

EFFICIENT AOA ESTIMATION TECHNIQUES FOR GPS SIGNAL

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Abstract

Global Positioning System (GPS) interference signals are suppressed using angle-of-arrival (AOA) techniques, while at the same time the power of the GPS signal is enhanced. After estimating all AOAs from the received signal, we must determine which AOA corresponds to the GPS signal of interest, and in the presence of high-power interference signals. In this paper, we describe an algorithm for selecting the GPS AOA by first comparing all AOAs derived from the received signals before despreading. Although this approach has excellent performance, it has a high computational complexity. In order to overcome this drawback, we introduce a modification that yields an efficient GPS AOA estimation algorithm, which is based on a modified despreader and the constant modulus (CM) array cost function. The CM array is capable of selecting signals that have a constant modulus while rejecting non-CM interference signals. The modified despreader is the mechanism that allows this to be achieved, where unlike the interference signals, the GPS signal of interest maintains a constant modulus.

I. INTRODUCTION

Although GPS is used for a variety of commercial and military applications, its performance is affected by extremely high power interference signals because the GPS signal has a very low power [1]. Using a beamforming approach based on the GPS AOA, the power of the GPS signal can be enhanced while the interference signals are suppressed. However, it is usually difficult to estimate this AOA without some tracking process because the receiver operates in an environment with an extremely low signal-to-noise ratio (SNR) and a very high jamming-to-signal ratio (JSR). Even if the AOAs of all received signals are estimated, we must be able to determine which one corresponds to the GPS signal of interest [2].

In this paper, we introduce an algorithm for selecting the GPS AOA from all estimated AOAs in the presence of high-power interference signals. This algorithm compares the AOAs derived from the received signal before despreading to those estimated at the output of the despreader [2]. The AOA found in the latter result that is not present in the former result is the AOA estimate of the GPS signal. In order to suppress the interference signals and enhance the GPS signal, this AOA information can be applied to efficient interference suppression techniques such as the

minimum-variance distortionless-response (MVDR) beamformer [3] or a generalized sidelobe canceler (GSC) [4]. Although this approach has excellent performance, it has a high computational complexity because it performs AOA estimation twice using, for example, the Multiple Signal Classification (MUSIC) algorithm [5] or Estimation of Signal Parameter via Rotational Invariance Techniques (ESPRIT) [6].

In order to overcome this disadvantage, we use an efficient GPS AOA estimation algorithm based on the modified despreader in [7]. This approach has a low computational complexity because it does not require a singular value decomposition (SVD) unlike conventional AOA estimation algorithms, and it directly estimates only the AOA of the GPS signal. The modified despreader is designed to transform CM interference signals into non-CM signals while retaining the CM property of the GPS signal, and its output is applied to the CM array cost function. Since only the GPS signal has the CM property at the output of the modified despreader, only the GPS AOA is estimated by the CM array.

Table 1: Summary of Received Signal Model

| Symbol (Vector/Matrix) | Size | Definition |
|------------------------|--------------|--|
| $\mathbf{x}(k)$ | $M \times 1$ | Received signal vector |
| \mathbf{a}_c | $M \times 1$ | Array response vector for GPS signal |
| \mathbf{A} | $M \times L$ | Array response matrix for interference signals |
| $\mathbf{s}(k)$ | $L \times 1$ | Interference signals vector |
| $\mathbf{n}(k)$ | $M \times 1$ | AWGN vector with i.i.d components |

II. RECEIVED SIGNAL MODEL

We focus on estimating and selecting the AOA for the coarse acquisition (C/A) code and assume a grid antenna array of size $P \times Q$ ($M = PQ$) as described in [8]. For discrete-time index k , the received signal vector is modeled as

$$\mathbf{x}(k) = \mathbf{a}_c c_i(k) b(k) + \mathbf{A} \mathbf{s}(k) + \mathbf{n}(k), \quad (1)$$

where $c_i(k)$ is an element of the cyclostationary pseudorandom noise (PRN) code (of length $N = 20 \times 1023$) for the i th satellite and $b(k)$ is the GPS data bit which remains constant for the duration of one cycle of the PRN code. Other quantities and sizes of the vectors and the matrix in (1) are summarized in Table 1. The columns of \mathbf{A} are the AOA array response vectors for the interference signals, L is the number of interference signals, and $\mathbf{n}(k)$ is additive white Gaussian noise (AWGN) with zero mean and variance σ^2 .

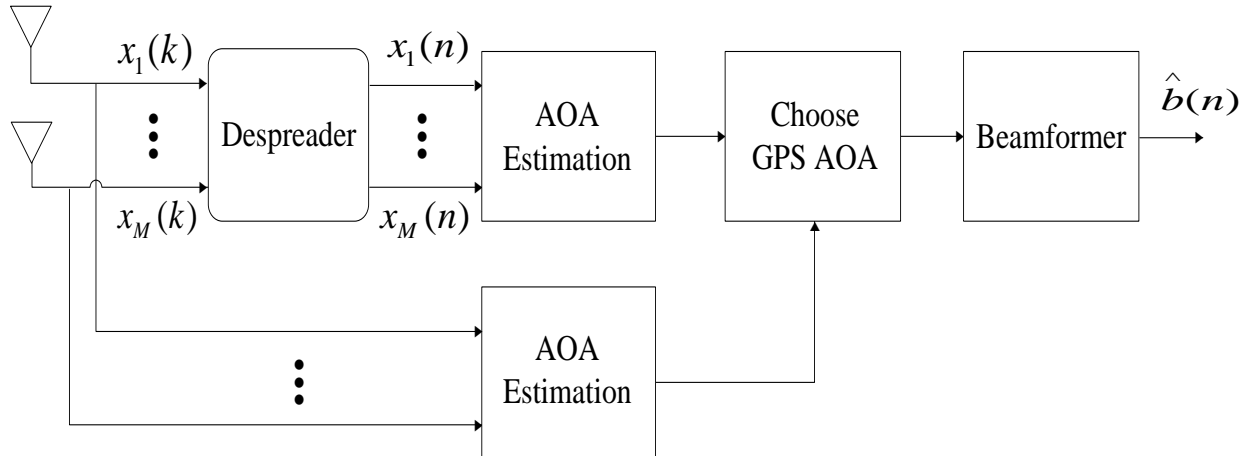


Figure 1: Architecture for GPS AOA selection.

III. GPS AOA SELECTION

In this section, we introduce the AOA estimation algorithm for selecting the GPS signal from all estimated AOAs using a conventional technique such as MUSIC. Figure 1 shows the architecture of the algorithm consisting of AOA estimators based on the received signals before despreading, whose results are compared to the output signals of the despreader. The time index of the despreader output n differs from k , and $\hat{b}(n)$ is the estimated GPS data bit at discrete time instant index n .

A. AOA ESTIMATION

In order to choose the GPS AOA from all estimated AOAs, we need two AOA estimation results based on the received signals before despreading and the output signals of the despreader. The first set of estimated AOAs correspond to those of the interference signals, and they do not include the GPS signal AOA because it has a very low power and thus appears in the noise subspace.

In order to increase the power of the GPS signal above the noise power level, the received signals are applied to the despreader. Since the power of the GPS signal after despreading is higher than the noise power level, it now appears in the signal subspace, and so its AOA is estimated at the output of the despreader. However, this AOA cannot be easily found because of the high-power interference signals.

Table 2: Summary of the GPS AOA Selection Algorithm

- | |
|---|
| <ol style="list-style-type: none"> 1. Estimate AOAs using the received signal before despreading. 2. Estimate AOAs using the output signal of the despreader. 3. The AOA in result 1 but not in result 2 is the AOA of the GPS signal. |
|---|

B. SELECTION OF GPS AOA

Since the AOAs of the interference signals before and after despreading are identical, we can determine the GPS signal AOA because it appears only at the output of the despreader, as summarized in Table 2. The selected AOA of the GPS signal using this algorithm is applied to an adaptive beamformer such as MVDR or GSC in order to suppress the interference signals and enhance the quality of the GPS signal. Although this algorithm has good performance, it has a high computational complexity because it requires two conventional AOA estimators using the received signals before and after despreading.

IV. GPS AOA ESTIMATION BASED ON A MODIFIED DESPREADER

In the previous section, we introduced the AOA selection algorithm for the GPS signal of interest, which has good performance but a high computational complexity. In order to overcome this drawback, we present an efficient GPS AOA estimator based on the modified despreader. The system architecture for this algorithm, shown in Figure 2, consists of the modified despreader and an AOA estimator based on the CM array cost function.

A. MODIFIED DESPREADER

Since the modified despreader transforms all CM interference signals into non-CM signals, while the GPS signal retains CM property, only the GPS signal has the CM characteristic at the output of the modified despreader. The modified despreader output is then processed to generate the GPS AOA estimate using the CM array cost function.

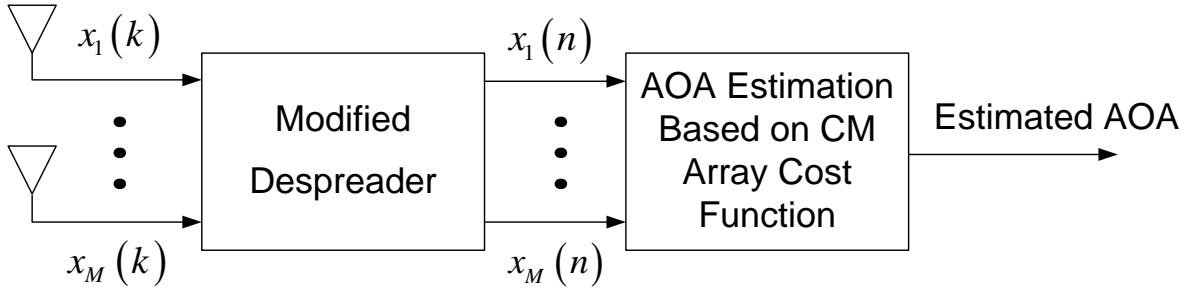


Figure 2: System architecture for GPS AOA estimation.

The modified despreading code is defined as

$$\bar{\mathbf{c}}_i \triangleq \mathbf{c}_i - \tilde{\mathbf{c}}_i, \quad (2)$$

where

$$\mathbf{c}_i = [\mathbf{ca}_i, \dots, \mathbf{ca}_i]^T, \quad (3)$$

\mathbf{ca}_i is the C/A code which is a row vector (length: 1023) for the i th satellite,

$$\tilde{\mathbf{c}}_i = [\mathbf{c}_{a_{i,1}}, \dots, \mathbf{c}_{a_{i,20}}]^T, \quad (4)$$

and $\mathbf{c}_{a_{i,l}}$ ($l = 1, \dots, 20$) is randomly chosen from the C/A codes, excluding \mathbf{c}_{a_i} . The output of the modified despreader is given by

$$\begin{aligned} \mathbf{x}(n) &\triangleq \mathbf{X}(n) \bar{\mathbf{c}}_i \\ &= \mathbf{a}_c(N+20)b(n) + \mathbf{A}\bar{\mathbf{s}}(n) + \bar{\mathbf{n}}(n), \end{aligned} \quad (5)$$

where $\mathbf{X}(n) \triangleq [\mathbf{x}(k), \dots, \mathbf{x}(k+N-1)]$. The interference signal vector and noise vector after the modified despreader are defined as

$$\bar{\mathbf{s}}(n) \triangleq \mathbf{S}(n) \bar{\mathbf{c}}_i, \quad (6)$$

and

$$\bar{\mathbf{n}}(n) \triangleq \mathbf{N}(n) \bar{\mathbf{c}}_i, \quad (7)$$

respectively, where $\mathbf{S}(n) \triangleq [\mathbf{s}(k), \dots, \mathbf{s}(k+N-1)]$ and $\mathbf{N}(n) \triangleq [\mathbf{n}(k), \dots, \mathbf{n}(k+N-1)]$. Since only the GPS signal term $\mathbf{a}_c(N+20)b(n)$ in (5) has the CM property, the AOA of the GPS signal can be estimated using the CM cost function.

B. CM COST FUNCTION

The CM cost function [9] for the modified despreader output is defined as

$$\mathfrak{J}(\mathbf{a}(\theta, \phi)) \triangleq E \left[\left(1 - \left\| (N+20)^2 \mathbf{a}^H(\theta, \phi) \mathbf{R}_x^{-1} \tilde{\mathbf{x}}(n) \right\|^2 \right)^2 \right], \quad (8)$$

where $\mathbf{a}(\theta, \phi)$ is the array response vector corresponding θ and ϕ , which are the elevation and azimuth angles, respectively, $\mathbf{R}_x \triangleq E[\mathbf{x}(n)\mathbf{x}^H(n)]$ is the autocorrelation matrix of the modified despreader output, and $\tilde{\mathbf{x}}(n) = \mathbf{x}(n)/(N+20)$ is the modified despreader output normalized by the amplitude of the GPS signal. In order to estimate the GPS AOA, we must find the minimum value of $\mathfrak{J}(\mathbf{a}(\theta, \phi))$ and the corresponding values for θ and ϕ . Since the CM array cost function in (8) is minimized for the CM signal, $\mathfrak{J}(\mathbf{a}(\theta, \phi))$ is minimized for the GPS signal.

In general, conventional AOA estimation methods such as MUSIC and ESPRIT have a high computational complexity because they utilize an SVD and select the AOA of the GPS signal from all estimated AOAs in the presence of high-power interference signals. As an alternative, the algorithm should employ the GPS AOA selection approach discussed in Section III, which has a lower complexity because it does not require an SVD.

V. COMPUTER SIMULATIONS

In this section, we present computer simulation examples to demonstrate the performance of the AOA selection algorithms for the GPS. We assume that $M = 8$ antenna array elements are utilized in the receiver, and the received signal consists of one GPS signal of interest, six interferences signals, and AWGN. The SNR of the GPS signal is -30 dB, the JSR for each jammer (interference signal) is 60 dB, and the frequency modulated (FM) jammers have modulation index $\beta = 0.05$ and normalized modulation frequency $f_m = 0.001$.

Table 3: Computer Simulation Scenario 1

| Signal | Azimuth (°) | Elevation (°) | Center Frequency |
|--------|-------------|---------------|------------------|
| GPS | 25 | -82 | - |
| CW | -85 | -82 | 0.41 |
| FM | 54, -42 | -82, -82 | 0.07, 0.19 |
| WB | 87 | -82 | 0.29 |
| Pulsed | 39, -16 | -82 | - |

A. GPS AOA SELECTION

For this simulation (Scenario 1), we assume one continuous wave (CW) jammer, two FM jammers, one wideband (WB) noise jammer, and two pulsed jammers. The pulsed jammers are periodic on/off signals with periods of 50 and 500. The other parameters of these signals are summarized in Table 3. Figure 3 shows the MUSIC cost function defined in [10] for the received signal before despreading. Observe that there are six peaks corresponding to the six interference signal AOAs (with azimuth angles -85° , 54° , -42° , 87° , 39° , -16° , and elevation angle -81°). Figure 4 shows the MUSIC function for the despreader output, which has seven peaks corresponding to the AOAs of the GPS signal (with azimuth angle 25° and elevation angle -81°) and the six interference signals. In order to select the AOA of the GPS signal, we compare the results in Figures 3 and 4, and select the AOA with azimuth angle 25° and elevation angle -81° for the GPS angle estimate.

Table 4: Computer Simulation Scenario 2

| Signal | Azimuth (°) | Elevation (°) | Center Frequency |
|--------|-------------|---------------|------------------|
| GPS | 67 | -28 | - |
| CW | 52, -41 | 71, 71 | 0.15, 0.25 |
| FM | -71, 38 | 71, 71 | 0.20, 0.45 |
| WB | 10, -12 | 71, 71 | 0.05, 0.35 |

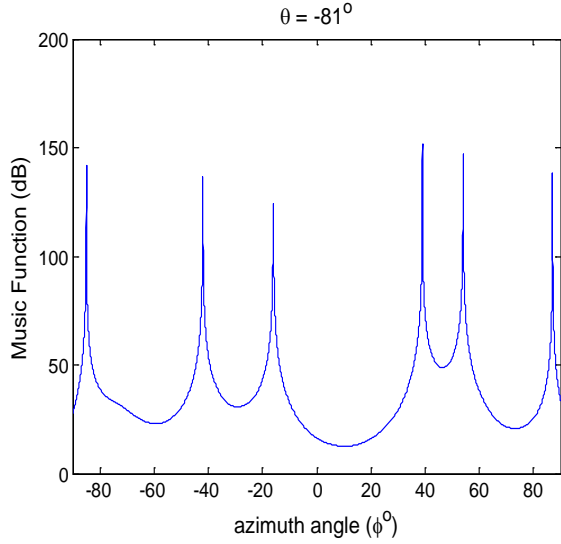


Figure 3: MUSIC function before despreading for Scenario 1.

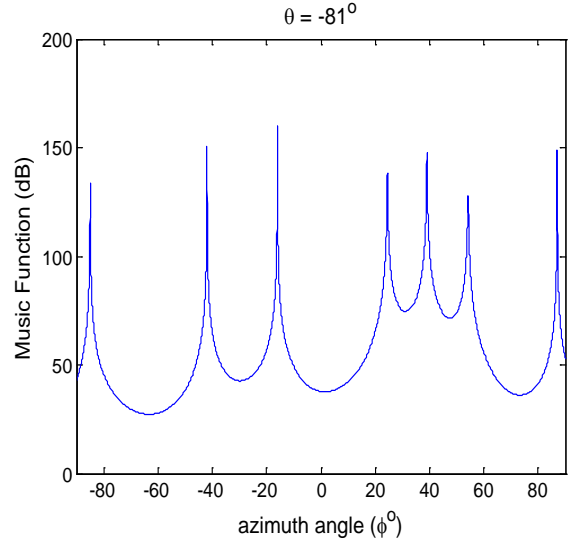


Figure 4: MUSIC function after despreading for Scenario 1.

B. MODIFIED DESPREADER

For this simulation (Scenario 2), we assume two CW jammers, two FM jammers as CM interferences signals, and two WB noise jammers. The other parameters for these signals are summarized on Table 4. The inverse of the CM array cost function and the MUSIC function at each azimuth and elevation angle are shown in Figures 5(a) and 5(b), respectively. From Figure 5(a), we observe a global maximum indicated by the arrow, corresponding to the GPS azimuth (67°) and elevation (-28°) angles. Since the GPS signal still has the CM characteristic after despreading, we can easily estimate those angles. Although Figure 5(b) includes seven peaks corresponding to the AOAs of the GPS signal and the interference signals, it is difficult to observe them in the figure. Therefore, we plot the MUSIC function versus the azimuth angle to better locate the peaks. Figures 6(a) and 6(b) show the results for the GPS signal with elevation angle $\theta = -28^\circ$, and the interference signals with elevation angle $\theta = 71^\circ$. We observe a peak for the azimuth angle of the GPS signal in Figure 6(a) and six peaks for the azimuth angles of interference signals in Figure 6(b). In order to estimate the GPS AOA using the MUSIC algorithm, we must know the number of interference signals and be able to select the GPS AOA from all estimated AOAs since there are multiple peaks.

VI. CONCLUSION

In order to efficiently suppress high-power interference signals and recover the GPS signal of interest, accurate AOA estimation is generally required in the receiver. Since the power of the GPS signal is much lower than that of the interference signals and noise, it is not possible to

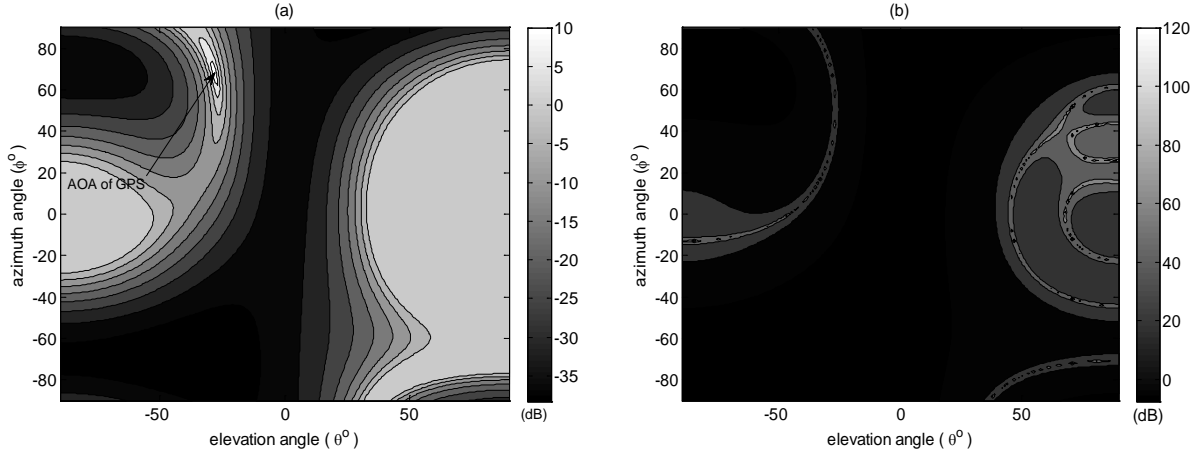


Figure 5: Inverse of functions for Scenario 2. (a) CM cost function (b) MUSIC function

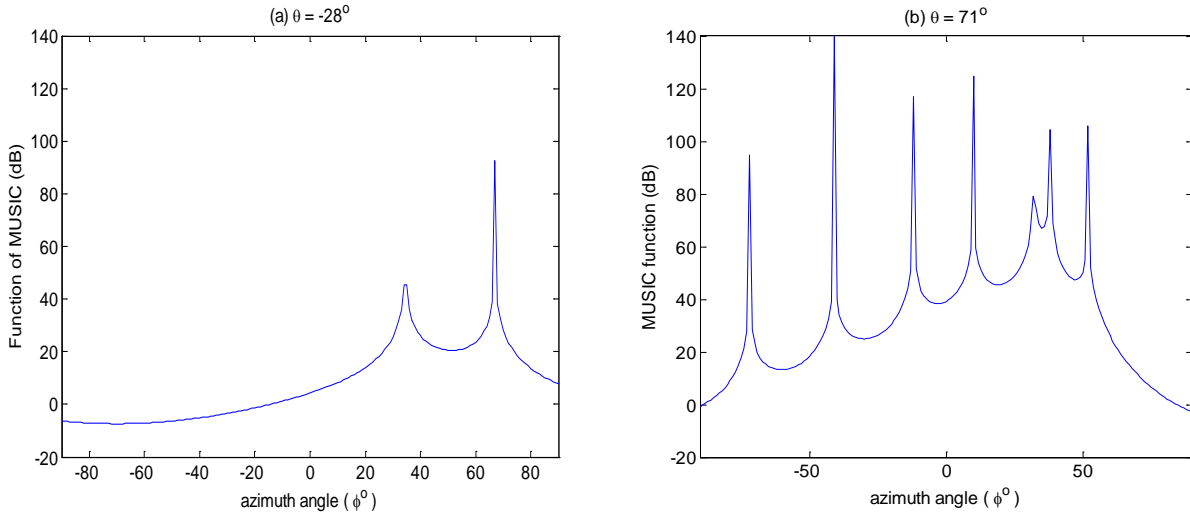


Figure 6: Inverse of functions for the azimuth angle for Scenario 2. (a) MUSIC function (elevation angle $\theta = -28^\circ$, peak of the GPS signal). (b) MUSIC function (elevation angle $\theta = 71^\circ$, peaks of the jamming signals)

estimate the GPS AOA using the received signal before despreading. Even if we estimate the GPS AOA using the depreader output, we must still select the GPS AOA from all estimated AOAs because of the interference signals. In this paper, we presented an AOA selection algorithm for the GPS signal based on a comparison of the AOA estimators using the received signals before and after despreading. Although this approach has good performance, it has high computational complexity. In order to avoid this drawback, we also introduced an efficient AOA estimation algorithm for the GPS signal based on the modified despreader and the CM array cost function. Unlike conventional AOA estimators such as the MUSIC and ESPRIT algorithms, this scheme does not require an SVD, and so it has a lower computational complexity. In addition, it does not require an AOA selection process for the multiple estimated AOAs because it only estimates the AOA of the GPS signal.

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