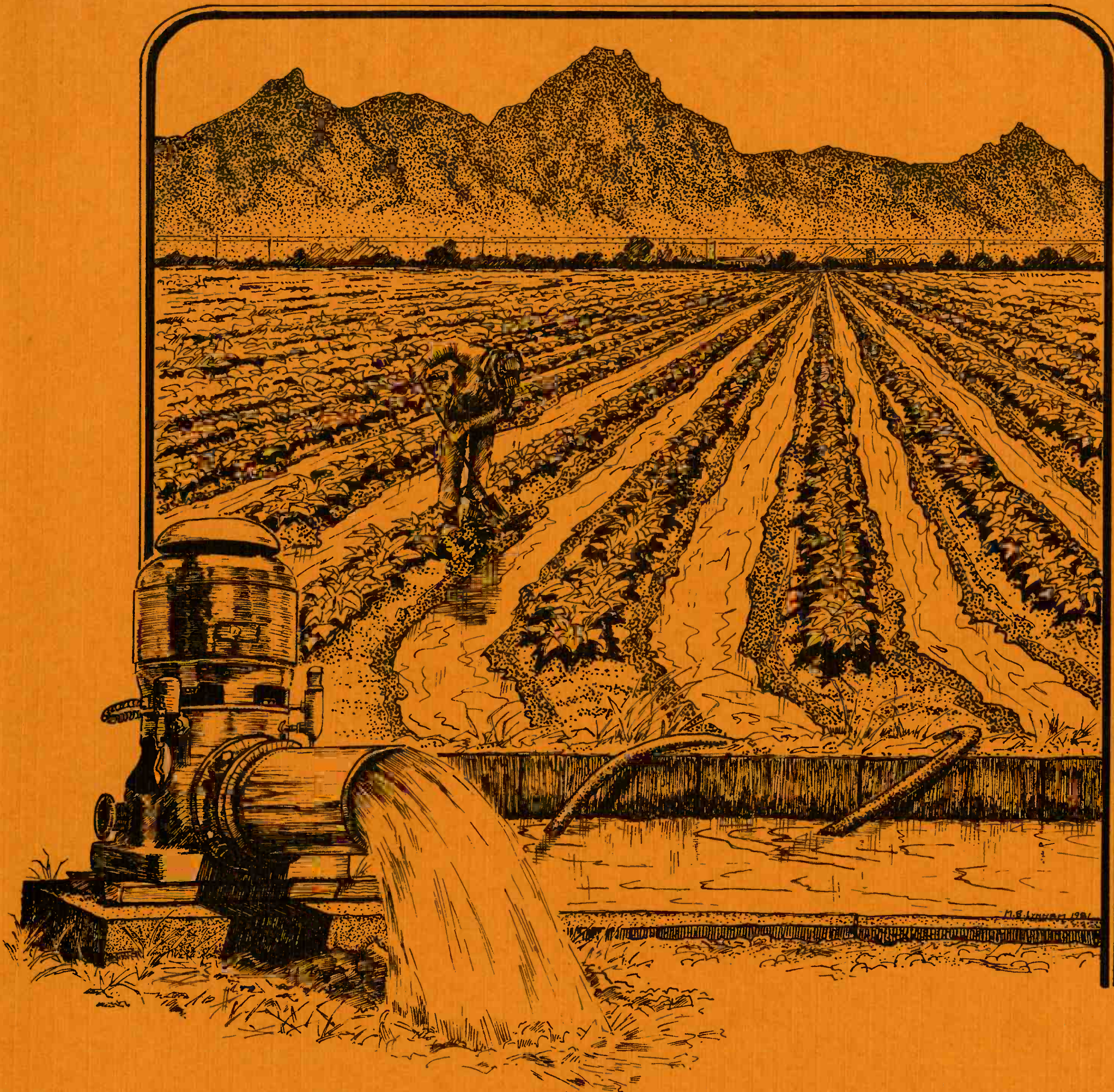

Crop-Water Production Functions: Economic Implications for Arizona

Harry W. Ayer and Paul G. Hoyt

Technical Bulletin Number 242, Agricultural Experiment Station, September 1981
College of Agriculture, The University of Arizona, Tucson, Arizona 85721

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Preface and Acknowledgements

Water is increasingly scarce in many areas of the western United States. Because irrigation consumes over 85% of all water consumed in arid areas, much of the attention of policy makers and competing sectors focuses on irrigated agriculture—the impact of increasing water scarcity on that sector, and, concomitantly, that sector's impact on water use and conservation.

In 1978, we initiated research to estimate crop-water production functions and use them in an economic analysis of water problems and conservation alternatives for the major crops in each of the principal crop producing areas of the West. Information from the study is intended to be useful particularly to agencies and private consultants who offer irrigation management services to farmers and to those who formulate water conservation policy.

This report focuses on the first portion of the overall research—the crop-water production functions and economic analysis for crops grown in Arizona. Future reports will cover other western states. The Arizona study, however, provides a good example of the type of analysis and policy implications to be drawn from the research. In many respects the varieties of conditions in Arizona cover the range of conditions to be found in other states. For example, three soil textures, fine, medium and coarse are used in the Arizona study. Also, the price of water, an important variable in the economic analysis, varies greatly in Arizona—from less than 50 cents per acre inch to over $5 per acre inch, depending on the source and lift height. The principal crops and irrigation efficiencies of Arizona are often similar to those of other western irrigation regions.

In spite of these similarities, the Arizona study can only serve as a rough guide to those who provide irrigation management services to farmers and those who guide water policy in other areas of the West. Many conditions in other areas are different: water law and institutions, climate and particular crops often vary from region to region and require separate analyses.

The research reported here was sponsored by the Natural Resource Economics Division of the Economic Research Service of the USDA, and by the Soil Conservation Service, also of the USDA.

Special thanks are due the following people for providing agronomic data on crop response to water: Dale Bucks, Leonard Erie and Orrin French of the U.S. Water Conservation Laboratory, Phoenix; Alan Halderman, The University of Arizona, Tucson; Eldon Hanson, New Mexico State University, Las Cruces; Ernest Jackson, University of Arizona Experimental Farm, Yuma; David Kittock, SEA-AR, USDA, Phoenix; and Fred Turner, Jr., University of Arizona Experimental Farm, Safford.

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Views expressed are those of the authors.

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Table of Contents

List of Figures

List of Tables

Introduction

The Problem

Water is becoming an increasingly scarce resource in Arizona and the arid West. Agriculture consumes over 85% of all water consumed in most of the region. Present estimates indicate that Arizona consumes 1.7 times annual ground and surface water replenishment. At least three factors have and will continue to increase the scarcity of water: water tables are often deep and declining; energy prices are increasing rapidly; and other sectors, besides agriculture, are competing for a limited water supply.

Approximately one half of all cropland in Arizona is irrigated with groundwater. Some 58% of the cropland irrigated with groundwater is pumped from a depth of 300 feet or more. Table 1 indicates the portion of irrigation groundwater pumped from each depth.

The price of electricity, the principal source of power for irrigation pumps, as well as the price of other energy, has increased sharply in recent years. As an example, from 1976 to 1979 the annual, compounded rate of electricity price increases in the important Maricopa areas not under long-term hydroelectric power contracts was 14%. Although many farmers in some irrigated areas have had long-term hydroelectric power contracts at low, fixed rates, most of these contracts expire in the mid-1980s, and prices will increase dramatically.

These two factors, deep pumping depths and high energy prices, imply high energy costs and therefore high irrigation costs, since energy is by far the largest component of variable irrigation costs. Table 2 shows the variable irrigation costs as a percent of total variable costs for the principal crops in the most important irrigated areas. In most areas and for most crops, variable irrigation costs are over 30% of total variable costs. Even for medium size (500 acres) farms, annual electricity bills are over $25,000 in many areas.

Competition from nonagricultural sectors is the third factor which increases the scarcity of irrigation water. Populations of the Phoenix and Tucson areas increased at an annual rate of 4.4% between 1968 and 1978, compared to the U.S. rate of 0.8 %. During the next decade, this difference is expected to be even greater as Arizona's urban areas share in the projected growth of the Sun Belt. Urban industrial water users can afford to pay greater prices for water than agricultural users, even to the point of purchasing agricultural land to acquire water rights. Further, the urban-industrial sector of Arizona is increasing in political influence and may eventually change existing water laws, adversely affecting the supply of water for agriculture. Mexico is also increasing pressure to receive a larger supply of low saline water delivered by the Colorado River, which forms Arizona's western boundary, and Indian reservations may alter the current allocation of water through legal acquisition of additional water rights and subsequent development of irrigated land.

Table 2. Variable Irrigation Costs[a] as a Percent of Total Variable Costs, by Crop, for Typical Arizona Farms, 1979.

County	Alfalfa	Cotton	Sorghum	Wheat
Cochise	77%	37%	52%	52%
Graham	45	15	27	34
Maricopa	38	18	42	22
Pima	63[b]	24	38	39
Pinal	45[b]	30	52	42
Yuma	24	7	24	24

a. Variable irrigation costs include water pumping and irrigation labor costs.

b. Water as a percent of total variable costs of alfalfa production is higher in Pima than Pinal County because of a methodological problem in computing total variable costs. In Pinal County, most alfalfa is custom harvested (and counted as a variable cost), while in Pima County most harvesting is done with the farmers' own machinery (and therefore most machinery costs are not counted as variable costs), according to the sources listed below. Variable water costs for alfalfa production in Pima County are $115 per acre and in Pinal County are $165 per acre. If equal machinery and custom harvesting costs are assumed for each county ($56 per acre), then water as a percent of total variable costs for Pima and Pinal Counties are 59% and 71%, respectively. The methodological problem of custom vs. own machinery costs is unimportant for all except alfalfa in Pima and Maricopa Counties.

Sources: Hathorn and Sullivan; Hathorn and Cluff; Hathorn and Farr; Hathorn and Armstrong; Hathorn, Little and Stedman; Hathorn, Howell and Hazlitt.

Conservation and Efficiency Alternatives

Various government agencies, as well as farmers, are reacting to conserve irrigation water and increase efficiency in view of increasing water scarcity. Several policies are currently in effect or proposed: water management, financial policies and water rationing schemes. Better water management is encouraged through the "best management practices" of the U.S. Soil Conservation Service, the Irrigation Management Service (IMS) of the U.S. Bureau of Reclamation, scheduling services provided by the Salt River Project, and by the Agricultural Extension Service. Typically, these practices include the scheduling and application of water to

Table 1. Depths to Water under Croplands Irrigated Only by Pumped Groundwater, Arizona.

Depth (feet)	0-99	100-199	200-299	300-399	400-499	500 +	No Data
Acres	40,000	92,000	97,500	115,200	158,500	63,700	19,300
% of total	6.8	15.7	16.6	19.7	27.0	10.9	3.3

Sources: Arizona Crop and Livestock Reporting Service; Mann, *et. al.*; Ross; Stulik and Laney; Wilkins.

minimize the effect of water stress on yield and yet avoid water applications above the amount recommended. The Penman, Blaney-Criddle or Jensen-Haise evapotranspiration models are used to estimate plant water "requirements" under various soil and climatic conditions. Another management practice is aimed not only at conserving water, but also at reducing salt loads to stream flows after the irrigation water has passed through the land under irrigation. Proposals have been made to cut irrigation rates on the Colorado River Indian Reservation in Arizona to evapotranspiration plus a minimum leaching fraction. In theory, this practice will conserve water and decrease the amount of salt leached from the soil and returned to the Colorado River, which carries it into Mexico. Other management strategies to conserve water or increase irrigation efficiency are to alter crop mix, level flood irrigated fields, change application technologies or increase irrigation delivery efficiency.

Financial policies to conserve water or increase the efficiency of water use include those to raise the price of water in federal irrigation districts, raise the effective price of pumped water through pumping taxes, or to subsidize the application of water saving technology such as laser leveling of land as carried out through the Agricultural Stabilization and Conservation Service (ASCS) cost sharing program.

Water rationing schemes are also in effect or have been proposed by various government agencies. For example, the Colorado River Indian Reservation is restricted by law to 717,148 acre feet of water annually.

The success of these various conservation and efficiency measures will depend upon farmers' acceptance of them; *i.e.*, upon the profitability or, more generally, upon both the expected profitability and riskiness of alternative irrigation management decisions. Profits and risk depend directly upon the underlying crop-water production function—*i.e.*, the yield of a particular crop associated with alternative levels of water application. To date, this crop-water relationship has not been well defined for most crops in most areas.

Purpose of the Report

The purpose of the research reported here is to estimate crop-water production functions for four of Arizona's most important crops—cotton, wheat, sorghum and alfalfa—and to provide an economic analysis, based upon the production functions, useful both to those who formulate water conservation policy and agencies and private consultants who offer water management services to irrigation farmers. More specifically, the objectives are to:

1. Estimate the crop-water response functions for cotton wheat, sorghum and alfalfa on different Arizona soils,
2. Estimate the profit maximizing levels of water applied for each crop under varying soil texture, water price and crop price conditions,
3. Compare the profit maximizing level of water applied with that level suggested by yield maximizing, common practice and other models,
4. Estimate the effects of increasing pump efficiency on water applied and crop profits,
5. Estimate the effects of increasing irrigation delivery efficiency on water applied and crop profits,
6. Estimate the effects of water quantity restrictions on water applied and crop profits,
7. Estimate the elasticity of demand for water by each crop and for alternative crop prices.
8. Draw implications for farm level irrigation management, aggregate level water policy, and research.

The estimated crop-water production functions and associated economic analyses are based upon agronomic crop-water experiments which stressed the crop with respect to water applied (irrigation plus rainwater). The experiments, however, did not investigate the day-to-day decisions of when and how much to irrigate and were conducted on a limited range of soil types and included only part of the climatic conditions which exist statewide. We recognize that many day-to-day decisions on particular farms will require other information, such as continuous climatic information, to determine more precisely when and how much to irrigate. However, we believe the production functions reflect the general nature of crop response to alternative levels of total water and that there are important farm and policy implications to be drawn from the seasonal crop-water production functions.

Methods and Data

Methods

Production and Related Functions

Production functions are estimated from agronomic crop-water data and used in the economic analysis. Figure 1A shows a classical, hypothetical production function. The function indicates the yield associated with each level of water. The change in yield associated with each succeeding unit of water, called the marginal physical product, is shown in Figure 1B. The dollar value of the yield associated with each succeeding unit of water, called the value of the marginal product (VMP), is depicted in 1C.

Often it is necessary to express crop yield as a function of not only water applied, but also other factors of pro-

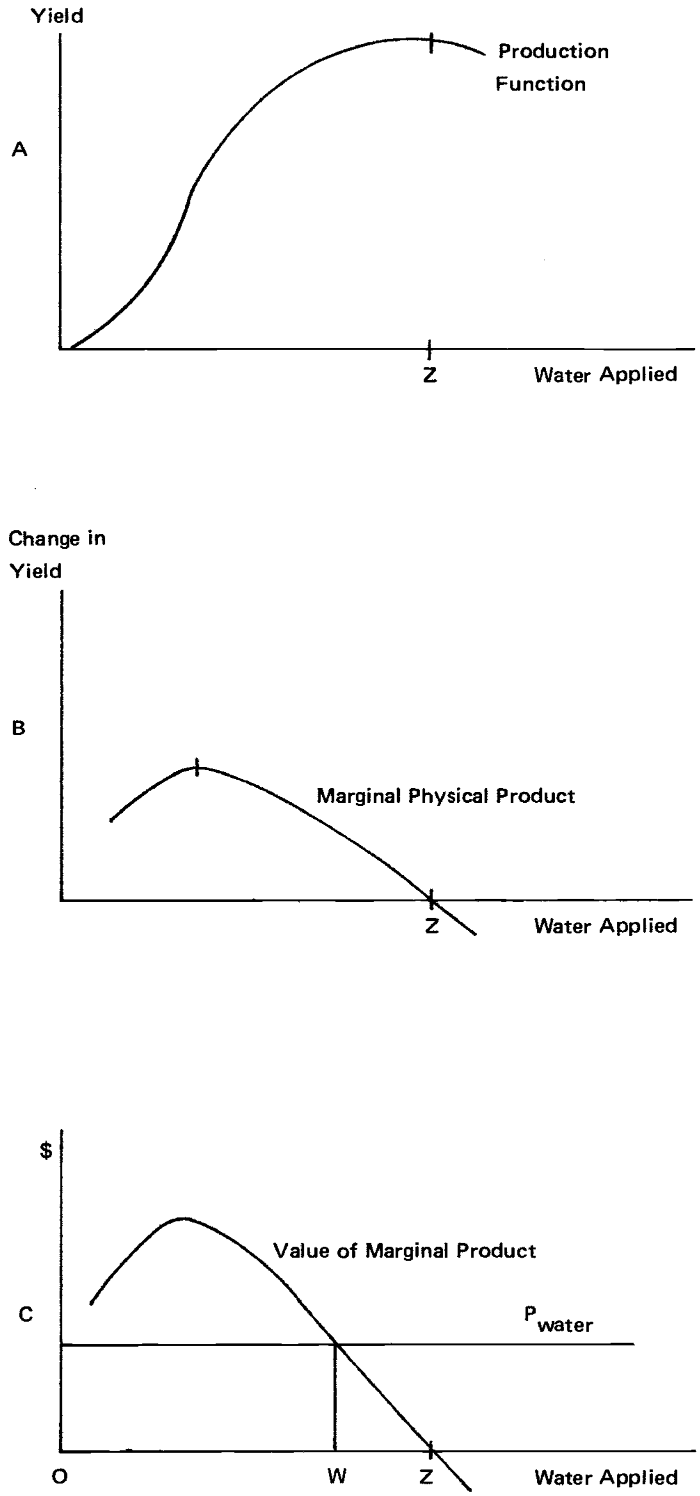

Figure 1. Production Function Marginal Physical Production and Value of Marginal Product.

duction. In several of the agronomic experiments upon which the current research is based, the level of nitrogen, climatic conditions (expressed in terms of pan evaporation), and water-soil chemistry did vary among plots. The relationship between these other variables and yield can be expressed in mathematical terms, but are difficult or impossible to show graphically. Accordingly, in the remainder of the discussion pertaining to theory, only crop-water relationships are graphed and discussed, even though the complete analysis is based on multi-input production functions which included nitrogen, pan-evaporation, and water-soil chemistry factors, when appropriate.

Other factors may also influence yield, but in the experiments these have either been held constant or are assumed to occur randomly and therefore not to bias the estimated water-applied, yield relationship. For example, irrigation scheduling may influence yield. In the Arizona experiments, the timing of irrigations was done without regard to plant growth stage. Rather, irrigation treatments were based upon soil moisture depletion (explained later). Or, as another example, irrigation field efficiency may affect yield *vis-a-vis* water applied. In the Arizona experiments, field efficiency averaged about 65%, and differences in field efficiency among experimental plots for any given level of water are assumed to be random.

Profit and Yield Maximization

To maximize profits per acre, water is applied until the value of the marginal product just equals the price of water. The price of water, as used in this report, refers to the variable cost per unit. Accordingly, the price of pumped water refers to the variable cost to pump and apply a unit of water. In Figure 1C, amount OW is applied to maximize profits. If less than OW is applied, then the value of output from the last unit of water is greater than the cost of the last unit of water, and clearly that unit, and more, should be applied to increase profits. If more than OW is applied the value of the output from the last unit of water is less than its price, and water applications should be cut. The rule for profit maximization is: water should be applied until the value of the marginal product equals the price of water.

The rule for maximizing profits may be contrasted with the rule for maximizing yield. Yield is maximized at the apex of the production function, point A in Figure 1A, or correspondingly, where the marginal physical product and value of marginal product equal zero. As shown, yield is maximized when quantity OZ of water is applied. Only when the price of water is zero will the profit maximizing and yield maximizing level of water coincide. The amount by which the profit maximizing and yield maximizing level of water diverge depends on the shape of the underlying crop-water production function and upon the price of water: in Figure 1C, the difference is WZ.

The distinction between profit maximizing and yield maximizing levels of water applied is relevant for both aggregate and farm level water policy and management. In the past, most water policy and management information have been based on yield maximizing amounts. For example, surface water allocations to particular areas have been based on estimates of consumptive water use of particular crops—the amount of water needed for a particular soil and climate to minimize plant stress. Similarly, on-farm irrigation recommendations have usually been based on these same consumptive use estimates. Consumptive use prescriptions have usually been designed to maximize yield—not profit. Theory suggests that, in general, maximizing yield will both increase water use and decrease farm profits in comparison to maximizing profit.

The profit maximizing criteria change only slightly if water applied, as discussed above and shown in Figure 1, includes rainfall. If rainfall is significant, then somewhat more water than OW would be applied since rain is at zero price. In Arizona, rainfall is usually of no consequence during the growing season.

The preceding analysis has assumed that land is fixed, and that enough water is available to irrigate until the value of the marginal product of water (VMP) equals the price of water. In some situations, however, the total amount of water available to a region or to a farm is limited, and there may not be enough water to bring the VMP into equilibrium with the price of water on each acre. In these situations, profits for the farm or to the region can be maximized if the limited water is applied to each acre in an amount that equalizes the VMP on each acre.

The limited water, variable land situation is depicted in Figure 2. Supppose that total water available for distribution to the three-acre region is 12 units. All 12 units of water could be applied to Acre 1 and the VMP of water would equal the price of water on that acre. However, inspection of Figure 2 suggests that subtracting one unit of water from Acre 1 and adding it to either Acre 2 or Acre 3 results in a greater increase in the value of output (VMP) than the decrease from Acre 1. As long as the VMP is higher on one acre than another, clearly it is profitable to take water from the acre with the lower and add it to the acre with the higher VMP. Profits to the limited water will be maximized when the value of the marginal product is equalized. In Figure 2, the VMP's from 12 units of water are equalized when four units are applied to Acre 1, Acre 2 and Acre 3 respectively.

Elasticity of Demand

The demand function for irrigation water shows the amount of water which will be demanded at each water price if profits are to be maximized. Knowledge of this relationship is, therefore, useful to agencies which aim to conserve water though changing water prices.

The demand for water is directly related to the underlying crop-water production function. In its simplest form, the demand function is the value of marginal product curve depicted in Figure 1C. The VMP is the demand curve because it indicates the amount of water that will be used, at each water price, in order to maximize profits. Of course, if the price of the product changes, then the VMP curve will also change and thereby affect the demand for water.

Elasticities of demand are derived directly from the demand function. Elasticity of demand for water indicates the percentage change in the quantity of water demanded with a 1% change in the price of water. If the price of water increases 1% and the amount of water demanded decreases by more than 1%, demand is said to be elastic: farmers are relatively responsive to price changes. If, on the other hand, a 1% price rise results in less than a 1% decrease in water demanded, demand is said to be inelastic: farmers are relatively unresponsive to water price changes. If a percentage change in water price is met by an equal percentage change in water demanded, demand is of unitary elasticity.

As an example, suppose that the elasticity of demand for irrigation water is found to be -0.05. The elasticity indicates that for each 1% change in the price of water, there is only 0.05% change in quantity of water demanded. Such a policy to conserve water by increasing water price will not succeed.

Elasticities may change as the price of the product changes and even as the price of water itself changes. The sensitivity of elasticities of demand for water to product and water price changes are examined.

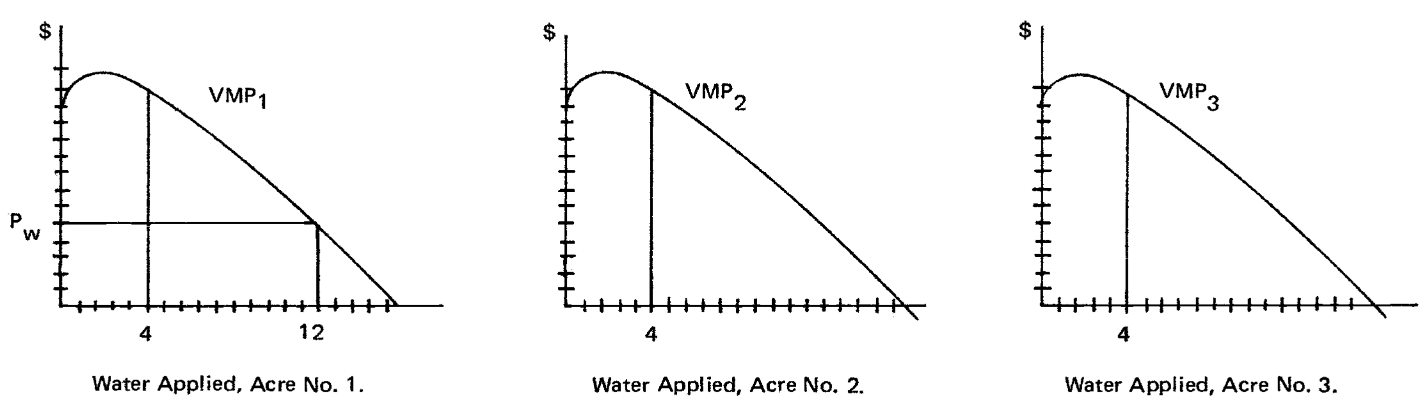

Figure 2. Value of Marginal Product Curves on Each of Three Acres within a Region.

Data

Agronomic Data

The production function data used in the research reported here are from agricultural experiment stations in Arizona and New Mexico. All relevant agronomic data from current and past experiments in Arizona were collected and evaluated. The cotton and wheat data are primarily from a multistate study by Hexem and Heady (1978) funded by the Bureau of Reclamation. The sorghum data are from an unpublished set of experiments by Erie, French and Bucks conducted at the University of Arizona's experimental farm at Mesa. Alfalfa data are from the Agricultural Experiment Station of New Mexico State University at Las Cruces, because no data are available from an Arizona site. However, the New Mexico site is in an area very similar in climate to agricultural areas of Arizona.

When possible, production functions are estimated for each of three soil textures; fine, medium and coarse. Fine texture soils are found at the Yuma Valley and Safford Experiment Stations, medium texture soils at the Mesa, Tempe and Phoenix stations, and coarse texture soils at the Yuma Mesa station. Texture classifications are based on SCS (1964) and Fuller's criteria for desert soils and consider composition, profile and available water holding capacity.

In almost all agronomic experiments, water was applied by surface irrigation. Surface irrigation is used on over 90% of Arizona's irrigated acreage. Water measurements for the experiments are for gross water applied (irrigation plus rainfall) to the field or plot: field efficiency was approximately 65% but varied by level of irrigation and other factors. Field efficiency was similar to that on farms with similar soil textures.

The cotton and wheat experiments are incomplete block design with factorials. Five irrigation treatments, based on soil moisture depletion of from 45% to 90% depletion were used. Preplant irrigations brought water levels to 100% of field capacity, so that antecedent soil moistures were standardized among plots. Rainfall is included in water measurements. Subsequent irrigations brought the soil to 100% of its water-holding capacity in most experiments. Five nitrogen treatments were used in the experiments. Native soil fertility and differences in soil fertility due to prior cropping mix were not found to be statistically different among blocks for most experiments (Hexem and Heady, pp. 133-135).

Cotton experiments were conducted at Yuma Mesa, Yuma Valley, Tempe, and Safford, Arizona experimental farms during 1970 and 1971. All three soil textures—coarse, medium and fine—are represented among these sites. Cotton experiments at the Cotton Research Center in Phoenix during 1975 and 1976 are not of the Hexem and Heady design, but included varying degrees of moisture stress and are used also.

Wheat experiments were conducted at Mesa (1970-75), Yuma Mesa (1970-72), Yuma Valley (1970-72) and Safford (1970-71). All of these are of the Hexem and Heady design (incomplete blocks with factorials). As with cotton, all three soil textures are found among these sites.

The sorghum data are from experiments conducted at the Mesa experimental farm in 1957 and 1958 on varieties which are still in use today. These experiments were designed to test the effect on grain yield of irrigations at various stages of plant growth. As many as 12 irrigation schedules were used. Each treatment had a different combination of irrigations among sorghum's five basic stages of growth. Antecedent soil moisture and rainfall are accounted for in the same way as in the cotton and wheat experiments. Soils at the Mesa station are classed as medium-fine texture.

The alfalfa data are from experiments at Las Cruces, New Mexico. These experiments used three different irrigation levels with soil moisture depletion rates of 50% to 65%, 70% to 80% and 90%-plus over three years. Antecedent soil moisture and rainfall are accounted for in the same way as in the cotton and wheat experiments. A fourth experiment conducted at Las Cruces in 1978 was also included, although it contained different irrigation treatments than the original experiments. Soils at the Las Cruces station are very fine in texture.

Price Data

Economic analysis of the impact of water management, water pricing, and water restriction policies on water conservation and farm profits requires data on input and product prices. Expected product prices (late summer 1979) are taken from the *Wall Street Journal* and various government documents. These prices are respectively $75 per ton of alfalfa, $0.65 per pound of grain sorghum and $0.07 per pound of wheat. In testing the sensitivity of various irrigation policies to product prices, historic high and low prices during the past decade are used and are specified in the tables of results.

Typical water prices for three water source situations are estimated—prices for surface water and prices for water pumped from 300 and 600 feet. Prices used in the analysis are $.50, $2.50 and $5 per acre inch. Surface water prices are similar to those paid by irrigators in the Salt River Project and in the Yuma areas. Typical water prices for the two lift depths are estimated from Hathorn (1978) and the following equation.

$$\text{Price of water per acre foot} = \left[\left(\begin{array}{l}\text{KWH to lift}\\ \text{one acre foot}\\ \text{of water one}\\ \text{foot}\end{array} \times \begin{array}{c}\text{lift depth}\\ \text{in feet}\end{array} \times \begin{array}{c}\text{power cost}\\ \text{per KWH}\end{array} \div \begin{array}{c}\text{pump}\\ \text{efficiency}\end{array}\right) + \begin{array}{l}\text{cost of repairs,}\\ \text{maintenance, lub-}\\ \text{rication and}\\ \text{attendance}\\ \text{per foot of lift}\end{array} \times \begin{array}{c}\text{lift depth in}\\ \text{feet}\end{array}\right] \div \begin{array}{c}\text{irrigation}\\ \text{delivery}\\ \text{efficiency}\end{array}$$

As shown in the equation, the effective price of pump water is determined by lift depth, the price of energy, pump efficiency and irrigation delivery efficiency. The lift depth varies by area, the price of energy is expected to increase over time, and, of special interest here, on-farm water management practices may change pump or delivery efficiencies. Energy prices represent typical 1979 prices for electricity. Pump and delivery efficiencies are initially assumed to be 50% and 95%, respectively (Halderman, 1979), but, as specified in the later analysis, are changed to show the effect of alternative management practices.

Results and Analysis

Statistical production functions for cotton, wheat, sorghum and alfalfa are given in Table 3 and graphed in Figure 3. Separate functions are shown for each of three soil textures–coarse, medium and fine–in accordance with soil textures at experiment locations. Functions given are considered the "best"–in terms of goodness of fit, expected signs, significance of coefficients, and in making both agronomic and economic sense–among the dozens of functional forms and variable definitions investigated and estimated. The graphed functions are two dimensional–yield is a function of gross water applied (including rainfall), all other variables held constant at their mean. The graphs illustrate the shape of the crop-water relationship and, in particular, show that there are diminishing marginal returns to added irrigation water. The graphs also depict the range of data upon which the statistical production functions are based. Projections of yield beyond the range of the data must be made with caution, and often should not be made. Production functions for each particular crop are plotted on the same graph: one function is plotted for each soil texture. For both cotton and wheat (the crops grown on different soil textures) the graphs illustrate that the shapes of the functions are very similar regardless of soil texture. Differences in the height of the functions cannot be attributed solely to differences in soil texture because each function is plotted with other variables fixed at their mean level at the site upon which the statistical production function is based. The mean levels of nitrogen used in Figure 3A, the cotton production functions, are 300 pounds per acre for cotton grown on coarse soil (C_c) and 125 pounds per acre for cotton grown on fine texture soil (C_f). Nitrogen levels per acre for the wheat production functions vary from 50 pounds for fine texture soil (W_f) to 150 pounds for medium texture soil (W_m) to 200 pounds for coarse texture soil (W_c).

Pan evaporation (EVAP) also varies by crop (different season lengths) and site (different climatic conditions) and affects the height of the functions. The cotton production functions are based on mean seasonal pan evaporation values of 89 inches for medium texture soil (C_m) and 81.4 inches for fine texture soil (C_f). Seasonal pan evaporation used in the wheat production functions (Figure 3B) are 34 inches, 40 inches and 46.8 inches for the coarse (W_c), medium (W_m) and fine texture (W_f) soils respectively. The alfalfa production function is based on a pan evaporation value of 93.5 inches while the grain sorghum function utilizes a pan evaporation of 38 inches.

The production functions are reproduced in Appendix A with their 95% confidence intervals.

Coarse Texture Soils

The production function for cotton explains 58% of the total variation in yield ($\bar{R}^2$ = 0.58). The function includes both water and fertilizer inputs, plus a water-nitrogen interaction term. All coefficients have the expected sign and are statistically significant. The positive sign on the interaction term (WN) indicates the complementary relationship of water and nitrogen. The crop-water production function for cotton is graphed in Figure 3A and is labeled C_c (cotton grown on coarse soil). The solid portion of the curve indicates the range of the water applications in the underlying agronomic experiments. The function exhibits diminishing marginal returns and a third stage of production (diminishing total physical product) within the range of data.

The production function for wheat grown on coarse soil explains 73% of the variance in yields. Because the experiment was conducted for two seasons at the Yuma Mesa site, pan evaporation is included as an explanatory variable and is very significant. All variables except the nitrogen polynomial term are significant at the 1% level and all coefficients exhibit the expected sign. Water and nitrogen fertilizer complement each other as shown by the positive sign of the interaction term. The wheat function is graphed in Figure 3B and is labeled W_c. The function shows a diminishing marginal product and a maximum yield just beyond the range of the data.

Medium Texture Soil

Some 85% of the variance in cotton yields are explained by the water and pan evaporation variables. All variables are significant at the 1% level. The signs for the water coefficients are as expected. The negative sign on the water-evaporation interaction term is contrary to initial expectations because cotton is, up to a point, a heat-loving plant. Inspection of the data reveals that one year of the cotton experiments at Phoenix was exceedingly hot, and that yields were lower that year. Data on pan evaporation and yields from other years and other sites indicate the usual production function relationship between yield and pan evaporation–yield rises as pan evaporation rises, but at some point the relationship turns negative. We hypothesize that the very high pan evaporation and limited other observations used in computation of the cotton function are

Table 3. Crop-Water Production Functions by Soil Texture.

Coarse Texture Soil

Cotton

$$Y_c = \underset{(2532.92)}{\overset{***}{-7209.473}} - \underset{(40.91)}{\overset{***}{135.807W}} + \underset{(651.97)}{\overset{***}{2006.207W^{.5}}} - \underset{(1.72)}{\overset{***}{5.341N}} + \underset{(51.17)}{\overset{**}{106.82N^{.5}}} + \underset{(.01)}{\overset{***}{.041WN}}$$

$\bar{R}^2 = .58$
$F = 7.82$

Wheat

$$Y_w = \underset{(1826.44)}{\overset{***}{12083.008}} + \underset{(143.98)}{\overset{***}{372.87W}} - \underset{(19.06)}{\overset{**}{44.014W^{1.5}}} + \underset{(5.47)}{\overset{***}{15.237N}} - \underset{(.32)}{\overset{***}{1.368N^{1.5}}} + \underset{(.11)}{\overset{***}{.571WN}} - \underset{(51.09)}{\overset{***}{424.168EVAP}}$$

$\bar{R}^2 = .73$
$F = 32.87$

Medium Texture Soil

Cotton

$$Y_c = \underset{(259.26)}{\overset{**}{-636.919}} + \underset{(16.56)}{\overset{***}{128.359W}} - \underset{(.16)}{\overset{***}{.503W^2}} - \underset{(.09)}{\overset{***}{.697WEVAP}}$$

$\bar{R}^2 = .85$
$F = 68.04$

Wheat

$$Y_w = \underset{(1501.11)}{288.111} + \underset{(2.65)}{\overset{**}{5.834WEVAP}} - \underset{(1.93)}{3.119W^2} + \underset{(4.96)}{\overset{***}{16.784N}} - \underset{(.01)}{\overset{**}{.028N^2}}$$

$\bar{R}^2 = .62$
$F = 15.18$

Sorghum

$$Y_s = \underset{(1.07)}{\overset{*}{7.57}}\, \underset{(.08)}{\overset{***}{W^{.312}}}\, \underset{(.27)}{\overset{***}{EVAP^{1.38}}}$$

$\bar{R}^2 = .64$
$F = 18.08$

Fine Texture Soil

Cotton

$$Y_c = \underset{(177.83)}{\overset{**}{-380.377}} + \underset{(13.01)}{\overset{***}{35.032W}} - \underset{(.14)}{\overset{***}{.499W^2}} + \underset{(.14)}{\overset{**}{.307WEVAP}} + \underset{(.48)}{\overset{***}{2.262N}} - \underset{(.002)}{\overset{***}{.007N^2}}$$

$\bar{R}^2 = .93$
$F = 147.52$

Wheat

$$Y_w = \underset{(1521.32)}{\overset{***}{-4385.796}} + \underset{(94.05)}{\overset{***}{495.812W}} - \underset{(1.28)}{\overset{***}{3.752W^2}} - \underset{(.28)}{\overset{***}{2.778WEVAP}} + \underset{(2.06)}{\overset{***}{5.819N}} - \underset{(.007)}{\overset{**}{.016N^2}}$$

$\bar{R}^2 = .71$
$F = 42.8$

Sorghum

$$Y_s = \underset{(1.07)}{\overset{*}{7.57}}\, \underset{(.08)}{\overset{***}{W^{.312}}}\, \underset{(.27)}{\overset{***}{EVAP^{1.38}}}$$

$\bar{R}^2 = .64$
$F = 18.08$

Alfalfa

$$Y_a = \underset{(1.62)}{\overset{**}{-4.4285}} + \underset{(.0007)}{\overset{***}{.00297WEVAP}} - \underset{(.0005)}{\overset{*}{.000997W^2}}$$

$\bar{R}^2 = .95$
$F = 147.30$

Where:

Y_c = cotton yield in pounds of lint per acre.
Y_w = wheat yield in pounds per acre.
Y_s = sorghum yield in pounds of grain per acre.
Y_a = alfalfa yield in tons per acre.
W = gross water applied and effective rainfall from preplant irrigation to harvest in acre inches.
N = nitrogen applied in pounds per acre.
EVAP = total pan evaporation for the season in inches.
WEVAP = W · EVAP.
WN = W · N.
$\bar{R}^2$ = coefficient of determination adjusted for degrees of freedom.
*** = coefficient is statistically significant at the one percent level.
** = coefficient is statistically significant at the two percent level.
* = coefficient is statistically significant at the ten percent level.
Numbers in parenthesis are the standard error of the estimate.

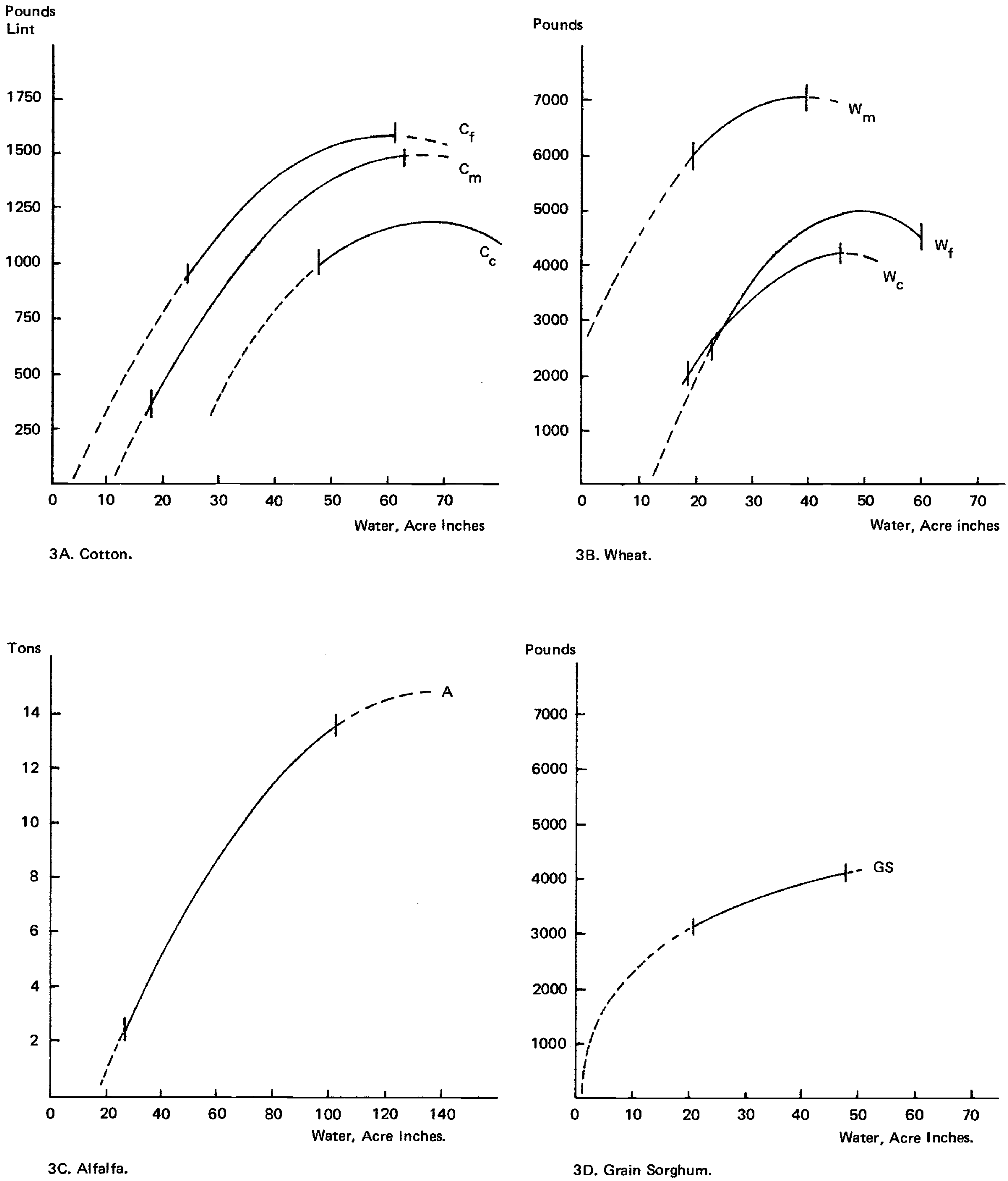

Figure 3. Crop-Water Production Functions for Cotton (C), Wheat (W), Alfalfa (A), and Grain Sorghum (GS), Grown on Coarse (c), Medium (m) and Fine (f) Soils.

revealing just the third stage of the total yield-pan evaporation relationship. No nitrogen term is included in the equation because nitrogen was not varied at the Phoenix site and was not statistically significant in the Tempe experiment. The cotton-water function is graphed in Figure 3A and is labled C_m. The function exhibits diminishing marginal productivity and is based on a wide range of water levels.

The wheat equation explains 62% of the variance in yields. All coefficients are statistically significant at the 10% level or better except the coefficient for the water squared term which is statistically significant at the 12% level. The water-evaporation term (WEVAP) is positive. Since pan evaporation had little variation (between 38 and 41 inches) in the experiments, this term primarily reflects the effect of water on yield. In equations where the water term is included as a separate variable, the evaporation term is expected to have a negative sign because wheat responds better to cooler weather. In another equation estimated from the same data, water and evaporation variables were separated: the water term was positive and the evaporation term negative (though not statistically significant). Both the reported and unreported equations had the same $\bar{R}^2$ and exhibit essentially identical two-dimensional water-yield production functions. The water-wheat function is graphed in Figure 3B and is labeled W_m. Marginal productivity diminishes as water use increases. Although the range of water levels is somewhat limited, the data include the water level which produces the maximum yield as given by the statistical production function.

The sorghum production function explains 64% of the variance in yields. Both the water and the pan evaporation variables are significant at the 1% level and have the expected signs. Nitrogen was not varied in the agronomic experiments. The water yield relationship for sorghum is graphed in Figure 3D. The underlying experiments did not provide water levels which pushed production into the third stage–*i.e.*, decreasing yield. Projections beyond the data range are questionable.

Fine Texture Soil

About 93% of the variance in cotton yields is explained by the production function. All variables are significant at the 1% or 5% levels, and all signs are reasonable. The function (C_f) is graphed in Figure 3A and is labeled C_f. The function exhibits diminishing marginal returns and a third stage of production within the range of the data.

All variables in the wheat production function are significant at or near the 1% level and all signs are as expected. The function explains 71% of the variance in yields. The function (W_f) is graphed in 3B. Both a second and third stage of production are shown within the range of data.

The production function for alfalfa explains 95% of the variance in yields. All variables are significant and signs are as expected. The function is shown graphically in Figure 3C. The alfalfa function must be applied with caution in economic analyses. The function indicates considerably higher yields than exist on average in Arizona counties. It indicates, for example, a yield of nearly 12 tons per acre when 84 acre inches (a typical farm application) of water are applied. A 12-ton yield is approximately double county average yields. This function, then, should not be used directly in farm crop budget comparisons which attempt to depict typical farms.

The equation may be defended and used for other analyses, however. First, the validity of the equation for the best farms is supported by the fact that the best farms do obtain 10-12 ton yields. Second, the slope of the function (in contrast to its height) may also be defended and used in economic analyses. The equation indicates a nearly linear relationship between water applied and yield within the range of the data. That is, the function indicates only very slight decreases in marginal productivity as water is increased. The validity of the near linear production function is supported by the very high $\bar{R}^2$ of .95. The validity is also supported by the fact that there has been no trend toward decreased yields per acre in those counties where water prices are high and have been forced sharply upward in recent years. If the marginal physical product diminished significantly, higher water prices should elicit decreased water rates and lower yields. In the economic analysis which follows, implications will be based on the evidence that the slope (marginal productivity) of the alfalfa yield-water relationship does not diminish with water applications in normal ranges.

The sorghum production function is the same as that for medium texture soil since the data are from a site with medium-fine soils.

Profit-Maximizing Quantity of Irrigation Water

Profit-maximizing quantities of irrigation water are estimated from the production functions in Table 3 and expected pan evaporation for Yuma, Mesa and Safford, Arizona for coarse, medium and fine texture soils, respectively. A sensitivity analysis is performed by including low, medium and high water and product prices. Results are on a per-acre basis.

Coarse Texture Soil

Table 4 indicates the amount of water to apply over an entire season for cotton and wheat in order to maximize profits, given one of three prices for water and cotton or wheat. For low priced water ($.50 per acre inch), about the same level of water is applied, regardless of the price received for the product. Cotton shows less sensitivity to increases in water prices than does wheat. At low product prices, cotton irrigation water is reduced by 13 inches when the price of water is increased from $.50 to $5 per acre inch while water for wheat is reduced by 20 inches with the same increase in water prices. At high product prices, water is reduced by only 5 inches for cotton and 9 inches for wheat when water prices increase tenfold.

Table 4. Profit-Maximizing Quantity of Water[a] with Varying Water and Product Prices, Coarse Texture Soil (acre inches per acre).

Price of Water $/Ac.In.	Cotton Price ($/lb.)[b]			Wheat Price ($/lb.)		
	.30	.65	.80	.04	.06	.08
.50	64	64	65	46	47	47
2.50	58	62	63	36	40	42
5.00	51	58	60	26	33	36

a. Based on production functions from Table 3 and expected pan evaporation for Yuma Valley, Arizona.

b. The price of cotton is the price of lint plus seed per pound of lint.

Medium Texture Soil

Table 5 gives profit-maximizing levels of water for crops produced on medium texture soil. As on coarse soils, wheat appears to be more sensitive to water prices than cotton.

Sorghum is the crop most sensitive to both product and water prices. At high water prices, sorghum could not be grown at the water levels indicated by profit-maximization criteria.

Fine Texture Soil

Results for fine soils are given in Table 6. Profit-maximizing levels of water for cotton and wheat are similar in sensitivity to changes in the price of water: water decreases by five to 15 acre inches per acre as water prices are increased from $.50 to $5 per acre inch and product prices are held at specific levels.

Profit-maximizing quantities of water for sorghum vary similarly to sorghum grown on medium texture soils where different conditions of pan evaporation pertain. At low water prices, water quantities are constrained to common practice levels while sorghum could not be grown using water quantities shown for high priced water.

The alfalfa production function is constrained to the common practice level (84 inches in the Safford area) because of the almost linear relationship between water and yield shown in the agronomic data. The slope of the underlying production function is such that all profit-maximizing quantities of water are estimated to be in excess of common practice quantities. Therefore, common practice quantities are indicated for alfalfa in Table 6.

Table 5. Profit-Maximizing Quantity of Water[a] with Varying Water and Product Prices, Medium Texture Soil (acre inches per acre).

Price of Water $/Ac. In.	Cotton Price ($/lb.)			Wheat Price ($/lb.)			Sorghum Price ($/lb.)		
	.30	.65	.80	.04	.06	.08	.03	.05	.07
.50	64	64	65	35	36	36	36[c]	36[c]	36[c]
2.50	57	61	62	27	31	32	9[d]	19	31
5.00	49	58	59	17	24	27	3[d]	7[d]	11[d]

a. Based on production functions from Table 3 and expected pan evaporation for Mesa, Arizona.

b. The price of cotton is the price of lint plus seed per pound of lint.

c. The production function indicates an excessive quantity of water and an amount beyond the range of the data used to estimate the statistical production function. Therefore, standard practice water quantity is shown.

d. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

Table 6. Profit-Maximizing Quantity of Water[a] with Varying Water and Product Prices, Fine Texture Soil (acre inches per acre).

Price of Water $/Ac. In.	Cotton Price ($/lb.)[b]			Wheat Price ($/lb.)			Sorghum Price ($/lb.)			Alfalfa Price ($/ton)		
	.30	.65	.80	.04	.06	.08	.03	.05	.07	50	75	100
.50	58	59	59	47	48	48	44[c]	44[c]	44[c]	84	84	84
2.50	52	56	57	40	43	45	8[d]	18	29	84	84	84
5.00	43	52	54	32	38	40	3[d]	7[d]	11[d]	84	84	84

a. Based on production functions from Table 3 and expected pan evaporation for Safford, Arizona.

b. The price of cotton is the price of lint plus seed per pound of lint.

c. The production function indicates an excessive quantity of water and an amount beyond the range of the data used to estimate the statistical production function. Therefore, standard practice water quantity is shown.

d. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

Comparison of Soil Texture Models with Other Models

The profit-maximizing quantities of water may be compared with quantities suggested by other irrigation models. Here, the profit-maximizing levels of water, derived from the soil texture models, are compared with water applications suggested by common practice, yield-maximizing models, and a site specific and an aggregate (over sites) model given by Hexem and Heady. The common practice models show typical irrigation quantities for areas with the same soil type as the profit-maximizing models. The yield-maximizing models are based on the soil texture production functions, but without regard for water or product prices. The yield-maximizing models correspond, as indicated previously, to models upon which current irrigation recommendations are based. The Hexem and Heady site models are estimated from data from sites which make up part of the data used in the estimation of the soil texture models. Hexem and Heady models are also profit-maximizing models, but differ from the profit-maximizing models of the current work in that supplemental soil and agronomic data, plus alternative functional forms, are used in the current work. Hexem and Heady's aggregate models are based on data from several sites, but with soil and climatic variables to account for differences in yield among sites. Again, their aggregate models are also profit-maximizing models based upon their estimates of the crop-water production functions.

For illustrative purposes and because comparative models are not available for all crops, comparisons are made only for cotton and wheat. The results for the cotton models are shown in Table 7. The principle findings may be summarized: 1) The profit-maximizing level of water is near or within the range of common practice application for fine and medium soils, but the profit model does imply substantially less water than commonly applied on coarse soils. 2) Models which maximize yield imply a four-to-eight acre inch increase in water over that implied by profit-maximizing models. 3) The Hexem and Heady site models produce unreasonable results for both fine and medium soils. The underlying production functions upon which the water estimates are based are not well specified. 4) The Hexem and Heady aggregate models are not sensitive to soil texture differences when used to derive profit-maximizing quantity of water because of the mathematical form of production function estimated. Their aggregate model implies unreasonably high levels of water for all soil types and water prices.

Table 8 indicates the water applications implied by the various models for wheat grown on different soil textures. The key findings are: 1) The profit-maximizing model implies water applications which are, in general, lower than common practice. Lower water applications are especially implied when water is high priced and for all water price situations on coarse soils. But even for low and medium water price situations on fine and medium soils, the profit-maximizing quantities of water tend to be less than or at the very lowest end of the common practice range. 2) Although the difference between the yield-maximizing models and the profit-maximizing models does not tend to be great at low water prices, at medium and high water prices the difference is usually six or more inches—an amount which by many standards is a significant water savings. 3) The Hexem and Heady site models produce results not greatly different than the results of the profit-maximizing models for medium and coarse soils. For wheat produced on fine soil, however, the Hexem and Heady site model implies water applications which are excessive in comparison with both the profit model and common practice. 4) The Hexem and Heady aggregate models again yield results which are insensitive to soil texture and are, by any standard, unreasonably high.

Table 7. Water Application Implied by Five Cotton "Models"[a]: Profit-Maximizing Model, Common Practice, Yield-Maximizing Model, and Two Hexem and Heady Models[b] (acre inches per acre).

	Fine Soil Texture					Medium Soil Texture					Coarse Soil Texture				
Price of Water $/Ac.In.	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model
.50	59	48-60	60	c	83	64	42-60	66	37	83	64	72-84	66	66	83
2.50	56	48-60	60	c	74	61	42-60	66	37	74	62	72-84	66	63	74
5.00	52	48-60	60	c	65	58	42-60	66	37	65	58	72-84	66	59	65

a. Based on: production functions in Table 3; Hathorn and Cluff; Hathorn, Howell and Hazlitt; Hathorn and Farr; Hazlitt; Hexem and Heady (1978).

b. In the profit-maximizing and Hexem and Heady models, the price of cotton is assumed to be that expected in 1979—$.65 for fiber and seed per pound of cotton fiber.

c. The model implies irrationally high profit-maximizing quantities of water and which are beyond the range of the data. Coefficients on water variables in the underlying production function are not statistically significant.

Table 8. Water Applications Implied by Five Wheat "Models"[a]: Profit-Maximizing Model, Common Practice, Yield-Maximizing Model, and Two Hexem and Heady Models[b] (acre inches per acre).

	Fine Soil Texture					Medium Soil Texture					Coarse Soil Texture				
Price of Water $/Ac.In.	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model	Profit Model	Common Practice	Yield-Maximizing Model	Hexem and Heady Site Model	Hexem and Heady Aggregate Model
.50	48	39-50	49	60	c	36	36-40	37	30	c	47	72-84	49	49	c
2.50	44	39-50	49	54	c	32	36-40	37	29	c	41	72-84	49	43	c
5.00	41	39-50	49	46	c	26	36-40	37	28	c	35	72-84	49	37	c

a. Based on: production functions in Table 3; Hathorn and Cluff, Hathorn, Howell and Hazlitt; Hathorn and Farr; Hazlitt; Hexem and Heady (1978).

b. In the profit-maximizing and Hexem and Heady models, the price of wheat is that expected in 1979—$.07 per pound.

c. The model implies irrationally high profit-maximizing quantities of water and which are beyond the range of the data.

Irrigation Scheduling and Yield

Agronomists have long known that yield response to water differs for different stages of plant development. With a precise knowledge of yield response by growth stage, the value of each irrigation in a farmer's schedule could be determined. However, most of the agronomic experiments upon which the analysis given here is based were not designed to provide data on yield response by growth stage. The experiments were designed to vary water levels through the implementation of a particular water scheduling rule. That is, lower water treatments were achieved by allowing soil water to drop to a lower fraction of soil water holding capacity before water was applied; higher water rates were achieved by irrigating when soil water had not fallen as far below soil water holding capacity. The scheduling rule, therefore, implies that water deficits, below some specified amount, were distributed over the growing season. Whether or not this scheduling is the best way to distribute water deficits for these crops cannot be answered from our data. A study by Stewart, Hagan and Pruitt on corn response to scheduling supports the contention that distributing deficits over the growing season is most efficient. However, information from other sources indicates that water deficits might be more optimally scheduled by deleting entire irrigations rather than spreading the deficit over a number of irrigations. Table 9 summarizes this outside information.

The present report can only draw attention to the scheduling issue and indicate that future research based on other agronomic crop-water-scheduling experiments will analyze the economics of scheduling.

Table 9. Common Practice Irrigation Schedules[a] with Sequence for Deletion[b] of Irrigations to Minimize Yield Loss, Safford, Arizona.

Crop	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Cotton						xx	xx	x^3x^2	x^1			
Wheat			x^2x	xx	x^3x^1							
Sorghum							x^3	xx	x^2x^1			
Alfalfa		x	x	x	xx	x^3x	x^2x	x^1x	xx	x		

a. Preplant irrigations are not shown. Each irrigation is approximating six acre inches of water per acre.

b. Numbers in the table represent the order in which irrigations should be eliminated, to minimize reduction in yields.

Sources: Hathorn and Cluff; Dennis (1979); Dennis, *et al.* (1977); Kittock and Farr; Willett, Taylor and Buxton; Erie, French and Bucks.

Effects of Irrigation Delivery and Pump Efficiencies on Profits and Water Use

Delivery Efficiencies

Irrigation delivery efficiency, defined as the ratio of water reaching the head of the field to water at the wellhead, has a great effect on the costs and hence the profits of irrigated farming and may affect the profit-maximizing quantity of water to apply. It should be noted that most

Arizona farms now have lined irrigation ditches with delivery efficiencies averaging more than 90%. However, this section is applicable to the less than 10% of Arizona farms which still operate with unlined ditches (Parsons) and has implications outside Arizona where ditches are unlined.

Table 10 illustrates the effect of irrigation delivery efficiency on returns over total variable costs. The difference between returns over total variable costs at different delivery efficiencies is the change in profits. Differences between numbers in a row indicate how profits are affected as delivery efficiency changes. Columns illustrate the effect of water price on profits for any particular delivery efficiency. With low-priced water, profits are not greatly affected by delivery efficiency. However, with higher-priced water, profits drop dramatically. Sorghum and wheat are unprofitable to grow with higher water prices and low irrigation delivery efficiency.

Profit-maximizing quantities of irrigation water pumped, shown in Table 11, drop considerably for all crops except alfalfa as delivery efficiency increases and water prices increase. Sorghum cannot be grown with profit-maximizing water levels at medium and high water prices and low and medium irrigation delivery efficiencies.

Pump Efficiencies

Another, often overlooked, efficiency is that of the irrigation pump. Pumps are never 100% efficient, and usually don't exceed 70% - 75% according to Hathorn (1979). Pumps have a tendency to wear rather rapidly due to sediments in the water. Wear reduces efficiency to about 50% in three years. Although this wear may not be obvious without constant metering, it can result in considerable losses to the farmer.

Table 12 indicates the difference between total revenues and total variable costs for four crops and three pump efficiencies. This table is based on the production functions

Table 10. Returns over Total Variable Costs[a] with Varying Irrigation Delivery Efficiencies, Fine Texture Soil[b] ($ per acre).

Price of Water	Cotton			Wheat			Sorghum			Alfalfa		
	Irrigation Delivery Efficiency											
$/Ac.In.	.50	.75	.90	.50	.75	.90	.50	.75	.90	.50	.75	.90
.50	619	640	646	151	167	172	66	81	86	646	674	684
2.50	392	485	517	-26	47	72	c	c	5	310	450	497
5.00	141	314	372	-224	-87	-42	c	c	c	-110	170	263

a. Variable costs include those for water, feed, fertilizer, pesticides, herbicides, labor and seed.

b. Based on: Hathorn and Cluff; production functions from Table 3; expected 1979 product prices and expected pan evaporation for Safford, Arizona.

c. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

Table 11. Profit-Maximizing Quantity of Irrigation Water,[a] Pumped and Applied; with Varying Irrigation Delivery Efficiencies, Fine Texture Soil (acre inches per acre).

Price of Water	Cotton			Wheat			Sorghum			Alfalfa		
	Irrigation Delivery Efficiency											
$/Ac.In.	.50	.75	.90	.50	.75	.90	.50	.75	.90	.50	.75	.90
	Acre Inches Pumped											
.50	116	79	66	94	64	53	88	59	49	168	112	93
2.50	104	72	62	78	56	48	16[b]	20	22	168	112	93
5.00	90	67	57	60	48	42	6[b]	8[b]	8[b]	138	112	93
	Acre Inches Applied											
.50	58	59	59	47	48	48	44	44	44	84	84	84
2.50	52	54	56	39	42	43	8[b]	15[b]	20	84	84	84
5.00	45	50	51	30	36	38	3[b]	6[b]	7[b]	69	84	84

a. Based on: production functions in Table 3, expected 1979 product prices and expected pan evaporation for Safford, Arizona.

b. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

and expected 1979 crop prices previously reported and an electrical irrigation pump with a 300-foot lift in Graham County. As pump efficiency is cut in half, from 72% to 36%, alfalfa profits are reduced over 30% and cotton profits decline by 16%. Both sorghum and wheat become unprofitable and should not be grown because even variable costs are not covered by gross returns.

Decreasing pump efficiency increases water costs (because of the additional energy required to lift a given volume of water) and thus reduces profit-maximizing quantities of water for most crops. However, as shown in Table 13, only the profit-maximizing quantity of water for sorghum is greatly affected by the changing pump efficiencies when pumping from medium depths. Differences in optimum amounts of water would, of course, be greater at lower depths, as reflected in Table 6.

The decision to replace or renovate pumps should be based upon an investment analysis in which investment costs are compared with decreased pumping costs over time.

Table 12. Returns over Total Variable Costs[a] with Varying Irrigation Pump[b] Efficiencies, by Crop, Fine Texture Soil ($ per acre).

Pump Efficiency	Cotton	Wheat	Sorghum	Alfalfa
.72	570	115	26	548
.50	521	77	8	475
.36	461	31	c	382

a. Based on: Hathorn and Cluff; production functions from Table 3; expected pan evaporation for Safford, Arizona; expected 1979 product prices and the 1979 price of electricity in the Safford Valley of $.0557 per KWH. See Table 10 for the definition of variable costs.

b. Electric pump and 300-foot lift depth assumed.

c. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

Table 13. Profit-Maximizing Quantity[a] of Irrigation Water with Varying Irrigation Pump Efficiencies, Fine Texture Soil (acre inches per acre).

Pump Efficiencies	Cotton	Wheat	Sorghum	Alfalfa
.72	57	45	28	84
.50	55	43	17	84
.36	54	41	11[b]	84

a. Based on: Hathorn and Cluff; production functions from Table 3; expected 1979 product prices; electric irrigation pump with a 300-foot lift and expected pan evaporation for Safford, Arizona.

b. Insufficient water to produce a crop. Agronomists indicate water applications less than 12-18 inches, depending on weather conditions, are insufficient to produce a crop. Although the estimated water applications, based on the statistical production functions, are below the range of the experimental data, the outside information from agronomists suggests that applications more than a few inches below the experimental range are simply not sufficient for crop production.

Effects of Water Constraints on Profits

Constraints may be placed on the amount of surface water allocated to particular irrigation districts and to individual farms. Constraints may be imposed as a means to conserve water, to allocate a limited supply among users or to reduce salinity problems. Along the Colorado River, for example, the Colorado River Indian Reservation is restricted by law to a specified annual amount of water from the almost totally allocated waters of the lower Colorado River. In the same area, some specialists have proposed that on-farm irrigation practices restrict water applications in order to reduce the amount of salts leached through the soil and later deposited in the Colorado River. In other areas, drought conditions have constrained the amount of surface water available for crop production and will continue to do so. The effects of water constraints on profits depend upon the underlying crop-water production function. The production functions reported earlier are here used to illustrate the impact of water constraints on crop profits. These production functions are derived from irrigation schedules based on varying levels of soil moisture depletion. Profit-maximizing water levels usually result from a 50% soil moisture depletion schedule while a 10% and 20% water reduction imply irrigating with about a 70% and 80% moisture depletion schedule, respectively.

The effects of a 10% and a 20% per-acre, per-crop restriction on water to Yuma Valley irrigators are shown in Table 14. Alfalfa returns over total variable costs are the most sensitive to water restrictions: a 20% decrease in water use, from the profit-maximizing level, cuts per acre profits by $198. Sorghum profits are least sensitive to reductions in water. Cotton profits per acre decrease $108 with a 20% decrease in available water while wheat profits per acre are reduced by $83.

Another type of water constraint limits water per farm (rather than per acre). In this situation, the farmer may ad-

Table 14. Returns over Total Variable Costs[a] for Alternative Per-Acre Water Restrictions, by Crop, Fine Texture Soil ($ per acre).

Water Restrictions	Cotton	Wheat	Sorghum	Alfalfa
0	557	298	150	658
-10%	511	257	145	552
-20%	449	215	139	460

a. Based on Hathorn, Howell and Hazlett; price of water = $.50/ac.in.; production functions from Table 3; expected 1979 product prices and expected pan evaporation for Yuma Valley, Arizona.

just water applied per acre, the number of acres irrigated and crop mix. Suppose, for example, a 300-acre farm which can produce cotton, wheat and sorghum on fine textured soil. The prices of cotton, wheat, sorghum and water are assumed to be $0.38 per pound, $0.07 per pound, $0.06 per pound and $0.50 per acre-inch, respectively. Assume also that a restriction is placed upon the amount of water available to the farm—10,800 acre-inches. Two irrigation strategies which might be followed are: 1) Apply water at the rate which maximizes profits per acre until all water is used. This strategy is appropriate for maximizing whole farm profits when the quantity of water is not limited. This strategy (or a similar one) has also been recommended by various irrigation management services when a water constraint is in effect. 2) The second strategy applies that amount of water per acre which maximizes profits to the whole farm.

Table 15 shows the results of these two strategies and illustrates the importance of recognizing the impact of a water-quantity restriction on irrigation decisions. The strategy to maximize whole-farm profits results in lower profits per acre, but since more acreage is planted, whole-farm profits are higher.

The underlying crop-water production function explains the difference in per-acre and whole-farm profits for the two strategies. At lower per-acre irrigation rates, the marginal increase in output is greater than at higher water rates. Spreading limited water over acre units of land until the marginal physical products from each unit of water are equal maximizes whole-farm prices.

Table 15. Irrigation Strategies and Their Impacts for a 300-Acre Farm with Limited Water.

Strategy	Crops and Acres Planted	Water Applied Per Acre (acre inches)	Returns Over Total Variable Costs Per Acre ($)	Whole Farm Returns Over Variable Costs ($)
1. Apply water at level which maximizes profits per acre until all water is used.	Wheat 225 Ac.	48	216	48,600
2. Apply water at level which maximizes whole farm profits.	Wheat 300 Ac.	36	178	53,400

a. Fine texture soil and water limited to 10,800 acre-inches are assumed. Product and water prices are as specified in the text. The crop-water production functions used in the analysis are those for fine texture soil. Linear programming was used to simultaneously determine the whole farm profit-maximizing level of water per acre and number of acres, subject to the 10,800 acre-inch water constraint.

Elasticity of Demand for Irrigation Water

In general, the crop-water production functions imply an inelastic demand for water. Tables 16, 17 and 18 show the estimated elasticity of demand for irrigation water for coarse, medium and fine soils, respectively, with low and expected 1979 product prices. For all soil textures, cotton has the most inelastic demand for irrigation water, while sorghum exhibits the most elastic demand. In all cases, except sorghum which remains constant because of the mathematical form of the production function, the demand for irrigation water becomes more elastic as product prices decline.

For all crops, except sorghum produced at low ($.50 per acre inch) or medium ($2.50 per acre inch) water prices, the elasticity of demand is so low that efforts to conserve water through marginal changes in the price of water will not be successful. For example, wheat produced on medium texture soil at a low product price and water priced at $2.50 per acre inch has an elasticity of demand of -.372. The elasticity indicates that a 1% increase in the price of water will diminish water use by .372%. Roughly, then a 10% increase in price will be met by a 3.72% decrease in water use. From Table 5, the profit-maximizing level of water for wheat grown with water priced at $2.50 per acre inch is 35 acre inches per acre. Therefore, if the price of water were to be increased from $2.50 to $2.75 per acre inch, water use would decline from 35 to 33.7 acre inches per acre (35 x .0372 = 1.3 and 35 - 1.3 = 33.7).

In a similar fashion, it might be illustrated that even doubling or tripling the price of surface water (often priced near $.50 per acre inch), water use will not be decreased significantly on most crops. The initial price of water is simply too low and the underlying production functions are so shaped that to maximize profits, water use remains nearly unchanged.

A caveat is in order. The elasticity analysis is based on a given irrigation application efficiency. At medium- and

Table 16. Elasticity of Demand[a] for Irrigation Water, by Crop, Coarse Texture Soil.

Price of Water $/Ac.In.	Cotton Price ($/lb.) .30	Cotton Price ($/lb.) .65	Wheat Price ($/lb.) .04	Wheat Price ($/lb.) .07
.50	-.027	-.013	-.050	-.034
2.50	-.126	-.060	-.272	-.198
5.00	-.239	-.118	-.618	-.504

a. Based on production functions from Table 3 and expected pan evaporation for Yuma Valley, Arizona.

Table 17. Elasticity of Demand[a] for Irrigation Water, by Crop, Medium Texture Soil.

Price of Water $/Ac.in.	Cotton Price ($/lb.) .30	Cotton Price ($/lb.) .65	Wheat Price ($/lb.) .04	Wheat Price ($/lb.) .07	Sorghum Price ($/lb.) All Prices
.50	-.026	-.012	-.057	-.032	-1.45
2.50	-.147	-.061	-.372	-.181	-1.45
5.00	-.340	-.132	-1.176	-.442	-1.45

a. Based on production functions from Table 3 and expected pan evaporation for Mesa, Arizona.

Table 18. Elasticity of Demand[a] for Irrigation Water, by Crop, Fine Texture Soil.

Price of Water $/Ac.In.	Cotton Price ($/lb.) .30	Cotton Price ($/lb.) .65	Wheat Price ($/lb.) .04	Wheat Price ($/lb.) .07	Sorghum Price ($/lb.) All Prices	Alfalfa Price ($/ton) 50	Alfalfa Price ($/ton) 75
.50	-.029	-.013	-.052	-.020	-1.45	-.037	-.027
2.50	-.160	-.069	-.304	-.108	-1.45	-.218	-.134
5.00	-.386	-.148	-.761	-.232	-1.45	-.592	-.328

a. Based on production functions from Table 3 and expected pan evaporation for Safford, Arizona.

high-priced water, it is possible that increases in the price of water would induce water-saving technologies, such as laser or dead leveling fields, to be adopted. The adoption of water-saving technologies implies a more elastic demand for water. Estimates of the profitability of water-saving technologies are to be investigated in follow-up studies.

Implications

Results of the analysis have implications for farm level irrigation management, regional water conservation policy and needed research. The farm level implications are necessarily generalizations; specific recommendations for a particular farm need to account for peculiarities of the individual farm.

Farm Level

1) Perhaps the most important implication at the farm level is that in areas of medium-to-high water prices, profits can be increased if water applications are cut below common applications which typically are intended to maximize yield. The amount by which water should be cut is frequently six acre inches (one normal irrigation) or more on cotton, wheat and sorghum, grown on a variety of soil textures and under a wide range of product prices.

2) In those areas of low water prices, farmers can maximize profits by applying nearly the same amount of water as that required to maximize yield. Current recommendations of irrigation management services should be valid in both an economic and technical sense.

3) Irrigation delivery efficiency has very little impact on short-run profits when water is low priced, but has a very large impact on short-run profits for medium- and high-priced water. Accordingly, there is little incentive for the farmer to line irrigation canals and improve delivery efficiency in other ways when water is priced low. However, it may be economically rational to improve delivery efficiency if water is higher priced. For wheat, sorghum and alfalfa, a combination of high water prices ($5 per acre inch) and low delivery efficiency (50%) implies net financial losses. A long-run, benefit-cost analysis of capital investments in delivery efficiency is required to assess the net benefits of particular investments.

4) Irrigation pump efficiency can greatly affect short-run profits of all crops in medium and deep lift areas. Good management, then, requires that pump efficiencies be checked and the long-run benefits compared to long-run costs of repairing or replacing pumps.

5) If water available to a farm is limited, whole-farm profits can be maximized by irrigating a relatively large number of acres at relatively low per-acre irrigation rates, rather than applying high water rates to fewer acres.

Water Conservation Policy

1) At the regional level and in areas where water recharge is not great, the results imply that large amounts of water may be conserved (cut per acre application) by changing from yield- to profit-maximizing levels of water

in medium- to high-price water areas. In Arizona, for example, 200,000 to 250,000 acre feet of water could be saved by cutting water applications to an amount which maximizes profits. This water savings is approximately twice the annual municipal-industrial use in Tucson—a city of nearly one half million people.

2) Attempts to conserve surface water through the price mechanism are unlikely to succeed unless water prices are raised several fold. Current established prices are minimal, about $0.50 per inch. Estimated elasticities of demand for water on cotton, wheat and alfalfa are so low for all soil textures and a wide range of product prices that even doubling the price of surface water will not significantly affect the water use.

3) Efforts to conserve water through quantity restrictions on surface water can greatly affect farm profits in the short run. On cotton, wheat, and sorghum grown on fine soil, profits are cut by $80 to $100 per acre as water is restricted to 20% below profit-maximizing levels. However, in the long run water quantity restrictions could encourage more efficient irrigation systems or practices which would reduce or negate the short run reduction in profits.

Research

There are several agronomic and economic research needs.

Agronomic Research

1) Future agronomic experiments should focus on yield response to plant stress, in comparison to most past research which has determined the amount of water needed to maximize yield. If water is scarce, then water can be conserved and profits increased by stressing the plant. But to determine the economical amount of water, from both a farm management and policy viewpoint, information is needed on the crop-water response function when water is restricted.

2) The scheduling of irrigations is usually perceived to affect yield as much as the level of water applied. Still, relatively little agronomic research has been designed to stress plants differently during each of the stages of plant growth. Again, both farm level management and policy can be improved if the relationship between yield and water applied by growth stage is estimated. One significant research effort along these lines might be used as a pattern for future experimental design. The research reported by Stewart, *et al.* was designed to determine the effect on corn yield of alternative levels of water applied during vegetative, pollination and maturation stages of corn growth.

Economic Research

1) A multi-crop analysis is needed to determine the total-farm level response to changing energy and water prices, water quantity restrictions and other factors which effect water use. The current research focused on a crop-by-crop analysis, and the production functions used in the single crop analysis can be incorporated in a multicrop analysis.

2) Scheduling also needs to be incorporated in the analysis in a more formal manner. The research reported could only use supplemental information on scheduling because of the underlying agronomic data upon which the research is based. When agronomic data permit, crop-water production functions should be estimated by growth stage. Economic analysis of multi-period production functions can provide more detailed information for farm level decisions and indicate the possibility of greater water savings on a regional basis.

3) Risk should be considered. An argument can be made that farm managers base their decisions not only on expected profits, but also on the riskiness of particular decisions. Quadratic programming techniques, along with the individual crop-water production functions which incorporate a stochastic weather term, can be combined to show the variance of profits and expected profits from alternative irrigation decisions. Related research can estimate how managers react to different risk-profit situations and the results combined with the quadratic programming results to indicate which irrigation decisions will be made.

4) The extremely low price of surface water raises the questions "is water truly scarce, as usually perceived, and if so, what is its opportunity cost and what is an efficient (and/or equitable) administered price?" As shown, the administered price of surface water is approximately $0.50 per acre inch, and compares with an implied price of water pumped from 600 feet at approximately $5 per acre inch. Is such a price spread economically rational? Policy and research needs to address this question.

Appendix A

Confidence Bands for Crop-Water Production Functions

Confidence bands showing the 95% prediction interval for the crop-water production functions are shown in Figures A-1 through A-4. The bands are obtained by extension of the Working-Hotelling interval estimation for multiple regression equations (Neter and Wasserman) and use the same levels of the nitrogen and pan evaporation variables as used in plotting the production functions in Figure 3. Confidence bands for sorghum could not be estimated because of the Cobb-Douglas form of the equation.

Most of the confidence bands appear "reasonable." That is, within the range of the profit-maximizing levels of water, the bands usually are relatively close together. The widest confidence intervals, and ones which may make decisions based on the production functions suspect, occur where crops are grown on the coarse texture soils. In Arizona, areas of coarse texture soils, as exist at the Yuma Mesa experimental site, are not widespread and are therefore of little interest from a management or policy standpoint.

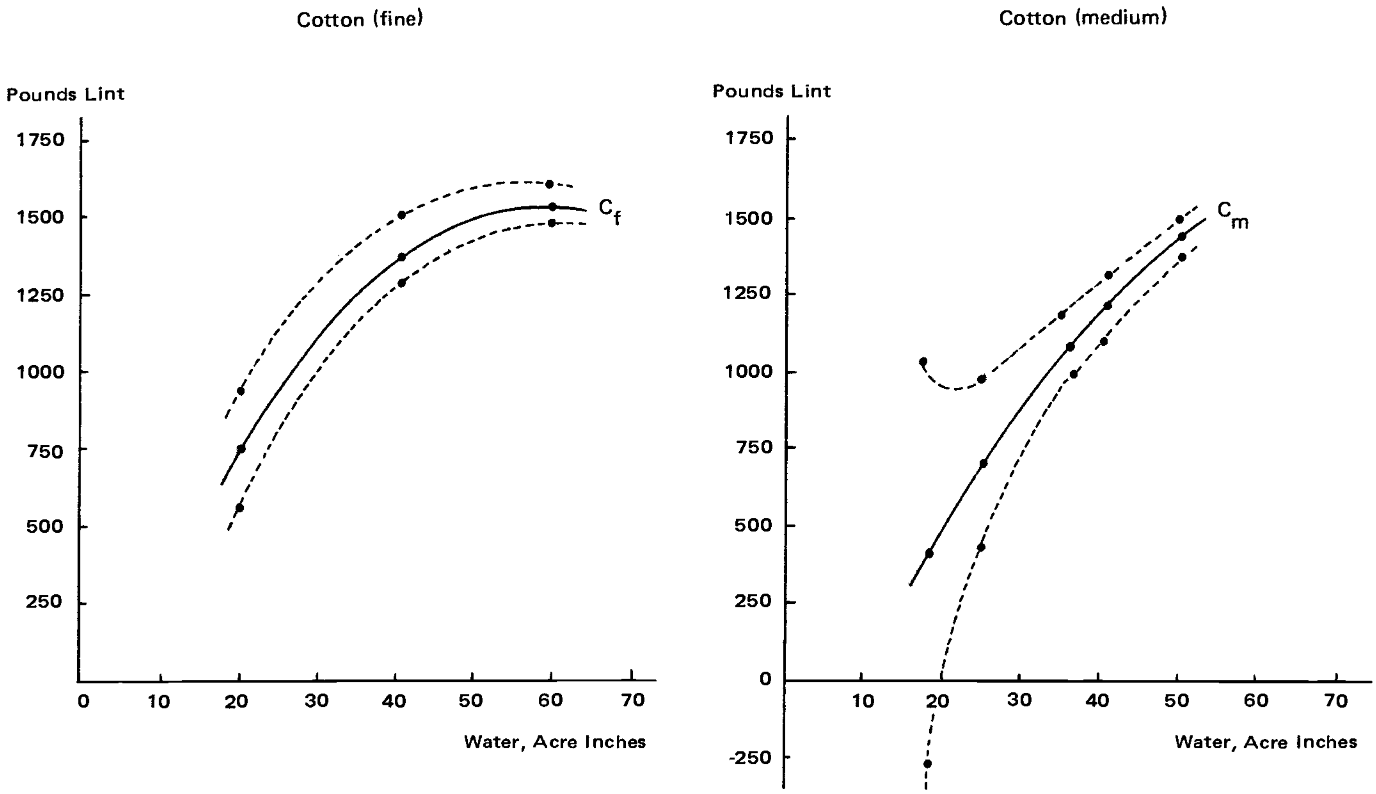

Figure A-1. Crop-Water Production Functions and 95 Percent Confidence Bands for Cotton Grown on Medium and Fine Soils.

Author's note: Jane Prentzel performed the statistical analysis. Her assistance is appreciated.

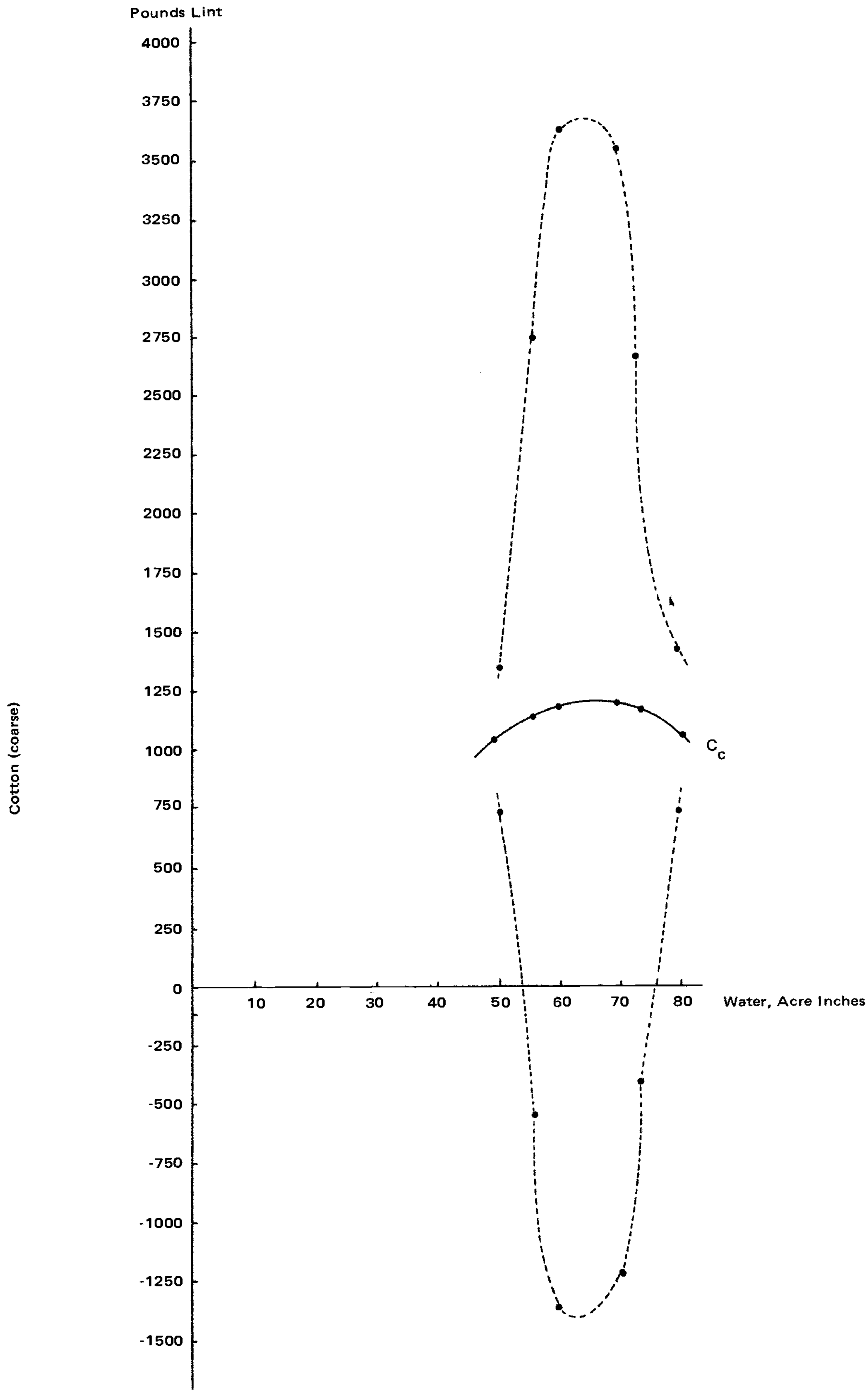

Figure A-2. Crop-Water Production Function and 95 Percent Confidence for Cotton Grown on Coarse Soil.

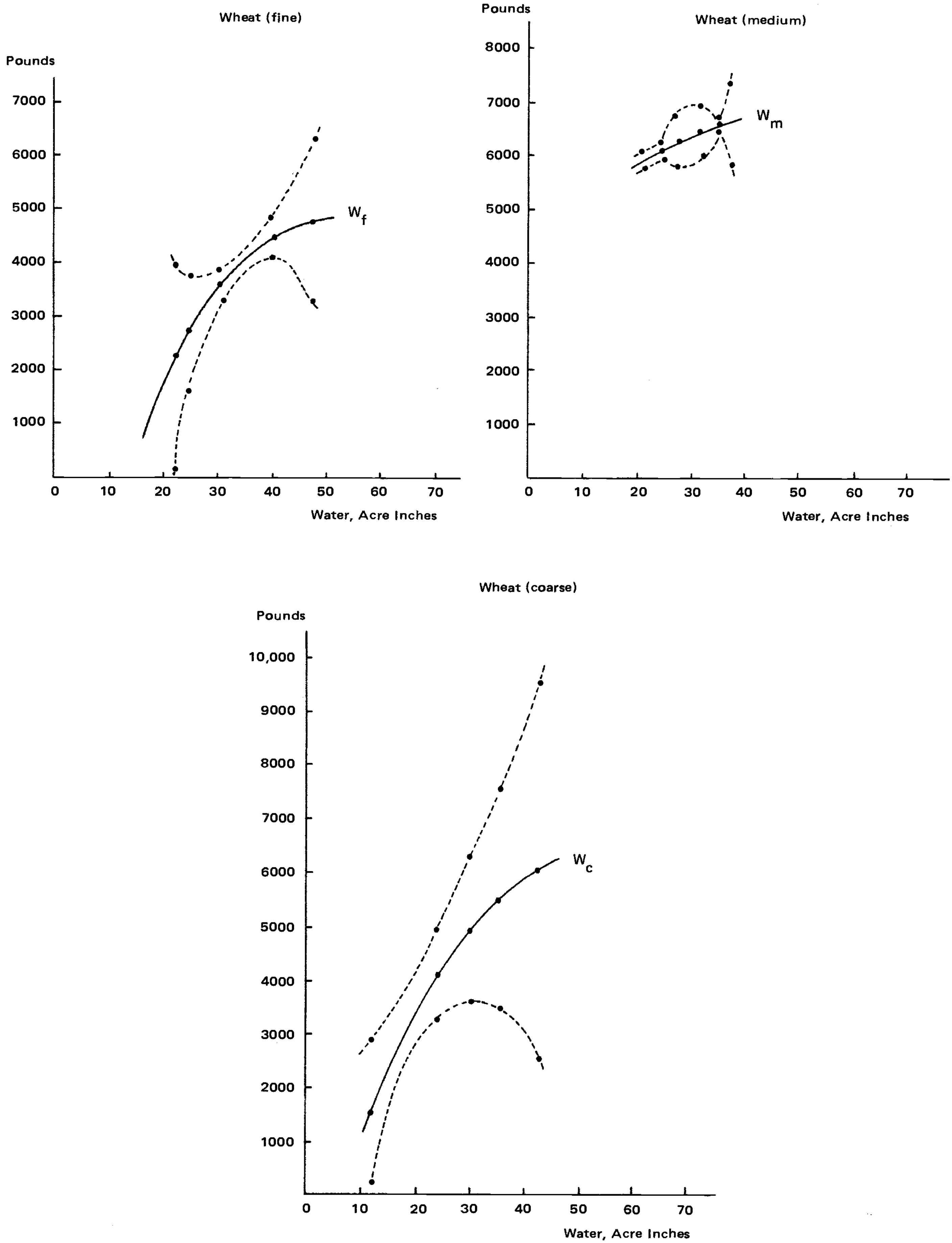

Figure A-3. Crop-Water Production Functions and 95 Percent Confidence Bands for Wheat Grown on Coarse, Medium and Fine Soils.

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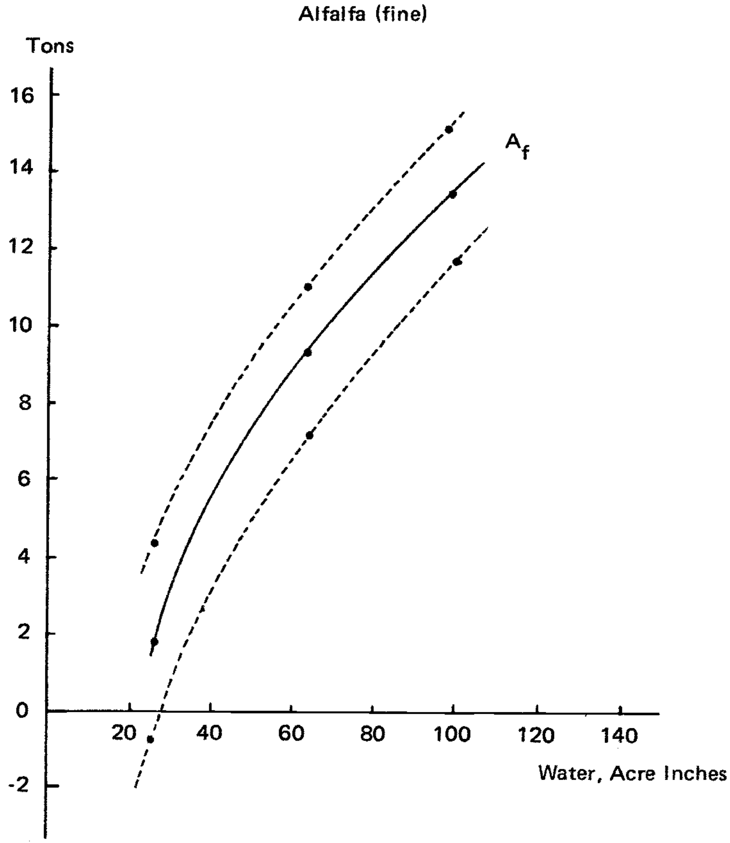

Figure A-4. Crop-Water Production Functions and 95 Percent Confidence Bands for Alfalfa Grown on Fine Soils.