12)$$

In this example, the best channel at the MLBD input has a probability of error 10^{-4} whereas the BSS output has a probability of error approximately 10^{-5} . This is an example where the MLBD output has a lower probability of error than any of the inputs. Furthermore, the MLBD output has a lower probability of error than the best source selector. \square

Special Case: Equal Channels For the case of equal channels, $p_n = p$ for all n and the MLBD reduces to the majority voter. Assuming $x = 0$, an error occurs if a majority of the y_n are 1. The probability this occurs is

$$P(E|0) = \sum_{n=\lceil N/2 \rceil}^N \binom{N}{n} p^n (1-p)^{N-n} \quad (13)$$

where $\lceil z \rceil$ is the smallest integer greater than or equal to z . A plot of the MLBD output probability of error for $N = 3, 5, 7$ is shown in Figure 3. The plot shows a remarkable improvement in the probability of error. \square

The probability of error performance of the MLBD has been illustrated for a simple example and for the special case of equal channels. Generalizing beyond these examples is difficult. At first glance it is not clear that the MLBD generally improves the situation. In the next section we prove that the bit error probability at the output of the MLBD is *always* the same as or better than the best input channel. The implication is that MLBD is the same as or better than best source selection.

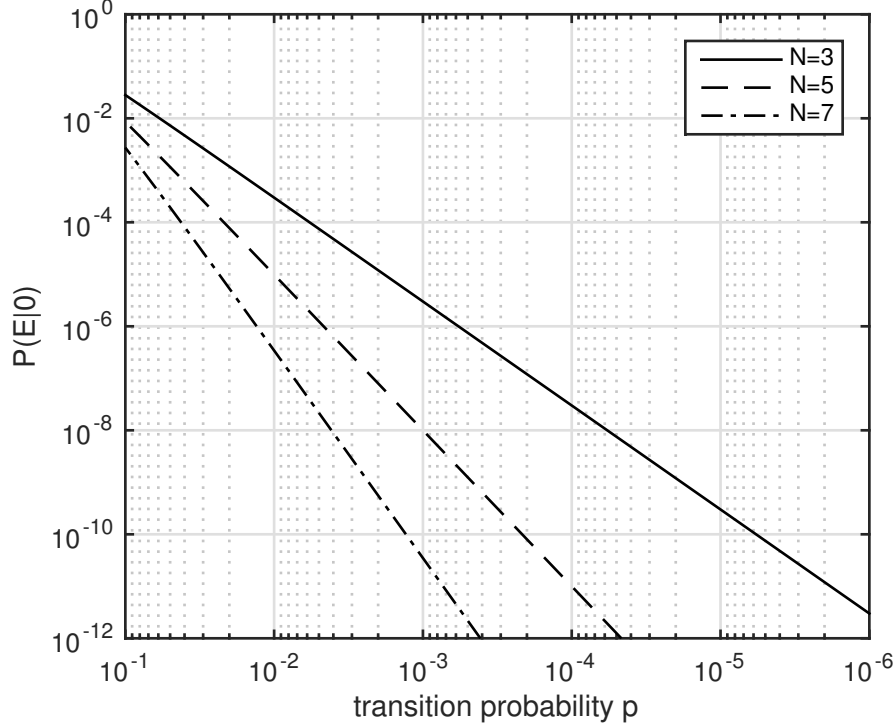


Figure 3: A plot of the probability of bit error at the MLBD output for the special case of equal channels each with transition probability p .

PROOF THAT THE MLBD IS EQUAL TO OR BETTER THAN BEST SOURCE SELECTION

We introduce the concept of the *error pattern* to prove that the MLBD is equal to or better than best source selection. The BSC output symbols are $y_n \in \{0, 1\}$. We model these as $y_n = x \oplus e_n$, where $x \in \{0, 1\}$ is the information bit that is common to all BSCs, $e_n \in \{0, 1\}$ is an error bit, and \oplus is modulo-2 addition. Due to the nature of the BSC model, $e_n = 1$ occurs with probability p_n and $e_n = 0$ occurs with probability $1 - p_n$. There are 2^N possible length- N binary error patterns, $\mathbf{e}[i]$, where $i \in \{1, \dots, 2^N\}$. The elements of the column vector $\mathbf{e}[i]$ are $e_n[i] \in \{0, 1\}$, where $n \in \{1, \dots, N\}$. We define $\mathcal{N}_0[i]$ as the set of indexes n for which $e_n[i] = 0$, and $\mathcal{N}_1[i]$ as the set of indexes n for which $e_n[i] = 1$.

The i -th error pattern occurs with probability

$$P[i] = \prod_{n \in \mathcal{N}_0[i]} (1 - p_n) \prod_{n \in \mathcal{N}_1[i]} p_n. \quad (14)$$

and $\sum_i P[i] = 1$ by virtue of $P[i]$ being a probability mass function. We define the MLBD metric

for the i -th error pattern as

$$M[i] = \sum_{n \in \mathcal{N}_1[i]} \log \left(\frac{1-p_n}{p_n} \right) - \sum_{n \in \mathcal{N}_0[i]} \log \left(\frac{1-p_n}{p_n} \right). \quad (15)$$

Let \mathcal{I}_+ be the set of indexes i for which $M[i] > 0$, and let \mathcal{I}_- be the set of indexes i for which $M[i] < 0$. Let $\mathbf{e}[j]$ be the complement of $\mathbf{e}[i]$. Because complementing each $e_n[i]$ switches the set to which n belongs, for each $i_+ \in \mathcal{I}_+$ there is an $i_- \in \mathcal{I}_-$ for which $M[i_+] = -M[i_-]$. Therefore, \mathcal{I}_+ and \mathcal{I}_- each contain half of the indexes in $\{1, \dots, 2^N\}$, i.e. $|\mathcal{I}_+| = |\mathcal{I}_-| = 2^{N-1}$, and these two sets of indexes are mutually exclusive.

We will now show that the largest $M[i]$ has the smallest $P[i]$. The first step is to show that for the positive–negative index pairs, i_+ and i_- , we have

$$P[i_-] > P[i_+], \quad \text{or, equivalently,} \quad \frac{P[i_-]}{P[i_+]} > 1 \quad (16)$$

Because the ones and zeros for $\mathbf{e}[i_+]$ and $\mathbf{e}[i_-]$ are reversed, it follows that $\mathcal{N}_0[i_+] = \mathcal{N}_1[i_-]$ and $\mathcal{N}_1[i_+] = \mathcal{N}_0[i_-]$. Therefore, we begin with the the definition that $M[i_+]$ is positive and proceed from that point:

$$\sum_{n \in \mathcal{N}_1[i_+]} \log \left(\frac{1-p_n}{p_n} \right) - \sum_{n \in \mathcal{N}_0[i_+]} \log \left(\frac{1-p_n}{p_n} \right) > 0 \quad (17)$$

$$\sum_{n \in \mathcal{N}_1[i_+]} \log \left(\frac{1-p_n}{p_n} \right) + \sum_{n \in \mathcal{N}_0[i_+]} \log \left(\frac{1-p_n}{p_n} \right) > 0 \quad (18)$$

and raising both sides to the power of e we get

$$\frac{P[i_-]}{P[i_+]} > 1 \implies P[i_-] > P[i_+]. \quad (19)$$

Next, we consider two positive MLBD metrics with the relationship $M[i_+] > M[j_+]$. Using the identity $M[i_+] = \log(P[i_-]/P[i_+])$ that was established above, we see that

$$M[i_+] > M[j_+] \iff \frac{P[i_-]}{P[i_+]} > \frac{P[j_-]}{P[j_+]} \quad (20)$$

Ordinarily, the relationship $a/b > c/d$ would not imply that $a > c$ and $b < d$. However, due to the relationship $q_n = 1 - p_n$, when $P[i_+]$ diminishes, $P[i_-]$ grows. Therefore, the only way (20) can be satisfied is for $P[i_-] > P[j_-]$ and $P[i_+] < P[j_+]$. Combining (19) and (20) completes the

proof that the largest values of $M[i]$ have the smallest values of $P[i]$.

The probability of error for the MLBD scheme is equivalent to the probability that a positive error metric $M[i]$ occurs, i.e.

$$P_e = \sum_{i \in \mathcal{I}_+} P[i] \quad (21)$$

Likewise, the probability of being correct is equivalent to the probability that a negative error metric $M[i]$ occurs, i.e.

$$P_c = \sum_{i \in \mathcal{I}_-} P[i] \quad (22)$$

with the expected result of $P_e + P_c = 1$. The MLBD scheme yields the minimum probability of error, because the probability of error calculation in (21) sums the 2^{N-1} terms with the smallest values.

The consequence of the minimality of (21) is profound. Let \mathcal{I}' be any set of 2^{N-1} indexes. Then, the minimality of (21) means that

$$\sum_{i \in \mathcal{I}_+} P[i] \leq \sum_{i \in \mathcal{I}'} P[i] \quad (23)$$

where equality holds when $\mathcal{I}' = \mathcal{I}_+$.

Now assume without loss of generality that $p_1 = \min_n \{p_n\}$. The best source selector relies solely on the input from the best channel (BSC 1). The probability of bit error of the best source selector is obtained from (14) by summing over i for which $e_1[i] = 1$. Let \mathcal{I}_1 be the set of i for which $e_1[i] = 1$. Then we have

$$p_1 = \sum_{i \in \mathcal{I}_1} P[i]. \quad (24)$$

But by (23), we must have

$$P_e = \sum_{i \in \mathcal{I}_+} P[i] \leq \sum_{i \in \mathcal{I}_1} P[i] = p_1. \quad (25)$$

This establishes the relationship

$$P_e \leq \min_n \{p_n\} \quad (26)$$

and shows that the bit error rate of the MLBD (5) is *always* equal to or better than the bit error rate of the best input channel. Because the best source selector ideally selects the bit from the best input channel, we conclude that the bit error probability of the MLBD is equal to or better than that of the best source selector.

Special Case: Dominant Channel In the case of a dominant channel, $\mathcal{I}_+ = \mathcal{I}_1$ and the relationships (25) and (26) are satisfied with equality. \square

We close by pointing out that $N = 2$ is always the dominant channel case when $p_1 \neq p_2$ (otherwise, it is the equal channel case). The consequence is that for $N = 2$, best source selection (when $p_1 \neq p_2$) or majority voting (when $p_1 = p_2$) is optimal.

CONCLUSIONS

We derived the maximum likelihood bit detector (MLBD) based on the assumption that each input bit is accompanied by a value proportional to its probability of error. The MLBD reduces to the majority voter when the bits from all channels have the same probability of error, and the MLBD reduces to the best source selector when there is a dominant channel in the sense of condition (8). We concluded with a mathematical proof that the bit error probability of the MLBD is the same as or lower than the bit error probability of the best source selector.

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