PERFORMANCE OF UNITARY SPACE TIME CODES GENERATED BY GIVENS ROTATION MATRICES IN MULTIPLE-INPUT, MULTIPLE-OUTPUT COMMUNICATION CHANNELS

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ABSTRACT

Multiple-Input, Multiple-Output (MIMO) communication systems promise to provide significantly higher data rates at no increase in transmitted power or bandwidth. Unfortunately it is challenging to locate space-time codes which achieve these gains. It was recently shown that codes based on Givens Rotation Matrices (GRM) out perform many of the more conventional space-time codes at extreme values of signal to noise ratio (SNR). This paper investigates the performance of GRM codes over a wider range of SNR, to determine their usefulness in MIMO applications of interest to the telemetry community.

INTRODUCTION

The use of unitary space-time codes in Multi-Input Multi-Output (MIMO) communication systems offer improved data rates over Single-Input Single-Output systems. One promising method for generating these codes is by using Givens Rotation Matrices (GRM) [5]. It has been shown that codes generated using GRM are beneficial because their weak group property promises reduced code complexity, while producing codes with performance near, or better, than many other codes. However, the performance of these codes were measured using diversity product and diversity sum metrics. These measures are useful at extremely high, and extremely low, signal-to-noise ratios (SNR) – but they do not guarantee high performance over the full range of SNR a system may operate over. This paper discusses methods, and simulation results, which measure the performance of the GRM generated codes over the range of SNR more typically seen in many applications.

The need to examine a wider range of SNR, means that we must consider far more codes, than simply those with superior diversity sum and diversity product measures. If we examine the average performance of all possible codes generated using the GRM, we can guarantee that our performance will be sub-optimal at all SNR. Assuming there is at least one code which is worse than average (and it is trivial to find such a code), then we know that at least one code must have performance superior to the average. In this way, the average forms a lower bound on the performance of the best possible GRM code. This technique will work over a wide range of SNR.

There are other metrics which can be used to determine code performance. One other choice (in addition to diversity sum and diversity product) is the union bound design metric [6]. This bound can be used to optimize codes for any SNR. However, a code optimized for a specific SNR, does not guarantee improved performance at all SNR. Therefore, it is useful to simulate the performance of new codes.

Why then, are GRM codes useful over other types of space time codes? Often with unitary space-time block codes, the performance difference between two types of codes can be quite significant. Previous work has shown that the cyclic codes reported by Hochwald et al in [1] have a significantly lower diversity sum and diversity product, and thus significantly inferior performance, then more recently proposed codes. As, was show in [5], GRM codes provide performance near, or better, than many of the best known codes at that time. Also, some codes are defined only for a specific number of transmit antennas. The codes proposed in [3] are only defined for an even number of transmit antennas. Those proposed in [4] expand this to allow an odd antennas, but is still only for three to six transmit antennas. Some codes, such as those in [2] are design for only a specific number of transmit antenna. GRM codes are useful since they can be defined for an arbitrary number of transmit antennas.

The next section describes the simulation parameters used to evaluate the new coding bound, followed by the performance that was actually measured. The code that was used to conduct these simulations is available from the authors upon request. In addition to the performance graphs and tables presented in the text, an appendix illustrates histograms of the performance for a variety of cases.

SIMULATION PARAMETERS

We wish to design our simulation in a way that we can make comparisons to what we know as best codes. To our knowledge, the best known codes were reported in [4]. For this reason, we will consider the same slow, land mobile fading channel using Jakes' Model that was used in [4]. We will use the same normalized fade rates of $f_d T_s = 3.56 \times 10^{-3}$ and $f_d T_s = 7.1 \times 10^{-3}$. We will consider constellations of size $L = 32$ using for $M = 3$ to $M = 6$ transmitted antennas and $N = 1$ receive antenna with a block length of $T = 2M$. To decode the GRM codes, we will use the maximum likelihood detection method given in [1].

We have determined above that our simulation must consider all possible codes. Because we cannot consider all possible codes, we will randomly generate codes then average the performance across all these codes. For each code, 10000 symbols will be simulated and the symbol error rate will be calculated and averaged. As the number of codes simulated increases, our average error rate across randomly generated codes will approach the average error rate across all possible codes. Either way, we achieve a sub-optimal performance curve with which we can guarantee at least one code that will improve our performance for any SNR that we choose.

SIMULATED PERFORMANCE OF GIVENS ROTATION MATRICES CODES

Figures 1 and 2 show the performance curves for the GRM codes at the fade rates of $f_d T_s$ = 3.56x10^{-3} and $f_d T_s$ = 7.1x10^{-3}, respectively. Using Figures 1 and 2 and [5], tables 1 and 2 were compiled with performances of the GRM code and the best known codes. The performance difference, in dB, was also given in these tables. In most cases, it can be seen the performance of our Givens code was near, but slightly less than the best known codes. We see for M = 3 transmit antennas at a normalized fade rate of $f_d T_s$ = 3.56x10^{-3} that our performance is approximately 2 - 3 dB less for most of the error probability cases we consider. For M = 4 transmit antennas, we see that our performance is approximately 2.5 dB less. For the normalized fade rate $f_d T_s$ = 7.1x10^{-3}, we see that our performance is only about 1 dB less for both M = 3 and M = 4 transmit antennas. We also see at this fade rate that the Givens code has an improved high-SNR asymptote for the error probability. For both the M = 3 and M = 4 transmit antenna cases, we see that at error probably of 10^{-4} and 10^{-5}, our code has improved performance, especially at an error probability of 10^{-5}. We have also included the cases for M = 5 and M = 6 transmit antennas, however, there were no plots in [5] with which to compare them.

M	Error Probability	Code in [2]	Givens Code	Performance Difference
3	10^{-1}	10.5 dB	11.5 dB	-1 dB
	10^{-2}	16 dB	18 dB	-3 dB
	10^{-3}	20.5 dB	23 dB	-2.5 dB
	10^{-4}	25 dB	28 dB	-2 dB
	10^{-5}	31.5 dB	32.5 dB	-1 dB
4	10^{-1}	8 dB	8.5 dB	-0.5 dB
	10^{-2}	12 dB	14.5 dB	-2.5 dB
	10^{-3}	15 dB	17.5 dB	-2.5 dB
	10^{-4}	18.5 dB	21 dB	-2.5 dB
	10^{-5}	22 dB	24.5 dB	-2.5 dB
5	10^{-1}	N/A	7 dB	N/A
	10^{-2}	N/A	11.5 dB	N/A
	10^{-3}	N/A	14.5 dB	N/A
	10^{-4}	N/A	17.5 dB	N/A
	10^{-5}	N/A	20.5 dB	N/A
6	10^{-1}	N/A	6 dB	N/A
	10^{-2}	N/A	9.5 dB	N/A
	10^{-3}	N/A	12.5 dB	N/A
	10^{-4}	N/A	15 dB	N/A
	10^{-5}	N/A	17 dB	N/A

Table 1: Performance of Codes at Normalized Fade Rate of $f_d T_s$ = 3.56x10^{-3}.

M	Error Probability	Code in [2]	Givens Code	Performance Difference
3	10^{-1}	11 dB	12 dB	-1 dB
	10^{-2}	17 dB	18 dB	-1 dB
	10^{-3}	23 dB	24 dB	-1 dB
	10^{-4}	N/A	31 dB	N/A
	10^{-5}	N/A	N/A	N/A
4	10^{-1}	8 dB	8 dB	0 dB
	10^{-2}	12.5 dB	13 dB	-0.5 dB
	10^{-3}	16.5 dB	17.5 dB	-1 dB
	10^{-4}	23 dB	21.5 dB	1.5 dB
	10^{-5}	N/A	25 dB	N/A
5	10^{-1}	N/A	7 dB	N/A
	10^{-2}	N/A	11.5 dB	N/A
	10^{-3}	N/A	14.5 dB	N/A
	10^{-4}	N/A	17.5 dB	N/A
	10^{-5}	N/A	20.5 dB	N/A
6	10^{-1}	N/A	6 dB	N/A
	10^{-2}	N/A	9.5 dB	N/A
	10^{-3}	N/A	12.5 dB	N/A
	10^{-4}	N/A	15 dB	N/A
	10^{-5}	N/A	17 dB	N/A

Table 2: Performance of Codes at Normalized Fade Rate of $f_dT_s = 7.1 \times 10^{-3}$.

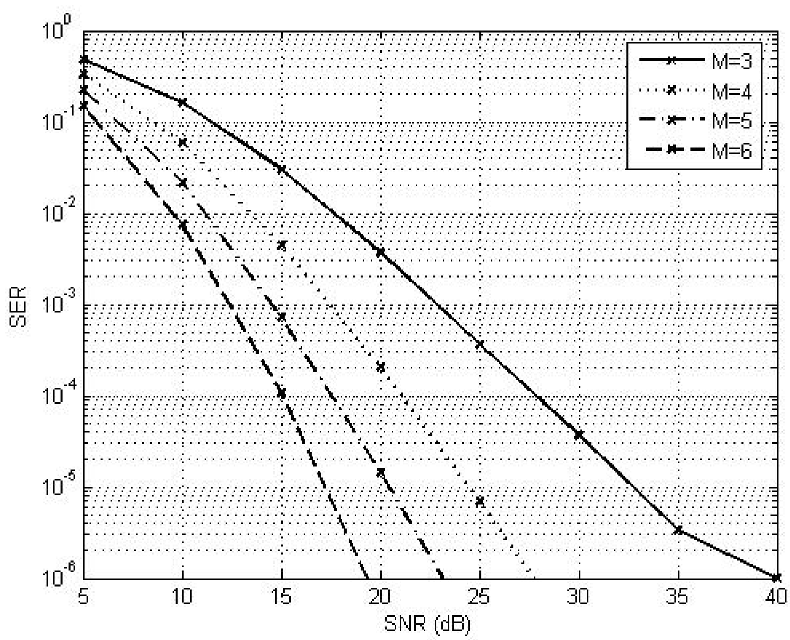

Figure 1: Symbol error rate for L = 32, $f_dT_s = 3.56 \times 10^{-3}$ Givens Rotation Matrices Code

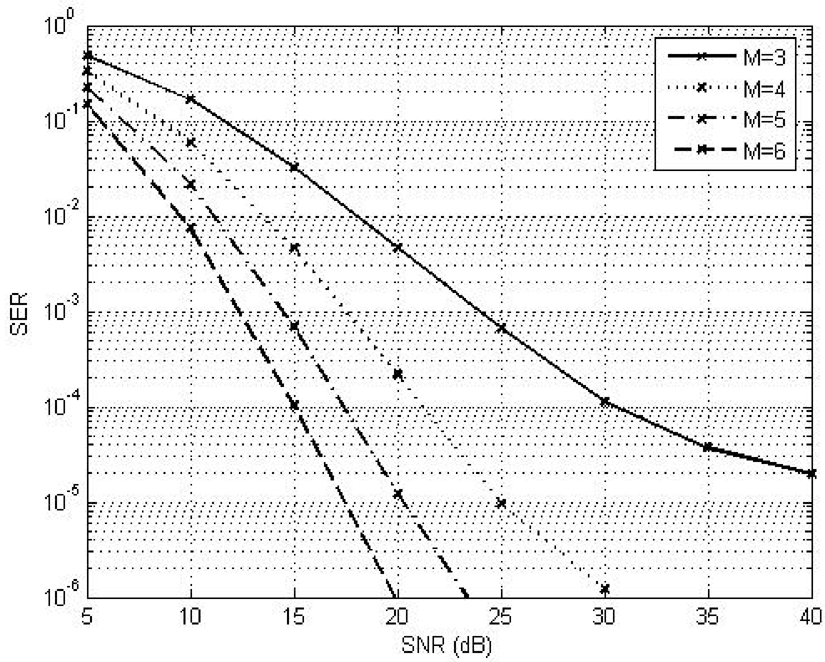

Figure 2: Symbol error rate for L = 32, $f_dT_s = 7.1\text{x}10^{-3}$ Givens Rotation Matrices Code

Contained in the appendix is a collection of histograms for the GRM codes at a normalized fade rate $f_dT_s = 3.56\text{x}10^{-3}$. From these, we can see that the distributions of the performance of our GRM codes. At 5 dB, we see that the distributions for all transmit antenna cases are approximately normally distributed. At 10 dB the distributions begin to look Rican, and then for values of SNR greater than 20 dB the distributions look Rayleigh. These histograms confirm our assumption that the performance curves are sub-optimal. We can see in the case of high SNR, most codes perform as good codes, since very few had any errors in the 10000 symbols which were transmitted. While in this case, there are few codes that will increase are performance, there is still are a few bad codes As the SNR decreases, there are many codes which have improved performance over our sub-optimal performance curves. Considering this, we see that choosing codes optimized for low SNR may be more beneficial in a system in which a wider range of SNR is present.

CONCLUSION

Our simulated data has shown that for a sub-optimal case, the performance is slightly less than that of the best known codes. However, we can guarantee that for any SNR there is at least one code that will improve the performance. For the asymptotically high and low SNR cases, this is in agreement with what was reported in [1], since the diversity product and diversity sum report either match or are an improvement over the best known codes. The performance curves generated show than for asymptotically high case at a normalized fade rate of $f_dT_s = 7.1\text{x}10^{-3}$, there is improvement in the performance of the high SNR asymptote over the best known codes. We also can see from our collection of histograms, that most GRM codes, at least for a normalized fate rate of $f_d T_s = 3.56x10^{-3}$, generate few errors at high SNR. For a system in which we see a wider range of SNR, we can conclude that codes optimized for low SNR are more beneficial than codes optimized for high SNR.

References

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[5] A. Panagos, K. Kosbar, and A. Mohammad, "Weak Group Unitary Space-Time Codes," *GLOBECOM 2005*, Nov 2005.

[6] A. Panagos, and K. Kosbar, "A New Design Metric for Unitary Space-Time Codes," 2006.

APPENDIX – HISTOGRAMS OF THE PERFORMANCE OF GIVENS ROTATION MATRICES CODES

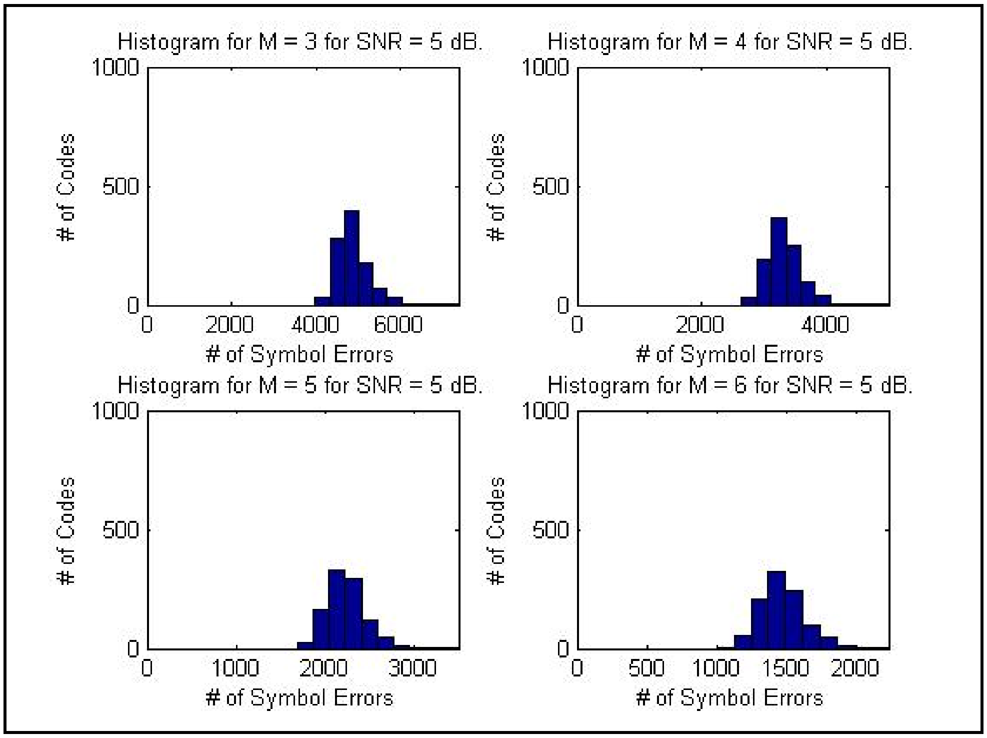

Figure A.1 – Histogram of Givens Rotation Matrices Code for SNR = 5 dB.

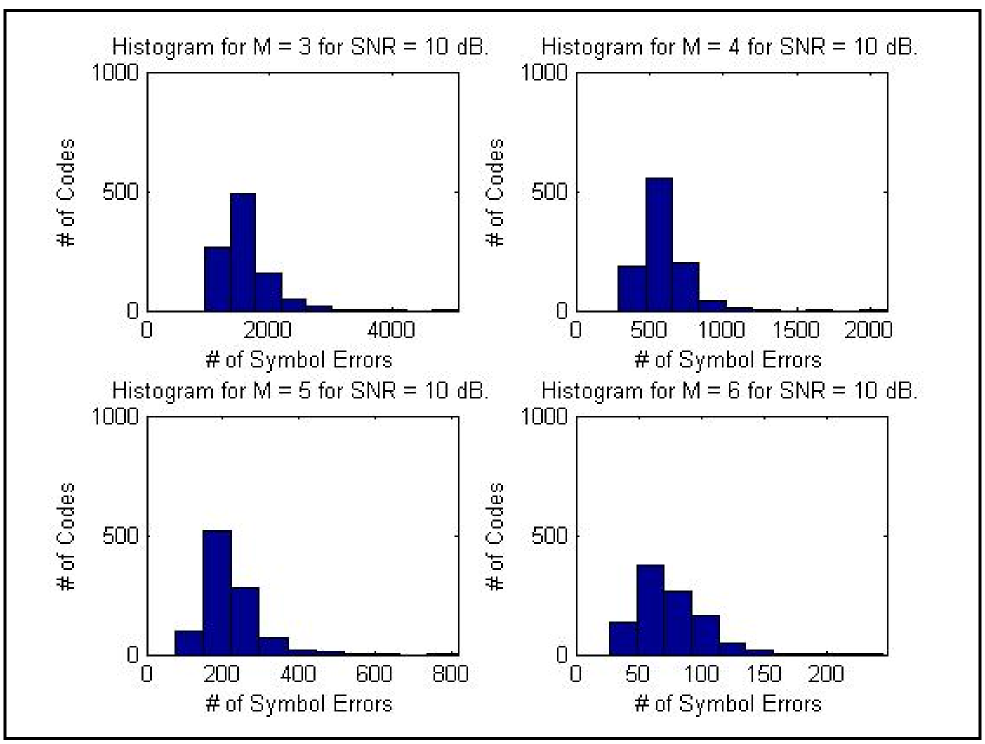

Figure A.2 – Histogram of Givens Rotation Matrices Code for SNR = 10 dB.

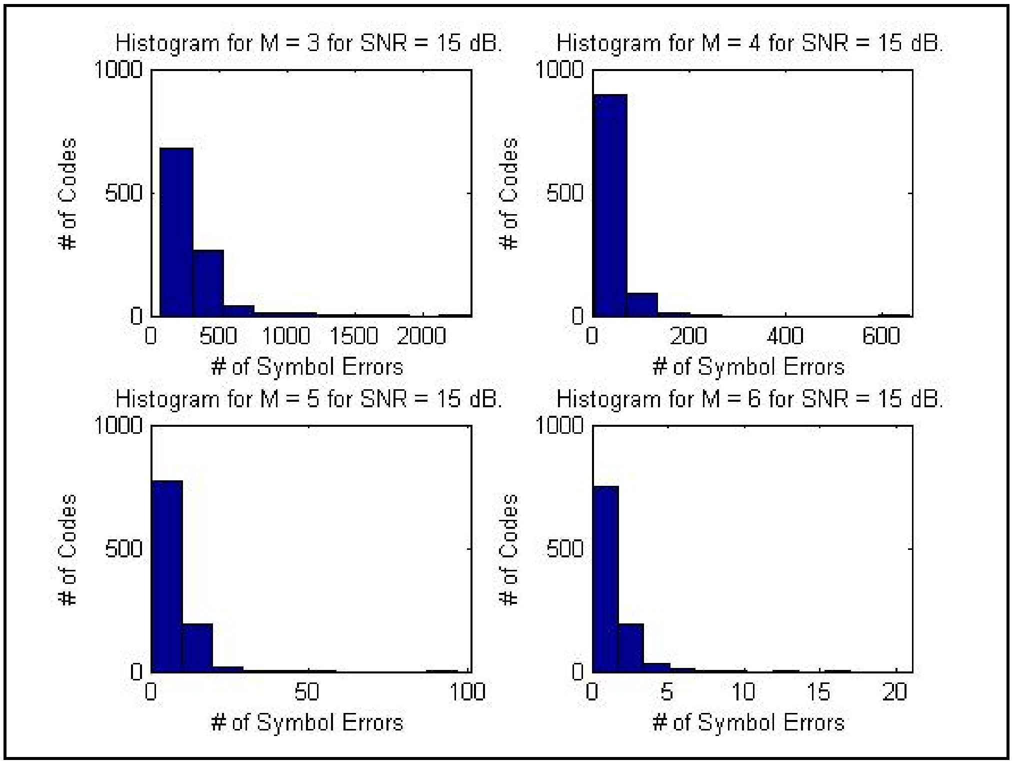

Figure A.3 – Histogram of Givens Rotation Matrices Code for SNR = 15 dB.

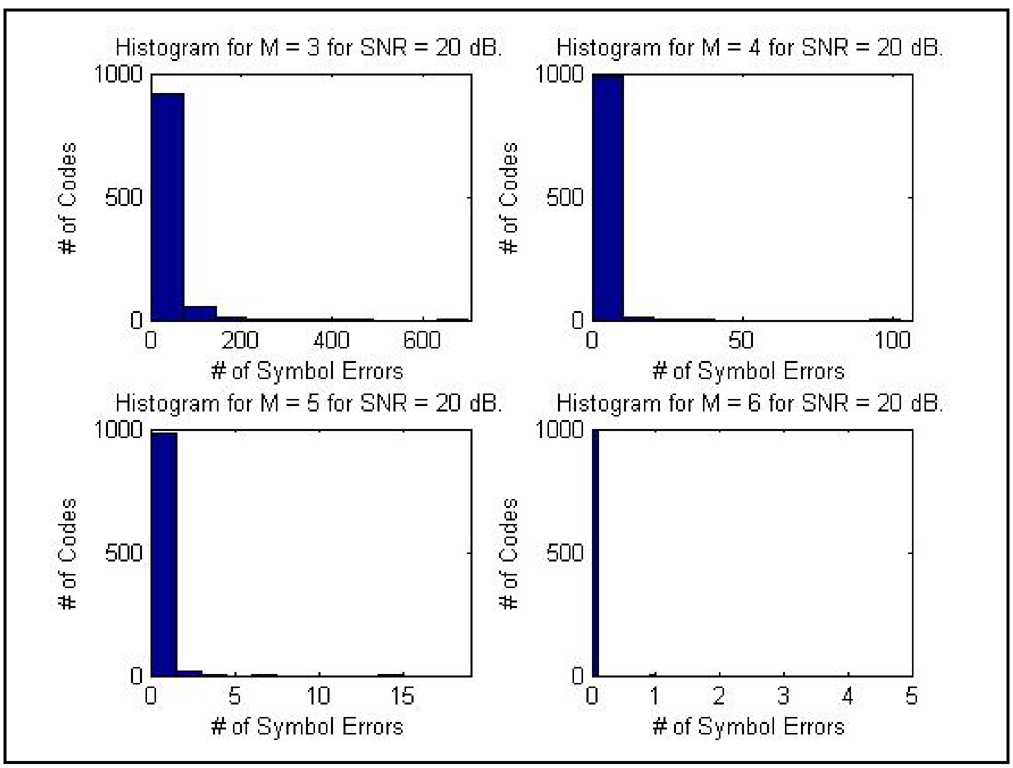

Figure A.4 – Histogram of Givens Rotation Matrices Code for SNR = 20 dB.