SERIALLY CONCATENATED HIGH RATE CONVOLUTIONAL CODES WITH CONTINUOUS PHASE MODULATION

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ABSTRACT

We propose serially concatenated convolutional codes with continuous phase modulation for aeronautical telemetry. Such a concatenated code has an outer encoder whose code words are permuted by an interleaver, and a modulation, which is viewed as a code and takes the interleaved words as its input and produces the modulated signal. Since bandwidth expansion is a concern when coding is introduced, we focus on high rate punctured codes of rates $2/3$ through $9/10$. These are obtained by puncturing the basic rate $1/2$ convolutional codes with maximal free distance. At the receiver end we use a reduced complexity iterative decoding algorithm which is essentially a soft input soft output decoding algorithm. These simple highly powerful concatenated codes produce high coding gains with minimum bandwidth expansion.

INTRODUCTION

In the modern world of communication systems, channel coding has become the most vital means of satisfying power and bandwidth constraints. For a given spectral efficiency, large coding gains can be achieved by encoding large blocks of information sequences. The idea is to find a good code with large block sizes that achieves a required error probability. As stated by most coding theorists, most codes are good (i.e.) the task of finding a code with an encoder that performs efficiently is simple. The problem revolves around the complexity associated with decoding the randomly generated code.

The ideal solution to the above setback would be to use concatenated codes. The increase in complexity

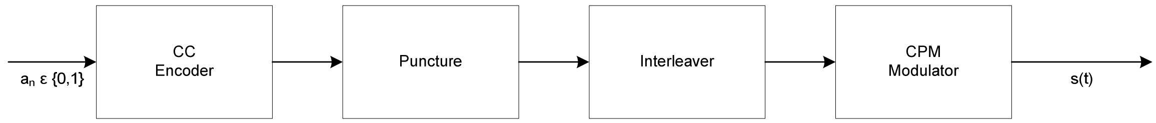

Figure 1: Serially Concatenated Convolutional Encoder with CPM

with the quest to find large coding gains and spectral efficiency is overcome by concatenating encoders, and using an iterative decoding algorithm whose complexity increases only linearly with the block size. In reality these concatenated codes have become the most popular structures in communication systems.

In this paper we present a serially concatenated convolutional code (SCCPM) [1, 2]. The SCCPM has an outer encoder serially concatenated to an inner encoder. A basic $g_1 = 5$, $g_2 = 7$ convolutional encoder is used at the outer part which takes the information sequence as its input and convolves them. A S-random interleaver [2] is used to interleave the output of the outer encoder. The interleaved sequence of bits is fed into a continuous phase modulator (CPM) [3], which is viewed as an inner encoder.

As an enhancement to the system built, high rate serially concatenated codes are derived from the basic rate SCCPM. Puncturing of convolutional codes [4] is employed to significantly simplify the decoding process. The technique apart from simplifying the decoding process, allows us to implement a rate selectable SCCPM. These turbo-like serially concatenated convolutional codes can be maximum likelihood decoded using Viterbi algorithm(VA). But because of the large decoding complexity involved, an iterative decoding technique is employed, where the decoded information symbols are reiterated between the two decoders. Hence the information passed between the two decoders and the interleaver/deinterleaver is the additional information associated with the information symbols, because of this standard decoding procedures like VA cannot be used. Instead an algorithm which calculates the *a posteriori* probabilities of the associated symbols [5] based on the knowledge of the received sequences is employed.

In this paper we will guide the reader through the design of high-rate (turbo-like) serially concatenated convolutional codes with continuous phase modulation. We adopt the iterative decoding scheme from [5] and simulate the performance of high-rate SCCPM.

SERIALLY CONCATENATED CONVOLUTIONAL ENCODERS

The convolutional codes are highly suitable due to their strong coding gains, simplicity in structure and rate flexibility. The serial concatenation of the convolutional encoder [1] consists of an outer encoder separated from an inner modulator by an interleaver, [2] as shown in Figure 1. Ideally a rate $R = k/n$ convolutional encoder is used as an outer encoder. This takes 'k' bits as its input and produces symbols of length 'n' at the output. The generator polynomial of this code are $g_1 = 5$ and $g_2 = 7$, octal representation [3]. The order of the symbols produced at the output of the outer encoder is permuted by the interleaver before they are coded again by a modulation scheme.

The concept of interleaving is very important in forward error correction as interleaving the encoded symbols provides a form of time diversity to guard against localized corruptions and burst of errors. In this paper we use S-random interleaver [2] which is used to scramble the order of sequence of numbers (i.e.)

Table 1: Map of Deleting Bits for High Rate Punctured Codes

Coding Rate	Constraint Length = 2	N	S
1/2	1 (5)	2048	32
	1 (7)		
2/3	1 0	1536	28
	1 1		
3/4	1 0 1	1364	26
	1 1 0		
4/5	1 0 1 1	1280	25
	1 1 0 0		
5/6	1 0 1 1 1	1230	25
	1 1 0 0 0		
6/7	1 0 1 1 1 1	1197	25
	1 1 0 0 0 0		
7/8	1 0 1 1 1 1 1	1168	24
	1 1 0 0 0 0 0		
8/9	1 0 1 1 1 1 1 1	1152	24
	1 1 0 0 0 0 0 0		
9/10	1 0 1 1 1 1 1 1 1	1140	24
	1 1 0 0 0 0 0 0 0		

the sequence of the symbols. This ensures an excellent bit rate and frame error rate performances. With a serially concatenated convolutional code, the interleaver has some important functions. The interleaver mainly aims at increasing the hamming weight of the code word which increases the code strength. As shown in Figure 1 the interleaver permutes the information from the outer encoder to the inner encoder, thereby associating a weaker (lesser weight) outer code word with a stronger (larger weight) inner code word. The interleaver also serves to reduce the correlation in information between the outer and inner decoders which helps to improve the iterative decoding procedure. In this case, the interleaving parameters are usually carefully selected to match the error correcting capabilities of the codes involved. The size of a block and the interleaver for a rate $1/2$ code are $N = 2048$ and $S = 32$ respectively. For higher rate codes these sizes are given in Table 1. The inner modulator which is viewed as a code takes the interleaved code words and produces the modulated signal [6].

PUNCTURING

The concept of puncturing allows us to implement a rate selectable encoder/decoder. The punctured high rate convolutional codes can be obtained from the basic rate $1/2$ convolutional codes by periodically deleting certain bits as per Table 1. Moreover it is a well known fact that the Viterbi decoding is much simpler for punctured high rate convolutional codes [4]. In this paper we use Table 1 to puncture the basic rate $1/2$ convolutional codes to obtain high rate codes from $2/3$ through $9/10$. The table gives us a map which specifies the bits that are to be deleted in order to obtain the high rate punctured codes from the rate $1/2$ convolutional code. As an example, let us consider Table 1. For the rate $1/2$ code, both the bits of g_1 and g_2 are transmitted. With rate $2/3$ code being generated from the basic rate $1/2$ code, we transmit the first bit of g_1 and g_2, but delete the second bit of g_1. We continue to transmit the second bit of g_2 and proceed in this manner. This lets us to generate rate $2/3$ code from a basic rate $1/2$ code. Table 1 also includes the total number of channel bits N (i.e.) the number of bits being transmitted over the channel with the corresponding code rate. The important fact that allows puncturing is that even though the coding gain of these high rate codes decrease with the increase in the code rate, the coding gain is still higher compared to the uncoded system.

SERIALLY CONCATENATED SYSTEM WITH ITERATIVE DECODING

The concatenated coding schemes yield a sub-optimum decoding scheme based on the iterative use of *a posteriori* probability. The block diagram for the serial concatenation of convolutional codes with SOQPSK and PCM/FM is shown in Figure 2. With reference to the kind of modulation scheme used, the modulator block in Figure 2 can be either SOQPSK or PCM/FM modulation.

These SISO modules which are shown, are the max log versions of the ones in [5, 7]. The scale factors of K_1 and K_2 are used to improve the system performance. These constants $K_1 = 0.75$ and $K_2 = 0.75$ was determined by trail and error method to suit the SOQPSK SISO Decoding. K_1 and K_2 for PCM/FM were also determined based on trail and error method. For a full state PCM/FM K_1 and K_2 takes a value

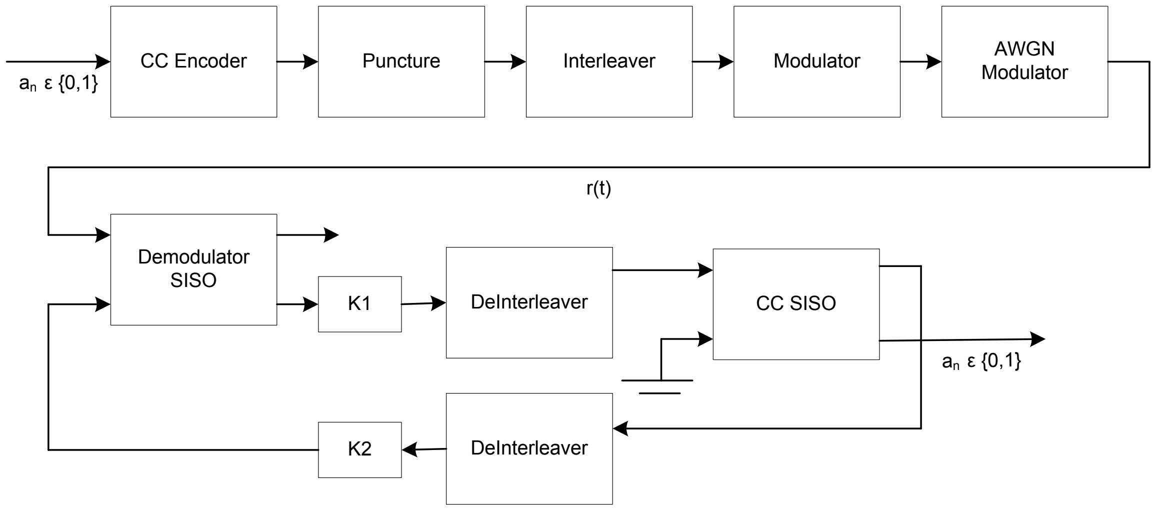

Figure 2: Serially Concatenated Convolutional Code with Iterative Decoding

Figure 3: The SISO Module

of 0.65 respectively with rate $1/2$ codes. Continuous simulation has proved that a value of $K_1 = 0.8$ and $K_2 = 0.8$ works well with a rate $2/3$ code and higher rate codes for full state PCM/FM.

As seen in Figure 2 there are two SISO modules and the decoded soft decisions are exchanged back and forth between them for a predetermined number of iterations [5, 6]. The decoded state metrics are initialized to zero and there are no termination bits which are added in the simulation. The outer codes are similar to the codes used in [8], which is done to reduce the complexity involved. We perform 5 iterations before we decode the bits at the output. As seen in Figure 2 the lower input given to the inner CC SISO module is grounded, in the sense it is made equal to zero with the reason being that the inputs 0 and 1 are equiprobable. Simlarly the lower input connected to the outer demodulator SISO is also initialized to zero for the first iteration. From the next iteration onwards the inner SISO is fed back with the soft decisions from its outer counterpart.

As it can be seen in Figure 3, the SISO is a four port device. It accepts two inputs and produces two outputs. The inputs are probability distributions of the information and code symbols and the outputs are the updates of these distributions [5]. In an iterative decoding scheme when a bit interleaver is used instead of a symbol interleaver, then the *a priori* probability distributions of symbols can be represented as the product of marginal distributions of bits.

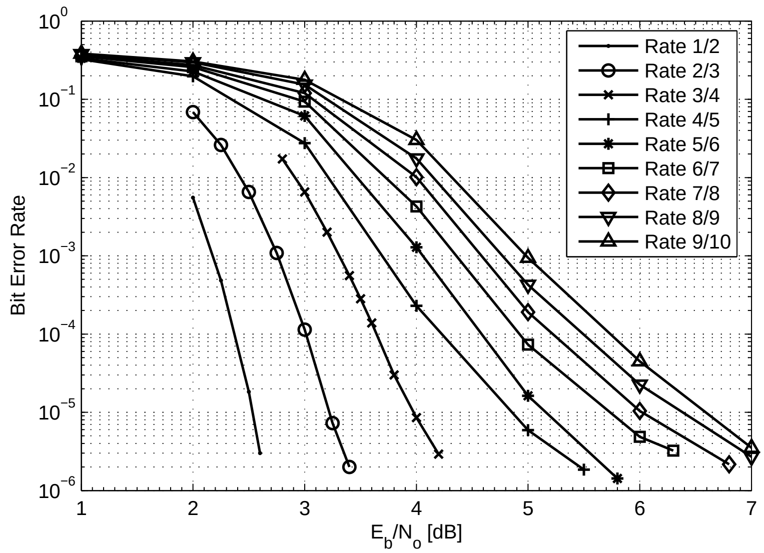

Figure 4: BER plot for High Rate Serially Concatenated convolutional Code with SOQPSK

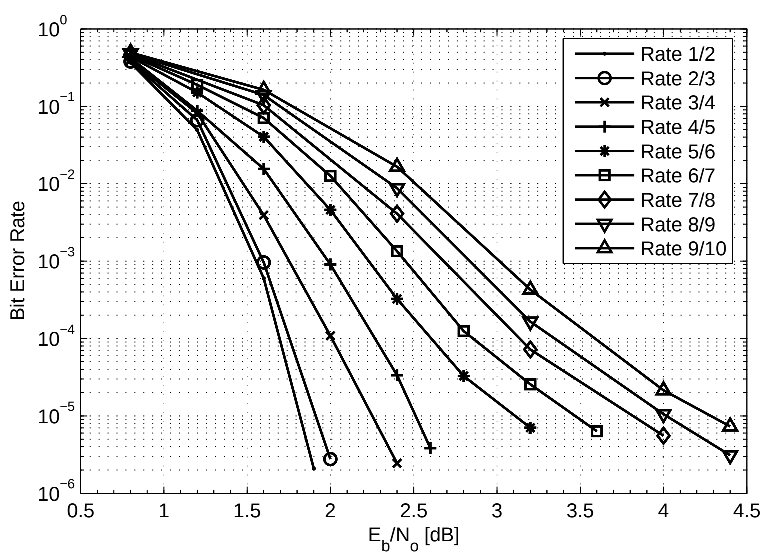

Figure 5: BER plot for High Rate Serially Concatenated convolutional Code with PCM/FM

SIMULATION RESULTS

The bit error rates simulated are plotted against the signal to noise ratio for all the high rate codes. The basic BER plot for rate $1/2$ code and rate $3/4$ code are identical to those found in [7]. The other plots for high rate codes are obtained with lower complexity and their coding gains are compared from the simulated bit error rate performance. From Figure 4 it is obvious that as the rate of the system increases the coding gain decreases. This reiterates the fact that rate $1/2$ code has the best performance, but high rate codes gives us the much needed spectral efficiency along with good coding gains. It is to be noted that even though we say that the coding gain decreases, the gain obtained is still higher when compared to the uncoded system. PCM/FM being a spectrally efficient modulation technique, when coupled serially with convolutional codes provides better spectral performance. As seen in Figure 5 bit error rate of about 10^-5 are reached well within 2 dB for rate $1/2$ as well as rate $2/3$ code.

CONCLUSION

Serially concatenated convolutional codes were developed for higher rates with continuous phase modulation. The two various forms of CPM namely SOQPSK and PCM/FM were coupled along with a basic convolutional encoder to form the SCCPM. In this paper we initially described the concept of serial concatenation of convolutional encoders and later discussed interleaving which was essentially done to improve code strength and to spread out the code words between the two decoders. Later puncturing was introduced and a map for deleting suffice bits to obtain high rate codes from basic rate $1/2$ was provided. An iterative decoding algorithm similar to the algorithm explained in [5] was adopted. We have shown with simulation results that the high rate coded systems achieves optimal performance with TG-STD SOQPSK and PCM/FM systems.

ACKNOWLEDGMENT

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