Iterative Decoding and Sparse Channel Estimation for an Underwater Acoustic Telemetry Modem [*]

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ABSTRACT

An acoustic modem employing direct-sequence spread-spectrum (DSSS) signaling is considered with LDPC coding. The underwater acoustic channel is tracked using a Kalman filter which requires accurate data decisions. To improve KF performance and reduce the overall error rate, joint iterative LDPC decoding and channel estimation is proposed based on a factor graph and sum-product algorithm approximation. In this scheme, the decoder posterior log likelihood ratios (LLRs) provide data decisions for the KF. Decoder extrinsic LLRs are similarly incorporated into the detector LLRs to yield improved priors for decoding. Error rate simulations of the overall modem are provided for a shallow-water channel model with Ricean/Rayleigh fading.

KEYWORDS

Underwater acoustic communications, Kalman filtering, iterative decoding, channel estimation.

INTRODUCTION

Previously, we have developed a DSSS underwater acoustic modem employing a Matching Pursuits (MP) channel estimator, and subsequently using Kalman and sparse-Bayesian Kalman filters (KF, SB-KF) [1, 2, 3]. The AquaNode version of the modem [4] with MP

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channel estimation has also been implemented in the TI C6713 device and is currently being tested at the Goleta Pier. DSSS has also been studied by other researchers for acoustic telemetry [5, 6, 7], however with the emphasis on conventional matched-filter and RAKE receivers for channel estimation/detection.

Recently, iterative decoding and estimation using Low Density Parity Check (LDPC) codes has been pursued for terrestrial RF communications [8, 9]. Here, we seek to combine LDPC coding with Kalman-filter channel estimation for the AquaNode DSSS modem. Due to the lower data rates (133 bps) and narrow bandwidth (10 kHz) of our DSSS format, iterative LDPC decoding and estimation is practical in current programmable signal processing hardware, and will allow combinations of increased range and decreased transmit power. However, previous approaches to designing iterative decoders and estimators are somewhat Ad hoc. It has been shown [9, 10] that these iterative approaches can be viewed as approximations to the sum-product algorithm [11] (SPA) implemented on a factor graph. However, cycles in the graph make the SPA suboptimal for joint decoding/estimation. Further, the messages passed on the graph must be simplified to avoid complex integrations of joint density functions. Thus, this paper considers appropriate approximations to make iterative decoding/estimation in the SPA framework practical for the acoustic telemetry application.

SIGNAL MODEL AND OPTIMAL JOINT DECODER/ESTIMATOR

In the AquaNode modem, an 8-ary truncated Gold sequence waveform is transmitted every $T_{sym} = 11$ msec., followed by a $T_{sym} = 11$ msec. time-guard band for channel clearing. The chip rate is $1/T_s = 5$ kHz and the center frequency is 25 kHz, with a Nyquist sampling rate of 10 kHz. The result is an 133 bps data rate spread-spectrum signal. Following synchronization as discussed in [1, 3], received vectors of $2N_s$ Nyquist samples over $2T_{sym}$ sec. are collected. The resulting received signal vector in the DSSS modem is given by [1, 3]

$$\mathbf{r}(k) = \mathbf{S}(\mathbf{c}(k))\mathbf{f}(k) + \mathbf{n}(k). \tag{1}$$

In (1), $\mathbf{c}(n) = [c_1(n), c_2(n), c_3(n)]^T$ represents the binary code symbols, $c_i(n) = \pm 1$ at the output of the LDPC encoder. The matrix $\mathbf{S} \in \mathbb{C}^{2N_s \times N_s}$ corresponds to N_s delayed versions of the truncated Gold sequence specified by $\mathbf{c}(n)$. The channel vector $\mathbf{f}(n) \in \mathbb{C}^{N_s}$ thus corresponds to a Nyquist sample-spaced tapped-delay line multipath model. The noise $\mathbf{n}(k)$ has covariance matrix $\mathbf{R} = 2N_0/T_s$, where the combination of ambient and amplifier noise is modeled as white circular Gaussian with spectral density $2N_0$.

It is useful to observe that the optimal joint estimator/decoder for the received signal (1) has a closed-form solution. For a known LPDC codeword $\mathbf{c}^k = \{\mathbf{c}(k)\mathbf{c}(k-1)\dots\mathbf{c}(1)\}$, the optimal estimator ignoring sparsity of the channel is a Kalman filter corresponding to the measurement function $\mathbf{S}(\mathbf{c}(k))$. Standard Bayesian analysis yields the structure of the

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optimum channel estimator as a weighted sum of Gaussian densities. At any time k,

$$p(\mathbf{f}(k)|\mathbf{r}^k) = \sum_{\mathbf{c}^k \in \mathcal{G}^k} p(\mathbf{f}(k)|\mathbf{r}^k, \mathbf{c}^k)p(\mathbf{c}^k|\mathbf{r}^k),$$

$$p(\mathbf{f}(k)|\mathbf{r}^k, \mathbf{c}^k) = \mathcal{N}(\mathbf{f}(k); \hat{\mathbf{f}}^{c^k}(k|k), \mathbf{P}^{c^k}(k|k)), \tag{2}$$

$$p(\mathbf{c}^k|\mathbf{r}^k) = \frac{1}{c} \prod_{l=1}^{k} \mathcal{N}(\mathbf{r}(l); \hat{\mathbf{r}}^{c^l}(l|l-1), \Sigma^{c^l}(l|l-1)).$$

In (2), $\mathcal{G}^k$ is the set of possible $2^{R_c k}$ truncated codewords, with R_c the code rate. Each summand represents a Kalman filter conditioned on a code subsequence c^k with estimate $\hat{\mathbf{f}}^{c^k}(k|k) \in \mathbb{C}^{N_s}$ and covariance matrix $\mathbf{P}^{c^k}(k|k) \in \mathbb{C}^{N_s \times N_s}$.

The a-posteriori probabilities (APPs) of a cumulative sequence denoted by $p(\mathbf{c}^k|\mathbf{r}^k)$ are given as a product of Kalman innovations likelihoods, defined in terms of means and covariances

$$\hat{\mathbf{r}}^{c^l}(l|l-1) = \mathbf{S}(\mathbf{c}(l))\hat{\mathbf{f}}^{c^l}(l|l-1)$$

$$\Sigma^{c^l}(l|l-1) = \mathbf{S}(\mathbf{c}(l))\hat{\mathbf{P}}^{c^l}(l|l-1)\mathbf{S}(\mathbf{c}(l))^H + \mathbf{R}. \tag{3}$$

From (2) it is clear that the optimal estimator/decoder is intractable due to the exponentially growing number of Kalman filters, $2^{R_c k}$ required with time k. However, it is interesting to note that the optimum estimator/decoder can be implemented purely in terms of Kalman filters, without requiring the more complicated smoothers arising in a factor-graph approach [10]. Furthermore, the Bayesian formula (2) yields the exact symbol APPs by appropriate marginalization, whereas the SPA solution is only approximate due to the cycles on the factor graph.

ITERATIVE DECODING AND CHANNEL ESTIMATION

Following [10, 12, 9], a factor graph for the joint decoding/estimation problem defined by the received signal (1) is given in Fig. 1. The function blocks are defined by

$$p(n) = p(\mathbf{r}(n)|\mathbf{f}(n), \mathbf{c}(n)) = \mathcal{N}(\mathbf{r}(n); \mathbf{S}(\mathbf{c}(k))\mathbf{f}(k), \mathbf{R})$$

$$p(\mathbf{f}(n)|\mathbf{f}(n-1)) = \mathcal{N}(\mathbf{f}(n); \mathbf{F}\mathbf{f}(n-1), \mathbf{Q}), \tag{4}$$

where $\mathbf{F}$ and $\mathbf{Q}$ are the state-transition and process noise covariance matrices in a first-order AR model for the channel vector $\mathbf{f}(n)$. The variable nodes are $\mathbf{f}(k)$ and $\mathbf{c}_i(k)$ where the latter are the binary code symbols of the LDPC code. The function blocks $\mathbf{H}_k$ represent the rows $k = 1, \ldots, m$ of the LDPC parity check matrix [13].

To obtain a practical algorithm, simplifications to the SPA message passing must be made, as suggested in [10, 12]. First, we ignore the backward messages on the graph transitions from $\mathbf{f}(n) \rightarrow \mathbf{f}(n-1)$. Message passing to and between the nodes $\mathbf{f}(k)$ will thus correspond only to Kalman filtering. First note that messages $\mu_{c_i(k)\rightarrow p(k)}$ are the extrinsic LDPC decoder

probabilities (or equivalently LLRs), denoted by $p_e(c_i(k))$. These are computed via the SPA [13]. Then the exact message $\mu_{p(k)\to\mathbf{f}(k)}$ and its approximation are

$$\mu_{p(k)\to\mathbf{f}(k)} = \sum_{\mathbf{c}_i(k)=\pm 1} p(\mathbf{r}(k)|\mathbf{f}(k),\mathbf{c}(k)) \prod_{i=1}^{3} \mu_{c_i(k)\to p(k)}$$
$$\approx \mathcal{N}(\mathbf{r}(k); \mathbf{S}(\hat{\mathbf{c}}(k))\mathbf{f}(k), \mathbf{R}),$$
(5)

where $\hat{\mathbf{c}}(k)$ corresponds to hard decisions based on $p_{ap}(c_i(k))$. These last probabilities are the APPs computed by the LDPC decoder on the previous iteration. The approximation (5) is justified by noting that as the LDPC decoder converges, the extrinsic probabilties in $\mu_{c_i(k)\to p(k)}$ approach unity only for a single vector $\mathbf{c}(k)$. Furthermore, the most significant summand is that for which the likelihood $p(\mathbf{r}(k)|\mathbf{f}(k)\mathbf{c}(k))$ is large, *and* for which the extrinsic probabilities dominate. Hence, the dominating summand in (5) should be that for which the total decoder APPs $p_{ap}(c_i(k))$ are largest. Since (5) is now a Gaussian density with mean linearly dependent on $\mathbf{f}(k)$, the top row of the factor graph corresponds to a Kalman filter.

The second major approximation is to the messages $\mu_{\mathbf{f}(k)\to p(k)}$ as follows.

$$\mu_{\mathbf{f}(k)\to p(k)} \approx \mathcal{N}(\mathbf{f}(k); \hat{\mathbf{f}}(k|k-1), \mathbf{P}(k|k-1)),$$
(6)

where $\hat{\mathbf{f}}(k|k-1), \mathbf{P}(k|k-1)$ are Kalman filter updates conditioned on the decisions $\hat{\mathbf{c}}(k-1),\ldots,\hat{\mathbf{c}}(1)$. Given approximation (6), the last message required is $\mu_{p(k)\to c_i(k)}$ for $i=1,2,3$. This is

$$\mu_{p(k)\to c_i(k)} = \sum_{c_j(k)=\pm 1, j\neq i} \int d\mathbf{f}(k) p(\mathbf{r}(k)|\mathbf{c}(k),\mathbf{f}(k))\mu_{\mathbf{f}(k)\to p(k)} \prod_{j\neq i} \mu_{c_j(k)\to p(k)}$$
$$= \sum_{c_j(k)=\pm 1, j\neq i} \mathcal{N}(\mathbf{r}(k); \mathbf{S}(\mathbf{c}(k))\hat{\mathbf{f}}(k|k-1), \Sigma(k|k-1)) \prod_{j\neq i} \mu_{c_j(k)\to p(k)},$$
(7)

where $\Sigma(k|k-1)$ is the KF innovations covariance matrix. The implicit message-passing schedule and iterative receiver are fully defined by the algorithm in Table 1.

RESULTS AND CONCLUSIONS

The iterative receiver described by Table 1 was simulated with an underwater acoustic channel model corresponding to Rayleigh or Ricean fading. A (200,100) LDPC code based on a random parity check matrix with $t=3$ ones per column was employed. The multipath corresponded to five rays, including bottom and surface bounces for the receiver and transmitter at 20 m depth, and a distance of 500 m. Fig. 2 shows the uncoded and coded BERs for a Doppler spread of $f_D = .1$ Hz, and a Ricean k-factor of 5. Overall improvement in BER is seen from from 1 to 3 outer iterations of the receiver, with an improvement over an uncoded system of 2.5 dB at an error rate of $< 10^{-3}$. Fig. 3 shows BER when the Doppler spread is increased to $f_D = .5$ Hz and the fading is Rayleigh. In this case, the improvement is 3 dB at a BER of 10^{-2} for the iterative receiver over an uncoded system.

Given previous decoder extrinsic and total APPs $p_e(c_i(k)), p_{ap}(c_i(k))$

For $k = 1, \ldots, n$

    Compute $\hat{\mathbf{c}}(k) = \arg\max_{\mathbf{c}(k)} \prod_{i=1}^{3} p_{ap}(c_i(k))$

    Update Kalman filter using $\hat{\mathbf{c}}(k)$ (or $\mathbf{c}(k)$ for training preamble)

        $\hat{\mathbf{f}}(k|k-1) \to \hat{\mathbf{f}}(k|k) \to \hat{\mathbf{f}}(k+1|k)$

        (transmission of $\mu_{p(k)\to\mathbf{f}(k)}$.)

    Compute new messages to code symbol variables

    $\mu_{p(k)\to c_i(k)} =$
$$\sum_{c_j(k)=\pm 1, j\neq i} \mathcal{N}(\mathbf{r}(k); \mathbf{S}(\mathbf{c}(k))\hat{\mathbf{f}}(k|k-1), \Sigma(k|k-1)) \prod_{j\neq i} p_e(c_j(k))$$

Next k

Run LDPC decoder using $\mu_{p(k)\to c_i(k)}$ as inputs and generate next

    $p_e(c_i(k)), p_{ap}(c_i(k))$.

Table 1: Iterative DSSS receiver corresponding to factor graph message approximations.

To conclude, an iterative DSSS acoustic modem was designed with LDPC decoding. A joint channel estimator and decoder was obtained via approximations to SPA message passing on a factor graph. For the fairly weak (200,100) LDPC code employed, significant BER improvements of 2.5 to 3 dB were obtained. More powerful codes and more extensive simulations will be considered to explore the potential of this iterative receiver design.
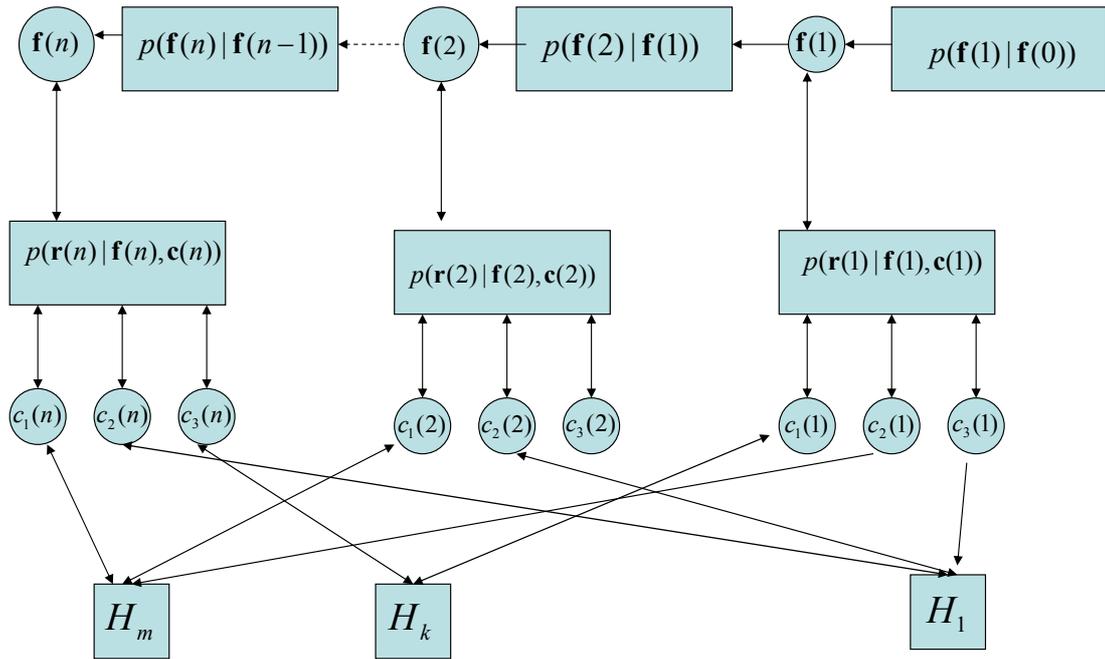
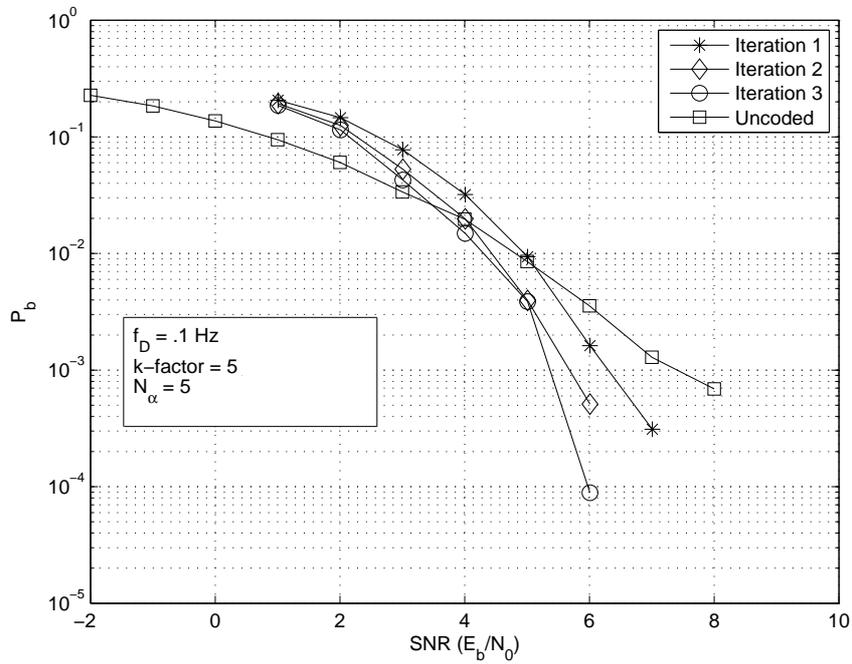Figure 1: Factor graph for DSSS modem joint estimator/LDPC decoder.

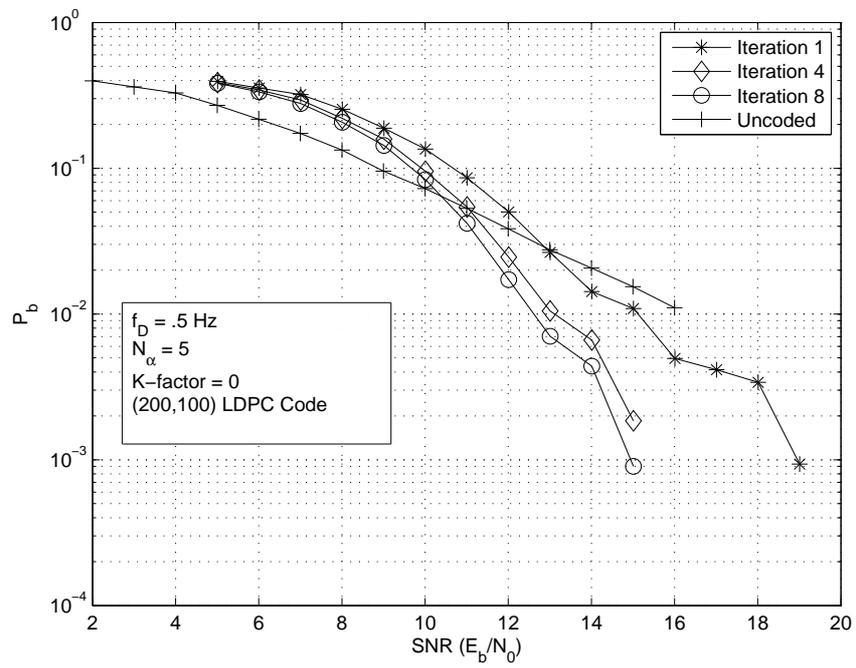Figure 2: BER in Ricean fading with $f_d = .1$ Hz Doppler spread.

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Figure 3: BER in Rayleigh fading with .5 Hz Doppler spread.

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