JOINT INTERFERENCE SUPPRESSION AND QRD-M DETECTION FOR SPATIAL MULTIPLEXING MIMO SYSTEMS IN A RAYLEIGH FADING CHANNEL *

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ABSTRACT

Spatial multiplexing (SM) systems have received significant attention because the architecture offers high spectral efficiency. However, relatively little research exists on optimization of SM systems in the presence of jamming. In a spatially uncoded SM system, such as V-BLAST, the channel state information is assumed to be unavailable *a priori* at both transmitter and receiver. Here, Kalman filtering is used to estimate the Rayleigh fading channel at the receiver. The spatial correlation of the jammer plus noise is also estimated, and spatial whitening to reject the jammers is employed in both the Kalman channel estimator and detector. To avoid the exponential complexity of maximum-likelihood (ML) detection, the QRD-M algorithm is employed. In contrast to sphere decoding, QRD-M has

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fixed decoding complexity of order $\mathcal{O}(M)$, and is thus attractive for hardware implementation. The performance of the joint Kalman filter channel estimator, spatial whitener and QRD-M detector is verified by simulations.

KEYWORDS

MIMO systems, Kalman Filtering, channel estimation, QRD-M.

INTRODUCTION

The Vertical Bell Labs Layered Space-Time (V-BLAST) spatial multiplexing system was proposed to achieve high spectral efficiency in rich scattering environments [1] [2]. Many receiver designs have been introduced to detect data symbols in SM systems including linear decoders, MMSE, zero-forcing, and sphere decoding [3] [4] [5] [6] [7]. However, these latter algorithms assume additive white Gaussian noise as the only interference, and result in unacceptably high error rates in strong jamming. Here, we propose to use a spatial whitening filter as a pre-filter in the receiver to suppress interference. Then, a near-ML decoding technique based on the QR-decomposition and M-algorithm (QRD-M) [8] [9] [10] [11], is applied to detect the data symbols. QRD-M offers greatly reduced complexity compared to ML, and moderate levels of M, e.g. $M = 16$ have been shown to give near-ML performance. Furthermore, QRD-M implementation complexity is fixed for a given search size, which is attractive in practical applications. To accurately detect data sequences, channel estimation is required. Here, we use Kalman filtering based on the second-order autoregressive model to estimate the Rayleigh fading channels [12][13]. Spatial whitening is implicitly built into the KF to further improve estimator performance in jamming.

In the Signal Model section, the SM signal and autoregressive channel process are described. The joint interference suppression and channel tracker algorithm is given in the section Channel Estimation using Kalman Filtering. We review the QRD-M algorithm and its performance analysis in the Data Symbol Detection section. Finally, symbol error rate results and conclusions are given in the last section.

Vectors and matrices are all in boldface. Vectors are shown in lower case while matrices are in upper case. The notation $()^H$ refers to conjugate transpose and $\mathbf{I}$ refers to an identity matrix. $E\{\}$ stands for expectation.

SIGNAL MODEL

In a spatial multiplexing MIMO system with N_t transmit and N_r receive antennas, a received signal vector is $\mathbf{r}(k) \in \mathbb{C}^{N_r}$

$$\mathbf{r}(k) = \sqrt{\frac{E_s}{N_t}}\mathbf{H}(k)\mathbf{b}(k) + \mathbf{j}(k) + \mathbf{n}(k) \tag{1}$$

where E_s is the average energy per symbol, $\mathbf{b}(k) = [b_1(k), b_2(k), ..., b_{N_t}(k)]^T$ is the uncoded data symbol vector with $\mathbf{b}_i(k) \in \mathbb{C}$ taken from a unit-energy QAM constellation. We assume data symbols are uncorrelated, i.e., $E\{\mathbf{b}(k)\mathbf{b}(k)^H\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The channel matrix $\mathbf{H}(n)$ consists of $N_r \times N_t$ uncorrelated Rayleigh fading channel coefficients. Its $(i,j)^{th}$ entry denotes the complex gain from transmit antenna j to receive antenna i. The thermal noise, $\mathbf{n}(k)$, is zero-mean complex white Gaussian noise and $E\{\mathbf{n}(k)\mathbf{n}(k)^H\} = N_0\mathbf{I}$. The jammer is assumed to be zero-mean wide-sense stationary with correlation function

$$E\{\mathbf{j}(k)\mathbf{j}(k-m)^H\} = \mathbf{R}_J\delta_{m,0}, \tag{2}$$

where $\delta_{k,0}$ is the Kronecker delta function.

CHANNEL ESTIMATION USING KALMAN FILTERING

The well-known Jakes' model has been used to simulate Rayleigh fading channels [14] [15]. The Jakes' model has been previously approximated by a second-order autoregressive process (AR2) and incorporated into the Kalman filter in [12][13]. This AR approximation is used here for channel estimation as follows. To estimate the channel matrix $\mathbf{H}(k)$ with the KF, we rewrite (1) as the measurement model

$$\mathbf{r}(k) = \mathbf{J}(k)\mathbf{x}(k) + \mathbf{j}(k) + \mathbf{n}(k) = \mathbf{J}(k)\mathbf{x}(k) + \mathbf{e}(k) \tag{3}$$

The state vector $\mathbf{x}(k) \in \mathbb{C}^{2N_rN_t}$ is defined as

$$\mathbf{x}(k) = \begin{bmatrix} vec\{\mathbf{H}(k)\} \\ vec\{\mathbf{H}(k-1)\} \end{bmatrix}, \tag{4}$$

where vec is the column-wise vectorization operator. The effective Jacobian matrix $\mathbf{J}(k) \in \mathbb{C}^{N_r \times 2N_rN_t}$ is defined by

$$\mathbf{J}(k) = \begin{bmatrix} \mathbf{b}(k)^T \otimes \mathbf{I}_{N_r \times N_r} & \mathbf{0}_{N_r \times N_rN_t} \end{bmatrix}, \tag{5}$$

where $\otimes$ denotes the Kronecker product operator. The covariance matrix of the measurement noise is

$$\mathbf{R}_e = E\left\{\mathbf{e}(k)\mathbf{e}(k)^H\right\} = \mathbf{R}_J + N_0\mathbf{I} \tag{6}$$

We can estimate the covariance matrix $\mathbf{R}_e$ by the moving average algorithm as below

$$\hat{\mathbf{R}}_e(k) = \frac{k-1}{k}\hat{\mathbf{R}}_e(k-1) + \frac{1}{k}\left[\mathbf{r}(k) - \sqrt{\frac{E_s}{N_t}}\hat{\mathbf{H}}(k|k-1)\hat{\mathbf{b}}(k)\right]\left[\mathbf{r}(k) - \sqrt{\frac{E_s}{N_t}}\hat{\mathbf{H}}(k|k-1)\hat{\mathbf{b}}(k)\right]^H, \tag{7}$$

where $\hat{\mathbf{H}}(k|k-1)$ is the Kalman one-step prediction and $\hat{\mathbf{b}}(k)$ will be obtained from the QRD-M step. To simplify notation, we denote $N_x = N_r N_t$ in the sequel.

¿From [12] [13], each independently fading channel coefficient $\mathbf{H}_{i,j}(k)$ evolves as

$$\mathbf{H}_{i,j}(k) = -a_1\mathbf{H}_{i,j}(k-1) - a_2\mathbf{H}_{i,j}(k-2) + w(k). \tag{8}$$

The AR2 parameters a_1 and a_2 are defined by

$$a_1 = -2r_d\cos(2\pi f_p T) \qquad a_2 = r_d^2, \tag{9}$$

where T is the symbol sampling rate. $f_p = f_d/\sqrt{2}$ and f_d is the maximum Doppler frequency defined by

$$f_d = \frac{v}{c} f_c, \tag{10}$$

where v is the vehicle speed in m/sec, c is the propagation velocity (3×10^8 m/sec), and f_c is the central carrier frequency. The radius, r_d, is chosen very close to 1 to model the spectral peaks at the maximum Doppler frequency of the fading process.

Hence, the process model for estimating a Rayleigh fading channel in the Kalman filter is expressed by

$$\begin{aligned} \mathbf{x}(k+1) &= \left[\begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \otimes \mathbf{I}_{N_x \times N_x}\right]\mathbf{x}(k) + \begin{bmatrix} \mathbf{I}_{N_x \times N_x} \\ \mathbf{0}_{N_x \times N_x} \end{bmatrix}\mathbf{w}(k) \\ &= \mathbf{A}\mathbf{x}(k) + \mathbf{G}\mathbf{w}(k), \end{aligned} \tag{11}$$

where $\mathbf{x}(k)$ is defined in Eq. (4). $\mathbf{A}$ and $\mathbf{G}$ are $\in \mathbb{C}^{2N_x \times 2N_x}$, and $\mathbf{w}(k)$ is $\in \mathbb{C}^{N_x}$. The covariance matrix of the process noise is

$$\mathbf{Q} = \begin{bmatrix} \sigma_{w_0}^2 \mathbf{I}_{N_x \times N_x} & \mathbf{0}_{N_x \times N_x} \\ \mathbf{0}_{N_x \times N_x} & \mathbf{0}_{N_x \times N_x} \end{bmatrix} \tag{12}$$

The Kalman channel estimator in information form is described in Table 1

DATA SYMBOL DETECTION

The receiver design is shown in Figure 1. In order to recover the data $\mathbf{b}(k)$, $N_r \geq N_t$ receive antennas are thus needed. The maximum number of jammers that can be spatially rejected

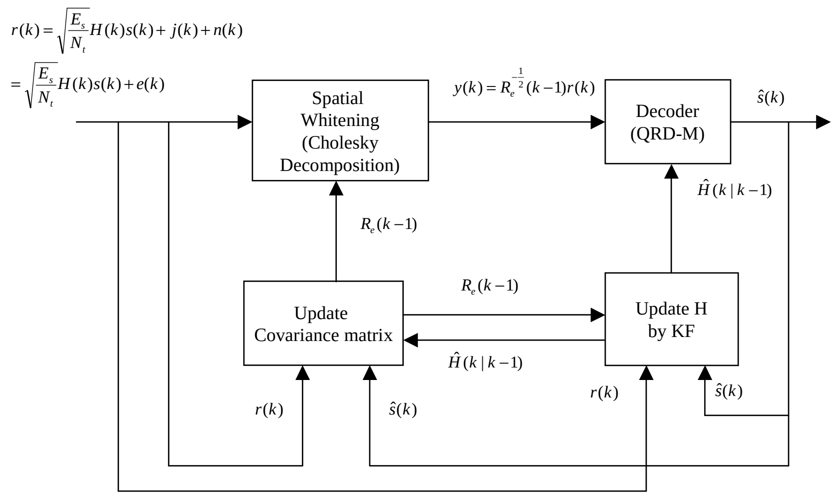


Figure 1: Block diagram of the spatial multiplexing receiver

is $N_J = N_r - 1$, hence for $N_J < N_r - 1$, additional temporal jammer suppression may be unnecessary. Before decoding the data symbols, a spatial whitening filter is first applied to the received signal vector. Let $\hat{\mathbf{L}}(k-1)$ be the Cholesky decomposition square root of the estimated covariance matrix of jammer and noise, $\hat{\mathbf{R}}_e(k-1)$ in eq. (7).

Define the transformed signal as

$$\begin{aligned}
\mathbf{y}(k) = \hat{\mathbf{L}}^{-1}(k-1)\mathbf{r}(k) &= \sqrt{\frac{E_s}{N_t}}\hat{\mathbf{L}}^{-1}(k-1)\mathbf{H}(k)\mathbf{b}(k) + \mathbf{v}(k) \\
&= \sqrt{\frac{E_s}{N_t}}\mathbf{H}_w(k)\mathbf{b}(k) + \mathbf{v}(k), \qquad (13)
\end{aligned}$$

where $\mathbf{H}_w(k) = \hat{\mathbf{L}}^{-1}(k-1)\mathbf{H}(k)$ and $E\{\mathbf{v}(k)\mathbf{v}(k)^H\} \approx \mathbf{I}$. The exact ML decision based on the channel estimate is given by

$$\hat{\mathbf{b}}_{ML}(k) = \arg\min_{\mathbf{b}(k)\in\mathcal{A}^{N_t}} \left\| \mathbf{y}(k) - \sqrt{\frac{E_s}{N_t}}\mathbf{H}_w(k)\mathbf{b}(k) \right\|^2 \qquad (14)$$

The size $\mathcal{M}$ QAM signal constellation is $\mathcal{A} \subset \mathbb{CZ}^{\mathcal{M}}$.

At this point, we could separate $\mathbf{y}(k)$ and $\mathbf{H}_w(k)$ into real and imaginary parts and apply the sphere decoding algorithms in a straightforward manner [16][6][5][7][17]. However, the QRD-M algorithm is considered as an alternative, which like SD, uses the QR decomposition of $\hat{\mathbf{H}}_w$, but without the need to convert to real vectors first. The complex QR decomposition is $\mathbf{H}_w(k) = \mathbf{QR}$ where $\mathbf{R}$ is upper triangular and has real diagonal entries, and $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$. Then define

$$\mathbf{y}_{QR}(k) = \mathbf{Q}^H\mathbf{y}(k) \approx \mathbf{R}\mathbf{b}(k) + \mathbf{v}'(k). \qquad (15)$$

Now the ML decision is expressed as

$$\hat{\mathbf{b}}_{ML}(k) = \arg\min_{\mathbf{b}(k)\in\mathcal{A}} \sum_{i=1}^{N_t} \left|\left|\mathbf{y}_{QR,i}(k) - \sum_{j=i}^{N_t} \mathbf{R}_{i,j}\mathbf{b}_j(k)\right|\right|^2 . \quad (16)$$

The metric decomposition in (16) is similar to that used for sphere decoding in [6], but the QRD-M algorithm again uses complex measurements directly, and has fixed search complexity.

The idea in the QRD-M algorithm is to first reorder the columns of $\hat{\mathbf{H}}_w$ so as to heuristically increase the reliability of earlier decisions. Thus define the re-ordered matrix

$$\mathbf{H}_w^{\pi} = [\mathbf{H}_w(:,(1))\ldots\mathbf{H}_w(:,(N_t))], \quad (17)$$

where $||\mathbf{H}_w(:,(N_t))||^2 > \ldots > ||\mathbf{H}_w(:,(1))||^2$. The vector $\mathbf{y}_{QR}(k)$ is then redefined as $\mathbf{Q}^H\mathbf{H}_w^{\pi}$. The QRD-M algorithm for $M = 1$ reduces to decision-feedback, or an $M = 1$ tree search, with the first decision

$$\hat{\mathbf{b}}_{(N_t)} = \arg\min_{b\in\mathcal{A}} ||\mathbf{y}_{QR,N_t} - \mathbf{R}_{N_t,N_t}b||^2, \quad (18)$$

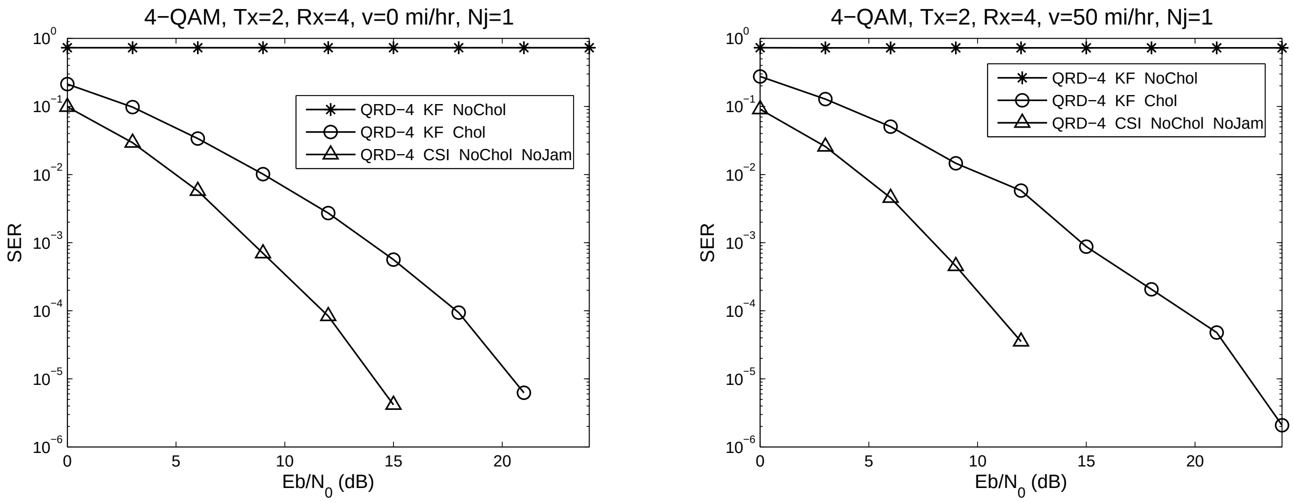
and for $i = N_t - 1, \ldots, 1$

$$\hat{\mathbf{b}}_{(i)} = \arg\min_{b\in\mathcal{A}} ||\mathbf{y}_{QR,i} - \mathbf{R}_{i,i}b - \sum_{j=i+1}^{N_t} \mathbf{R}_{i,j}\hat{\mathbf{b}}_{(j)}||^2.$$

For arbitrary M, the QRD-M algorithm corresponds to a tree search with M branches retained at each level. For convenience, $\min^M\{x_1,\ldots,x_N\}$ indicates the M smallest elements of a set. Each symbol $b \in \mathcal{A}$ takes on one of $|\mathcal{A}|$ values in $\mathbb{C}$, with k-th value denoted by b^k. The set $\tilde{\mathbf{b}}^j$ indicates the tentative decisions corresponding to the j-th of M paths through the tree. At level i, the cumulative path j is thus $\tilde{\mathbf{b}}_i^j, \tilde{\mathbf{b}}_{i+1}^j, \ldots, \tilde{\mathbf{b}}_{N_t}^j$. The extension of path j to symbol b^k is denoted by (k,j). The metrics $m(1),\ldots,m(M)$ correspond to the M smallest Euclidean metrics at a given level. The QRD-M algorithm is summarized in Table 2.

SIMULATION RESULTS AND CONCLUSION

The spatial multiplexing system has N_r receive and N_t transmit antennas, where $N_r \geq N_t$. There is one narrowband jammer throughout the simulation. The generation of Rayleigh fading channels is based on the Jakes model [15] with parameters f_c and $1/T$ corresponding to a carrier frequency 800 MHz and symbol sampling rate 25 kb/s. For the AR2 model in the Kalman filter, r_d is chosen to be 0.998 as specified in [13]. Pilot symbols are inserted to reduce filter divergence. The symbol error rate (SER) at each SNR value is averaged over



(a) (b)

Figure 2: One jammer, QPSK, T2R4 (a) velocity = 0 mi/hr (b) velocity = 50 mi/hr

200,000 Monte-Carlo runs. Figure 2 (a) presents simulation results when the vehicle is static, Figure 2(b) shows the result when the vehicle moves at a speed of 50 mi/hr. The symbol error rate is also seen to approach 1/2 for the Kalman filter/QRD-M algorithm without the spatial whitening filter. Our proposed approach mitigates jamming signal and makes data detection possible. The benchmark error rate for the QRD-M algorithm and known channel state information at the receiver is also given.

In summary, the proposed SM receiver design has robust detection performance in high-power interference. For large numbers of transmit elements, and/or large QAM constellations, the QRD-M algorithm provides an effective tradeoff between decoding performance and computational complexity.

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Given the state prediction $\hat{\mathbf{x}}(k|k-1)$,

and the error covariance matrix $\mathbf{P}(k|k-1)$.

The state prediction is defined by

$$\hat{\mathbf{x}}(k|k-1) = \begin{bmatrix} vec\{\hat{\mathbf{H}}(k|k-1)\} \\ vec\{\hat{\mathbf{H}}(k-1|k-2)\} \end{bmatrix}$$

The received signal vector $\mathbf{r}(k)$

Use training sequence $\mathbf{b}(k)$ or

the decision $\hat{\mathbf{b}}(k)$ from the QRD-M algorithm

The measurement noise $\mathbf{e}(k)$ is approximated by

$$\mathbf{e}(k) = \mathbf{r}(k) - \sqrt{\frac{E_s}{N_t}}\hat{\mathbf{H}}(k|k-1)\hat{\mathbf{b}}(k)$$

Update noise covariance estimate

$$\hat{\mathbf{R}}_e(k) = \frac{k-1}{k}\hat{\mathbf{R}}_e(k-1) + \frac{1}{k}\mathbf{e}(k)\mathbf{e}(k)^H$$

Compute the effective Jacobian matrix

$$\mathbf{J}(k) = \begin{bmatrix} \hat{\mathbf{b}}(k)^T \otimes \mathbf{I} & \mathbf{0} \end{bmatrix}$$

Update KF error covariance matrix

$$\mathbf{P}(k|k)^{-1} = \mathbf{P}(k|k-1)^{-1} + \mathbf{J}(k)^H\hat{\mathbf{R}}_e(k)^{-1}\mathbf{J}(k)$$

Measurement update

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{P}(k|k)\mathbf{J}(k)^H\hat{\mathbf{R}}_e(k)^{-1}\mathbf{e}(k)$$

One-step predictions

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{A}\hat{\mathbf{x}}(k|k)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}\mathbf{P}(k|k)\mathbf{A}^H + \mathbf{Q}$$

Extract Channel Estimates

$$vec(\hat{\mathbf{H}}(k+1|k)) = \hat{\mathbf{x}}(k+1|k)(1:N_rN_t, 1)$$

Table 1: KF Channel Estimator using AR2

Given $\hat{\mathbf{H}}(k|k-1)$ from KF and $\hat{\mathbf{R}}_e(k-1)$

Let $\hat{\mathbf{L}}^{-1}(k-1)$ be the Cholesky decomposition

  square root of $\hat{\mathbf{R}}_e(k-1)$

Receive $\mathbf{r}(k)$. Spatially whiten $\mathbf{r}(k)$

  $\mathbf{y}(k) = \hat{\mathbf{L}}^{-1}(k-1)\mathbf{r}(k)$

Compute and reorder to obtain $\hat{\mathbf{H}}_w^{\pi}(k|k-1)$

  $\hat{\mathbf{H}}_w(k|k-1) = \hat{\mathbf{L}}^{-1}\hat{\mathbf{H}}(k|k-1)$

  Order in terms of increasing column norm to get $\hat{\mathbf{H}}_w^{\pi}$

Perform QR decomposition $\hat{\mathbf{H}}_w^{\pi} = \mathbf{QR}$

Set $i = N_t$

  $\{mp(1), \ldots, mp(M)\} = \min_{b\in\mathcal{A}}^{M} ||\mathbf{y}_{QR,N_t} - \mathbf{R}_{N_t,N_t} b||^2$

  $\{\tilde{\mathbf{b}}_{N_t}^{1}, \ldots, \tilde{\mathbf{b}}_{N_t}^{M}\} = \arg\min_{b\in\mathcal{A}}^{M} ||\mathbf{y}_{QR,N_t} - \mathbf{R}_{N_t,N_t} b||^2$

For $i = N_t - 1, \ldots, 1$

  For path $j = 1, 2, \ldots, M$

    For symbol $k = 1, 2 \ldots, |\mathcal{A}|$

      $mc(k,j) = ||\mathbf{y}_{QR,i} - \mathbf{R}_{i,i} b^k - \sum_{q=i+1}^{N_t} \mathbf{R}_{i,q} \tilde{\mathbf{b}}_q^j||^2 + mp(j)$

      $\tilde{\mathbf{b}}^{k,j} = \{b^k, \tilde{\mathbf{b}}_{i+1}^j, \ldots, \tilde{\mathbf{b}}_{N_t}^j\}$

    Next k

  Next path j

  Prune to M paths with smallest metrics

    $\{mp(1) \ldots mp(M)\} = \min^{M}\{mc(k,j)\}$

    $\{(k',j')_1, \ldots, (k',j')_M\} = \arg\min_{k,j}^{M}\{mc(k,j)\}$

    $\{\tilde{\mathbf{b}}^1, \ldots,$ “$\tilde{\mathbf{b}}^M\} = \{\tilde{\mathbf{b}}^{(k',j')_1} \ldots \tilde{\mathbf{b}}^{(k',j')_M}\}$

Next level i

Table 2: QRD-M Algorithm

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