The Sum-Rate Capacity of a Cognitive Multiple Access Sensor Network

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ABSTRACT

This paper investigates the sum-rate capacity of a cognitive multiple access (MAC) sensor network. The multiple access network consists of K sensors communicating to a common base station. Outside of the network exists another user of the radio spectrum. Each sensor of the MAC network is aware (i.e. cognitive) of this user, denoted the *primary user*, and transmits in a manner to avoid any interference to this user. No interference transmission is achieved using the *dirty-paper coding* technique. The sum-rate capacity is the theoretical maximum of the sum of the simultaneously achievable rates of each sensor within the network. Using a recently derived iterative algorithm, we quantify the sum-rate capacity of this network and investigate its behavior as a function of the number of sensors, cognitive signal-to-noise ratio (CSNR) and primary SNR (PSNR) in a Rayleigh fading environment. We also derive bounds and scaling results for the ergodic sum-rate capacity.

KEY WORDS

Cognitive systems, channel capacity, wireless communications, sensor network

1 Introduction

Cognitive radio is a developing field of research in the area of wireless communication theory. A cognitive radio monitors its environment to identify portions of the spectrum that are desirable for transmission. It then adapts its modulation to take advantage of the identified band. The ability to dynamically alter transmission to match the environment makes cognitive radio a desirable technology for military applications with unknown or hostile propagation environments, as well as typical consumer applications where available spectrum is scarce.

The concept of cognitive radio was first introduced in [1] as a technology that would enable dynamic and distributed spectrum allocation, as opposed to static and centralized policies currently enforced by regulatory agencies. Under the cognitive radio paradigm, local users, not governmental agencies, would decide their operating frequency band. A primary goal of cognitive radio is to allow users that observe an unused frequency band to use it if needed, and therefore improve overall system efficiency. Essentially, unused spectrum is considered wasteful if other users need an operating band to transmit on.

Allowing users to determine their operating band does reduce the amount of "wasted" spectrum, but introduces a host of other technical challenges [2]. One common way of approaching this problem is to assign users of the network to different classes. Primary users are the class of users that "own" the band and have priority in accessing the spectrum. A primary user is often the legacy user of the band and does not have cognitive sensing capabilities. Secondary users are the class of cognitive users that are allowed to operate as long as they do not create interference to the primary user. Thus, secondary users must continually monitor (i.e. be cognitive of) the environment to ensure the primary user has not started transmitting again. Designing distributed protocols for secondary users is a challenging area of research.

This paper investigates fundamental limits of a cognitive sensor network operating in the presence of a primary user. The cognitive sensor network is a multiple-access channel (MAC) where K sensors transmit to a single base station. The sensors are "cognitive" as they are aware of the data being transmitted by the primary user and transmit their own data to avoid interference. We use the iterative algorithm first presented in [3] to quantify the sum-rate capacity of this network as a function of cognitive sensor signal-to-noise ratio (SNR), primary user SNR, and number of sensors. Section 2 introduces the system model and capacity regions for the primary and secondary networks. Section 3 derives ergodic sum-rate capacity upper and lower bounds and investigates the scaling behavior of the cognitive sensor network. Numerical results for the sum-rate capacity are presented in Section 4 and a conclusion is given in Section 5.

2 System Model and Capacity

The system under consideration consists of two distinct networks. A set of K cognitive sensors transmitting to basestation B_c form the *secondary network*. The sensors operate in the presence of a primary user transmitting to a receiving station B_p that forms the *primary network*. Sensors of the secondary network operate under two special constraints. They are required to transmit without reducing the capacity of the primary user, and transmit without requiring the primary user to modify any behavior. These constraints ensure the primary network is not effected by the cognitive sensor network. The primary user transmits messages $m_p \in \{0, 1, \ldots, 2^{nR_p}\}$ at rate R_p, and the secondary users transmit messages $m_i \in \{0, 1, \ldots, 2^{nR_i}\}$ at rate R_i for $i = 1, \ldots, K$. The primary user has average power constraint P_p and the secondary users have average power constraint P_i for $i = 1, \ldots, K$.

The following sections describe the primary and secondary network channel models and capacity regions. This system model and capacity characterization was first provided in [3]. We follow their development closely but expound on several points. We adopt their discrete time notation here as well.

2.1 Secondary Network Channel Model

The received signal at B_c at time n is

$$Y^n = \sum_{k=1}^{K} h_k \cdot X_k^n + f \cdot X_p^n + Z^n, \tag{1}$$

where X_k^n for $k = 1, \ldots, K$ is the codeword transmitted by the kth sensor, h_k is the path gain between the kth sensor and B_c, X_p^n is the codeword transmitted by the primary user, f is the path gain between the primary user and B_c, and Z^n is additive white Gaussian noise (AWGN) of variance σ_c^2.

Recall that communication by users of the secondary network must not impact the primary network. As shown in [4], the optimal strategy for the secondary users is a mixture of dirty paper coding and *cooperative communication*. Dirty paper coding [5] is a coding technique for known interference environments. The analogy of "writing on dirty paper" is used to demonstrate that when interference is known, the transmitted signal can be adapted to avoid interference.

We assume the primary user messages are available to the secondary users. This enables the secondary users to communicate without interfering with the primary user via dirty paper coding. In practice, the primary user messages are not known to the secondary users, thus, the capacity region calculated in the following section represents an upper bound. Other techniques for achieving sum-rate capacity may be interesting to pursue [6]. The transmitted signal of the kth cognitive sensor is

$$X_k^n = \hat{X}_k^n + \gamma_k \sqrt{\frac{P_k}{P_p}} X_p^n \tag{2}$$

where $\hat{X}_k^n = f_{dpc}(m_p, m_k)$ is the dirty-paper coded message of the kth cognitive sensor based on the current message of the primary user and the second part of equation (2) is the cooperative transmission term. Each cognitive sensor re-transmits the primary users transmitted signal X_p^k to ensure the rate of the primary transmission is met. The amount of cooperation is controlled by the *cooperation coefficient* γ_k for each user. Recall the power transmitted by cognitive sensor k is P_k. Thus, the power of $\hat{X}_k^n$ is $(1 - \gamma_k^2)P_k$.

2.2 Secondary Network Channel Capacity

Dirty-paper coding of the cognitive sensor messages ensures information transmitted by the cognitive sensor network does not interfere with the primary network. Thus, no interference is created by the primary user in the secondary network. The channel model of the secondary network is simply a Gaussian multiple-access channel with sum-rate capacity of

$$C_{sum} = \frac{1}{2}\log\left(1 + \frac{\sum_{k=1}^{K}(1-\gamma_k^2)h_k^2 P_k}{\sigma_c^2}\right). \tag{3}$$

The sum-rate capacity is maximized for $\gamma_k = 0$ for $k = 1, \ldots, K$. However, these cooperation coefficients must be chosen properly to ensure the primary user capacity is not impacted. Constraints on γ_k are developed in the following section.

2.3 Primary Network Channel Model

The received signal at B_p at time n is

$$Y_p^n = h_p \cdot X_p^n + \sum_{k=1}^{K} g_k \cdot X_k^n + Z_p^n, \tag{4}$$

where X_p^n is the codeword transmitted by the primary user, h_p is the path gain of the primary user, g_k for $k = 1 \ldots, K$ are the path gains between the sensors and B_p, and Z_p^n is additive white Gaussian noise of variance σ_p^2.

Substituting equation (2) into equation (4) yields

$$Y_p^n = X_p^n\left(h_p + \sum_{k=1}^{K}\sqrt{\frac{P_k}{P_p}}\gamma_k g_k\right) + \sum_{k=1}^{K} g_k \hat{X}_k^n + Z_p^n, \tag{5}$$

which will be a useful form for determining the capacity of this channel in the following section.

2.4 Primary Network Channel Capacity

With $K = 0$ sensors, the channel model of (4) reduces to the standard additive white Gaussian noise (AWGN) channel with fading, i.e.

$$Y_p^n = h_p \cdot X_p^n + Z_p^n. \tag{6}$$

The channel capacity for a given channel h_p is easily calculated as

$$C_P^{K=0} = \frac{1}{2}\log_2\left(1 + \frac{h_p^2 P_p}{\sigma_p^2}\right). \tag{7}$$

The capacity of the channel for $K > 0$ can be written from inspection from equation (5). The power of the received codeword is $\left(h_p\sqrt{P_p} + \sum_{k=1}^{K}\gamma_k g_k\sqrt{P_k}\right)^2$. The noise component has power $\sum_{k=1}^{K} g_k^2(1-\gamma_k^2)P_k + \sigma_p^2$. The capacity of the primary network for $K > 0$ is

$$C_p^{K>0} = \frac{1}{2}\log_2\left(1 + \frac{\left(h_p\sqrt{P_p} + \sum_{k=1}^{K}\gamma_k g_k\sqrt{P_k}\right)^2}{\sum_{k=1}^{K} g_k^2(1-\gamma_k^2)P_k + \sigma_p^2}\right). \tag{8}$$

The cognitive sensors are not allowed to impact the capacity region of the primary user, thus, we equate $C_P^{K=0} = C_P^{K>0} \equiv C_P$ yielding the desired equality

$$\frac{h_p^2 P_p}{\sigma_p^2} = \frac{\left(h_p\sqrt{P_p} + \sum_{k=1}^{K} g_k\gamma_k\sqrt{P_k}\right)^2}{\sigma_p^2 + \sum_{k=1}^{K} g_k^2(1-\gamma_k^2)P_k}. \tag{9}$$

Our goal is to maximize the sum-rate capacity while satisfying this constraint.

3 Sum-Rate Capacity Bounds

The algorithm proposed in [3] calculates the cooperative coefficients, γ_k for $k = 1,\ldots,K$, to maximize the sum-rate capacity while satisfying (9). The iterative nature of this algorithm makes it difficult to characterize the behavior of the cooperation coefficients, and thus provide analytic results for the sum-rate capacity. In this section we derive lower and upper bounds for the sum-rate capacity. We assume each secondary user has the same transmitted power, i.e. $P_k = P_c$ for $k = 1,\ldots,K$. The cognitive SNR (CSNR) and primary SNR (PSNR) are defined as $\rho_c = \frac{P_s}{\sigma_c^2}$ and $\rho_p = \frac{P_p}{\sigma_p^2}$ respectively. Functions and definitions not formally defined in this section are included in the appendix.

In deriving the bounds we define a *cooperative user* as any cognitive sensor operating in a fully cooperative manner, i.e. $\gamma_k = 1$. These sensors contribute to the data rate of the primary user and ensure his capacity is supported. We define *non-cooperative* users as cognitive sensors that do not cooperate, i.e. $\gamma_k = 0$. These sensors contribute to the sum-rate capacity. We define K_1 as the number of cooperative users and K_2 as the number of non-cooperative users. We begin by stating the definition of outage capacity.

Definition 3.1. *The ϵ-outage capacity is the data rate R that can be supported $1-\epsilon$ percent of the time, i.e. we have*

$$P(C \leq R) = 1 - \epsilon \tag{10}$$

The strategy for calculating the sum-rate capacity bounds is outlined as follows: 1) Determine the outage capacity of the primary user. 2) Determine the number of cooperative users (K_1) required to support a desired primary user outage capacity. 3) Calculate the sum-rate capacity distribution for $K_2 = K - K_1$ non-cooperative users. The following lemma characterizes the primary users outage capacity.

Lemma 3.2. *The ϵ-outage capacity of the primary user is*

$$C_{p,\epsilon} = \frac{1}{2}\log_2(1 - \log(\epsilon)\rho_p) \tag{11}$$

Proof.

$$P(C_P \le C_{p,\epsilon}) = P\left(\frac{1}{2}\log_2\left(1+h_p^2\rho_P\right) \le C_{p,\epsilon}\right) = P\left(h_p^2 \le \frac{2^{2C_{p,\epsilon}}-1}{\rho_p}\right). \quad (12)$$

The random variable h_p^2 ~Exp(1) according to Remark 6.1. Thus,

$$P\left(h_p^2 \le \frac{2^{2C_{p,\epsilon}}-1}{\rho_p}\right) = E\left(\frac{2^{2C_{p,\epsilon}}-1}{\rho_p}, 1\right) = 1 - e^{-\frac{2^{2C_{p,\epsilon}-1}}{\rho_p}} \equiv 1-\epsilon. \quad (13)$$

$\Rightarrow e^{-\frac{2^{2C_{p,\epsilon}-1}}{\rho_p}} = \epsilon \Rightarrow 2^{2C_{p,\epsilon}} - 1 = \log(\epsilon)\rho_p \Rightarrow C_{p,\epsilon} = \frac{1}{2}\log_2(1-\log(\epsilon)\rho_p)$. □

We now wish to calculate how many cooperative users are needed to accomodate some $C_{p,\epsilon}$. The following theorem provides this information.

Theorem 3.3. *The number of cooperative users required to support the primary user capacity is at most, the smallest K_1 such that the following inequality is satisfied*

$$1 - \frac{(2^{2C_{p,\epsilon}}-1)^{K_1}\Gamma(K_1)_2\tilde{F}_1(K_1, K; K_1+1; 1-2^{2C_{p,\epsilon}})}{B(K_1, K-K_1)} \ge \zeta, \quad (14)$$

for values of ζ close to but less than one.

Proof. The primary user capacity can be approximated as

$$C_P^{K>0} = \frac{1}{2}\log_2\left(1 + \frac{\left(h_p\sqrt{P_p} + \sqrt{P_c}\sum_{i=1}^{K_1} g_i\right)^2}{P_c\sum_{j=1}^{K_2} g_j^2 + \sigma_p^2}\right) \quad (15)$$

$$\approx \frac{1}{2}\log_2\left(1 + \frac{0 + P_c\sum_{i=1}^{K_1} g_i^2}{P_c\sum_{j=1}^{K_2} g_j^2 + 0}\right) = \frac{1}{2}\log_2\left(1 + \frac{\sum_{i=1}^{K_1} g_i^2}{\sum_{j=1}^{K_2} g_j^2}\right). \quad (16)$$

where the channel gains are partitioned according to cooperative ($i = 1, \ldots, K_1$) and non-cooperative ($j = 1, \ldots, K_2$) cognitive sensors. Also, it is assumed that the primary user does not contribute to the rate by setting $h_p\sqrt{P_p} = 0$ in the approximation, and that cross terms of the squared-summation are also zero. Both of these approximations increase K_1 needed to support the primary user capacity.

The distributions of $W \equiv \sum_{i=1}^{K_1} g_i^2 \sim$ Gamma$(K_1, 1)$, $Z \equiv \sum_{j=1}^{K_2} g_j^2 \sim$ Gamma$(K_2, 1)$, and W/Z ~BetaPrime(K_1, K_2) all follow from Remark 6.1. Then,

$$\begin{aligned}
P(C_p^{K>0} \ge C_{P,\epsilon}) &= 1 - P(C_p^{K>0} \le C_{p,\epsilon}) \\
&= 1 - P\left(\frac{1}{2}\log_2(1+W/Z) \le C_{p,\epsilon}\right) \\
&= 1 - P\left(W/Z \le 2^{2C_{p,\epsilon}} - 1\right) \\
&= 1 - F(2^{2C_{p,\epsilon}} - 1, K_1, K_2) \\
&= 1 - \frac{(2^{2C_{p,\epsilon}}-1)^{K_1}\Gamma(K_1)_2\tilde{F}_1(K_1, K; K_1+1; 1-2^{2C_{p,\epsilon}})}{B(K_1, K-K_1)} \quad (17)
\end{aligned}$$

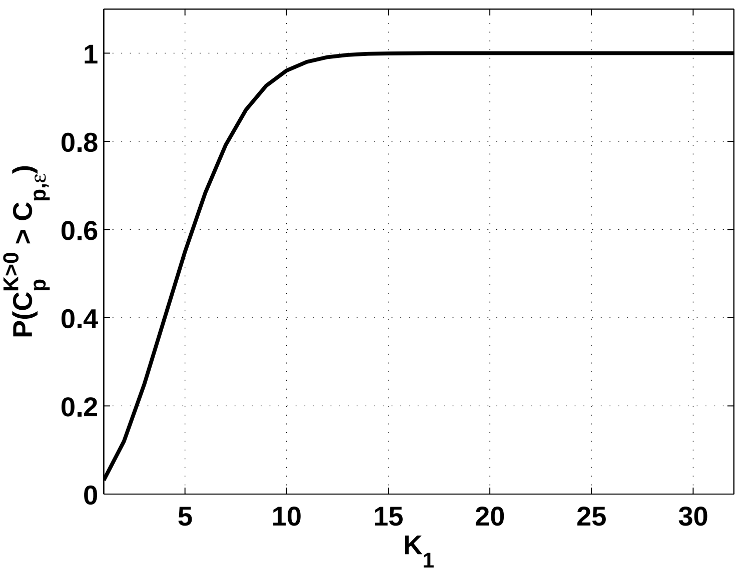

Figure 1: $P(C_p^{K>0} \geq C_{P,\epsilon})$ versus K_1 for $K = 32$, $\epsilon = 0.01$, $\zeta = 0.99$, $\rho_p = \rho_c = 10$ dB.

This expression can then be evaluated to determine the value of K_1 such that

$$P(C_p^{K>0} \geq C_{P,\epsilon}) \geq \zeta \tag{18}$$

□

The number of cooperative users required to satisfy (9) with some arbitrary probability can now be calculated. If the constraint is satisfied with $K_1 < K$ cooperative users, then the remaining non-cooperative $K_2 = K - K_1$ users contribute to the sum-rate capacity of (3). Once again, the cognitive users contributing to the sum-rate capacity all have $\gamma_k = 0$, so this is in general a sub-optimal solution. However, results in the following section show this lower bound is reasonably tight.

As an example consider Figure 1 where Equation (17) versus K_1 is plotted for $K = 32, \rho_c = 10$ dB, $\rho_p = 10$ dB, $\epsilon = 0.01$, and $\zeta = .99$. This figure shows that for $K_1 \geq 15$, the probability of supporting the primary users 0.01-outage capacity 99% of the time is approximately one. Thus, approximately $K_2 = 32 - 15 = 17$ cognitive users can contribute to the sum-rate capacity. Figure 2 shows the behavior of K_1 and K_2 as a function of K for the same parameters. This figure shows the approximate linear increase in the number of cognitive users as K increases for sufficiently large K. The next theorem characterizes the distribution of the sum-rate capacity for N non-cooperative users.

Theorem 3.4. *For N non-cooperative users the sum-rate capacity cumulative distribution function is*

$$F^N_{C_{sum}}(x) = G\left(\frac{2^{2x} - 1}{\rho_c}, N, 1\right). \tag{19}$$

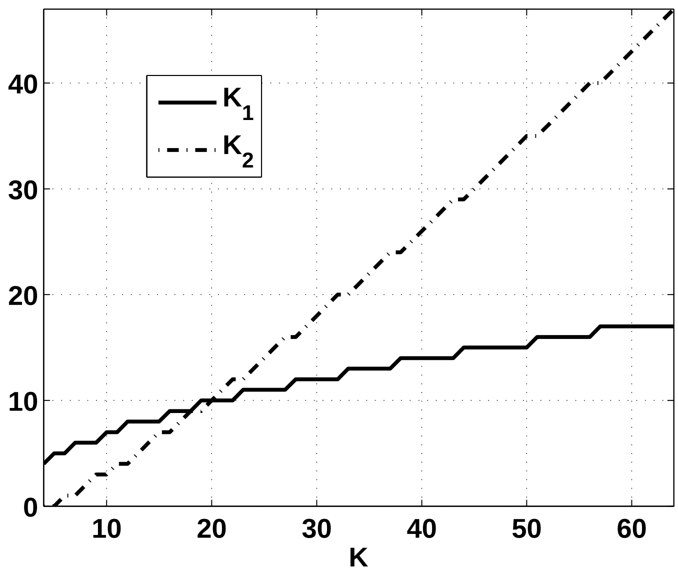

Figure 2: Cooperative (K_1) and Non-Cooperative (K_2) Users vs. K Calculated from Theorem 3.3.

Proof. The sum-rate capacity of equation (3) with N non-cooperative users reduces to

$$C_{sum} = \frac{1}{2}\log\left(1 + \rho_c \sum_{i=1}^{N} h_i^2\right). \tag{20}$$

$$P(C_{sum} \le x) = P\left[\frac{1}{2}\log\left(1 + \rho_c \sum_{i=1}^{N} h_i^2\right) \le x\right] = P\left(\sum_{i=1}^{N} h_i^2 \le \frac{2^{2x}-1}{\rho_c}\right). \tag{21}$$

From Remark 6.1 we have $\sum_{i=1}^{N} h_i^2 \sim Gamma(N, 1)$. So,

$$P\left(\sum_{i=1}^{N} h_i^2 \le \frac{2^{2x}-1}{\rho_c}\right) = G\left(\frac{2^{2x}-1}{\rho_c}, N, 1\right) \tag{22}$$

Figure 3 shows the sum-rate capacity distribution as a function of the number of non-cooperative users. □

Remark 3.5. *A lower bound on the sum-rate capacity distribution can be calculated by letting $N = K_2 = K - K_1$ where K_1 is calculated from Theorem 3.3. Recall that K_1 calculated from this theorem is an upper bound on the number of cooperative users, thus providing a lower bound on the K_2 non-cooperative users that contribute to the sum-rate capacity. An upper bound on the sum-rate capacity distribution can be calculated by letting $N = K$.*

Theorem 3.6. *The ergodic (i.e. mean) sum-rate capacity for K cognitive sensors with CSNR ρ_c is bounded by*

$$\frac{1}{2}\log_2(\rho_c(K - K_1) + 1) \le C_{sum} \le \frac{1}{2}\log_2(\rho_c K + 1) \tag{23}$$

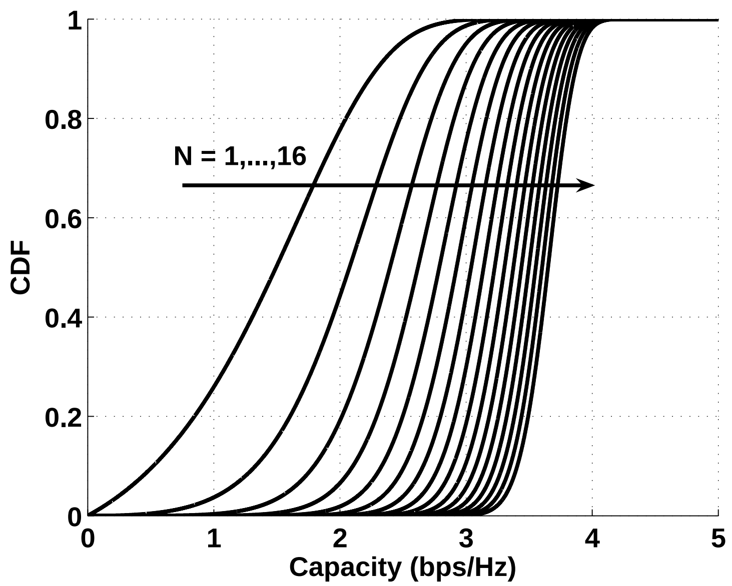

Figure 3: Non-Cooperative User Sum-Rate Capacity Distribution as Function of N. $\rho_c = \rho_p = 10$ dB, $K = 16$ $N = 1, 2 \ldots, 16$.

where K_1 is calculated according to Theorem 3.3.

Proof. From Theorem 3.4 we have

$$F^N_{C_{sum}}(x) = G\left(\frac{2^{2x}-1}{\rho_c}, N, 1\right) \tag{24}$$

$$= \frac{\gamma(N, \frac{2^{2x}-1}{\rho_c})}{\Gamma(N)} \sim \frac{1}{2}\text{erfc}(-\eta\sqrt{N/2}). \tag{25}$$

where $\text{erfc}(\cdot)$ is the complementary error function, $\eta = \pm\sqrt{2(\lambda - 1 - \log(\lambda))}$, $\lambda = \frac{2^{2x}-1}{N\rho_c}$, the positive value of η corresponds to $\lambda > 1$ and the negative value of η corresponds to $\lambda \leq 1$ [7].

For the mean value of the distribution, we need to know the value of x such that $F^N_{C_{sum}}(x) = 0.5$, or, using the error function approximation the value of x such that $\text{erfc}(-\eta\sqrt{N/2}) = 1$. This occurs when the argument of the error function is zero, i.e., when $\lambda - 1 - \log(\lambda) = 0 \Rightarrow \lambda = 1$. Setting $\frac{2^{2x}-1}{N\rho_c} = 1$ and solving for x yields $x = \frac{1}{2}\log_2(\rho_c N + 1)$. The value of N can be selected from Remark 3.5 to yield the stated bounds. □

4 Results

An iterative algorithm for calculating the sum-rate capacity of the cognitive MAC is provided in [3]. This algorithm was used to compute the sum-rate capacity of the cognitive multiple access channel described in Section 2. Numerous Monte Carlo simulations were performed and the capacity was calculated for each random channel draw. The following figures examine the ergodic sum-rate capacity in a Rayleigh fading environment for different values of K, ρ_c, and ρ_p.

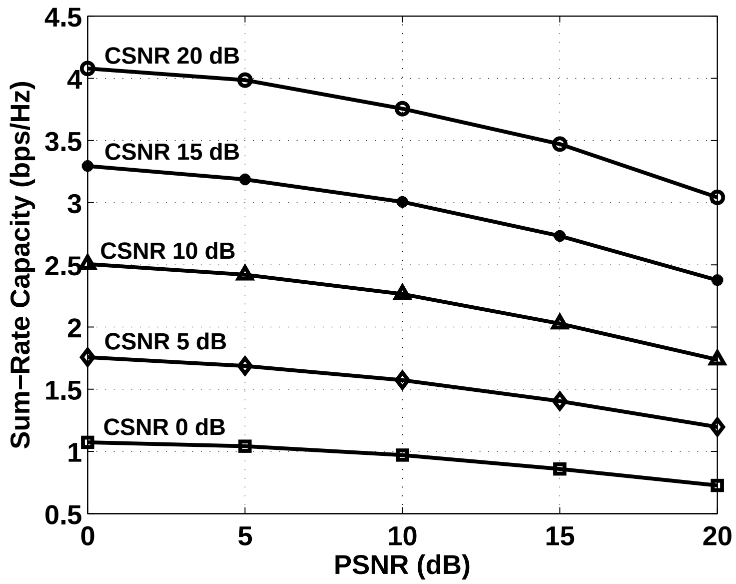

Figure 4: Sum-Rate Capacity As Function of PSNR

Figure 4 plots the sum-rate capacity versus PSNR for $K = 4$ and various CSNR. This figure shows that increasing the primary user PSNR decreases the sum-rate capacity and that sum-rate capacity increases with CSNR. Results for different values of K are similar.

Figure 5 plots the sum-rate capacity versus CSNR for $K = 4$ and various PSNR. This figure shows the sum-rate capacity linear increase in CSNR as predicted by Theorem 3.6.

Figure 6 plots the sum-rate capacity versus K for $\rho_p = 0$ dB and various ρ_c. This figure demonstrates the sum-rate capacity logorithmic increase with K as predicted by Theorem 3.6. Results for different values of ρ_p show similar trends.

Figure 7 plots the ergodic sum-rate capacity obtained via simulation, the exact bounds by using $G(\cdot)$ and Remark 3.5, and the approximate bound of Theorem 3.6. This figure shows that the bounds are within approximately 10% of the true value and the error function approximation used in Theorem 3.6 introduces little error.

5 Conclusion

This paper has investigated the sum-rate capacity of a cognitive multiple access channel. A recently proposed iterative algorithm [3] was implemented and used to quantify the sum-rate capacity as a function of the number of sensors and signal-to-noise ratio in a Rayleigh fading environment. A variety of new results have been derived including bounds for the ergodic sum-rate capacity. Numerical simulation results demonstrating the capacity of the cognitive MAC channel have been presented. These results demonstrate behavior of the sum-rate capacity as a function of the various channel paramters and validate the derived bounds.

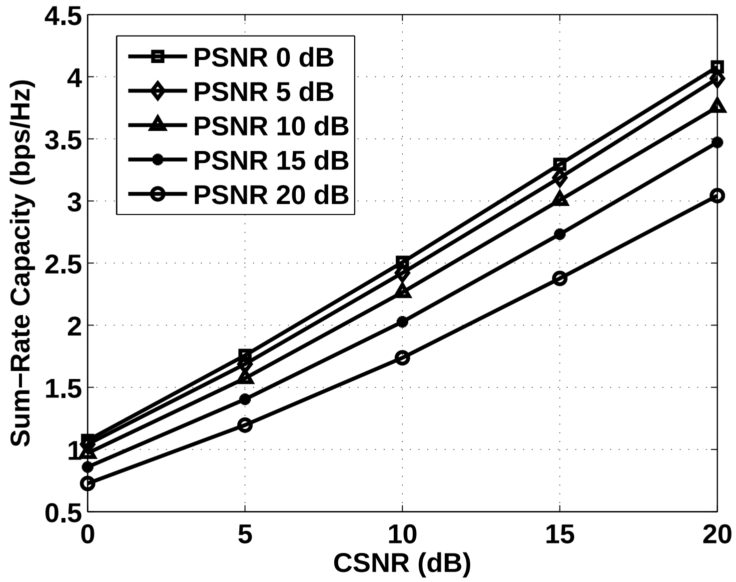

Figure 5: Sum-Rate Capacity As Function of CSNR

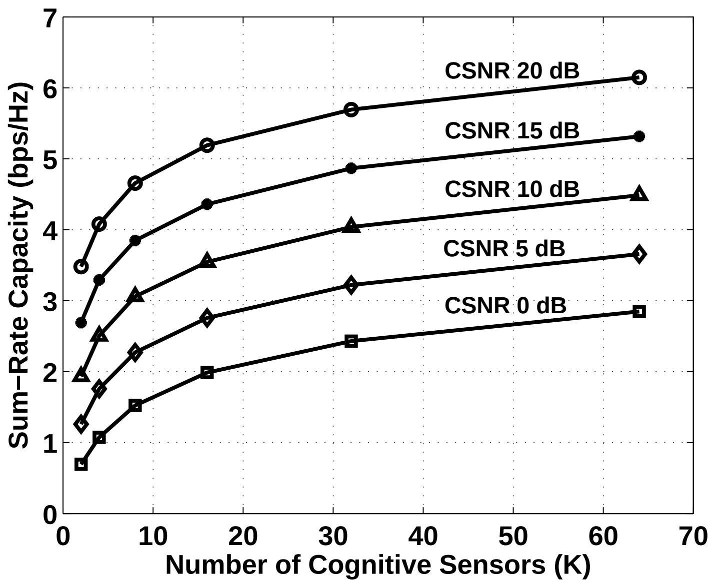

Figure 6: Sum-Rate Capacity As Function of K

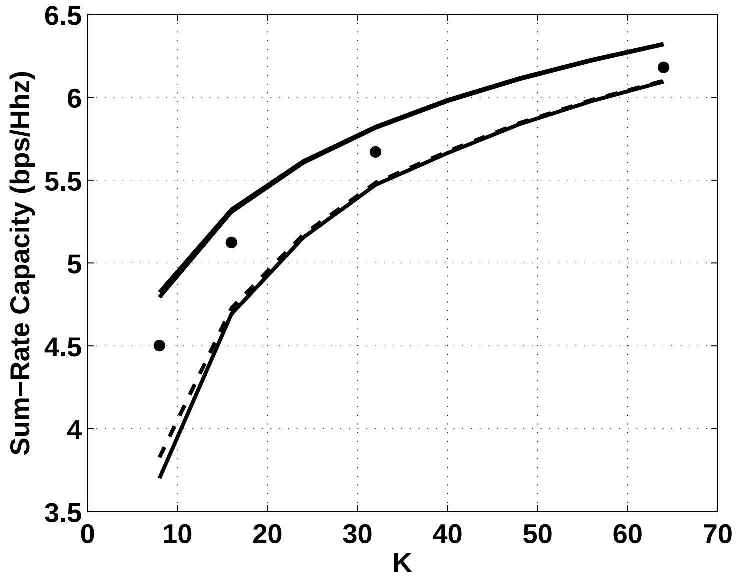

Figure 7: Sum-Rate Capacity Bounds (solid), Approximation (dashed), and Simulated (dots). CSNR = 20 dB, PSNR = 10 dB.

6 Appendix

Remark 6.1 (Distribution Relationships)**.** *Let $R = \sqrt{X^2 + Y^2}$ where $X, Y \sim N(0, \sigma^2)$. Then $R \sim$ Rayleigh(σ) and $R^2 \sim$Exponential$\left(\frac{1}{2\sigma^2}\right)$. Let $X_1, \dots, X_N$ be independent $\sim$Exponential(σ). Then $\sum_{i=1}^{N} X_i \sim$ Gamma(N, σ). Let $W \sim$Gamma(α, σ) and $Z \sim$Gamma(β, σ) be independent. Then $X/Y \sim$ BetaPrime(α, β).*

Definition 6.2. *The incomplete Gamma function is defined as*

$$\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt \tag{26}$$

Definition 6.3. *The Gamma function is defined as*

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt = \gamma(\infty, x) \tag{27}$$

and $\Gamma(n) = (n-1)!$ for positive integers, n.

Definition 6.4. *The cumulative distribution function for an Exponential random variable with parameter λ is defined for $x \geq 0$ as*

$$E(x, \lambda) = 1 - e^{-\lambda x}. \tag{28}$$

Definition 6.5. *The cumulative distribution function for a Gamma random variable with parameters a and b is defined for $x \geq 0$ as*

$$G(x, a, b) = \frac{\gamma(a, x/b)}{\Gamma(a)}. \tag{29}$$

Definition 6.6. *The probability density function for a Beta Prime random variable with parameters a and b is defined for $x \geq 0$ as*

$$f(x) = \frac{x^{a-1}(1+x)^{-a-b}}{B(a, b)} \tag{30}$$

where $B(x, y)$ is the Beta function defined as

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt. \tag{31}$$

Lemma 6.7. *The cumulative distribution function for a Beta Prime random variable with parameters a and b is defined for $x \geq 0$ as*

$$F(x, a, b) = \frac{x^a \Gamma(a) {}_2\tilde{F}_1(a, a+b; a+1; -x)}{B(a, b)} \tag{32}$$

where $\Gamma(\cdot)$ is the Gamma function, $B(\cdot, \cdot)$ is the Beta function, and ${}_2\tilde{F}_1$ is a regularized hypergeometric function.

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