ROBUST ADAPTIVE BEAMFORMING WITH BROAD NULLS

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ABSTRACT

Robust adaptive beamforming using worst-case performance optimization is developed in recent years. It had good performance against array response errors, but it cannot reject strong interferences. In this paper, we propose a scheme for robust adaptive beamforming with broad nulls to reject strong interferences. We add a quadratic constraint to suppress the power of the array response over a spatial region of the interferences. The optimal weighting vector is then obtained by minimizing the power of the array output subject to quadratic constrains on the desired signal and interferences, respectively. We derive the formulations for the optimization problem and solve it efficiently using Newton recursive algorithm. Numerical examples are presented to compare the performances of the robust adaptive beamforming with no null constrains, sharp nulls and broad nulls. The results show its powerful ability to reject strong interferences.

KEY WORDS

Newton recursive algorithm, robust adaptive beamforming, worst-case performance optimization, broad nulls

1. INTRODUCTION

Adaptive beamforming is a spatial filter approach for signal processing. It is used to enhance a desired signal while suppressing the noise and interferences at the output of a sensor array. It is likely that it will be widely used in radar, wireless communications and other fields [1]. Traditional adaptive beamforming depends on the exact knowledge of array response to the desired signal. It is very sensitive to the array error. In practical systems, the performance is known to degrade greatly due to the mismatch between assumptions and actual array responses. Performance degradation can also be caused by small sample size even if the array responses are exactly known. So, the robustness of the beamforming algorithm against array errors is a key research point.

In the past decades, many robust adaptive beamforming algorithms have been proposed [2, 3, 4, 5, 6]. Robust adaptive beamforming using worst-case performance optimization, which is theoretically rigorous, is one of the most promising techniques [7, 8]. It defines uncertainty sets for the presumed steering vector and estimated covariance matrix. When the worst-case performance is optimized, good performance of the beamformer is preserved as long as the actual steering vector and covariance matrix are in the uncertainty sets. But when there are strong interferences, its performance will degrade severely. A simple way to suppress strong interferences is adding a set of linear constrains to form sharp nulls in the direction of interferences. When the knowledge of array response on interferences is not exact, the performance improvement is limited. So, robust adaptive beamforming with broad nulls may be a good choice [9, 10].

In this paper, we propose a scheme for robust adaptive beamforming with broad nulls to reject strong interferences. We add a quadratic constraint to suppress the power of the array response over a spatial region of the interferences. The optimal weighting vector is then obtained by minimizing the power of the array output subject to quadratic constrains on the desired signal and interferences, respectively.

The paper is organized as follows. In Section 2, robust adaptive beamforming using worst-case performance optimization is introduced. In Section 3, the proposed robust adaptive beamforming with broad nulls is presented. The optimization problem is derived and then solved using Newton recursive algorithm. Some numerical techniques for Newton recursive algorithm are also presented. In Section 4, numerical examples are given to demonstrate its performance. The performances of the robust adaptive beamforming with no null constrains, sharp nulls and broad nulls are compared. And finally, in Section 5, conclusions are presented.

2. ROBUST ADAPTIVE BEAMFORMING USING WORST-CASE PERFORMANCE OPTIMIZATION

For an array with M sensors, the output of the narrowband beamformer is given by

$$y(k) = \mathbf{w}^{\mathbf{H}}\mathbf{x}(k) \quad (1)$$

where w is the M*1 complex weighting vector of the beamformer, x(k) is the M*1 array observation vector, and $(\bullet)^H$ is the Hermitian transpose. The array observation vector is given by

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \quad (2)$$

where s(k), i(k), and n(k) are the desired signal, interferences, and sensor noise respectively.
For the array output, the signal-to-interferences-plus-noise (SINR) is given by

$$\mathbf{SINR} = \frac{\mathbf{w}^{\mathbf{H}}\mathbf{R}_s\mathbf{w}}{\mathbf{w}^{\mathbf{H}}\mathbf{R}_{i+n}\mathbf{w}} \quad (3)$$

where

$$\mathbf{R}_s = \mathbf{E}\left\{\mathbf{s}(k)\cdot\mathbf{s}^{\mathbf{H}}(k)\right\} \quad (4)$$

and

$$\mathbf{R}_{i+n} = \mathbf{E}\left\{\left(\mathbf{i}(k)+\mathbf{n}(k)\right)\cdot\left(\mathbf{i}(k)+\mathbf{n}(k)\right)^{\mathbf{H}}\right\} \quad (5)$$

The optimal weighting vector is the vector that maximizes the SINR of the array output. So the beamforming problem can be expressed as

$$\mathbf{w}_{opt} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^{\mathbf{H}}\mathbf{R}_s\mathbf{w}}{\mathbf{w}^{\mathbf{H}}\mathbf{R}_{i+n}\mathbf{w}} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^{\mathbf{H}}\mathbf{R}_s\mathbf{w}}{\mathbf{w}^{\mathbf{H}}\mathbf{R}\mathbf{w}} \quad (6)$$

where

$$\mathbf{R} = \mathbf{R}_s + \mathbf{R}_{i+n} \quad (7)$$

This approach is usually referred to as Minimum Variance Distortionless Response beamforming. The solution to (7) is given by

$$\mathbf{w}_{opt} = \rho\left\{\mathbf{R}^{-1}\mathbf{R}_s\right\} \quad (8)$$

where $\rho(\cdot)$ is the operator that gets the principal eigenvector of a matrix.

In fact, we can only get the presumed signal and training data covariance matrices. There is always a mismatch between the presumed and actual covariance matrices. So, we can get

$$\mathbf{R}_s = \hat{\mathbf{R}}_s + \Delta_1 \quad (9)$$

$$\mathbf{R} = \hat{\mathbf{R}} + \Delta_2 \quad (10)$$

Where $\mathbf{R}_s$ and $\hat{\mathbf{R}}_s$ represent the actual and presumed signal covariance matrices respectively. $\mathbf{R}$ and $\hat{\mathbf{R}}$ represent the actual and presumed training data covariance matrices respectively. Δ_1 and Δ_2 represent the uncertainty of signal and training data covariance matrices respectively. Usually Δ_1 and Δ_2 are limited as follows

$$\left\|\Delta_1\right\| \le \varepsilon \quad (11)$$

$$\|\Delta_2\| \le \gamma \quad (12)$$

where $\|\cdot\|$ denotes the Frobenius norm. In order to provide robustness against covariance mismatch, the worst-case performance is optimized. It can be expressed as

$$\max_{\mathbf{w}} \min_{\Delta_1,\Delta_2} \frac{\mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s+\Delta_1\right)\mathbf{w}}{\mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}+\Delta_2\right)\mathbf{w}} = \max_{\mathbf{w}} \frac{\min_{\Delta_1} \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s+\Delta_1\right)\mathbf{w}}{\max_{\Delta_2} \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}+\Delta_2\right)\mathbf{w}}, \quad \|\Delta_1\| \le \varepsilon, \|\Delta_2\| \le \gamma \quad (13)$$

It can be rewritten as

$$\max_{\mathbf{w}} \frac{\mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s-\varepsilon\mathbf{I}\right)\mathbf{w}}{\mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}+\gamma\mathbf{I}\right)\mathbf{w}} \quad (14)$$

The solution to (14) is

$$\mathbf{w}_{opt} = \rho\left\{\left(\hat{\mathbf{R}}+\gamma\mathbf{I}\right)^{-1}\left(\hat{\mathbf{R}}_s-\varepsilon\mathbf{I}\right)\right\} \quad (15)$$

3. ROBUST ADAPTIVE BEAMFORMING WITH BROAD NULLS

3.1. Formulation derivation

The robust adaptive beamforming using worst-case performance optimization is expressed as (14). It is equivalent to a constrained optimization problem

$$\min_{\mathbf{w}} \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}+\gamma\mathbf{I}\right)\mathbf{w}, \ s.t.\ \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s-\varepsilon\mathbf{I}\right)\mathbf{w} = 1 \quad (16)$$

Because the estimated training data covariance matrix is not ideal, and the there are no special constrains on the interferences, the beamformer has little ability to suppress strong interferences. Here, to form broad nulls in the interferences' directions, we integrate the power response of the array over the spatial region of the interferences, it is expressed as

$$\rho = \sum_{k=1}^{L} \int_{\theta_k-\frac{\Delta\theta_k}{2}}^{\theta_k+\frac{\Delta\theta_k}{2}} \mathbf{w}^{\mathbf{H}}\mathbf{a}(\theta)\mathbf{a}^{\mathbf{H}}(\theta)\mathbf{w}\,d\theta \quad (17)$$

where $\mathbf{a}(\theta)$ is the array steering vector in the direction θ, $\theta_k, k=1,2,\cdots L$ is the direction of the interference, and $\Delta\theta_k$ is the corresponding null width to the interference direction.

It can be rewritten as

$$\rho = \mathbf{w}^{\mathbf{H}}\mathbf{G}\mathbf{w} \quad (18)$$

where

$$\mathbf{G} = \sum_{k=1}^{L} \int_{\theta_k - \frac{\Delta\theta_k}{2}}^{\theta_k + \frac{\Delta\theta_k}{2}} \mathbf{a}(\theta)\mathbf{a}^{\mathbf{H}}(\theta) d\theta \quad (19)$$

We then constraint the quantity ρ to be equal to a small positive quantity δ^2, that is

$$\mathbf{w}^{\mathbf{H}}\mathbf{G}\mathbf{w} = \delta^2 \quad (20)$$

where δ^2 presents the mean null depth over the spatial region of interferences.

Then the robust adaptive beamforming problem can be expressed as following constrained optimization problem,

$$\min_{\mathbf{w}} \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}} + \gamma \mathbf{I}\right)\mathbf{w}, \ s.t. \ \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w} = 1 \quad (21)$$
$$\mathbf{w}^{\mathbf{H}}\mathbf{G}\mathbf{w} = \delta^2$$

3.2. Newton recursive algorithm

Using Lagrange multiplier method, we can convert (21) to an unconstrained optimization problem

$$\min_{\mathbf{w},\lambda,\mu} f(\mathbf{w},\lambda,\mu), \ f(\mathbf{w},\lambda,\mu) = \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}} + \gamma \mathbf{I}\right)\mathbf{w} + \lambda\left(1 - \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w}\right) + \mu\left(\delta^2 - \mathbf{w}^{\mathbf{H}}\mathbf{G}\mathbf{w}\right) \quad (22)$$

So, the beamforming vector satisfies the following equations

$$\nabla f(\mathbf{w},\lambda,\mu) = \begin{pmatrix} \left(\hat{\mathbf{R}} + \gamma \mathbf{I}\right)\mathbf{w} - \lambda\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w} - \mu \mathbf{G}\mathbf{w} \\ 1 - \mathbf{w}^{\mathbf{H}}\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w} \\ \delta^2 - \mathbf{w}^{\mathbf{H}}\mathbf{G}\mathbf{w} \end{pmatrix} = \mathbf{0} \quad (23)$$

The nonlinear equations (23) can be efficiently solved by Newton recursive algorithm.

We let $\mathbf{z} = \left[\mathbf{w}^{\mathbf{H}}, \lambda^{\mathbf{H}}, \mu^{\mathbf{H}}\right]^{\mathbf{H}}$, and then Newton recursive formulation is given by

$$\mathbf{z}_k = \mathbf{z}_{k-1} - \alpha\left(\nabla^2 f(\mathbf{z}_{k-1})\right)^{-1} \cdot \nabla f(\mathbf{z}_{k-1}), 0 < \alpha \le 1 \quad (24)$$

where

$$\nabla^2 f(\mathbf{z}) = \begin{pmatrix} \left(\hat{\mathbf{R}} + \gamma \mathbf{I}\right) - \lambda\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right) & -\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w} & -\mathbf{G}\mathbf{w} \\ -\left(\left(\hat{\mathbf{R}}_s - \varepsilon \mathbf{I}\right)\mathbf{w}\right)^{\mathbf{H}} & 0 & 0 \\ -\left(\mathbf{G}\mathbf{w}\right)^{\mathbf{H}} & 0 & 0 \end{pmatrix} \quad (25)$$

Therefore, the Newton recursive algorithm can be summarized as follows:

1) Initialize $\mathbf{w}_0 = [1, 0, \cdots, 0]^{\mathbf{H}}$, $\lambda_0 = 0$, $\mu_0 = 0$

2) For k=1,2…

compute $\nabla f\left(\mathbf{z}_{k-1}\right)$ and $\left(\nabla^2 f\left(\mathbf{z}_{k-1}\right)\right)^{-1}$;

compute $\mathbf{z}_k = \mathbf{z}_{k-1} - \alpha\left(\nabla^2 f\left(\mathbf{z}_{k-1}\right)\right)^{-1} \cdot \nabla f\left(\mathbf{z}_{k-1}\right)$;

if residual error $\left\|\nabla f\left(\mathbf{z}_k\right)\right\|^2 \big/ M \leq 10^{-6}$, then stop;

else go on;

3.3. Numerical techniques

It seems that matrix inversion should be carried out once per iteration. In fact, we only have to solve the following linear equations instead.

$$\mathbf{Ax} = \mathbf{b} \quad (26)$$

where $\mathbf{A} = \nabla^2 f\left(\mathbf{z}_{k-1}\right)$, and $\mathbf{b} = \nabla f\left(\mathbf{z}_{k-1}\right)$.

Then Newton recursive formulation is expressed as

$$\mathbf{z}_k = \mathbf{z}_{k-1} - \alpha\mathbf{x}, 0 < \alpha \leq 1 \quad (27)$$

Numerical techniques for solving the linear equations (26) will be presented in the following.
Matrix A can be factorized as A=QR, where Q is a unitary matrix, and R is an upper triangle matrix. Then equations (26) can be rewritten as

$$\mathbf{Rx} = \mathbf{Q}^{\mathbf{H}}\mathbf{b} \quad (28)$$

when A is non-singular, R is non-singular. Equations (28) can be quickly solved using back-substitution.
During the iteration steps of Newton recursive algorithm, matrix A may be almost singular. At this time, linear equations (26) have many solutions. We can do column-swap operations on upper triangle matrix R, so as to

$$\begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} = \mathbf{RP} \quad (29)$$

where P is a column-swap matrix, which satisfies $\mathbf{PP}^{\mathbf{T}} = \mathbf{I}$, and $\mathbf{R}_{11}$ is a non-singular upper triangle matrix.

We let $\mathbf{y} = \mathbf{P}^{\mathbf{T}}\mathbf{x} = \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix}$ and $\mathbf{Q}^{\mathbf{H}}\mathbf{b} = \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{pmatrix}$. Then linear equations (28) can be converted to

$$\mathbf{RPP}^{\mathbf{T}}\mathbf{x} = \mathbf{Q}^{\mathbf{H}}\mathbf{b} \quad (30a)$$

That is

$$\begin{pmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{pmatrix} \quad (30b)$$

The least square solution to (30b) is

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{R}_{11}^{-1}(\mathbf{c}_1 - \mathbf{R}_{12}\mathbf{y}_2) \\ \mathbf{y}_2 \end{pmatrix} \quad (31)$$

Then the solution to (26) is given by

$$\mathbf{x} = \mathbf{P}\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \mathbf{P}\begin{pmatrix} \mathbf{R}_{11}^{-1}(\mathbf{c}_1 - \mathbf{R}_{12}\mathbf{y}_2) \\ \mathbf{y}_2 \end{pmatrix} \quad (32)$$

Especially, let $\mathbf{y}_2 = \mathbf{0}$, we get a basic solution expressed as

$$\mathbf{x} = \mathbf{P}\begin{pmatrix} \mathbf{R}_{11}^{-1}\mathbf{c}_1 \\ \mathbf{0} \end{pmatrix} \quad (33)$$

Using these numerical techniques, the computation complexity can be reduced to O(M^2) per iterations.

4. NUMERICAL EXAMPLES

Newton recursive formulations have been derived for robust adaptive beamforming with broad nulls. In this section, numerical examples are given to demonstrate its performance. We assume a uniform linear array with M=10 omni-directional elements spaced half-wavelength apart. In the simulations, there are two signals impinging on the array, one is the desired signal and the other is interference. They come from the directions of 0^0 and 30^0, respectively. The presumed directions of arrival of the desired signal and interference are 2^0 and 32^0, with a look direction mismatch of 2^0, respectively. The null width of the broad null is $\Delta\theta = 5^\circ$ to make sure that the actual direction of the interference is in the region of the null. The factor α in the Newton recursive algorithm is $\alpha = 0.1$ which can be selected to be a little larger so that the algorithm may converge faster. The interference-to-noise ratio is set to be 30db, and the diagonal loading factors for signal and training data covariance matrices are $\varepsilon = 3$ and $\gamma = 30\sigma_n^2$, respectively.

Firstly, we demonstrate the convergence performance of the Newton recursive algorithms for robust adaptive beamforming with broad nulls when the signal-to-noise ratio is 0db, and the sample size is N=512. Fig. 1 and 2 display the residual error and output SINR of the beamformer versus the iteration index, respectively. Fig. 3 shows the final array pattern when the algorithm has converged.

Secondly, Fig. 4 shows the output SINR of the beamformer versus input SNR at a single sensor with the sample size N=512. The performances of robust adaptive beamforming with no null constrains, sharp nulls and broad nulls are compared.

Finally, the output SINR of the beamformer versus sample size is shown in Fig. 5 with the input SNR=0db at a single sensor. The performances of robust adaptive beamforming with no null constrains, sharp nulls and broad nulls are also compared.

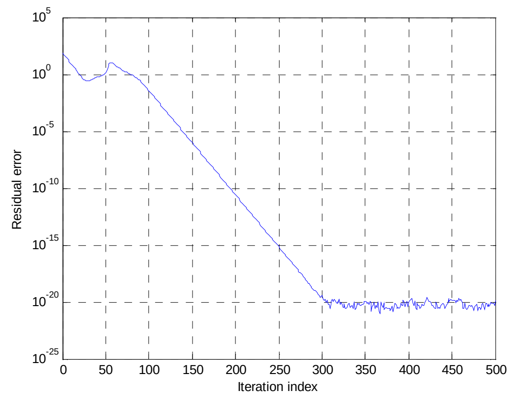

Fig. 1 Residual error versus iteration index

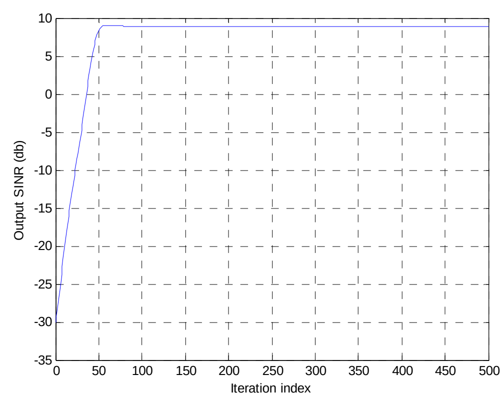

Fig. 2 Output SINR versus iteration index

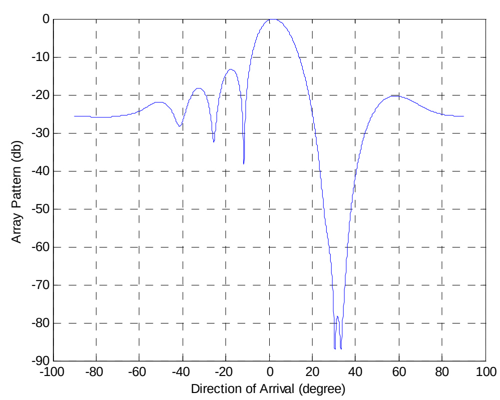

Fig. 3 the final array pattern

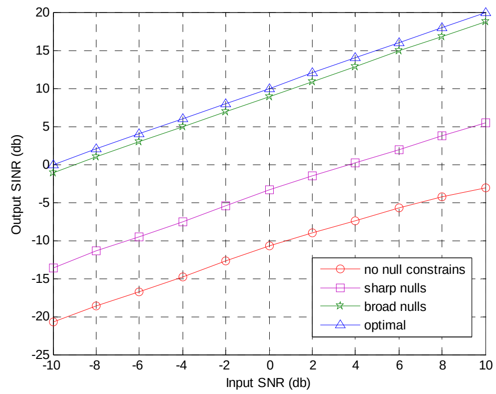

Fig. 4 Output SINR versus input SNR

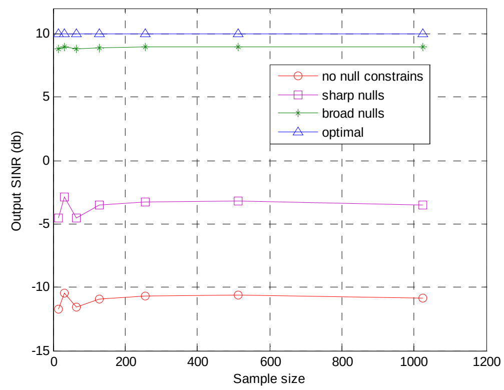

Fig. 5 Output SINR versus sample size

5. CONCLUSIONS

Robust adaptive beamforming using worst-case performance optimization is a theoretically rigorous method. Many papers have demonstrated its good performance under the condition of arbitrary array response errors and small sample size. In this paper, we propose a scheme with broad nulls to improve its performance when there are strong interferences. It can be expressed as an optimization problem with two quadratic constrains on the desired signal and interferences. It is efficiently solved using Newton recursive algorithm. When some numerical techniques are used, the computation complexity can be reduced to $O(M^2)$ per iteration. Numerical examples are presented to compare the performances of the robust adaptive beamforming with no null constrains, sharp nulls and broad nulls. The results show the powerful ability of the proposed scheme to reject strong interferences.

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